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Capital Markets Integration and Cointegration: Testing for the Correct Specification of Stock Market Indices

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Abstract: In this paper we develop a comprehensive Vector Autoregression Model consisting of five variables; the stock market and price indices of pairs of countries, as well as their bilateral nominal exchange rate. Then, we show that under certain long-run restrictions, our approach encompasses a large number of specifications encountered in the voluminous literature on testing for capital integration with cointegration techniques. This approach minimizes the risk of accepting the null of no cointegration between the equity price indices because of the introduction of additional stochastic trends through the transformation of those indices on a "real or nominal US dollar" basis. Furthermore, other interesting long run specifications emerge either with $I(1)$ only stochastic shocks or with the presence of some $I(2)$ disturbances characterizing the system. We apply the testing methodology on monthly data for the US, UK, Germany, and Japan for the period January 1980–May 2019. The main findings provide partial support in favor of cointegration, and therefore for capital markets integration, among stock market indices when proper attention is given to issues like the identification and temporal stability of the cointegration vectors as well as the choice of units that the stock indices are expressed in.

Keywords: International stock markets; $I(2)$ cointegration analysis; common trends; identification; purchasing power parity; temporal stability

JEL Classification: G15; G12; G32

1. Introduction

As an outcome of the various stock market crashes that the world has experienced over the last 40 years a voluminous literature has been produced with the intension to examine the possible existence of long-run co-movements among the major stock markets of the world. The results of this examination were of obvious interest to both academic researchers and investment managers. If the evidence was favorable to these co-movements, then the diversification of portfolios could not be risk reducing for investors with a holding period longer than the time needed for the markets to adjust to their equilibrium path. Moreover, this evidence would constitute an empirical rejection of the efficient markets hypothesis under which two (or more) asset prices that are determined in efficient markets cannot be cointegrated (Granger 1986, p. 218)[1]. It has also been claimed that the existence of cointegration is linked to the concept of capital markets integration. This can be explained by the increased synchronization of macroeconomic policies and / or the reduction of equity risk premia as a result of the international diversification of equity portfolios, which limits the number of undiversifiable risks to be remunerated. Capital market integration would therefore imply that the same model of required returns applies to all assets, regardless of where they are traded (Taylor and Tonks 1989)[2].

The main analytical framework within which the investigation of the above problem has been conducted is provided by cointegration theory[3]. The initial evidence supplied by Kasa (1992) was strongly in favor of the presence of a single common stochastic trend that drove the quarterly stock price indices of five countries, namely, the U.S., Canada, Germany, Japan, and the United Kingdom, for the period 1974:1 to 1990:3. Those findings came under criticism because they had been derived by an arbitrary increase of the number of lags in the VAR model with no adjustment of the critical values of the relevant tables (Richards 1995). If that had been done, then there would have been no evidence of a common stochastic trend. Later studies produced conflicting evidence on the issue, where the number of cointegrating vectors among the stock market indices varied substantially, meaning that no identification of the system was possible. For instance, Jeon and Chiang (1991) examine aggregate stock market indices for the U.S., the U.K., Germany and Japan over the period 1975:1–1990:3 and are able to show the existence of a single common stochastic trend, especially for sub-samples covering most of the 1980s. In stark contrast, Francis and Leachman (1998) study produced a single cointegrating vector among the stock market indices of Germany, Japan, the U.K., and the U.S. for the period 1974:1–1990:8. This result is not easily interpretable since it implies three stochastic trends that are not readily identifiable, given that the data have been converted into their real U.S. dollars equivalent. This conversion introduces stochastic trends into the system that might be associated with the exchange rates, while on the other hand it removes stochastic trends that are associated with domestic growth components that manifest themselves through the price level. Serletis and King (1997) have studied the problem for the case of ten European Union stock markets for the period 1971:1–1992:1. They supply evidence in favor of eight cointegrating vectors instead of nine that the condition for multi-country convergence would imply. They attributed this failure to the fact the Greek capital market had not been integrated with the rest of the European Union prior to that point. Crowder and Wohar (1998) employed the same data base as Kasa (1992) and showed that the system is better characterized by four common stochastic trends. This result has been rationalized not just on the evidence of the Johansen type cointegration tests, but also from the recursive analysis of the alternative cointegrating systems. Fraser and Oyefeso (2005) also examine the long-run interrelationships of the European capital markets and they conclude that although cointegration exists, the gains from diversification are short-lived since the adjustment to the common trend is very fast. Moreover, Ceylan (2006) assesses the long-term stock market integration with the European Union and the US in the cases of Eastern European countries over the period 2000–2005. The Johansen cointegration tests he applies provide no support for the existence of a long-run equilibrium. Worthington et al. (2003) have also used cointegration analysis

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1 This argument follows from the evidence linking cointegration and error correction that leads to the predictability of at least one of the asset prices. Those results apply strictly however on total returns (i.e., cum dividend) or otherwise on non-interest/dividend paying assets (Richards 1995). The efficiency hypothesis can still be preserved, in the presence of cointegration, if the error correction is a proxy for a risk premium.

2 The general tendency is to draw a distinction between tests for capital integration and those for the existence of common trends. The important implication of integrated capital markets is the equalization among countries of marginal rates of substitution in consumption both inter-temporally and across states of nature (Lucas 1982; Kasa 1995). But even then the existence of a common model of required returns is not sufficient to generate cointegrating relationships. The cumulative stochastic errors, if they are found to be $I(1)$, of each one of the assets return series must be cointegrated as well (Richards 1995).

3 In this paper we focus on results derived from the multivariate cointegration analysis of Johansen (1988). However, the first strand of papers examining cointegration between stock prices employed the Engle and Granger (1987) methodology (see e.g., Arshanapalli and Doukas 1993; Chan et al. 1992).

in order to assess the degree of integration between European financial markets before and after the introduction of the single currency. They conclude that there has been an increase in European financial integration, both within and outside the single currency area, immediately after the implementation of the Maastricht treaty. Caporale et al. (2016) use an extended sample that covers the recent global financial crisis and apply fractional cointegration techniques on the S&P 500 and the Euro Stoxx 50 indices. They conclude that fractional cointegration and mean reverting errors can be found on all sub-samples before March 2009. After that date the pattern of co-movement that existed before the crisis disappeared, and the authors attribute this finding to the divergent monetary and fiscal policies that U.S. and E.U. have adopted in order to contain the consequences of the crisis.

A common feature of several of the above mentioned studies has been the transformation of the data into their "real or nominal dollar" equivalent value using the spot exchange rates and the U.S. consumer price index.[4] This has been rationalized on the grounds that returns must be "covered" against exchange rate risk or else a model for the pricing of the exchange rate risk would be needed to test the "integration" hypothesis (Kasa 1995). However, it is worth noting that the use of data on a "real or nominal dollar" basis for the cointegration analysis presupposes the satisfaction of certain restrictions on the variables appearing in the cointegrating space. If this is not supported empirically, then the use of transformed data might introduce additional stochastic trends which drive the obtained results. Therefore, the failure to obtain meaningful long run relationships among the equity indices can be attributed to the adopted transformation and not to the intrinsic properties of those indices.

In the present paper, we provide a systematic way of testing for common trends where we estimate the vector autoregressive (VAR) model in its most general form and then we test it down to the specifications employed by most researchers in the area. It is shown that the transformation of the indices into their "real dollar" equivalent is superfluous for the long-run properties of the model, and that what is needed is a mere transformation to a "*nominal US dollar*" basis. Moreover, it is shown that this is just one of the possible specifications of the model. Another interesting specification, within the $I(1)$ environment, would imply that if the *Purchasing Power Parity* is a valid model for the exchange rate determination, then we should test for cointegration among the indices in *nominal domestic currency* terms. The set of possible alternative specifications increases substantially if we allow for the presence of $I(2)$ components in the model.

The analysis is conducted within the context of cointegration and therefore we examine the existence of long-run relationships between the stock price indices, the bilateral exchange rates and the corresponding consumer price indices of the U.S., the U.K., Germany and Japan. Our testing approach departs from previous ones in a number of ways. First, we provide an analysis which allows us to reveal the existence of $I(2)$ and $I(1)$ components in a multivariate context and is based on a testing methodology suggested by Johansen (1992a, 1995a, 1997) and extended by Paruolo (1996) and Rahbek et al. (1999) This analysis is conducted by testing successively less and less restricted hypotheses according to the Pantula (1989) principle. Additionally, we apply an approach suggested by Juselius (1995) that is based on the roots of the companion matrix and allows us to make firmer conclusions about the rank of the cointegration space. Second, since in a multivariate framework a vector error correction model may contain multiple cointegrating vectors, following Johansen and Juselius (1994) and Johansen (1995b), we impose independent linear restrictions on the coefficients of the accepted cointegrating vectors in order to identify them. Third, given that at least one statistically significant cointegrating vector has been found we examine the stability of the long-run relationships through time. The evidence that two or more stock indices are cointegrated, might be exploitable by the investors only if this evidence is sample independent. In the literature this issue has not been examined formally until now with the

[4] Serletis and King (1997) have transformed their data into "real deutschemark" units. In other studies, the stock indices are expressed in local, nominal, currency terms (e.g., Calvi 2010).

exception of the aforementioned study by Crowder and Wohar (1998), and the papers by Rangvid (2001) and Garcia Pascual (2003).

There are several interesting findings that stem from our estimation approach. First, for each bilateral case, we find evidence of one cointegrating vector between the domestic and U.S. price indices, the exchange rate, and the corresponding stock price indices. On the face of this evidence, we conclude that four (or alternatively three) $I(1)$ stochastic trends drive the system while the presence of $I(2)$ stochastic trends lacks any statistical support. Second, for each bilateral exchange rate, we reject the scenario that differences of stock indices require the $I(1)$ deviations from PPP in order to be stationary. Third, we reject the scenario that stock markets are cointegrated when converted to nominal U.S. dollars (i.e., that prices are long-run excluded). Four, for all three cases under examination we were unable to reject the over-identified restrictions for the scenario that exchange rates are long-run excluded and stock market indices in real terms are cointegrated. Five, we were also unable to reject the over-identified restriction that prices and exchange rates are long-run excluded and that indices in nominal domestic currencies are cointegrated. Finally, the application of the stability tests shows that cointegration is established in the early 1990s that might be attributed to the existence of some coordination policy among the countries involved in the aftermath of the October 1987 stock market crisis.

The rest of the paper is organized as follows. In Section 2 we present various models, with an economic reasoning, that are encompassed by our generic modeling approach. Section 3 presents the econometric methodology. Section 4 discusses the data and presents the empirical results while our conclusions are given in Section 5.

2. Testing for Common Trends in an Integrated Framework

2.1. Treating the Variables as I(1)

Consider a 5-dimensional vector autoregressive (VAR) model that in error correction form is given by

$$\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + \gamma D_t + \mu + \varepsilon_t,\ t = 1,\ \ldots,\ T \tag{1}$$

where, $z_t = \left[p_t, i_t, e_t, p_t^f, i_t^f,\right]$ and p (p^f) stands for the logarithm of the consumer price index of the domestic economy (the country of the base currency, i.e., the U.S. dollar in our case), e is the logarithm of the units of the local currency per one dollar, and $i(i^f)$ is the domestic (the base country, i.e., the U.S.) stock market index expressed in nominal domestic currency terms. Also, we consider that $z_{k+1}, \ldots, z_0$ are fixed and $\varepsilon_t \sim Niid_p(0, \Sigma)$. The adjustment of the variables to the values implied by the steady-state relationship is not immediate for a number of reasons like imperfect information or costly arbitrage. Therefore, the correct specification of the dynamic structure of the model, as expressed by the parameters $(\Gamma_1, \ldots, \Gamma_{k-1}, \gamma)$, is important in order for the equilibrium relationship to be revealed. The matrix $\Pi = \alpha\beta'$ defines the cointegrating relationships, β, and the rate of adjustment, α, of the endogenous variables to their steady-state values. Both matrices, α and β, are of dimension $(5 \times r)$ where r stands for the number of cointegrating vectors. D_t is a vector of non-stochastic variables, such as centered seasonal dummies which sum to zero over a full year by construction and are necessary to account for short-run effects which could otherwise violate the Gaussian assumption, and/or intervention dummies; μ is a drift and T is the sample size.

The interesting problem is to determine the restrictions that model (1) should satisfy in order to derive, if possible, long-run specifications which are common in the literature. Furthermore, in anticipation of the empirical results, those restrictions should comply with interesting economic scenarios that are better understood through the moving average (MA) representation of z_t. This representation of model (1) for the case of $I(1)$ variables is given by:

$$z_t = C\sum_{i=1}^{t} \varepsilon_i + C\mu t + C\Phi\sum_{i=1}^{t} D_i + C*(L)(\varepsilon_t + \mu + \Phi D_t) + Z_0 \quad (2)$$

where $C = \beta_\perp(\alpha'_\perp \Gamma\beta_\perp)^{-1}\alpha'_\perp, C*(L)$ is a polynomial in the lag operator L, Z_0 is a function of the initial values and $\alpha_\perp, \beta_\perp$ are both $(5 \times (5 - r))$ matrices orthogonal to α and β, respectively (Johansen 1991). In the MA representation $(\alpha'_\perp \sum \varepsilon)$ determine the $(5 - r)$ stochastic trends while $\beta_\perp$ determines the variables that are being influenced by them. The realization at time t of the variables in z_t is determined by a stochastic trend component, described by the first term of Equation (2), a deterministic trend component, the cumulated value of the non-stochastic variables D_t, a stationary component and the initial values.

2.1.1. Scenario I: One Cointegrating Vector, Four Common Stochastic Trends

This scenario could be associated with the model where there is a co-movement in the long run between the stock market indices in nominal dollar terms. This specification implies a single cointegrating vector (0, 1, −1, 0, −1) among the variables $\left[p_t, i_t, e_t, p_t^f, i_t^f\right]$ that is exactly equivalent, from the long-run point of view, to having that $(i - e - p^f)$ and $(i^f - p^f)$ cointegrated with a vector (1, −1), i.e., the indices in real U.S. dollars, since the U.S. price index, (p^f) appears in both variables. In this scenario, when the variables are $I(1)$, the transformation in the real dollar terms of the indices is considered to be superfluous. In this specification the model is driven by four common stochastic trends, $(\alpha'_\perp \sum \varepsilon)$, that can be associated with disturbances in the exchange rate as well as the domestic and U.S. price and the stock market indices, $(u_{it} = \alpha'_{\perp i}\varepsilon_{jt}, i = 1, \ldots, 4, j = 1, \ldots, 5)$. The system in (2) then can be written as:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \\ c_{51} & c_{52} & c_{53} & c_{54} \end{bmatrix} \begin{bmatrix} \sum_{k=1}^{t} u_{1k} \\ \sum_{k=1}^{t} u_{21k} \\ \sum_{k=1}^{t} u_{3k} \\ \sum_{k=1}^{t} u_{4k} \end{bmatrix} + \text{stationary components.} \quad (3)$$

The single cointegrating vector of this case is consistent with the satisfaction of the restrictions $c_{21} = c_{31} + c_{51}, c_{22} = c_{32} + c_{52}, c_{23} = c_{33} + c_{53}, c_{24} = c_{34} + c_{54}$. Those restrictions imply that the effect, for example, of the first common stochastic trend, $\sum_{k=1}^{t} u_{1k}$, on the level of the domestic and the U.S. stock market indices will differ by an amount that has been absorbed by changes in the level of the exchange rate.[5] In a similar way we can interpret the effects on the stock markets indices of the other three shocks. The situation we examine here might be representative of two economies with insignificant bilateral trade relationships which would make the existence of PPP less likely to exist.[6] On the other hand, the long run specification is consistent with considerable integration of the capital markets in the sense that the equity market indices, denominated in the same currency, cannot deviate in the long run.

Another interesting scenario that is often encountered in the literature requires that the stock market indices, in local currency terms, be cointegrated while the exchange rate and the price indices are missing from the cointegrating vector, i.e., (0, 1, 0, 0, −1), (Corhay et al. 1993; Arshanapalli and

[5] In this case it might be more realistic to restrict the impact on the U.S. price index and have the exchange rate absorb all of the shock (i.e., $c_{51} = 0, c_{21} = c_{31}$). As a matter of fact, the common trends in (2) are over-identified and up to three restrictions can be imposed without changing the likelihood function.

[6] Since the two prices indices are excluded from the cointegrating vector. Therefore, deviations from PPP have no impact, in the long-run, on the behavior of the two equity market indices.

Doukas 1993). In this case the effect of any shock on each one of the two stock markets will be the same while the exchange rate is not a determinant, in the long run, of this relationship. In this situation the imports and exports of each country are not dependent, in a crucial way, on the other country and therefore deviations from Purchasing Power Parity are not expected to influence the relative stock market valuations of the two countries. Moreover, the companies listed in the two stock indices share common international activities and are therefore affected by the same shocks (Bracker et al. 1999). A variation of this model would be one where cointegration exists between the stock price indices expressed in real domestic currency terms. In this case the cointegrating vector would be (−1, 1, 0, 1, −1) and the exchange rate would be long-run excluded.

In the same vein, there are some other specifications that have an interesting economic interpretation. For instance, there might be a case where no variable is long-run excluded and therefore the cointegrating vector would be (−1, 1, 1, 1, −1) or that $\left(i - i^f\right) - \left(p - e - p^f\right) \sim I(0)$. This specification implies that stock indices' deviation from stationarity is fully captured by the deviation from stationarity of the "law of one price". Therefore, a persistent positive deviation of the local stock market index, from the U.S. one, is being explained by the "loss" of the purchasing power of the domestic currency, relative to the U.S. dollar.

2.1.2. Scenario II: Two Cointegrating Vectors, Three Common Stochastic Trends

This model is consistent with a co-movement, in the long run, on the one hand of the domestic and the U.S. price indices, i.e., the PPP holds, and on the other hand of the nominal value of the two stock price indices. This case might exemplify two economies with similar production and consumption patterns that compete internationally in both the goods and the capital markets so that their price indices are equalized. Under this specification the two cointegrating vectors could be (1, 0, −1, −1, 0) and (0, 1, 0, 0, −1) among the variables $\left[p_t, i_t, e_t, p_t^f, i_t^f\right]$ where we have three common stochastic trends that drive the system. The law of one-price guarantees that the competitiveness of the companies in the two countries cannot change since a positive, for example, productivity shock in the domestic economy which lowers the prices will be accompanied by an equi-proportionate appreciation of its currency vis-à-vis the U.S. dollar. Similarly, a positive demand shock in the U.S. will leave the international competitiveness of its companies unaltered, since an increase in the U.S. price level will be accompanied with a devaluation of the U.S. dollar. Therefore, part of the increase in the demand will be satisfied from foreign companies and consequently the stock market boost will be spread among the U.S. and its competitors. The system in (2) can then be written in this case as:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{52} & c_{53} \end{bmatrix} \begin{bmatrix} \sum_{k=1}^{t} u_{1k} \\ \sum_{k=1}^{t} u_{21k} \\ \sum_{k=1}^{t} u_{3k} \end{bmatrix} + \text{stationary components}. \quad (4)$$

The two cointegrating vectors in this case are consistent with the satisfaction of the restrictions $c_{11} = c_{31} + c_{41}, c_{12} = c_{32} + c_{42}, c_{13} = c_{33} + c_{53}$ and $c_{21} = c_{51}, c_{22} = c_{52}, c_{23} = c_{53}$.

Another possibility under this scenario is that the two cointegrating vectors are (1, 0, −1, −1, 0) and (0, 1, −1, 0, −1) among the variables $\left[p_t, i_t, e_t, p_t^f, i_t^f\right]$. The first one represents the presence of the PPP in absolute terms, as before, but the second one implies that the two stock market indices are cointegrated when expressed in nominal U.S. dollar terms. This is the case we referred to at the beginning of this section of the paper where it is equivalent to testing for cointegration between the two stock market indices when they are expressed in real U.S. dollar currency terms. Finally, a variation of this model could have been one where the two cointegrating vectors are (1, 0, −1, −1, 0) and (−1, 1, 0, 1, −1). In this specification, the second vector implies cointegration between the two stock market indices in real local currency terms.

2.2. Treating Prices as I(2)

The characterization of the stochastic properties of the data as being integrated of order two, one or zero is an issue that can be settled through empirical investigation rather than on "theoretical" grounds (Juselius 1999). If the chosen data set spans a short period of time, then it is more likely for some series, like the price level, to be characterized as $I(2)$ processes since there are not enough turning points in the sample. The same series when studied for its stochastic properties over a longer sample period will be more likely characterized as an *I(1)* process.

The problem of cointegration within this more general framework was initially studied by Johansen (1997) who has shown that the moving average representation is given by:

$$z_t = C_2 \sum_{s=1}^{t} \Sigma_{i=1}^{s} \varepsilon_i + C_2 \tfrac{1}{2} \mu t^2 + C_2 \Phi \Sigma_{s=1}^{t} \Sigma_{i=1}^{s} D_i + C_1 \Sigma_{i=1}^{t} (\varepsilon_i + \Phi D_t) + C^*(L)(\varepsilon_t + \mu + \Phi D_t) + B_0 + A_0 t \tag{5}$$

where $C_2 = \beta_\perp^2 \Psi^{-1} \alpha_\perp^{2''}$ and Ψ, C_1 are functions of the parameters of the model. $\alpha_\perp^{2''}$ represents the matrix of the coefficients of the common $I(2)$ trends while $\beta_\perp^2$ define the sensitivity of the variables in z_t to the common trends. The space spanned by the (5×1) vector z_t can be decomposed into r stationary directions, β, and $(5 - r)$ nonstationary directions, $\beta_\perp$, and the latter into the directions $(\beta_\perp^1, \beta_\perp^2)$, where $\beta_\perp^1 = \beta_\perp \eta$ is of dimension $(5 \times xs_1)$ and $\beta_\perp^2 = \beta_\perp (\beta'_\perp \beta_\perp)^{-1} \eta_\perp$ is of dimension $(5 \times xs_2)$ and $s_1 + s_2 = 5 - r$. If $(r > s_2)$ then the r cointegrating vectors can be decomposed into $(r - s_2)$ directly cointegrating vectors, β_0, which according to Engle and Granger (1987) definition are *CI*(2,2), and s_2 multicointegrating vectors β_1. The properties of the process are summarized in Table 1.

Table 1. Long-term relations in the $I(2)$ framework.

Direction	Dimension	Stationary Process
$\beta_0' z_t \sim I(0)$	$r - s_2$, if $(r > s_2)$	
$\beta_1' z_t \sim I(1)$	s_2	$\beta_1' z_t + k' \Delta z_t \sim I(0)$
$\beta_{\perp 1}' z_t \sim I(1)$	s_1	$\beta_{\perp 1}' \Delta z_t \sim I(0)$
$\beta_{\perp 2}' z_{tt} \sim I(2)$	s_2	$\beta_{\perp 2}' \Delta^2 z_{tt} \sim I(0)$

Under this statistical specification two models appear likely to occur. In both of them the aggregate demand shock accumulates twice to form the $I(2)$ common stochastic trend of the prices. As far as the *I(1)* level is concerned, one expects to find either three stochastic trends in which case there will be two cointegrating vectors one of which will be a multicointegrating one, or four stochastic trends which are consistent with a single multicointegrating relationship.

2.2.1. Scenario I: One Cointegrating Vector, Three *I(1)* and One *I(2)* Common Stochastic Trends

Under the first economic scenario the model in (5) can be written more clearly as follows:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} C_{2,1} \\ 0 \\ 0 \\ C_{2,4} \\ 0 \end{bmatrix} \Sigma_{s=1}^{t} \Sigma_{i=1}^{s} \varepsilon_i + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{52} & c_{53} \end{bmatrix} \begin{bmatrix} \Sigma_{k=1}^{t} u_{1k} \\ \Sigma_{k=1}^{t} u_{2k} \\ \Sigma_{k=1}^{t} u_{3k} \end{bmatrix} + stat.and\ detr.components \tag{6}$$

One plausible multicointegrating vector could be $\left\{ i - i^f - (p - e - p^f) - \beta(\Delta p - \Delta p^f) \right\} \sim I(0)$, where the following coefficient restrictions should be satisfied: $c_{2i} - c_{5i} - (c_{1i} - c_{3i} - c_{4i}) + \beta(c_{2,1} - c_{2,4}) = 0$, $i = 1, 2, 3$. This characterization implies a dynamic long-run steady state relation where stock price indices differ by the deviation of prices from their Purchasing Power Parity level as well as the inflation rates differential. This could be exemplified by the case where the two countries follow different macroeconomic policies which are reflected to their corresponding inflation rates. Since stock market prices incorporate future expectations, the argument is that inflation rates differentials are reflected in stock market valuations.

2.2.2. Scenario II: Two Cointegrating Vectors, Two $I(1)$ and One $I(2)$ Common Stochastic Trends

The stochastic trend components of this case can be seen from:

$$\begin{bmatrix} p_t \\ i_t \\ e_t \\ p_t^f \\ i_t^f \end{bmatrix} = \begin{bmatrix} C_{2,1} \\ 0 \\ 0 \\ C_{2,4} \\ 0 \end{bmatrix} \Sigma_{s=1}^{t} \Sigma_{i=1}^{s} \varepsilon_i + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \\ c_{41} & c_{42} \\ c_{51} & c_{52} \end{bmatrix} \begin{bmatrix} \Sigma_{k=1}^{t} u_{1k} \\ \Sigma_{k=1}^{t} u_{2k} \end{bmatrix} + stat.and\ detr.components \tag{7}$$

A strong candidate for the directly cointegrating vector of this case is the PPP hypothesis $(p - e - p^f) \sim I(0)$. The multicointegrating relationship could relate the stock market indices to the inflation rates differential, i.e., $(i - i^f - \beta(\Delta p - \Delta p^f))$, and the coefficient restrictions can be derived in a similar way to the ones presented for the other cases. The explanation for the presence of the inflation rates differential resembles to the one given above, but in this case, it directly influences the stock market indices which are not affected by deviations from the Purchasing Power Parity.

3. Econometric Methodology

We will briefly discuss the estimation procedure starting from the most general case within the framework of Johansen (1988, 1991) multivariate cointegration methodology as it was extended in Johansen (1992a, 1995a, 1997) and Paruolo (1996) and Rahbek et al. (1999) to take into account the stochastic properties of $I(2)$ variables.

If we allow the parameters of model (1), $\theta = (\Gamma_1, \ldots\ldots, \Gamma_{k-1,} \Pi, \gamma, \mu, \Sigma)$, to vary unrestrictedly, this model then corresponds to the $I(0)$ model. The $I(1)$ and $I(2)$ models are obtained if certain restrictions are satisfied. Thus, the higher-order models are nested within the more general $I(0)$ model. It has been shown (Johansen 1991) that if $z_t \sim I(1)$, then that matrix Π has reduced rank $r < p$, and there exist $p \times r$ matrices α and β such that $\Pi = \alpha\beta'$. Furthermore, $\Psi = \alpha'_\perp(\Gamma)\beta_\perp$ has full rank, where $\Gamma = -\sum_{i=1}^{k-1} \Gamma_i$ and $a_\perp$ and $\beta_\perp$ are $p \times (p - r)$ matrices orthogonal to α and β, respectively. Following this parameterization, there are r linearly-independent stationary relations given by the cointegrating vectors β and $p - r$ linearly-independent non-stationary relations that define the common stochastic trends of the system.

The $I(2)$ model is defined by the first reduced rank condition of the $I(1)$ model and $\Psi = \alpha'_\perp \Gamma \beta_\perp = \varphi\eta'$ is of reduced rank s_1, where φ and η are $(p - r) \times s_1$ matrices and $s_1 < (p - r)$. Following Rahbek et al. (1999), we outline a representation of the restricted VAR model (2) which allows the observed process z_t to have (at most) linear deterministic trends and some or all components $I(2)$[7]. Johansen (1991) shows how the model can be written in a moving average form, while Johansen (1997) derives the FIML solution to the estimation problem for the $I(2)$ model. Furthermore, Johansen (1995a) provides an asymptotically equivalent two-step procedure that computationally is simpler.

In a multivariate context a vector error correction model may contain multiple cointegrating vectors, and in such a case the individual cointegrating vectors are *under-identified* in the absence of

[7] In general if $z_t \sim I(2)$ then the unrestricted linear regressor, $\mu_1 t$, allows for cubic trends while the constant regressor, μ_0, allows for quadratic ones. Rahbek et al. (1999) show that to guarantee linear trends in all linear combinations of z_t we must impose restrictions on both μ_1 and μ_0. Finally, Rahbek et al. (1999) provide a likelihood ratio (LR) test, which is asymptotically $\chi^2(r)$ distributed, to examine whether the linear trend significantly enters the cointegrating vector.

sufficient linear restrictions on *each* of the vectors. The issue of identification in cointegrated systems has been addressed by Johansen and Juselius (1994) and Johansen (1995b). Consider again the long-run matrix $\Pi = \alpha\beta'$ and let Φ be any $r \times r$ matrix of a full rank. Then $\Pi = \alpha\Phi^{-1}\Phi\beta' = \alpha^{*}\beta^{*\prime}$, where $\alpha^{*} = \alpha\Phi^{-1}$ and $\beta^{*} = \Phi\beta'$ without imposing restrictions on α and β so that to limit the admissible matrices, Φ, the cointegrating vectors are not unique.

The necessary and sufficient conditions for identification in a cointegrated system in terms of linear restrictions on the columns of β are analogous to the classical identification problem that we face in the simultaneous equations problem. Thus, the order condition for identification of each of the r cointegrating vectors is that we can impose at least $r-1$, just identifying restrictions and one normalization on each vector without changing the likelihood function. This is a necessary condition. The necessary and sufficient condition for identification of the *ith* cointegration vector, *the Rank* condition, is that the rank $(R_i'H_1, \ldots, R_i'H_k) \geq k$, where i and $k = 1, \ldots, r-1$ and $k \neq i$ (Johansen and Juselius 1994). The linear restrictions of the model are of the form $R_i\beta_i = 0$, where R_i is a $(p \times k_i)$ matrix, or equivalently by $R_i'H_i = 0, i = 1, \ldots, r$, where H_i is a known $(p \times s_i)$ design matrix which satisfies $\beta_i = H_i'\tau_i$ and τ_i is a $(s_i \times 1)$ vector of freely varying parameters $(k_i + s_i = p)$.

An equally important issue, along with the existence of at least one cointegration vector, is the issue of the stability of such a relationship through time as well as the stability of the estimated coefficients of such a relationship. Hansen and Johansen (1993, 1999) have suggested methods for the evaluation of parameter constancy in cointegrated VAR models, formally using estimates obtained from the Johansen FIML technique. Three tests have been constructed under the two VAR representations. In the "*Z*-representation" all the parameters of model (2) are re-estimated during the recursions while under the "*R*-representation" the short-run parameters $\Gamma_i = 1, \ldots, k-1$ are fixed to their full sample values and only the long-run parameters α and β are re-estimated.

The first test is called the *Rank* test and it examines the null hypothesis of sample independence of the cointegration rank of the system. The trace test statistics, scaled by the corresponding 95% critical values, are plotted against time and calculated for each sub-sample during the recursive analysis. An interesting result in the recursive analysis is that the rank test as a function of time will be upward sloping for chosen rank $r < r^{*}$ and approximately constant for $r > r^{*}$ (Johansen 1988; Hansen and Johansen 1993). The second test for the constancy of the cointegration space considers the hypothesis $H : sp(b) = sp(\beta)$ where b is a known $p \times r$ matrix and is chosen so that $b = \hat{\beta}(T)$, i.e., the full—sample estimate of β. In the recursive analysis, we perform a sequence of likelihood ratio tests

$$-2\ln(Q(H/\hat{\beta}(t)) = t\sum_{i=1}^{r}\ln\left[\frac{1-\hat{\rho}_i(t)}{1-\hat{\lambda}_i(t)}\right], t = T_0, \ldots T.$$

where λ are the roots of the unrestricted problem and ρ are the equivalents of the restricted one.

For each estimation period, the LR test has the same form and Johansen and Juselius (1992) have shown that it is asymptotically χ^2 distributed with $(p-r) \times r$ degrees of freedom. In the third test we examine the constancy of the eigenvalues, λ, which are related to the cointegration vectors, β, and the loadings, α, through the relationship $\hat{\beta}'S_{10}S_{00}^{-1}S_{01}\hat{\beta} = \hat{\alpha}'S_{00}^{-1}\alpha = diag(\hat{\lambda}_1, ..\hat{\lambda}_r)$, under the normalization $\hat{\beta}'S_{00}^{-1}\hat{\beta} = I$ for the eigenvectors (S_{01}, S_{00} are product moment matrices, Johansen 1988). Thus, an evaluation of the time path for λ_i, $(i = 1, \ldots, r)$ can be seen as an evaluation of the ith column of β or α, and structural changes in α and β will be reflected in the estimated eigenvalues. The asymptotic distribution of the estimator for λ has been derived by Hansen and Johansen (1993, 1999).

4. Empirical Evidence

In this paper we study four stock markets, those of the U.S., Germany, the U.K., and Japan. The time period of the analysis extends from January 1980 through May 2019. The stock price data are the *Capital International* indices constructed by Morgan Stanley. These are end-of-month value-weighted indices of a large sample of firms in each market and they are taken from *Datastream*. The *Capital International* indices correspond quite closely to the standard published indices, such as the NYSE index, the Topix, Commerzbank and FT-industrial 'Ordinary', but they have the advantages of being constructed on a consistent basis across countries and of netting out cross-listed securities. The data for the consumer price indices as well as the exchange rate series of pound sterling/U.S. dollar, Deutschemark-Euro/U.S. dollar and Yen/U.S. dollar have been obtained from the *International Financial Statistics* of the International Monetary Fund. Table 2 provides another interesting feature of the stock price data, which is the contemporaneous correlation between monthly changes of the various markets. It is clear that the U.S. and U.K. are much more correlated with each other than they are with Germany and Japan.

Table 2. Contemporaneous correlations between stock returns monthly changes.

	USA	**UK**	**GER**	**JAP**
USA	1.0			
UK	0.750	1.0		
GER	0.705	0.709	1.0	
JAP	0.422	0.445	0.419	1.0

Notes: The stock price indices are in nominal terms in domestic currency.

Determination of the Cointegration Rank and the Order of Integration

The first step in the analysis is the determination of the cointegration rank index, r, and the order of integration of the variables. We begin by considering the three bilateral cases, those of U.S.–U.K., U.S.–Germany, and U.S.–Japan. As a first check for the statistical adequacy of model (1) we report some univariate misspecification tests in Table 3, in order to ascertain that the estimated residuals do not deviate from being Gaussian white noise errors.

Table 3. Residual misspecification tests of the model with $k = 4$.

U.S.–U.K.							
Eq.	σ_ε	**LB(36)**	**ARCH(4)**	η_3	η_4	**NORM(4)**	R^2
Δp	0.006	28.14	9.71	0.39	1.19	100.71 *	0.703
Δi	0.054	23.12	6.43	−0.68	4.19	86.44 *	0.363
Δe	0.039	34.09	1.59	−0.05	0.88	9.59 *	0.285
Δp^f	0.007	24.81	2.94	−0.28	1.76	102.25 *	0.603
Δi^f	0.045	23.65	5.22	−0.71	2.15	73.61 *	0.362
U.S.–Germany							
Eq.	σ_ε	**LB(36)**	**ARCH(4)**	η_3	η_4	**NORM(4)**	R^2
Δp	0.004	31.28	3.51	−0.78	6.78	59.39 *	0.446
Δi	0.024	14.31	3.37	−0.76	5.87	55.79 *	0.365
Δe	0.010	29.44	5.15	−0.08	3.56	7.04	0.402
Δp^f	0.003	31.16	8.15	−1.36	1.55	102.21 *	0.667
Δi^f	0.018	30.14	10.38	−0.97	5.01	73.33 *	0.209
U.S.–Japan							
Eq.	σ_ε	**LB(36)**	**ARCH(4)**	η_3	η_4	**NORM(4)**	R^2
Δp	0.003	22.39	8.62	−0.35	0.01	19.45 *	0.553
Δi	0.023	31.35	11.39	−0.37	0.08	18.97	0.288
Δe	0.011	28.16	7.32	−0.31	0.33	8.62 *	0.346
Δp^f	0.003	32.11	6.73	−1.37	0.01	105.22 *	0.591
Δi^f	0.018	24.12	10.07	−1.02	0.05	73.02 *	0.371

Notes: σ_ε is the standard error of the residuals, η_3 and η_4 are the skewness and kurtosis statistics. LB is the Ljung-Box test statistic for residual autocorrelation, ARCH is the test for heteroscedastic residuals, and NORM the Jarque-Bera test for normality. The ARCH and NORM statistics are distributed as χ^2 with 4 and 2 degrees of freedom, respectively and the LB statistic is distributed as χ^2 with 36 degrees of freedom. *(**) denotes significance at the 5% (1%) level.

A structure of four lags for each case was chosen based on these misspecification tests. We note that our conditional VAR model is well specified in each case, except for the presence of non-normality. Normality can be rejected as a result of skewness (third moment) or excess kurtosis (fourth moment). Since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis, we report the univariate Jarque-Bera test statistics together with the third and fourth moment around the mean. It turns out that the rejection of normality is essentially due to excess kurtosis, and hence is not so serious for the estimation results. The ARCH(4) tests for fourth order autoregressive heteroscedasticity and is rejected for all equations but again cointegration estimates are not very sensitive to ARCH effects.[8] The R^2 measures the improvement in explanatory power relative to the random walk (with drift) hypothesis, i.e., $\Delta x_t = \mu + \varepsilon_t$. They show that with this information set we can explain quite a large proportion of the variation in the inflation rates, but also the variation in the bilateral exchange rates and the stock price indices to a much lesser extent.

The Johansen—Juselius multivariate cointegration technique, as explained in Section 3, is applicable only in the presence of variables that are realizations of $I(1)$ and $I(2)$ processes and/or a mixture of $I(1)$ and $I(0)$ processes. Until recently the order of integration of each series was determined via the standard unit root tests. However, it has been made clear by now that if the data are being determined in a multivariate framework, a univariate model is at best a bad approximation of the multivariate counterpart, while at worst, it is completely mis-specified leading to arbitrary conclusions. Thus, in the presence of $I(1)$ series, Johansen and Juselius (1990) developed a multivariate stationarity test which has become the standard tool for determining the order of integration of the series within the multivariate context.

Additionally, when the data are $I(2)$, one also has to determine the number of $I(2)$ trends, s_2, among the $p - r$ common trends. The two-step procedure discussed in Section 3 is used to determine the order of integration and the rank of the matrix Π. The hypothesis that the number of $I(1)$ trends = s_1 and the rank = r is tested against the unrestricted H_0 model based on a likelihood ratio test procedure discussed in Johansen (1992a, 1995a, 1997) and extended by Paruolo (1996) and Rahbek et al. (1999).

Table 4 reports the trace test statistics for all possible values of r and $s_1 = p - r - s_2$, under the assumption that the data contain linear but no quadratic trends. The 95% critical test values reported in italics below the calculated test values are taken from the asymptotic distributions reported in Rahbek et al. (1999, Table 1). Starting from the most restricted hypothesis $\{r = 0, s_1 = 0, s_2 = 5\}$ we tested successively less and less restricted hypotheses according to the Pantula (1989) principle. The last column of Table 4 reports the standard Johansen trace test, Q_r. Therefore, the first hypothesis that we were unable to reject was $\{r = 1, s_1 = 3, s_2 = 1\}$ for the U.S./U.K., U.S./Germany and U.S./Japan cases, which implies that there are $I(2)$ components. However, when we also apply a small sample adjustment the first hypothesis that we were unable to reject was $\{r = 1, s_1 = 4, s_2 = 0\}$ which means that there is one linear cointegrating relation and four $I(1)$ common trends in the multivariate framework.[9,10]

8 Gonzalo (1994) shows that the performance of the maximum likelihood estimator of the cointegrating vectors is little affected by non-normal errors. Lee and Tse (1996) have shown similar results when conditional heteroskedasticity is present.

9 The calculations of all tests as well as the estimation of the eigenvectors have been performed using the program CATS 2.0 in RATS 9.0 developed by Katarina Juselius and Henrik Hansen, Estima Inc. Illinois, 1995.

10 A small sample adjustment has been made to the Trace test statistics, Q_r, for the $I(1)$ analysis $-2\ln Q = -(T - kp)\sum_{i=r_{0+1}}^{k} \ln(1 - \hat{\lambda}_i)$ as suggested by Reimers (1992).

Table 4. Testing the Rank of the $I(2)$ and the $I(1)$ Model.

Testing the Joint Hypothesis $H(s_1 \cap r)$ **U.S.–U.K**							
p-r	r			$Q(s_1 \cap r/H_0)$			Q_r
5	0	901.2 *206.1*	691.0 *174.3*	486.9 *146.4*	346.8 *123.1*	216.5 *103.8*	126.7 *88.6*
4	1		483.6 *141.5*	302.8 *115.8*	166.5 *94.2*	73.4 *76.8*	59.8 *63.7*
3	2			164.7 *89.0*	68.6 *69.4*	50.9 *53.9*	37.2 *42.7*
2	3				49.7 *48.5*	29.7 *34.98*	20.8 *25.7*
1	4					18.5 *20.0*	8.28 *12.44*
s_2		5	4	3	2	1	0
U.S.–Germany							
p-r	r			$Q(s_1 \cap r/H_0)$			Q_r
5	0	867.1 *206.1*	603.1 *174.3*	493.1 *146.4*	327.2 *123.1*	229.6 *103.8*	112.4 *88.6*
4	1		444.1 *141.5*	299.6 *115.8*	172.3 *94.2*	64.3 *76.8*	64.1 *63.7*
3	2			171.5 *89.0*	66.4 *69.4*	51.7 *53.9*	38.7 *42.7*
2	3				41.2 *48.5*	28.3 *34.98*	17.8 *25.7*
1	4					15.3 *20.0*	6.59 *12.44*
s_2		5	4	3	2	1	0
U.S.–Japan							
p-r	r			$Q(s_1 \cap r/H_0)$		Q_r	
5	0	802.4 *206.1*	598.9 *174.3*	452.2 *146.4*	319.7 *123.1*	208.8 *103.8*	107.7 *88.6*
4	1		426.7 *141.5*	287.1 *115.8*	170.6 *94.2*	68.1 *76.8*	63.3 *63.7*
3	2			167.1 *89.0*	68.6 *69.4*	35.0 *53.9*	27.9 *42.7*
2	3				34.7 *48.5*	25.2 *34.98*	13.2 *25.7*
1	4					12.4 *20.0*	4.16 *12.44*
s_2		5	4	3	2	1	0

Notes: p is the number of variables, r is the rank of the cointegration space, s_1 is the number of $I(1)$ components and s_2 is the number of $I(2)$ components. For each case a structure of four lags was chosen according to a likelihood ratio test, corrected for the degrees of freedom (Sims 1980) and the Ljung-Box Q statistic for detecting serial correlation in the residuals of the equations of the VAR. A model with an unrestricted constant in the VAR equation and a linear trend restricted in the cointegration space is estimated for all three cases according to the Johansen (1992a, 1992b) testing methodology. The numbers in italics are the 95% critical values (Rahbek et al. 1999, Table 1). A small sample adjustment based on Reimers (1992) has been applied.

In addition to the formal test, Juselius (1995) offers further insight into the $I(2)$ and $I(1)$ analysis as well as the correct cointegration rank. She argues that the results of the trace and maximum eigenvalue test statistics of the $I(1)$ analysis, i.e., from the estimation of the model without allowing for $I(2)$ trends, should be interpreted with some caution for two reasons. First, the conditioning on intervention dummies and weakly exogenous variables is likely to change the asymptotic distributions to some (unknown) extent. Second, the asymptotic critical values may not be very close approximations in small samples. Juselius (1995) suggests the use of the additional information contained in the roots of the characteristic polynomial. Table 4 also provides the five first, in size, $p \times k$ roots of the companion matrix. If there are $I(2)$ components in the vector process, then the number of unit roots in the characteristic polynomial is $s_1 + 2s_2$. Hence, if $r = 1$, implying one cointegrating vector, there should be four unit roots in the process, all of which are $I(1)$ components, and if $r = 2$ there should also be four unit roots if one of them was an $I(2)$ process. The evidence from the estimated roots of the companion matrix is consistent with that provided by the formal tests for the cointegration rank for the U.S./U.K. and U.S./Japan cases under which $\{r = 1, s_1 = 4, s_2 = 0\}$ while for the US/Germany case there is a disagreement. The formal test indicates two cointegrating vectors and three unit roots, while the companion matrix is also consistent with one cointegrating vector and four unit roots. The overall evidence leads us to concentrate on the $I(1)$ case under which the preferred model is $\{r = 1, s_1 = 4, s_2 = 0\}$.

Finally, we allow for the presence of a linear trend. Dornbusch (1989) has suggested that due both to differing productivity trends in the tradeable and non-tradeable goods sectors and to inter-country differences in consumption patterns, a decline in domestic prices relative to foreign prices could appear as a linear trend in the purchasing power parity relationship underlying the model. We tested for the significance of the deterministic trend in the multicointegrating relation by applying the likelihood ratio statistic of Rahbek et al. (1999). The test statistic in the U.S.–U.K. case is 24.53 with a p-value (0.00), in the U.S.–Germany case it is 11.29 with a p-value (0.00) and in the U.S.–Japan case it is 9.30 with a p-value (0.00). Therefore, we reject the null hypothesis that the linear trend does not enter significantly in any one of the estimated cointegrating spaces.

The next stage of the cointegration analysis involves the stability analysis of our cointegration results. The overall conclusion drawn from the three tests is mixed. Specifically, for all three cases it is shown that the rank of the cointegration space is dependent on the sample size from which it has been estimated, since the null hypothesis of a constant rank, one or two in our case, is rejected (Figure 1). It is worth noticing however that this evidence is consistent with previous findings in the literature according to which cointegration is established on samples extending after 1990. This has been partly attributed to the coordination policies pursued by the G-7 countries in the aftermath of the 1987 crisis (Arshanapalli and Doukas 1993; Leachman and Francis 1995; Francis and Leachman 1998)[11]. From the second test (Figure 2) we obtain overwhelming evidence in favor of constancy of the estimated coefficients, since we are unable to reject the null hypothesis for the sample independence of the cointegration space for a given cointegration rank. Finally, the last test (Figure 3) also provides substantial evidence in favor of the constancy of the cointegrating vectors and the speed of adjustment coefficients, since no substantial drift was detected on the time paths of the two largest in size eigenvalues apart from a spike in 2012 for the case of the U.S./Japan). If we combine the evidence from the second and the third tests, then the time dependency should be rather attributed to the adjustment coefficients to the error correction terms than the equilibrium relationships themselves. The exception appears to be the U.S.–Germany case where both eigenvalues are relatively constant especially after 1987, and this possibly manifests in the presence of some form of intervention in the market.

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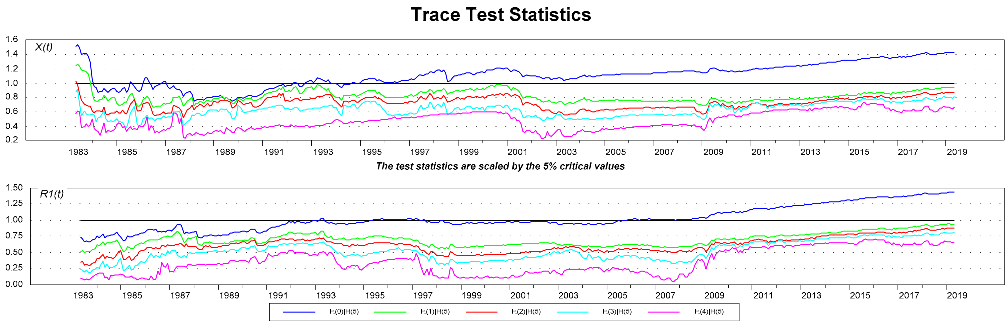
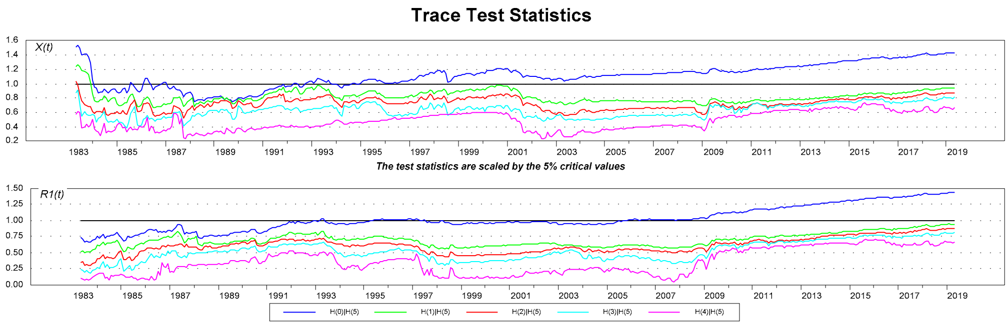
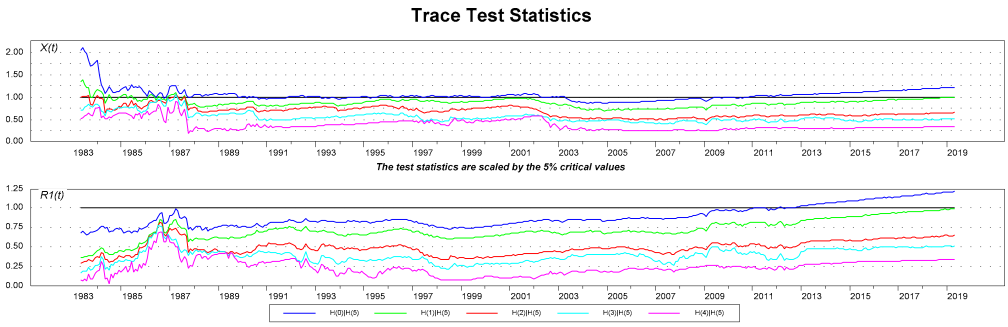
[11] It should be noted however that the falling number, over time, of the underlying stochastic trends governing the system can be attributed to the convergence of the trace statistics to their long-run values as the sample expands (i.e., the power of the test increases). Garcia Pascual (2003) presented evidence on estimates of the trace statistic obtained from constant sample sizes that are being rolled over each time to the next period. It is shown there that the trace statistics do not present any upward trend and this is interpreted as running against the proposition that an increased number of cointegrating vectors among the stock price indices is an indicator of an increasing integration among the capital markets (Rangvid 2001).



(**a**)

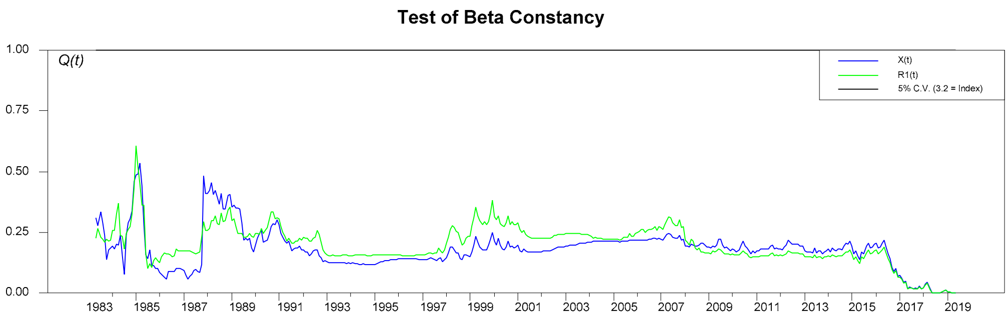


(**b**)



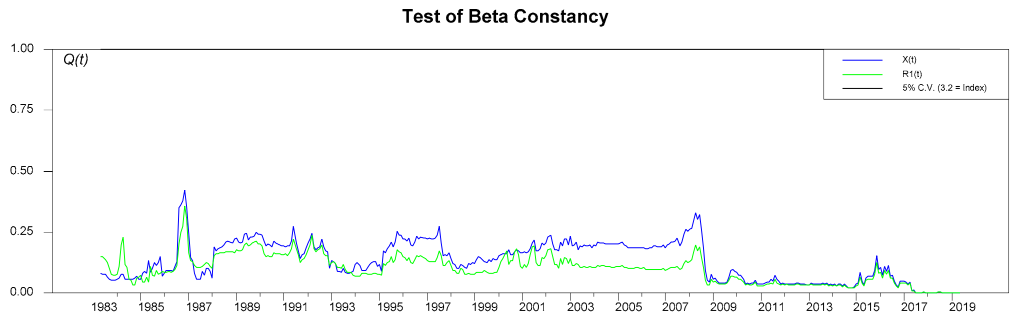
(**c**)

Figure 1. Trace Test 1 is the 5% significance level. (**a**) U.K.–U.S. case. (**b**). Germany–U.S. case. (**c**) Japan–U.S. case.

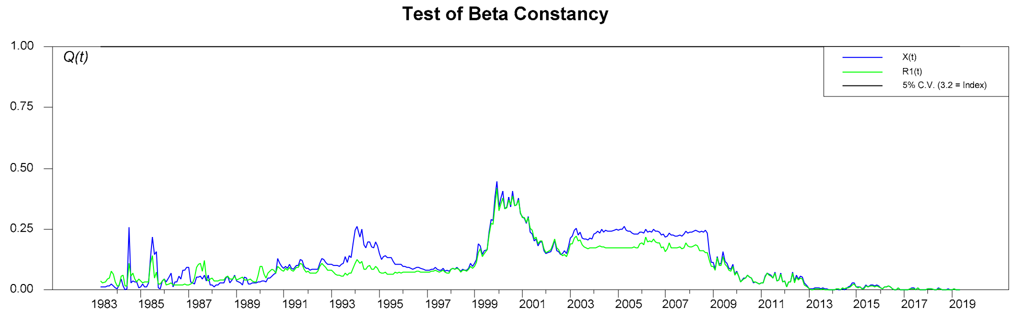


(**a**)

Figure 2. *Cont.*

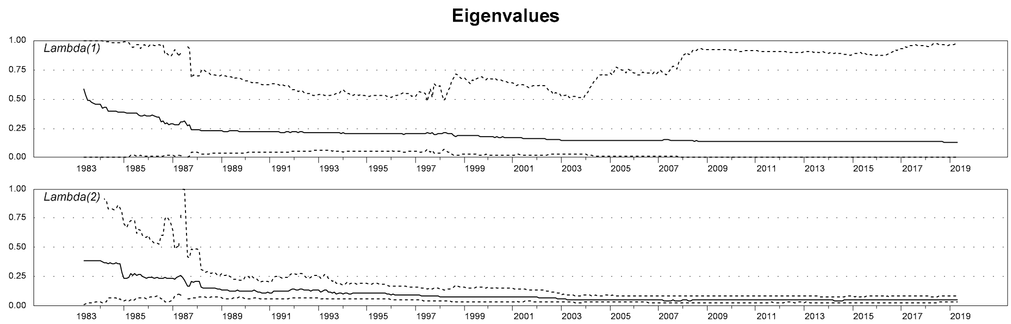


(**b**)

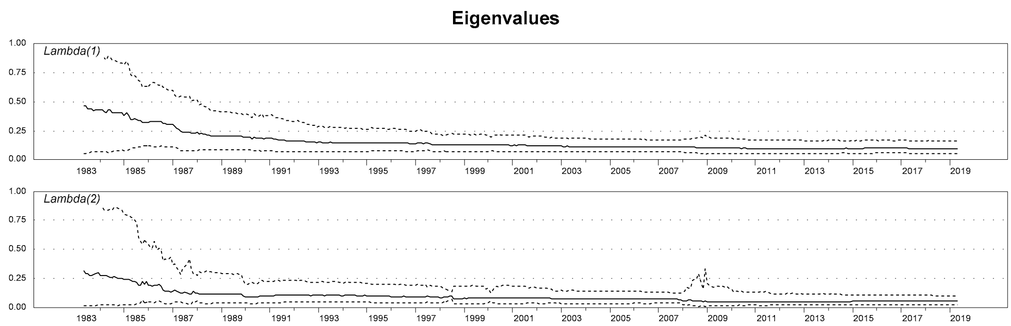


(**c**)

Figure 2. The Test of Constancy of Beta 1 is the 5% significance level. (**a**) U.K.–U.S. case. (**b**) Germany–U.S. (**c**) Japan–U.S.

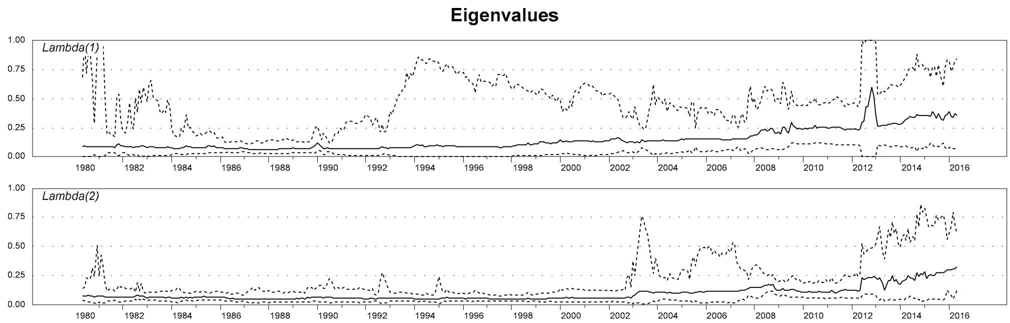


(**a**)



(**b**)

Figure 3. *Cont.*



(**c**)

Figure 3. The Eigenvalue Test U.K.–U.S. case. (**a**) Test for lambda 1 and for lambda 2 Germany–U.S. case. (**b**) Test for lambda 1 and for lambda 2 Japan–U.S. case. (**c**) Test for lambda 1 and for lambda 2.

To assess the statistical properties of the chosen variables the test statistics reported in Table 5 and Table 6 are useful. The test of long-run exclusion is a check of the adequacy of the chosen measurements and shows that none of the variables can be excluded from the cointegration space. The test for stationarity indicates that none of the variables can be considered stationary under any reasonable choice of r. Finally, the test of weak exogeneity shows that the U.K. stock price and consumer price indices for the U.S./U.K. case, the U.S. stock price index for the Germany/U.S. case and the Yen/dollar exchange rate and the Japanese consumer price index can be considered weakly exogenous for the long-run parameters β. All three tests are χ^2 distributed and are constructed following Johansen and Juselius (1990, 1992). Furthermore, Tables 5 and 6 presents diagnostics on the residuals from the cointegrated VAR model that indicates that they are *i.i.d.* processes since no evidence of serial correlation was detected in each bilateral case. This provides further support for the hypothesis of a correctly specified model.

The final stage of our analysis deals with the issue of the economic identification of our system. We test all four scenarios which we fully analyzed in Section 2.1 for the *I*(*1*) case. The results of the estimated restricted vectors along with the likelihood ratio test for the acceptance of the overidentifying restrictions, for the U.S./U.K., the U.S./Germany, and the U.S./Japan case, are given in Table 7. For each bilateral exchange rate, we reject the scenario that differences of stock indices require the difference from PPP to be stationary. In the same vein, we also reject the scenario that stock markets are cointegrated when converted to nominal USD (prices are long-run excluded). On the contrary for all three cases under examination, we were unable to reject the over-identified restrictions for the scenario that exchange rates are long-run excluded and stock market indices in real terms are cointegrated. Furthermore, we were also unable to reject the over-identified restriction that prices and exchange rates are long-run excluded and that indices in nominal domestic currencies are cointegrated. According to the evidence, we reject the joint restrictions on the single cointegrating vector that is contradictory to the specification and results derived in studies like the ones by Arshanapalli and Doukas (1993) and Richards (1995). Those results bring on the forefront the issue of identification in the cointegration analysis; having found a number of cointegrating vectors has little implication for the statistical determination of co-movements between two or more stock market indices if it is not identified with the theoretical structure.

Table 5. Statistical Properties and Misspecification Tests of the Model. (a) Tests for long-run exclusion, stationarity, and weak exogeneity.

	(a) Tests for Long-Run Exclusion, Stationarity, and Weak Exogeneity								
	Long-Run Exclusion			**Stationarity**			**Weak Exogeneity**		
	US/UK	**US/GE**	**US/JP**	**US/UK**	**US/GE**	**US/JP**	**US/UK**	**US/GE**	**US/JP**
p	3.39 *	15.32 *	*	20.60 *	22.47 *	11.71 *	2.77	4.69 *	0.62
i	0.97 *	11.61 *	6.63 *	14.77 *	23.01 *	24.11 *	9.08 *	4.43 *	14.78 *
e	5.84 *	0.05 *	14.02 *	40.83 *	25.88 *	15.52 *	3.30	7.27 *	1.52
p^f	1.08 *	16.73 *	7.22 *	11.76 *	18.50 *	12.72 *	13.33 *	9.43 *	*
i^f	2.47 *	0.16 *	9.29 *	25.16 *	19.525 *	14.73 *	1.67	3.49	8.72 *

Notes: The long-run exclusion restriction and the weak exogeneity tests are χ^2 distributed with two degrees of freedom and the 5% critical level is 5.99, and the stationarity test is a χ^2 distributed with four degrees of freedom and the 5% critical level is 9.49. An (*) denotes statistical significance at the 5 percent critical level.

Table 6. Statistical Properties and Misspecification Tests of the Model. (b) Multivariate Residuals Diagnostics.

(b) Multivariate Residuals Diagnostics				
Case	**L-B(117)**	**LM(1)**	**LM(4)**	χ^2 **(10)**
U.S–U.K	1960.46(0.00)	34.39(0.69)	40.92(0.26)	1023.49(0.00)
U.S.–Germany	1784.33(0.00)	36.95(0.41)	38.12(0.31)	490.69(0.00)
U.S.–Japan	1000.49(0.00)	35.19(0.62)	29.84(0.46)	351.53(0.00)

Notes: L-B is the multivariate version of the Ljung-Box test for autocorrelation based on the estimated auto- and cross-correlations of the first [T/4 = 117] lags distributed as a χ^2 with 2825 degrees of freedom. LM(1) and LM(4) are the tests for first- and fourth-order autocorrelation distributed as χ^2 with 25 degrees of freedom and χ^2 is a normality test which is a multivariate version of the Shenton-Bowman test. Numbers in parentheses refer to marginal significance levels.

Table 7. Tests of overidentified restrictions on $z_t = \left[p_t, i_t, e_t, p_t^f, i_t^f, cons\tan t\right]$. Scenario *I*: one cointegrating vector, four *I(1)* common stochastic trends. Case 1(a).

U.S.–U.K.	
$\beta' = [1, 1, -1, -1, -1, 0.002]$,	Q(5) = 53.91(0.0000)
$\beta' = [0, 1, -1, 0, -1, -0.009]$	Q(5) = 54.31(0.0000)
$\beta' = \left[\ -1, 1, \quad 0 \quad 1 \quad -1 \quad 0.05\ \right]$	Q(5) = 9.51 (0.0642)
$\beta' = [0, 1, 0, 0, -1, 0.08]$	Q(5) = 8.46 (0.1752)
U.S.–Germany	
$\beta' = [1, 1, -1, -1, -1, 0.010]$,	Q(5) = 29.83 (0.0000)
$\beta' = [0, 1, -1, 0, -1, -0.015]$	Q(5) = 33.35 (0.0000)
$\beta' = \left[\ -1, 1, \quad 0 \quad 1 \quad -1 \quad 0.056\ \right]$	Q(5) = 7.31 (0.0941)
$\beta' = [0, 1, 0, 0, -1, 0.03]$	Q(5) = 9.02 (0.0833)
U.S.–Japan	
$\beta' = [1, 1, -1, -1, -1, 0.012]$,	Q(5) = 44.18(0.0000)
$\beta' = [0, 1, -1, 0, -1, -0.022]$	Q(5) = 41.29(0.0000)
$\beta' = \left[\ -1, 1, \quad 0 \quad 1 \quad -1 \quad 0.058\ \right]$	Q(5) = 6.38 (0.1348)
$\beta' = [0, 1, 0, 0, -1, 0.011]$	Q(5) = 10.28 (0.057)

Notes: Q denotes a likelihood ratio test for overidentifying restrictions as suggested by Johansen and Juselius (1994) and is distributed as a χ^2 with the corresponding degrees of freedom given in parentheses. The last column refers to the estimate of the trend. Numbers in brackets denote marginal significance levels.

5. Conclusions

In this paper, we analyze the implications for the identification of common stochastic trends among stock price indices by using a model where the transformation of the data to a domestic or U.S. dollar—nominal or real—basis is decided statistically and not imposed a priori. By applying a general VAR model where all the relevant variables (stock indices, consumer price indices and the exchange rate) are included, we show that the expected results from the cointegration analysis differ substantially.

By examining the MA representation of a VAR system among the exchange rate, the stock market and consumer price indices we attempt to offer a structural interpretation of alternative long-run relationships. Also, by allowing for the presence of $I(2)$ variables, we enrich the set of possible specifications of the long-run co-movements among the stock price indices.

The analysis was conducted using monthly data for the U.S., the U.K., Germany, and Japan for the period January 1980 to May 2019. Several developments in the econometrics of non-stationarity and cointegration were applied and a number of novel results stem from our analysis. First, this paper tested for the existence of $I(2)$ and $I(1)$ components in a multivariate context and the joint hypothesis of either one or two cointegrating vectors and the presence of a significant deterministic trend in the cointegrating vector could not be rejected although the hypothesis of at least one $I(2)$ component was rejected in all three cases. Second, we tested for parameter stability and it was shown that both the dimension of the cointegration rank as well as the loadings to the cointegrating vectors and possibly the vectors themselves were sample dependent. Furthermore, the stability analysis revealed that cointegration was established during the early 1990s in all three cases, which provides support to the argument that the stock markets have become more integrated since a smaller number of stochastic trends drives the system. Finally, for a given cointegration rank we formally imposed independent and linear restrictions on each vector in order to identify our system. The test statistics of the overidentified restrictions were partially positive to the existence of some form of capital market integration, in the sense that cointegration existed between the stock market indices expressed in nominal terms or in real domestic currency terms, for the U.S./U.K., U.S./Germany, and U.S. /Japan cases.

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