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# Geometric Aspects of Assessing the Amount of Material Consumption in the Construction of a Designed Single-Family House

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Received: 31 August 2020; Accepted: 9 October 2020; Published: 15 October 2020



**Abstract:** In this paper, we present a new approach for the analysis of the dependence of construction costs on the geometric shape of a building. Instead of difficult or even impossible-to-establish uniform prices and costs, we propose a cost analysis concerning the amount of materials needed for construction. We show that the basic parameters are the base area of the building (plan), assumed in the study as the building area, and the area of the external walls of the building. The amount of consumption of most materials is proportional to the base area and the area of the external walls. The materials required for construction consume large amounts of energy during their manufacture. Therefore, shape optimization is not only economically significant for the investor but is also important in terms of the energy consumption, i.e., embodied energy. We propose a set of indicators to help a designer optimize the shape of the building at the initial design stage.

**Keywords:** cost of building; embodied energy; building shape; correlation; design of a building; geometric efficiency of a building; compactness indicator; cuboid; relative defect of perimeter; relative defect of area

## 1. Introduction

Not so long ago, the common belief was that the amount of embodied energy was small compared to the operational energy needed to operate a building throughout its lifetime. Hence, measures were mainly taken to reduce the operating energy by increasing the energy efficiency of the building envelope. This hypothesis has been challenged after extensive research.

For 1996–2019, Hu and Milner [1] found 320 publications in the Web of Science on embodied energy and its environmental impact in the building and construction industry. In particular, the analysis of these works showed that buildings consume huge amounts [2], around 40% [3], over 50% [4], over 33% [5], and almost 50% [6] of the annual global energy and, thus, increase the concentration of carbon dioxide in the atmosphere. The total energy consumption for the entire life cycle of a building consists of embodied and operational energy. Embodied energy comprises all the energy needed to extract raw materials; produce and transport building materials; and to construct, maintain, and demolish a building. Embodied energy can be the equivalent of many years of operational energy. Research by the Australian National Research Agency Commonwealth Scientific and Industrial Research Organization (CSIRO) showed that the average house contains about 1000 GJ of energy hidden in the materials used to build it. This corresponds to approximately 15 years of normal energy consumed during operation. In the case of a 100-year-old house, this is more than 10% of the energy

used during its life [7]. Cement production is estimated to account for 7% of the global greenhouse gas emissions and is expected to increase continuously [8].

Researchers in countries rich in energy resources are making efforts to offer their governments new solutions to reduce the energy demands in construction [9]. In other countries, researchers are constantly looking for building materials that require less energy to produce [10,11].

The building envelope, through which heat is lost to the outside, is important in reducing carbon dioxide emissions. This also requires thermal insulation cladding, the production of which consumes significant amounts of energy [12]. The relatively small amount of scientific research on the geometric form of a building does not reflect the importance of this problem [13].

The search for cheap buildings in construction and energy consumption led to the concept of geometric building efficiency [14] and then to the geometric compactness of the building structure—the basic parameter characterizing this efficiency. Many different indicators have been formulated in the literature regarding the geometric shape of a building, including compactness indicators, and many of them are used to describe and estimate construction costs and energy consumption [15–19]. Most often, the parameters (indicators) of the shape of the building in the analyzed works are a consequence of the relationship of the perimeter of a flat figure with its area or area side surface (envelope) with its volume (see Section 3.1). Banks defined a length/breadth index [15,16,19]. Aksoy and Inalli used a similar shape factor (SF), which is the ratio of the building's length to its width [20]. Cooke defined the wall/floor ratio [16,17] (cf. (2)). Markus and Morris defined the ratio of change (ROC). The ROC is calculated by comparing the building area to cubic volume ratio with a similar ratio for a cube of the same volume [21].

Mahdavi and Gurtekin used the relative compactness ( $RC$ ) for a sphere (cube), determined as the proportion  $(A/V)_{\text{sph}}$  and  $(A/V)_{\text{build}}$  ( $(A/V)_{\text{cube}}$  and  $(A/V)_{\text{build}}$ ). Using this indicator, objects of various shapes were studied [22–24]. Bostancioglu used the index  $EWA/FA$  (the ratio of the external walls area to the floor area). Ourghi et al. used the relative compactness ( $RC$ ) coefficient, which expresses the ratio between  $(A/V)_{\text{des}}$  (a designed building) and  $(A/V)_{\text{ref}}$  (a reference building) [25]. Parasonis et al. proposed several improvements to the last ratio [26]. Instead of the  $A/V$  ratio,  $A/S$  is assumed, where  $A$  is the external envelope area of a building and  $S$  is the heated area. In [14], a whole family of compactness indicators was introduced; some of these are described in Section 3.

The geometric efficiency of a building [14] that meets the assumed size parameters (cubic capacity, usable area) is a set of geometric features that makes the building functional, economical (with a low energy demand) in construction and maintenance, safe in use, and aesthetic. An important geometric feature of the building is its compactness. By building compactness, we mean the compactness of the solid, which is an isometric geometrical model of the building envelope or its part. The geometric compactness of a rigid solid is the relationship between the enclosing surface and volume. The classical measure of compactness is defined by the dimensionless ratio  $(\text{area})^3/(\text{volume})^2$  [27]. In this work, as a measure of compactness, compactness indicators, defined by us or quoted from the literature, will be used.

## 2. The Objective of the Work

The dependence of construction costs on the shape of a building is within our area of expertise and research [15,17,18,28,29]; textbooks have devoted considerable space to this field, enough to fill nine monograph editions [16,19]. The designer of a building should be aware from the beginning of the design process of the extent to which the adopted first compositions concerning the geometrical shape (plan) of the building will result in the structure, construction costs, operating costs, aesthetics, functionality, etc. The designer should not compromise any of the above-mentioned criteria according to a formally or intuitively made multicriteria analysis. The literature [2–11] advises that an important criterion should be the limitation of the consumption of building materials, resulting, in particular, in the reduction of the embodied energy, regardless of the need to constantly search for low-energy technologies for the production of building materials [30–33].

In our opinion, the best solution is to provide the designer with as simple as possible mechanisms to check the dependence of construction costs on the shape of the building after adopting the projection of the designed house. Previous studies known from the cited literature concerned the analyses of several formulated theoretical models of buildings. In this study, we examined 30 ready-made designs of single-family houses and formulated a short set of indicators, helpful for the designer, which are easy to determine at the beginning of the design process based only on the base (floor plan) area, perimeter, and height of the story.

### 3. Compactness Indicators Used in the Analysis of Material Consumption in the Designed Single-Family Building

#### 3.1. Relationships between the Building Compactness Indicators Known in the Literature

For  $A$ —the area of the plan (building projection) of the building (floor area),  $P$ —the perimeter of the plan of the building ( $P_s$ —the perimeter of a square of the same area  $A$ ), and  $h$ —the height of the external walls, we can define the indicators:

$$EWA/FA = \frac{Ph}{A}, \quad (1)$$

which is the ratio of external wall area to floor area [17],

$$JC(= W/F) = \frac{P - P_s}{P_s}, \quad (2)$$

which is the Cooke wall/floor ratio index [17,19],

$$RC_{sq} = \frac{P}{4\sqrt{A}}, \quad (3)$$

which is the relative compactness indicator with respect to a square [14],

$$RC_{cd} = \frac{2A + Ph}{2A + 4\sqrt{Ah}}, \quad (4)$$

which is the relative compactness indicator with respect to the cuboid [14], and

$$LBI = \frac{P + \sqrt{P^2 - 16A}}{P - \sqrt{P^2 - 16A}}, \quad (5)$$

which is the Banks length/breadth index [17,19].

The  $LBI$  index is defined as the quotient of the elements of the pair ( $a$  and  $b$ ) of the solution of the system of equations:  $a \cdot b = A$ ,  $2a + 2b = P$ . This is the ratio of the sides of the rectangle with dimensions  $a$  and  $b$ .

Note that:

$$JC(= W/F) = \frac{P}{4\sqrt{A}} - 1. \quad (6)$$

Thus,

$$JC(= W/F) = RC_{sq} - 1. \quad (7)$$

Furthermore,

$$\lim_{h \rightarrow \infty} \frac{\frac{2A}{h} + P}{\frac{2A}{h} + 4\sqrt{A}} = \frac{P}{4\sqrt{A}}, \quad (8)$$

i.e.,

$$\lim_{h \rightarrow \infty} RC_{cd} = RC_{sq}. \quad (9)$$

The indicators (1), (2), (3), and (4) are linearly dependent. However, there is no simple relationship between  $EWA/FA$  and the other indicators. Namely,

$$RC_{sq} = \frac{P}{4\sqrt{A}} = \frac{\frac{Ph}{A}}{\frac{4h}{\sqrt{A}}} = \frac{EWA/FA}{\frac{4h}{\sqrt{A}}}, \quad (10)$$

$$RC_{cd} = \frac{2A + Ph}{2A + 4\sqrt{A}h} = \frac{2 + EWA/FA}{2 + \frac{4h}{\sqrt{A}}}. \quad (11)$$

However, if, in the design process, we assume that area  $A$  is fixed (building size) and the height  $h$  is determined by nature, then the dependencies ( $RC_{sq}$  and  $EWA/FA$ ) and ( $RC_{cd}$  and  $EWA/FA$ ) are linear, because then  $\frac{4h}{\sqrt{A}}$  is a constant value. Thus, the  $RC_{sq}$  and  $RC_{cd}$  ratios can serve as a measure of the  $EWA/FA$  ratio. These indicators express a clear measure: values of the indicator greater than 1 (the building plan is a rectangular polygon, other than a square) and equal to 1 are ideal (the building plan is square), and multiplied by 100% indicates the percentage deviation from the ideal dimensions in terms of the measure of the compactness of the object. The relative defects of an area ( $RDA$ ) ( $RDA'$ ) and  $RDP$  indicators show the percent loss of deviation from the rectangle.

### 3.2. Indicators Characterizing Building Blocks on a Rectangular Polygon Plan

Most buildings, especially single-family houses, are built on a polygonal plan with right convex or concave angles. A polygon with this property is called a rectangular polygon; it has an even number of sides, and the difference between the number of convex and concave angles is 4 (see Figures 1 and 2).  $A$ —the area of the plan of a building,  $P$ —the perimeter of the plan of building,  $A_R$ —the area of the rectangle circumscribed on the plan of a building, and  $P_R$ —the perimeter of the rectangle circumscribed on the plan of a building can be defined by the indicators:

$$RDA = \frac{A_R - A}{A_R}, \quad (12)$$

$$RDA' = \frac{A_R - A}{A}, \quad (13)$$

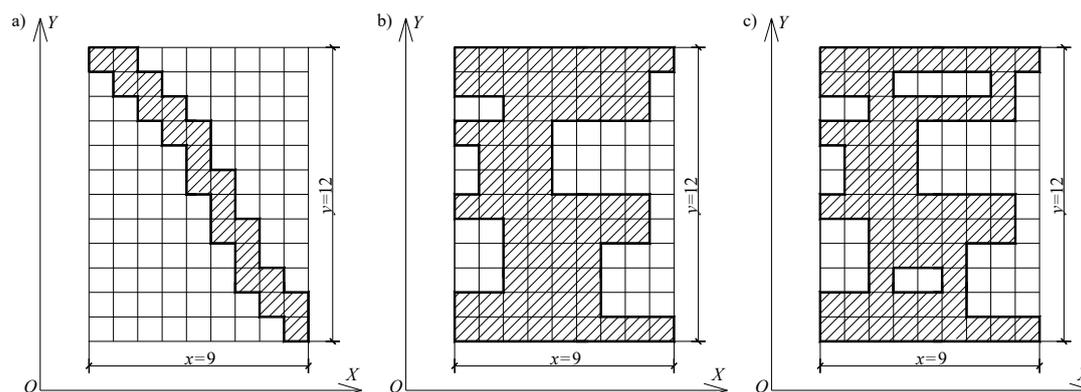
which are the relative defects of an area (Figures 1 and 2), and

$$RDP = \frac{P - P_R}{P_R}, \quad (14)$$

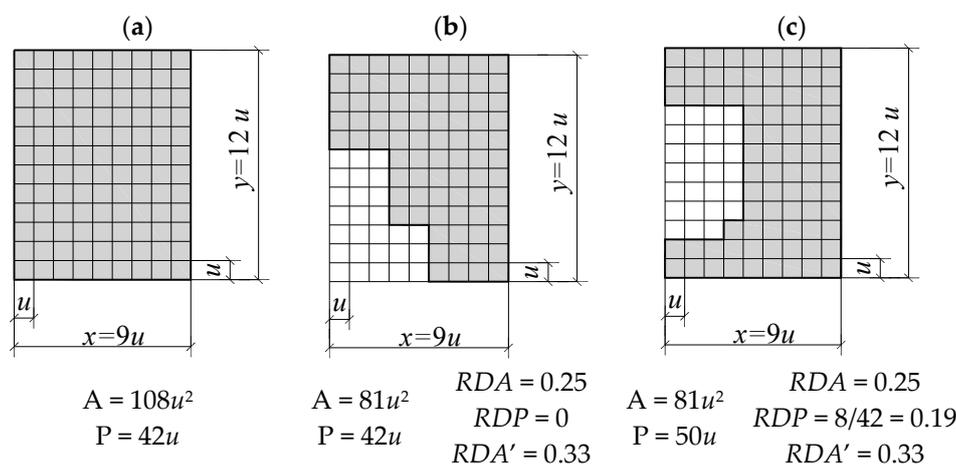
which is the relative defect of the perimeter (Figures 1 and 2).

It is worth paying attention to the values of the  $RC_{sq}$  and  $LBI$  indicators in the examples in Figure 1: (a)  $\rightarrow RC_{sq} = 2.35$ ;  $LBI = 20$ , (b)  $\rightarrow RC_{sq} = 1.97$ ;  $LBI = 13.48$ , and (c)  $\rightarrow RC_{sq} = 2.56$ ;  $LBI = 24.22$ . The dimensions of the equivalent rectangles (in units of  $u$ ) are respectively equal to: (a)  $20 \times 1$ , (b)  $30.72 \times 2.27$ , and (c)  $39.37 \times 1.63$ . Thus, the values of the  $RC_{sq}$  and  $LBI$  indicators are very large.

A rectangular polygon with more than four sides must have concave angles and  $RDA > 0$ .



**Figure 1.** Rectangular polygons ( $x \times y$ , where  $x = 9 u$  and  $y = 12 u$ ) and their associated rectangles: (a) with a large 81% area defect,  $A_R - A = 88 u^2$ ,  $RDA = 0.81$  and zero's perimeter defect,  $RDP = 0$ , (b) with a 57% perimeter defect,  $P - P_R = 24 u$ ,  $RDP = 0.57$  and, with a 35% area defect,  $A_R - A = 38 u^2$ ,  $RDA = 0.35$ , and (c) with a 95% perimeter defect,  $P - P_R = 40 u$ ,  $RDP = 0.95$  and, with a 41% area defect,  $A_R - A = 44 u^2$ ,  $RDA = 0.41$  (taken from [14] with permission from the authors).



**Figure 2.** The base rectangle and a rectangular polygon inscribed therein: (a) a reference rectangle, (b) a rectangular polygon with a perimeter defect equal to zero, and (c) a rectangular polygon with a perimeter defect greater than zero (Figure taken from [14] with approval from the authors).

#### 4. Analysis of Projects of 30 Single-Family Houses

##### 4.1. Selected Designs of Single-Family Houses—The General Characteristics

We researched 30 single-family house designs [34], which were subjected to a comparative analysis, according to the following criteria:

- usable area of 100–150 m<sup>2</sup> (data according to the Central Statistical Office—the most statistically popular range of selected houses in Poland and Europe);
- free-standing buildings with a rectangular plan;
- single-story (one-story) buildings with no usable attic, no basement, and no garage;
- basic building materials for the housing: H + H blocks, concrete, and steel;
- parameters of windows and external doors the same for all projects;
- and roof shape—gable or hipped with a wooden roof truss structure (allowing for possible installation of a photovoltaic installation—roof slope angle 30–45°).

The above-mentioned projects, H1–H30, are listed in Appendix A (Tables A1–A3 and Figures A1 and A2), where they are arranged in ascending order to the built-up area. The building

plans are listed in Figures A1 and A2. Each building plan (in the form of a rectangular polygon) is accompanied by a drawing of a rectangle described on the polygon of the base of the building. After some adaptations, Tables A1–A3 contains data on the dimensions of the buildings, costs, quantities of selected materials, calculated values of parameters, and indicators discussed in Section 3.

#### 4.2. Analysis of Selected Designs of Single-Family Houses

These considerations do not include the area of the plot on which the project is implemented, the location (climatic zones), the location in relation to the directions of the world (insolation), the annual distribution of external temperatures, etc. The cost was calculated based on the available data ([www.archon.pl](http://www.archon.pl)) and the minimum prices for the second quarter of 2020 [35,36]. However, due to major changes in the Polish, European, and world economy, the presented calculations should be adopted as estimates.

Before further analyses, it is worth paying attention to how the indicators work together on the data from the H1–H30 projects. Here are their mutual correlation coefficients:

$$\begin{aligned} \text{cor}(RC_{sq}, JC) &= 1.0000 \text{ (obvious, full correlation between } RC_{sq}, JC \text{ (see (6))}; \\ \text{cor}(RC_{cd}, RC_{sq}) &= 0.9964; \text{cor}(RC_{cd}, LBI) = 0.9958; \text{cor}(RDA, LBI) = 0.9993; \text{ and} \\ \text{cor}(RDA, RDP) &= -0.0848. \end{aligned}$$

The last dependence does not have to hold. The *RDP* index, where  $RDP > 0$ , shows the existence of a “niche” in the plan of the building. In the H1–H30 project set, only nine buildings (H1, H2, H6, H12, H16, H23, H27, H28, and H29) have a small niche in their plan. For others,  $RDP = 0$ .

Comparing the shell construction costs of many buildings in terms of the shape of the body makes sense with many limitations: the same building area; the same materials and technologies; the same percentage of individual materials (e.g., reinforced steel and concrete in the same ratio); and the same prices of materials, equipment, and labor. However, considering prices means that the analysis of the shape of the building is no longer universal. The prices of materials, equipment, and labor depend on so many factors that this type of comparative analysis no longer makes sense.

As can be seen from the preliminary analysis of the indicators in Tables A1 and A3, there is no relationship between the raw state costs per square meter of surface area (the total minimum cost per  $1 \text{ m}^2$  (TMCpm<sup>2</sup>), Table A1, row 9) and the analyzed indicators. This is evidenced by low values of the correlation coefficient between costs and the values of indicators  $RC_{cd}$ ,  $RC_{sq}(JC)$ ,  $RDA$ ,  $RDA'$ ,  $LBI$ :  $\text{cor}(RC_{cd}, \text{TMCpm}^2) = -0.1688$ ,  $\text{cor}(RC_{sq}, \text{TMCpm}^2) = -0.1444$ ,  $\text{cor}(RDA, \text{TMCpm}^2) = -0.0444$ ,  $\text{cor}(RDA', \text{TMCpm}^2) = -0.0352$ , and  $\text{cor}(LBI, \text{TMCpm}^2) = -0.1441$ , which indicate the lack of correlation.

These results confirm that including costs in the considerations distorts any dependence of these costs on shape indicators (see Figure 3).

For similar reasons as the prices and costs, we basically omitted the problem of the dependence of the amount of labor in the considerations. This is because it is quite well-characterized by the *A* parameter and the shape of the base polygon (plan) of the building (including the *RDA* and *RDP* parameters and the number of vertices of the base polygon). Due to the inability to separate the steel and concrete used in the construction of horizontal and vertical elements, these materials were also omitted from the considerations. With these data, such an analysis is possible, and similar results should be expected.

Therefore, it remained to analyze the consumption of individual materials depending on the shape of the building, which was performed separately for each type of material. It is a completely sufficient analysis to show the close relationship between the consumption of materials and the values of indicators discussed in Section 3. The consumption of many materials essential in the adopted technology depends on the area of the building. These include building blocks, ready-mixed concrete, reinforcing steel, insulation materials, etc., and, therefore, all the most important building

materials. Some materials (concrete, reinforcing steel, insulating foils, etc.) concern horizontal elements, such as the floor and ceiling; others (e.g., masonry blocks) concern external walls; others (concrete and reinforced steel) concern vertical elements (columns, lintels, and window sills).

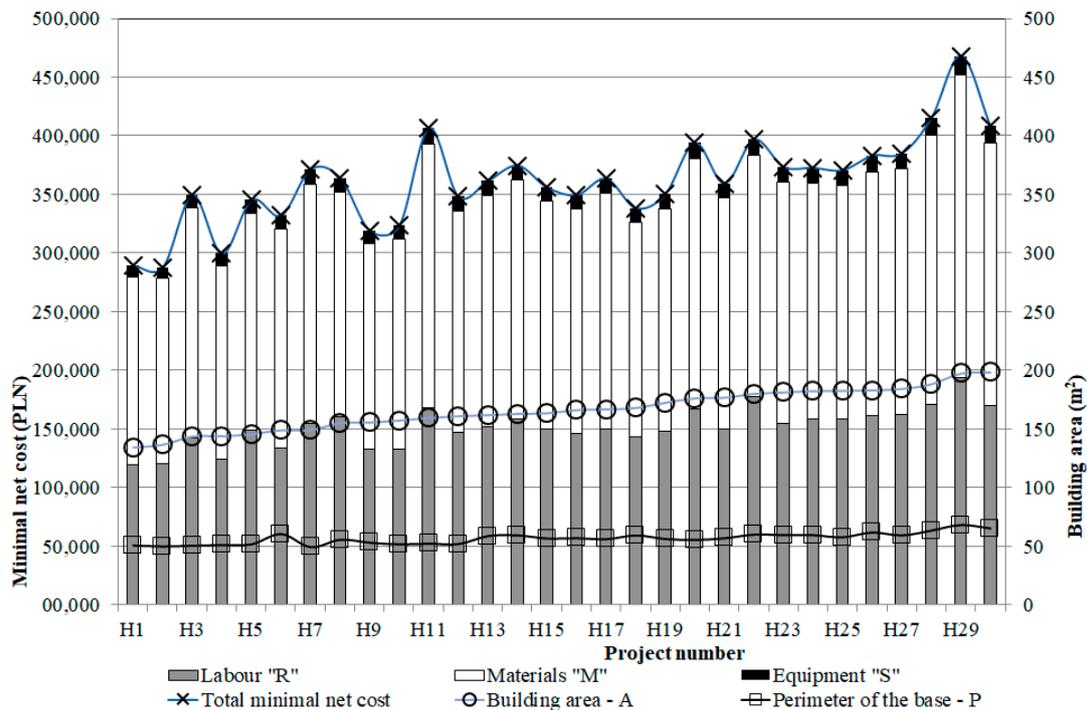


Figure 3. Summary of the share of selected materials and costs.

The analysis discussed above is also possible for various building areas, but then, the number of materials used per 1 m<sup>2</sup> of building area should be analyzed. This approach also shows the possibility of calculating the number of materials per 1 m<sup>2</sup> of the flat area and, thus, for the collective estimation of the cost of 1 m<sup>2</sup> of the flat area. In the presented set of buildings H1–H30, we had the opportunity to analyze the size of, among others, the consumption of concrete blocks for the construction of external walls of buildings. The analysis of the consumption of concrete or steel was not possible due to the lack of information: how much concrete (steel) was used for the construction of horizontal elements (broken down into floor and ceiling) and for the construction of vertical elements (columns and lintels).

Therefore, the number of materials required to construct individual elements of the building should be considered as broken down into horizontal elements, depending on the size of the building area (floor, ceiling, and roof) and vertical elements (building walls), depending on the ratio of the wall area to the building area (plan). The built-up area A (Section 3) adopted to describe the shape of the building allows for the formulation of a direct correlation with the number of materials needed to make horizontal elements, such as the floor and ceiling, and considers the slope angle and the roof.

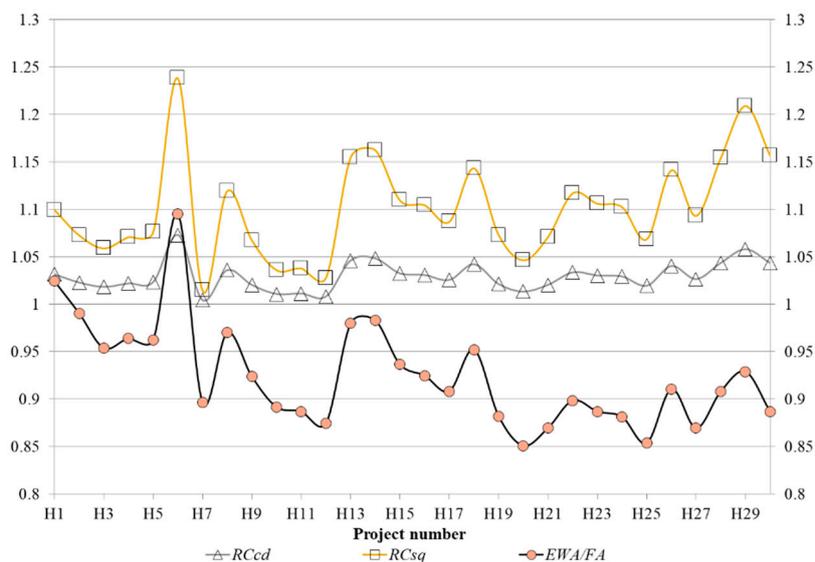
Derived dependencies (10) and (11), assuming  $h = \text{const}$  and  $A = \text{const}$ , define the linear relationship between the  $EWA/FA$  index and the  $RC_{cd}$ ,  $RC_{sq}$ , and  $JC$  indices, respectively. In the range of H1–H30, the area varies from 134.41 to 198.51 m<sup>2</sup>; thus, a high correlation coefficient cannot be expected (Table 1, Figure 4); in the H1–H15 range, the area varies from 134.41 to 163.73 m<sup>2</sup>, and the correlation coefficient is much higher (Figure 5); after selecting the range H10–H17, the area changes from 159.74 to 167.03 m<sup>2</sup>, and then, the relationship is almost linear (correlation coefficient close to 1.0000). If  $A = \text{const}$ , the dependence of  $RC_{cd}$ ,  $RC_{sq}$ , and  $JC$  on  $EWA/FA$  is exactly linear (see (10), (11), and Figure 6). The  $RC_{cd}$  index is most closely correlated with the  $EWA/FA$  parameter (and with the “H + H blocks per 1 m<sup>2</sup>” parameter), due to its three-dimensional nature. Thus, it can serve as the best measure of the  $EWA/FA$  parameter with a fixed (or slightly changing) A. The smaller (closer to

one) the  $RC_{cd}$  value, the closer the building shape to the ideal shape of a square cuboid with the greatest compactness.

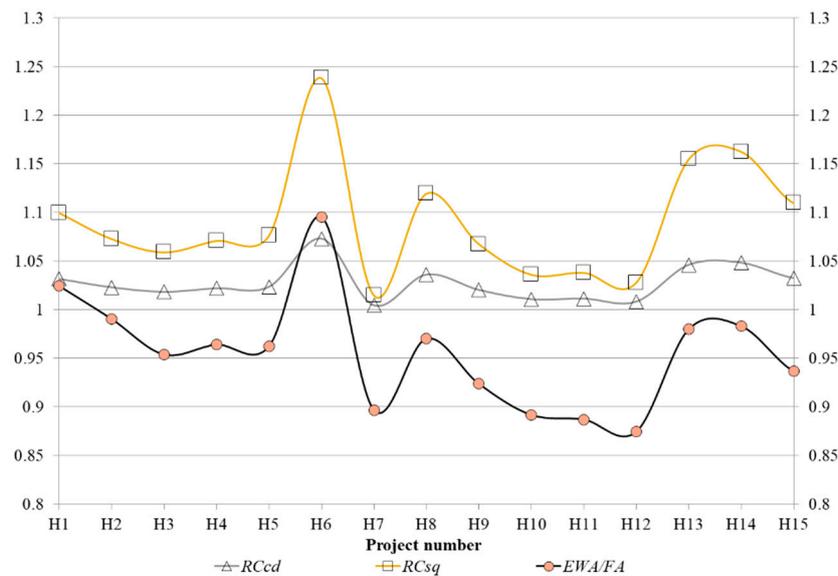
Considering the sensitivity of the  $RC_{cd}$  and  $EWA/FA$  parameters in relation to the shape of the building (building plan), the sensitivity of the  $EWA/FA$  and  $RC_{cd}$  parameters to the change in the shape of the building base is visible in Figures 5 and 6 in comparison with the shapes of buildings H6, H8, H13, and H14 (Tables A1 and A3 and Figure A1). The complexity of the shape (discrepancy with the square shape) translates into the values of the parameters in Figures 5 and 6. The  $EWA/FA$  parameter indicates the material consumption per  $m^2$ , and  $RC_{cd}$  shows the measure of this material consumption as a deviation from the 1 (reference) value.

**Table 1.** The dependence of the correlation coefficient of the relative compactness indicator with respect to the cuboid ( $RC_{cd}$ ), the relative compactness indicator with respect to a square ( $RC_{sq}$ ), Banks length/breadth index ( $LBI$ ), and relative defects of an area ( $RDA$ ) indices on the range of changes in the built-up area of buildings belonging to H1–H30.  $EWA/FA$ : the ratio of the external walls area to the floor area.

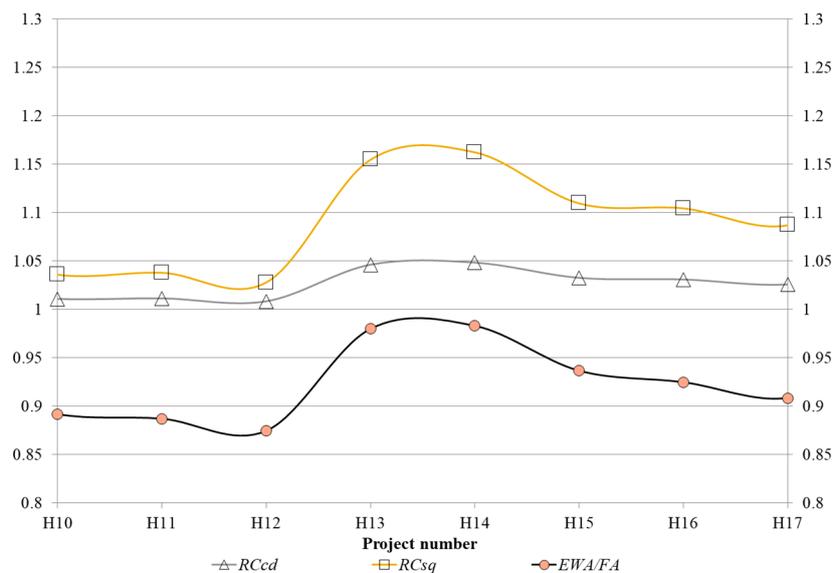
No.	Correlation Coefficients	Ranges		
		H1–H30	H1–H15	H10–H17
1	$cor(RC_{cd}, H+H \text{ blocks per } 1 \text{ m}^2)$	0.5897	0.8424	0.9690
2	$cor(RC_{cd}, EWA/FA)$	0.5870	0.8665	0.9832
3	$cor(RC_{sq}, H+H \text{ blocks per } 1 \text{ m}^2)$	0.5262	0.8236	0.9671
4	$cor(RC_{sq}, EWA/FA)$	0.5222	0.8488	0.9816
5	$cor(LBI, H+H \text{ blocks per } 1 \text{ m}^2)$	0.5258	0.8307	0.9621
6	$cor(LBI, EWA/FA)$	0.5202	0.8536	0.9780
7	$cor(RDA, H+H \text{ blocks per } 1 \text{ m}^2)$	0.2802	0.4372	0.9753
8	$cor(RDA, EWA/FA)$	0.2541	0.4496	0.9855



**Figure 4.** Visible correlation between the relative compactness indicator with respect to the cuboid ( $RC_{cd}$ ) and ratio of the external walls area to the floor area ( $EWA/FA$ ) indicators at the level  $cor(RC_{cd}, EWA/FA) = 0.5870$  in the full range of H1–H30 area variation (Table 1).  $RC_{sq}$ : relative compactness indicator with respect to a square.



**Figure 5.** The clear correlation between the  $RC_{cd}$  and  $EWA/FA$  indicators at the level correlation coefficient  $\text{cor}(RC_{cd}, EWA/FA) = 0.8665$  in the limited range of the H1–H15 area variation (Table 1).



**Figure 6.** An almost perfect correlation between the  $RC_{cd}$  and  $EWA/FA$  indicators at the level correlation coefficient  $\text{cor}(RC_{cd}, EWA/FA) = 0.9832$  in the limited range of the H10–H17 area variation (Table 1).

#### 4.3. Indicators Helpful in the Initial Geometric Analysis of the Project

The results of the research demonstrated that the parameter that best characterizes the amount of material consumed in the shell home is the area of the external envelope of the building, i.e., the sum of the areas of the horizontal elements (floor and ceiling) and vertical elements (external walls of the building) divided into horizontal and vertical elements. In accordance with the adopted notations, these are, therefore, the area  $A$  and the  $EWA/FA$  ratio. We know from the considerations in Section 3.2, the value of  $EWA/FA$  parameter is best characterized by the  $RC_{cd}$  index (Table 1 and Figures 4 and 5 and formula (11)). According to the definition of the  $RC_{cd}$  index, in terms of the building compactness, the area of the entire building envelope is best characterized by the  $RC_{cd}$  index [14].

The  $RC_{cd}$  indicator has clear scaling. The reference value is equal to 1, and the product  $(1 - RC_{cd}) \times 100\%$  shows the percentage deviation of the shape from the reference value. Due to the

relationship between the  $EWA/FA$  and  $RC_{cd}$  parameters, the  $RC_{cd}$  indicator describes the amount of material consumption and, thus, the expected construction cost. The size of the  $RC_{cd}$  indicator allows the designer to optimize the geometric efficiency of the building. In addition, a helpful indicator is the  $RDA$  indicator, showing the loss on the surface of the house with the given materials used for the execution of external walls. The  $LBI$  indicator informs the shape of a rectangle with the area and perimeter equal to the area and perimeter of the planned building plan.

In summary, the  $RC_{cd}$ ,  $LBI$ , and  $RDA$  indicators easily obtained based on  $A$ ,  $P$ , and  $h$  data will be helpful to the designer in the first stage the building geometry. Additionally, the  $RDP$  indicator can be helpful, especially when  $RDP > 0$ . Determining the  $RDA$  and  $RDP$  indices requires simple calculations of the  $A_R$  area value and the  $P_R$  perimeter of the rectangle described on the polygon of the building plan (Section 3). The number of sides (vertices) of the building base polygon is important. The larger the number of vertices, the more complicated the shape generated, among others, increases the labor.

Finally, many new results were obtained in terms of the influence of the shape of the building on the consumption of materials and, thus, on the energy consumption and the size of costs. Namely, the previously known  $EWA/FA$  index and the  $RC_{cd}$  geometric compactness indicator introduced in [14] were used to analyze real designs, not model diagrams as before. Thus, a reliable method of assessing material consumption in the initial phase of the building design was constructed. A linear relationship between the  $EWA/FA$  and  $RC_{cd}$  parameters was demonstrated. The  $RC_{cd}$  index has a readable and natural scale (value 1 for a building with a square base and natural height) and, thus, measures the  $EWA/FA$  index values well (material consumption per 1 m<sup>2</sup> of building area). Thus, both indicators become more useful for designing buildings than those proposed so far. Research showed that the cost can be objectively analyzed only after characterizing the relationship between the amount of material consumption and the value of the compactness index. This ensures the independence of the analysis from material prices, as well as the direct measurement of embodied energy after using the appropriate tables of embodied energy materials.

The extensive literature on the subject so far lacked simple procedures for the preliminary analysis of the influence of the shape of the building on the consumption of materials and energy in the first phase of building design. The considerations on this subject did not provide unambiguous constructive and simple conclusions for the designer. We hope that this work will fill this gap and that the results of further research on the effect of building shape on operational energy demand will provide a complete solution to the problem.

## 5. Conclusions

Research on the relationship between the shape of a building and the costs of its erection can be reduced to the analysis of the consumption of materials. Then, the amount of material consumption is independent of the prices, costs, and, thus, of the country and its currency.

The geometric shape of the building, which can be determined by the system of building compactness indicators proposed in the article, has a significant impact on the amount of material consumption when erecting a building. The amount of building materials directly translates to the amount of embodied energy.

The proposed system of indicators ( $RC_{cd}$ ,  $LBI$ ,  $RDA$ , and  $RDP$ ) may be helpful for a designer in terms of the shape optimization, in particular, at the initial stage of building design. This should be understood so that the architect chooses the option with the lowest  $RC_{cd}$  ( $LBI$ ,  $RDA$ , and  $RDP$ ) value from among the acceptable shapes of buildings in terms of the functionality, aesthetics, maintenance, etc.

Reducing the consumption of materials results in a reduction in the embodied energy consumption.

**Author Contributions:** E.K., first conceptualization and methodology and prepared and wrote Section 3; K.B., prepared the data and calculations; formulated Tables 1, A1 and A2; and prepared Figures 3–6; E.K. and K.B. together discussed and perfected the methodology, conducted the analyses, and wrote the manuscript. All authors have read and agreed to the published version of the manuscript.

**Funding:** This work was supported by the Bialystok University of Technology grant WZ/WBiS/6/2019, financed from the funds for the science of MNiSW, and an agreement on the implementation of the scientific grant in 2020 for the employees of the Warsaw University of Technology supporting scientific activities in the discipline of Civil Engineering and Transport—504/04505/1110/43.070004.

**Acknowledgments:** The authors would like to thank the anonymous reviewers for their remarks, suggestions, and comments that have improved the manuscript.

**Conflicts of Interest:** The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

## Appendix A

**Table A1.** Selected parameters of the buildings of the H1–H30 building set (Part I): (1) Unfinished state closed (minimal cost) (PLN); (2) Finishing works in building construction (minimal cost) (PLN); (3) Total minimal cost of construction works; (PLN); (4) Total minimal cost of construction works per 1 m<sup>2</sup> (PLN); (5) Breakdown of the costs of works (national minimum rates) (PLN) Labor: working hour “R”, (6) Materials “M”, (7) Equipment “S”; (8) Total minimal cost of construction works (economic system) (PLN); (9) Total minimal cost per 1 m<sup>2</sup> (economic system) (PLN); (10) Estimated installation cost (14% of construction costs) (PLN).

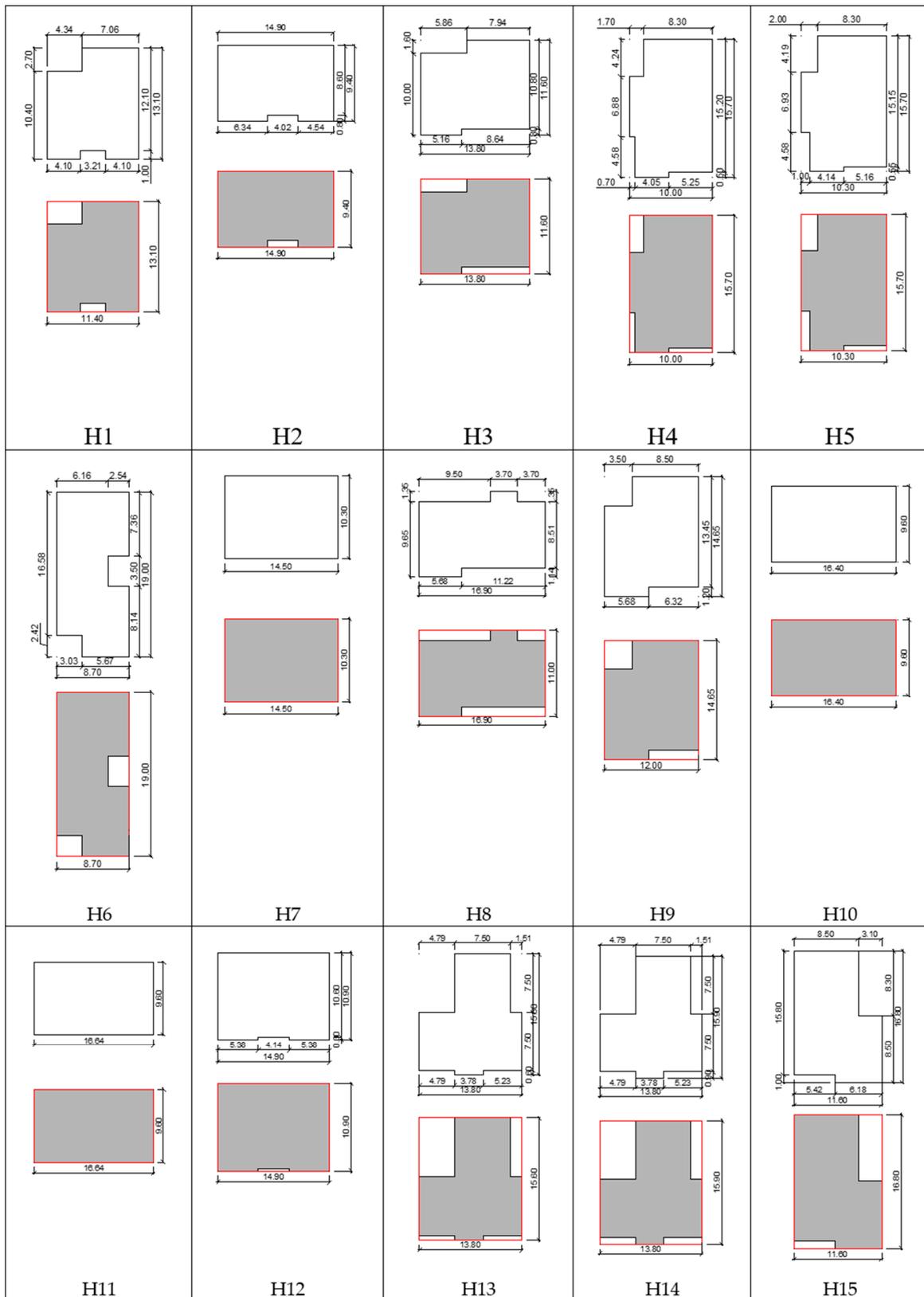
	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13	H14	H15
1	186,700	173,300	215,600	172,200	196,100	215,400	215,900	238,400	211,000	200,200	255,800	225,400	224,200	228,700	229,900
2	102,700	114,700	133,800	127,500	149,200	116,600	155,500	125,700	108,200	123,100	150,600	122,800	137,300	145,900	126,100
3	289,400	288,000	349,400	299,700	345,300	332,000	371,400	364,100	319,200	323,300	406,400	348,200	361,500	374,600	356,000
4	2,153.11	2,104.65	2,429.93	2,081.83	2,366.53	2,226.99	2,486.78	2,344.65	2,048.91	2,053.48	2,544.13	2,160.45	2,231.21	2,296.05	2,174.31
5	119,600	120,700	142,100	124,000	148,500	133,800	154,900	160,300	132,500	132,400	168,100	147,200	151,900	158,400	150,000
6	160,100	157,600	196,200	165,000	185,500	187,100	204,000	191,300	175,500	179,800	224,700	188,900	197,500	203,700	194,400
7	9,700	9,700	11,100	10,700	11,300	11,100	12,500	12,500	11,200	11,100	13,600	12,100	12,100	12,500	11,600
8	193,720	191,440	235,720	200,400	226,500	224,960	247,380	235,760	213,200	217,380	271,920	230,340	239,980	247,780	236,000
9	1441.26	1399.01	1639.34	1392.05	1552.33	1508.99	1656.38	1518.19	1368.51	1380.72	1702.27	1429.17	1481.18	1518.73	1441.40
10	40,500	40,300	48,900	41,900	48,300	46,500	52,000	51,000	44,700	45,300	56,900	48,700	50,600	52,400	49,800
	H16	H17	H18	H19	H20	H21	H22	H23	H24	H25	H26	H27	H28	H29	H30
1	223,100	235,300	219,200	227,400	250,400	219,700	261,200	237,100	234,300	236,200	246,100	250,500	264,200	281,500	249,400
2	126,500	128,200	119,300	122,900	143,500	139,600	135,500	136,400	138,400	134,500	136,900	134,400	150,800	186,500	158,800
3	349,600	363,500	338,500	350,300	393,900	359,300	396,700	373,500	372,700	370,700	383,000	384,900	415,000	468,000	408,200
4	2,601.00	2,656.39	2,354.13	2,433.31	2,699.61	2,410.12	2,656.18	2,405.18	2,392.32	2,354.55	2,397.65	2,388.16	2,561.41	2,868.53	2,493.13
5	145,900	149,800	142,800	148,300	166,600	150,300	177,200	154,900	158,300	158,000	161,600	161,900	170,900	193,700	169,600
6	191,500	201,200	183,800	189,700	213,700	196,600	205,900	206,000	201,200	199,800	207,700	209,800	229,900	258,400	224,500
7	12,200	12,500	11,900	12,300	13,600	12,400	13,600	12,700	13,200	12,900	13,700	13,200	14,200	15,900	14,100
8	232,780	243,660	224,260	231,560	260,620	238,960	254,940	249,680	246,060	244,200	253,720	255,380	278,180	313,040	272,520
9	1398.75	1458.78	1333.37	1341.13	1477.44	1350.36	1414.21	1376.25	1347.61	1336.03	1384.78	1384.55	1475.91	1584.05	1372.83
10	48,900	50,900	47,400	49,000	55,100	50,300	55,500	52,300	52,200	51,900	53,600	53,900	58,100	65,500	57,100





**Table A3.** The calculated values of the indicators of the H1–H30 building set: (30)  $RC_{cd}$ ; (31)  $RC_{sq}$ ; (32)  $RDP$ ; (33)  $RDA$ ; (34)  $RDA'$ ; (35)  $WWR$ ; (36)  $A/V$ ; (37)  $JC(=W/F)$ ; (38)  $JC(=W/F)*100\%$ ; (39)  $EWA/FA$ ; (40)  $LBI$ .

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13	H14	H15
30	1.0317	1.0230	1.0184	1.0220	1.0235	1.0731	1.0045	1.0361	1.0204	1.0109	1.0114	1.0083	1.0461	1.0483	1.0326
31	1.0998	1.0728	1.0591	1.0710	1.0762	1.2383	1.0147	1.1194	1.0676	1.0361	1.0381	1.0279	1.1549	1.1626	1.1097
32	0.0408	0.0329	0.0000	0.0000	0.0000	0.0917	0.0000	0.0000	0.0000	0.0000	0.0000	0.0116	0.0000	0.0000	0.0000
33	0.1000	0.0230	0.1018	0.0831	0.0977	0.0981	0.0000	0.1647	0.1138	0.0000	0.0000	0.0076	0.2474	0.2564	0.1598
34	0.1111	0.0235	0.1133	0.0906	0.1083	0.1088	0.0000	0.1971	0.1284	0.0000	0.0000	0.0077	0.3287	0.3449	0.1903
35	0.24	0.23	0.27	0.18	0.18	0.27	0.33	0.27	0.28	0.25	0.31	0.26	0.26	0.28	0.29
36	1.1202	1.1076	1.0940	1.0978	1.0971	1.1464	1.0728	1.1001	1.0829	1.0710	1.0693	1.0646	1.1037	1.1048	1.0877
37	0.0998	0.0728	0.0591	0.0710	0.0762	0.2383	0.0147	0.1194	0.0676	0.0361	0.0381	0.0279	0.1549	0.1626	0.1097
38	9.98%	7.28%	5.91%	7.10%	7.62%	23.83%	1.47%	11.94%	6.76%	3.61%	3.81%	2.79%	15.49%	16.26%	10.97%
39	1.02	0.99	0.95	0.96	0.96	1.10	0.90	0.97	0.92	0.89	0.89	0.87	0.98	0.98	0.94
40	2.43	2.14	1.98	2.12	2.17	3.88	1.41	2.63	2.08	1.71	1.73	1.60	3.00	3.08	2.53
	H16	H17	H18	H19	H20	H21	H22	H23	H24	H25	H26	H27	H28	H29	H30
30	1.0309	1.0257	1.0422	1.0213	1.0135	1.0206	1.0336	1.0304	1.0293	1.0196	1.0403	1.0266	1.0436	1.0581	1.0435
31	1.1046	1.0871	1.1435	1.0731	1.0466	1.0712	1.1172	1.1062	1.1027	1.0688	1.1414	1.0934	1.1545	1.2093	1.1569
32	0.0440	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0420	0.0000	0.0000	0.0000	0.0241	0.0326	0.0180	0.0000
33	0.0936	0.0772	0.2185	0.1118	0.0000	0.0478	0.1375	0.0893	0.1171	0.1144	0.2318	0.0580	0.1976	0.2908	0.2515
34	0.1032	0.0836	0.2796	0.1259	0.0000	0.0502	0.1594	0.0980	0.1326	0.1292	0.3017	0.0616	0.2463	0.4100	0.3360
35	0.25	0.34	0.16	0.21	0.36	0.31	0.25	0.24	0.22	0.24	0.24	0.24	0.30	0.28	0.20
36	1.0832	1.0772	1.0934	1.0674	1.0559	1.0628	1.0736	1.0693	1.0672	1.0570	1.0780	1.0628	1.0771	1.0848	1.0692
37	0.1046	0.0871	0.1435	0.0731	0.0466	0.0712	0.1172	0.1062	0.1027	0.0688	0.1414	0.0934	0.1545	0.2093	0.1569
38	10.46%	8.71%	14.35%	7.31%	4.66%	7.12%	11.72%	10.62%	10.27%	6.88%	14.14%	9.34%	15.45%	20.93%	15.69%
39	0.92	0.91	0.95	0.88	0.85	0.87	0.90	0.89	0.88	0.85	0.91	0.87	0.91	0.93	0.89
40	2.48	2.29	2.88	2.14	1.84	2.12	2.61	2.49	2.46	2.09	2.86	2.36	3.00	3.57	3.02



**Figure A1.** Building plans of the H1–H15 set and rectangles described on rectangular polygons of the plans of these buildings.



Figure A2. Building plans of the H16–H30 set and rectangles described on rectangular polygons of the plans of these buildings.

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