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Magnetically Nonlinear Dynamic Models of Synchronous Machines and Experimental Methods for Determining Their Parameters

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Abstract: This paper deals with rotary and linear synchronous reluctance machines and synchronous permanent magnet machines. It proposes a general method appropriate for determining the two-axis dynamic models of these machines, where the effects of slotting, mutual interaction between the slots and permanent magnets, saturation, cross-saturation, and—in the case of linear machines—the end effects, are considered. The iron core is considered to be conservative, without any losses. The proposed method contains two steps. In the first step, the dynamic model state variables are selected. They are required to determine the model structure in an arbitrarily chosen reference frame. In the second step, the model parameters, described as state variable dependent functions, are determined. In this way, the magnetically nonlinear behavior of the machine is accounted for. The relations among the Fourier coefficients of flux linkages and electromagnetic torque/thrust are presented for the models written in dq reference frame. The paper presents some of the experimental methods appropriate for determining parameters of the discussed dynamic models, which is supported by experimental results.

Keywords: synchronous machines; dynamic models; nonlinear magnetics; parameter estimation

1. Introduction

Rotary electric machines perform rotary motion or rotation, the origin of which is the electromagnetic torque produced in the machine. Linear electric machines perform linear motion or translation. It is caused by the thrust produced in the machine. In the linear and rotary permanent magnet synchronous machines (PMSMs), the torque or thrust that causes motion appears due to a magnetic field which results from the interaction between magnetic excitations caused by the winding currents and the permanent magnets. In synchronous reluctance machines (SRMs) the origin of motion is a force or torque caused by the magnetic field resulting from the interaction of winding currents and variable reluctance. Modern PMSMs make use of both phenomena in order to increase the thrust or torque.

In the modern modeling of electric machines, the contribution of G. Kron [1–3], where he set a solid and general theoretical background for modeling of electric machines, is often neglected. The generality of Kron's tensor-based approach was reduced by introducing matrices [4–6], where electric machines are mostly treated as magnetically linear systems with neglected magnetically nonlinear properties. However, these properties have to be included in the dynamic models of electric machines when a good agreement between the measured and calculated results is required, or when these models are applied in nonlinear control design for demanding applications.

A constant saliency ratio and a variable inductance are applied in [7–9] to describe the effects of saturation while the authors in [10] use two parameters to describe the magnetically nonlinear properties of a synchronous machine. The authors in [11] show that only using one parameter seems to be insufficient for a proper description of magnetically nonlinear properties of a PMSM. The characteristics of flux linkages are applied in [12] to describe the magnetically nonlinear properties of a linear SRM. The model proposed in [12] is used in the realization of nonlinear input-output linearizing control in [13], where the magnetically nonlinear properties of the machine are considered. To represent a conservative or loss-less system, the characteristics of flux linkages must fulfill the conditions described in [14,15]. An energy function-based magnetically nonlinear model of an SRM is described in [16]. It is applied in the sensorless control realizations reported in [17,18]. The magnetically nonlinear dynamic models of PMSMs based on the characteristics of flux linkages are presented in [19–21]. The authors in [22–24] demonstrate the use of magnetically nonlinear PMSM models in different control applications. On the contrary, the authors in [25–32] use mainly magnetically linear models of PMSMs for similar purposes. Different methods that can be used to determine the parameters of magnetically nonlinear PMSM and SRM models are presented in [11,12], [16], [19–21] and [33–40].

The authors of some of the aforementioned papers often use the magnetically nonlinear dynamic models of synchronous machines without explaining how these models were derived and how their parameters were determined. Therefore, this paper proposes a general procedure for determining the magnetically nonlinear dynamic models of rotary and linear PMSMs and SRMs, which is presented in Section 2. The proposed procedure is completed by the descriptions of experimental methods that can be applied for determining parameters of the obtained models. They are presented in Section 3. The paper ends with the presentation of experimental results given in Section 4, and the Conclusion given in Section 5. The novelties in this paper are related to the proposed straightforward method for determining the structure of the magnetically nonlinear two-axis dynamic models of rotary and linear PMSMs and SRMs written in dq references frame, to the presented relations among Fourier coefficients of flux linkages and electromagnetic force/thrust, and partially to the experimental methods applied for determining required parameters of the obtained dynamic models.

2. Dynamic Models

This section first explains the differences between the magnetically nonlinear properties of ferromagnetic materials, and the magnetically nonlinear properties of an entire electric machine observed from its terminals. Then the three-phase rotary PMSM is described with its three-phase magnetically nonlinear dynamic model in a general form, where the effects of slotting, interaction between the slots and permanent magnets, saturation, and cross-saturation are considered. The iron core is considered to be a conservative, or loss-less, system [14,15], which means that the effects of iron core losses are neglected. The three-phase model is then transformed into a model written in the $dq0$ reference frame, where the d -axis is aligned with the flux linkage vector due to the permanent magnets. In this way, only the structure of the magnetically nonlinear model written in the $dq0$ reference frame is obtained, whereas the model parameters must be determined either experimentally or, e.g., by finite element analysis (FEA). Some of the suitable experimental methods are presented in Section 3. The model flux linkages and the electromagnetic torque are expressed in the form of Fourier series. The relations between the Fourier coefficients of flux linkages and the electromagnetic torque are presented. The obtained magnetically nonlinear PMSM dynamic model written in the $dq0$ reference frame is modified in order to be suitable for a proper description of PMSM performing linear motion, where the end effects are an integral part of this model. By neglecting the effects of permanent magnets in the obtained dynamic models of rotary and linear PMSMs, the magnetically nonlinear dynamic models of rotary and linear SRMs are determined.

2.1. Magnetically Nonlinear Behavior of an Electric Machine

When dealing with a ferromagnetic material, its magnetically nonlinear behavior can be described by the $B(H)$ characteristics, where B is the magnetic flux density while H is the magnetic field strength [41]. The $B(H)$ characteristics are normally determined experimentally for a specimen of the material [39]. The differential permeability, given in the form of the partial derivative $\partial B/\partial H$, can be used to describe the local changes in the properties of an isotropic material along the $B(H)$ characteristic. Similarly, in the case of an anisotropic material, the changes in material properties influence the relation between the flux density vector $\mathbf{B}$ and the magnetic field strength vector $\mathbf{H}$, which can be described with the differential permeability tensor $\partial \mathbf{B}/\partial \mathbf{H}$. Unfortunately, the approach appropriate to describe the magnetically nonlinear behavior of material is not appropriate to describe the resultant magnetically nonlinear behavior of an entire electric machine, especially in cases where only those variables available on the machine's terminals can be used to describe the machine's magnetically nonlinear behavior.

In an electric machine, different kinds of material can be combined [42–47] in order to reach different goals. Different kinds of material, together with the machine's geometry, influence the resultant magnetically nonlinear behavior of the entire machine as it can be observed through the variables measured on the machine's terminals. These variables are the currents and voltages, while the flux linkages can be determined by the integration of corresponding voltages. If the machine contains only one winding, its magnetically nonlinear behavior can be described by the relation between the flux linkage ψ and the current i , given in the form of $\psi(i)$ characteristic. The local magnetically nonlinear behavior of the machine along the $\psi(i)$ characteristic is described with the partial derivative $\partial \psi/\partial i$. In the case of an electric machine that contains more windings, the currents and corresponding flux linkages of all windings can be arranged in the current vector $\mathbf{i}$ and flux linkage vector $\boldsymbol{\psi}$. The use of linearly independent variables in both vectors is recommendable. The magnetically nonlinear behavior of the machine can be described by the $\boldsymbol{\psi}(\mathbf{i})$ characteristics, while the partial derivative $\partial \boldsymbol{\psi}/\partial \mathbf{i}$ can be used to describe the local magnetically nonlinear behavior along the $\boldsymbol{\psi}(\mathbf{i})$ characteristics.

2.2. Dynamic Model of a Rotary Three-Phase PMSM

A schematic presentation of the two-pole, three-phase PMSM is given in Figure 1 where the model phase windings are placed in the magnetic axes of actual phase a, b and c stator windings.

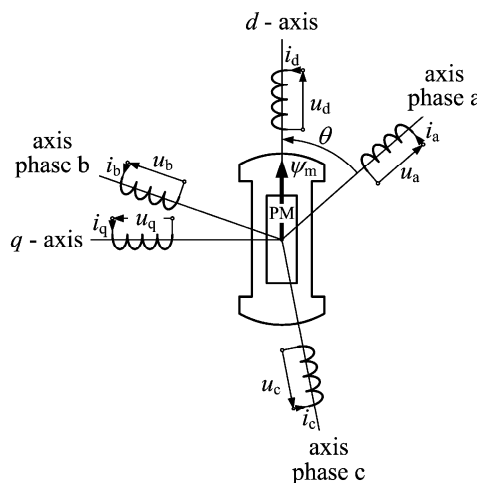


Figure 1. Schematic presentation of a two-pole, three-phase PMSM.

The d -axis is aligned with the magnetic axis of the flux linkage vector $\boldsymbol{\psi}_m$ due to the permanent magnets placed on the rotor, while ψ_m denotes the length of $\boldsymbol{\psi}_m$. The q -axis leads the d -axis for the electric angle of $\pi/2$. The d -axis is displaced with respect to the phase a axis for an electric angle θ which represents the (angular) rotor position.

The mathematical description of the PMSM shown in Figure 1, written in its most general form in the abc reference frame, defined with the magnetic axes of the phase a, b and c windings, is given by (1) to (4):

$$\mathbf{u}_{abc} = \mathbf{R}\mathbf{i}_{abc} + \frac{d}{dt}\boldsymbol{\psi}_{abc} + \frac{d}{dt}\boldsymbol{\psi}_{mabc}, \quad (1)$$

$$J\frac{d^2\theta}{dt^2} = t_e(i_a, i_b, i_c, \psi_m, \theta) - t_l - b\frac{d\theta}{dt}, \quad (2)$$

$$\mathbf{u}_{abc} = \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}, \quad \mathbf{i}_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} R_a & & \\ & R_b & \\ & & R_c \end{bmatrix}, \quad (3)$$

$$\boldsymbol{\psi}_{abc} = \begin{bmatrix} \psi_a(i_a, i_b, i_c, \theta) \\ \psi_b(i_a, i_b, i_c, \theta) \\ \psi_c(i_a, i_b, i_c, \theta) \end{bmatrix}, \quad \boldsymbol{\psi}_{mabc} = \begin{bmatrix} \psi_{ma}(\theta) \\ \psi_{mb}(\theta) \\ \psi_{mc}(\theta) \end{bmatrix}, \quad (4)$$

where u_a , u_b and u_c are the phase a, b and c voltages, i_a , i_b and i_c are the phase a, b and c currents, R_a , R_b and R_c are the phase a, b and c resistances, J is the moment of inertia, t_e is the electromagnetic torque, t_l is the load torque while b is the coefficient of friction. The phase a, b and c flux linkages caused by the permanent magnets are marked with ψ_{ma} , ψ_{mb} and ψ_{mc} , respectively. Similarly, ψ_a , ψ_b and ψ_c are the phase a, b and c flux linkages caused by the magnetic excitation due to the stator currents.

The three-phase magnetically nonlinear PMSM dynamic model, given in a general form by (1) to (4), is not intended to be used in the design of electric machines, but in dynamic simulations and control design. It is given for a two-pole machine or two poles of a multi-pole machine. For PMSMs with a higher number of the pole pairs ($p > 1$), the electromagnetic torque in (2) must be multiplied by p and the relation between the mechanical angle θ_m and the electric angle $\theta = p\theta_m$ must be considered. The electromagnetic torque t_e in (2) depends on the stator currents i_a , i_b , i_c , flux linkages due to the permanent magnets ψ_m and the position θ . The presumption that the flux linkages due to the permanent magnets are position-dependent, while the flux linkages due to the magnetic excitation with the stator currents are current- and position-dependent, seems to be reasonable as a first approximation. Both the position-dependent flux linkages due to the permanent magnets as well as the current- and position-dependent flux linkages due to the stator current excitation are used in model (1)–(4) to consider the effects of slotting, interactions between the slots and permanent magnets, saturation, and cross-saturation. The characteristics of the current- and position-dependent flux linkages can be determined either experimentally or by the FEA.

Most of the PMSMs used in electric drives are wye connected, which means that the currents i_a , i_b and i_c are linearly dependent. Since only the models with independent state variables can be used in the control design, the model (1)–(4), written in the abc reference frame, is transformed into another reference frame, where the state variables are independent while the obtained model is appropriate for the control design. A usual choice is the $dq0$ reference frame, where the d - and q -axes shown in Figure 1 are orthogonal, while the 0-axis is orthogonal to both of them. Since the flux linkages (4) are nonlinearly dependent on currents and position in order to consider the magnetically nonlinear behavior of the PMSM, the well-known methods for derivation of magnetically linear PMSM models [4,5] cannot be applied.

The procedure described in this work makes it possible only to determine the structure of the magnetically nonlinear PMSM model written in the $dq0$ reference frame. The corresponding model parameters in the form of current- and position-dependent characteristics of flux linkages have to be determined separately using experimental or FEA-based methods.

Independently of (1) and (2), which describe the voltage balances and motion of the PMSM, the voltage and current vectors $\mathbf{u}_{abc}$ and $\mathbf{i}_{abc}$, as well as the position-dependent vector of flux linkages due to the permanent magnets $\boldsymbol{\psi}_{mabc}$, can be always written in a new reference frame. The relation

between the vector written in the reference frame abc ($\bullet_{abc}$) and the one written in the reference frame $dq0$ ($\bullet_{dq0}$) is given by the transformation matrix $\mathbf{T}$ (5) and Equations (6) and (7):

$$\mathbf{T} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & \frac{\sqrt{2}}{2} \\ \cos(\theta + \frac{4}{3}\pi) & -\sin(\theta + \frac{4}{3}\pi) & \frac{\sqrt{2}}{2} \\ \cos(\theta + \frac{2}{3}\pi) & -\sin(\theta + \frac{2}{3}\pi) & \frac{\sqrt{2}}{2} \end{bmatrix} \quad (5)$$

$$\mathbf{u}_{abc} = \mathbf{T}\mathbf{u}_{dq0}; \mathbf{i}_{abc} = \mathbf{T}\mathbf{i}_{dq0}; \boldsymbol{\psi}_{mabc} = \mathbf{T}\boldsymbol{\psi}_{mdq0}; \boldsymbol{\psi}_{abc} = \mathbf{T}\boldsymbol{\psi}_{dq0}; \quad (6)$$

$$\mathbf{u}_{dq0} = \begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix}, \quad \mathbf{i}_{dq0} = \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}, \quad \boldsymbol{\psi}_{mdq0} = \begin{bmatrix} \psi_{md}(\theta) \\ \psi_{mq}(\theta) \\ \psi_{m0}(\theta) \end{bmatrix}, \quad \boldsymbol{\psi}_{dq0} = \begin{bmatrix} \psi_d(i_d, i_q, i_0, \theta) \\ \psi_q(i_d, i_q, i_0, \theta) \\ \psi_0(i_d, i_q, i_0, \theta) \end{bmatrix} \quad (7)$$

where indices d , q and 0 denote the currents, voltages and flux linkages written in the $dq0$ reference frame. Again, $\boldsymbol{\psi}_{mdq0}$ is the position-dependent flux linkage vector due to the permanent magnets, while $\boldsymbol{\psi}_{dq0}$ is the current and position vector dependent flux linkage vector due to magnetic excitation of the stator currents. Before the relation $\boldsymbol{\psi}_{abc} = \mathbf{T}\boldsymbol{\psi}_{dq0}$ can be written, the currents i_a , i_b and i_c in $\boldsymbol{\psi}_{abc}(i_a, i_b, i_c, \theta)$ (4) must be replaced with the currents i_d , i_q and i_0 . This means that the flux linkage vector $\boldsymbol{\psi}_{abc}$, that describes magnetically nonlinear behavior of the PMSM, is expressed as a nonlinear function $\boldsymbol{\psi}_{abc}(i_d, i_q, i_0, \theta)$.

Considering (6) in (1) yields (8), while (11) is obtained after simple mathematical manipulations in (9) and (10).

$$\mathbf{T}\mathbf{u}_{dq0} = \mathbf{R}\mathbf{T}\mathbf{i}_{dq0} + \frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{dq0}\} + \frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{mdq0}\} \quad (8)$$

$$\mathbf{u}_{dq0} = \mathbf{T}^{-1}\mathbf{R}\mathbf{T}\mathbf{i}_{dq0} + \mathbf{T}^{-1}\frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{dq0}\} + \mathbf{T}^{-1}\frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{mdq0}\} \quad (9)$$

$$\mathbf{u}_{dq0} = \mathbf{T}^{-1}\mathbf{R}\mathbf{T}\mathbf{i}_{dq0} + \mathbf{T}^{-1}\frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{dq0}\} + \mathbf{T}^{-1}\mathbf{T}\frac{d}{dt}\{\boldsymbol{\psi}_{dq0}\} + \mathbf{T}^{-1}\frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{mdq0}\} + \mathbf{T}^{-1}\mathbf{T}\frac{d}{dt}\{\boldsymbol{\psi}_{mdq0}\} \quad (10)$$

$$\mathbf{u}_{dq0} = \mathbf{T}^{-1}\mathbf{R}\mathbf{T}\mathbf{i}_{dq0} + \mathbf{T}^{-1}\frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{dq0}\} + \frac{d}{dt}\{\boldsymbol{\psi}_{dq0}\} + \mathbf{T}^{-1}\frac{d}{dt}\{\mathbf{T}\boldsymbol{\psi}_{mdq0}\} + \frac{d}{dt}\{\boldsymbol{\psi}_{mdq0}\} \quad (11)$$

The form of the matrix equation that describes voltage balances in the magnetically nonlinear PMSM dynamic model, written in the $dq0$ reference frame, is given by (11). Considering the balanced stator resistances $R = R_a = R_b = R_c$ in (3) and (5), (7), and (9) in (11) gives (12):

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_d}{\partial i_d} & \frac{\partial \psi_d}{\partial i_q} & \frac{\partial \psi_d}{\partial i_0} \\ \frac{\partial \psi_q}{\partial i_d} & \frac{\partial \psi_q}{\partial i_q} & \frac{\partial \psi_q}{\partial i_0} \\ \frac{\partial \psi_0}{\partial i_d} & \frac{\partial \psi_0}{\partial i_q} & \frac{\partial \psi_0}{\partial i_0} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \frac{d\theta}{dt} \cdot \left\{ \begin{bmatrix} \frac{\partial \psi_{md}}{\partial \theta} \\ \frac{\partial \psi_{mq}}{\partial \theta} \\ \frac{\partial \psi_{m0}}{\partial \theta} \end{bmatrix} + \begin{bmatrix} -\psi_q \\ \psi_d \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_{md}}{\partial \theta} \\ \frac{\partial \psi_{mq}}{\partial \theta} \\ \frac{\partial \psi_{m0}}{\partial \theta} \end{bmatrix} + \begin{bmatrix} -\psi_{mq} \\ \psi_{md} \\ 0 \end{bmatrix} \right\} \quad (12)$$

where the time derivatives are expressed by the partial derivatives using the chain rule. For the wye connected PMSM, the current i_0 from (7) can be expressed using (6) and the inverse $\mathbf{T}^{-1}$ of the transformation matrix $\mathbf{T}$ (5), which gives (13).

$$i_a + i_b + i_c = 0 \Rightarrow i_0 = \sqrt{\frac{2}{3}} \frac{1}{\sqrt{2}} (i_a + i_b + i_c) = 0 \quad (13)$$

Since i_0 equals 0, all terms containing i_0 in (12) can be eliminated. The machine's neutral point voltage u_0 is caused by the locally changing saturation level inside the machine. Considering (12), u_0 can be expressed by (14) as:

$$u_0 = \frac{\partial \psi_0}{\partial i_d} \frac{di_d}{dt} + \frac{\partial \psi_0}{\partial i_q} \frac{di_q}{dt} + \frac{\partial \psi_0}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial \psi_{m0}}{\partial \theta} \frac{d\theta}{dt} \quad (14)$$

where on the right-hand side of (14) the dominant last term represents the changes of the flux linkage vector component ψ_{m0} , caused by the changing position of the permanent magnets with respect to the stator windings. The first three normally recessive terms appear due to the changes in ψ_0 caused by the changing i_d , i_q and θ . In the cases when iron core losses are important, the changing flux linkages ψ_0 and ψ_{m0} can contribute to the increase of iron core losses. The effects of changing neutral point voltage u_0 are often neglected, since they are implicitly compensated by the closed-loop current controllers, which force the currents i_d and i_q to follow their references. Moreover, according to (12), u_0 cannot directly influence the d - and q -axis currents, voltages and flux linkages. After neglecting all terms in (12) that contribute to u_0 , (12) changes to (15). Equation (15) describes the voltage balances in the magnetically nonlinear two-axis dynamic model of PMSM written in the dq reference frame. The model is completed by (16), describing motion while the torque equation is still missing.

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_d}{\partial i_d} & \frac{\partial \psi_d}{\partial i_q} \\ \frac{\partial \psi_q}{\partial i_d} & \frac{\partial \psi_q}{\partial i_q} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d\theta}{dt} \left\{ \begin{bmatrix} \frac{\partial \psi_d}{\partial \theta} \\ \frac{\partial \psi_q}{\partial \theta} \end{bmatrix} + \begin{bmatrix} -\psi_q \\ \psi_d \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_{md}}{\partial \theta} \\ \frac{\partial \psi_{mq}}{\partial \theta} \end{bmatrix} + \begin{bmatrix} -\psi_{mq} \\ \psi_{md} \end{bmatrix} \right\} \quad (15)$$

$$J \frac{d^2 \theta}{dt^2} = t_e - t_l - b \frac{d\theta}{dt} \quad (16)$$

The electromagnetic torque t_e can be determined as a partial derivative of coenergy W_c [5] (17).

$$t_e(i_d, i_q, \theta) = \frac{\partial W_c(i_d, i_q, \theta)}{\partial \theta} \quad (17)$$

To consider the effects of cogging torque in the cases when the stator currents i_d, i_q are not flowing, it is wise to describe the flux linkages due to the permanent magnets ψ_{md} and ψ_{mq} as position-dependent functions of a fictive excitation current. In this way, the coenergy can change with the position even when i_d, i_q are not flowing, while its partial derivative gives a position-dependent term in the electromagnetic torque which represents the cogging torque. The calculation of coenergy could be quite a demanding task especially in the cases when the flux linkages due to the permanent magnets are position-dependent while the flux linkages due to the stator current excitation are current- and position-dependent. Therefore, the expression for electromagnetic torque is determined from the power balance (18), where the products of induced voltages e_d, e_q and currents i_d, i_q in d - and q -axis are given by (19).

$$\frac{d\theta}{dt} t_e = e_d i_d + e_q i_q \quad (18)$$

$$\begin{aligned} e_d i_d &= \frac{d\theta}{dt} \left(\frac{\partial \psi_d}{\partial \theta} + \frac{\partial \psi_{md}}{\partial \theta} - \psi_q - \psi_{mq} \right) i_d \\ e_q i_q &= \frac{d\theta}{dt} \left(\frac{\partial \psi_q}{\partial \theta} + \frac{\partial \psi_{mq}}{\partial \theta} + \psi_d + \psi_{md} \right) i_q \end{aligned} \quad (19)$$

The comparison of the left-hand side and the right-hand side of (18), considering products (19), gives (20).

$$t_e = \left(\frac{\partial \psi_d}{\partial \theta} + \frac{\partial \psi_{md}}{\partial \theta} - \psi_q - \psi_{mq} \right) i_d + \left(\frac{\partial \psi_q}{\partial \theta} + \frac{\partial \psi_{mq}}{\partial \theta} + \psi_d + \psi_{md} \right) i_q \quad (20)$$

Equation (20) represents a very good approximation for electromagnetic torque calculation. It gives acceptable results with the exception of no current condition, where the cogging torque cannot be calculated properly.

The magnetically nonlinear two-axis PMSM model is given by (15), (16) and (20). To consider the magnetically nonlinear behavior of the PMSM, which means the effects of slotting, the interaction

between the slots and permanent magnets as well as the saturation and cross-saturation, the characteristics of flux linkages must be determined properly, which is discussed in the next section. The next subsection describes the relations between individual harmonic components of the flux linkages and electromagnetic torque.

2.3. Fourier Analysis of Flux Linkages and Torque

The position- and current-dependent flux linkages ψ_d and ψ_q , caused by the stator currents, are given in the form of Fourier series (21) and (22) for different constant values of i_d and i_q :

$$\psi_d = \psi_{d0} + \sum_{h=1}^N (\psi_{dch} \cos(h\theta) + \psi_{dsh} \sin(h\theta)) \quad (21)$$

$$\psi_q = \psi_{q0} + \sum_{h=1}^N (\psi_{qch} \cos(h\theta) + \psi_{qsh} \sin(h\theta)) \quad (22)$$

where h is the harmonic order, N is the highest harmonic order while ψ_{d0} , ψ_{q0} , ψ_{dch} , ψ_{qch} and ψ_{dsh} , ψ_{qsh} are the Fourier coefficients. Similarly, ψ_{md0} , ψ_{mq0} , ψ_{mdch} , ψ_{mqch} and ψ_{mdsh} , ψ_{mqsh} are the Fourier coefficients of the position-dependent flux linkages ψ_{md} and ψ_{mq} , caused by the permanent magnets and given in the form of Fourier series (23) and (24).

$$\psi_{md} = \psi_{md0} + \sum_{h=1}^N (\psi_{mdch} \cos(h\theta) + \psi_{mdsh} \sin(h\theta)) \quad (23)$$

$$\psi_{mq} = \psi_{mq0} + \sum_{h=1}^N (\psi_{mqch} \cos(h\theta) + \psi_{mqsh} \sin(h\theta)) \quad (24)$$

The electromagnetic torque t_e can be expressed in the form of Fourier series (25), with the Fourier coefficients T_{d0} , T_{ch} and T_{sh} .

$$t_e = T_{e0} + \sum_{h=1}^N (T_{ch} \cos(h\theta) + T_{sh} \sin(h\theta)) \quad (25)$$

Considering (21) to (24) in (20) leads to the relations between the Fourier coefficients of electromagnetic torque and flux linkages described by (26) to (28), where h is the harmonic order.

$$T_{e0} = ((\psi_{d0} + \psi_{md0})i_q - (\psi_{q0} + \psi_{mq0})i_d) \quad (26)$$

$$T_{ech} = (\psi_{dch} + \psi_{mdch})i_q - (\psi_{qch} + \psi_{mqch})i_d + h(\psi_{dsh} + \psi_{mdsh})i_d + h(\psi_{qsh} + \psi_{mqsh})i_q \quad (27)$$

$$T_{esh} = (\psi_{dsh} + \psi_{mdsh})i_q - (\psi_{qsh} + \psi_{mqsh})i_d - h(\psi_{dch} + \psi_{mdch})i_d - h(\psi_{qch} + \psi_{mqch})i_q \quad (28)$$

The well-known equation for electromagnetic torque of a PMSM is obtained when only the DC torque component (26) is applied. Such a description of electromagnetic torque is normally sufficient for application at higher speeds, where the losses are not of primary interest, while the higher harmonic order torque pulsation is filtered out through the mechanical subsystem of the PMSM and does not influence the speed and position trajectories. However, in low-speed applications, the higher harmonic order torque pulsation cannot be filtered out through the mechanical subsystem of the PMSM and directly influence the speed and position trajectories. In such cases, the terms (27) and (28), containing only a few dominant torque harmonics, can be substantial for a proper, often nonlinear, control design and smooth tracking of position and speed references.

2.4. Dynamic Model of Linear PMSM

In the case of linear PMSM, the angular position θ is expressed with the pole pitch τ_p and the position x (29), while the electromagnetic torque t_e (20) and the moment of inertia J in (2) are replaced with the thrust f_e (31) and the mass m in (32). Considering these changes, the magnetically nonlinear two-axis dynamic model of a linear PMSM is given by (29) to (32):

$$\theta = \frac{\pi}{\tau_p} x \quad (29)$$

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_d}{\partial i_d} & \frac{\partial \psi_d}{\partial i_q} \\ \frac{\partial \psi_q}{\partial i_d} & \frac{\partial \psi_q}{\partial i_q} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\pi}{\tau_p} \frac{dx}{dt} \left\{ \begin{bmatrix} \frac{\partial \psi_d}{\partial x} \\ \frac{\partial \psi_q}{\partial x} \end{bmatrix} + \begin{bmatrix} -\psi_q \\ \psi_d \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_{md}}{\partial x} \\ \frac{\partial \psi_{mq}}{\partial x} \end{bmatrix} + \begin{bmatrix} -\psi_{mq} \\ \psi_{md} \end{bmatrix} \right\} \quad (30)$$

$$f_e = \frac{\pi}{\tau_p} \left(\frac{\partial \psi_d}{\partial x} + \frac{\partial \psi_{md}}{\partial x} - \psi_q - \psi_{mq} \right) i_d + \frac{\pi}{\tau_p} \left(\frac{\partial \psi_q}{\partial x} + \frac{\partial \psi_{mq}}{\partial x} + \psi_d + \psi_{md} \right) i_q \quad (31)$$

$$m \frac{d^2 x}{dt^2} = f_e - f_l - f_f \quad (32)$$

where f_l and f_f are the load force and friction force, respectively. Equation (30) describes voltage balances, (31) is the thrust expression, while (32) describes linear motion (translation). The expressions for the flux linkages (21) to (24) preserve their forms while θ is replaced with $x \pi / \tau_p$ (29). Similarly, the expressions for electromagnetic torque (25) to (28) change to (33) to (36), describing the position-dependent thrust produced by the linear PMSM, where F_0 , F_{ch} and F_{sh} are the Fourier coefficients of the thrust f_e .

$$f_e = F_0 + \sum_{h=1}^N \left(F_{ch} \cos \left(h \frac{\pi}{\tau_p} x \right) + F_{sh} \sin \left(h \frac{\pi}{\tau_p} x \right) \right) \quad (33)$$

$$F_0 = \frac{\pi}{\tau_p} \left((\psi_{d0} + \psi_{md0}) i_q - (\psi_{q0} + \psi_{mq0}) i_d \right) \quad (34)$$

$$F_{ch} = \frac{\pi}{\tau_p} (\psi_{dch} + \psi_{mdch}) i_q - \frac{\pi}{\tau_p} (\psi_{qch} + \psi_{mqch}) i_d + \frac{\pi}{\tau_p} h (\psi_{dsh} + \psi_{mdsh}) i_d + \frac{\pi}{\tau_p} h (\psi_{qsh} + \psi_{mqsh}) i_q \quad (35)$$

$$F_{sh} = \frac{\pi}{\tau_p} (\psi_{dsh} + \psi_{mdsh}) i_q - \frac{\pi}{\tau_p} (\psi_{qsh} + \psi_{mqsh}) i_d - \frac{\pi}{\tau_p} h (\psi_{dch} + \psi_{mdch}) i_d - \frac{\pi}{\tau_p} h (\psi_{qch} + \psi_{mqch}) i_q \quad (36)$$

The magnetically nonlinear dynamic model of the linear PMSM is given by (29) to (31). It is completed by (33) to (36), required for the thrust calculation. In addition to the effects of slotting, interaction between the slots and permanent magnets, saturation and cross-saturation, this model implicitly also considers the end effects which are specific for the linear machines.

2.5. Dynamic Models of Rotary and Linear Synchronous Reluctance Machines

When the schematic presentations of the PMSM shown in Figure 1 and the one showing synchronous reluctance machine (SRM) in Figure 2 are compared, and neglecting the actual designs of the machines, the only substantial difference that can be pointed out is the missing permanent magnets in the case of SRMs. Thus, the d -axis is defined with the axis of the lowest reluctance.

The magnetically nonlinear two-axis dynamic models of the rotary and linear SRMs can be described with the same sets of equations as the models of corresponding PMSMs, omitting the flux linkages due to the permanent magnets and their partial derivatives. Considering $\psi_{md} = 0$, $\psi_{mq} = 0$, $\partial \psi_{md} / \partial \theta = 0$, and $\partial \psi_{mq} / \partial \theta = 0$ in (15) and (20) leads to the magnetically nonlinear two-axis dynamic model of a rotary SRM given by (37) describing voltage balances, (38) describing electromagnetic torque, and (16) describing motion.

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_d}{\partial i_d} & \frac{\partial \psi_d}{\partial i_q} \\ \frac{\partial \psi_q}{\partial i_d} & \frac{\partial \psi_q}{\partial i_q} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d\theta}{dt} \left\{ \begin{bmatrix} \frac{\partial \psi_d}{\partial \theta} \\ \frac{\partial \psi_q}{\partial \theta} \end{bmatrix} + \begin{bmatrix} -\psi_q \\ \psi_d \end{bmatrix} \right\} \quad (37)$$

$$t_e = \left(\frac{\partial \psi_d}{\partial \theta} - \psi_q \right) i_d + \left(\frac{\partial \psi_q}{\partial \theta} + \psi_d \right) i_q \quad (38)$$

Similarly, if $\psi_{md} = 0$, $\psi_{mq} = 0$, $\partial \psi_{md}/\partial x = 0$, and $\partial \psi_{mq}/\partial x = 0$ are considered in (30) and (31), the magnetically nonlinear two-axis dynamic model of a linear SRM can be given by (39) describing voltage balances, (40) describing thrust, and (32) describing motion.

$$\begin{bmatrix} u_d \\ u_q \end{bmatrix} = R \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{\partial \psi_d}{\partial i_d} & \frac{\partial \psi_d}{\partial i_q} \\ \frac{\partial \psi_q}{\partial i_d} & \frac{\partial \psi_q}{\partial i_q} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{\pi}{\tau_p} \frac{dx}{dt} \left\{ \begin{bmatrix} \frac{\partial \psi_d}{\partial x} \\ \frac{\partial \psi_q}{\partial x} \end{bmatrix} + \begin{bmatrix} -\psi_q \\ \psi_d \end{bmatrix} \right\} \quad (39)$$

$$f_e = \frac{\pi}{\tau_p} \left(\frac{\partial \psi_d}{\partial x} - \psi_q \right) i_d + \frac{\pi}{\tau_p} \left(\frac{\partial \psi_q}{\partial x} + \psi_d \right) i_q \quad (40)$$

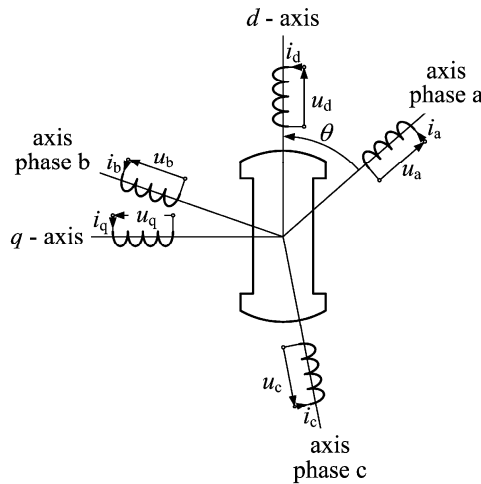


Figure 2. Schematic presentation of a two-pole, three-phase SRM.

3. Procedures for Determining Parameters of Dynamic Models

The magnetically nonlinear two-axis dynamic models of rotary and linear PMSMs and SRMs, written in the dq reference frame, are given in Section 2. The magnetically nonlinear properties of the discussed machines are described in the form of position-dependent characteristics of flux linkages due to the permanent magnets and in the form of current- and position-dependent characteristics of flux linkages due to the magnetic excitation of stator currents. These characteristics complete the models. They can be determined using FEA or experimental methods. Some of the experimental methods that can be applied for the rotary and linear machines are described in the next subsections. The other parameters required in the model, like the stator resistance R , the moment of inertia J , the mass m , and the coefficient of viscous friction b , are not discussed in this paper since they can be measured directly or determined by some of the well-known methods.

3.1. Flux Linkages Caused by the Permanent Magnets

The tested three-phase PMSM is driven by another speed-controlled machine at the constant speed. The position θ and the speed $\omega = d\theta/dt$ are measured together with the waveforms of the three-phase back electromotive forces (EMFs) e_a , e_b and e_c available on the open PMSM terminals. Considering (5)

to (7), the waveforms of e_d , e_q and e_0 are calculated. Neglecting e_0 and ψ_{m0} , and considering $i_d = 0$, $i_q = 0$, and $\omega = d\theta/dt$, (15) reduces to (41) and (42).

$$\frac{e_d}{\omega} = -\psi_{mq} + \frac{\partial\psi_{md}}{\partial\theta} \quad (41)$$

$$\frac{e_q}{\omega} = \psi_{md} + \frac{\partial\psi_{mq}}{\partial\theta} \quad (42)$$

After performing partial derivation of (41) and (42) with respect to θ , (43) and (44) are obtained. The partial derivatives $\partial\psi_{md}/\partial\theta$ and $\partial\psi_{mq}/\partial\theta$ expressed from (41) and (42) are considered in (43) and (44), which gives (45) and (46) [19–21].

$$\frac{\partial}{\partial\theta}\left(\frac{e_d}{\omega}\right) = -\frac{\partial\psi_{mq}}{\partial\theta} + \frac{\partial^2\psi_{md}}{\partial\theta^2} \quad (43)$$

$$\frac{\partial}{\partial\theta}\left(\frac{e_q}{\omega}\right) = \frac{\partial\psi_{md}}{\partial\theta} + \frac{\partial^2\psi_{mq}}{\partial\theta^2} \quad (44)$$

$$\frac{\partial^2\psi_{md}}{\partial\theta^2} + \psi_{md} = \frac{\partial}{\partial\theta}\left(\frac{e_d}{\omega}\right) + \frac{e_q}{\omega} \quad (45)$$

$$\frac{\partial^2\psi_{mq}}{\partial\theta^2} + \psi_{mq} = \frac{\partial}{\partial\theta}\left(\frac{e_q}{\omega}\right) - \frac{e_d}{\omega} \quad (46)$$

The back EMFs e_d and e_q , as well as the flux linkages ψ_{md} and ψ_{mq} , are expressed in the form of Fourier series (47) to (50):

$$e_d = e_{d0} + \sum_{h=1}^N (e_{dch} \cos(h\theta) + e_{dsh} \sin(h\theta)) \quad (47)$$

$$e_q = e_{q0} + \sum_{h=1}^N (e_{qch} \cos(h\theta) + e_{qsh} \sin(h\theta)) \quad (48)$$

$$\psi_{md} = \psi_{md0} + \sum_{h=1}^N (\psi_{mdch} \cos(h\theta) + \psi_{mdsh} \sin(h\theta)) \quad (49)$$

$$\psi_{mq} = \psi_{mq0} + \sum_{h=1}^N (\psi_{mqch} \cos(h\theta) + \psi_{mqsh} \sin(h\theta)) \quad (50)$$

where h is the harmonic order and N is the highest harmonic order. The Fourier coefficients of the back EMFs are denoted with e_{d0} , e_{dch} , e_{dsh} and e_{q0} , e_{qch} , e_{qsh} , while the ones describing the flux linkages are denoted with ψ_{md0} , ψ_{mdch} , ψ_{mdsh} and ψ_{mq0} , ψ_{mqch} , ψ_{mqsh} . After inserting (47) to (50) into (45) and (46), and performing the required partial derivations of (47) to (50), (51) and (52) are obtained. They describe the relations among individual Fourier coefficients of the flux linkages and back EMFs.

$$(1 - h^2)(\psi_{mdch} \cos(h\theta) + \psi_{mdsh} \sin(h\theta)) = \frac{he_{dsh} + e_{qch}}{\omega} \cos(h\theta) + \frac{-he_{dch} + e_{qsh}}{\omega} \sin(h\theta) \quad (51)$$

$$(1 - h^2)(\psi_{mqch} \cos(h\theta) + \psi_{mqsh} \sin(h\theta)) = \frac{he_{qsh} - e_{dch}}{\omega} \cos(h\theta) + \frac{-he_{qch} - e_{dsh}}{\omega} \sin(h\theta) \quad (52)$$

The comparison of the terms that multiply the same harmonic function on the left- and right-hand side of (51) and (52) gives (53) and (54), where the Fourier coefficients of flux linkages are expressed with the Fourier coefficients of back EMFs.

$$\psi_{mdch} = \frac{he_{dsh} + e_{qch}}{(1-h^2)\omega}, \quad \psi_{mdsh} = \frac{-he_{dch} + e_{qsh}}{(1-h^2)\omega} \quad (53)$$

$$\psi_{mqch} = \frac{he_{qsh} - e_{dch}}{(1-h^2)\omega}, \quad \psi_{mqsh} = \frac{-he_{qch} - e_{dsh}}{(1-h^2)\omega} \quad (54)$$

Thus, the position-dependent characteristics of flux linkages ψ_{md} and ψ_{mq} are completely defined with the Fourier coefficients of the back EMFs measured on the open terminals of the tested PMSM. To achieve acceptable results, it is often sufficient to consider the DC component supplemented by a few dominant higher order harmonics.

3.2. Flux Linkages Caused by the Stator Currents

The characteristics of current- and position-dependent flux linkages ψ_d and ψ_q , caused by the magnetic excitation of the stator currents, can be determined with tests performed at the locked rotor of the PMSM, where $d\theta/dt = 0$. The tested PMSM is supplied with a controlled voltage source inverter (VSI). The current in the one axis is closed loop controlled in order to keep the constant value, while the voltage in the orthogonal axis is changing in a stepwise manner. Considering the described conditions and a constant value of the current i_q , the first row of (15) is reduced to (55).

$$u_d = Ri_d + \frac{\partial \psi_d}{\partial i_d} \frac{di_d}{dt} = Ri_d + \frac{d\psi_d}{dt} \quad (55)$$

The stepwise changing voltage u_d and the responding current i_d , measured during the test, are applied to determine the time behavior of the flux linkage $\psi_d(t)$ by numerical integration (56):

$$\psi_d(t) = \psi_d(0) + \int_0^t (u_d(\tau) - Ri_d(\tau)) d\tau \quad (56)$$

where $\psi_d(0)$ is the initial d -axis flux linkage due to the permanent magnets and remanent flux. If $\psi_d(0)$ is considered to have a value of 0, the flux linkage $\psi_d(t)$ (56) contains only changes around pre-magnetization $\psi_d(0) = \psi_{md}$ caused by the permanent magnets. $\psi_d(t)$ can be presented as a current $i_d(t)$ dependent function given in the form of a hysteresis loop $\psi_d(i_d)$ at the constant values of i_q and θ . A family of hysteresis loops $\psi_d(i_d)$ is obtained by repeating the described procedure for different amplitudes of the stepwise changing voltages u_d . The end points of individual hysteresis loops are used to define a unique characteristic $\psi_d(i_d)$. If the described procedure is repeated first for different constant values of the closed-loop controlled current i_q , and after that for different positions of the locked rotor θ , the current- and position-dependent characteristics $\psi_d(i_d, i_q, \theta)$, required in (15) and (20), can be determined over the entire range of operation. Similar procedure is applied to determine the characteristics $\psi_q(i_d, i_q, \theta)$. It must be pointed out that $\psi_d(i_d, i_q, \theta)$ and $\psi_q(i_d, i_q, \theta)$ are the characteristics of flux linkages caused by the magnetic excitation of the stator current. Implicitly, these characteristics also contain the effects of pre-magnetization caused by the permanent magnets, considered as $\psi_d(0) = \psi_{md}$ (56) and similarly $\psi_q(0) = \psi_{mq}$.

The proposed method for determining the characteristics of flux linkages is robust, if the resistance R in (56) is determined and updated from the steady state current and voltage after each voltage step change

4. Experiments and Results

In this section, the descriptions of the applied experimental set-up, tested objects and presented results are given.

4.1. Experimental Set-Up and Tested Machines

The applied experimental set-up is schematically presented in Figure 3. It consists of the tested rotary (or linear) PMSM or SRM, controlled VSI, current measurement chains based on LEM current sensors, voltage measurement chains based on differential probes, torque sensor and measurement chain, external source of torque in the form of speed or torque controlled machine acting as an active load, and control system dSPACE 1103 PPC, which is used for the open-loop voltage control and closed-loop current control of the tested machine performed in the dq reference frame.

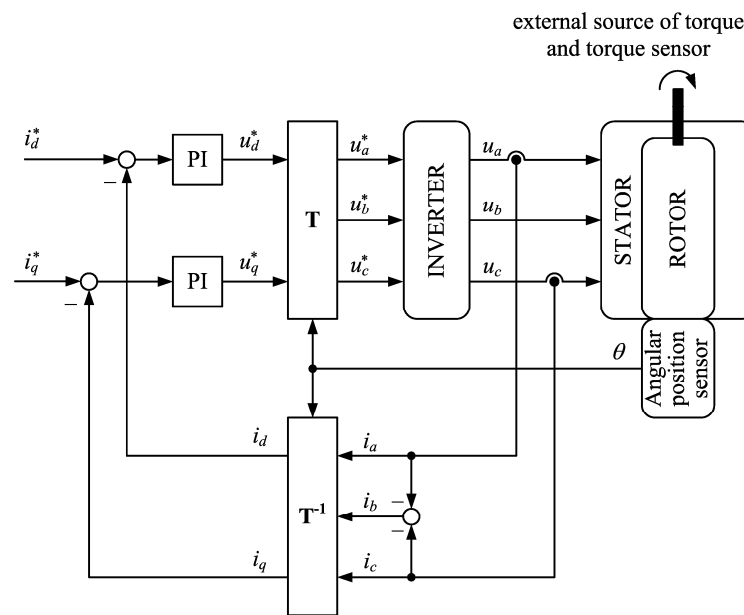


Figure 3. Schematic presentation of applied experimental set-up.

A photograph of the experimental set-up applied to determine the magnetically nonlinear characteristics of the tested rotary SRM and PMSM at locked rotor is shown in Figure 4. The tested synchronous machine shaft is connected to the mechanical brake with a clutch as shown in the photograph on the right-hand side in Figure 4. However, when the back-EMFs are measured, as is described in the next subsection, the mechanical brake is replaced by a speed-controlled driving motor, providing a constant angular speed. The stator terminals with connected differential probes are open which enables the measurement of the back-EMFs with differential probes.



Figure 4. Experimental set-up for testing rotary synchronous machines at locked rotor.

Similarly, Figure 5 shows the experimental set-up applied for experimental work on linear synchronous machines [12,48].

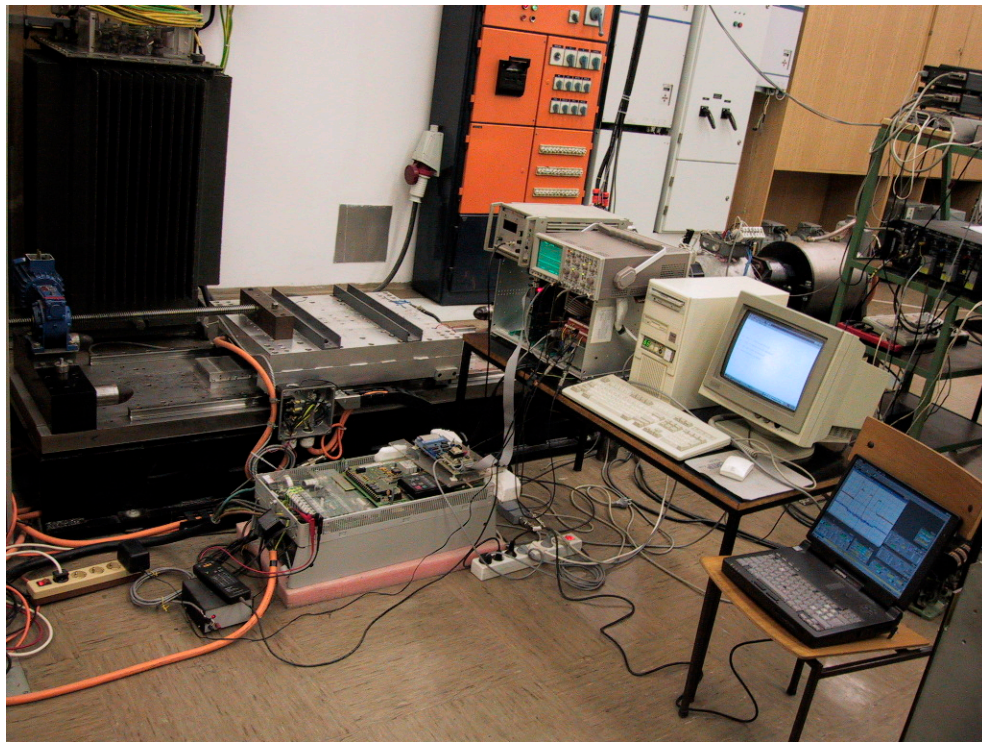


Figure 5. Experimental set-up for testing linear synchronous machines.

Two prototypes of rotary machines were built, with the aim of confirming the here-proposed methods experimentally. The stator of a standard 1.1 kW induction motor (frame size 90) with a four-pole three-phase winding in wye connection was employed. Two equal reluctance rotors with flux barriers were constructed. Permanent magnets of Nd-Fe-B type were inserted in the flux barriers of one of the rotors, thus producing a pure SRM and an interior PMSM; these were employed in the experimental study. The cross-sections of the aforementioned rotors are schematically represented in Figure 6.

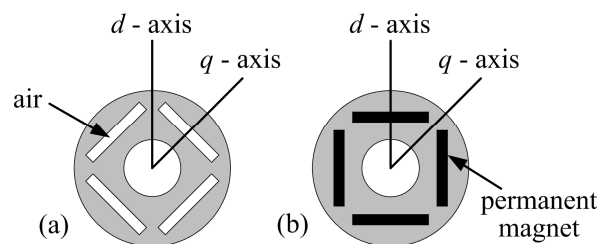


Figure 6. Schematic representation of rotor cross-sections of the SRM (a) and PMSM (b).

The third tested object is a prototype of a linear SRM with a short moving primary and a long secondary as shown in Figure 7 [48].

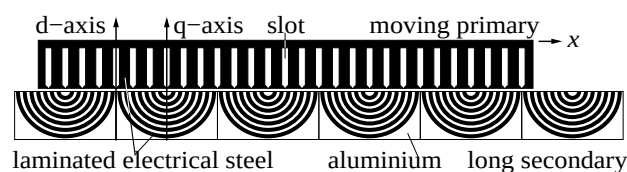


Figure 7. Schematic presentation of linear SRM prototype.

4.2. Determining Flux Linkages Caused by the Permanent Magnets

The procedure for determining these flux linkages is described in Section 3.1. Figure 8 shows the position-dependent three-phase back EMFs e_a , e_b and e_c measured on the open terminals of the tested PMSM driven by another speed-controlled machine. Figure 9 shows the back EMFs e_d , e_q and e_0 determined from e_a , e_b and e_c shown in Figure 8, by (6) considering the inverse of the transformation matrix (5). After performing the procedure described in Section 3.1, considering only the dominant harmonics up to the harmonic order 30, the position-dependent characteristics of flux linkages shown in Figure 10 are determined.

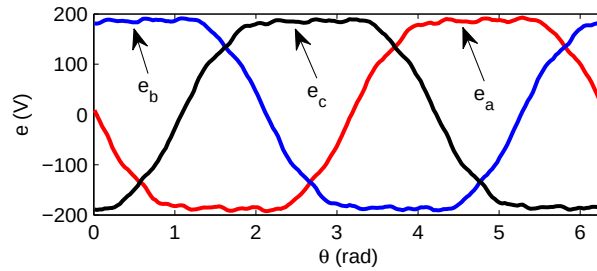


Figure 8. Back EMFs e_a , e_b and e_c measured on the open terminals of the tested PMSM.

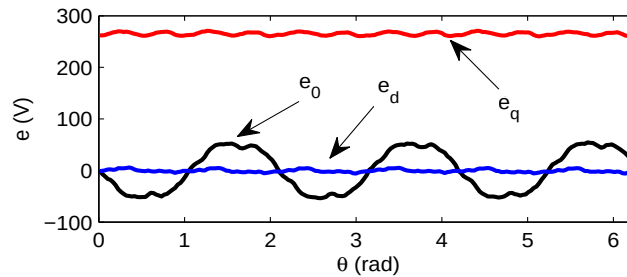


Figure 9. Back EMFs e_d , e_q and e_0 calculated from the measured back EMFs e_a , e_b and e_c .

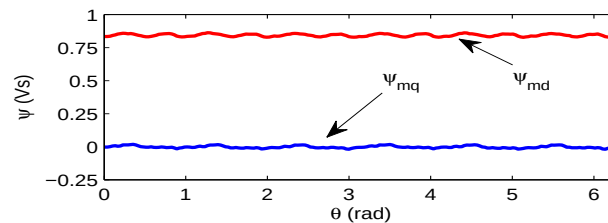


Figure 10. Position-dependent characteristics of flux linkages $\psi_{md}(\theta)$ and $\psi_{mq}(\theta)$.

Figure 9 clearly shows that the back EMFs e_a , e_b and e_c are transformed into e_d , e_q and e_0 , where $e_0 \neq 0$. In the given case the machine terminals are open, and no currents are flowing. Thus, e_0 equals the last term on the right-hand side of (14) and changes the neutral point voltage ($u_0 = e_0$). In the wye connected machines i_0 (13) cannot flow and cannot influence the voltages u_d and u_q in (12), although $u_0 \neq 0$. Therefore, (12) can be reduced to (15), which means that u_0 is neglected, although it exists. The model obtained in this way is correct. However, in the case of control scheme realization (Figure 3), the neutral point voltage u_0 changes all the time, which means that the current controllers compensate the influence of u_0 implicitly when minimizing the error between reference and measured d - and q -axis current trajectories.

From the results shown in Figure 10, it is evident that both $\psi_{md}(\theta)$ and $\psi_{mq}(\theta)$, change with the position θ . The flux linkage $\psi_{md}(\theta)$ changes around its DC component, while $\psi_{mq}(\theta)$ changes around the value 0.

Figure 11 show the comparison of measured and finite element analysis (FEA) determined phase a back EMF e_a at a constant angular speed of the rotor. The calculated results were obtained by an in

house developed program solution for 2D FEA, which is a further development of the one applied in [11].

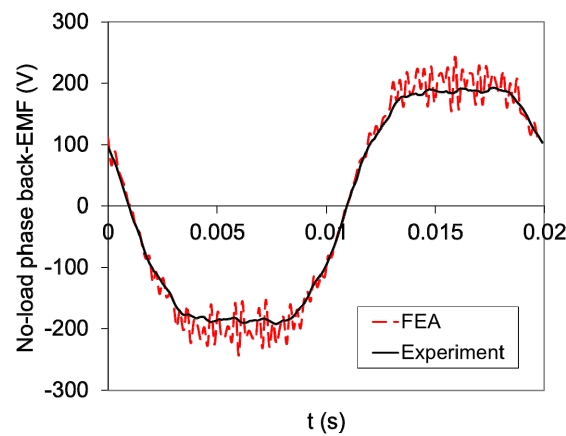


Figure 11. PMSM: measured and FEA calculated phase a back EMF e_a .

4.3. Determining Flux Linkages Caused by the Stator Currents

This procedure, which can be applied to determine the current- and position-dependent characteristics of flux linkages, caused by the stator currents, is described in Section 3.2.

The rotor is locked at a given position, while the current i_d is closed-loop controlled to keep a constant value. Figure 12 shows the applied voltage u_q and the responding current i_q measured during the experiment. Figure 13 shows the flux linkage $\psi_q(t)$ determined by numerical integration (56). The corresponding characteristic $\psi_q(i_q)$ in the form of a hysteresis loop is shown in Figure 14. The hysteresis loops determined for different amplitudes of the stepwise changing voltages u_q are shown in Figure 15, while Figure 16 shows a unique $\psi_q(i_q)$ characteristic for the constant value of $i_d = 0$ A. Figures 17 and 18 show the characteristics $\psi_d(i_d)$ at $i_q = 0$ A in the form of a hysteresis loop and in the form of a unique characteristic, respectively. The comparison of characteristics $\psi_d(i_d)$ and $\psi_q(i_q)$ determined for the SRM and PMSM is shown in Figures 19 and 20.

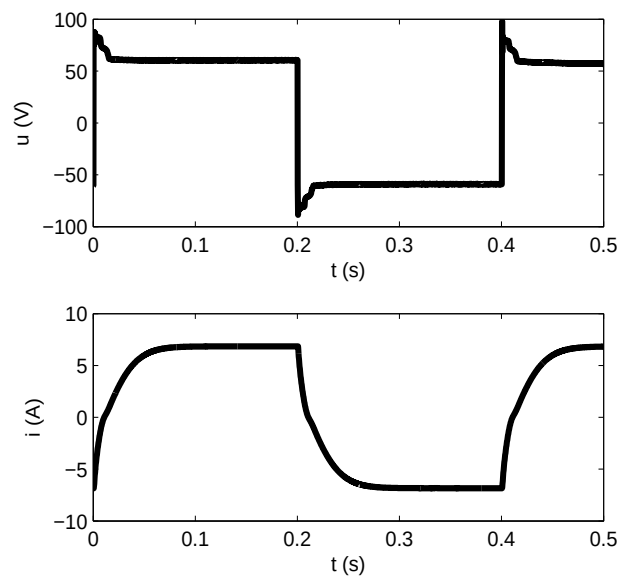


Figure 12. Stepwise changing voltage $u_q(t)$ and responding current $i_q(t)$ measured on the tested SRM for $i_d = 0$ A.

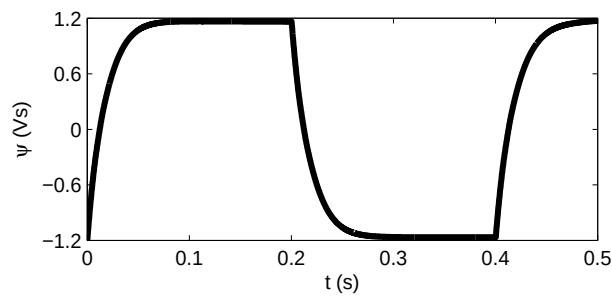


Figure 13. SRM: flux linkage $\psi_q(t)$ calculated using $u_q(t)$ and $i_q(t)$ shown in Figure 9.

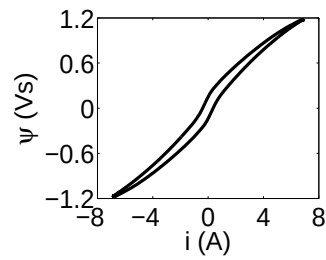


Figure 14. SRM: flux linkage characteristic $\psi_q(i_q)$ at $i_d = 0$ A given in the form of a hysteresis loop.

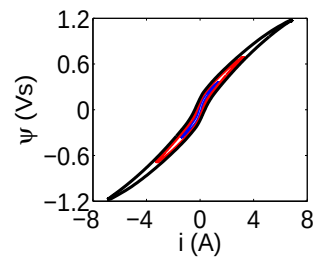


Figure 15. SRM: flux linkage characteristics $\psi_q(i_q)$ at $i_d = 0$ A given in the form of hysteresis loops determined for different amplitudes of applied voltage u_q .

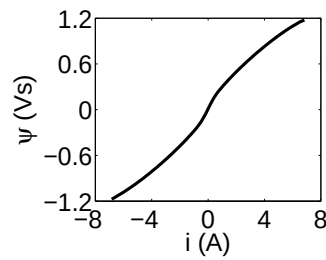


Figure 16. SRM: unique flux linkage characteristics $\psi_q(i_q)$ at $i_d = 0$ A.

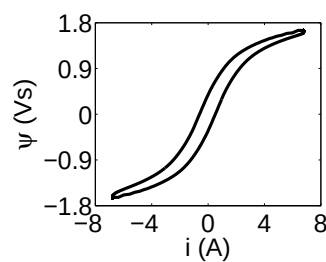


Figure 17. SRM: flux linkage characteristics $\psi_d(i_d)$ at $i_q = 0$ A given in the form of a hysteresis loop.

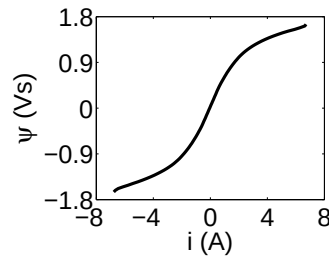


Figure 18. SRM: unique flux linkage characteristics $\psi_d(i_d)$ at $i_q = 0$ A.

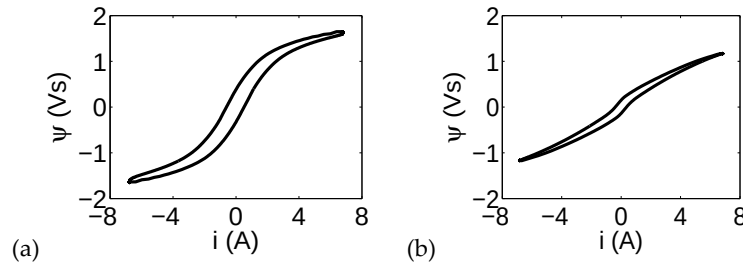


Figure 19. SRM: hysteresis loops. $\psi_d(i_d)$ at $i_q = 0$ A (a) and $\psi_q(i_q)$ at $i_d = 0$ A (b).

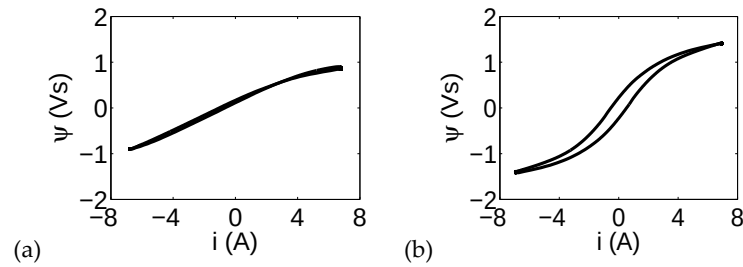


Figure 20. PMSM: hysteresis loops. $\psi_d(i_d)$ at $i_q = 0$ A (a) and $\psi_q(i_q)$ at $i_d = 0$ A (b).

The characteristics $\psi_d(i_d)$ and $\psi_q(i_q)$ shown in Figure 16 are centered. They are given for the SRM. The same characteristics are shown in Figure 17 for the PMSM. They are not centered, due to the pre-magnetization in the form of $\psi_{md}(\theta)$ and $\psi_{mq}(\theta)$ shown in Figure 8. However, this pre-magnetization is not shown explicitly in Figure 17. As mentioned before, the initial flux linkage due to the remanent flux and permanent magnets is considered with the value 0. Thus, the pre-magnetization is shown in Figure 17 rather indirectly through the increased saturation level in comparison to the SRM, shown in Figure 16. The increased saturation level decreases the slope of the $\psi_d(i_d)$ and $\psi_q(i_q)$ characteristics shown in Figure 17.

The flux linkages from Figure 20 are shown in Figure 21 as unique magnetically nonlinear characteristics determined in the same way as in [12]. Please note, that the flux linkage due to the permanent magnets $\psi_d(0) = \psi_{md}$, which appears in (56), is in this case considered with the value 0.

Based on the unique flux linkage characteristics $\psi_d(i_d)$ and $\psi_q(i_q)$ from Figure 21, the dynamic inductances $L_{sd} = \partial\psi_d/\partial i_d$ and $L_{sq} = \partial\psi_q/\partial i_q$ are calculated numerically. The comparison of experimentally and FEA determined dynamic inductances presented in Figure 22 shows an acceptable agreement.

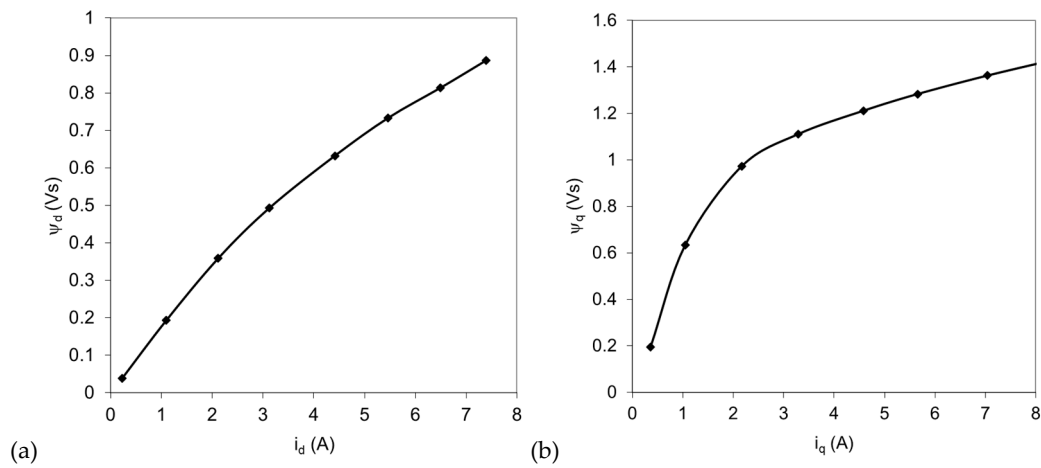


Figure 21. PMSM: unique magnetically nonlinear characteristics $\psi_d(i_d)$ at $i_q = 0$ A (a) and $\psi_q(i_q)$ at $i_d = 0$ A (b).

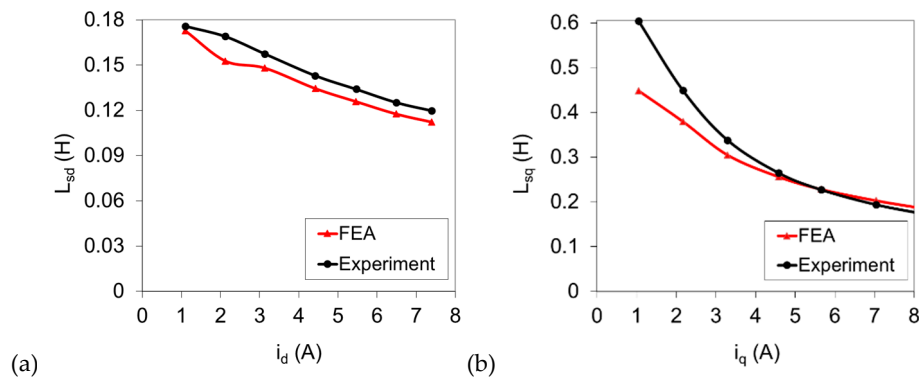


Figure 22. PMSM: FEA and experimentally determined dynamic inductances L_{sd} (a) and L_{sq} (b).

4.4. Low-Speed Kinematic Control of Linear SRM

The proposed models, as well as the experimental methods applied to determine model parameters in the form of position- and current-dependent flux linkages [12], are confirmed by the comparison of measured (Figure 23) and dynamic model calculated (Figures 24 and 25) trajectories of individual variables in the case of linear SRM kinematic control. The next set of results demonstrates the importance of higher order harmonic components in the thrust equation (33) in the case of low speed kinematic control. The tested linear SRM is shown in Figure 7. Its position x is changed for one pole pitch τ_p . The position x , current i_d , speed $v = dx/dt$, and current i_q , measured during the experiment, are shown in Figure 23. The calculations are performed with the proposed dynamic model of linear SRM given by (39), (32), (33) under the same conditions as the experiment shown in Figure 23. Figure 24 shows the calculated speed v and current i_q for the case when 24 higher order harmonic components are considered in (33). The same calculated variables are given in Figure 25 for the case when only DC components are considered in (33). The results presented clearly show that the higher order harmonic components of the thrust can substantially influence the calculated trajectories of the speed v and current i_q , which can be important in the case of low-speed kinematic control.

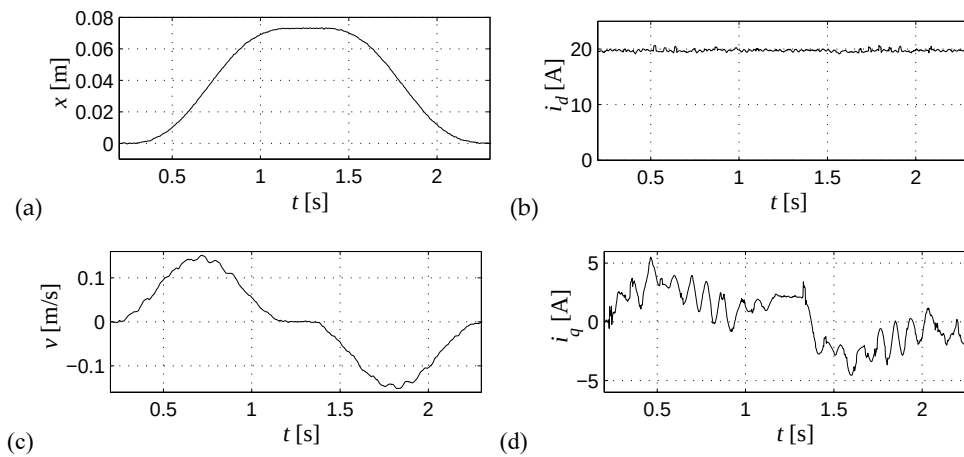


Figure 23. Kinematic control of linear SRM: measured position x (a), current i_d (b), speed $v = dx/dt$ (c) and current i_q . (d).

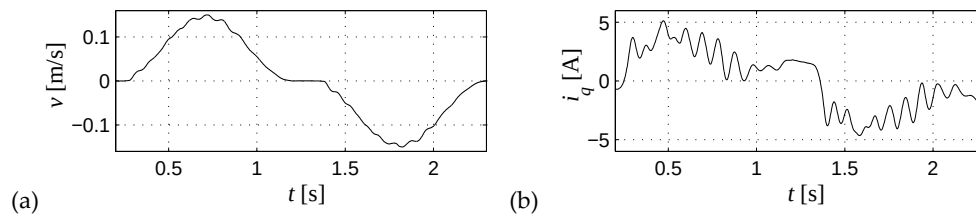


Figure 24. Kinematic control of linear SRM: calculated speed $v = dx/dt$ (a) and current i_q (b) considering $N=24$ higher order harmonics in thrust calculation (33).

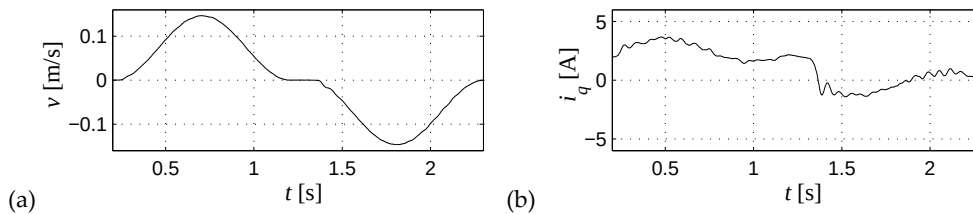


Figure 25. Kinematic control of linear SRM: calculated speed $v = dx/dt$ (a) and current i_q (b) considering only DC component in thrust calculation (33).

5. Conclusion

The main contribution of this paper is a proposed general method for deriving magnetically nonlinear two-axis dynamic models of synchronous machines, which is applied to derive the dynamic models of rotary and linear PMSMs and SRMs.

The proposed method consists of two steps. In the first step, only the structure of the magnetically nonlinear dynamic model is determined, while in the second step, the current- and position-dependent characteristics of flux linkages are determined experimentally by measurements on the machine's terminals. With these characteristics, the effects of slotting, interactions between the slots and permanent magnets, saturation, cross-saturation as well as the linear machine specific end effects are considered; and this without any prior knowledge of the machine properties required in FEA-based methods. Some of the experimental methods suitable for determining the aforementioned characteristics of flux linkages are presented in the paper. The flux linkages, electromagnetic torque and thrust are described in the form of Fourier series, where the relations among their Fourier coefficients are given in the form of equations.

The proposed approach to the modelling of synchronous machines, together with the experimental methods applied for determining magnetically nonlinear characteristic of flux linkages, is confirmed

through the comparison of measured and calculated results, given for the low speed kinematic control of a linear SRM. The results presented clearly show the impact of considered order of higher harmonics in the thrust calculation on current and speed trajectories.

Author Contributions: G.Š. performed derivations, build up experimental system for linear machines, performed experiments and contributed in paper preparation. B.Š. designed rotary machines, provided prototypes of rotary machines and results of FEA. T.M. performed experimental work on rotary machines and contributed in the analysis of obtained results and in paper preparation.

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