

Quasi-Zero Stiffness-Based Synchronous Vibration Isolation and Energy Harvesting: A Comprehensive Review

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Abstract: In recent years, the advantages of nonlinearity in vibration isolation and energy harvesting have become increasingly apparent. The quasi-zero stiffness (QZS) of the nonlinear term provided by the negative stiffness element can achieve vibration isolation under low-frequency environments while improving the efficiency of energy harvesting. The QZS provides a new research idea for simultaneous vibration isolation and energy harvesting. The main purpose of this paper is to review past research results, summarize possible problems, and discuss trends. After briefly analyzing the basic principle of QZS vibration isolation, the progress of QZS in vibration isolation and energy harvesting are also discussed. Finally, according to the existing QZS challenges, the future development trend of QZS is proposed. This paper would provide a quick guide for future newcomers to this field.

Keywords: nonlinearity; QZS; vibration isolation; energy harvesting; low-frequency vibrations



Citation: Chen, Z.; Chen, Z.; Wei, Y. Quasi-Zero Stiffness-Based Synchronous Vibration Isolation and Energy Harvesting: A Comprehensive Review. *Energies* **2022**, *15*, 7066. https://doi.org/10.3390/en15197066

Academic Editor: Paul Stewart

Received: 1 August 2022 Accepted: 21 September 2022 Published: 26 September 2022

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1. Introduction

As a common phenomenon in nature, vibration always exists in various forms, such as the swing of leaves, the surging of waves, the vibration of mechanical equipment, and so on [1,2]. Generally speaking, vibrations can have beneficial or destructive effects in practice [3–5]. On one hand, vibrations can help us to produce musical sounds, transmit signals and generate electricity [6–8]. However, on the other hand, once the vibration exceeds a certain threshold, it will bring adverse consequences [9–11]. For example, vibration may decrease the accuracy of mechanical equipment, cause mechanical damage, and shorten the useful life of mechanical equipment [12–16]. Therefore, accurate vibration isolation is very necessary for controlling such vibrations within a limited range on many occasions [17–19].

Vibration isolators are widely used to control the dynamic response of a structure, which are usually composed of dampers and elastic elements [20]. To date, different dampers have been rapidly developed, and the corresponding vibration isolation performance has also been improved greatly. Vibration isolation technologies can be divided into passive [21–24], semi-active [25–28], and active [29–31] methods according to the damping mechanism. Passive vibration isolators function without external power sources and they are driven by the vibration itself. In most cases, they have the simplest structure and require little or no maintenance. However, they still have some shortcomings in practical applications. The first one is that the stiffness and damping are not adjustable so that good vibration isolation performance cannot be achieved under various working conditions. The second is that the adaptability and stability cannot be balanced simultaneously. The third one is that they cannot work effectively under vibrations of high frequencies and low magnitudes. Active vibration isolators reduce vibrations by the additional actuator and are most effective under vibrations of low frequencies. However, they require power for operating the sensors, processing the sensors' signals, and driving the actuator. Meanwhile,

they have a higher cost and possibility of failure. Semi-active vibration isolators are intermediate between active and passive ones, which have variable stiffness and damping according to the characteristics of vibrations. However, they also require external power.

As stated above, semi-active and active vibration isolators are promising in vibration isolations. However, the disadvantage is that both of them need external electric energy to achieve the damping effect. At the same time, vibration energy in structures is dissipated into the air. Therefore, there is a strong conflict from the viewpoint of energy conservation. In order to overcome this issue, a valuable way is to harvest vibration energy and convert it into electrical energy. By this way, simultaneous vibration isolation and energy harvesting (SVIEH) can be carried out. Moreover, the harvested electrical energy can be used for semi-active or active dampers. In recent years, SVIEH has been widely studied by many researchers. Relevant professionals have done some work on the above research, which is also known as vibration isolation and energy harvesting, vibration suppression and energy harvesting. According to the above keywords, the relevant literature in the past 20 years was obtained from the Web of Science document retrieval system. As shown in Figure 1, it can be seen from the figure that both China and the United States have carried out a lot of research in the field of vibration isolation and energy harvesting. Adhikari et al. [32] first proposed the idea of energy harvesting using dynamic vibration absorbers, which was carried out by placing a piezoelectric stack between the primary and secondary vibration isolators. Brennan et al. [33] investigated SVIEH under both random and harmonic excitations. The results demonstrated that enough energy was harvested under random excitation and the maximum vibration isolation ability was achieved at the same time. While under harmonic excitations, the excitation frequency should be equal to the natural frequency of the system in order to achieve both optimal goals. Bardaweel et al. [34] developed a new type of low-frequency vibration isolation and energy harvesting device, which is composed of a negative stiffness honeycomb spring and a positive stiffness magnetic spring. Nitin et al. [35] proposed an energy-harvesting shock absorber for vehicle suspension, which consisted of a linear generator and a vibration amplification unit. Shen et al. [36] designed an electromagnetic damper for SVIEH in bridge diagonal cables, which could harvest vibration energy and then provide sufficient damping for the cable.



Figure 1. Search results by Web of Science. (a) Distribution by time; (b) Distribution by country.

The main influencing factor of the vibration isolation frequency band of the vibration isolation system is the system stiffness [37]. Reducing the system stiffness can reduce the natural frequency of the system. The vibration isolation frequency band of the system can be extended to low frequency and ultra-low frequency, so as to obtain a wider vibration isolation frequency band and isolate the harm of vibration [38]. In order to reduce the stiffness of the system, there are usually two technical means: (1) structural optimization design of vibration isolation elements to reduce its stiffness; (2) a negative stiffness element

is introduced to reduce the overall stiffness of the system through a quasi-zero stiffness (QZS) mechanism. Due to the limitation of physical size, it is very difficult to effectively reduce the stiffness of the system through structural optimization design [39]. At the same time, the bearing capacity and stability of the system will be limited under low stiffness. The dynamic stiffness of the QZS isolator at the static equilibrium position is zero, which can make the isolator have low stiffness and show excellent low-frequency vibration isolation performance on the premise of ensuring the bearing capacity [40]. Therefore, research on QZS has become a hot topic in the international academic community. The concept of QZS, which is a kind of nonlinear vibration isolator with positive and negative stiffness, was first proposed by Alabuzhev. By optimizing the geometry and stiffness parameters of the negative stiffness mechanism reasonably, the QZS can realize the large bearing capacity of the vibration isolation system and the superior low frequency vibration isolation performance [41,42]. Compared with the traditional passive vibration isolation system, the QZS has the following three advantages: (1) Because of the nonlinear stiffness term, the static bearing capacity is good, and the structural deformation is small. (2) The natural frequency of the system is reduced, and the vibration isolation frequency band is widened. (3) The stiffness characteristics of the system can be flexibly adjusted according to different working conditions. The bearing capacity is determined by the positive stiffness spring, and the negative stiffness element is used to reduce the dynamic stiffness of the system. The positive and negative stiffness in parallel has the characteristics of high static and low dynamic stiffness, which can significantly reduce the system stiffness, broaden the vibration isolation band of the system, and improve the vibration isolation performance of the system on the premise of ensuring the bearing capacity of the system. At the same time, QZS provides a feasible solution for low-frequency vibration energy harvesting due to the very low dynamic stiffness. Therefore, the QZS characteristic can not only widen the vibration isolation region, but also facilitate low-frequency energy harvesting, which has received extensive attention from researchers [43]. In recent years, QZS has been widely studied for simultaneous vibration isolation and energy harvesting. However, there are still many challenges in engineering applications to be overcome. Thus, it is very significant to review and summarize QZS-based simultaneous vibration isolation and energy harvesting. The main purpose of this paper is to review present progress and discuss future directions. The rest of this article is organized as follows. Section 2 describes the development history and basic principles of QZS vibration isolation. Section 3 mainly summarizes the classification of QZS structures. Section 4 summarizes the research progress of QZS synchronous vibration isolation and energy harvesting. Section 5 reviews the challenges and advantages in QZS simultaneous vibration isolation and energy harvesting. Section 6 discusses the shortcomings of current research and provides an outlook for future research. Section 7 makes a brief conclusion.

2. Description of QZS Development

2.1. Basic Principle of QZS

The development of QZS, which is realized by the parallel combination of positive and negative stiffness elements, can be traced back to the 1980s. When a force (ΔF) is applied to a system, the corresponding deformation is denoted as ΔX . Then, if $\Delta F / \Delta X$ is positive, we always call it 'positive stiffness'. Otherwise 'negative stiffness'. The drawback of negative stiffness elastic elements is their instability, so they are rarely used alone in engineering applications. However, it is not the case when positive and negative stiffness elements are connected in parallel. The basic principle of QZS is shown in Figure 2 and the total stiffness of the system can be expressed as $k_0 = k^+ + k^-$. In practice, the positive stiffness k^+ can be adjusted so that $|k^+| \approx |k^-|$ and $|k^+| > |k^-|$. In this case, the total stiffness of the system is infinitely close to zero, so it is called as 'QZS'.

According to the vibration isolation principle, linear vibration isolation devices can only play a role when the excitation frequency is larger than $\sqrt{2}$ times natural frequency of the system. Therefore, the performance of linear vibration isolators is not good under

low-frequency or ultra-low frequency vibrations. In order to deal with this problem, it is often necessary to reduce the stiffness of the system. However, reducing the stiffness will reduce the system stability and the loading capacity. To resolve this conflict, nonlinearity is introduced into linear vibration isolators. QZS is just one kind of new nonlinearity, which attracts more and more attention nowadays in the field of vibration isolation due to near-zero dynamic stiffness and strong loading capacity.



Figure 2. Schematic diagram of QZS.

A simplified schematic diagram of QZS-based vibration isolation is shown in Figure 3. When the mass m is excited by the external force F, the system will generate forced responses, leading to vibration. During the vibration, the positive stiffness element in the QZS plays the key role of loading and damping. The negative stiffness element can adjust the system stiffness according to different working conditions. When the mass is at static equilibrium position, the dynamic stiffness of the system is zero. At the same time, the vibration isolation device has the advantages of low natural frequency and large bandwidth. According to the literature, the current research on QZS-based vibration isolation mainly focuses on two aspects: one is static analysis and the other is stability analysis.



Figure 3. Schematic diagram of QZS-based vibration isolator.

2.2. Development of QZS-Based Vibration Isolation

In 1989, Alabuzhev et al. [44] first presented a comprehensive introduction to the zero stiffness vibration isolation theory. Since then, many implementation schemes of QZS have been proposed. The whole development of QZS-based vibration isolation can be summarized in Figure 4, which is divided into three stages: vibration isolation, vibration isolation and noise reduction, and vibration isolation and energy harvesting. In 1994, Trimboli et al. [45] proposed a new concept of vibration isolation using a magnetic spring

and a feedback adjustment system. The magnetic spring exhibits negative stiffness in the limited movement region. The use of magnetic springs in parallel with linear springs leads to ideal vibration isolation characteristics. In 1999, Platus et al. [46] first established a compression spring-rod structure with negative stiffness to counteract the positive stiffness in the suspension. Compared with the state-of-the-art vibration isolation systems, the performance of this new system was greatly improved. In 2003, Takasaki et al. [47] first proposed to use linear actuators for negative stiffness active control in the field of vibration isolation. In 2004, Lee et al. [48] performed dynamic analysis on an elastic device with zero stiffness and then carried out an optimal design of the instability caused by negative stiffness in the system. In 2006, Lee et al. [49] first combined variable structural negative stiffness air damping with positive stiffness spring and applied it for actual seat pneumatic control suspension.



Figure 4. Development of QZS-based vibration isolation.

After 2006, the study of QZS isolation gradually and significantly increased. Carrella et al. [50] analyzed the dynamics of QZS passive vibration isolators by establishing an inclined spring model, and here the concept of high static and low dynamic stiffness was first introduced. In previous studies, it was found that although QZS could be generated at the equilibrium position, the system stiffened once a disturbance occurred. To address this problem, Kovacic et al. [51] used a nonlinear, pre-stressed inclined spring to explore the possible effects of system nonlinearity in 2007. In 2008, Brennan et al. [52] compared the force transmissibility of the QZS vibration isolator with the linear vibration isolator. It was concluded that as long as the system parameters were selected correctly, the QZS vibration isolation performance would be better than that of the linear one. In 2011, Peng et al. [53] first analyzed the stability of a spring-linked QZS isolation device using the L-P regression method. In 2012, Carrella et al. [54] carried out static analysis on nonlinearity in the QZS. Based on this, many researchers subsequently proposed and optimized many kinds of QZS vibration isolators. To date, QZS vibration isolation has been gradually developed to become a research hot-spot in engineering fields. In summary, the utilization of QZS is very promising for vibration isolation and energy harvesting in future [55].

So far, QZS vibration isolators have been widely used in different fields, as shown in Figure 5, such as transportation engineering, marine engineering, aerospace, civil engineering, and so on. As for new energy vehicles, ride comfort, road handling, and battery endurance are main challenges in future. To address these problems, Thanh et al. [56] designed a QZS vibration isolator for a car seat, as shown in Figure 5. Zhou et al. [57] presented a QZS vibration isolator for medical infant cabins. Hu et al. [58] proposed a QZS vibration isolator consisting of three linear springs for pipeline vibration suppression in the field of civil engineering. As for transportation engineering, bridges are threatened by low-frequency vibrations. Attary et al. [59–61] designed a QZS vibration isolation device to reduce the low-frequency vibration of bridges and the simulation results validated its excellent vibration isolation performance. Ishida et al. [62] proposed an origami-type QZS vibration isolator for earthquakes and conducted experimental validations in a simulated seismic zone. In order to protect industrial equipment (e.g., generators, compressors and pipes) from vibration hazards in engineering applications, Burian et al. [63] applied QZS properties to design proper supporting bases for engineering equipment, which could effectively reduce vibration transmission.



Figure 5. Engineering applications of QZS-based vibration isolation. Reprinted/adapted with permission from Ref. [56]. 2013 Elsevier Ltd. All rights reserved.Thanh Danh Le and Kyoung Kwan Ahn.Reprinted/adapted with permission from Ref. [57].Copyright The Author(s) 2017 Reprints and permissions. Jiaxi Zhou and Kai Wang. Reprinted/adapted with permission from Ref. [59]. Copyright A. S. Elnashai, N.ATTARY and M.SYMANS.

3. Classification of QZS-Based Vibration Isolation Structures

The key point of QZS-based vibration isolation is to design proper negative stiffness structures. Up to now, many scholars have designed various negative stiffness structures for vibration isolation, which can be summarized as the following four classes.

3.1. Classical Structures with Positive and Negative Stiffness

Carrella et al. [50] built a nonlinear vibration isolation system consisting of a vertical spring and two oblique springs connected in parallel. The two inclined springs were used to generate negative stiffness [64–66]. The behaviors of vibration transmission and isolation were studied and the QZS vibration isolation characteristic with high static and

low dynamic stiffness was found. At the same time, it was proven that there was a special relationship between the geometry and the QZS stiffness, which provided a theoretical foundation for follow-up research. Later, Xu et al. [67] further increased the number of inclined springs and built a structure consisting of four inclined springs and one vertical spring. Similar to the inclined spring structure, many researchers also studied other related structures. Thanh et al. [56] proposed a novel QZS vibration isolation device shown in Figure 6a, which is composed of two parts. The positive stiffness was provided by the vertical spring and the negative stiffness was generated by two symmetrical structure, as shown in Figure 6b. The isolator was composed of a vertical spring and two identical pre-compressed horizontal springs, where the combination of horizontal spring, slider, and rod generated negative stiffness and the vertical spring generated positive stiffness. The advantages of this kind of isolator include high stability and loading capacity. Finally, a simplified mathematical model of the QZS vibration isolation was established and analyzed.



Figure 6. Schematic diagram of spring-rod structures in (**a**) Spring connecting rod structure. Reprinted/adapted with permission from Ref. [56]. Copyright 2013 Elsevier Ltd., Kyoung KwanAhn and Thanh DanhLe. and (**b**) Three dimensional quasi zero stiffness isolator. Reprinted/adapted with permission from Ref. [57]. Copyright Springer Nature B.V. 2020, Chaoran Liu and Kaiping Yu.

Different from the inclined spring, Liu et al. [69] presented a large negative stiffness under small displacement by utilizing the Euler buckling beam. As shown in Figure 7a, the structure consisted of a linear spring with positive stiffness and two Euler buckling beams with negative stiffness. Huang et al. [70] studied a negative stiffness corrector based on Euler buckling beams. As shown in Figure 7b, two pairs of buckling beams were symmetrically placed at both ends of the mass to provide negative stiffness. Vertical springs provided positive stiffness. This nonlinear isolator performed well under low excitation amplitudes in applications. Carrella et al. [71] proposed a QZS vibration isolation device composed of a magnetic element and a coil spring in parallel. As shown in Figure 7c, it consisted of two linear springs separated by a permanent magnet. Two permanent magnets were symmetrically placed at both ends of the spring and the magnets acted as a negative stiffness. Zheng et al. [72] proposed a QZS shaft coupling, and the repulsive force generated by two coaxial magnetic rings generated negative torsional stiffness.

3.2. Negative-Stiffness Structures with Active Control

A negative stiffness structure with active control is generally established by adding a controller, a sensor, and an external circuit to traditional QZS element. The principle is to introduce a control strategy to adjust dynamic response of the system. Good vibration isolation and stability can be achieved by continuously supplying external electrical energy to the controller.



Figure 7. Classical structures with positive and negative stiffness in (**a**) QZS isolator structure of euler buckling beam, (**b**) Double Euler Buckling Beam Symmetrical Distribution QZS Vibration Isolator and (**c**) QZS magnetic spring structure.Reprinted/adapted with permission from Ref. [69]. Copyright 2013 Elsevier Ltd., Xiuchang Huang and Xingtian Liu. Reprinted/adapted with permission from Ref. [70]. Copyright 2013 Elsevier Ltd., Xingtian Liu and Xiuchang Huang. Reprinted/adapted with permission from Ref. [71]. Copyright 2008 Elsevier Ltd. All rights reserved, A. Carrella and M.J. Brennan.

Sun et al. [73] proposed a QZS vibration isolator with simple linear and time-delay active control strategy. As shown in Figure 8a, the structure is connected by two pairs of springs and a rod with length a in the horizontal direction. Each pair of springs is composed of linear stiffness k_1 and nonlinear stiffness k_2 . In the vertical direction, the mass is connected to the base by a spring with a stiffness of k_0 and an active control unit. The control unit is composed of a vertical spring and a servo motor, where the motor generates active feedback force. Zhou et al. [74] presented a QZS device with active control, where the positive stiffness was provided by mechanical springs and the negative stiffness was provided by electromagnetic springs. As shown in Figure 8b, the beam acted as a spring and the negative stiffness was determined by the current through the electromagnet. Xu et al. [75] proposed a novel QZS actuator with active control. The actuator was composed of a negative stiffness electromagnetic spring and a positive stiffness flexible link. Thanh et al. [76] proposed a negative stiffness vibration isolator with active control for low-frequency seat vibrations. The isolator introduced an adaptive intelligent controller to obtain high performance of vibration isolation. As shown in Figure 8c, the whole model consisted of a negative stiffness structure, vertical springs, dampers, and a low-friction cylinder controlled by a proportional valve.

3.3. Geometrically Nonlinear Structures

The realization scheme of geometrically nonlinear QZS is supposed to make the forcedisplacement relation nonlinear due to structural deformation, variable disturbances, etc. The nonlinearity due to geometric instability can be used to generate negative stiffness.



So far, existing QZS vibration isolators with geometric nonlinearity can be classified into cam-like structures and shear-type structures.

Figure 8. Negative-stiffness structures with active control in (**a**) Simplified schematic diagram of active control, (**b**) Experimental device for active control of negative stiffness structure and (**c**) Negative stiffness structure of active seat control. Reprinted/adapted with permission from Ref. [73]. Copyright 2014 Elsevier Ltd., Xiuting Sun and Jian Xu. Reprinted/adapted with permission from Ref. [74]. Copyright 2009 Elsevier Ltd., N. Zhou and K. Liu. Reprinted/adapted with permission from Ref. [76]. Copyright 2013 Elsevier Ltd., Le Thanh Danh and Kyoung Kwan Ahn.

Li et al. [77] proposed a passive QZS vibration isolator by using a cam structure and the structural profile was designed to achieve the force-displacement relation of QZS. As shown in Figure 9a, the sliding rod and the coil spring connected by the bearing were symmetrically placed in the horizontal direction. The sliding rod and the cam were connected by the bearing in the vertical direction. The experiments demonstrated that the QZS isolator had a low resonance frequency. Zhou et al. [78] proposed a QZS vibration isolator by using a cam-roller-spring structure, as shown in Figure 9b. During vibrations, the cam moved vertically and compressed the rollers connected by the coil spring in the horizontal direction, leading to produce restoring force in different directions. In this structure, the cam-roller-spring generated negative stiffness and the vertical coil springs generated positive stiffness.

Zou et al. [79] proposed a QZS vibration isolator by using a shear structure, as shown in Figure 9c. Then, negative stiffness was provided with by two scissor-like structures and springs. Each scissor-like structure included a coil spring and a linear damper. The geometrically nonlinear stiffness was calculated by parametric analysis and the isolator could be designed to obtain a large frequency band.

3.4. Multi-Direction/Multi-Degree-of-Freedom QZS Structures

In many cases, the vibrations are multi-direction, instead of single-direction. Therefore, it is necessary to design multi-direction QZS vibration isolation devices. Liu et al. [80] proposed an in-plane QZS vibration isolation device, as shown in Figure 10. The device consisted of two concentric magnetic rings and eight cables connected inside the mag-

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netic ring. Interactions between two magnetic rings generated negative stiffness and the cables generated positive stiffness. Vibration in the horizontal direction could be arbitrarily decomposed into orthogonal vibrations, so that vibrations in any direction could be reduced effectively.



Figure 9. Geometrically nonlinear vibration isolation structure in (**a**) QZS isolator structure of shear type structure, (**b**) QZS isolator for cam mechanism and (**c**) Cam-roller-spring QZS isolator structure. Reprinted/adapted with permission from Ref. [77]. Copyright Springer-V erlag GmbH Germany, part of Springer Nature 2020, Wei Zou and Chun Cheng. Reprinted/adapted with permission from Ref. [78]. Copyright 2020 Published by Elsevier Ltd., Ming Li and Wei Cheng. Reprinted/adapted with permission from Ref. [79]. Copyright 2015 Elsevier Ltd., Jiaxi Zhou and Xinlong Wang.



Figure 10. Schematic diagram of multi-direction QZS vibration isolation structure. Reprinted/ adapted with permission from Ref. [80]. Copyright 2021 Elsevier Inc., Chaoran Liu and Rui Zhao.

Wu et al. [81] proposed a six-degree-of-freedom passive vibration isolator, which could lead to QZS by adjusting the structural parameters. The whole structure is similar to the Stewart platform, which can achieve passive vibration isolation in six directions. Zhou et al. [82] constructed a six-degree-of-freedom QZS vibration isolation platform by using QZS support poles. Compared with linear vibration isolation devices, this platform had both a wider frequency band of vibration isolation and better isolation performance in the low frequency range.

In summary, the above four types of QZS vibration isolation structures are compared in Tables 1 and 2. It can be seen that: (1) Classical structures with positive and negative stiffness advantages represent the simplest approach, but their stability is poor. (2) The stability of negative stiffness structures with active control is high and their resonance frequencies are low. However, they require a large amount of external electrical energy to maintain, leading to high cost and technical obstacles. (3) Geometrically nonlinear QZS vibration isolators can be more compact than other structures. but they have the disadvantage of limited displacement range and loading capacity. (4) Superior to other

structures, multi-direction or multi-degree-of-freedom QZS vibration isolators can reduce

QZS Structure	Representative Literature	Excitation Frequency	Peak Transmissibility
Classical structures with positive and negative stiffness	Thanh et al. [56]	1.5 Hz	9 dB
Negative-stiffness structures with active control	Zhou and Liu et al. [74]	7 Hz	4 dB
Geometrically nonlinear structures	Li and Cheng [77]	10 Hz	17 dB
Multi-direction/Multi-degree-of- freedom structures	Liu et al. [80]	7.23 Hz	11.22 dB

Table 1. Comparisons of four QZS implementation structures.

multi-direction vibrations and adapt to different working conditions.

Table 2. Comparisons of four QZS implementation structures.

QZS Structure	Advantages	Disadvantages
Classical structures with positive and negative stiffness	Simple structure	Poor stability
Negative-stiffness structures with active control	low resonance frequency rapid response strong stability	Require external electrical energy High cost
Geometrically nonlinear structures	Compact structure	Limited displacement range Limited loading capacity
Multi-direction/Multi-degree-of-freedom structures	Multi-direction vibration isolation	Complex structure Large space

4. Current Progress of QZS-Based SVIEH

From the viewpoint of energy flow, vibration energy harvesting can also be considered as the counterpart of vibration isolation. As shown in Figure 11, according to different mechanisms, the research of QZS-based SVIEH is mainly composed of electromagnetic, piezoelectric, and hybrid methods.



Figure 11. Classification of QZS-based SVIEH [83]: (a) Electromagnetic, (b) Piezoelectric and (c) Hybrid.

4.1. QZS-Based Electromagnetic SVIEH

Electromagnetic vibration energy harvesting was based on the Faraday's law of electromagnetic induction. In recent years, QZS-based electromagnetic SVIEH devices have attracted more and more attention. Yang et al. [83] first proposed a high-order quasi-zero stiffness (HQZS)-based SVIEH structure. As shown in Figure 12a, by adjusting structural parameters, the HQZS can achieve arbitrarily small stiffness at the equilibrium position. Compared with the QZS four-stable system, the HQZS structure had more advantages in vibration isolation and energy harvesting. Li et al. [84] proposed a novel QZS-based bistable electromagnetic energy harvesting device for ocean wave energy. As shown in Figure 12b, the structure consisted of two biomimetic X structures and a power take-off (PTO) system. The new biomimetic X-structure had QZS and bistable properties. There was a mechanical rectifier and electromagnetic DC generator in the PTO system. Compared with linear energy harvesting devices, its energy harvesting efficiency was significantly enhanced [85]. Lu et al. [86] proposed a QZS electromagnetic Stewart platform to achieve SVIEH. As shown in Figure 12c, for various mechanical and electrical parameters, the analytical and numerical results both demonstrate that the frequency band of vibration isolation extended to lower frequencies and produces considerable power output. Moreover, the increase in energy harvesting led to reduced vibration transmissibility under varying some parameters. Yang et al. [87] proposed a multistable and multidirectional (MMD) QZS energy harvesting system based on the former. As shown in Figure 12d, the structure was guided by the slider to realize the relative movement between the permanent magnet and the coil. At the same time, the multi-stable behavior was analyzed. It was verified by experiments that the MMD QZS energy harvesting device can easily realize low-frequency energy harvesting.



Figure 12. QZS-based SVIEH by using (**a**) the HQZS structure, (**b**) X structure electromagnetic wave energy converter, (**c**) the Stewart Platform, and (**d**) the multistable and multidirectional structure. Reprinted/adapted with permission from Ref. [40]. Copyright 2021 Elsevier Ltd. All rights reserved, Tao Yang and Qingjie Cao. Reprinted/adapted with permission from Ref. [84]. Copyright 2021 Elsevier Ltd. All rights reserved, Meng Li and Xingjian Jing. Reprinted/adapted with permission from Refs. [85,86]. Copyright 2020 Published by Elsevier Inc., Zeqi Lu and Dao Wu. Reprinted/adapted with permission from Ref. [87]. Copyright 2020 Elsevier Ltd. All rights reserved, Tao Yang and Qingjie Cao.

4.2. QZS-Based Piezoelectric SVIEH

Piezoelectric vibration energy harvesting is achieved through the positive piezoelectric effect. When piezoelectric material is deformed by an external force, polarization occurs inside. Then, positive and negative ions in the electrolyte move along the direction of the deformation. By this way, charges will be generated on the two opposite surfaces of the material. Piezoelectric vibration energy harvesting has the advantages of high energy density, simple structure, and easy miniaturization.

Many scholars have combined the advantages of QZS with piezoelectric materials for vibration energy harvesting. Liu et al. [88] proposed piezoelectric beams with QZS for simultaneous low-frequency vibration isolation and energy harvesting. As shown in Figure 13a, the electrical energy output by the deformed piezoelectric material was very high. At the same time, the piezoelectric buckling beam also exhibited negative stiffness. Therefore, the structure had a good vibration isolation effect in the low frequency environment. Compared with traditional linear vibration isolators and cantilever beam piezoelectric devices, it achieved better vibration isolation and energy harvesting in lower frequency environments. Liu et al. [89] further proposed a novel QZS vibration isolation and piezoelectric energy harvesting device for vibration localization in previous research. As shown in Figure 13c, the device consisted of a QZS device and six piezoelectric cantilevers. The device can not only achieve a good low-frequency vibration isolation effect, but also achieve piezoelectric energy harvesting in multiple directions. Sun et al. [90] proposed and analyzed a nonlinear, adjustable, and continuous structure. As shown in Figure 13e, the structure can have large amplitude vibration with adjustable output frequency. Iwaniec et al. [91] designed a QZS dual potential energy well and piezoelectric structure energy harvesting device. As shown in Figure 13b, the QZS effect resonator was able to generate larger amplitude and higher voltage output. The advantage of this device was the wider distribution of potential wells and lower potential barriers. Lu et al. [92] proposed a QZS device of a bistable piezoelectric composite plate structure for SVIEH in a low-frequency environment. As shown in Figure 13d, a feasible analytical model was established through theoretical analysis. The approximate solution of displacement transmissibility and voltage amplitude was obtained. Finally, the accuracy of the approximate solution was judged by numerical simulation. The result show that the energy harvesting increases with the decrease of displacement transfer rate in a certain frequency band range.

4.3. QZS-Based Hybrid SVIEH

Due to the low conversion efficiency of the single-form vibration energy conversion device, researchers designed a hybrid vibration energy harvesting device to overcome the above technical shortcomings [93]. According to current research hybrid QZS vibration energy harvesting devices are not limited to conventional energy conversion methods. In recent years, triboelectric nanogenerators have exhibited excellent energy harvesting performance in low-frequency and ultra-low frequency environments. Therefore, many scholars have proposed coupling triboelectric nanogenerators with electromagnetic generators for energy harvesting in low-frequency environment [94–96]. Wang et al. [97] proposed a nonlinear dual QZS hybrid energy harvesting device. As shown in Figure 14a, the energy conversion part was coupled with a traditional electromagnetic conversion form and a triboelectric nanogenerator to form a hybrid vibration energy conversion device. The effects of QZS and bistable system on energy harvesting efficiency are verified by numerical analysis. Yang et al. [98] designed a time-delay QZS piezoelectric-electromagnetic hybrid energy harvester. As shown in Figure 14b, the parameters that can affect the energy output by external excitation were optimized by designing different types of time-delay control techniques. In this way, the optimal parameters of the energy output efficiency can be obtained. Most importantly, time-delay control techniques may enhance common amplitude values under either random or harmonic excitation. Energy output was significantly enhanced.



Figure 13. Typical structures based on piezoelectric QZS-SVIEH in (**a**) Simplified diagram of QZS synchronous vibration isolation and energy harvesting, (**b**) Schematic Diagram of Piezoelectric Cantilever Beam Energy Harvesting Structure, (**c**) QZS energy harvesting physical structure, (**d**) Schematic Diagram of Vibration Isolation and Energy Harvesting Integration of Bistable Piezoelectric Composite Plate and (**e**) Piezoelectric energy harvesting structure. Reprinted/adapted with permission from Ref. [88]. Copyright 2021 Elsevier Ltd. All rights reserved, Chaoran Liu and Rui Zhao. Reprinted/adapted with permission from Ref. [91]. Copyright 2021 by the authors. Licensee MDPI, Basel, Switzerland, Joanna Iwaniec and Grzegorz Litak. Reprinted/adapted with permission from Ref. [89]. Copyright 2021 Elsevier Inc. All rights reserved, Chaoran Liu and Rui Zhao. Reprinted/adapted with permission from Ref. [92]. Copyright Journal of Vibration and Control 2019, Vol. 0(0) 1–11, Zeqi Lu and Dong Shao. Reprinted/adapted with permission from Ref. [90]. Copyright Korean Society for Precision Engineering 2019, Xiuting Sun and Feng Wang.



Figure 14. Typical structures based on hybrid QZS-SVIEH in (**a**) Multi directional composite QZS energy harvesting structure and (**b**) Hybrid energy harvesting structure. Reprinted/adapted with

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To sum up, the energy density generated by energy harvesting based on QZS is different due to different energy conversion methods. The energy data generated by different energy conversion methods are shown in Table 3.

Energy Conversion Mode	Representative Literature	Incentive Type	Energy Harvesting Value
Electromagnetic	Yang and Cao et al. [3]	harmonic excitation	0.5 W
	Li and jing et al. [84]	Random excitation	0.5~1 W
	Lu and Wu et al. [86]	harmonic excitation	1.74 W
Piezoelectric -	Liu and Zhao et al. [89]	harmonic excitation	2 W
	Sun and Wang et al. [90]	Random excitation	100 mW
Hybrid.	Wang and Zhou et al. [97]	Random excitation	10 mW

Table 3. Comparisons of four QZS implementation structures.

5. Key Challenges of QZS-Based SVIEH

Up to now, QZS has been widely studied for vibration isolation and much progress has been achieved [99]. In particular, QZS began to be explored for SVIEH in recent years. It should be pointed out that QZS-based SVIEH is still in its infancy and faces some problems to be solved in engineering applications [100]. According to the literature, several key challenges of QZS-based SVIEH are summarized as follows.

5.1. Improve Energy Harvesting/Weight Ratio

Current negative stiffness is mainly implemented by means of oblique springs, Eulerian flexural beams [101–103], spring-linked bars [104,105], and geometric structures [106]. These structural dimensions make it difficult to carry out the miniaturization of a QZS element. Compared with negative stiffness elements, positive stiffness elements are always realized by using linear helical springs, which also play a major role of load bearing. As a result, the entire QZS device hardly becomes small by changing the positive stiffness element [107,108] and then the power/volume ratio is indeed low. In recent years, magnetic springs have been proposed to realize negative stiffness, which are superior to other structures in miniaturization. However, magnetic springs are prone to magnetic degeneration, so it is difficult keep the bearing capacity of magnetic springs. Therefore, it is urgent to develop QZS-based SVIEH device with simple structure and strong carrying capacity.

5.2. Potential Instability

Most current research on the QZS focuses on the way how to carry out negative stiffness [109,110]. However, negative stiffness itself also leads to the instability of QZS [111]. A QZS system can only have static equilibrium position within a small displacement range. Once the QZS system shifts from its static equilibrium position, the negative stiffness element is most likely to show hardening characteristics, which will cause unstable phenomena, such as jumping [112–114], chaos [115–117], and voltage bifurcation [118,119]. In this case, it is possible that vibration isolation and energy harvesting cannot be achieved simultaneously.

As for QZS-based SVIEH, most studies mainly adopt dynamic modeling, response analysis, sensitivity analysis, performance comparison, and so on [120,121]. The instability in QZS-based SVIEH has rarely been studied widely [122,123]. Nowadays, researchers mainly adjust system parameters to achieve the best performance of the QZS-based SVIEH device and the effects due to the instability of negative stiffness instabilities are ignored. Compared with traditional energy harvesting systems, negative stiffness elements are key

components to generate instability. Although multi-stable energy harvesting device can improve the harvesting efficiency, it is prone to jumping back and forth between different energy wells.

5.3. Multi-Stable Phenomenon Due to Nonlinear Negative Stiffness

The characteristic of nonlinear negative stiffness makes a QZS system easy to generate multi-stable phenomenon, so QZS-based energy harvesting system can achieve high harvesting efficiency among a broad frequency band [124,125]. It breaks the limitation that linear energy harvesting devices can only achieve the maximum performance in the resonance region. However, nonlinear QZS systems exhibit multiple resonances among certain frequency ranges. In this case, the advantage of broad band is missing. Especially in the multi-solution region, it is difficult to realize the motion between high-energy wells. At the same time, the distance between potential wells will become narrow and the barrier will become high under ultra-low frequency vibrations, such that it is not easy to transit the barrier.

5.4. Small Range of QZS

In recent years, QZS-based systems have demonstrated good low-frequency vibration isolation characteristics due to high static and low dynamic stiffness. However, there are still low-frequency resonant regions, which receive much attention in the field of low-frequency vibration energy harvesting [126,127]. The researchers obtained the minimum stiffness by adjusting the quasi-zero stiffness structural parameters. The energy output between traps in a low frequency environment is achieved near the static equilibrium position. When the QZS energy harvesting device is excited by low intensity or ultra-low frequency, the realization range of QZS becomes smaller. It is difficult to obtain high-energy inter-well oscillations for QZS devices. Therefore, the small range of QZS realization is still a challenge to be solved in the future.

5.5. Limited Implementation Method

As a key technology to improve QZS-based SVIEH devices, nonlinearity has been recognized by more and more researchers. At present, most studies on the performance of QZS-based SVIEH mainly focus on how to optimally design of nonlinear negative stiffness. Then, the nonlinear negative stiffness element is used in parallel with linear helical springs to achieve high static and low dynamic stiffness. By this way, linear helical springs with positive stiffness are limited to compensate for negative stiffness. Thus, it is difficult to innovatively design QZS-based SVIEH structures and new ideas are much desirable.

6. Future Trend

As for the above challenges of QZS-based SVIEH, new theories, new structures, and new designs should be explored to deal with them. In this paper, the following trends will be discussed.

6.1. Nonlinear Positive Stiffness Compensation

Existing QZS structures for SVIEH are mainly based on symmetrically distributed nonlinear negative stiffness and the positive stiffness is mainly realized by linear helical spring [128]. However, using linear helical springs to compensate for negative stiffness is not perfect for all kinds of QZS [129,130]. From the perspective of dynamics, the QZS is often realized through combining positive and negative stiffness elements. However, the nonlinearity is not limited to only negative stiffness elements. It is also a new idea to realize QZS-based SVIEH by using nonlinear positive stiffness elements to compensate negative stiffness with nonlinear positive stiffness for QZS-based vibration isolation and then analyzed its feasibility. In the future, more nonlinear positive stiffness elements should be studied for QZS-based SVIEH.



Figure 15. Nonlinear positive stiffness compensates for negative stiffness. (**a**) Force-displacement curve of QZS. (**b**) Analysis Diagram of QZS Realization. Reprinted/adapted with permission from Ref. [131]. Copyright 2022 Elsevier Ltd. All rights reserved, Ge Yan and Zhiyuan Wu.

6.2. QZS Molecular Spring

In recent years, molecular spring isolators have received much attention due to their high static and low dynamic stiffness characteristics. Molecular spring isolators are composed of water and nano hydrophobic microporous materials. This new solid–liquid mixture is like a mechanical spring, which can easily achieve variable stiffness characteristics [132]. Li et al. [133] first proposed a molecular spring isolator for heavy loads and analyzed the vibration isolation performance by means of energy transfer. Nie et al. [134] applied a molecular spring isolator for a car suspension and simulated the smoothness by using MATLAB/Simulink simulation. The results showed that the perturbation of molecular spring isolators was better than that of linear suspensions under external fixed-frequency excitation. Up to now, the QZS molecular spring has been rarely studied for SVIEH, which deserves further study in the future.

6.3. QZS-Based SVIEH Using New Material

In recent years, some metamaterials or synthetic materials have been introduced into vibration energy harvesting. The biggest difference between these new materials and traditional materials is that the new materials easily demonstrate variable stiffness, light weight, and compact structure [135]. For QZS-based SVIEH, structural miniaturization, low-frequency and broad band, nonlinearity, and the efficiency are still the key points in future research. However, natural materials hardly have the above characteristics simultaneously [136]. Thus, the integration of new materials and energy harvesting provides a new idea for QZS-based SVIEH [137]. Although the related studies have not been reported to date, it is worth using them to develop QZS-based SVIEH in the future.

6.4. The Effects of External Circuit Parameters

In recent years, researchers have focused on the effects of structural parameters on QZS-based SVIEH devices, such as the system damping ratio [138], the electromechanical coupling coefficient [139], and the excitation amplitude [140]. When a QZS-based SVIEH system is connected to an external circuit, the external resistance will also affect the performance [141]. Zhang et al. [142] used cantilever piezoelectric structural parameters as quantification. The resistance value with the best energy capture effect was obtained by using the structural parameters. Yan et al. [143] proposed a low-frequency broadband piezoelectric tri-stable external resistive-inductive resonant (RL) circuit energy harvesting device. The results show that the energy harvesting device with the resonant circuit significantly improves the energy harvesting efficiency. At the same time, the vibration response is reduced. Resonant circuits have ushered in new technical means in the field of improving vibration energy capture. Zhou et al. [144] summarized the research on multistable energy harvesting devices while presenting the key challenges for energy

harvesting by external circuits. The importance of the conversion circuit is expounded when the AC power obtained by the energy harvesting device under the external excitation is used to supply the DC equipment. For example, circuits, such as synchronous charge extraction techniques [145–147], parallel synchronous switched capture inductor (P-SSHI) circuits [148], and SSHI electronic interface circuits [149], have shown significant improvements in vibration energy harvesting efficiency. At present, much research in the field of QZS vibration isolation and energy harvesting (QZS-VI-EH) mainly focuses on structural design and low-frequency broadband. However, the effect of external circuits on QZS-VI-EH is rarely studied. It is not difficult to find that external circuitry may also have an impact on the QZS-VI-EH device. For future research, the coupling between the external circuit and QZS-VI-EH should be closely combined. An optimal coupling condition is sought to improve the energy harvesting performance.

6.5. QZS-Based SVIEH Double Optimization

Currently, QZS-based SVIEH regards vibration isolation and energy harvesting as two conflicting indicators, and its main work focuses on theoretical modeling, numerical simulation, experimental testing, etc. A tradeoff optimization can be achieved by sacrificing the performance of vibration suppression and energy harvesting, respectively. Therefore, it will be an important direction to realize the dual optimization of vibration isolation and energy harvesting in the future, and the following two aspects can be considered: (1) Mechanism research, according to the application scenario of QZS-based SVIEH, to reveal the relationship between the internal vibration isolation and energy harvesting of the structure, how the performance of QZS-based SVIEH is constrained in structure, etc. (2) A composite QZS-based SVIEH structure is established to realize the decoupling of vibration isolation and energy harvesting from the structure.

7. Conclusions

The QZS devices can not only achieve a good vibration isolation effect in the low frequency or ultra-low frequency environment, but also improve the energy harvesting efficiency. This presents wide applications in the fields of vehicle transportation, aerospace engineering, civil engineering, marine energy equipment engineering, environmental monitoring, and micro-electromechanical engineering. The QZS perfectly fits the two goals of low-frequency vibration isolation and vibration energy harvesting. This paper aims to summarize the research progress of nonlinear QZS in vibration isolation and energy harvesting in recent decades. The theoretical status, basic principle, structural status, and engineering application of zero stiffness vibration isolation are analyzed. The research status of QZS-VI-EH is discussed from the perspective of energy conversion. Finally, according to the problems existing in QZS-VI-EH, the future development trend of QZS-VI-EH is prospected.

Author Contributions: Z.C. (Zhiwen Chen): investigation, methodology, writing—original draft. Z.C. (Zhongsheng Chen): conceptualization, supervision, writing—original draft, funding acquisition. Y.W.: investigation. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant number 51911530202 and 51577189.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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