

Supporting Information for “Antarctic Basal Water Storage Variations Inferred from Multi-source Satellite Observation and Relevant Model”

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S1. Gravity forward modelling

Gravity forward modelling is used to convert the volume variations and associated density data into gravity variations in each layer of Antarctic ice sheets (AIS). In this study, we adopt a triangular prism division method that fits topography without gaps. For this purpose, surface of ellipsoid is divided into many triangles with sides approximately 50 km. The geodetic heights and their variations, as well as the associated density in each layer of AIS, are sampled according to the location (latitude and longitude) of corner points of each triangular (Figure. S1). The sampled height geodetic heights (AIS’s basal and surface heights are available from BEDMAP2 [1]) and their variations of each triangular corner points can be approximated as a triangular prism whose three sides are perpendicular to the local ellipsoidal plane, while the upper and lower faces are nonparallel to each other (Figure S2(A)).

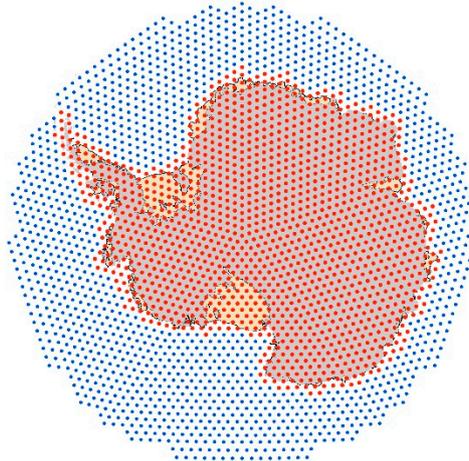


Figure S1. Locations of corner points of triangles on surface of ellipsoid. Grey regions are AIS, yellow regions are Antarctic ice shelves. Red points are corner points of 10314 triangles used for calculating gravity variations in each layer of AIS, and these corner points are extended 1° outward for reducing possible leakage-out errors of gravity signals.

For the accuracy of the conversion of volume variations to gravity variations, we perform a coordinate transformation in each gravity calculation point before the gravity forward modelling calculation. The specific steps are: 1. converting the geodetic coordinates of the corner points of each triangular to geocentric coordinates; 2. shifting the Z-axis of the geocentric coordinate system ($O_g-X_gY_gZ_g$ in Figure S2(A)) to the normal direction in each triangular prism; 3. moving the coordinate origin of the geodetic coordinate system to the location of gravity calculation point, to obtain the new local coordinate system ($O-XYZ$ in Figure S2(B)); 4. for the triangular prisms at distance greater than 100 km from the gravity calculation point, we calculate the angle between the ellipsoidal normal directions of each triangular prism and the gravity calculation point, to address the ellipsoidal corrections.

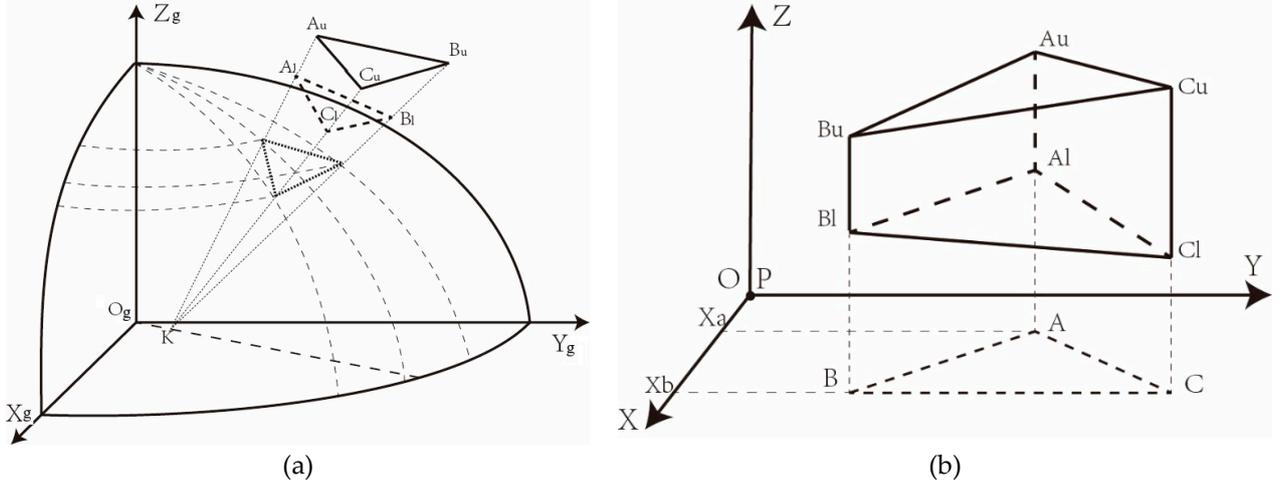


Figure S2. (A) Volume variation of single triangular prism AuBuCu-AlBIcl in geocentric coordinate system $O_g-X_gY_gZ_g$ and (B) in local coordinate system $O-XYZ$. AuBuCu and AlBIcl are upper and lower surfaces of the triangular prism. Au-Al, Bu-BI, Cu-Cl are sampled height variations. Z-direction in (B) is perpendicular to the local ellipsoidal plane. The gravity calculation point P in (B) is placed at the local coordinate system's origin O.

In local coordinate system, gravity variations of P is expressed by the modified method of Zhou, *et al.* [2]:

$$dg_p = -G\rho \cos \theta \iiint \frac{z}{(x^2 + y^2 + z^2)^{3/2}} dx dy dz \quad (S1)$$

where dg_p are gravity variations in AIS layers, G is the gravitational constant, ρ are densities of the triangular prism, θ are angles between the ellipsoidal normal directions of each triangular prism and gravity calculation point, x, y, z are coordinate of the triangular prism in local coordinate system.

Volume integral in Equation S1 can be converted into surface integral, accordingly, gravity variations components in different Antarctic layers are expressed as the sum of gravity variations of triangular prism [2]:

$$dg_p = \sum_{k=1}^N G \cos \theta_k \rho_k \left[\iint \frac{1}{(x_k^2 + y_k^2 + z_{k,l}^2)^{3/2}} dx_k dy_k - \iint \frac{1}{(x_k^2 + y_k^2 + z_{k,u}^2)^{3/2}} dx_k dy_k \right] \quad (S2)$$

where N is the number of triangular prisms within the integrated radius (500km), $z_{k,l}$ and $z_{k,u}$ are lower/upper surface height of each triangular prisms.

S2. Layered gravity density inversion based on the forward modelling

The layered gravity density inversion method is developed to estimate the basal mass balance (BMB) of AIS. For simplification, BMB are assumed to occur within a fixed thickness layer in Antarctic ice-bed interface, and the associated densities are variables that need to be estimated. According to Equation S2, BMB-induced gravity variations dg_{BMB} can be expressed as follow:

$$dg_{BMB} = \sum_{k=1}^N G \cos \theta_k \rho_k \left[\iint \frac{1}{(x_k^2 + y_k^2 + z_k^2)^{\frac{1}{2}}} dx_k dy_k - \iint \frac{1}{(x_k^2 + y_k^2 + (z_k + H)^2)^{\frac{1}{2}}} dx_k dy_k \right] \quad (S3)$$

where H is the thickness (0.01m in this study) of the layer that occurring BMB, ρ_k are associated densities that need to be estimated, z_k and $z_k + H_k$ are lower/upper surface height of each triangular prisms.

BMB are known to be mainly caused by basal water migration beneath AIS, and can be expressed by the equivalent water thickness (EWH) [3]:

$$\rho_k H = \rho_w H_k \quad (S4)$$

where ρ_w is water density, H_k are EWH of BMB.

Then, Equation S3 & S4 could be modified as follows:

$$\frac{dg_{BMB} H}{G \rho_w} = \sum_{k=1}^N H_k \cos \theta_k \left[\iint \frac{1}{(x_k^2 + y_k^2 + z_k^2)^{\frac{1}{2}}} dx_k dy_k - \iint \frac{1}{(x_k^2 + y_k^2 + (z_k + H)^2)^{\frac{1}{2}}} dx_k dy_k \right] \quad (S5)$$

For M gravity points ($M=10314$), Equation S5 can be expressed as:

$$d = BX \quad (S6)$$

where

$$d = \begin{bmatrix} \frac{dg_{BMB_1} H}{G \rho_w} \\ \vdots \\ \frac{dg_{BMB_M} H}{G \rho_w} \end{bmatrix}; B = \begin{bmatrix} T_{11} & \dots & T_{1N} \\ \vdots & \ddots & \vdots \\ T_{M1} & \dots & T_{MN} \end{bmatrix}; X = \begin{bmatrix} H_1 \\ \vdots \\ H_N \end{bmatrix}$$

$$T_{MN} = \cos \theta_{mn} \left[\iint \frac{1}{(x_{mn}^2 + y_{mn}^2 + z_{mn}^2)^{\frac{1}{2}}} dx_{mn} dy_{mn} - \iint \frac{1}{(x_{mn}^2 + y_{mn}^2 + (z_{mn} + H)^2)^{\frac{1}{2}}} dx_{mn} dy_{mn} \right]$$

Then the conjugate gradient method is employed to obtain the EWH of BMB (that is, X in Equation S6).

S3. Surface and bedrock elevation and basal hydraulic potential for the Antarctic continent

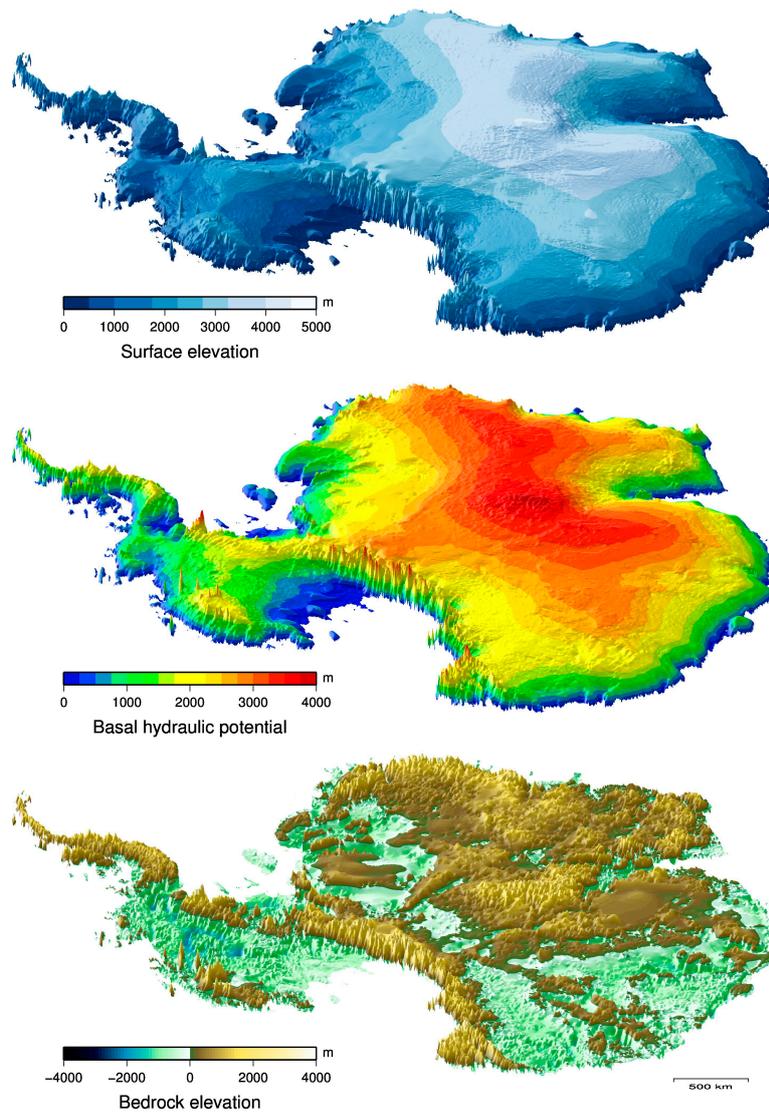


Figure S3. Surface and bedrock elevation and calculated basal hydraulic potential for the Antarctic continent [4]. The hydraulic potential is mainly influenced by the surface elevation but shows the attenuated imprint of the bedrock topography. The hydraulic potential is reflecting the combined influence of bedrock elevation and ice pressure and lies well defined above sea level. Its elevation is mainly governed by the elevation of the ice sheet but it does not show the smoothness of the ice surface.

References

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4. Göeller, S. Antarctic subglacial hydrology-interactions of subglacial lakes, basal water flow and ice dynamics. State and University Library of Bremen, 2014.