

Article

# Optimizing Bus Line Based on Metro-Bus Integration

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**Abstract:** Metros are usually built and added on the basis of a completed bus network in Chinese cities. After the metro construction, it is faced with the problem of how to adjust and optimize the original bus lines based on the new metro system. This research mainly proposes a bus line optimization method based on bus and metro integration. In the consideration of the geographical space, the cooperation and competition relationship between bus and metro lines is qualitatively introduced according to the geographical location and service range of metro (800 m radius) and bus (500 m radius) stations. The competition and cooperation indexes are applied to define the co-competition relationship between bus and metro lines. The bus line optimization model is constructed based on the co-competition coefficient and Changsha Metro Line Number 2 is chosen as a case study to verify the optimization model. The results show that the positive competition, efficient cooperation, and travel efficiency between metro and bus has been significantly enhanced after optimization. Moreover, this paper provides a reasonable reference for public transport network planning and resource allocation.

**Keywords:** bus line; metro station; competition; cooperation; co-competition

## 1. Introduction

Due to their large capacity reliability, metros have been constructed in many big cities to alleviate serious traffic congestion in China. In most cases, metros were built and added upon the completed and complex bus network. The interactions will come out on the operation of the original public transport network after the metro construction. In order to improve the overall operating efficiency, metro and bus lines should be integrated into one unified system. This research discusses the development of bus and metro integration in urban cities, which belongs to the field of sustainable transportation network planning and facilities optimization. The achievements can not only improve the service level of urban public transportation, but also enhance the urban transportation sustainability.

These two modes of transportation play quite different roles in the modern public transport system. Generally, the main features and challenges of metro-based bus line adjustment include:

- Because of its high speed and capacity, the goal of planning and constructing metro is to construct a rapid and large-capacity transport corridor in the city, which is more pre-leading and independent than the bus line.
- Related bus lines along the metro stations can be divided into lines of feeder, supplementation, and competition. It is faced with adjusting those buslines to maximize the comprehensive network efficiency.

- In the optimization of metro-based bus lines, it is usually conducted from the perspective of the entire network, which has a massive workload and is not specific; or it is conducted from a separate bus line, which has less comprehensiveness. There are methods such as truncation, merging, and encountering, which lack both theoretical basis and standard guidance.
- People believe that there is always a perfect coordination between metro and bus, but it is difficult to achieve in practice.
- The competition and cooperation between metro and public transportation is of vital importance in the integrated public transportation network. However, most researchers only qualitatively describe the competitive relationship and mechanism of metro and bus transit. How to quantify the competition and cooperation relationship between these two systems requires further development research.

Bus and metro lines are connected to each other at the metro station in practice. Thus, we believe that the geographical space of metro station is the most basic and important element for metro and bus integration. This research focuses on metro station-based bus line optimization instead of redesigning a new entire network, which can reduce workload and improve operability simultaneously. The main contribution of this study lies in proposing a quantitative assessment for the competition and cooperation indexes between bus and metro, and developing a mathematical modeling framework for optimizing localized integration between metro and bus system based on the co-opetition coefficient. For demonstrating the practical significance of the proposed method, a case study based on the Changsha Metro Line 2 shows that the service level and total travel cost can be improved considerably by the proposed localized integration between metro and bus systems.

This remainder of the paper is organized as follows. Section 2 reviews relevant papers in the literature and highlights the research gap. Section 3 defines the cooperation and competition between metro and bus lines. Section 4 provides a quantitative way to calculate the cooperation and competition indexes. Section 5 develops a modeling framework for optimizing bus line based on metro stations. A case study based on a real-world metro system is conducted in Section 6. Finally, Section 7 draws discussions and conclusions, as well as suggests future research directions.

## 2. Literature Review

### 2.1. Competition and Cooperation in Public Transport Network

As the two common modes of transportation, it is particularly important to study the relationship between metro transit and public transport. There are many studies focus on bus line optimization models based on the metro network. Chowdhury et al. [1–3] significantly reduced the transfer time of metro lines and branch bus lines. Akiva et al. [4–6] pointed out that metro and bus with similar service will attract the same passengers, which means that high-performance bus services are likely to become alternatives to metro services. Upon transport mode selection, Ivanova [7] constructed the model which maximized the indirect personal utility of the initial place based on the way passengers travel to their destination. Verma [8,9] developed a model for developing the best bus lines on feeder lines within the framework of integrated public transport planning for urban metro transit stations, then Verma proposed another model for developing the best integrated plan for urban metro and transit operation, including train dispatching sub-models and dispatching coordination sub-models. Kuan et al. [10–12] evaluated the efficiency and quality of heuristic methods that applied to several randomly generated test problems. The results showed that heuristic methods can provide a good solution for the optimal design of the metro transit bus network in a reasonable time. Schmöcker et al. [13,14] introduced the “boarding failure” probability processing line capacity limitation problem and proposed a bus scheduling model based on dynamic frequency. In a multi-to-multi-demand urban traffic corridor, Chien et al. [15,16] developed an optimization model for regional transportation systems between the joint metro transit and feeder bus system. In the perspective of technical and economic characteristics.

Nayeem et al. [17–19] has established a model based on population growth that still satisfied the maximum passengers, the minimum transfers, and the minimum travel time.

## 2.2. Coordination and Optimization of Rail and Public Transport

Researchers have achieved rich results in the field of coordination and optimization. Shrivastava et al. [20,21] applied heuristic algorithms to develop rail transit bus lines to meet the demand from rail stations, while meeting the minimum break-even point for economic operations. Bielli et al. [22,23] proposed a new method to calculate fitness function value for bus network optimization by using a genetic algorithm to generate a new iterative population (bus network set). Lam et al. [24–27] put forward the analysis of rail transit and bus transfer service optimization system. The results show that the joint implementation of intermodal public transport and other plans can significantly improve the performance of the social multimodal transport system. Mohaymany et al. [28–31] enabled the network design to have various capacity and performance, and then introduced an ant colony algorithm to find better routes, which can attract more private car users to use public transport. Alshalafah et al. [32–36] studied the feasibility and advantages of using flexible route services instead of fixed connecting bus lines, and selected three lines in the suburb of Toronto with regional commuter rail lines for simulation. The result showed that it is significant to allocate appropriate slack time to provide effective flexible line service. Dijosephl et al. [37–39] developed an optimization algorithm to search for the maximal-profits optimal solution by considering the public transport network and passenger demand, as well as optimized the regional public transport system and financially sustainable operation.

To sum up, the research on bus route optimization based on rail transit has attracted the attention of traffic researchers and managers. In terms of rail station classification, researchers paid less attention to the classification based on the co-opetition performance between rail and bus transit. Most researchers focused on the cooperation, while the research on the competition is rare, or only from the macro perspective to discuss and compare the attractiveness between rail and bus transit.

## 3. Definitions of Bus-Metro Cooperation and Competition

Cooperation and competition are common in urban metro and bus systems. When they compete for limited public resources, they will form a competitive relationship. When they assist with each other, they will form a cooperative relationship. In the consideration of the geographical space and route layout of metro and bus lines, this paper mainly studies the network integration based on the competition and cooperation relationship between metro and bus systems.

### 3.1. Cooperation

Due to the large space and the limited radiation range of each metro station, it is still necessary to configure a bus line  $B$  with a short distance across or partially parallel along the metro line  $M$ . The main function of these bus lines is to provide convenience for the passengers' last miles. In addition, these buses can also play the role of diversion to alleviate the problem of peak flow and congestion in metro transit. The bus line  $B$  is to provide passenger flow for each metrostation transit, and it can connect metro stations in suburban areas with a low radiation range of metro network. It is mainly to provide a shuttle service for passengers from the residential area to the nearby metro station, and it can also be used as a community bus to meet the needs of some passengers for short-distance travel. Therefore, in Figure 1, there is a cooperation between the metro line  $M$  and bus line  $B$ .

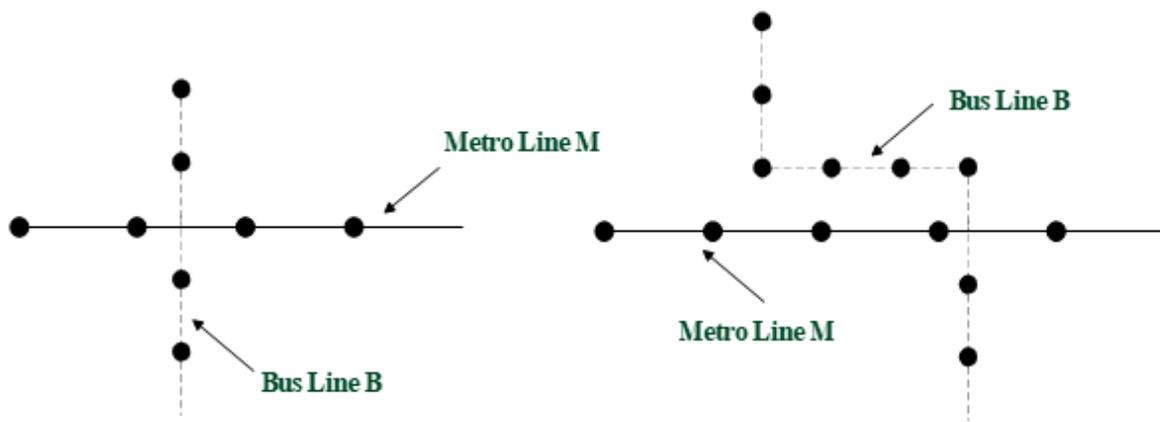


Figure 1. Cooperation diagram.

### 3.2. Competition

Generally, the service radius of the metro station is 800 m, and the service radius of the bus station is 500 m. In Figure 2, the origin and destination of bus line are located near the metro station within the 800 m range, and they are completely parallel to metroline *M*. The bus line *B* also undertakes a large number of medium and long-distance transportation with a large travel span. As bus line *B* has accumulated a large number of passenger flow, after the operation of metro line *M*, these two lines will compete with each other in passenger flow if there is a lack of coordination.

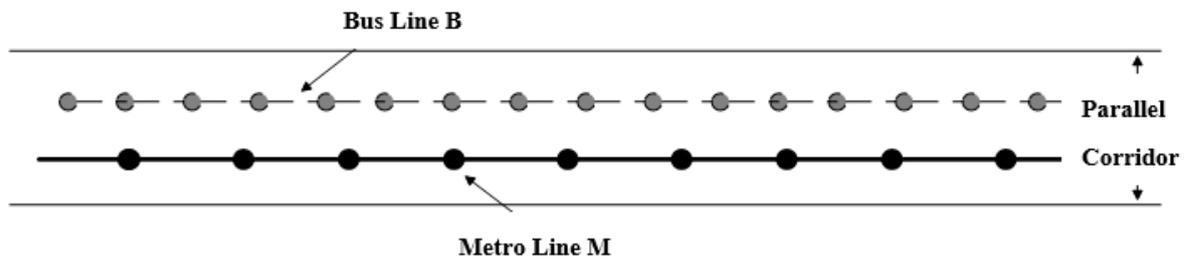


Figure 2. Competition diagram.

## 4. Cooperation and Competition Indexes

### 4.1. Cooperation Index

To explore the relationship between metro and bus lines, we should focus on the service scope of metro stations, such as the number of bus stations, the number of bus lines passing through the metro station, and the density of bus lines within the service scope of metro stations. Those factors have an inevitable relationship with attracting and competing for passenger flow. In this paper, the service radius of the metro station is 800 m, and the service radius of the bus station is 500 m. Therefore, the number of overlap stations and overlap areas can be calculated. The cooperation index can quantitatively describe the cooperation between metro and bus lines. The number of cooperation stations between metro and bus lines is used to measure the transfer possibility between two lines. Therefore, the ratio of the actual cooperation station number to the total station number can be defined as the index of cooperation, which is expressed as follows:

$$T_{coo} = (T_M - N)(T_B - N) \tag{1}$$

$$T_{BM} = (T_M - 1)(T_B - 1). \tag{2}$$

Based on Equations (1) and (2), the cooperation index can be expressed as:

$$coo = \frac{T_{coo}}{T_{BM}} = \frac{(T_M - N)(T_B - N)}{(T_M - 1)(T_B - 1)} \quad (3)$$

where  $coo$  is the cooperation index between bus line  $B$  and metro line  $M$ ;  $T_{coo}$  is the total number of actual cooperation stations;  $T_{BM}$  is the total number of bus and metro stations;  $N$  is the number of overlap stations;  $T_B$  is the total number of bus line stations;  $T_M$  is the total number of metro line stations.

#### 4.2. Competition Index

The ratio of the actual competition station number to the total number of stations, is defined as the competition index, which is expressed as follows:

$$com = \frac{T_{com}}{T_{BM}}, \quad (4)$$

where  $com$  is the competition index between bus line  $B$  and metro line  $M$ ;  $T_{com}$  is the number of actual competition stations;  $T_{BM}$  is the total number of bus and metro stations. For a more complex competitive line that has both complete overlap stations and partial overlap stations, assuming that the number of overlap station is  $N$ , according to the direction of the track line, denoted overlap station as  $i$ , where  $i = 1, 2, \dots, n$ . Based on Equation (4), the number  $T_{com}$  of competition stations in complex competitive lines can be calculated, that is,

$$T_{com} = \sum_{i=1}^N \sum_{j=1}^{N-i} \left[ \frac{\Delta S_i}{S_b} \frac{\Delta S_{i+j}}{S_b} \frac{\min(L_b^{i,i+j}, L_m^{i,i+j})}{\max(L_b^{i,i+j}, L_m^{i,i+j})} \right] \quad (5)$$

$$com = \frac{T_{com}}{T_{BM}} = \frac{2 \sum_{i=1}^N \sum_{j=1}^{N-i} \left[ \frac{\Delta S_i}{S_b} \frac{\Delta S_{i+j}}{S_b} \frac{\min(L_b^{i,i+j}, L_m^{i,i+j})}{\max(L_b^{i,i+j}, L_m^{i,i+j})} \right]}{(T_M - 1)(T_B - 1)} \quad (6)$$

where  $\Delta S_i$  is overlap area of overlap station number  $i$ ;  $\Delta S_{i+j}$  is overlap area of overlap station number  $(i+j)$ ;  $S_b$  is service area of a single bus station;  $L_b^{i,i+j}$  is the bus station travel distance between station number  $i$  and  $(i+j)$ ;  $L_m^{i,i+j}$  is metro station travel distance between station number  $i$  and  $(i+j)$ .

#### 4.3. Co-Opetition Index

The information of Changsha Metro Line 2 stations has collected through the AMap application interface, which include the station longitudes and latitudes, bus station information within 800 m range, and the specific bus lines passing through the metro station. The indexes of competition and cooperation are quantitatively analyzed based on these data. The 2D Figure is illustrated for the competition and cooperation indexes between 245 bus lines and Metro Line 2 (see Figure 3). In Figure 3, there are 12 bus lines have competition with Changsha Metro Line 2, which are represented by red bubbles; 76 bus lines have both competition and cooperation with Changsha Metro Line 2, which are represented by black bubbles; the remaining 157 bus lines have cooperation with Changsha Metro Line 2, which are represented by green bubbles.

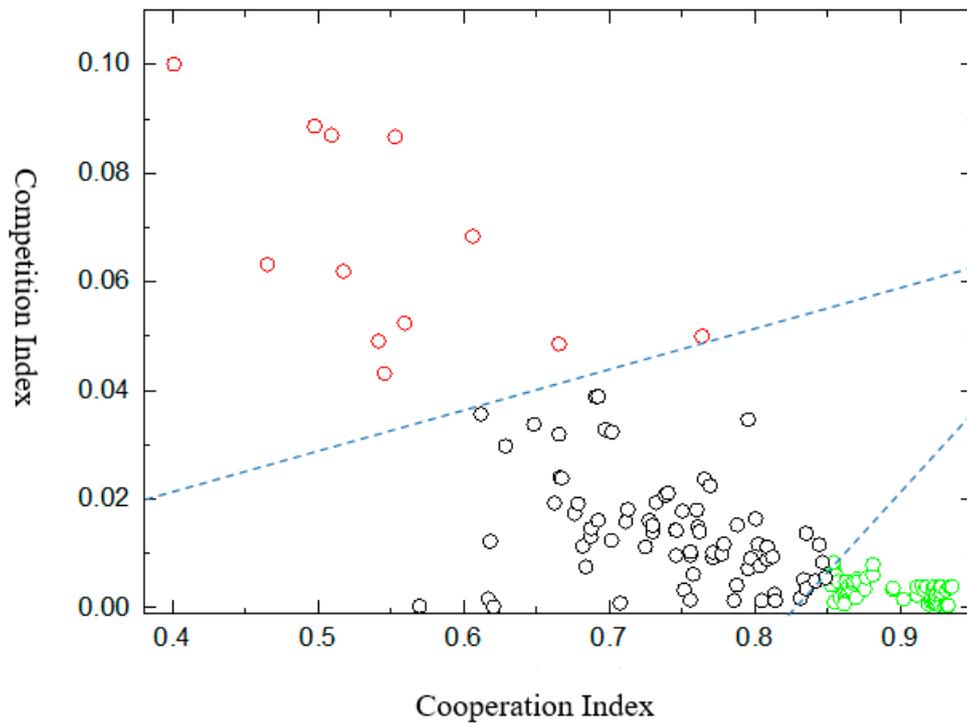


Figure 3. Competition and cooperation between 245 bus lines and Changsha Metro Line 2.

### 5. Bus Line Optimization Based on Integration

#### 5.1. Optimization Strategy and Objective

The optimization method proposed in this paper is based on the metro and bus competition and cooperation indexes in the previous chapter. The total passenger travel cost (travel cost + travel time \* time value) is taken as the objective function. The length of the bus line and the non-linear coefficient are taken as constraints. The genetic algorithm is applied in the optimization model. The basic flow chart is shown in Figure 4.

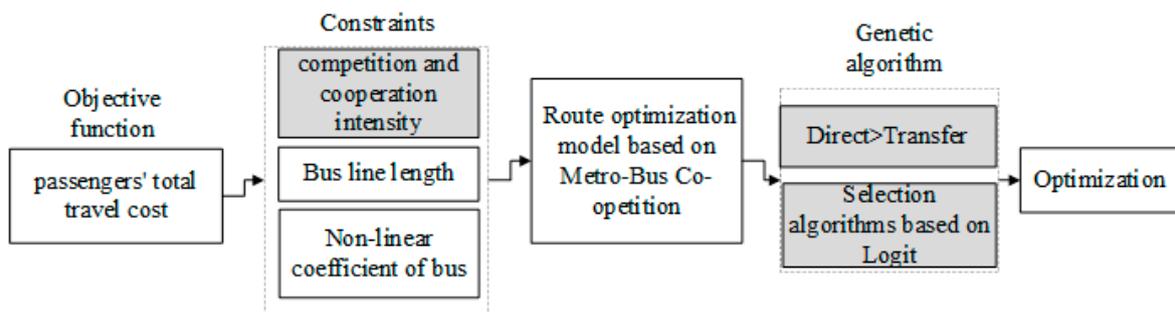


Figure 4. Flow chart of the optimization method.

Passengers give priority to travel time and travel cost when traveling, so these two factors are mainly concerned. The dual-objective optimization problem is also converted into one single-objective optimization problem. The generalized cost function  $U$  is used to establish passenger travel costs, including direct costs and indirect costs, i.e., travel economic costs  $F$  and travel time costs  $P$ . Shown as follow:

$$U = \alpha_1 F + \alpha_2 \gamma P \tag{7}$$

$$P = \frac{D}{V_0} + P_h + \frac{L_b}{V_b} + P_{hc} + \frac{L_m}{V_m} \tag{8}$$

$$P = \begin{cases} \frac{D}{V_0} + P_h + \frac{L_b}{V_b} & \text{if passengers take the directbus} \\ \frac{D}{V_0} + P_h + \frac{L_m}{V_m} & \text{if passengers take the directmetro} \end{cases} \quad (9)$$

$$F = \begin{cases} 2 + k + f \times (L_m - L_0) & \text{bus + metro} \\ 2 & \text{bus} \\ k + f \times (L_m - L_0) & \text{metro} \end{cases} \quad (10)$$

Here,  $\alpha_1, \alpha_2$  is the weight coefficient, which can be obtained through investigation;  $\gamma$  is travel time value;  $D$  is walking distance at both destinations of the passenger. Considering the service range of bus and metro, which are 500 m and 800 m.  $V_0$  is walking speed 4.4 km/h;  $P_h$  is passengers' waiting time, which is half the departure frequency (bus departure frequency is 10 mins, metro departure frequency is 6 mins).  $L_b$  is the distance between passenger and the bus station;  $V_b$  is average bus speed 20 km/h;  $P_{hc}$  is passenger transfer time;  $L_m$  is the distance between passenger and the metro station;  $V_m$  is average metro speed;  $k$  is ticket price;  $f$  is ticket price level;  $L_0$  is travel distance covered by the price.

Constraints

(1) Co-opetition Coefficient

After transposing the horizontal and vertical coordinates of Figure 3, the connection between the bubble (bus line) and the original coordinate on the horizontal axis (competition) forms an angle  $\alpha$ , which reflects the relative index of competition and cooperation (see Figure 5). The cosine of angle  $\alpha$  represents the co-opetition coefficient between the integrated metro and bus lines.

$$\omega_{BM} = \frac{com}{\sqrt{(com)^2 + (coo)^2}} \quad (11)$$

where  $\omega_{BM}$  is the co-opetition coefficient between bus line **B** and metro line **M**; *com* is competition index between bus line **B** and metro line **M**; *coo* is cooperation index between bus line **B** and metro line **M**. A box chart is a statistical chart used to display scattered data. The chart has five statistics values: minimum, first quartile, median, third quartile and maximum. The third quartile ( $Q_3$ ) and the first quartile ( $Q_1$ ) of the relevance from all lines are selected as the upper and lower limits.

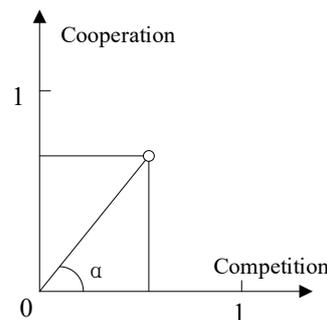


Figure 5. Co-opetition coefficient of metro and bus lines.

(2) Non-linear coefficient of the bus line

$$\left[ \frac{l_B}{d_B} \right] \leq \left[ \frac{l}{d} \right]_{max} = 1.4 \quad (12)$$

Here,  $l_B$  is the length of bus line **B** (km);  $d_B$  is the straight-line distance between the bus line **B** origin and destination (km).

(3) Bus line length

$$l_{min} \leq l_B \leq l_{max} \quad (13)$$

Here,  $l_{min}$  is the lower limit of line length 8 km;  $l_{max}$  is the upper limit of line length 12 km.

## 5.2. Optimization Model

Taking the passenger's total travel cost as the objective function, thus, the minimum value is the optimal travel plan. Metro line is considered as a fixed line in the calculation because of its limited adjustment capacity. The lines between OD points are taken as elements to form a set. According to the set of matrix  $X$  combination, the assorted pattern is marked as  $r$ , and the  $s$  short line in the assort pattern is marked as  $s$ , so that the matrix columns are composed of  $r$  and  $s$ .

$$X_{rs} = \begin{cases} 1 & \text{there is a bus on the shortest path} \\ 0 & \text{there is not a bus on the shortest path} \end{cases} \quad (14)$$

Passengers' total travel cost  $F$  is:

$$F = \sum_k \sum_w q_{kw} F_{kw}(X) \quad (15)$$

Here,  $q_{kw}$  is passenger volume between area  $k$  and  $w$ ;  $F_{kw}(X)$  is bus travel cost between area  $k$  and  $w$  in co-opetition network. Therefore, the line optimization model is established:

$$\text{s.t. } 8 \text{ km} \leq l_B \leq 12 \text{ km} \quad (16)$$

$$\left[ \frac{l_B}{d_B} \right] \leq 1.4 \quad (17)$$

$$Q_1 \leq \omega_{BM} \leq Q_3 \quad (18)$$

The genetic algorithm is chosen to optimize the localized road network which enclosed by the metro and bus lines. The fitness function is determined according to the transformation of the objective function. Regardless of the secondary transfer, the priority of the line is set as: direct line > transfer line. Meanwhile, metro transit is regarded as a fixed line and participates in the calculation of passenger volume distribution. When there are multiple travel lines with the same priority level, it is judged according to the travel efficiency. The Logit model is applied to calculate the passenger route selection probability of passenger selected line.

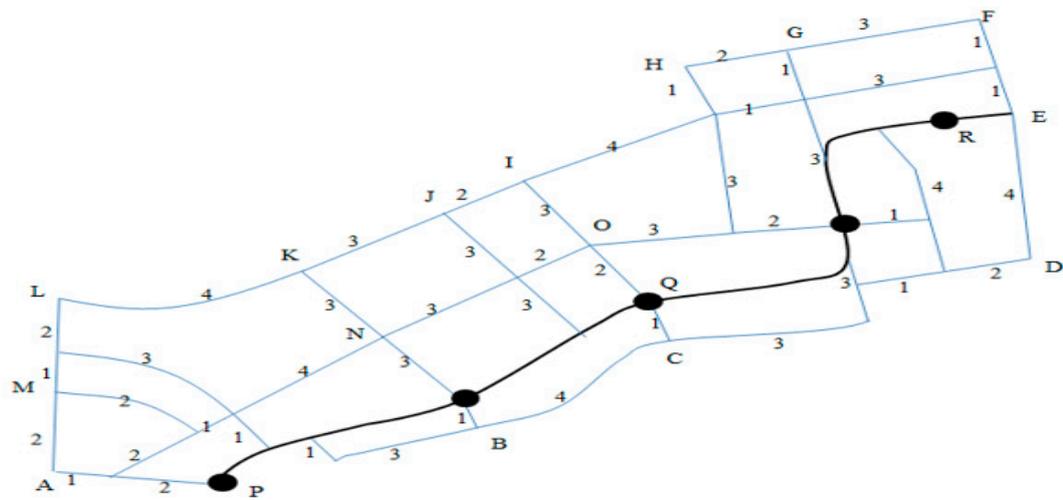
$$P_{BM}^n = \frac{\exp\left(-\frac{\theta F_n}{F}\right)}{\sum_{c=1}^m \exp\left(-\frac{\theta F_c}{F}\right)} \quad (19)$$

Here,  $P_{BM}^n$  is passenger route selection probability;  $F_n$  is travel cost of line  $n$ ;  $F$  is average travel cost;  $\theta$  is distribution parameter;  $m$  is the total number of travel lines.

## 6. Case Study

### 6.1. Basic Data

Changsha Metro Line 2 has a total length of 21.68 km with 23 stations. The line starts at West Meixi Lake Station and ends at Guangda Station, which is going from east to west. According to the metro station service range mentioned above, this paper takes the local road network between Wangchengpo Station, Culture-Art Center Station, West Meixi Lake Station and the bus line within 800 m service range as an example. Figure 6 shows the road network of the selected section. The black line (P-Q-R) in the diagram represents Metro line 2 and the number indicates the travel time (min) of the bus. According to the survey results of the resident's average annual income, the value of travel time is 0.6 Yuan/min.



- A - Dongfanghong RD    B - Huanhu RD    C - Meixi Lake RD    D - West Second Ring    E - 2<sup>nd</sup>Fenglin RD
- F - West Xianjia Lake RD    G - Lujing RD    H - Fenglin RD    I - Lugu Avenue    J - Luhong RD
- K - Luyun RD    L - 3<sup>rd</sup>Fenglin RD    M - Mufeng RD    N - Xiufeng RD    O - Linshui RD
- Q - Cultural Arts Center    R - West Xiangjiang River Comprehensive Transportation Hub

Figure 6. Network of Wangchengpo Station to West Meixi Lake Station.

According to the land using type around Wangchengpo and Meixi Lake Station, and the index of trip rate in “Technical Guidelines for Traffic Impact Assessment of Construction Projects in Changsha City”, the OD matrix is obtained by using TransCAD software. In the example, the C, F, K, M, and R areas are set as the origin and destination station. Because the P–R has a metro line, the terminal pairing processing is not necessary to perform. According to the pairing test through route length and non-linear coefficient, the results are shown in Table 1.

Table 1. Current situation of areas.

| Pair | Route       | Linear Distance (km) | Route Length (km) | Non-Linear Coefficient | Shortest Path Sequence |
|------|-------------|----------------------|-------------------|------------------------|------------------------|
| C-F  | C-Q-R-E-F   | 2.49                 | 3.48              | 1.4                    | 1                      |
|      | C-D-E-F     | 2.49                 | 3.58              | 1.44                   | 2                      |
|      | C-O-I-G-F   | 2.49                 | 3.77              | 1.51                   | 3                      |
| C-K  | C-B-N-K     | 1.62                 | 2.27              | 1.4                    | 1                      |
|      | C-O-I-K     | 1.62                 | 2.31              | 1.43                   | 2                      |
| C-M  | C-B-P-A-M   | 2.84                 | 3.31              | 1.17                   | 1                      |
|      | C-Q-O-N-M   | 2.84                 | 3.57              | 1.25                   | 2                      |
|      | C-Q-P-A-M   | 2.84                 | 3.95              | 1.39                   | 3                      |
|      | C-I-J-L-M   | 2.84                 | 4.21              | 1.48                   | 4                      |
| C-R  | C-Q-R       | 1.97                 | 2.58              | 1.31                   | 1                      |
|      | C-D-E-R     | 1.97                 | 3.05              | 1.55                   | 2                      |
| F-K  | F-I-K       | 3.27                 | 3.57              | 1.1                    | 1                      |
|      | F-H-I-K     | 3.27                 | 3.62              | 1.11                   | 2                      |
|      | F-E-O-N-K   | 3.27                 | 4.6               | 1.41                   | 3                      |
|      | F-E-R-Q-N-K | 3.27                 | 5.46              | 1.67                   | 4                      |

Table 1. Cont.

| Pair | Route       | Linear Distance (km) | Route Length (km) | Non-Linear Coefficient | Shortest Path Sequence |
|------|-------------|----------------------|-------------------|------------------------|------------------------|
| F-M  | F-I-K-L-M   | 4.7                  | 5.47              | 1.16                   | 1                      |
|      | F-H-I-K-L-M | 4.7                  | 5.52              | 1.17                   | 2                      |
|      | F-E-Q-P-A-M | 4.7                  | 6.84              | 1.46                   | 3                      |
| F-R  | F-E-R       | 0.65                 | 0.84              | 1.29                   | 1                      |
|      | F-G-R       | 0.65                 | 3.11              | 4.8                    | 2                      |
| K-M  | K-L-M       | 1.48                 | 1.92              | 1.3                    | 1                      |
|      | K-N-M       | 1.48                 | 2.36              | 1.6                    | 2                      |
|      | K-N-P-A-M   | 1.48                 | 3.2               | 2.16                   | 3                      |
| K-R  | K-I-E-R     | 2.93                 | 3.72              | 1.27                   | 1                      |
|      | K-N-O-R     | 2.93                 | 3.72              | 1.27                   | 2                      |
|      | K-N-Q-R     | 2.93                 | 4.7               | 1.6                    | 3                      |
| M-R  | M-N-O-R     | 4.29                 | 4.99              | 1.16                   | 1                      |
|      | M-A-P-Q-R   | 4.29                 | 5.8               | 1.35                   | 2                      |
|      | M-L-I-E-R   | 4.29                 | 5.69              | 1.33                   | 3                      |

## 6.2. Results

Trough the OD matching mode test of line length and non-linear coefficient, the maximum length of bus line is 6 km, the minimum length is 2 km, the distance of metro transit P-O is 2.6 km, and the distance of O-R is 2.3 km. The average speed is 35 km/h, so the average driving time of P-O is 4 min and O-R is 4 min too. In the example, the average transfer time is set to 3 min. the number of buses on the road network is 20, the initial value of the population is 30. Meanwhile, the crossover probability is set to 0.95 and the mutation probability is set to 0.1 for genetic operation by using Java language. The number of iterations is determined by formula 5.22–5.26. The initial population is determined by a random generation method, the roulette method is selected to perform the operation, and the iteration is repeated 20 times. The program is run five times under the same conditions, and the average is taken as the optimization result. The optimized scheme is shown in Table 2.

Table 2. Line optimization scheme.

| Combination | Original Line | Original Non-Linear Coefficient | Optimized Line  | K Shortest Line | Optimized Non-Linear Coefficient |
|-------------|---------------|---------------------------------|-----------------|-----------------|----------------------------------|
| C-F         | C-O-I-G-F     | 1.51                            | C-Q-R-E-F       | 1               | 1.4                              |
| C-M         | C-Q-P-A-M     | 1.39                            | C-Q-P-A-M       | 1               | 1.21                             |
| C-R         | C-D-E-R       | 1.55                            | C-Q-R           | 1               | 1.31                             |
| F-K         | F-E-R-Q-N-K   | 1.67                            | F-I-K           | 1               | 1.1                              |
| F-M         | F-E-R-Q-P-A-M | 1.46                            | F-I-K-L-M       | 1               | 1.16                             |
| F-R         | F-E-R         | 1.29                            | Cancel the line | /               | /                                |
| K-M         | K-N-P-A-M     | 2.16                            | K-L-M           | 1               | 1.3                              |
| K-R         | K-N-Q-R       | 1.6                             | K-I-E-R         | 1               | 1.27                             |
| M-R         | M-L-I-E-R     | 1.33                            | M-N-O-R         | 1               | 1.16                             |

## 6.3. Comparative Analysis

With an average travel cost of 9 Yuan, the total travel cost of passengers is 16,9236.18 Yuan after optimization, which is 37,608.28 Yuan less than before. In this case, the length of metroline is 4.9 km, the repetition length before the optimization is 12.1 km, and the repetition coefficient is 2.47. After optimization, the repetition distance is reduced to 1.6 km, and the repetition coefficient is reduced to

0.32. Table 3 gave the changes to other specific indicators. All indicators have been improved after optimization, and the overall benefit of the local road network has been upgraded.

**Table 3.** Comparison of indicators before and after optimization.

| Indicators                | Before Optimization | After Optimization                                  | Result       |
|---------------------------|---------------------|---|--------------|
| Service Level             | Comfortable         | More comfortable and enhanced connection efficiency | Improved     |
| Co-opetition Coefficient  | 0.2341              | 0.2154  | Decreased 8% |
| Total length of bus lines | 37.5 km             | 29.68 km  | Decrease 21% |
| Repeated length           | 12.1 km             | 4.6 km  | Decrease 62% |
| Total Travel Cost         | 206844.5 Yuan       | 169236.2 Yuan                                       | Decrease 19% |

According to the result, service level has improved, co-opetition relevance has decreased by 8%, total length of bus lines has decreased by 21%, repeated length has decreased by 62%, total travel expenses of passengers has decreased by 19%, passengers volume during peak hours has decreased, transfer ratio has increased, and the overall efficiency of local road networks has improved.

## 7. Discussion and Conclusions

It is an important task for urban public transportation to optimize and integrate the comprehensive transportation network after the metro construction, so that bus and metro can cooperate and give full play to their advantages. Most of the public transportation systems in China's big cities are complex large-scale networks. It is a very complex question to optimize the public transportation network after the completion of the metro. Researches on the traditional optimization of public transport network mostly consider the entire public transport network as a whole to pursue the overall benefits of the network by using a single time cost as the evaluation of the route benefits. However, it lacks the expression of the micro-level of the competition and cooperation between the bus and the metro lines, which leads to problems such as malignant competition between multi-bus lines and unreasonable competition between metro and bus lines in the actual public transport network. In order to solve these problems, this paper proposes a bus line optimization method based on metro-bus competition and cooperation, and it develops a quantitative assessment to evaluate the competition and cooperation between metro and bus. The optimization model is constructed based on the optimization method, and then it is applied to Changsha Metro Line 2 and related bus lines. For the evaluation of metro-bus competition and cooperation, we propose a mathematical modeling framework, which is more operational than the traditional qualitative analysis. Meanwhile, the competition and cooperation indexes are introduced into the bus line optimization, which can provide a guarantee for the coordinated operation of metro and bus, as well as a new optimization method for bus lines adjustment under the influence of metro.

The main contributions of this paper include:

First, this paper develops the cooperation and competition indexes between metro and bus lines. The indexes are applied to the case study of Changsha Metro Line 2, and it is found that there are 245 related bus lines with Metro Line 2, which include 12 lines with the competition, 157 lines with cooperation, and 76 lines with both competition and cooperation.

Furthermore, a bus line optimization model based on metro and bus co-opetition is proposed to find the best integration approach. It is capable of improving the flexibility of metro networks, as well as the complementary capacity of existing bus service. Different from the traditional network optimization method, we introduce the co-opetition coefficient into the network optimization model, which improves the single index only based on time efficiency.

Last but not least, the proposed optimization procedure has been tested by using the data of Changsha Metro Line 2. The results obtained from the case study suggest that the integration of metro and bus service can not only enhance significantly the performance of public transport service level and co-opetition degree, but also decrease the total length of bus lines, repeated length and total travel

cost. Moreover, the optimized integration network helps to reduce resource waste as well as giving reasonable passenger distribution.

Theoretically, the factors that affect the co-opetition degree between metro and bus include: the geographical space relation, the transport capacity, passenger demand distribution, etc. Therefore, the calculation becomes very complex and requires massive basic data. Considering the feasibility and data availability, this paper only calculates the co-opetition degree from the attributes of geographical space. Although the passenger flow demand and service capacity are not considered, it can still meet the needs of network layout planning and optimization.

The competition and cooperation model proposed in this paper is mainly based on the geographical space, which lacks consideration of bus line passenger flow demand and service capacity. For further research, we are interested in extending the model by introducing more parameters. Meanwhile, the current results are only limited to simulation experiments, which will be applied in engineering practice in the future.

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## Nomenclature

Sets:

|           |                               |
|-----------|-------------------------------|
| $T_B$     | set of bus line stations      |
| $T_M$     | set of metro line stations    |
| $T_{BM}$  | set of bus and metro stations |
| $T_{coo}$ | set of cooperation stations   |
| $T_{com}$ | set of competition stations   |
| $N$       | set of overlap stations       |

Parameters:

|                      |  |
|----------------------|--|
| $coo$                | cooperation index between bus line $B$ and metro line $M$        |
| $com$                | competition index between bus line $B$ and metro line $M$        |
| $\omega_{BM}$        | co-opetition coefficient between bus line $B$ and metro line $M$ |
| $\Delta S_i$         | overlap area of overlap station number $i$                       |
| $\Delta S_{i+j}$     | overlap area of overlap station number $(i+j)$                   |
| $L_b^{i,i+j}$        | travel distance between bus station number $i$ and $(i+j)$       |
| $L_m^{i,i+j}$        | travel distance between metro station number $i$ and $(i+j)$     |
| $P$                  | passenger travel time  |
| $P_h$                | passenger waiting time   |
| $P_{hc}$             | passenger transfer time  |
| $F$                  | passenger travel cost  |
| $\alpha_1, \alpha_2$ | weight coefficient   |
| $\gamma$             | travel time value  |
| $D$                  | passenger walking distance at both destinations                  |

|       |  |
|-------|--|
| $V_0$ | walking speed                                |
| $V_b$ | average bus speed                            |
| $V_m$ | average metro speed                          |
| $L_b$ | distance between passenger and bus station   |
| $L_m$ | distance between passenger and metro station |
| $L_0$ | travel distance covered by price             |
| $k$   | ticket price                                 |
| $f$   | ticket price level                           |

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