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Supplementary Information:

Stand Development Monitoring Data – Equations for Domain Analysis

The Stand Development Monitoring program uses a two staged cluster design to sample stands from a target population. The target population is all managed stands between the ages of 15 and 40 years that are greater than or equal to five hectares in size within a management unit.

In the first stage, stands are randomly selected from a list of the target population. Selection is without replacement and all stands have an equal probability of selection. In the second stage, ten randomly located 3.99 meter radius plots are located within each stand selected for sampling.

Domain analyses are used when survey data is subdivided into subpopulations of interest that were not part of the original survey design. This could include size classes, species, tree status (dead or alive) or any combination of subdivisions. The number of observations that fall into a domain is unknown until sampling has been completed. As such, the sample size of a domain is a random variable with a value that is unknown at the time the survey was designed (Lohr, 2009). To account for the random sample size, all observations are used to calculate standard errors, with observations not in the domain given a value of zero (Cochran, 1977).

Since the secondary stage of sampling uses cluster sampling, the total variance includes the variance between stands and the variance within stands. In the equations given below, the within stand variance is not included as it is assumed that it does not contribute much to the total variance. This is a common assumption (Lohr, 2009).

Notation and equations are provided for:

- Domain totals, such as stems, basal area and volume, for individual strata (management units)
- Domain totals for combined strata
- Proportions, such as the proportion of trees affected by a particular disturbance agent, for individual strata
- Proportions for combined strata.

Domain Total

Notation

w_{hi}	Tree level weight for trees in a stand (*i*) in a stratum (*h*) to account for the probability of selection in the second stage of sampling
m_{hi}	Number of plots in a stand (*i*) in a stratum (*h*)
a	Area, in hectares, of the fixed radius plots. For the SDM survey, it is 0.005 ha.
A_{hi}	Area, in hectares, of a stand (*i*) in a stratum (*h*)
$y_{hij}(d)$	Variable denoting whether a tree (*j*) is in the domain (*d*) for a stand (*i*) in a stratum (*h*). Examples of variables are tree count, tree basal area or tree volume.
$\hat{Y}_{hi}(d)$	Estimated population total of the variable (tree count, basal area or volume) in the domain (*d*) for a stand (*i*) in a stratum (*h*)
N_h	Number of stands in a stratum (*h*) that belong to the target population
n_h	Number of stands in a stratum (*h*) that belong to the target population that were sampled
$\hat{Y}_h(d)$	Estimated population total of the variable (tree count, basal area or volume) in the domain (*d*) for a stratum (*h*)
$var\left(\hat{Y}_h(d)\right)$	Variance of the population total (tree count, basal area or volume) in the domain (*d*) for a stratum (*h*)
$s^2(d)$	Sample variance of the population total (tree count, basal area or volume) in the domain (*d*)
$se\left(\hat{Y}_h(d)\right)$	Standard error of the population total (tree count, basal area or volume) in the domain (*d*) for a stratum (*h*)
$\hat{y}_h(d)$	Per hectare value of the population total (tree count, basal area or volume) in the domain (*d*) for a stratum (*h*)
A_h	Area, in hectares, of a stratum (*h*)
$var\left(\hat{y}_h(d)\right)$	Variance of the per hectare value of the population total (trees per hectare, basal area per hectare or volume per hectare) in the domain (*d*) for a stratum (*h*)
$se\left(\hat{y}_h(d)\right)$	Standard error of the per hectare value of the population total (trees per hectare, basal area per hectare or volume per hectare) in the domain (*d*) for a stratum (*h*)
$\hat{Y}(d)$	Estimated population total of the variable (tree count, basal area or volume) in the domain (*d*) for all strata combined
$var\left(\hat{Y}(d)\right)$	Variance of the population total (tree count, basal area or volume) in the domain (*d*) for all strata combined
$se\left(\hat{Y}(d)\right)$	Standard error of the population total (tree count, basal area or volume) in the domain (*d*) for all strata combined

$\hat{y}(d)$	Per hectare value of the population total (tree count, basal area or volume) in the domain (*d*) for all strata combined
$var(\hat{y}(d))$	Variance of the per hectare value of the population total (trees per hectare, basal area per hectare or volume per hectare) in the domain (*d*) for all strata combined
$se(\hat{y}(d))$	Standard error of the per hectare value of the population total (trees per hectare, basal area per hectare or volume per hectare) in the domain (*d*) for all strata combined

Individual Management Units

Equation (1) provides a tree value weight for trees within a stand to account for the probability of selection for trees in the second stage of sampling.

(1)
$$w_{hi} = \frac{A_{hi}}{(m_{hi} * a)}$$

Equation (2) provides the population total for a variable of interest within a domain for a stand within a stratum.

(2)
$$\hat{Y}_{hi}(d) = w_{hi} \sum_{j}^{m_{hi}} y_{hij}(d)$$

Equation (3) provides the population total for a variable of interest within a domain for the stratum.

(3)
$$\hat{Y}_{h}(d) = \left(\frac{N_h}{n_h}\right) \sum_{i}^{n_h} \hat{Y}_{hi}(d)$$

Equation (4) provides the variance for the population total derived with Equation (3).

(4)
$$var\left(\hat{Y}_{h}(d)\right) \cong \left(\frac{N_h^2}{n_h}\right)\left(1 - \frac{n_h}{N_h}\right) s^2(d)$$

Equation (5) provides the sample variance, for use in Equation (4).

$$s^2(d) = \frac{\left(\hat{Y}_{hi}(d) - \left(\frac{\hat{Y}_h(d)}{N_h}\right)\right)^2}{(n_h - 1)} \quad (5)$$

Equation (6) provides the standard error for the population total derived with Equation (3).

$$se\left(\hat{Y}_h(d)\right) = \sqrt{var\left(\hat{Y}_h(d)\right)} \quad (6)$$

Equation (7) provides the per hectare value for the population total derived with Equation (3).

$$\hat{y}_h(d) = \frac{\hat{Y}_h(d)}{A_h} \quad (7)$$

Equation (8) provides the variance for the per hectare value derived with Equation (7).

$$var(\hat{y}_h(d)) \cong \frac{var\left(\hat{Y}_h(d)\right)}{A_h^2} \quad (8)$$

Equation (9) provides the standard error for the per hectare value derived with Equation (7).

$$se(\hat{y}_h(d)) = \sqrt{var(\hat{y}_h(d))} \quad (9)$$

Combined Management Units

Equation (10) provides the population total for a variable of interest within a domain for all strata combined.

$$\hat{Y}(d) = \sum_h^H \hat{Y}_h(d) \quad (10)$$

Equation (11) provides the variance for the population total derived with Equation (10).

(11) $$var\left(\hat{Y}(d)\right) \cong \sum_{h}^{H} var\left(\hat{Y}_h(d)\right)$$

Equation (12) is the sample variance, for use in Equation (11).

(12) $$se\left(\hat{Y}(d)\right) = \sqrt{var\left(\hat{Y}(d)\right)}$$

Equation (13) provides the per hectare value for the population total derived with Equation (10).

(13) $$\hat{y}(d) = \frac{\sum_{h}^{H} \hat{Y}_h(d)}{\sum_{h}^{H} A_h}$$

Equation (14) provides the variance for the per hectare value derived with Equation (13).

(14) $$var\left(\hat{y}(d)\right) \cong \frac{var\left(\hat{Y}(d)\right)}{\left(\sum_{h}^{H} A_h\right)^2}$$

Equation (15) provides the standard error for the per hectare value derived with Equation (13).

(15) $$se(\hat{y}(d)) = \sqrt{var(\hat{y}(d))}$$

Domain Proportion

Notation

w_{hi}	Tree level weight for trees in a stand (*i*) in a stratum (*h*) to account for the probability of selection in the second stage of sampling
m_{hi}	Number of plots in a stand (*i*) in a stratum (*h*)
a	Area, in hectares, of the fixed radius plots. For the SDM survey, it is 0.005 ha.
A_{hi}	Area, in hectares, of a stand (*i*) in a stratum (*h*)
$\hat{X}_{hi}(d)$	Population total of the variable (tree count, basal area or volume) in the domain (*d*) for a stand (*i*) for a stratum (*h*)
$\hat{Y}_{hi}(d)$	Population total of the variable (tree count, basal area or volume) with a property of interest, such as a particular disturbance agent or dead/live, in the domain (*d*) for a stand (*i*) for a stratum (*h*)

N_h	Number of stands in a stratum (*h*) that belong to the target population
n_h	Number of stands in a stratum (*h*) that belong to the target population that were sampled
$\hat{X}_h(d)$	Population total of the variable (tree count, basal area or volume) in the domain (*d*) for a stratum (*h*)
$\hat{Y}_h(d)$	Population total of the variable (tree count, basal area or volume) with a property of interest in the domain (*d*) for a stratum (*h*)
$\hat{r}_h(d)$	Proportion of trees with a property of interest in the domain (*d*) for a stratum (*h*)
$var(\hat{r}_h(d))$	Variance of the proportion of trees with a property of interest in the domain (*d*) for a stratum (*h*)
$s_h^2(d)$	Sample variance of the ratio estimate in the domain (*d*) for a stratum (*h*)
$se(\hat{r}_h(d))$	Standard error of the proportion of trees with a property of interest in the domain (*d*) for a stratum (*h*)
$\hat{X}(d)$	Population total of the variable (tree count, basal area or volume) in the domain (*d*) for all strata combined
$\hat{Y}(d)$	Population total of the variable (tree count, basal area or volume) with a property of interest in the domain (*d*) for all strata combined
$\hat{r}_c(d)$	Proportion of trees with a property of interest in the domain (*d*) for all strat combined
$var(\hat{r}_c(d))$	Variance of the proportion of trees with a property of interest in the domain (*d*) for all strata combined
$e_{hi}(d)$	Difference for a variable between the actual property of interest and the property of interest predicted with the combined ratio estimator for all trees (*i*) in the domain (*d*) for a stratum (*h*)
$\bar{e}_h(d)$	Average difference for a variable between the actual property of interest and the property of interest predicted with the combined ratio estimator for all trees (*i*) in the domain (*d*) for a stratum (*h*)
$se(\hat{r}_c(d))$	Standard error of the proportion of trees with a property of interest in the domain (*d*) for all strata combined

Individual Management Units

Equation (16) provides a tree value weight for trees within a stand to account for the probability of selection for trees in the second stage of sampling. This is the same as Equation (1) for domain totals.

$$w_{hi} = \frac{A_{hi}}{(m_{hi} * a)} \tag{16}$$

Equation (17) provides the population total for a variable of interest (stem count, basal area or volume) in a domain for a stand for a stratum.

$$\hat{X}_{hi}(d) = w_{hi} \sum_{j}^{m_{hi}} X_{hij}(d) \tag{17}$$

Equation (18) provides the population total for a variable of interest (stem count, basal area or volume) with a particular attribute of interest (disturbance agent or dead/live) in the domain for a stand for a stratum.

$$\hat{Y}_{hi}(d) = w_{hi} \sum_{j}^{m_{hi}} Y_{hij}(d) \tag{18}$$

Equation (19) provides the population total for a variable of interest in the domain for a stratum.

$$\hat{X}_{h}(d) = \left(\frac{N_h}{n_h}\right) \sum_{i}^{n_h} \hat{X}_{hi}(d) \tag{19}$$

Equation (20) provides the population total for a variable of interest with a particular attribute of interest in the domain for a stratum.

$$\hat{Y}_{h}(d) = \left(\frac{N_h}{n_h}\right) \sum_{i}^{n_h} \hat{Y}_{hi}(d) \tag{20}$$

Equation (21) provides the domain estimate for a ratio, such as the proportion of trees with a particular disturbance agent, within a domain for a stratum.

$$\hat{r}_{h}(d) = \frac{\hat{Y}_{h}(d)}{\hat{X}_{h}(d)} \tag{21}$$

Equation (22) provides the variance estimate for the ratio derived with Equation (21).

$$var\left(\hat{r}_h(d)\right) \cong \left(\frac{1}{\hat{X}_h(d)}\right)^2 * \left(1 - \frac{N_h^2}{n_h}\right) * s_h^2(d) \quad (22)$$

Equation (23) provides the sample variance of the ratio estimate for use in Equation (21).

$$s_h^2(d) = \frac{\sum_i^{n_h} \left(\hat{Y}_{hi}(d) - \left(\hat{r}_h(d) * \hat{X}_{hi}(d)\right)\right)^2}{n_h - 1} \quad (23)$$

Equation (24) provides the standard error for the ratio derived with Equation (21).

$$se(\hat{r}_h(d)) = \sqrt{var\left(\hat{r}_h(d)\right)} \quad (24)$$

Combined Management Units

Equation (25) provides the population total for a variable of interest (stem count, basal area or volume) in a domain for a stand for all strata combined.

$$\hat{X}(d) = \sum_h^H \hat{X}_h(d) \quad (25)$$

Equation (26) provides the population total for a variable of interest (stem count, basal area or volume) with a particular attribute of interest (disturbance agent or dead/live) in the domain for a stand for all strata combined.

$$\hat{Y}(d) = \sum_h^H \hat{Y}_h(d) \quad (26)$$

Equation (27) provides the domain estimate for a combined ratio estimator, such as the proportion of trees with a particular disturbance agent, within a domain for all strata combined.

$$\hat{r}_c(d) = \frac{\hat{Y}_h(d)}{\hat{X}_h(d)} \quad (27)$$

Equation (28) provides the variance estimate for the combined ratio estimator derived with Equation (27).

(28) $$var\left(\hat{r}_c(d)\right) \cong \frac{1}{\hat{X}(d)^2}\sum_{h}^{H}\frac{N_h^2}{n_h}\left(1-\frac{n_h}{N_h}\right)s_h^2(d)$$

Equation (29) provides the sample variance of the combined ratio estimate for use in Equation (28).

(29) $$s_h^2(d) = \frac{\sum_i^{n_h}\left(e_{hi}(d)-\bar{e}_h(d)\right)^2}{n_h-1}$$

Equation (30) provides the difference for a variable between the actual property of interest and the property of interest predicted with the combined ratio estimator for all trees (*i*) in the domain (*d*) for a stratum (*h*) for use in Equation (29).

(30) $$\bar{e}_h(d) = \frac{1}{n_h}\sum_{i}^{n_h}e_{hi}(d)$$

Equation (31) provides the difference for a variable between the actual property of interest and the property of interest predicted with the combined ratio estimator for all trees (*i*) in the domain (*d*) for a stratum (*h*) for use in Equation (29).

(31) $$e_{hi}(d) = \hat{Y}_{hi}(d) - \hat{r}_c(d)\cdot\hat{X}_{hi}(d)$$

Equation (32) provides the standard error for the combined ratio estimator derived with Equation (27).

(32) $$se(\hat{r}_c(d)) = \sqrt{var\left(\hat{r}_c(d)\right)}$$

References

Cochran WG. (1977) *Sampling techniques – third edition*. John Wiley and Sons Inc.

Lohr SL, (2009) *Sampling: design and analysis – second edition*. Pacific Grove CA. Brooks/Cole.