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Modeling the Dependency between Extreme Prices of Selected Agricultural Products on the Derivatives Market Using the Linkage Function

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Abstract: The purpose of the article is to identify and estimate the dependency model for the extreme prices of agricultural products listed on the Chicago Mercantile Exchange. The article presents the results of the first stage of research covering the time interval 1975–2010. The selected products are: Corn, soybean and wheat. The analysis of the dependency between extreme price values on the selected futures was based on the estimation of five models of two-dimensional extreme value copulas, namely, the Galambos copula, the Gumbel copula, the Husler–Reiss copula, the Tawn asymmetric copula and the *t*-EV copula. The next stage of the analysis was to test whether the structure of the dependency described with the estimated copulas is a sufficient approximation of reality, and whether it is suitable for modeling empirical data. The quality of matching the estimated copulas to empirical data of return rates of agricultural products was assessed. For this purpose, the Kendall coefficient was calculated, and the methodology of the empirical combining function was used. The conducted research allowed for the determination of the conduct for this kind of phenomena as it is crucial in the process of investing in derivatives markets. The analyzed phenomena are highly dependent on e.g., financial crises, war, or market speculation but also on drought, fires, rainfall, or even crop oversupply. The conducted analysis is of key importance in terms of balancing agricultural production on a global scale. It should be emphasized that conducting market analysis of agricultural products at the Chicago Mercantile Exchange in the context of competition with the agricultural market of the European Union is of significant importance.

Keywords: agricultural product; price; modeling; management

1. Introduction

Sustainable agricultural production is crucial for balancing the needs of current and future populations. The benefits of modern agriculture are significant, allowing food production go hand in hand with an increase in the population [1,2]. Cooperation between business and the agricultural production circles are of particular interest at both the national and international level [3,4].

Demographic forecasts indicate that by 2050, the world's population will reach 9.7 billion. This will lead to an increase in demand for food, further aggravating environmental problems related to intensive agricultural production. For this reason, one of the main challenges is to achieve global food security and sustainable agriculture. While food security aims at ensuring sustainable and healthy food supply over time, sustainable agriculture plays a key role in maintaining resilient agro-ecosystems [5]. The above-mentioned concept of sustainable farming is fostered by ecological farming. Its development is accompanied by a high growth rate of the organic food market and a significant growth of sales of organic food. Even with strict adherence to production practices and increasing their availability, most consumers still are unaware of organic production methods. The customers' awareness of organic products does not necessarily translate into real consumption [6,7].

According to the United States Department of Agriculture (USDA), the projected global wheat trade in the 2019/2020 season is expected to increase by 6.7 million tonnes, i.e., by 4% and reach 184.6 million tonnes in the case of larger export deliveries and lower expected export prices.

The global perspective of the feed grain market in the 2019/2020 season relates to record production and consumption, as well as lower final inventories. Global production of corn is projected to increase, mostly in the USA, South Africa, Russia, Canada, India, and Brazil, compensating for slightly lower crops in China and Ukraine. The global use of corn is expected to increase by 1%, while global corn imports are expected to increase by 2%. Global soy production in the 2018/2019 season increased in relation to the May forecast by 0.7 million tonnes, reaching 355.2 million tonnes [8–11].

In the decision-making processes, as well as in many scientific disciplines, the study of the dependency of extreme values is of great importance. In many situations, their proper identification enables avoiding wrong investment decisions. Studying the dependency between extreme prices of agricultural products is of key importance in the process of analyzing the structure of dependency. This is especially true when it is important to model the dependency between maintaining the maximum and minimum values of the analyzed observations. Dependencies for extremely small or extremely large values may have different characteristics than those determined based on the entire sample. The reason for fluctuations in the prices of agricultural products are, e.g., financial crises, wars, market speculation, but also droughts, fires, rains, or even crop oversupply. For this reason, the difference in price behavior during periods of extreme change, i.e., unusual and rarely observable market events require careful monitoring because it can lead to above-average losses or profits in the process of investing on derivatives markets.

A very useful tool in the process of modeling dependencies between extreme values of time series is the functions of relations of extreme values, also called extreme value copulas. Their main advantage is the possibility of analyzing above-average losses or profits in the field of finance, insurance, but also in the case of examining futures for agricultural products.

One of the first applications of the theory of extreme value based on a two-dimensional copula appeared in the subject literature in 1964 in the study of Gumbel and Goldstein [12]. These copulas were also used in many other fields of science, e.g., in the process of analyzing the return rates of exchange rates [13], the study of capital markets characterized by high volatility [14], in the portfolio analysis and portfolio risk estimation [15] or in the field of insurance [16], and in hydrology [17].

Extreme value copulas were created as possible limit copulas in relation to the maximum i.i.d. (independent and identically distributed) distribution, i.e., a distribution in which all random variables are independent and have the same distribution of probability. Since two-dimensional copulas were used in the empirical part of this article, the basic definitions and conclusions regarding extreme value copulas presented below will be demonstrated for the two-dimensional case. More detailed information on extreme value copulas, together with an extension for a multidimensional case can be found, e.g., in a monograph by R.B. Nelsen [18], a monograph by H. Joe [19], books by J. Segers [20] and G. Gudendorf and J. Segers [21], or in the following papers: C. Genest and J. Seger [22], M. Ribatet and M. Sedki [23].

As a supplier of raw food materials, the agricultural market is of strategic importance from the point of view of food security. Therefore, it is so important to strive towards risk-reducing solutions, including models, and inference based on the statistical analyzes, as exemplified by those proposed by the authors.

In order to tackle the growing challenges of sustainable agricultural production, a better understanding of complex agricultural ecosystems is needed. This can be done using modern digital technologies that constantly monitor the physical environment, producing large amounts of data at an unprecedented rate. The analysis of the big data will enable farmers and enterprises to leverage its value and improve their efficiency. Although the analysis of large data sets leads to progress in various industries, it is not yet widely used in agriculture [24,25].

2. Materials and Methods

The source of data used for the analysis in the empirical part of the paper is the Chicago Mercantile Exchange. The choice of this exchange was motivated by its importance in the international market of agricultural products. The research sample was created using daily quotations of nominal prices of futures for three agricultural products, i.e., corn, wheat and soybean. The value of the contract is expressed in the price of a bushel, the transaction unit of a commodity in US dollars. The data includes the closing price of the contract with the shortest expiration date, so that a series of quotations could be treated as a forward price with the shortest possible execution time. The choice of products was dictated by the importance of trading on the futures market and the availability of appropriately long time series. Empirical data come from the years 1975–2010. A single time series includes 9070 observations. They have been checked for possible discontinuities and errors. In order to minimize the impact of arbitrary interference on the obtained results, no procedures have been applied to correct or supplement empirical data. The selected combining function is one, for which the distance from the empirical combining function is the smallest.

2.1. Theoretical Models of Extreme Value Copulas, the Inference Functions for Margins (IFM) Estimation Method, and the Empirical Combining Function Method

The notion of the linkage function, also often called the copula function, appeared first in subject literature in 1959 in the work of A. Sklar [26]. However, it was not until the 1990s and the monograph of H. Joe [19] and R.B. Nelsen [18] that these functions have gained immense popularity, mainly due to the presentation of their broad practical application in dependence modeling. In the general, and at the same time the simplest view, the copula is a function that allows distinguishing the component describing only the structure of dependence from the cumulative distribution function of the total distribution of a random vector. In other words, it can be said that the copula functions are functions that combine a multidimensional cumulative distribution function of a random variable with its one-dimensional limit cumulative distribution function [19–26].

Among the copula functions, one can distinguish their particularly important class, namely, the copula of extreme values [27–33]. The use of two-dimensional extreme value copulas in empirical studies has been greatly simplified by using the representation introduced into the subject literature by J. Pickands in 1981 [34]. This issue was also an essential element of research conducted in 1977 by A.A. Balkema and S.I. Resnick [35] and L. de Haan and S. I. Resnick [36]. Observations of the extremes were classified in accordance with the Extreme Value Theory (EVT), which is used to describe the behavior of limit properties of extreme values. It consists of analyzing the tails of distributions, which constitute only a small part of the entire distribution examined. Its purpose is not to describe the usual behavior of stochastic phenomena, but the unusual and rarely observed events. The Extreme Value Theory is widely used wherever modeling relationships between the behaviors of the maximum and minimum values of the analyzed observations is particularly important [37–39]. The classic approach for modeling extreme values is based on the block maxima model. This method is used for a large number of observations that have been selected from a large sample. Modeling the behavior of the extreme values of independent random variables with an identical probability distribution is in fact using the maxima or minima of the observations in fixed time blocks. These blocks are designated by means of separate time intervals of equal length, most often months, quarters or years [30].

The next part of this article will present extreme values copulas used in empirical research.

2.1.1. Theory of Extreme Copulas

Among the copula functions, one can distinguish a particularly important class, i.e., the extreme value copula. A copula is a function that allows to distinguish a component describing only the structure of dependence from a total random vector distribution. In other words, the copula functions are functions that combine a multidimensional cumulative distribution of a random variable with its one-dimensional limit distributors [40].

The definition of a copula for a two-dimensional case is as follows:

A two-dimensional copula (2-copula) is each function $C : [0, 1]^2 \to [0, 1]$ that meets the following conditions:

For each $u_1, u_2 \in [0, 1]$ there is:

$$C(u_1, 0) = C(0, u_2) = 0 \tag{1}$$

1. For each $u_1, u_2 \in [0, 1]$ there is:

$$C(u_1, 1) = u_1 \text{ and } C(1, u_2) = u_2 \tag{2}$$

2. For each $u_1, u_2, v_1, v_2 \in [0, 1]$, such that $u_1 \leq u_2$ and $v_1 \leq v_2$, there is:

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \tag{3}$$

For basic information in terms of modeling the price dependence of futures on agricultural products listed on the Chicago Mercantile Exchange using the copula function, let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be independent pairs of random variables with the same distributions and common copula C. Moreover, let $C_{(n)}$ be a copula with respect to the maximum components, namely $X_n = \max\{X_i\}$, $Y_n = \max\{Y_i\}$, $i = 1, \ldots, n$. From Sklar's theorem [26], it follows that $C_{(n)}(u, v) = C^n\left(u^{\frac{1}{n}}, v^{\frac{1}{n}}\right)$ for every $u, v \in [0, 1]$ The limit of the sequence $\left\{C_{(n)}\right\}$ naturally leads to the concept of an extreme value copula. Therefore, the copula C_E is called an extreme value copula if there is a copula C, such that:

$$\forall\, u, v \in [0, 1] \;\; C_E(u, v) = \lim_{n \to \infty} C^n\left(u^{\frac{1}{n}}, v^{\frac{1}{n}}\right) \tag{4}$$

2.1.2. Galambos' Copula

This copula was introduced to subject literature in 1975 by J. Galambos [41]. It is an example of a symmetrical copula, described with the formula below:

$$C_\theta^{Ga}(u, v; \theta) = uv \exp\left\{\left[(-\ln u)^{-\theta} + (-\ln v)^{-\theta}\right]^{-1/\theta}\right\} \tag{5}$$

where $\theta > 0$.

The dependence function of the discussed copula is presented using the following formula:

$$A^{Ga}(t) = 1 - \left(t^{-\theta} + (1 - t)^{-\theta}\right)^{-1/\theta} \tag{6}$$

2.1.3. The Gumbel Copula

This copula was introduced to literature in 1960 by E.J. Gumbel and is also called the Gumbel–Hougaard copula [42]. It is described by the following formula:

$$C_\theta^{Gu}(u, v; \theta) = \exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}\right\} \tag{7}$$

where $\theta \geq 1$.

The independence function is described by the formula:

$$A^{Gu}(t) = \left(t^\theta + (1-t)^\theta\right)^{1/\theta} \tag{8}$$

It can be observed that for $\theta \to \infty$ there is an excellent dependence, while independence occurs in the case of $\theta = 1$.

2.1.4. The Husler–Reiss Copula

The name of the copula comes from its authors, who presented it for the first time in a paper in 1989 [43]. The Husler–Reiss copula is defined by the following formula:

$$C_\theta^{HR}(u, v; \theta) = \exp\left\{-\widetilde{u}\Phi\left[\frac{1}{\theta} + \frac{1}{2}\theta\ln\left(\frac{\widetilde{u}}{\widetilde{v}}\right)\right] - \widetilde{v}\Phi\left[\frac{1}{\theta} + \frac{1}{2}\theta\ln\left(\frac{\widetilde{v}}{\widetilde{u}}\right)\right]\right\} \tag{9}$$

wherein $\theta \geq 0$, $\widetilde{u} = -\ln u$, $\widetilde{v} = -\ln v$. On the other hand, Φ is a cumulative distribution function of a standardized normal distribution.

The dependence function of the discussed copula is determined by the formula:

$$A^{HR}(t) = t\Phi\left[\theta^{-1} + \frac{1}{2}\theta\ln\left(\frac{t}{1-t}\right)\right] + (1-t)\Phi\left[\theta^{-1} - \frac{1}{2}\theta\ln\left(\frac{t}{1-t}\right)\right] \tag{10}$$

while the parameter θ for the Husler–Reiss copula measures the degree of dependence. This means that starting from $\theta = \infty$, one is dealing with independence until the situation in which $\theta = 0$, which brings total dependence.

2.1.5. Tawn Copula

Introduced in the subject literature in 1988 by J. Tawn, this copula is an asymmetric extension of the Gumbel cupola [44]. The model of the discussed copula is presented below:

$$C_\theta^{Ta}(u, v; \theta) = uv\exp\left\{\theta\frac{(\ln u)(\ln v)}{\ln u + \ln v}\right\} \tag{11}$$

where $0 \leq \theta \leq 1$.

In turn, the independence function of the Tawn copula is given by the formula:

$$A^{Ta}(t) = \theta t^2 - \theta t + 1 \tag{12}$$

2.1.6. The *t*-EV Copula

This copula is known as the *t*-Student extreme copula. For a two-dimensional case, the λ degrees of freedom and the correlation coefficient $\rho \in (-1, 1)$, the *t*-EV copula is determined by the following formula:

$$C_{\lambda,\rho}^{tEV}(u, v; \rho) = \exp\left\{T_{\lambda+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u}{\ln v}\right)^{1/\lambda}\right]\ln u + T_{\lambda+1}\left[-\frac{\rho}{\theta} + \frac{1}{\theta}\left(\frac{\ln u}{\ln v}\right)^{1/\lambda}\right]\ln v\right\} \tag{13}$$

where T_λ is a cumulative *t*-Student distribution function with λ degrees of freedom, while the parameter θ depends on the value of the correlation coefficient ρ and the degrees of freedom λ, and is described by the equation: $\theta^2 = \frac{1-\rho^2}{\lambda+1}$ [40].

The function of *t*-Student extreme dependence copula is presented using the following formula:

$$A^{tEV}_{\lambda,\rho}(t) = tT_{\lambda+1}\left(\frac{\left(\frac{t}{1-t}\right)^{1/\lambda} - \rho}{\sqrt{1-\rho^2}}\sqrt{1+\lambda}\right) + (1-t)T_{\lambda+1}\left(\frac{\left(\frac{1-t}{t}\right)^{1/\lambda} - \rho}{\sqrt{1-\rho^2}}\sqrt{1+\lambda}\right) \tag{14}$$

Estimation of five models of two-dimensional extreme value copulas was carried out based on the two-stage method of maximum likelihood, i.e., the IFM method. It is the inference function method for limit distributions, also called the two-stage maximum likelihood estimation method due to the two-stage estimation process. It was proposed for the first time in the works of H. Joe and J.J. Xu [27] and H. Joe [19]. The authors suggested to divide the set of estimated parameters into two subsets, so as to first estimate the parameters associated with the limit distributions, and then find the estimator responsible for the combined distribution, i.e., the copula. The empirical combining function method is used to assess the quality of matching the parameters of the extreme value copula [35–37]. The principle of selecting an appropriate combining function boils down to selecting the best function from the finite set of candidates, which is a subset of the set of all possible combining functions. There are various ways to define the empirical combining function. In this paper, a practical formula based on the values of the combining function defined on the grid will be used [45]. Since the price series of financial instruments belong to the group of non-stationary processes, daily constant return rates were determined based on the price series to conduct statistical analysis between the prices of the agricultural products. The use of logarithmic return rates is meaningful for the properties of the data series under investigation. As one of the Box–Cox transformations, logarithmation is known to stabilize the variance of the series [38].

$$R_t = \ln\left(\frac{X_t}{X_{t-1}}\right) \tag{15}$$

where X_t is the value of the futures on the day t.

Based on the daily closing prices, the logarithmic return rates for individual agricultural products were calculated, in accordance with the formula presented above.

2.1.7. The Empirical Combining Function Method

This method is used to assess the quality of matching the parameters of the extreme value copula, based on the empirical link function [39–44]. Let the theoretical structure of the dependence be described by the combining function C_θ, dependent on the value of the parameter θ. It is then assumed that the null hypothesis H_0 means that the structure of the dependence is determined using the combining function C_θ, against the alternative hypothesis H_1, which is a negation of hypothesis H_0. The principle of selecting the appropriate combining function comes down to selecting the best function from the finite set of candidates C, which is a subset of the set of all possible combining functions. The combining function selected from the set C is that for which the distance from the empirical copula is the smallest. There are various ways to define the empirical copula. In this work a practical formula was used, based on the values of the combining function defined on the following grid:

$$L = \left\{\left(\frac{i}{m}, \frac{j}{m}\right) : i, j = 0, 1, \ldots, m\right\} \tag{16}$$

The empirical values of the combining function at the points of the L-grid are determined by the following formula:

$$C_m\left(\frac{i}{m}, \frac{j}{m}\right) = \frac{1}{m}\sum_{k=1}^{m} \mathbf{1}(R_k \le i)\mathbf{1}(S_k \le j) \tag{17}$$

where R_i, S_i are ranks of variables X and Y.

On the other hand, the distance measured between the combining functions is based on the standard L^2, and in the discrete version takes the form:

$$d_L(C_m, C) = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{m}\left(C_m\left(\frac{i}{m}, \frac{j}{m}\right) - C\left(\frac{i}{m}, \frac{j}{m}\right)\right)^2} \tag{18}$$

where:
C_m—empirical copula,
C—function of the copula from the distinguished set of copulas C.

2.1.8. Kendall's Correlation Coefficient

Empirical studies often use rank-based correlation coefficients. One of the most popular is the Kendall rank correlation coefficient. It is usually used when there are restrictions related to the use of Pearson's linear correlation coefficient. Its advantage is that, as a representative of the measure of compatibility, it does not depend on limit distributions. Although the limit distributions and the correlation matrix do not determine the form of the combined distribution, similar to the linear correlation coefficient, for any continuous edge distributions, a combined distribution with a given rank correlation coefficient can still be constructed from the entire range $[-1; 1]$.

In order to define the Kendall rank correlation coefficient, a less formal definition of compatibility should first be introduced, as follows [46].

Two different realizations (x_1, y_1) and (x_2, y_2) of the random vector (X, Y) are compatible if

$$(x_1 - x_2)(y_1 - y_2) > 0 \text{ (i.e., } x_1 > x_2 \text{ and } y_1 > y_2\text{, or } x_1 < x_2 \text{ and } y_1 < y_2\text{)}$$

and non-compatible, if

$$(x_1 - x_2)(y_1 - y_2) < 0 \text{ (i.e., } x_1 > x_2 \text{ and } y_1 < y_2 \text{ or } x_1 < x_2 \text{ and } y_1 > y_2\text{)}$$

Let (X_1, Y_1) and (X_2, Y_2) be independent random vectors with identical distribution. Kendall's correlation coefficient τ is defined as follows:

$$\tau(X_1, Y_1) = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0] \tag{19}$$

Using the above terminology, it can be concluded that the Kendall's coefficient for the vector (X_1, Y_1) is the probability of compatible realizations of the random vector (X_1, Y_1) minus the likelihood of realizing the non-compatible of the vector.

However, if $\{(x_1, y_1), \ldots, (x_n, y_n)\}$ is an n-element sample of a vector (X, Y) of continuous random variables, then there are $\binom{n}{2} = \frac{n(n-1)}{2}$ different pairs (x_i, y_i) and $\left(x_j, y_j\right)$ that are compatible or inconsistent. The Kendall factor τ for the sample version, which is designated to be distinguished from the theoretical coefficient through $\hat{\tau}$, can be calculated based on the following formula:

$$\hat{\tau} = \frac{c - d}{c + d} \tag{20}$$

where c is the number of compliant pairs, and d the number of incompatible pairs.

3. Results

Sustainability is a human-centered concept that comprises multiple aspects and objectives of different interest groups. It is not readily measurable, except as a compromise between different parts of society, of which some may try to represent future generations of mankind [47]. In order to analyze the dependence of extreme price values on selected futures listed on the Chicago Mercantile Exchange, five selected models of two-dimensional extreme values copulas defined in the previous chapter, were used. As said before, the following copulas were used in empirical research: the Gumbel copula, the Galambos copula, the Husler–Reiss copula, the *t*-EV copula and the Tawn asymmetric copula. The estimation of parameters of selected extreme values copulas was carried out based on the two-stage maximum likelihood estimation method, i.e., the IFM method for two different limit distributions, namely, for the normal distribution and the *t*-Student distribution. The selection of distributions was motivated by the results recommending their use, presented widely in the subject literature [48–52]. Results for each of the three agricultural product pairs in question are presented in Tables 1 and 2. In addition to the values of the estimated parameters and their average estimation errors given in brackets, the likelihood function (LLF) values can also be found there. In addition, the extreme t -Student dome has an additional parameter λ, responsible for the number of degrees of freedom, the estimations of which are also included in the tables.

Table 1. Results of the estimation of the parameters of extreme values copulas with the normal limit distribution.

Copula	Parameter	Maize-Soy	LLF	Corn-Wheat	LLF	Soybean-Wheat	LLF
Galambos	θ	0.897 *** (0.013)	1831	0.674 *** (0.011)	1031	0.571 *** (0.010)	693.3
Gumbel	θ	1.628 *** (0.013)	1871	1.411 *** (0.011)	1050	1.316 *** (0.010)	710.2
Hüsler–Reiss	θ	1.268 *** (0.018)	1711	1.039 *** (0.012)	981.9	0.924 *** (0.011)	660.9
Tawn	θ	0.928 *** (0.009)	1883	0.758 *** (0.013)	1033	0.633 *** (0.015)	686.4
t-EV	ρ	0.786 *** (0.012)	1885	0.672 *** (0.018)	1053	0.589 *** (0.016)	711.8
	λ	4.000 ** (1.876)		4.000 ** (1.882)		4.000 ** (1.843)	

* significance at the level of 10%, ** significance at the level of 5%, *** significance at the level of 1%. Source: [own study].

Table 2. Results of the estimation of the parameters of extreme values copulas with *t*-Student limit distribution.

Copula	Parameter	Corn–Soybean	LLF	Corn–Wheat	LLF	Soybean–Wheat	LLF
Galambos	θ	0.895 *** (0.013)	1824	0.673 *** (0.011)	1028	0.570 *** (0.010)	691.9
Gumbel	θ	1.626 *** (0.013)	1864	1.411 *** (0.011)	1047	1.315 ** (0.010)	708.9
Hüsler-Reiss	θ	1.267 *** (0.018)	1706	1.038 *** (0.012)	977.1	0.923 *** (0.011)	658.8
Tawn	θ	0.927 *** (0.009)	1875	0.757 *** (0.013)	1032	0.633 *** (0.015)	685.5
t-EV	ρ	0.785 *** (0.012)	1878	0.671 *** (0.018)	1051	0.588 *** (0.016)	710.9
	λ	4.000 ** (1.876)		4.000 ** (1.882)		4.000 ** (1.843)	

* significance at the level of 10%, ** significance at the level of 5%, *** significance at the level of 1%. Source: [own study].

The picture emerging from the analysis of the results presented in Tables 1 and 2 shows that very similar results have been obtained irrespective of the adopted limit distribution. The copula, which best describes the relationship between the extreme values of all researched agricultural product pairs, is the extreme *t*-Student copula. When comparing the obtained logarithm values of the likelihood function for individual copulas, it can also be observed that the Tawn copula may be useful for describing the relationship between extreme observations for the corn–soybean pair. A minor difference in the logarithm values of the credibility function between the *t*-EV copula and the Tawn copula, may suggest that the pair corn–soybean is characterized by a small degree of asymmetry, with more observations deviating in plus.

On the other hand, for pairs of agricultural products—corn–wheat and soybean–wheat—the second copula of extreme values, which may be helpful to describe the studied dependencies, with small differences in the value of the credibility function, turned out to be the Gumbel copula. Please note that in the case of an extreme values Gumbel copula, the parameter θ takes on values greater than or equal to 1; the higher the value of this parameter, the stronger the dependence of maximum losses between the analyzed observations. The results given in Tables 1 and 2 indicate a minor correlation between the extreme observations of the researched pairs of return rates of agricultural products quoted on the Chicago Mercantile Exchange. This means that unlike stock exchange indices for which, as it is well known and documented in extensive empirical studies, the occurrence of extremely negative return rates is much more likely than of extremely high rates. In the case of futures, one is not dealing with such strong dependence. The obtained results may rather point to a possible symmetry or very minor asymmetry in the tails of the return rates for the analyzed agricultural product pairs.

The next stage of the analysis was to test whether the structure of the relationship described with the estimated extreme values copulas is a sufficient approximation of reality, and whether it is suitable for modeling empirical data. In order to assess the quality of matching the estimated copulas to the empirical data on return rates of agricultural products, the methodology presented [37–44] was used. It says that one of the methods of checking the quality of matching the copula parameters is to compare the coefficients implied by the selected copula with empirical Kendall coefficients $\hat{\tau}$. Estimation of the Kendall coefficient τ was obtained for all extreme copulas using a simulation method. The results are presented in Tables 3 and 4. On the other hand, Table 5 contains the values of the Kendall correlation coefficient $\hat{\tau}$ calculated for the sample version. All estimated correlations are positive and statistically significant. Regardless of the pair of agricultural products, the strongest dependence is demonstrated by the pair corn–soy, while the weakest dependence is characterized by the pair soybean–wheat. Estimates of the Kendall coefficient for all extreme value copulas were obtained using the simulation method. The results are presented in Tables 3 and 4. Table 5, in turn, contains the values of the Kendall correlation coefficient calculated for the sample version.

Table 3. Values of Kendall's tau coefficient in the case of normal limit distribution.

Type of Copula	Corn–Soybean	Corn–Wheat	Soybean–Wheat
The Galambos' copula	0.382	0.289	0.236
The Gumbel copula	0.386	0.291	0.240
The Husler–Reiss copula	0.357	0.271	0.223
The Tawn copula	0.387	0.290	0.239
The *t*-EV copula	0.390	0.293	0.242

Source: [own study].

Table 4. Values of Kendall's tau coefficient in the case of *t*-Student distribution.

Type of Copula	Corn–Soybean	Corn–Wheat	Soybean–Wheat
The Galambos' copula	0.381	0.288	0.235
The Gumbel copula	0.385	0.291	0.239
The Husler-Reiss copula	0.356	0.270	0.222
The Tawn copula	0.386	0.289	0.239
The *t*-EV copula	0.388	0.293	0.241

Source: [own study].

Table 5. Sample Kendall correlation coefficients between the daily return rates of the surveyed agricultural products.

	Corn	Soybean	Wheat
Corn	1.000	0.417	0.332
Soybean		1.000	0.270
Wheat			1.000

Source: [own study].

The comparison of the empirical values of Kendall coefficients with the corresponding theoretical values demonstrates that for all three pairs of agricultural products, for both applied limit distributions, the best studied extreme dependences is described by the *t*-EV copula. In the case of the pair corn–soybean, the Tawn asymmetrical copula may also be helpful, while for the pairs corn–wheat and soybean–wheat, the Gumbel copula may also be useful. Please also note the Tawn copula, as the Kendall tau coefficient for this copula is quite satisfactory, and compared to the best-rated extreme *t*-Student copula, the differences are minimal.

An alternative method used to assess the quality of matching the parameters of the extreme value copula, is based on an empirical combining function. In order to choose the best copula function, the estimated parameters of the extreme value combining function were compared to the empirical combining function defined for the grid for $m = 303$. The selection of this particular grid size is a natural reference to the analyzed time series, which 303 sub-periods correspond to the subsequent months. The tables below demonstrate the distances of the estimated extreme value copulas from the empirical combining functions, depending on the selected limit distribution.

Analysis of the results listed in Tables 6 and 7 confirms the results obtained to assess the goodness of the adjustment of the extreme value copulas obtained with a methodology using the Kendall coefficient τ. The minor difference between the distances from the empirical joining function for the Tawn copula and the t-Student extreme copula proves that both functions may prove equally useful to describe the relationship between extreme values of all researched agricultural product pairs, regardless of the adopted limit distribution. In order to capture the dependence between extreme observations for the pair corn–soybean, the *t*-EV copula is the best, but the Gumbel copula may turn out equally important. On the other hand, for the pair corn–wheat and soybean–wheat, the Tawn copula seems the most accurate, while the *t*-EV copula ranks second. However, the differences are minor and they result from the selection of the size of the grid.

Table 6. Values of distance from the empirical combining function for selected extreme value copulas for normal limit distribution.

	Corn–Soybean	Corn–Wheat	Soybean–Wheat
The Galambos' copula	0.221	0.211	0.188
The Gumbel copula	0.209	0.197	0.174
The Husler–Reiss copula	0.285	0.252	0.219
The Tawn copula	0.218	0.180	0.163
The *t*-EV copula	0.203	0.195	0.174

Source: [own study].

Table 7. Values of distance from the empirical combining function for selected extreme value copulas for *t*-Student limit distribution.

	Corn–Soybean	Corn–Wheat	Soybean–Wheat
The Galambos' copula	0.222	0.211	0.188
The Gumbel copula	0.210	0.197	0.174
The Husler–Reiss copula	0.286	0.252	0.219
The Tawn copula	0.218	0.179	0.162
The *t*-EV copula	0.203	0.195	0.173

Source: [own study].

4. Conclusions

Sustainable agriculture is a key issue for environmentally friendly agriculture: Effective, economically viable and socially desirable. In addition, conservation of resources, environmental protection and agricultural stewardship, i.e., all requirements of sustainability, will increase and not reduce global food production. Other issues, such as the links between sustainable agriculture and the rest of the food and agri-food industry, as well as the consequences of sustainable development for rural communities and society as a whole, have not yet been finally resolved [53].

The empirical results demonstrated that the extreme values copula, which best describes the dependence between the extreme values of all researched pairs of agricultural products, irrespective of the limit distribution adopted, is the extreme *t*-Student, a.k.a. the *t*-EV copula. Moreover, in the case of the pair corn–soybean, the Tawn copula may be useful, and for corn–wheat and soybean–wheat, the Gumbel copula. The obtained results were confirmed by the assessment of the goodness of matching the parameters of the extreme value copulas. It can be observed that the results obtained in order to check the goodness of matching the estimated copula to empirical data of return rates of agricultural products reflect the results obtained in the estimation process. The final conclusion is that the return rate of the analyzed agricultural product pairs may be characterized by a minor degree of asymmetry, with the right tail being particularly heavy, which means higher probability of extreme observations than in the case of normal distribution.

The economic determinants of agricultural production have become the main reason for both producers and agri-food processors to search for exchange-based instruments to reduce the risk associated with adverse changes in product prices. A tool that is perfectly suited for this purpose is futures, which allow transferring the risk relatively cheaply to third parties. Thanks to their effectiveness, futures markets started to increase their turnover steadily. Participation of entities operating on the agricultural products market became common in countries with advanced stock exchange systems, at the same time becoming an irreplaceable link of production, processing, and trade in the agri-food industry. Analyzing agricultural markets is particularly important in terms of the need for ideation and creation of key development strategies for the agricultural sector, both at global and EU level, including the coordinated goals of the Common Agricultural Policy (CAP) [54–57].

In conclusion, our review highlights the vast possibilities of analyzing large data sets in agriculture towards optimizing management decisions based on statistical modeling. It demonstrates that the availability, techniques, and methods for analyzing large data sets, as well as the growing openness of large data sources, will encourage more academic research, public sector initiatives, and business ventures in the agricultural sector. This practice is still at an early stage of development and many barriers have to be overcome.

In practice, the results allow for rationalization of decisions of companies interested in intervention on the futures market for agricultural products, because investing there not only reduces the risk of financial losses, but also generates a high return rate.

The use of these models allows the simulation of the possible behavior of prices of agricultural products, including identification of extreme deviations. This allows programming decisions regarding possible public intervention in agricultural markets. In extraordinary situations—force majeure, weather anomalies, natural disasters—decision modeling can be used to assess the scale of damage for the agricultural market, and also to develop plans for the protection of agricultural producers within the framework of policies such as the CAP. As a consequence, they allow for securing food resources on deficit markets, e.g., with respect to geographical location.

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