Minimal example

To better understand PROMETHEE, based on a minimal example the mathematical relations are presented. The discussed case study is the same as in the analysis of this paper on industrial hydrogen production in three different countries (Germany, Austria, Spain), but with only three indicators, i.e. LCA impact categories. The results of the LCA for the considered indicators and the discussed locations are listed in Table 1.

Table 1. Indicator results for the minimal example.

Impact category	DE	AT	ES
Acidification, mmol H^+ eq	44.5	21.6	50.3
Climate change, kg CO_2 eq	29.8	10.2	16.2
Resource depletion – Water, m^3 world-eq	23.6	23.9	43.9

Based on the data deviations between the different options are calculated (step 1). For each indicator a matrix is build (Table 2).

Table 2. Deviation between the different options with regard to the discussed indicators.

Acidification mmol H+ eq			
	DE	**AT**	**ES**
DE	0	22.9	-5.8
AT	-22.9	0	-28.7
ES	5.8	**28.7**	0

Climate change kg CO_2 eq			
	DE	**AT**	**ES**
DE	0	**19.6**	13.6
AT	-19.6	0	-6.0
ES	-13.6	6.0	0

Water, AWARE m^3 world-eq			
	DE	**AT**	**ES**
DE	0	-0.3	-20.3
AT	0.3	0	-20.0
ES	**20.3**	20.0	0

As can be seen the deviation in the indicator Water are very small for Germany and Austria. In order to avoid distorting the overall result, preference function (P_k) no. 5 is used. Here as default values for Q 5 % of the maximum deviation is chosen and for P 10 %. This results in the following values for the three indicators k (Table 3). The application of these functions to the minimum value the indicator.

Table 3. Absolute values for the parameters p and q for the minimal example for the preference functions P_k.

Impact category	q	p
Acidification, mmol H^+ eq	1.1	2.2
Climate change, kg CO_2 eq	0.5	1.1
Water, AWARE m^3 world-eq	1.2	2.4

It can be detected that between Austria and Germany with regard to water indifference prevails. That means that both options are equally good or bad in this indicator. With this in mind, the outranking matrices for the indicators can be created (Table 4), which are the results of the application of the preference functions $P_k(a,b)$ with a, b etc symbolizing the different options.

Table 4. Results of the preference function $P_k(a,b)$ for the discussed indicators (Outranking matrices A).

Acidification			
	DE	**AT**	**ES**
DE	0	0	1
AT	1	0	1
ES	0	0	0

Climate change			
	DE	**AT**	**ES**
DE	0	0	0
AT	1	0	1
ES	1	0	0

Water, AWARE			
	DE	**AT**	**ES**
DE	0	0	1
AT	0	0	1
ES	0	0	0

Based on this in the 3rd step the overall preference index is calculated, which again can be summarized in a matrix. The calculation is conducted according to equation 1.

$$\pi(a,b) = \sum_{k=1}^{K} w_k * P_k\,(a,b) \tag{1}$$

w_k: weighting factor of indicator k

The weighting factor w must comply with the condition $\sum_k^K w_k = 1$. For this example, equal weighting is assumed resulting in $w_k = 0.33$. For the example, the results are listed in Table 5.

Table 5. Results of the preference indices π(a,b).

	DE	AT	ES
DE	0	0	0.66
AT	0.66	0	1
ES	0.33	0	0

To come to a result for PROMETHEE I in step 4 the outranking flows Φ^+ and Φ^- for the different options are calculated by summarizing the different preference indices according to equations 2 and 3.

$$\phi^+(a) = \frac{1}{n-1}\sum_{j=1}^{n} \pi\,(a,x) \tag{2}$$

$$\phi^-(a) = \frac{1}{n-1}\sum_{j=1}^{n} \pi\,(x,a) \tag{3}$$

The results of the outranking flows are listed for the discussed example in Table 6.

Table 6. Results of the preference indices π(a,b) and the outranking flows Φ^+ and Φ^-

	DE	AT	ES	**Φ^+**
DE	0	0	0.66	0.33
AT	0.66	0	1	0.83
ES	0.33	0	0	0.17
Φ^-	0.50	0	0.83	-

PROMETHEE II goes one step further and a net outranking flow Φ^{net} is defined (equation 4).

$$\phi^{net} = \phi^+(a) - \phi^-(a) \tag{4}$$

For a better understanding, the three outranking flows are graphically depicted in Figure 1.

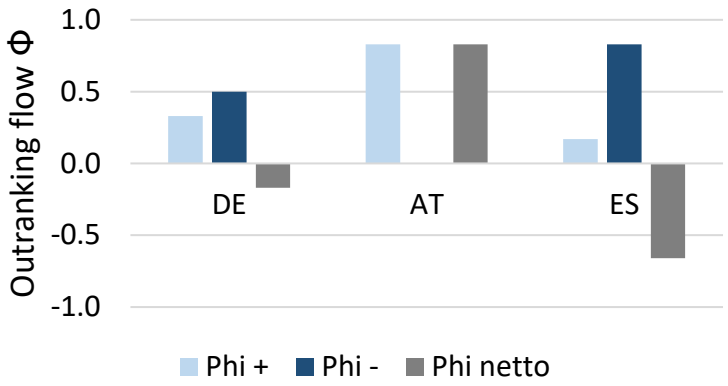
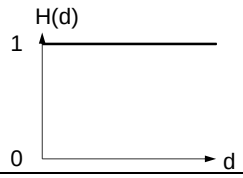
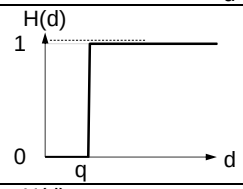
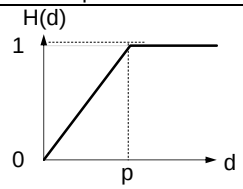
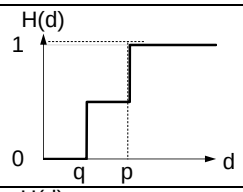
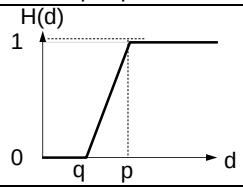
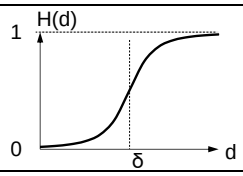


Figure 1. PROMETHEE outranking flows for the minimal example.

The results show that Austria is the best location to produce industrial hydrogen. PROMETHEE I as well as PROMETHEE II come to this conclusion. The best option is indicated by a high Φ^{+} and a low Φ^{-} (PROMETHEE I) or a high Φ^{net} (PROMETHEE II). Spain shows the worst results. Looking at the result for Φ^{net} in more detail, the Austrian option is significantly better than the other two options. The distance between Φ^{net}(AT) and Φ^{net}(DE) is even larger than between Φ^{net}(DE) and Φ^{net}(ES).

Table 7. Six types of preference functions for criteria [1].

1.	Usual criterion	
2.	Quasi-criterion	
3.	Criterion with linear preference	
4.	Level criterion	
5.	Criterion with linear preference and indifference area	

6.	Gaussian criterion	
q: indifference threshold ; *p*: strict preference threshold; *δ:* tipping point		

1 Brans, J.P.; Vincke, P.; Mareschal, B. How to select and how to rank projects: The Promethee method. *European Journal of Operational Research* **1986**, *24*, 228-238, DOI 10.1016/0377-2217(86)90044-5