sustainability

Article

Research on Green Supply Chain Formation and Government Subsidy Pricing Strategy Considering an Online Trading Platform

Wentao Gao [1], Hao Zhang [1,2], Jianfeng Lu [1,2,*] and Tiaojuan Han [1]

1 CIMS Research Center, School of Electronic and Information Engineering, Tongji University, Shanghai 201804, China; wentaogao@tongji.edu.cn (W.G.); hzhang@tongji.edu.cn (H.Z.); 1810855@tongji.edu.cn (T.H.)
2 Engineering Research Center of Enterprise Digital Technology, Ministry of Education, Shanghai 201804, China
* Correspondence: lujianfeng@tongji.edu.cn; Tel.: +86-181-2112-6937

Abstract: Low-carbon production is currently the development direction of enterprises. The government provides low-carbon subsidies to low-carbon manufacturing enterprises to promote the formation of a green supply chain. To better encourage the development of the green supply chain by determining government subsidies and platform authentication fees, this paper constructs a Stackelberg game model that consists of a supplier and a manufacturer, and solves the optimal decisions of each entity using the backward induction method. This study uses the matching method to analyze the payoff matrix for supply chain members when choosing between low-carbon and traditional production. After introducing the online trading platform, the impact of the platform on the profit of supply chain members is studied, considering whether the authentication service provided by the platform is completely accurate. The results show that (1) in the green supply chain, there is a relationship between the enterprises' profit and the level of emissions reduction, which increases first and then decreases. (2) The online trading platform is conducive to achieving green supply chain coordination. The government can pay fewer subsidies to prompt the formation of a green supply chain and the optimal emissions reduction level of the enterprise is higher. (3) When the authentication mechanism of the platform is imperfect, the authentication fee of the platform and the subsidy coefficient of the government need to be set within a reasonable range to avoid the enterprise adopting traditional production pretending to be the low-carbon type. This study contributes to research on the green supply chain and provides insights into pricing strategies for governments and platforms.

Keywords: green supply chain; online trading platform; Stackelberg game; matching analysis; pricing strategy

Citation: Gao, W.; Zhang, H.; Lu, J.; Han, T. Research on Green Supply Chain Formation and Government Subsidy Pricing Strategy Considering an Online Trading Platform. *Sustainability* **2023**, *15*, 4290. https://doi.org/10.3390/su151914290

Academic Editor: Tamás Bányai

Received: 11 August 2023
Revised: 23 September 2023
Accepted: 25 September 2023
Published: 27 September 2023

1. Introduction

With the increase in industrial activity, issues such as global warming have become increasingly prominent, and people have begun to pay more and more attention to environmental protection issues. It has gradually become a consensus to establish a green supply chain and reduce environmental pollution from manufacturing. At the government level, many countries around the world have introduced relevant policies in recent years [1,2]. China aims to reach its carbon dioxide emissions peak before 2030 and strives to achieve carbon neutrality before 2060. In 2021, the Council of the European Union passed the "European Climate Law", turning the commitment to achieve carbon neutrality by 2050 into a legally enforceable constraint [3]. In March 2023, the European Commission published the "Net Zero Industry Act" [4]. In addition, the U.S. government has used policies such as subsidizing renewable energy and pricing carbon to reduce the use of fossil fuels [5]. The Canadian federal government implemented a carbon tax policy aimed at encouraging the abandonment of coal-fired power plants in various provinces [6]. The "green economy" based on low energy consumption and low pollution has become a global focal point. Enterprises are an important part of the development of the green economy. Government subsidies to enterprises are often more effective incentives [7]. Meanwhile, enterprises are also actively implementing low-carbon measures. Walmart launched the "Carbon Disclosure Plan" in 2007 [8], and in 2017 it proposed the "One Billion Ton Project", urging suppliers to reduce carbon emissions in terms of energy and product design. In 2021, ABB Electric released the "Carbon Neutrality White Paper" to express its sustainable development vision and is committed to promoting green and low-carbon transformation through innovative technologies. In 2022, Huawei also proposed to integrate green environmental protection requirements into the whole process of business procurement. Low-carbon products are not only environmentally friendly but also more friendly to people's lives and health. Consumers have low-carbon preferences, and low-carbon products will promote their purchasing desire [9,10].

In a two-level supply chain consisting of low-carbon suppliers and low-carbon manufacturers, green, low-carbon, and environmentally friendly concepts and technologies are integrated to form a green supply chain [11]. The products can be considered low-carbon products, generating additional benefits such as government subsidies and increased sales brought about by consumers' low-carbon preferences [12]. Considering the whole life cycle of the production of an electronic product, when the raw materials provided by the supplier are degradable and the manufacturer adopts an environmentally friendly and green production method, the product will be considered green and the government will give subsidies, so low-carbon suppliers should be matched with low-carbon manufacturers. According to the literature [13], when the government gives the supplier and manufacturer equal low-carbon subsidies, the overall profit of the supply chain is the highest.

With the development of the internet and cloud technology, new changes have taken place in the manufacturing model. Platforms such as the Haier COSMO platform and American MFG.com have emerged to assist the supply and demand sides to achieve matching. Suppliers and manufacturers can conduct matching and resource interaction on the platform. Matching platforms like Catalant and Powerlinx1 help companies identify the partners they should be looking for and then connect them with high-quality business partners of that type around the world [14]. In today's globalized market, suppliers and manufacturers are eager to improve the efficiency of resource matching through the platform to find partners that bring higher benefits [15]. However, there is an issue in the matching process, where traditional companies may engage in "lying" and "free-riding" behaviors in order to obtain the government's low-carbon subsidy. To avoid such phenomena, guide public consumption choices, and encourage businesses to shift to low-carbon production models, a low-carbon product authentication system can be established. By granting a low-carbon logo to products, low-carbon product procurement and consumption patterns can be promoted to society. The development of online trading platforms is a trend and has become an important tool for improving efficiency in various industries. It provides a new environment for the matching of suppliers and manufacturers. The low-carbon authentication service provided by the online trading platform has helped to match supply and demand in the green supply chain.

In this context, this paper considers a two-level supply chain composed of suppliers and manufacturers, and constructs a game model between them under the government's low-carbon subsidy policy. The paper investigates the impact of government subsidy pricing on the production mode decision of the supply chain members. Then, considering the Stackelberg game dominated by suppliers, the profit functions under the optimal pricing of suppliers and manufacturers are analyzed. In addition, with the trend towards platformization of the supply chain [16], the platform can not only enhance the efficiency of supply–demand matching but also provide low-carbon authentication services to enterprises and charge authentication fees. Aiming at whether there is a probability of misjudgment in the low-carbon authentication mechanism provided by the online trading platform, the optimal pricing strategy of government subsidies and platform authentication fees are studied to solve the following problems:

1. How can government subsidies be set to promote both suppliers and manufacturers to adopt low-carbon production and form a green supply chain?
2. What is the impact of the introduction of the online trading platform on the decisions of supply chain members? What changes will it bring to the setting of government subsidies?

Therefore, the research objectives include the supply chain, the government, and the online trading platform, taking into account the game relationship between suppliers and manufacturers, and making a matching analysis based on the revenue sources and optimization goals of manufacturers and suppliers. The government aims to encourage both parties to take green and low-carbon behaviors and cooperate with each other. For the two situations of whether there is an online trading platform or not, it provides policy recommendations for the government's low-carbon subsidies and the pricing of online trading platform authentication fees.

The rest of this paper is structured as follows. In Section 2, a literature review is conducted on supply chain decision making and matching of supply and demand. In Section 3, the basic model of this paper and some assumptions are given. In Section 4, game models under three different situations are established, the optimal profit of suppliers and manufacturers under each strategy are analyzed, and the pricing range of government subsidies to promote the formation of green supply chains is given. The impact of the introduction of online trading platforms on supply chain decisions is also compared. In Section 5, numerical analysis is used to verify the obtained propositions and conclusions. In Section 6, the paper concludes with a summary and further discussions.

2. Literature Review

In the context of the rapid development of the green economy, some scholars have used econometric methods to study the important factors influencing the development of the green economy and provide suggestions to the government. These mainly focus on the following literature: Batrancea et al. [17,18] used an empirical approach to analyze the relevant variables affecting the green economy in different countries, providing solutions for the development of the green economy. In addition, they used the panel first-difference generalized method of moments to examine the energy production and its primary determinants, deriving factors influencing energy supply for the guidance of policymakers. Guliyev et al. [19] utilized panel data analysis with structural breaks and adopted a novel modeling method to study the dynamic impact of renewable energy consumption on economic growth in European countries. Andrea et al. [20] analyzed the main methods and models currently available for green economic assessment, studied whether various models are sufficient to meet green economic development, and provided recommendations for green economic policies.

However, more scholars have adopted the method of game theory to study the relationship between the government's low-carbon policy and factors such as consumer preferences, corporate profits, and corporate emissions reduction levels. Su et al. [16] studied the pricing problem under different government subsidies in the supply chain, and discussed the influence of government subsidy coefficients on the optimal decision making of the green supply chain. Wu et al. [13] studied the impact of the supply chain structure and the government's low-carbon subsidies on the profits of manufacturers and retailers, as well as the conditions that motivate manufacturers to voluntarily reduce carbon emissions. They concluded that if government agencies provide equal low-carbon subsidies to manufacturers and retailers, the effect will be better. Therefore, this paper only considers the situation where suppliers and manufacturers receive equal low-carbon subsidies. Wang et al. [21] found that government subsidies can increase the enthusiasm of developers and contractors to reduce carbon emissions and improve the profitability of the entire supply chain. Kang et al. [22] used the method of evolutionary game theory to analyze the impact of the government's low-carbon policy and consumer sensitivity on corporate strategy in the green supply chain. Meng et al. [23] used the Stackelberg game to construct a dual-channel green supply chain model considering the consumers' green preferences and channel preferences, and explored the coordinated pricing policy of products in the dual-channel green supply chain. Sun et al. [24] used the method of an evolutionary game to study the optimal green investment strategy of manufacturers and material suppliers in the two-level supply chain. Meng et al. [25] studied the positive impact of consumer heterogeneity and government subsidies on green innovation in the supply chain, and analyzed government subsidy strategies under different scenarios. In addition, some scholars have studied the impact of green credit and low-carbon publicity on government subsidies and supply chain profits, such as [26,27]. These studies are often focused on the relationships between government, manufacturers, and retailers, but with the rapid development of online bilateral market platforms and the emergence of the concept of "cloud manufacturing", platforms often play a connecting role in these relationships.

In service-oriented manufacturing based on online trading platforms, the two-sided market is an important concept. It was first proposed by Rochet and Tirole [28], and many scholars have conducted research on this issue since then. Zhang et al. [29] proposed four different service and pricing strategies for the platform by constructing a game theory model and analyzing the optimal decision making of the platform. Han et al. [30] established a bilateral matching model of service providers and demanders under the cloud manufacturing service platform, which solved the service matching problem between cloud manufacturing services and demands. Jung et al. [31] designed an online matching platform to improve participation and matching results by designing selection capabilities. Basu et al. [14,15] studied the interaction between the matching search process and the identity authentication process in online matching based on the game theory model, aiming at the matching problem in the two-sided market, and provided insights for the pricing strategy of search and authentication services on the platform. It can be seen that game-based methods are widely adopted in solving matching problems based on platforms, providing strategic suggestions for platforms and participants. The above literature also shows that there exist information quality issues in such two-sided markets.

Ghani et al. [32] pointed out that the information provided by the matchers on the platform may be untrue, and therefore, it is necessary to add an identity authentication mechanism to the two-sided market. Nie et al. [33] established a dynamic game model, considering the probability that the authentication mechanism misidentifies the credentials, and studied the optimal pricing strategy under a labor-sharing platform. After comparing the direct search mode and the online search mode based on the platform, Ref. [15] introduced the authentication service mechanism provided by the platform. They established a game model between the H-type and the L-type, and analyzed tthe impact of the proportion of different types of matchers in the market on the pricing of authentication services, providing recommendations for the platform's pricing decisions.

Currently, scholars mainly study the impact of government subsidies on supply chain profits or social welfare, providing subsidy pricing strategies for the government. However, they do not consider the impact of online trading platforms on suppliers, manufacturers, and government low-carbon subsidy policies. Furthermore, there are few studies that use the method of matching analysis to study the decision making of supply and demand entities in the supply chain. Therefore, in view of the research gaps in supply chain decision making under online trading platforms and the impact of joining platforms on the government's low-carbon policy, this paper constructs a game model between suppliers and manufacturers based on an online trading platform to analyze the matching of supply and demand. We study the minimum effective government subsidy to incentivize enterprises to reduce carbon emissions, and the impact of online trading platforms on the decisions of supply chain members and government subsidies. Table 1 shows the differences between this study and related studies.

Table 1. Comparison of relevant studies with this work.

Paper	Game Theory	Green Supply Chain	Pricing Decision	Platform	Matching Analysis	Authentication Service
Liu et al. [1]	✓	✓		✓		
Shi et al. [8]		✓	✓			
Su et al. [16]	✓	✓	✓			
Batrancea et al. [17,18]		✓	✓			
Sun et al. [24]	✓	✓	✓			
Liu et al. [34]	✓	✓	✓			
Jung et al. [31]				✓	✓	
Nie et al. [33]			✓	✓		
Zhang et al. [35]	✓		✓	✓		
Basu et al. [14,15]	✓			✓	✓	✓
Our paper	✓	✓	✓	✓	✓	✓

3. Model Hypotheses and Notation

Considering that suppliers and manufacturers form a two-level supply chain, there are two strategic choices for suppliers or manufacturers: low-carbon production and traditional production. In the matching process between manufacturers and suppliers, if there is no online trading platform to support direct offline matching, the matching process will be inefficient, and the matching success rate is μ ($0 < \mu < 1$). Considering the existing cloud manufacturing platforms and other matching platforms [14,35], it is certain that a search match can be found with the support of an online trading platform, but it cannot be guaranteed that the other party is of the expected type. The online trading platform can also provide low-carbon authentication services, and the authentication service fee is a. There are two cases here [33]: (1) the authentication mechanism can accurately identify low-carbon and traditional enterprises, which is called perfect authentication; (2) the authentication mechanism may make mistakes, and there is a probability of γ of certifying traditional enterprises as low-carbon enterprises, which is called imperfect authentication. This paper assumes that the goal of the platform is to encourage more low-carbon enterprises to participate in the platform.

In the market, the proportion of low-carbon suppliers is denoted as m, and the proportion of low-carbon manufacturers is denoted as n. That is, the probability for a manufacturer to be matched with a low-carbon supplier is m, and the probability for a supplier to be matched with a low-carbon manufacturer is n. This is common knowledge for both the government and the supply–demand entities. Only traditional-type enterprises may falsely claim themselves to be low-carbon, while there are no low-carbon-type enterprises that falsely claim themselves as traditional. When a low-carbon supplier is matched with a low-carbon manufacturer, the supply chain is recognized as green. In the green supply chain, as shown in Figure 1, suppliers provide manufacturers with degradable raw materials, and manufacturers provide green products to consumers after production using low-carbon technology [36].

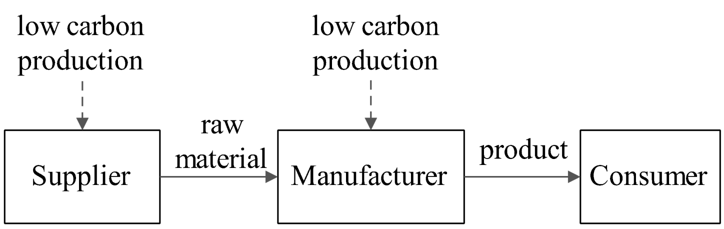

Figure 1. Supply chain operation mode.

At this time, the government will provide subsidies for the supply chain, and the subsidy coefficient per unit product is λ. In addition, the green supply chain also generates a low-carbon benefit δ, which is due to consumer preference for low-carbon products, leading to an increase in product sales in the green supply chain. Consumers' purchasing decisions are also affected by product value and price. Without affecting the conclusion, assuming that the product selling price is p, the consumer's evaluation of the product is v, which follows a uniform distribution on the interval [0, 1]. To characterize the influence of product price and emissions reduction level on consumers' purchasing decisions, this paper adopts the consumer utility function model: $U = v - p + \delta(e_s + e_m)$. Let e_0 be the initial carbon emission level of the entire supply chain, and e_s and e_m are the emissions reduction levels of the supplier and the manufacturer, respectively. All consumers with a value between $[p - \delta(e_s + e_m), 1]$ will buy the product. Then, the demand function [16,34,37] is as follows:

$$Q = 1 - p + \delta(e_s + e_m) \tag{1}$$

k_s and k_m are the emissions reduction cost coefficients of the supplier and the manufacturer, respectively.

The cost functions of the supplier and the manufacturer adopting low-carbon technologies [21,34] are, respectively,

$$C_s = \frac{1}{2}k_s e_s^2, C_m = \frac{1}{2}k_m e_m^2 \tag{2}$$

In actual supply chain decision making, decentralized decision making is often adopted. Moreover, in the supply chain operation process, the supplier is in the upstream position while the manufacturer is downstream. Therefore, we consider a Stackelberg game between the supplier and the manufacturer with the supplier being the leader [38,39]. The decision sequence diagram is shown in Figure 2.

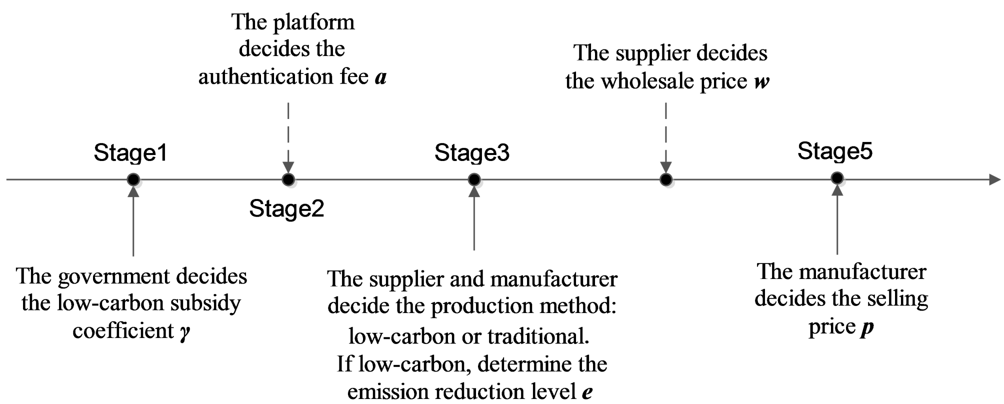

Figure 2. The sequence of the events.

The variable c represents the product cost, and w represents the wholesale price. When the government does not provide low-carbon subsidies, and neither the supplier nor the manufacturer is low-carbon, the supplier's profit is $\pi_s^{nl} = (w - c)(1 - p)$, and the manufacturer's profit is $\pi_m^{nl} = (p - w)(1 - p)$. The optimal wholesale price and the optimal selling price can be obtained using the backward induction method. For the variable p, the second derivative of the manufacturer's profit function is -2, indicating that the profit curve has a maximum value. Therefore, the manufacturer selects the selling price to maximize the profit. Let the first derivatives of the profits of the supplier and manufacturer are both 0. Therefore, when not considering low-carbon subsidies, the optimal wholesale price w_{nl} and the optimal product price p_{nl} are given by

$$w_{nl} = \frac{c+1}{2}, p_{nl} = \frac{c+3}{4} \tag{3}$$

After considering the government's low-carbon subsidy, let the unit product subsidy coefficient be λ [40,41], and the subsidy that the enterprises receive from the government is $\lambda[1-p+\delta(e_s+e_m)]$. The profit of the supplier in the green supply chain is

$$\pi_s^l = (w-c)[1-p+\delta(e_{\rm m}+e_s)] + \lambda[1-p+\delta(e_{\rm m}+e_s)] - \frac{1}{2}k_s e_s^2 \tag{4}$$

The profit of the manufacturer in the green supply chain is

$$\pi_m^l = (p-w)[1-p+\delta(e_{\rm m}+e_s)] + \lambda[1-p+\delta(e_{\rm m}+e_s)] - \frac{1}{2}k_{\rm m} e_{\rm m}^2 \tag{5}$$

Similarly, when considering low-carbon subsidies, the optimal wholesale price w_l and the optimal product selling price p_l are, respectively,

$$w_l = \frac{1+c+\delta(e_s+e_m)}{2}, p_l = \frac{3+3\delta(e_s+e_m)-2\lambda+c}{4} \tag{6}$$

Substituting this into Equations (4) and (5), then let $\frac{\partial \pi_s^l}{\partial e_s}=0$, $\frac{\partial \pi_m^l}{\partial e_m}=0$, the low-carbon emissions reduction levels e_s, e_m can be obtained by the supplier and the manufacturer when they decide to adopt low-carbon production:

$$e_s = \frac{2\delta k_m(1-c)}{8k_m k_s - 2\delta^2 k_m - \delta^2 k_s}, e_m = \frac{\delta k_s(1-c)}{8k_m k_s - 2\delta^2 k_m - \delta^2 k_s} \tag{7}$$

According to the conclusions from [33,37], this paper argues that the purpose of the government's low-carbon subsidies is to encourage enterprises to adopt low-carbon production, and assumes that after the supplier and manufacturer decide on the production mode, their decision-making variables are w and p. The level of low-carbon emissions reduction efforts e_s, e_m will be chosen once the supplier and manufacturer determine their respective strategies. Since the goal of this article is to provide pricing decision-making suggestions for the government and the platform, the impact of the enterprise's emissions reduction level on the model will be analyzed more through numerical simulation. Based on this, a game model between the supplier and the manufacturer is constructed, and a description of the symbols used in the model is shown in Table 2.

Table 2. Summary of parameter symbols.

Symbol	Meaning
δ	The low-carbon effect when the product is low-carbon
μ	Matching efficiency
p	Product selling price
w	Wholesale price
c	Cost of production
λ	Subsidy coefficient per unit product
v	Consumer evaluation of the product
π_s	Supplier profit
π_m	Manufacturer profit
k_s	Cost coefficient of supplier carbon emissions reduction
k_m	Cost coefficient of manufacturer carbon emissions reduction
e_0	Initial carbon emission level of supply chain
e_s	Supplier's emissions reduction level
e_m	Manufacturer's emissions reduction level
m	The proportion of green suppliers in the market
n	The proportion of green manufacturers in the market
a	Platform authentication fee
γ	Misjudgment probability of authentication

4. Model Description and Analysis

In this part, models are built for three scenarios: no online trading platform, a trading platform whose authentication is completely correct (perfect authentication), and a trading platform whose authentication has misjudgment (imperfect authentication). Effective subsidy coefficient setting and platform optimal authentication service pricing are studied, as well as the impact of the online trading platform on the formation of the green supply chain.

4.1. Model I: Matching of Suppliers and Manufacturers without an Online Trading Platform

When the online trading platform is not considered, the research problem is a supply chain matching problem. Assuming that suppliers or manufacturers are completely rational, they always make strategic choices with the goal of maximizing their own profits. In the matching process, a low-carbon manufacturer has a probability of m to match with a low-carbon supplier, forming a green supply chain, which will generate low-carbon benefits, and the government will provide subsidies for the supply chain. The profit of the supplier and manufacturer are, respectively,

$$\pi_s^{l-l} = \mu n(w - c + \lambda)Q - \frac{1}{2}k_s e_s^2 \tag{8}$$

$$\pi_m^{l-l} = \mu m(p - w + \lambda)Q - \frac{1}{2}k_m e_m^2 \tag{9}$$

A low-carbon manufacturer also has a probability of $1 - m$ to match a traditional supplier, then, the supply chain cannot be considered green, and neither the manufacturer nor the supplier will receive low-carbon subsidies. The manufacturer will also have to pay the low-carbon costs. Then, a 2 × 2 game payoff matrix is obtained in Table 3.

Table 3. Payoff matrix without the online trading platform.

		Manufacturer	
		Low-Carbon	**Traditional**
Supplier	Low-Carbon	$(\pi_s^{l-l}, \pi_m^{l-l})$	$(\pi_s^{nl} - C_s, \pi_m^{nl})$
	Traditional	$(\pi_s^{nl}, \pi_m^{nl} - C_m)$	(π_s^{nl}, π_m^{nl})

During the matching process between suppliers and manufacturers, there is a matching success rate. Traditional enterprises may pretend to be low-carbon enterprises, while low-carbon enterprises will not pretend to be traditional enterprises, and the types of matched enterprises are random. Therefore, when directly matched without the support of an online trading platform, the expected payoff of a supplier adopting a low-carbon production strategy is

$$\pi_s^l = \mu[n(w - c + \lambda)Q + (1 - n)(w - c)(1 - p)] - \frac{1}{2}k_s e_s^2 \tag{10}$$

The expected payoff of a manufacturer adopting low-carbon production is

$$\pi_m^l = \mu[m(p - w + \lambda)Q + (1 - m)(p - w)(1 - p)] - \frac{1}{2}k_m e_m^2 \tag{11}$$

The expected payoff of a manufacturer adopting low-carbon production is

$$\pi_s^l = \mu(w - c)(1 - p) \tag{12}$$

The expected payoff of a manufacturer adopting low-carbon production is

$$\pi_m^l = \mu(p - w)(1 - p) \tag{13}$$

In this game payoff matrix, when an enterprise adopting the low-carbon production strategy is matched with an enterprise adopting the traditional production strategy, it neither gains low-carbon subsidies nor pays additional low-carbon costs. Such a strategy profile is a strictly dominated strategy. For the enterprise adopting low-carbon production, it knows its expected payoff. Then, based on the backward induction method, the optimal wholesale price and optimal selling price in the Stackelberg game can be obtained as

$$w_l^* = \frac{1 + c + \lambda m - \lambda n + \delta(e_s + e_m)(2n - m)}{2} \tag{14}$$

$$p_l^* = \frac{3 + c + \lambda m - \lambda n + \delta(e_s + e_m)(2n - m)}{4} \tag{15}$$

Assuming that both the supplier and the manufacturer are risk-neutral, the optimal expected profits of low-carbon suppliers and low-carbon manufacturers are, respectively, as follows:

$$\begin{aligned} \pi_s^{l*} = {} & \mu n\left(\frac{\lambda m}{4} - \frac{c}{4} + \frac{\lambda n}{4} + \delta(e_m + e_s) - \frac{\delta m(e_m + e_s)}{4} - \frac{\delta n(e_m + e_s)}{2} + \frac{1}{4}\right) \\ & \times \left(\lambda - \frac{c}{2} + \frac{\lambda m}{2} - \frac{\lambda n}{2} - \frac{\delta m(e_m + e_s)}{2} + \delta n(e_m + e_s) + \frac{1}{2}\right) - \frac{{e_s}^2 k_s}{2} \\ & - \mu(n-1)\left(\frac{c}{2} - \frac{\lambda m}{2} + \frac{\lambda n}{2} + \frac{\delta m(e_m + e_s)}{2} - \delta n(e_m + e_s) - \frac{1}{2}\right) \\ & \times \left(\frac{c}{4} - \frac{\lambda m}{4} - \frac{\lambda n}{4} + \frac{\delta m(e_m + e_s)}{4} + \frac{\delta n(e_m + e_s)}{2} - \frac{1}{4}\right) \end{aligned} \tag{16}$$

$$\begin{aligned} \pi_m^{l*} = {} & \mu m\left(\frac{\lambda m}{4} - \frac{c}{4} + \frac{\lambda n}{4} + \delta(e_m + e_s) - \frac{\delta m(e_m + e_s)}{4} - \frac{\delta n(e_m + e_s)}{2} + \frac{1}{4}\right) \\ & \times \left(\lambda - \frac{c}{4} - \frac{3\lambda m}{4} + \frac{\lambda n}{4} + \frac{3\delta m(e_m + e_s)}{4} - \frac{\delta n(e_m + e_s)}{2} + \frac{1}{4}\right) - \frac{{e_m}^2 k_m}{2} \\ & - \mu(m-1)\left(\frac{c}{4} - \frac{\lambda m}{4} - \frac{\lambda n}{4} + \frac{\delta m(e_m + e_s)}{4} + \frac{\delta n(e_m + e_s)}{2} - \frac{1}{4}\right) \\ & \times \left(\frac{c}{4} + \frac{3\lambda m}{4} - \frac{\lambda n}{4} - \frac{3\delta m(e_m + e_s)}{4} + \frac{\delta n(e_m + e_s)}{2} - \frac{1}{4}\right) \end{aligned} \tag{17}$$

For enterprises adopting traditional production, substituting Equation (3) into (8) and (10), the optimal profits of the enterprises when adopting traditional production can be obtained:

$$\pi_s^{nl*} = \frac{\mu}{8}(1 - c)^2 \tag{18}$$

$$\pi_m^{nl*} = \frac{\mu}{16}(1 - c)^2 \tag{19}$$

To encourage suppliers and manufacturers to adopt low-carbon production, it is necessary to ensure that the expected profit of low-carbon enterprises is higher than that of traditional enterprises. That is, when $\pi_s^{l*} > \pi_s^{nl*}$ and $\pi_m^{l*} > \pi_m^{nl*}$, suppliers and manufacturers will tend to adopt low-carbon production to promote the formation of a green supply chain matching. The government subsidy coefficient encouraging suppliers to be low-carbon needs to satisfy $\lambda > \lambda_s^*$. The government subsidy coefficient encouraging manufacturers to be low-carbon needs to satisfy $\lambda > \lambda_m^*$. Then, the government's minimum effective subsidy coefficient is $\lambda^* = max[\lambda_s^*, \lambda_m^*]$.

Considering the relationship between the total profit in the green supply chain and the emissions reduction level of enterprises, the supply chain profit function is defined as follows:

$$\pi_{sc}^{l*} = \pi_s^{l*} + \pi_m^{l*} \tag{20}$$

Proposition 1. *From Equation (19), $\frac{\partial \pi_{sc}^{l*}}{\partial e_s}$, $\frac{\partial \pi_{sc}^{l*}}{\partial e_m}$ are obtained. It can be found that if the emissions reduction level satisfies $0 < e_s < e_s^*$, and $0 < e_m < e_m^*$, then $\frac{\partial \pi_{sc}^{l*}}{\partial e_s} > 0$, $\frac{\partial \pi_{sc}^{l*}}{\partial e_m} > 0$. Whereas, if $e_s > e_s^*$, $e_m > e_m^*$, then $\frac{\partial \pi_{sc}^{l*}}{\partial e_s} < 0$, $\frac{\partial \pi_{sc}^{l*}}{\partial e_m} < 0$. Since the analytical expressions of the optimal emissions reduction level e_s^*, e_m^* are too complex, these two variables will be studied in the numerical simulation in Section 5.*

Proposition 1 indicates that initially the supply chain profit increases as the emissions reduction level of enterprises slowly increases, because the emissions reduction brings more low-carbon effects, increasing product sales and the profit of the supply chain. However, after reaching e_s^*, e_m^*, due to the substantial increase in emissions reduction costs, emissions reduction costs become the main factor affecting the supply chain's profit, and the total profit of the supply chain decreases with the increase in emissions reduction level. The managerial insight brought about by this proposition is that when enterprises adopt a low-carbon production strategy, the determination of the emissions reduction level should not be just a minimal reduction for low-carbon, a moderate emissions reduction level is not only conducive to environmental benefits but can also increases supply chain profits. In addition, the government should also consider formulating policies to encourage enterprises to adopt moderate emissions reductions.

4.2. Model II: The Online Trading Platform with Perfect Authentication Service

In the previous subsection, we discussed the case where suppliers and manufacturers directly match under the situation of there being no online trading platform. However, this may lead to some problems, such as the inability to search for enterprises for matching due to information asymmetry or traditional enterprises falsely claiming that they are low-carbon. With the promotion and application of online trading platforms, a platform can improve the success rate of matching in the bilateral market and provide low-carbon authentication services for enterprises. This subsection considers the scenario where the online trading platform provides perfect authentication services and investigates the impact of the online trading platform on the green supply chain.

When a supplier or manufacturer purchases authentication services, it is assumed to prefer to cooperate with low-carbon types. In the case of no online trading platform, enterprises cannot accurately identify low-carbon types, so they will not reject the matching result. However, under the online trading platform, enterprises that purchase authentication services are more inclined to match with low-carbon types. Therefore, if they do not match their desired type in the first round, they will reject the matching result and try the second match to ensure their profits [14,15]. In addition, under perfect authentication, traditional enterprises will not choose to purchase authentication services because this will expose their true type. For the enterprises that do not purchase authentication services, their profits are the same as before. In this model, the decision sequence is as follows: stage 1, the cloud platform announces the authentication fee a; stage 2, the supplier decides the wholesale price w; stage 3, the manufacturer decides the selling price p. Therefore, the profit matrix is shown in Table 4.

Table 4. Payoff matrix under the online trading platform providing perfect authentication.

		Manufacturer	
		Low-Carbon + Perfect Authentication	**Traditional + Perfect Authentication**
Supplier	Low-Carbon + Perfect Authentication	$(\pi_{s+a}^{l-l}, \pi_{m+a}^{l-l})$	$(\pi_{s+a}^{l-nl}, \pi_{m+a}^{nl-l})$
	Traditional + Perfect Authentication	$(\pi_{s+a}^{nl-l}, \pi_{m+a}^{l-nl})$	$(\pi_{s+a}^{nl-nl}, \pi_{m+a}^{nl-nl})$

Analyzing the payoff matrix in the case of providing perfect authentication services on the online matching platform, for a traditional enterprise that does not purchase authentication services, any enterprises that cooperate with them will not generate additional benefits such as government subsidies. A low-carbon enterprise that has purchased the authentication service will receive more substantial benefits, but they also have to pay the extra platform authentication fee. The profit function can be obtained.

The expected payoff of low-carbon suppliers participating in the platform:

$$\pi_{s+a}^{l} = n(w-c+\lambda)Q + (1-n)[n(w-c+\lambda)Q + (1-n)(w-c)(1-p)] - \frac{1}{2}k_s e_s^2 - a \quad (21)$$

The expected payoff of low-carbon manufacturers participating in the platform:

$$\pi_{m+a}^{l} = m(p-w+\lambda)Q + (1-m)[m(p-w+\lambda)Q + (1-m)(p-w)(1-p)] - \frac{1}{2}k_m e_m^2 - a \quad (22)$$

When matching on the online trading platform, low-carbon enterprises have a higher degree of confidence in believing that the matched enterprises are also low-carbon, so the optimal wholesale price and selling price in the Stackelberg game satisfy Equation (6). So it can be simplified to obtain

$$\begin{aligned}\pi_{s+a}^{l*} = {} & \frac{n}{2}\left(\lambda - \frac{c}{2} + \frac{\delta(e_m+e_s)}{2} + \frac{1}{2}\right)^2 + \frac{n^2-n}{2}\left(\lambda - \frac{c}{2} + \frac{\delta(e_m+e_s)}{2} + \frac{1}{2}\right)^2 - \frac{1}{2}k_s e_s^2 \\ & - (n-1)^2\left(\frac{\delta(e_m+e_s)}{2} - \frac{c}{2} + \frac{1}{2}\right)\left(\frac{c}{4} - \frac{\lambda}{2} + \frac{3\delta(e_m+e_s)}{4} - \frac{1}{4}\right) - a\end{aligned} \quad (23)$$

$$\begin{aligned}\pi_{m+a}^{l*} = {} & m\left(\frac{2\lambda - c + \delta(e_m+e_s) + 1}{4}\right)^2 - \left(m^2 - m\right)\left(\frac{2\lambda - c + \delta(e_m+e_s) + 1}{4}\right)^2 \\ & - (m-1)^2\left(\frac{c + 2\lambda - \delta(e_m+e_s) - 1}{4}\right)\left(\frac{c - 2\lambda + 3\delta(e_m+e_s) - 1}{4}\right) - \frac{1}{2}k_m e_m^2 - a\end{aligned} \quad (24)$$

The profits of suppliers and manufacturers who choose traditional production on the platform are

$$\pi_{s+a}^{nl*} = \frac{\mu}{8}(1-c)^2 - a \quad (25)$$

$$\pi_{m+a}^{nl*} = \frac{\mu}{16}(1-c)^2 - a \quad (26)$$

For suppliers and manufacturers to participate in the online trading platform, the platform needs to satisfy the participation constraint and incentive compatibility constraint when setting the authentication fee [13]. Taking suppliers as an example, for an online trading platform, the participation constraint is that the profit after joining the platform is greater than the profit without joining the platform, i.e., $\pi_{s+a}^{l*} > \pi_s^{l*}, \pi_s^{nl*}$. The incentive compatibility constraint is that after the enterprise joins the online trading platform, it can obtain more benefits by adopting low-carbon production than traditional production, i.e., $\pi_{s+a}^{l*} > \pi_{s+a}^{nl*}$.

Proposition 2. *Considering the profit function relationship when the platform provides perfect authentication, where* $\pi_s^{nl} > \pi_{s+a}^{nl}$ *always holds. By solving this equation, it can be found that the authentication fee to encourage suppliers to participate in the platform needs to satisfy* $0 < a_s < a_s^*$*, where* a_s^* *is the minimum value of the solutions when* $\pi_{s+a}^{l*}(a) - \pi_s^{l*} = 0$ *and* $\pi_{s+a}^{l*}(a) - \pi_s^{nl*} = 0$*. The authentication fee to encourage low-carbon participation of manufacturers in the platform needs to satisfy* $0 < a_m < a_m^*$*, where* a_m^* *is the minimum value of the solutions when* $\pi_{m+a}^{l*}(a) - \pi_m^{l*} = 0$ *and* $\pi_{m+a}^{l*}(a) - \pi_m^{nl*} = 0$*. Therefore, the range of values for the platform's authentication fee is* $a^* \in (0, min[a_s^*, a_m^*])$*.*

Proposition 2 shows that in order to encourage enterprises to join the platform for matching and trading, the platform needs to consider the individual rationality constraints of enterprises and set appropriate authentication fees to attract enterprises to join the platform and produce low-carbon products.

When an enterprise joins the online trading platform, the government needs to set an appropriate low-carbon subsidy coefficient λ to encourage suppliers and manufacturers to adopt low-carbon strategies, satisfying the equations $\pi_{s+a}^{l} > \pi_s^{nl}, \pi_{s+a}^{nl}$ and $\pi_{m+a}^{l} > \pi_m^{nl}, \pi_{m+a}^{nl}$.

Proposition 3. *By solving these expressions, it can be obtained that the government subsidy coefficient for encouraging low-carbon production by suppliers needs to satisfy* $\lambda_s > \lambda_{s+a}^*$*;* λ_{s+a}^* *is the solution of* $\pi_{s+a}^{l}(\lambda) - \pi_s^{nl}(\lambda) = 0$*. The government subsidy coefficient for encouraging low-carbon production by manufacturers needs to satisfy* $\lambda_m > \lambda_{m+a}^*$*;* λ_m^* *is the solution of* $\pi_{m+a}^{l}(\lambda) - \pi_m^{nl}(\lambda) = 0$*. So, the minimum effective subsidy coefficient of the government* $\lambda_a^* = max[\lambda_{s+a}^*, \lambda_{m+a}^*]$*.*

Proposition 4. *When* $a^* \in (0, min[a_s^*, a_m^*])$*, we have* $\lambda_a^* < \lambda^*$*. And under this model, the optimal emissions reduction level satisfies* $e_{s+a}^* > e_s^*$*,* $e_{m+a}^* > e_m^*$*.*

Propositions 3 and 4 imply that the introduction of the online trading platform and the authentication service exerts a positive effect on reducing government subsidy expenditures. For the same level of emissions reduction by suppliers and manufacturers, the subsidy setting will be lower than without the online trading platform, which helps to improve the emissions reduction level of enterprises in the green supply chain and increase the total profit of the green supply chain. The management significance derived from this is that the government could consider collaborating with the platform to assist in its promotion, encouraging more enterprises to match through the online platform. This would be beneficial in reducing the government expenditure burden, boosting the platform's revenue, and enhancing the supply chain's operational efficiency.

4.3. Model III: The Online Trading Platform with Imperfect Authentication Service

In actual authentication services, the authentication mechanism often cannot achieve complete accuracy. Some enterprises disguised as low-carbon types will deceive the authentication mechanism, and traditional-type enterprises will be wrongly certified as low-carbon types, with a misjudgment probability of γ. In this case, traditional enterprises may purchase authentication services and be certified as low-carbon enterprises by the platform. Therefore, for the supplier, the probability of being certified as a low-carbon type by the online trading platform, and that the true type is also low-carbon is

$$P(l_s|T = l_s) = \frac{m}{m + \gamma(1 - m)} \tag{27}$$

The probability of being certified as a low-carbon type by the online trading platform, but the true type is traditional is

$$P(nl_s|T = l_s) = \frac{\gamma(1-m)}{m + \gamma(1-m)} \tag{28}$$

Similarly, for manufacturers, they are

$$P(l_m|T = l_m) = \frac{n}{n + \gamma(1-n)} \tag{29}$$

$$P(l_m|T = l_m) = \frac{\gamma(1-n)}{n + \gamma(1-n)} \tag{30}$$

Let $P_1 = P(l_s|T = l_s), P_2 = P(nl_s|T = l_s), P_3 = P(l_m|T = l_m), P_4 = P(nl_m|T = l_m)$.

Under model III, the decision-making sequence is as follows: stage 1, the platform announces the authentication fee a and the probability of misjudgment γ; stage 2, the supplier decides the optimal wholesale price w; stage 3, the manufacturer decides the optimal selling price p. The payoff matrix is constructed in Table 5.

Table 5. Payoff matrix under the online trading platform providing imperfect authentication.

		Manufacturer	
		Low-Carbon + Imperfect Authentication	**Traditional + Imperfect Authentication**
Supplier	Low-Carbon + Imperfect Authentication	$(\pi_{s+i}^{l-l}, \pi_{m+i}^{l-l})$	$(\pi_{s+i}^{l-nl}, \pi_{m+i}^{nl-l})$
	Traditional + Imperfect Authentication	$(\pi_{s+i}^{nl-l}, \pi_{m+i}^{l-nl})$	$(\pi_{s+i}^{nl-nl}, \pi_{m+i}^{nl-nl})$

In the case of imperfect authentication, traditional enterprises may also purchase authentication services. There is a probability that they will be misjudged as low-carbon enterprises and receive government subsidies, but they will not generate low-carbon benefits δ. Taking the supplier as an example, when adopting low-carbon production and participating in the platform, the profit of cooperating with a low-carbon manufacturer is

$$\pi_{s+i}^{l-l} = (w - c + \lambda)[1 - p + \delta(e_m + e_s)] - \frac{1}{2}k_s e_s^2 - a \tag{31}$$

$$\pi_{s+i}^{l-nl} = (1-\gamma)(w-c)(1-p) + \gamma(w - c + \lambda)(1-p) - \frac{1}{2}k_s e_s^2 - a \tag{32}$$

Therefore, in model III, the expected payoff of low-carbon suppliers participating in the platform:

$$\begin{aligned}\pi_{s+i}^{l} =& (n + \gamma(1-n))[P_3(w - c + \lambda)Q + P_4(w - c + \lambda)(1-p)] + \\ & (1 - n - \gamma(1-n))[n[(w - c + \lambda)Q] + (1-n)(w-c)(1-p)] - \frac{1}{2}k_s e_s^2 - a\end{aligned} \tag{33}$$

The expected payoff of low-carbon manufacturers participating in the platform:

$$\begin{aligned}\pi_{m+i}^{l} =& (m + \gamma(1-m))[P_1(p - w + \lambda)Q + P_2(p - w + \lambda)(1-p)] + \\ & (1 - m - \gamma(1-m))[m[(p - w + \lambda)Q] + (1-m)(p-w)(1-p)] - \frac{1}{2}k_m e_m^2 - a\end{aligned} \tag{34}$$

The authentication fee a and the misjudgment probability of authentication γ are exogenous variables for the supply chain. In this Stackelberg game, the optimal wholesale price and selling price also satisfy Equation (6). We can simplify the profit functions as follows:

$$\begin{aligned}\pi_{s+i}^{l*} &= \gamma(n-1)\left(\lambda - \frac{c}{2} + \frac{\delta(e_m+e_s)}{2} + \frac{1}{2}\right)\left(\frac{c}{4} - \frac{\lambda}{2} + \frac{3\delta(e_m+e_s)}{4} - \frac{1}{4}\right) - \frac{{e_s}^2 k_s}{2} - a \\ &+ \frac{n}{2}\left(\lambda - \frac{c}{2} + \frac{\delta(e_m+e_s)}{2} + \frac{1}{2}\right)^2 + \frac{(\gamma-1)(n^2-n)}{2}\left(\lambda - \frac{c}{2} + \frac{\delta(e_m+e_s)}{2} + \frac{1}{2}\right)^2 \\ &+ (\gamma-1)(n-1)^2\left(\frac{\delta(e_m+e_s)}{2} - \frac{c}{2} + \frac{1}{2}\right)\left(\frac{c}{4} - \frac{\lambda}{2} + \frac{3\delta(e_m+e_s)}{4} - \frac{1}{4}\right)\end{aligned} \tag{35}$$

$$\begin{aligned}\pi_{m+i}^{l*} &= \gamma(m-1)\left(\frac{\lambda}{2} - \frac{c}{4} + \frac{\delta(e_m+e_s)}{4} + \frac{1}{4}\right)\left(\frac{c}{4} - \frac{\lambda}{2} + \frac{3\delta(e_m+e_s)}{4} - \frac{1}{4}\right) - \frac{e_m^2 k_m}{2} - a \\ &+ m\left(\frac{\lambda}{2} - \frac{c}{4} + \frac{\delta(e_m+e_s)}{4} + \frac{1}{4}\right)^2 + (\gamma-1)(m^2-m)\left(\frac{\lambda}{2} - \frac{c}{4} + \frac{\delta(e_m+e_s)}{4} + \frac{1}{4}\right)^2 \\ &- (\gamma-1)(m-1)^2\left(\frac{c}{4} + \frac{\lambda}{2} - \frac{\delta(e_m+e_s)}{4} - \frac{1}{4}\right)\left(\frac{c}{4} - \frac{\lambda}{2} + \frac{3\delta(e_m+e_s)}{4} - \frac{1}{4}\right)\end{aligned} \tag{36}$$

For suppliers who adopt a traditional production strategy and also purchase authentication services, the profit of cooperating with low-carbon enterprises is

$$\pi_{s+i}^{nl-l} = (1-\gamma)(w-c)(1-p) + \gamma(w-c+\lambda)(1-p) - a \tag{37}$$

The profit of cooperating with traditional enterprises that have purchased authentication services is

$$\pi_{s+i}^{nl-nl} = \gamma^2(w-c+\lambda)(1-p) + (1-\gamma^2)(w-c)(1-p) - a \tag{38}$$

The expected profit of traditional suppliers who purchase authentication services is

$$\begin{aligned}\pi_{s+i}^{nl} &= (1-\gamma)(w-c)(1-p) + \gamma((n+\gamma(1-n))(w-c+\lambda)(1-p) + (1-n-\gamma(1-n)) \\ &\times [(n+\gamma(1-n))(w-c+\lambda)(1-p) + (1-n-\gamma(1-n)](w-c)(1-p)]) - a\end{aligned} \tag{39}$$

The profit function is explained as follows: traditional suppliers who purchase authentication services have a probability of $1-\gamma$ to deceive the authentication mechanism and be certified as low-carbon suppliers. After that, there is a probability of $n+\gamma(1-n)$ of encountering a manufacturer certified as low-carbon in the first match if they participate in the platform with low-carbon production strategies. If they are matched with a traditional manufacturer, they will reject the first match result and proceed to the second match. In the second match, there is still the same probability of matching with a manufacturer certified as low-carbon, and the second match result cannot be rejected.

The expected profit of traditional manufacturers who purchase authentication services is

$$\begin{aligned}\pi_{m+i}^{nl} &= (1-\gamma)(p-w)(1-p) + \gamma\Big((m+\gamma(1-m))(p-w+\lambda)(1-p) \\ &+ (1-m-\gamma(1-m))[(m+\gamma(1-m))(p-w+\lambda)(1-p) \\ &+ (1-m-\gamma(1-m))(p-w)(1-p)]\Big) - a\end{aligned} \tag{40}$$

Using the backward induction method to solve it, the optimal wholesale price w_{nl+i}^* and optimal selling price p_{nl+i}^* can be obtained. Then, π_{s+i}^{nl*} and π_{m+i}^{nl*} can also be obtained. Since the analytic expressions are too complex, we study these variables using numerical analysis in Section 5.

For the situation where the authentication service provided by the platform is imperfect, to enable low-carbon suppliers and manufacturers to participate in the online trading platform, it is necessary to satisfy $\pi_{s+i}^{l*} > \pi_{s}^{l*}$ and $\pi_{s+i}^{l*} > \pi_{s+i}^{nl*}$. Under this model, traditional enterprises may also join the platform and falsely claim that they are low-carbon. Taking traditional suppliers as an example, when $\pi_{s+i}^{nl*} > \pi_{s}^{nl*}$, traditional enterprises have the motivation to decide to purchase authentication services. To avoid this phenomenon, the authentication misjudgment probability and the government subsidy coefficient cannot be too high, and the authentication fee should not be too low.

Proposition 5. *By solving expressions, it can be obtained that the authentication fee for encouraging low-carbon production by suppliers needs to satisfy* $a_{s+i}^{nl*} < a_s < a_{s+i}^{l*}$; a_{s+i}^{l*} *is the solution of* $\pi_{s+i}^{l*}(a) - \pi_s^{l*} = 0$, *and* a_{s+i}^{nl*} *is the solution of* $\pi_{s+i}^{nl*}(a) - \pi_s^{nl*} = 0$. *The government subsidy coefficient for encouraging low-carbon production by manufacturers needs to satisfy* $a_{m+i}^{nl*} < a_m < a_{m+i}^{l*}$; a_{m+i}^{l*} *is the solution of* $\pi_{m+i}^{l*}(a) - \pi_m^{l*} = 0$, *and* a_{m+i}^{nl*} *is the solution of* $\pi_{m+i}^{nl*}(a) - \pi_m^{nl*} = 0$. *So, the value range of the authentication fee of the platform is* $a_i^* \in (max[a_{s+i}^{nl*}, a_{m+i}^{nl*}], min[a_{s+i}^{l*}, a_{m+i}^{l*}])$.

Proposition 5 indicates that the pricing of platform authentication fees should not be as low as possible. Instead, it should satisfy the range between the critical value for the participation of traditional manufacturing enterprises and the critical value for the participation of low-carbon production enterprises. The setting of a should both ensure that joining the platform does not become a dominant strategy for traditional enterprises, and make joining the platform a dominant strategy for low-carbon enterprises.

Proposition 6. $\frac{\partial min[a_{s+i}^{l*}, a_{m+i}^{l*}]}{\partial \gamma} < 0$, *the authentication cost is inversely proportional to the probability of misjudgment.*

Proposition 6 shows that the higher the false positive rate, the lower the authentication fee of the platform should be set. This is because when the probability of misjudgment γ is lower, P_1 and P_3 become larger, P_2 and P_4 become smaller, and the benefits brought by mutual cooperation between low-carbon enterprises in the platform become greater, and the authentication mechanism has a higher value. The authentication fee can be increased at this time, which is in line with our general perception. Managerial insights can be derived from this proposition: the platform should focus on improving its own authentication accuracy, investing in advanced technologies such as big data analytics to enhance the accuracy of authentication results, which can increase the economic benefits.

In model III, in order to encourage suppliers and manufacturers to be low-carbon, the government needs to set an appropriate low-carbon subsidy coefficient λ, satisfying $\pi_{s+i}^{l*} > \pi_s^{nl*}, \pi_{s+i}^{nl*}$ and $\pi_{m+i}^{l*} > \pi_m^{nl*}, \pi_{m+i}^{nl*}$.

Proposition 7. *By solving these expressions, it can be obtained that the government subsidy coefficient for encouraging low-carbon production by suppliers needs to satisfy* $\lambda_s > max[\lambda_s^*, \lambda_{s+i}^*]$; λ_{s+i}^* *is the solution of* $\pi_{s+i}^{l*}(\lambda) - \pi_{s+i}^{nl*}(\lambda) = 0$, *and* λ_s^* *is the solution of* $\pi_{s+i}^{l*}(\lambda) - \pi_s^{nl*}(\lambda) = 0$. *For manufacturers, the government subsidy coefficient needs to satisfy* $\lambda_m = max[\lambda_m^*, \lambda_{m+i}^*]$; λ_{m+i}^* *is the solution of* $\pi_{m+i}^{l*}(\lambda) - \pi_m^{l*}(\lambda) = 0$, *and* λ_m^* *is the solution of* $\pi_{m+i}^{nl*}(\lambda) - \pi_m^{nl*} = 0$. *So, the minimum effective subsidy coefficient of the government* $\lambda_i^* = (max[\lambda_s, \lambda_m])$.

5. Numerical Analysis

This section will conduct a numerical simulation analysis on the three game models established in the previous section, study the impact of changes in government subsidies on supply chain profits and enterprise decisions, and further validate the conclusions.

5.1. The Matching of the Supplier and Manufacturer without the Online Trading Platform

Model I mainly studies the influence of the government subsidy coefficient on the profit of members of the green supply chain and the optimal value of the emissions reduction level of the green supply chain. Considering the relationship between parameters, combined with Refs. [34,40], gives parameter values: $c = 0.2$, $\delta = 0.5$, $\mu = 0.8$, $m = 0.65$, and $n = 0.58$. These values all meet the assumptions and constraints of this paper. When a supplier of low-carbon production cooperates with a manufacturer of low-carbon production, the relationship between the total profit of the green supply chain and the enterprise's emissions reduction cost coefficient is studied. The simulation result is shown in Figure 3.

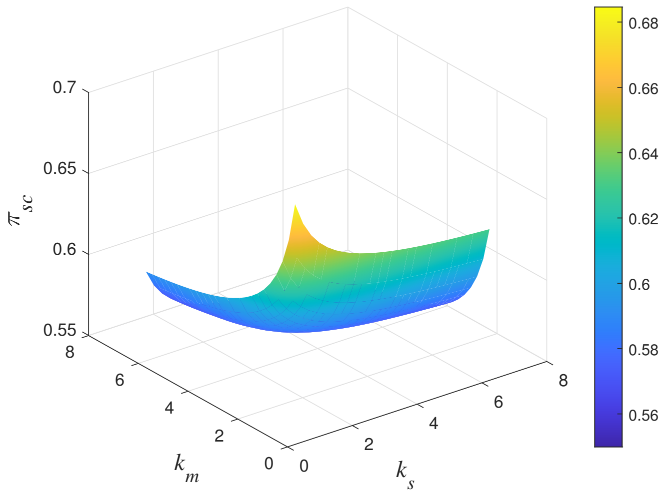

Figure 3. The relationship between the total profit of the supply chain and the emissions reduction cost coefficient of the supplier/manufacturer in model I.

Figure 3 shows the relationship between the total profit of the green supply chain and the emissions reduction cost coefficient under low-carbon production. According to Equation (7), the smaller the emissions reduction cost coefficient, the more efforts enterprises will make in emissions reduction, which will further promote the total profit of the supply chain. However, considering actual situations and Ref. [40], the emissions reduction cost coefficient often does not reach a very low value. Setting $k_s = 7$ and $k_m = 5$, in response to Proposition 1, the relationship between the total profit of the green supply chain and the enterprise's emissions reduction level is studied. The simulation result is shown in Figure 4.

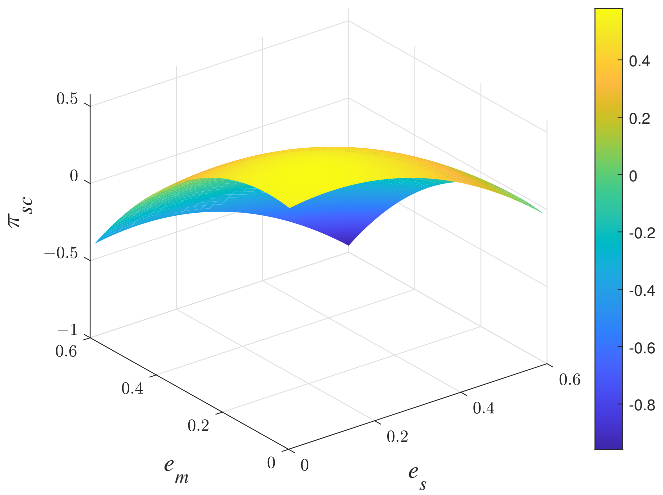

Figure 4. The relationship between the total profit of the supply chain and the emissions reduction level of the supplier/manufacturer in model I.

Figure 4 shows that in the absence of an online trading platform, the total profit of the green supply chain is affected by the emissions reduction levels of the enterprises. As the level of emissions reduction increases continuously, the total profit of the supply chain increases gradually. When $e_s = 0.1$ and $e_m = 0.08$, π_{sc} has a maximum value. Subsequently, the total profit of the supply chain decreases with an increase in emissions reduction level. Based on this conclusion, this paper investigates what value of government subsidy coefficient can encourage enterprises to adopt low-carbon production.

Figure 5 shows the profits of direct matching between suppliers and manufacturers without a platform. When the government subsidy coefficient is greater than 0.37, suppliers will obtain higher profits from low-carbon production than traditional production. When the government subsidy coefficient is greater than 0.66, manufacturers will be more willing to adopt low-carbon production, which is conducive to the formation of a green supply chain.

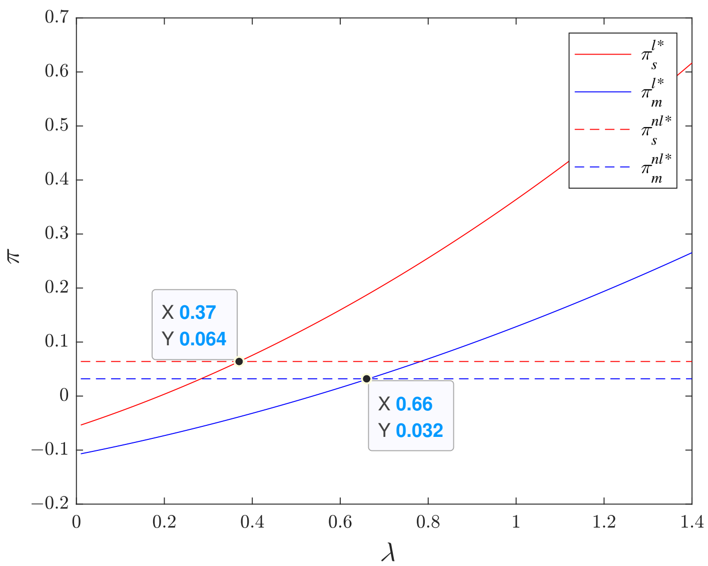

Figure 5. The impact of changes in government subsidy coefficients on supply chain profit in model I.

5.2. The Matching of the Supplier and Manufacturer without the Online Trading Platform

Model II mainly studies the changes in decision making and profits of the supply chain members after the introduction of an online trading platform. It also investigates how the government should set subsidies to better promote the formation of a green supply chain in this scenario.

To better compare with model I, this part uses the same numerical values as model I for simulation analysis. The government subsidy coefficient is set to $\lambda = 0.6$, and the impact of the pricing of platform authentication fees on the profits of the supply chain members is studied.

Figure 6 shows that the optimal authentication fee a for encouraging enterprises to participate in the platform is 0.08, which verifies Proposition 2.

For the authentication fee, considering the practical significance of the value and Ref. [15], let $a = 0.08$. In the matching platform, the relationship between the profit function of each member of the green supply chain and the government subsidy coefficient is studied.

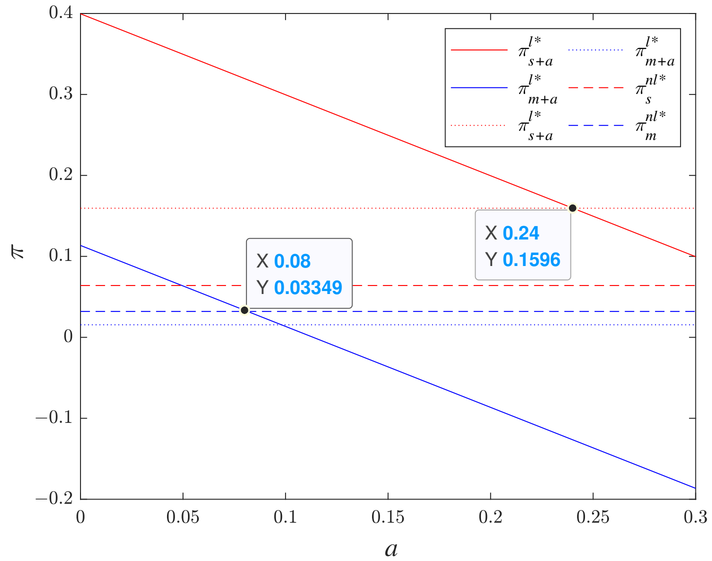

Figure 6. The impact of changes in authentication fee on supply chain profit in model II.

Figure 7 shows that the minimum effective subsidy coefficient that encourages enterprises to adopt low-carbon production is 0.60, which is significantly lower than 0.66 in model I, which verifies Proposition 3.

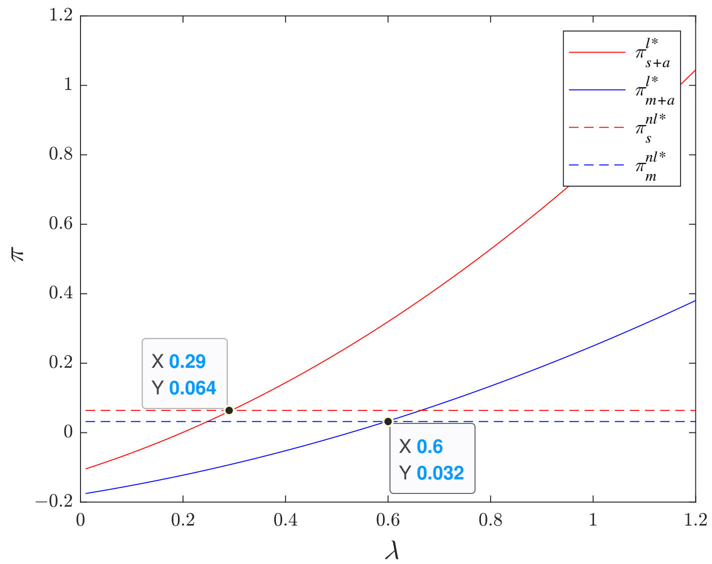

Figure 7. The impact of changes in government subsidy coefficients on supply chain profit in model II.

Under the green supply chain, the impact of the emissions reduction level on the setting of the government subsidy coefficient is studied, the proportion of the manufacturer's emissions reduction level is set to 0.10, and the interval of the supplier's emissions reduction level is set to (0, 0.25).

Figure 8 shows that no matter whether there is a platform or not, with the increase in emissions reduction level, the total profit of the green supply chain shows a trend of first increasing and then decreasing. Meanwhile, we find that the optimal emissions reduction level of the green supply chain in the matching platform is higher than that in model I, and the total profit of the supply chain is also higher.

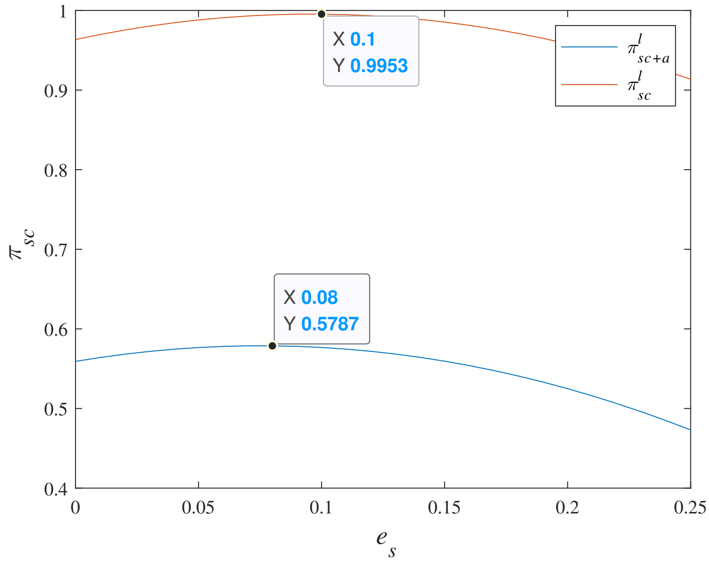

Figure 8. Comparison of optimal emissions reduction levels in green supply chains in model I and model II.

5.3. The Online Trading Platform Provides Imperfect Authentication Service

Model III studies the impact of changes in authentication fees and low-carbon subsidy coefficients on the profits of suppliers and manufacturers when the authentication mechanism of the platform is imperfect, and how to set authentication fees and government subsidies more effectively under this situation. We set the authentication misjudgment probability $\gamma = 0.2$ to study the impact of the platform authentication fee on suppliers' and manufacturers' profits.

Figures 9 and 10 show that the authentication fee should be within the range of $0.09 < a_s < 0.23$ in order to encourage low-carbon suppliers to participate in the platform. And to encourage low-carbon manufacturers to participate in the platform, the authentication fee should be within the range of $0.06 < a_m < 0.1$. So, the optimal range for the platform's authentication fee is in the range of (0.09, 0.10). This proves Proposition 5.

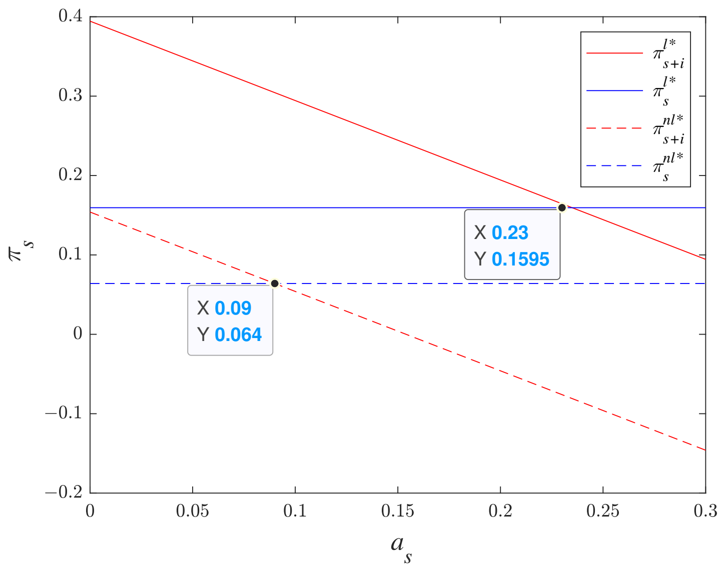

Figure 9. The impact of changes in authentication fees on supplier's profit in model III.

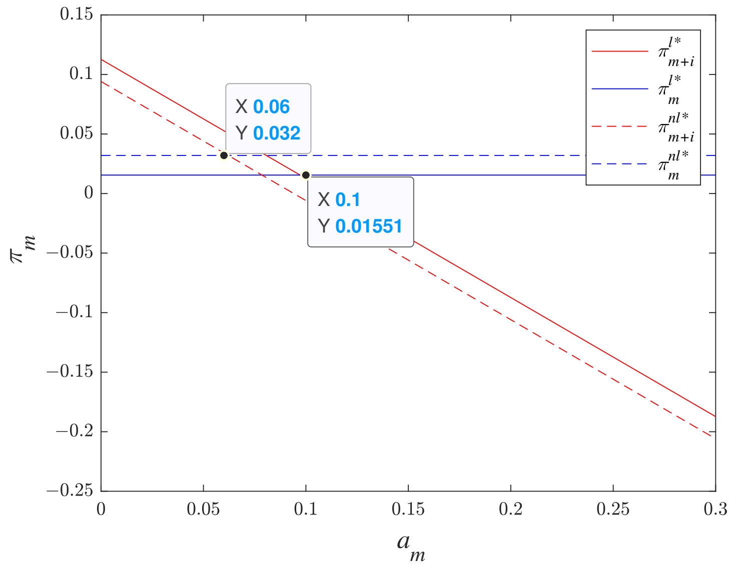

Figure 10. The impact of changes in authentication fees on manufacturer's profit in model III.

Comparing Figure 6 with Figures 9 and 10, it can be found that the optimal authentication fee of the platform has increased under imperfect authentication, which is conducive to achieving a "separating equilibrium" between traditional enterprises and low-carbon enterprises, providing incentives for low-carbon enterprises rather than traditional enterprises, so that they are more willing to participate in the platform.

The managerial implications are as follows: when the platform performs low-carbon authentication, the authentication mechanism is not always perfect. But this does not mean that the platform needs to lower its authentication fees. Inhibiting the "false claims" behavior of traditional enterprises and promoting environmental benefits should be the platform's primary considerations when setting authentication fees. Although a lower fee can attract more enterprises to join the platform, it may reduce the entry barriers for traditional enterprises. The platform needs to balance environmental and economic goals, set a reasonable authentication fee, and provide differentiated services. This can incentivize low-carbon enterprises, restrain traditional enterprises' misconduct, and promote the development of the green supply chain. Simultaneously, it is also conducive to establishing a good image for the platform.

Let the authentication fee $a = 0.08$, while keeping other parameters constant, to study the impact of the government subsidy on the profits of suppliers and manufacturers.

Figures 11 and 12 show that the government subsidy coefficient must satisfy $\lambda > 0.31$ in order to encourage low-carbon production by suppliers. And to encourage low-carbon production by manufacturers, the government subsidy coefficient must satisfy $\lambda > 0.6$. Therefore, the government's minimum effective low-carbon subsidy coefficient is 0.6. At the same time, it is also found that when the government subsidy is large enough, traditional enterprises can be encouraged to participate in the platform. This proves Proposition 7.

Comparing with Figures 5 and 7, it can be found that when the platform provides imperfect authentication, the government subsidy will be negligibly higher than under perfect authentication, but still lower than without the platform.

The managerial implications are as follows: the government should collaborate with the platform and help the development of the platform. This can help the government to encourage more enterprises to adopt low-carbon production, with less subsidy expenditure. Although imperfect authentication increases the subsidy required compared to perfect authentication, using an online platform is still beneficial overall. Therefore, the government should actively support the platform's technology advancement to improve authentication accuracy. Meanwhile, a reasonable subsidy setting needs to account for the misjudgment risk. With joint efforts from the government and platform, low-carbon development of the supply chain can be promoted in a more cost-effective manner.

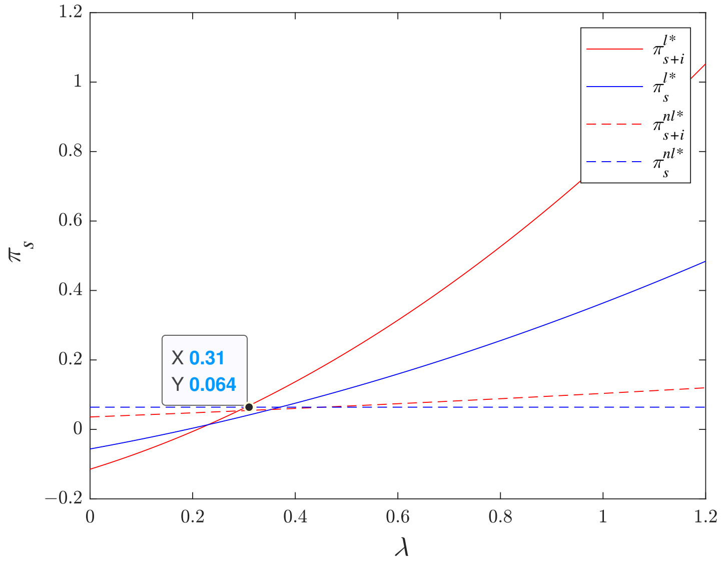

Figure 11. The impact of changes in the government subsidy coefficient on the supplier's profit in model III.

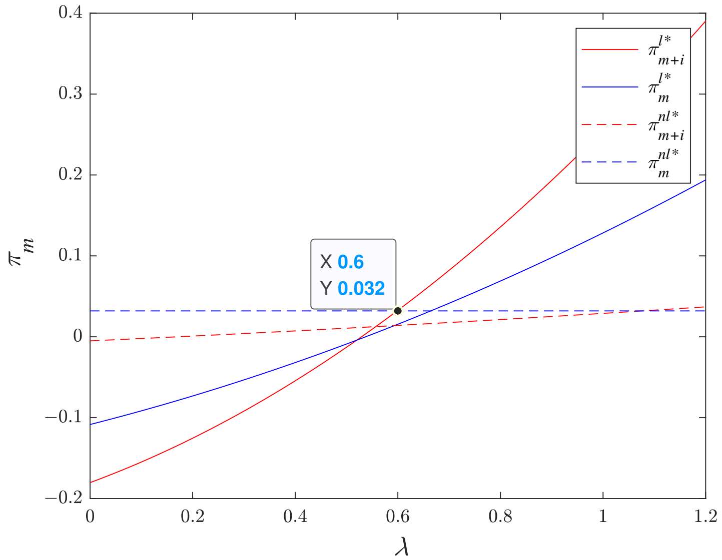

Figure 12. The impact of changes in the government subsidy coefficient on the manufacturer's profit in model III.

6. Conclusions and Future Research

For the two-level supply chain composed of a supplier and a manufacturer, this paper considers the three situations of no online trading platform, an online trading platform with perfect authentication, and an online trading platform with imperfect authentication, and establishes game models for these three scenarios. Each model analyzes the profit functions of suppliers and manufacturers when choosing different production methods, and considers the optimal expected profit under the Stackelberg game. At the same time, the impact of the introduction of the platform on the profit and decision making of supply chain members is compared, and how to set the government's low-carbon subsidy coefficient more effectively to motivate enterprises to low-carbon production. Through the comparative analysis of the game profit functions in these three situations, the following conclusions are drawn:

1. In the absence of an online trading platform, the government needs to give low-carbon subsidies to enterprises to encourage suppliers and manufacturers to adopt low-carbon strategies and cooperate to achieve Pareto improvement of corporate profits. When low-carbon suppliers and low-carbon manufacturers cooperate with each other to form a green supply chain, the higher emissions reduction levels of enterprises can bring higher low-carbon benefits, but also brings higher emissions reduction costs. Therefore, the emissions reduction level of suppliers and manufacturers should be moderate, which is more conducive to the overall benefits of the green supply chain.
2. In the case of an online trading platform that provides perfect authentication, the introduction of the platform makes it possible for low-carbon suppliers to match with low-carbon manufacturers, which can improve the expected profit of members of the green supply chain and help members of the supply chain to increase their emissions reduction level. The government can provide fewer subsidies than in model I, so that suppliers and manufacturers can choose the low-carbon production strategy as the dominant strategy, which is conducive to the formation of a green supply chain, which shows that the introduction of the platform can reduce the government subsidy expenditure.
3. When the online trading platform provides imperfect authentication, there is a special situation that needs to be considered: enterprises adopting traditional production may also participate in the platform. Therefore, when setting authentication fees, the platform should not only make more enterprises participate in the platform, but also prevent traditional enterprises from imitating low-carbon enterprises to falsely claim government subsidies.

Meanwhile, some management implications can be derived: while encouraging the development of the green supply chain, the government can consider cooperating with the online platform to achieve a win–win situation in both economic and environmental benefits. The government also needs to guide the platform to establish the consciousness of green environmental protection and build a more comprehensive green authentication mechanism. Enterprises should pay attention to investing in green innovation technologies to reduce emissions reduction costs and enhance emissions reduction efforts. A good environmental reputation will make them more favored by consumers, thus gaining economic benefits while fulfilling their social responsibilities.

This paper provides effective suggestions for the government to set subsidy coefficients in different situations, and provides a reference for the pricing of authentication fees for an online matching platform. Reasonable pricing can motivate enterprises to adopt low-carbon manufacturing and participate in the online trading platform to promote the development of the green supply chain. Both the government and the platform should take into account the triple impact of environment, society, and economy, implementing reasonable pricing and incentive methods to promote the low-carbon development of the supply chain. Enterprises should also proactively adopt appropriate emissions reduction measures to achieve a win–win situation for both economic and environmental benefits. However, there are certain deficiencies in the research process. This paper assumes that as long as enterprises are low-carbon, they can obtain government subsidies, but the emissions reduction levels of low-carbon enterprises is not considered. In the future, it will be considered whether the emissions reductions of low-carbon enterprises meet the government's requirements, and the levels of corporate emissions reductions is used as a decision variable for further research. At present, some studies consider the benefits of such platforms. This paper does not expand in detail because the research focus is on encouraging low-carbon production of enterprises. Furthermore, in recent years, the impact of the supply chain on social welfare and consumer surplus has also been a research focus. This paper only provides a partial discussion on this matter. In the future, we can conduct detailed research on social welfare, etc., and put forward better suggestions for the pricing of government subsidies and platform authentication fees, and further improve the decision-making model.

Author Contributions: Conceptualization, W.G. and J.L.; methodology, W.G.; software, W.G.; validation, W.G., J.L. and T.H.; formal analysis, W.G.; investigation, W.G.; writing—original draft preparation, W.G. and T.H.; writing—review and editing, H.Z. and J.L.; visualization, W.G.; supervision, H.Z.; project administration, J.L. and H.Z.; funding acquisition, J.L. and H.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant number 72171173, and Tongji University "Innovative Design and Intelligent Manufacturing Discipline Group" project, grant number F2206.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data sharing not applicable. No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Liu, Z.; Qian, Q.; Hu, B.; Shang, W.L.; Li, L.; Zhao, Y.; Zhao, Z.; Han, C. Government regulation to promote coordinated emission reduction among enterprises in the green supply chain based on evolutionary game analysis. *Resour. Conserv. Recycl.* **2022**, *182*, 106290. [CrossRef]
2. Jiang, Y.; Hu, Y.; Asante D.; Ampaw, E.M.; Asante, B. The Effects of Executives' low-carbon cognition on corporate low-carbon performance: A study of managerial discretion in China. *J. Clean. Prod.* **2022**, *357*, 132015. [CrossRef]
3. Olimid, D. A.; Olimid, A. P. EU Policies on Learning Environmental Sustainability and Climate Change: What Social Implications for Human and Organizational Development and Resilience? *Rev. Stiinte Politice* **2022** , *73*, 114–121.
4. Jones, C. *The Net-Zero Industry Act and the Reform of the Green Deal State Aid Rules: A Convincing Reaction to the Inflation Reduction Act*? European University Institute: Florence, Italy, 2023.
5. Szasz, J. Which approaches to climate policy decrease carbon dioxide emissions? Evidence from US states, 1997–2017. *Energy Res. Soc. Sci.* **2023**, *97*, 102969. [CrossRef]
6. Azargohar, R.; Dalai, A.; Hassanpour, E.; Moshiri, S. Agri-pellets as alternative fuels for coal-fired power plants in Canada. *Int. J. Energy Sect. Manag.* **2021**, *16*, 876–898. [CrossRef]
7. Han. J.; Kong, L.; Wang, W.; Xie, J. Motivating individual carbon reduction with saleable carbon credits: Policy implications for public emission reduction projects. *Ind. Manag. Data Syst.* **2022**, *122*, 1268–1305. [CrossRef]
8. Shi, X.; Qian, Y.; Dong, C. Economic and environmental performance of fashion supply chain: The joint effect of power structure and sustainable investment. *Sustainability* **2017**, *9*, 961. [CrossRef]
9. Ji, J.; Zhang, Z.; Yang, L. Carbon emission reduction decisions in the retail-/dual-channel supply chain with consumers' preference. *J. Clean. Prod.* **2017**, *141*, 852–867. [CrossRef]
10. Liu, M.; Li, Z.; Anwar, S.; Zhang, Y. Supply chain carbon emission reductions and coordination when consumers have a strong preference for low-carbon products. *Environ. Sci. Pollut. Res.* **2021**, *28*, 19969–19983. [CrossRef]
11. Dzikriansyah, M.A.; Masudin, I.; Zulfikarijah, F.; Jihadi, M.; Jatmiko, R.D. The role of green supply chain management practices on environmental performance: A case of Indonesian small and medium enterprises. *Clean. Logist. Supply Chain* **2023**, *6*, 100100. [CrossRef]
12. Zou, H.; Qin, J.; Zheng, H. Equilibrium pricing mechanism of low-carbon supply chain considering carbon cap-and-trade policy. *J. Clean. Prod.* **2023**, *407*, 137107. [CrossRef]
13. Zou, H.; Qin, J.; Zheng, H. Analysis of low carbon subsidies under different supply chain structures. In Proceedings of the 2015 12th International Conference on Service Systems and Service Management (ICSSSM), Guangzhou, China, 22–24 June 2015; Volume 1, pp. 1–6.
14. Basu, A.; Bhaskaran, S.R.; Mukherjee, R. Best or Right?—Positioning and Authentication in Online Matching Platforms 2021. Available online: https://questromworld.bu.edu/platformstrategy/wp-content/uploads/sites/49/2021/07/PlatStrat2021_paper_26.pdf (accessed on 1 August 2023).
15. Basu, A.; Bhaskaran, S.; Mukherjee, R. An analysis of search and authentication strategies for online matching platforms. *Manag. Sci.* **2019**, *65*, 2412–2431. [CrossRef]
16. Su, C.; Liu, X.; Du, W. Green supply chain decisions considering consumers' low-carbon awareness under different government subsidies. *Sustainability* **2020**, *12*, 2281. [CrossRef]
17. Batrancea, L.; Pop, M.C.; Rathnaswamy, M.M.; Batrancea, I.; Rus, M.I. An empirical investigation on the transition process toward a green economy. *Sustainability* **2021**, *13*, 13151. [CrossRef]

18. Batrancea, L.M.; Tulai, H. Thriving or Surviving in the Energy Industry: Lessons on Energy Production from the European Economies. *Build. Environ.* **2022**, *15*, 8532. [CrossRef]
19. Guliyev, H.; Tatoğlu, F.Y. The Relationship between Renewable Energy and Economic Growth in European Countries: Evidence from Panel Data Model with Sharp and Smooth Changes. *Renew. Energy Focus* **2023**, *46*, 185–196. [CrossRef]
20. Andrea, B. Using simulation models for green economy policy making: A comparative assessment. *Rev. Bus. Econ. Stud.* **2014**, *1*, 88–99.
21. Wang, W.; Hao, S.; He, W.; Mohamed, M.A. Carbon emission reduction decisions in construction supply chain based on differential game with government subsidies. *Build. Environ.* **2022**, *222*, 109149. [CrossRef]
22. Kang, K.; Zhao, Y.; Zhang, J.; Qiang, C. Evolutionary game theoretic analysis on low-carbon strategy for supply chain enterprises. *J. Clean. Prod.* **2019**, *230*, 981–994. [CrossRef]
23. Meng, Q.; Li, M.; Liu, W.; Li, Z.; Zhang, J. Pricing policies of dual-channel green supply chain: Considering government subsidies and consumers' dual preferences. *Sustain. Prod. Consum.* **2021**, *26*, 1021–1030. [CrossRef]
24. Sun, H.; Wan, Y.; Zhang, L.; Zhou, Z. Evolutionary game of the green investment in a two-echelon supply chain under a government subsidy mechanism. *J. Clean. Prod.* **2019**, *235*, 1315–1326. [CrossRef]
25. Meng, Q.; Wang, Y.; Zhang, Z.; He, Y. Supply chain green innovation subsidy strategy considering consumer heterogeneity. *J. Clean. Prod.* **2021**, *281*, 125199. [CrossRef]
26. Huang, S.; Fan, Z.P.; Wang, N. Green subsidy modes and pricing strategy in a capital-constrained supply chain. *Transp. Res. Part E Logist. Transp. Rev.* **2020**, *136*, 101885. [CrossRef]
27. Bian, J.; Zhang, G.; Zhou, G. Manufacturer vs. consumer subsidy with green technology investment and environmental concern. *Eur. J. Oper. Res.* **2020**, *287*, 832–843. [CrossRef]
28. Rochet, J.C.; Tirole, J. Platform competition in two-sided markets. *J. Eur. Econ. Assoc.* **2003**, *1*, 990–1029. [CrossRef]
29. Zhang, X.; Sui, R.; Dan, B.; Guan, Z. Bilateral value-added services and pricing strategies of the third-party platform considering the cross-network externality. *Comput. Ind. Eng.* **2021**, *155*, 107196. [CrossRef]
30. Han, T.; Lu, J.; Zhang, H. Two-sided matching model of service providers and demanders considering peer and synergy effects. *Heliyon* **2023**, *9*, e14732. [CrossRef]
31. Jung, J.; Lim, H.; Lee, D.; Kim, C. The secret to finding a match: A field experiment on choice capacity design in an online dating platform. *Inf. Syst. Res.* **2022**, *33*, 1248–1263. [CrossRef]
32. Ghani, E.; Kerr, W.R.; Stanton, C. Diasporas and outsourcing: Evidence from Desk and India. *Manag. Sci.* **2014**, *60*, 1677–1697. [CrossRef]
33. Nie, T.; Guan, S.; Du, S.; Zhu, S Pricing strategy selection for the sharing platform with heterogeneous laborers. *Comput. Ind. Eng.* **2022**, *169*, 108241. [CrossRef]
34. Liu, H.; Kou, X.; Xu, G.; Qiu, X.; Liu, H. Which emission reduction mode is the best under the carbon cap-and-trade mechanism? *J. Clean. Prod.* **2021**, *314*, 128053. [CrossRef]
35. Zhang, C.; Chen, J.; Raghunathan, S. Two-sided platform competition in a sharing economy. *Manag. Sci.* **2022**, *68*, 8909–8932. [CrossRef]
36. Zhu, G.; You, D. Ecological R&D Strategy and Coordination Mechanism in the Green Supply Chain Based on Differential Games. *Oper. Res. Manag. Sci.* **2017**, *26*, 62–69.
37. Wan, X.; Li, Q.; Qiu, L.; Du, Y. How do carbon trading platform participation and government subsidy motivate blue carbon trading of marine ranching? A study based on evolutionary equilibrium strategy method. *Mar. Pol.* **2021**, *130*, 104567. [CrossRef]
38. Vosooghidizaji, M.; Taghipour, A.; Canel-Depitre, B. Coordinating corporate social responsibility in a two-level supply chain under bilateral information asymmetry *J. Clean. Prod.* **2022**, *364*, 132627. [CrossRef]
39. Li, J.; Su, Q.; Lai, K.K. The research on abatement strategy for manufacturer in the supply chain under information asymmetry. *J. Clean. Prod.* **2019**, *236*, 117514. [CrossRef]
40. Chen, X.; Wang, X.; Chan, H.K. Manufacturer and retailer coordination for environmental and economic competitiveness: A power perspective. *Transp. Res. Part E Logist. Transp. Rev.* **2017**, *97*, 268–281. [CrossRef]
41. Cohen, M.C.; Lobel, R.; Perakis, G. The impact of demand uncertainty on consumer subsidies for green technology adoption. *Manag. Sci.* **2016**, *62*, 1235–1258. [CrossRef]