

Determining an Accurate and Cost-Effective Individual Height-Diameter Model for Mongolian Pine on Sandy Land

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Table S1 Akaike information criterion (AIC) and the Bayesian information criterion (BIC) of different variance function in fitting heteroscedasticity for mixed-effects models. Bold indicates the lowest AIC and BIC.

| Random parameters | Basic | | Generalized | |
|---------------------|----------------|----------------|----------------|----------------|
| | AIC | BIC | AIC | BIC |
| Exponential | 2754.74 | 2779.62 | 2609.42 | 2639.27 |
| Power | 2745.71 | 2770.59 | 2600.12 | 2629.97 |
| Constant plus power | 2747.71 | 2777.56 | 2602.12 | 2636.95 |

Table S2. Mean absolute error (MAE) and root mean squared error (RMSE) for different calibration designs of mixed-effects H-D models. The mean of words in tree size column are: FP for fixed-effects prediction, N for the thinnest tree, M for medium diameter tree, K for the thickest tree. The different letters in the tree size column indicate combinations of tree size. For example, the N1M1 means one thinnest and one medium diameter trees (in order), and N1M1K1 indicates one thinnest, one medium and one thickest diameter tree (in order). Asterisk in column of MAE indicates significant difference between predicted and observed height ($p < 0.05$).

| Tree size | Number of trees | MAE | | RMSE | |
|-----------|-----------------|----------|-------------|--------|-------------|
| | | Basic | Generalized | Basic | Generalized |
| | 0 | 1.9103 * | 0.6403 | 2.2613 | 0.8477 |
| N1 | 1 | 0.8983 | 0.6402 | 1.1805 | 0.8466 |
| M1 | 1 | 0.7652 * | 0.635 | 1.0333 | 0.8431 |
| K1 | 1 | 0.7892 * | 0.6423 | 1.0167 | 0.8493 |
| N2 | 2 | 0.7712 | 0.633 | 1.001 | 0.8405 |
| M2 | 2 | 0.742 * | 0.6329 | 0.9979 | 0.8417 |
| K2 | 2 | 0.6982 | 0.6419 | 0.9236 | 0.8494 |
| N1M1 | 2 | 0.7607 * | 0.6359 | 0.9775 | 0.8426 |
| M1K1 | 2 | 0.6783 * | 0.637 | 0.9088 | 0.8446 |
| N1K1 | 2 | 0.7619 | 0.6422 | 0.9874 | 0.8482 |
| N3 | 3 | 0.7105 * | 0.6329 | 0.9396 | 0.8406 |
| M3 | 3 | 0.6840 | 0.6303 | 0.8957 | 0.8355 |
| K3 | 3 | 0.6841 | 0.6415 | 0.9017 | 0.8487 |
| N1M2 | 3 | 0.7364 * | 0.6344 | 0.9535 | 0.8416 |
| M1K2 | 3 | 0.6531 * | 0.6372 | 0.8746 | 0.8448 |
| N1K2 | 3 | 0.7053 | 0.6417 | 0.923 | 0.8483 |
| N2M1 | 3 | 0.7328 * | 0.6301 | 0.9465 | 0.838 |
| M2K1 | 3 | 0.6786 * | 0.6349 | 0.9116 | 0.8427 |
| N2K1 | 3 | 0.7135 | 0.6354 | 0.9309 | 0.8422 |
| N1M1K1 | 3 | 0.6991 * | 0.6377 | 0.9072 | 0.8439 |
| N4 | 4 | 0.6796 | 0.6267 | 0.9011 | 0.8349 |

| Tree size | Number of trees | MAE | | RMSE | |
|-----------|-----------------|----------|-------------|--------|-------------|
| | | Basic | Generalized | Basic | Generalized |
| M4 | 4 | 0.6492 | 0.6278 | 0.8576 | 0.8321 |
| K4 | 4 | 0.6648 | 0.6408 | 0.8875 | 0.8486 |
| N1M3 | 4 | 0.6974 | 0.6316 | 0.8951 | 0.836 |
| M1K3 | 4 | 0.6577 * | 0.6371 | 0.8787 | 0.8447 |
| N1K3 | 4 | 0.6962 | 0.6414 | 0.9105 | 0.8481 |
| N2M2 | 4 | 0.7087 * | 0.6288 | 0.9243 | 0.8373 |
| M2K2 | 4 | 0.6501 * | 0.6348 | 0.8748 | 0.8426 |
| N2K2 | 4 | 0.6866 | 0.636 | 0.9032 | 0.843 |
| N3M1 | 4 | 0.6973 * | 0.6302 | 0.914 | 0.8382 |
| M3K1 | 4 | 0.6636 | 0.633 | 0.8761 | 0.8374 |
| N3K1 | 4 | 0.6767 * | 0.6346 | 0.8992 | 0.8418 |
| N5 | 5 | 0.6563 | 0.6249 | 0.8755 | 0.8326 |
| M5 | 5 | 0.6488 * | 0.626 | 0.8559 | 0.8283 |
| K5 | 5 | 0.6464 | 0.6391 | 0.8522 | 0.8437 |
| N2M3 | 5 | 0.6906 | 0.6277 | 0.8958 | 0.8332 |
| M2K3 | 5 | 0.6479 * | 0.6343 | 0.8704 | 0.8425 |
| N2K3 | 5 | 0.6852 | 0.6365 | 0.9013 | 0.8437 |
| N3M2 | 5 | 0.6930 * | 0.6297 * | 0.9126 | 0.8384 |
| M3K2 | 5 | 0.6516 | 0.6334 | 0.862 | 0.8379 |
| N3K2 | 5 | 0.6609 * | 0.6348 | 0.8803 | 0.8423 |
| N6 | 6 | 0.6521 | 0.6262 | 0.8687 | 0.8338 |
| M6 | 6 | 0.6464 * | 0.6251 | 0.8446 | 0.8259 |
| K6 | 6 | 0.6522 | 0.6385 | 0.8538 | 0.841 |
| N3M3 | 6 | 0.6660 * | 0.6267 | 0.8735 | 0.8327 |
| M3K3 | 6 | 0.6519 | 0.6335 | 0.8626 | 0.8385 |
| N3K3 | 6 | 0.6597 * | 0.6351 | 0.8784 | 0.8425 |
| N2M2K2 | 6 | 0.6601 * | 0.6309 | 0.8724 | 0.8387 |