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Article

A Two-Period Game Theoretic Model of Zero-Day Attacks with Stockpiling

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Abstract: In a two-period game, Player 1 produces zero-day exploits for immediate deployment or stockpiles for future deployment. In Period 2, Player 1 produces zero-day exploits for immediate deployment, supplemented by stockpiled zero-day exploits from Period 1. Player 2 defends in both periods. The article illuminates how players strike balances between how to exert efforts in the two periods, depending on asset valuations, asset growth, time discounting, and contest intensities, and when it is worthwhile for Player 1 to stockpile. Eighteen parameter values are altered to illustrate sensitivity. Player 1 stockpiles when its unit effort cost of developing zero-day capabilities is lower in Period 1 than in Period 2, in which case it may accept negative expected utility in Period 1 and when its zero-day appreciation factor of stockpiled zero-day exploits from Period 1 to Period 2 increases above one. When the contest intensity in Period 2 increases, the players compete more fiercely with each other in both periods, but the players only compete more fiercely in Period 1 if the contest intensity in Period 1 increases.

Keywords: game; cybersecurity; zero-days; stockpiling; production; attack; defense

1. Introduction

1.1. Background

Zero-day attacks are becoming increasingly common. The most well-known attack, utilizing the Stuxnet worm to exploit four zero-day vulnerabilities, is probably the 2010 attack on the Natanz nuclear facility in Iran [1]. A so-called zero-day vulnerability means that a defender's vulnerability in its computer system is known to the defender for zero days before it is discovered, most commonly through an attack. Zero-day attacks require resources and are challenging to produce. Once produced, the next challenge is whether to deploy them immediately or stockpile them for deployment at some suitable future point in time. Stockpiling can be useful for a player in providing security in the knowledge that threats posed by an opposing player can be ameliorated or eliminated. A more recent zero-day attack targeted Microsoft Windows in Eastern Europe in June 2019 [2]. The exploit abused a local privilege escalation vulnerability in Microsoft Windows pertaining to the NULL pointer dereference in the win32k.sys component (a NULL pointer dereference is an error causing a segmentation fault, which occurs when a program tries to read or write to memory with a NULL pointer). For other recent zero-day attacks, see PhishProtection [3].

1.2. Contribution

This article intends to capture the general aspects of this phenomenon, which are that a defender has an asset it seeks to defend, while the attacker seeks to attack the asset over two periods—by attacking and stockpiling in Period 1, and attacking and utilizing the stockpile in Period 2. A variety of reasons and justifications for stockpiling are illustrated. A two-player two-period game is analyzed. Player 1 is equipped with resources in Period 1, which can be utilized for producing zero-day exploits for immediate deployment in Period 1 or stockpiled for future deployment in Period 2. Player 2 defends against the attack in Period 1. Zero-day exploits may become more valuable if the stakes involved in their deployment increase, but this also entails the risk of becoming obsolete, e.g., if knowledge of their content leaks. We thus assume that Player 1's stockpiled zero-day exploits may appreciate or depreciate in value from Period 1 to Period 2, i.e., the stockpiled zero-day exploits may become more or less valuable. Such changes in value may be due to technological, economic, or societal factors, market conditions, or the players' preferences. In Period 2, Player 1 produces new zero-day exploits for immediate deployment in Period 2 and also deploys its stockpiled zero-day exploits. In Period 2, the defender defends against the attack, i.e., against both the zero-day exploits produced by Player 1 in Period 2 and the appreciated or depreciated zero-day exploits stockpiled from Period 1 to Period 2. The presence of Period 2 enables Player 1 to strike a balance between whether or not to stockpile in Period 1, and both players strike balances between how to exert efforts in both periods.

The research questions are how the attacking Player 1 allocates its resources between immediate zero-day attack in Period 1 and stockpiling for attack in Period 2, how the defender defends in both periods, and how the players' strategic choices in both periods depend on the model characteristics, i.e., Player 1's available resources, the contest intensities in both periods, the zero-day appreciation factor from Period 1 to Period 2, and both players' unit costs of effort, asset valuations, and time discount factors. Players in a cyberwar are always in a contest, regardless of the extent to which they understand the particulars of the contest, which justifies the use of the widely applied contest success function. The model in this article is applicable beyond zero-day vulnerabilities, assuming one attacking player and one defending player over two periods, where the attacking player can stockpile its capabilities from Period 1 to Period 2.

1.3. Literature

Aside from Hausken and Welburn [4] and, in part, Chen et al. [5], considered in Section 1.3.1, the literature has not directly considered the research questions in this article but has instead focused on various indirectly linked research questions, as shown in the subsequent subsections below. The literature on zero-day attacks is mostly concerned with detecting, mitigating, understanding, and simulating zero-day attacks. Most of the articles below have been identified by searching for the two words "zero-day" on the Web of Science database for the most recent years. Regarding zero-day vulnerabilities and their exploits, see Ablon and Bogart [6].

1.3.1. Game Theoretic Analyses

In earlier research, Hausken and Welburn [4] considered a one-period game theoretic model of zero-day cyber exploits, incorporating the benefit of stockpiling into the same period as when production and zero-day attack are determined. They found, for example, that decreasing Cobb Douglas output elasticity for a player's stockpiling causes its attack to increase and its expected utility to eventually reach a maximum, while the opposing player's expected utility reaches a minimum. Chen et al. [5] analyzed whether two countries should disclose or not disclose to the vendor the hardware/software vulnerabilities they discover in a repeated game. Disclosing may benefit the country if it gets exposed by the vulnerability. Not disclosing may benefit the country's defense given that the other country does not discover the vulnerability and is exposed by it. They develop an algorithm and find that countries benefit from discovering vulnerabilities quickly and from incurring low costs of developing exploits.

1.3.2. Detection, Prioritization, Ranking, and Classification

Singh et al. [7] realized the challenge in defending against zero-day attacks. They proposed a framework for detection and prioritization based on likelihood by identifying the zero-day attack path and ranking the severity of the vulnerability. [8] developed a detection model for crypto-ransomware zero-day attacks. The model is based on an anomaly-based estimator, which suffers from high rates of false alarms, supplemented by behaviorally-based classifiers. Venkatraman and Alazab [9] reviewed existing visualization techniques for zero-day malware and designed a visualization using a similarity matrix method for classifying malware.

1.3.3. Detection and Identification by Applying Probability Theory and Statistics

Sun et al. [10] acknowledged the information asymmetry between attackers and defenders and applied Bayesian networks for identifying zero-day attack paths probabilistically; this is intended to be superior to targeting individual zero-day exploits. Parrend et al. [11] presented a framework for characterizing zero-day attacks and multistep attacks and relevant countermeasures. They applied rule-based and outlier-detection-based statistical solutions and machine learning, which detects behavioral anomalies and tracks event sequences. Singh et al. [12] proposed a hybrid layered architecture framework for real-time zero-day attack detection based on statistics, signatures, and behavior techniques.

1.3.4. Detection Applying Learning

Kim et al. [13] proposed a method to detect zero-day malware. The method generates fake malware and learns to distinguish it from real malware. A deep autoencoder extracts appropriate features and stabilizes the generative adversarial network training. Gupta and Rani [14] observed that zero-day malware grows exponentially in terms of volume, variety, and velocity. They proposed a big data framework with scalable architecture and machine learning for detection.

1.3.5. Mitigation, Robustness, Recovery, and Simulation

Sharma et al. [15] presented a consensus framework for mitigating zero-day attacks, incorporating context behavior, an alert message protocol, and critical data-sharing protocol for reliable communication. Haider et al. [16] applied data sets based on the Windows Operating System to evaluate the robustness of host-based intrusion detection systems to zero-day and stealth attacks. Tran et al. [17] implemented an epidemiological model to combat zero-day attacks. They proposed a dynamic recovery model to combat the simulated attack and minimize disruptions. Tidy et al. [18] simulate previous and hypothetical zero-day worm epidemiology scenarios, accounting for susceptible populous and stealth-like behavior on the dynamic, heterogeneous internet.

1.3.6. Filtering, Protocol Context, Honeypots, and Signatures

Chowdhury et al. [19] proposed a multilayer hybrid strategy for zero-day filtering of phishing emails by using training data collected during an earlier time span. Duessel et al. [20] incorporated protocol context into payload-based anomaly detection of zero-day attacks, integrating syntactic and sequential features of payloads, thus proceeding beyond analyzing plain byte sequences. Chamotra et al. [21] suggested baselining high-interaction honeypots, i.e., identifying and whitelisting legitimate system activities in the honeypot attack surface. Subsequently, captured zero-day attacks are mapped to the vulnerabilities exposed by the honeypot. Afek et al. [22] presented a tool for extracting zero-day signatures for high-volume attacks, intended to detect and stop unknown attacks.

1.3.7. Cyber Security

More generally, for cybersecurity, Baliga et al. [23] identified opportunities for cyber deterrence with detection and the potential to undermine deterrence. Edwards et al. [24] considered a game theoretic model of blame, with an attacker and a defender, involving attribution, attack tolerance, and peace stability. Welburn et al. [25] found that although a cybersecurity defender prefers not to signal truthfully, the defender can enhance deterrence through signaling, which has implications for cyber deterrence policies. Nagurney and Shukla [26] considered three models for cybersecurity investment involving noncooperation, the Nash bargaining theory with information sharing, and system optimization with cooperation.

1.3.8. Information Security

Within information security, game theoretic research has focused on data survivability versus security in information systems [27], substitution and interdependence [28–30], returns on information security investment [31,32], and information sharing to prevent attacks [33–37]. See Do et al. [38], Hausken and Levitin [39], and Roy et al. [40] for reviews on game theoretic cybersecurity research.

1.4. Article Organization

Section 2 presents the model. Section 3 analyzes the model. Section 4 illustrates the solution. Section 5 discusses the results. Section 6 concludes.

2. The Model

Consider two players in a simultaneous move two-period game.

2.1. Period 1

Assume that Player 1 in Period 1 gets cyber resources R_{11} (e.g., capital, manpower, competence) from a national budget, which is allocated to develop zero-day exploits (zero-days, for short) Z_{11} deployed in Period 1 to exploit zero-day vulnerabilities for Player 2 at unit cost b_{11} and develop zero-day exploits S_1 stockpiled for use in Period 2 at unit cost b_{11}. The Nomenclature is shown before the reference list. Player 1's upper constraint R_{11} for resource allocation in Period 1 is

$$R_{11} \geq b_{11}Z_{11} + b_{11}S_1 = R_{11b} \tag{1}$$

where R_{11b} is the actual amount of resources used by Player 1 in Period 1. Player 2 exerts defense effort D_{21} in Period 1 at unit cost a_{21} to defend its asset, which it values as V_2 and Player 1 values as V_1. Figure 1 illustrates Period 1.

We apply the widely used ratio form contest success function [41], which is a plausible and widely used method for assessing two opposing players' success. See Hausken and Levitin [42], Hausken [43], and Congleton et al. [44] for the use of the contest success function. In Period 1, Player 1's expected contest success is p_{11} and Player 2's expected contest success is p_{21}, i.e.,

$$p_{11} = \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v}, p_{21} = \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} \tag{2}$$

where v, $v \geq 0$, is the contest intensity in Period 1. Expected contest success is usually interpreted as a probability between 0 and 1. It can also be interpreted as a guaranteed fraction of an asset one competes to obtain, which presumes that the asset is divisible. When $v = 0$, the contest is egalitarian, and efforts do not matter. When $v = 1$, efforts matter proportionally. When $v = \infty$, "winner-takes-all," so that exerting slightly more effort than one's opponent guarantees contest success. When $0 < v < 1$, a disproportional advantage exists of investing less than one's opponent. When $v > 1$, a disproportional advantage exists of investing more than one's opponent. In Equation (2), the ratios have a sum of two efforts in the denominator and one of the efforts in the numerator. That gives a number between zero and one, which specifies contest success.

With these assumptions, Player i's expected utility in Period 1 is

$$\begin{aligned} U_{11} &= p_{11}V_1 - b_{11}Z_{11} - b_{11}S_1 = \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v}V_1 - b_{11}Z_{11} - b_{11}S_1, \\ U_{21} &= p_{21}V_2 - a_{21}D_{21} = \frac{D_{21}^v}{Z_{11}^v + D_{21}^v}V_2 - a_{21}D_{21} \end{aligned} \tag{3}$$

where Equations (1) and (2) have been inserted. Player 1's two free-choice variables in Period 1 are Z_{11} and S_1, constrained by Equation (1). Player 1 obtains no utility in Period 1 for allocating S_1 to stockpiling. Player 2's one free-choice variable in Period 1 is D_{21}, constrained by $D_{21} \geq 0$.

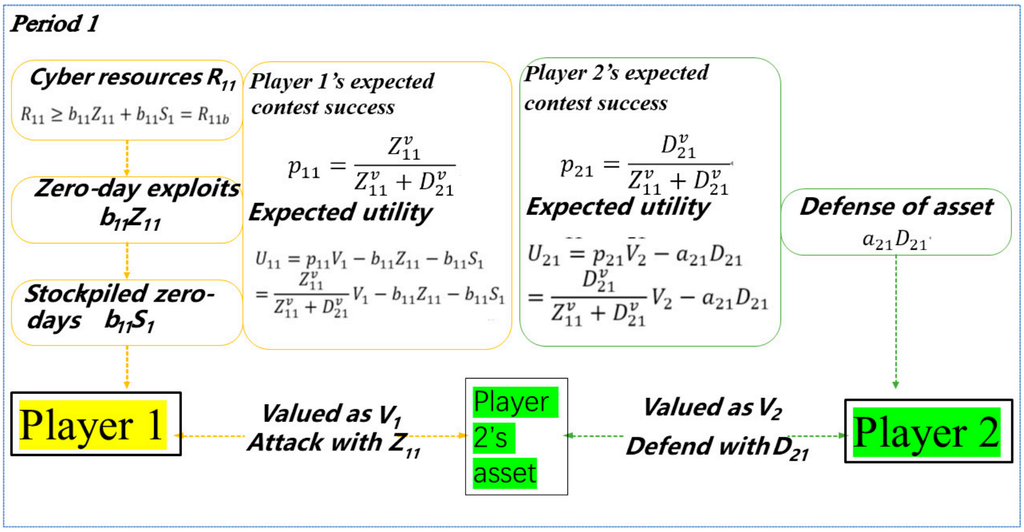

Figure 1. Illustrating Period 1.

2.2. Period 2

Figure 2 illustrates Period 2.

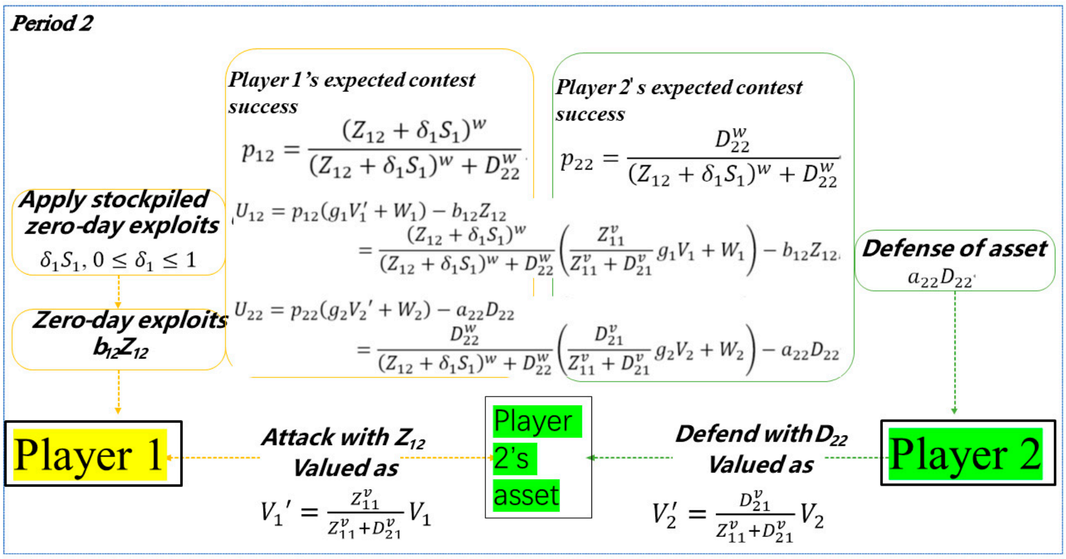

Figure 2. Illustrating Period 2.

In Period 2, Player 1 applies its stockpiled zero-day exploits S_1 from Period 1, if it has stockpiled. Additionally, in Period 2, Player 1 exerts effort Z_{12} at unit cost b_{12} to develop zero-day exploits, against which Player 2 exerts defense effort D_{22} at unit cost a_{22}. More specifically, assume that Player 1 in Period 2 applies its stockpiled zero-day exploits S_1 from Period 1, either keeping its same value with no appreciation if $\delta_1 = 1$, appreciating in value if $\delta_1 > 1$, or depreciating in value if $0 \leq \delta_1 \leq 1$. Appreciation of zero-day exploits over time occurs if technical, economic, or cultural circumstances change, making zero-day exploits more useful. In contrast, depreciation occurs if some aspects of the zero-day exploits leak or somehow becomes known or if technological or other developments make zero-day exploits less valuable over time. For example, increased competence may enable defenders against zero-day exploits to defend better, even though the nature of the zero-day exploit is unknown. 100% depreciation is expressed as $\delta_1 = 0$.

Player 1 in Period 2 exerts effort Z_{12} at unit cost b_{12} to develop zero-day exploits deployed in Period 2 to exploit zero-day vulnerabilities for Player 2. Player 2 exerts defense effort D_{22} in Period 2 at unit cost a_{22} to defend its asset, which it values as $V_2' = \frac{D_{21}^v}{Z_{11}^v + D_{21}^v} V_2$ and Player 1 values as $V_1{}' = \frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} V_1$. In Period 2, Player 1's expected contest success is p_{21} and Player 2's expected contest success is p_{22}, i.e.,

$$p_{12} = \frac{(Z_{12} + \delta_1 S_1)^w}{(Z_{12} + \delta_1 S_1)^w + D_{22}^w}, p_{22} = \frac{D_{22}^w}{(Z_{12} + \delta_1 S_1)^w + D_{22}^w} \tag{4}$$

where w, $w \geq 0$, is the contest intensity in Period 2, with the same interpretation as v for Period 1, and S_1 is determined by (1).

Assume that Player 2's asset, valued as V_i by Player i, $i = 1, 2$, grows with a growth factor g_i from Period 1 to Period 2; $g_i \geq 0$, with an interpretation similar to that of δ_1 for Player 1's stockpiling S_1. That is, an asset with value V_i grows if $g_i > 1$, keeps its value if $g_i = 1$, and loses value if $0 \leq g_i < 1$. Furthermore, assume that Player 2 in Period 2 gets injected with a new fresh asset valued as W_i by Player i, $i = 1, 2$. With these assumptions, Player i's expected utility in Period 2 is

$$\begin{aligned} U_{12} &= p_{12}\left(g_1 V_1' + W_1\right) - b_{12} Z_{12} = \frac{(Z_{12}+\delta_1 S_1)^w}{(Z_{12}+\delta_1 S_1)^w + D_{22}^w}\left(\frac{Z_{11}^v}{Z_{11}^v + D_{21}^v} g_1 V_1 + W_1\right) - b_{12} Z_{12}, \\ U_{22} &= p_{22}(g_2 V_2{}' + W_2) - a_{22} D_{22} = \frac{D_{22}^w}{(Z_{12}+\delta_1 S_1)^w + D_{22}^w}\left(\frac{D_{21}^v}{Z_{11}^v + D_{21}^v} g_2 V_2 + W_2\right) - a_{22} D_{22} \end{aligned} \tag{5}$$

Player 1's one free-choice variable in Period 2 is Z_{12}, constrained by $Z_{12} \geq 0$. Player 2's one free-choice variable in Period 2 is D_{22}, constrained by $D_{21} \geq 0$.

For the two-period game as a whole, with time discount factor β_i, $0 \leq \beta_i \leq 1$, Player i's expected utility over the two periods is

$$U_1 = Max(0, U_{11} + \beta_1 U_{12}), U_2 = U_{21} + \beta_2 U_{22} \tag{6}$$

The Max function is used for Player 1 since Player 1 will not use its entire budget R_{11} if that causes negative expected utility U_1.

3. Solving the Model

In Section 3.1.1, the game is solved with backward induction starting in Period 2. In Section 3.1.1, Period 1 is solved. Thereafter, various corner solutions have been determined. The 11 solutions in Table 1 have been identified for the game. All the solutions except Solution 9 have positive efforts $Z_{11} \geq 0$ and $D_{21} \geq 0$ in Period 1, which is the nature of the ratio form contest success function in (2) and (3), with simultaneous moves in Period 1. That is, a player may decrease its effort arbitrarily close to zero, but not to zero. In Solution 9, Player 1 withdraws to avoid negative expected utility, i.e., to ensure $U_1 \geq 0$.

Table 1. Characteristics of the 11 solutions. $Z_{11} \geq 0$ and $D_{21} \geq 0$ in Period 1 in all the solutions.

Sol.	Stockpiling	Budget Constraint	Period 2	Description	Section
1	$S_1 = 0$	$R_{11} \geq R_{11b}$	$Z_{12} \geq 0, D_{22} \geq 0$	Player 1 neither stockpiles nor utilizes entire budget	Section 3.1.2
2	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} \geq 0, D_{22} \geq 0$	Player 1 stockpiles and utilizes entire budget	Section 3.1.2
3	$S_1 = 0$	$R_{11b} = R_{11}$	$Z_{12} \geq 0, D_{22} \geq 0$	Player 1 does not stockpile and utilizes entire budget	Section 3.1.3
4	$S_1 \geq 0$	$R_{11} \geq R_{11b}$	$Z_{12} = D_{22} = 0$	Player 2 is deterred; Player 1 is superior	Section 3.2.1
5	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} = D_{22} = 0$	Player 2 is deterred; Player 1 utilizes entire budget	Section 3.2.2
6	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} = 0, D_{22} \geq 0$	$\frac{\partial U_1}{\partial S_1} = 0, Z_{11} = \frac{R_{11} - b_{11} S_1}{b_{11}}$, Player 2 is not deterred	Section 3.2.3
7	$S_1 \geq 0$	$R_{11b} = R_{11}$	$Z_{12} = 0, D_{22} \geq 0$	$\frac{\partial U_1}{\partial Z_{11}} = 0, S_1 = \frac{R_{11} - b_{11} Z_{11}}{b_{11}}$, Player 2 is not deterred	Section 3.2.3
8	$S_1 \geq 0$	$R_{11b} \geq R_{11}$	$Z_{12} = 0, D_{22} \geq 0$	Player 2 is not deterred, though Player 1 is superior	Section 3.2.3
9	$S_1 = 0$	$R_{11} \geq R_{11b}$	$Z_{11} = 0, D_{22} \geq 0$	Player 1 withdraws to ensure $U_1 \geq 0$	Section 3.3
10	$S_1 = 0$	$R_{11} = R_{11b}$	$Z_{11} = D_{21}, Z_{12} = D_{22}$	Equally matched players; $U_1 = U_2 = 0$	Section 3.4
11	$S_1 = 0$	$R_{11b} \geq R_{11}$	$Z_{12} = D_{22} = 0$	Player 2 is deterred; Player 1 does not stockpile	Section 3.5

3.1. *Solutions 1, 2, 3 ($Z_{12} \geq 0$, $D_{22} \geq 0$, $S_1 \geq 0$)*

3.1.1. Solving Period 2

Differentiating Player i's expected utility U_{i2} in (5) in Period 2 with respect to its one free-choice variable, i.e., Z_{12} for Player 1 and D_{22} for Player 2, and equating it with zero, gives the first-order conditions

$$\begin{aligned} \frac{\partial U_{12}}{\partial Z_{12}} &= \frac{w D_{22}^w P_{11} (Z_{12} + \delta_1 S_1)^{w-1}}{\left(Z_{11}^v + D_{21}^v\right)\left((Z_{12} + \delta_1 S_1)^w + D_{22}^w\right)^2} - b_{12} = 0, \\ \frac{\partial U_{22}}{\partial D_{22}} &= \frac{w D_{22}^{w-1} Q_{21} (Z_{12} + \delta_1 S_1)^w}{\left(Z_{11}^v + D_{21}^v\right)\left((Z_{12} + \delta_1 S_1)^w + D_{22}^w\right)^2} - a_{22} = 0, \\ P_{11} \equiv W_1 D_{21}^v &+ (g_1 V_1 + W_1) Z_{11}^v,\ Q_{21} \equiv W_2 Z_{11}^v + (g_2 V_2 + W_2) D_{21}^v \end{aligned} \tag{7}$$

which are solved to yield

$$Z_{12} = \frac{a_{22}/Q_{21}}{b_{12}/P_{11}} D_{22} - \delta_1 S_1, D_{22} = \frac{w Q_{21} A}{a_{22}\left(Z_{11}^v + D_{21}^v\right)(1 + A)^2}, A \equiv \left(\frac{a_{22}/Q_{21}}{b_{12}/P_{11}}\right)^w \tag{8}$$

The second-order conditions are

$$\begin{aligned} \frac{\partial^2 U_{12}}{\partial Z_{12}^2} &= -\frac{w D_{22}^w P_{11} (Z_{12} + \delta_1 S_1)^{w-2}\left((1+w)(Z_{12} + \delta_1 S_1) + (1-w) D_{22}^w\right)}{\left(Z_{11}^v + D_{21}^v\right)\left((Z_{12} + \delta_1 S_1)^w + D_{22}^w\right)^3}, \\ \frac{\partial^2 U_{22}}{\partial D_{22}^2} &= -\frac{w D_{22}^{w-2} Q_{21} (Z_{12} + \delta_1 S_1)^w\left((1-w)(Z_{12} + \delta_1 S_1) + (1+w) D_{22}^w\right)}{\left(Z_{11}^v + D_{21}^v\right)\left((Z_{12} + \delta_1 S_1)^w + D_{22}^w\right)^3} \end{aligned} \tag{9}$$

which are satisfied as negative when

$$\begin{aligned} (1+w)(Z_{12} + \delta_1 S_1) + (1-w) D_{22}^w \geq 0, \\ (1-w)(Z_{12} + \delta_1 S_1) + (1+w) D_{22}^w \geq 0 \end{aligned} \tag{10}$$

3.1.2. Solving Period 1

Inserting Equations (8) and (3) into Player *i*'s expected utility in Equation (6) over the two periods gives

$$\begin{aligned} U_1 &= \frac{Z_{11}^v V_1}{Z_{11}^v+D_{21}^v} - b_{11}Z_{11} - b_{11}S_1 + \frac{\beta_1 A}{1+A}\left(\frac{Z_{11}^v}{Z_{11}^v+D_{21}^v} g_1 V_1 + W_1\right) - \frac{\beta_1 w P_{11} A}{\left(Z_{11}^v+D_{21}^v\right)(1+A)^2} + \beta_1 b_{12}\delta_1 S_1, \\ U_2 &= \frac{D_{21}^v V_2}{Z_{11}^v+D_{21}^v} - a_{21}D_{21} + \frac{\beta_2}{1+A}\left(\frac{D_{21}^v}{Z_{11}^v+D_{21}^v} g_2 V_2 + W_2\right) - \frac{\beta_2 w Q_{21} A}{\left(Z_{11}^v+D_{21}^v\right)(1+A)^2} \end{aligned} \tag{11}$$

which is rewritten as

$$\begin{aligned} U_1 &= \frac{Z_{11}^v V_1}{Z_{11}^v+D_{21}^v} - b_{11}Z_{11} + \frac{\beta_1 P_{11}(A+1-w)A}{\left(Z_{11}^v+D_{21}^v\right)(1+A)^2} - (b_{11} - \beta_1 b_{12}\delta_1)S_1, \\ U_2 &= \frac{D_{21}^v V_2}{Z_{11}^v+D_{21}^v} - a_{21}D_{21} + \frac{\beta_2 Q_{21}(1+(1-w)A)}{\left(Z_{11}^v+D_{21}^v\right)(1+A)^2} \end{aligned} \tag{12}$$

which has three unknown variables: S_1, Z_{11}, and D_{21}. Using (12), Player 1's optimal stockpiling is

$$S_1 = \begin{cases} Min\left(\frac{D_{22}}{\delta_1}\frac{a_{22}/Q_{21}}{b_{12}/P_{11}}, \frac{R_{11}-b_{11}Z_{11}}{b_{11}}\right) if\ b_{11} \le \beta_1 b_{12}\delta_1 \\ 0\ otherwise, \end{cases} \tag{13}$$

where $\frac{D_{22}a_{22}/Q_{21}}{\delta_1 b_{12}/P_{11}}$ according to (8) is the amount of stockpiling S_1 that causes zero effort Z_{12} for Player 1 in Period 2, and $\frac{R_{11}-b_{11}Z_{11}}{b_{11}}$ according to (1) is the maximum stockpiling S_1 permitted by Player 1's budget constraint R_{11}. Player 1 chooses the lowest of these two values since excessive stockpiling S_1 in Period 1, which cannot be utilized in Period 2, is not preferable, since Player 1 cannot exceed its budget constraint R_{11}. We refer to $S_1 = 0$ in (13) when $b_{11} > \beta_1 b_{12}\delta_1$ and $R_{11} \ge R_{11\mathrm{b}}$ as Solution 1. If $b_{11} > \beta_1 b_{12}\delta_1$, Player 1 does not stockpile in Period 1, i.e., $S_1 = 0$, since its unit cost b_{11} of stockpiling exceeds the product of Player 1's unit cost b_{12} of exerting effort Z_{12} in Period 2, Player 1's time discount factor β_1, and Player 1's zero-day appreciation factor δ_1 from Period 1 to Period 2. We refer to $S_1 = \frac{R_{11}-b_{11}Z_{11}}{b_{11}}$ in (13) when $b_{11} \le \beta_1 b_{12}\delta_1$ and $R_{11} = R_{11\mathrm{b}}$ as Solution 2. Then, Player 1 chooses Z_{11}, optimally, and applies its remaining budget to stockpile $S_1 \ge 0$.

Differentiating each player's expected utility in (12) with respect to the two remaining free-choice variables, i.e., Z_{11} for Player 1 and D_{21} for Player 2, and equating it with zero, gives the first-order conditions

$$\begin{aligned} \frac{\partial U_1}{\partial Z_{11}} &= \frac{D_{21}^v v Z_{11}^{v-1}\left(Ag_2 P_{11} V_2 w(B-Cw)\beta_1 + Q_{21}V_1\left(B^3 + Ag_1\left(B^2 - Cw^2\right)\beta_1\right)\right)}{B^3 Q_{21}\left(Z_{11}^v+D_{21}^v\right)^2} - b_{11} = 0, \\ \frac{\partial U_2}{\partial D_{21}} &= \frac{D_{21}^{v-1} v Z_{11}^{v}\left(Ag_1 Q_{21} V_1 w(B+Cw)\beta_2 + P_{11}V_2\left(B^3 + g_2\left(B^2 + CAw^2\right)\beta_2\right)\right)}{B^3 P_{11}\left(Z_{11}^v+D_{21}^v\right)^2} - a_{21} = 0, \\ & B \equiv 1+A, C \equiv 1-A \end{aligned} \tag{14}$$

which are cumbersome to analyze analytically. Hence, we solve (14) numerically for Z_{11} and D_{21} and use (13) to determine S_1, which are both inserted into (8) to determine the free-choice variables Z_{12} and D_{22} in Period 2. We finally insert the result into (12) to determine the players' expected utilities U_1 and U_2 over the two time periods.

3.1.3. Solution 3 ($Z_{11} = R_{11}/b_{11}$)

Inserting $Z_{11} = R_{11}/b_{11}$ into (1) causes zero stockpiling, $S_1 = 0$. Thus, Player 1 in Period 1 allocates all its resources to exploit zero-day vulnerabilities for Player 2 and has no resources to stockpile zero-day exploits for use in Period 2. The solution follows from solving the second first-order condition in (14) when $Z_{11} = R_{11}/b_{11}$ and applying $Z_{11} = R_{11}/b_{11}$ instead of the first first-order condition in (14).

3.2. Solutions 4–8 ($Z_{12}=0$, $D_{22}\geq 0$, $R_{11}\geq R_{11b}$)

When $Z_{12}=0$, Player 1 exerts no effort to develop zero-day capabilities in Period 2; instead, it relies on the stockpiling S_1 from Period 1 to attack Player 2. Solving Player 2's first-order condition in (7) when $Z_{12}=0$ gives

$$D_{22}^{w}-\sqrt{D_{22}^{w-1}}\sqrt{\frac{wQ_{21}(\delta_1 S_1)^{w}}{a_{22}\left(Z_{11}^{v}+D_{21}^{v}\right)}}+(\delta_1 S_1)^{w}=0 \tag{15}$$

which is not analytically solvable for general w (since w appears multiplicatively under a root sign, appears as an exponent with two different bases, appears as an exponent under a root sign and without a root sign, and appears as an exponent $w-1$ under a root sign), but is, for $w=1$, conveniently solved to

$$D_{22}=\begin{cases}\left(\sqrt{\frac{Q_{21}}{a_{22}\left(Z_{11}^{v}+D_{21}^{v}\right)}}-\sqrt{\delta_1 S_1}\right)\sqrt{\delta_1 S_1}\ if\ \frac{Q_{21}}{a_{22}\left(Z_{11}^{v}+D_{21}^{v}\right)}>\delta_1 S_1\\ 0\ otherwise.\end{cases} \tag{16}$$

Inserting $Z_{12}=0$, $w=1$, and (3) into Player i's expected utility in (6) gives

$$\begin{aligned}U_1&=\frac{Z_{11}^{v}V_1}{Z_{11}^{v}+D_{21}^{v}}-b_{11}Z_{11}-b_{11}S_1+\beta_1\frac{\delta_1 S_1}{\delta_1 S_1+D_{22}}\left(\frac{Z_{11}^{v}}{Z_{11}^{v}+D_{21}^{v}}g_1V_1+W_1\right)\\ U_2&=\frac{D_{21}^{v}V_2}{Z_{11}^{v}+D_{21}^{v}}-a_{21}D_{21}+\beta_2\left(\frac{D_{22}}{\delta_1 S_1+D_{22}}\left(\frac{D_{21}^{v}}{Z_{11}^{v}+D_{21}^{v}}g_2V_2+W_2\right)-a_{22}D_{22}\right)\end{aligned} \tag{17}$$

where D_{22} follows from (16). Differentiating U_1 in (17) with respect to S_1 and equating with zero gives

$$\frac{\partial U_1}{\partial S_1}=\frac{\beta_1\sqrt{\delta_1}\sqrt{a_{22}}P_{11}}{2\sqrt{S_1}\sqrt{Z_{11}^{v}+D_{21}^{v}}\sqrt{Q_{21}}}-b_{11}=0\Rightarrow S_1=\frac{\beta_1^2\delta_1 a_{22}P_{11}^2}{4b_{11}^2\left(Z_{11}^{v}+D_{21}^{v}\right)Q_{21}} \tag{18}$$

The two remaining unknown variables Z_{11} and D_{21} in (17) are determined by solving $\frac{\partial U_1}{\partial Z_{11}}=0$ and $\frac{\partial U_2}{\partial D_{21}}=0$ together with (18) for Period 1.

3.2.1. Solution 4 ($Z_{12}=D_{22}=0$, $R_{11}\geq R_{11\mathrm{b}}$)

When $\frac{Q_{21}}{a_{22}\left(Z_{11}^{v}+D_{21}^{v}\right)}\leq\delta_1 S_1$ in (16), Player 2 is deterred from exerting effort in Period 2, i.e., $D_{22}=0$. Then, Player 1 wins the Period 2 contest since $S_1>0$. Inserting $Z_{12}=D_{22}=0$, $w=1$, and (3) into Player i's expected utility in (6) gives

$$\begin{aligned}U_1&=\frac{Z_{11}^{v}V_1}{Z_{11}^{v}+D_{21}^{v}}-b_{11}Z_{11}-b_{11}S_1+\beta_1\left(\frac{Z_{11}^{v}}{Z_{11}^{v}+D_{21}^{v}}g_1V_1+W_1\right),\\ U_2&=\frac{D_{21}^{v}V_2}{Z_{11}^{v}+D_{21}^{v}}-a_{21}D_{21}\end{aligned} \tag{19}$$

Differentiating (19) to determine the optimal efforts Z_{11} and D_{21} for Players 1 and 2, respectively, and equating with 0 gives

$$\begin{aligned}\frac{\partial U_1}{\partial Z_{11}}&=\frac{vV_1Z_{11}^{v-1}D_{21}^{v}(1+\beta_1 g_1)}{\left(Z_{11}^{v}+D_{21}^{v}\right)^2}-b_{11}=0,\\ \frac{\partial U_2}{\partial D_{21}}&=\frac{vD_{21}^{v-1}Z_{11}^{v}V_2}{\left(Z_{11}^{v}+D_{21}^{v}\right)^2}-a_{21}=0\end{aligned} \tag{20}$$

which are solved to yield

$$Z_{11} = \frac{a_{21}/V_2}{b_{11}/V_1(1+\beta_1 g_1)} D_{21}, D_{21} = \frac{vV_2\left(\frac{a_{21}/V_2}{b_{11}/V_1(1+\beta_1 g_1)}\right)^v}{a_{21}\left(1+\left(\frac{a_{21}/V_2}{b_{11}/V_1(1+\beta_1 g_1)}\right)^v\right)^2} \tag{21}$$

The second-order conditions are

$$\begin{aligned} \frac{\partial^2 U_1}{\partial Z_{11}^2} &= -\frac{vV_1 D_{21}^v Z_{11}^{v-2}(1+\beta_1 g_1)\left((1+v)Z_{11}^v+(1-v)D_{21}^v\right)}{\left(Z_{11}^v+D_{21}^v\right)^3}, \\ \frac{\partial^2 U_2}{\partial D_{21}^2} &= -\frac{vV_2 D_{21}^{v-2} Z_{11}^v\left((1-v)Z_{11}^v+(1+v)D_{21}^v\right)}{\left(Z_{11}^v+D_{21}^v\right)^3} \end{aligned} \tag{22}$$

which are satisfied as negative when

$$\begin{aligned} (1+v)Z_{11}^v + (1-v)D_{21}^v \geq 0, \\ (1-v)Z_{11}^v + (1+v)D_{21}^v \geq 0 \end{aligned} \tag{23}$$

To deter Player 2 in Period 2, Player 1 must choose sufficiently large stockpiling S_1 to make Player 2 indifferent between exerting and not exerting effort D_{22} in Period 2. Inserting $Z_{12} = D_{22} = 0$ and $w = 1$ into (3), that implies

$$\begin{aligned} \frac{D_{22}}{\delta_1 S_1 + D_{22}}\left(\frac{D_{21}^v}{Z_{11}^v+D_{21}^v} g_2 V_2 + W_2\right) - a_{22}D_{22} = 0 \; when \; D_{22} = 0 \\ \Leftrightarrow S_1 = \frac{1}{\delta_1 a_{22}}\left(\frac{D_{21}^v g_2 V_2}{Z_{11}^v+D_{21}^v} + W_2\right) \end{aligned} \tag{24}$$

where Z_{11} and D_{21} in (17) are determined in (21).

3.2.2. Solution 5 ($Z_{12} = D_{22} = 0$, $R_{11} = R_{11b}$)

The solution for Z_{11}, D_{21}, and S_1 in (17) and (24) presupposes that the budget constraint $R_{11} \geq b_{11}Z_{11} + b_{11}S_1 = R_{11b}$ in (1) is not exceeded. If it is exceeded, Player 1 must decrease either the effort Z_{11} or the stockpiling S_1 that deters Player 2 in Period 2. Let us analyze the event that Player 1 chooses stockpiling S_1 to deter, as in (24), and uses the budget constraint R_{11} in (1) to determine Z_{11} (which is then lower than the optimal Z_{11} with no budget constraint in (17)). Applying $\frac{\partial U_2}{\partial D_{21}} = 0$ in (20), S_1 in (24), and the budget constraint in (1) gives the three equations

$$\frac{vD_{21}^{v-1} Z_{11}^v V_2}{\left(Z_{11}^v + D_{21}^v\right)^2} = a_{21}, S_1 = \frac{1}{\delta_1 a_{22}}\left(\frac{D_{21}^v g_2 V_2}{Z_{11}^v + D_{21}^v} + W_2\right), b_{11}Z_{11} + b_{11}S_1 = R_{11}, \tag{25}$$

which are numerically solvable for Z_{11}, D_{21}, and S_1.

3.2.3. Solutions 6–8 ($Z_{12} = 0$, $D_{22} \geq 0$, $R_{11} = R_{11b}$)

If Player 1 chooses effort $Z_{12} = 0$ in Period 2 and Player 1's budget constraint $R_{11} = R_{11b}$ prevents sufficient stockpiling S_1 to deter Player 2 in Period 2, Player 2 will choose positive effort $D_{22} \geq 0$ in Period 2. Then, (16) applies for D_{22} and (17) applies for U_1 and U_2. Solution 6 follows from solving $\frac{\partial U_2}{\partial D_{21}} = 0$ in (17) together with S_1 in (18) and the budget constraint $Z_{11} = \frac{R_{11}-b_{11}S_1}{b_{11}}$. Solution 7 follows from solving $\frac{\partial U_1}{\partial Z_{11}} = 0$ and $\frac{\partial U_2}{\partial D_{21}} = 0$ in (17) together with the budget constraint $S_1 = \frac{R_{11}-b_{11}Z_{11}}{b_{11}}$. Solution 8, in which Player 1 does not utilize its entire budget $R_{11} \geq R_{11b}$, follows from solving $\frac{\partial U_1}{\partial Z_{11}} = 0$ and $\frac{\partial U_2}{\partial D_{21}} = 0$ in (17) together with S_1 in (18). Solution 8 has not been demonstrated in practice. It is distinguished from Solutions 6 and 7 in that Player 1 does not utilize its entire budget $R_{11} \geq R_{11b}$, while still not deterring Player 2. It is also distinguished from Solutions 4 and 5, where Player 2 is indeed deterred, either by the player being superior (Solution 4) or by Player 1 utilizing its entire budget $R_{11} \geq R_{11b}$.

3.3. Solution 9 ($S_1 = Z_{11} = 0$)

Player 1's budget constraint $R_{11} \geq b_{11}Z_{11} + b_{11}S_1$ in (1) may prevent Player 1 from an optimal exertion of efforts. Hence, we require that Player 1 should always receive positive expected utility $U_1 \geq 0$ and otherwise assume that Player 1 chooses zero efforts $Z_{11} = Z_{12} = 0$ in both periods and that Player 2 keeps its asset by exerting arbitrarily small defense efforts $D_{21} = D_{22} = \epsilon > 0$, where ϵ is arbitrarily small but strictly positive. Inserting into (3), (5) and (6), the players' expected utilities are thus $U_1 = U_{11} = U_{12} = 0$, $U_{21} = V_2$, $U_{22} = g_2V_2 + W_2$, $U_2 = V_2 + \beta_2 g_2 V_2 + W_2$.

3.4. Solution 10 ($S_1 = 0$, $Z_{11} = R_{11}/b_{11} = D_{21}$)

A solution is possible, where the players are equally matched (equally advantaged) and Player 1 chooses Period 1 effort $Z_{11} = R_{11}/b_{11} = D_{21}$, which equals Player 2's Period 1 effort D_{21}. Furthermore, if the players are equally matched in Period 2 and exert equal and high Period 2 efforts $Z_{12} = D_{22}$, a solution can emerge where they both receive zero expected utilities since their efforts in both periods outweigh the benefits they receive from the asset values, i.e., $U_1 = U_{11} = U_{12} = U_2 = U_{21} = U_{22} = 0$.

3.5. Solution 11 ($Z_{12} = D_{22} = S_1 = 0$)

When Player 2 is deterred in Period 2, $D_{22} = 0$, and Player 1 does not stockpile in Period 1, $S_1 = 0$, what remains for Period 1 is for Player 1 to choose effort Z_{11} and Player 2 to choose effort D_{21}. In order to deter Player 2 in Period 1, so that Player 2 chooses zero effort $D_{21} = 0$, (19) for Player 2 implies

$$U_2 = \frac{D_{21}^v V_2}{Z_{11}^v + D_{21}^v} - a_{21}D_{21} \leq 0 \Leftrightarrow Z_{11} \geq \left(\frac{D_{21}^{v-1}(V_2 - a_{21}D_{21})}{a_{21}}\right)^{1/v} \quad (26)$$

Equation (26) needs to be analyzed for each combination of parameter values to determine whether Player 1's budget R_{11} enables it to choose Z_{11}/b_{11} to deter Player 2 so that $D_{21} = 0$ or whether deterrence is impossible. Solution 11 has not been demonstrated in practice. It is distinguished from Solutions 4 and 5, where Player 2 is also deterred, $D_{22} = 0$, in Period 2, but Player 1 stockpiles $S_1 \geq 0$.

4. Illustrating the Solution

Figure 3 illustrates the solution, i.e., the efforts Z_{11}, D_{21}, Z_{12}, D_{22}, stockpiling S_1, the actual amount R_{11b} (dependent variable) of resources used by Player 1 in Period 1, and the expected utilities U_1, U_2, U_{11}, U_{21}, U_{12}, U_{22} for Players 1 and 2 with the 16 benchmark parameter values $R_{11} = a_{2j} = b_{1j} = g_i = v = w = \delta_1 = \beta_i = 1$, $V_i = 2$, $W_i = 0$, $i, j = 1, 2$. We have chosen unitary parameter values whenever possible. We also plot as functions of $a_{21} = a_{22}$ and $b_{11} = b_{12}$. In each of the 16 + 2 = 18 double panels, one parameter value varies, while the other parameter values are kept at their benchmarks. The upper part of each panel shows which solution is plotted for the various ranges along the horizontal axis. The benchmark solution (which is Solution 1) is $Z_{11} = D_{21} = R_{11b} = 0.875$, $Z_{12} = D_{22} = 0.25$, $S_1 = 0$, $U_1 = U_2 = 0.375$, $U_{11} = U_{21} = 0.125$, $U_{12} = U_{22} = 0.25$.

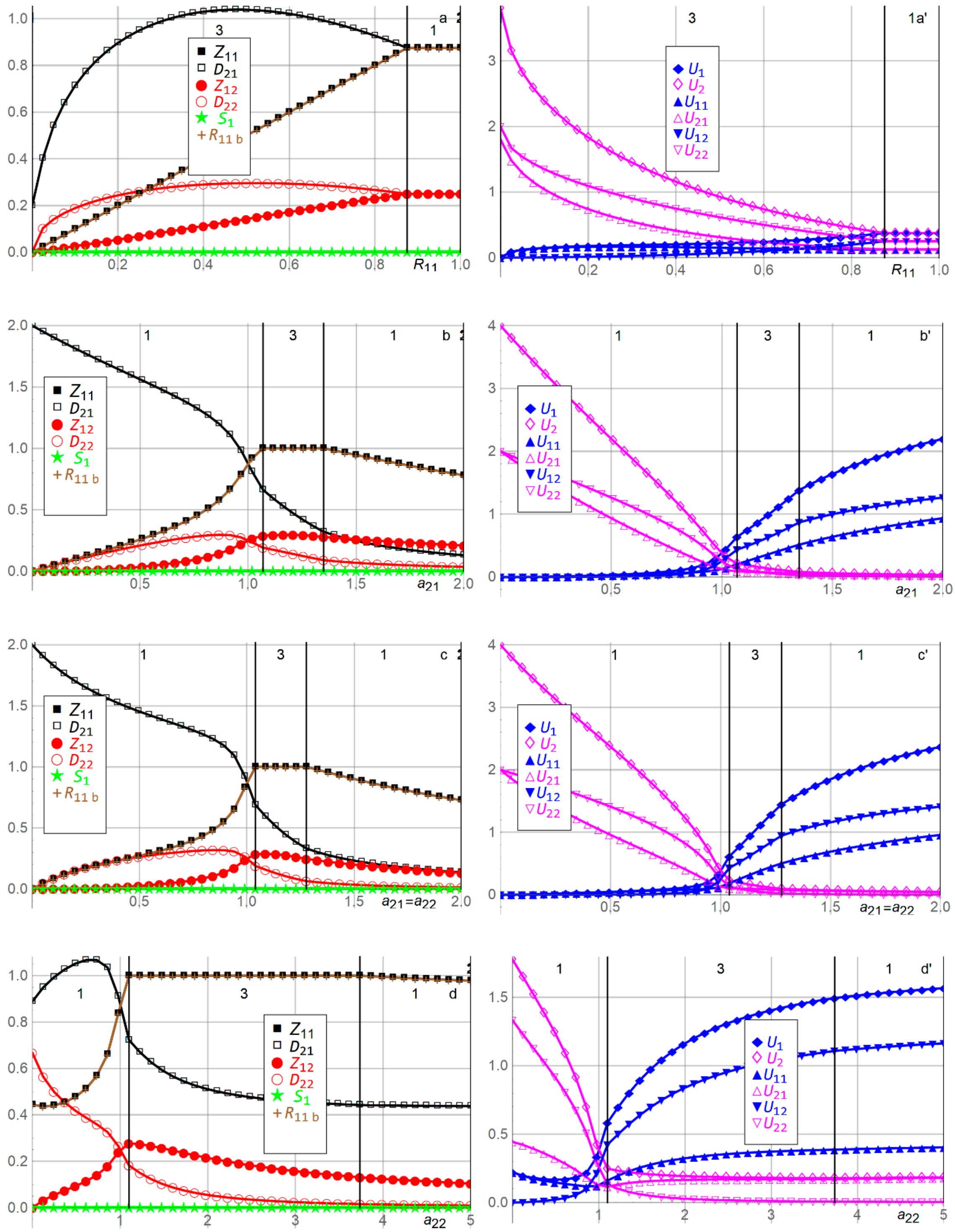

Figure 3. *Cont.*

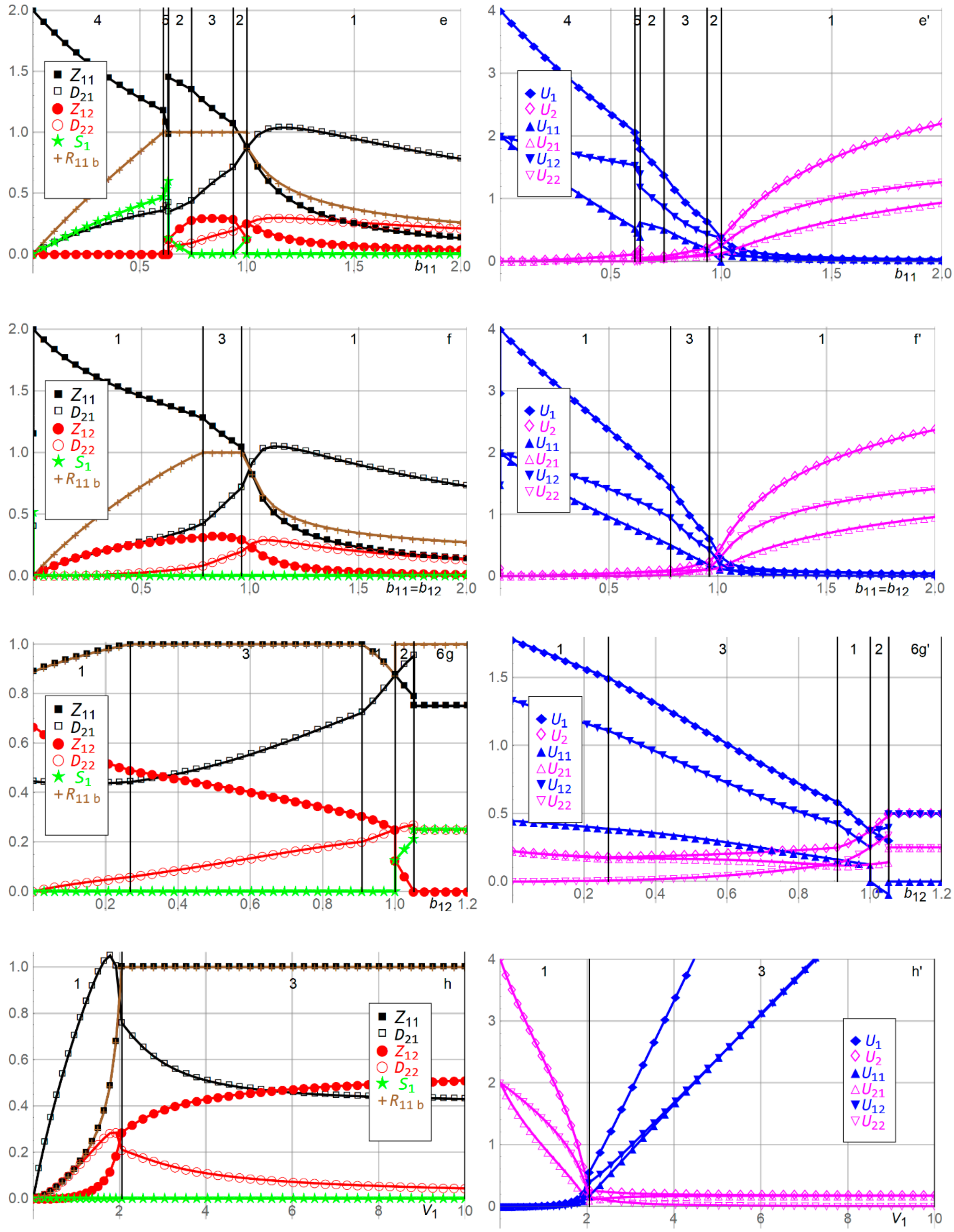

Figure 3. *Cont.*

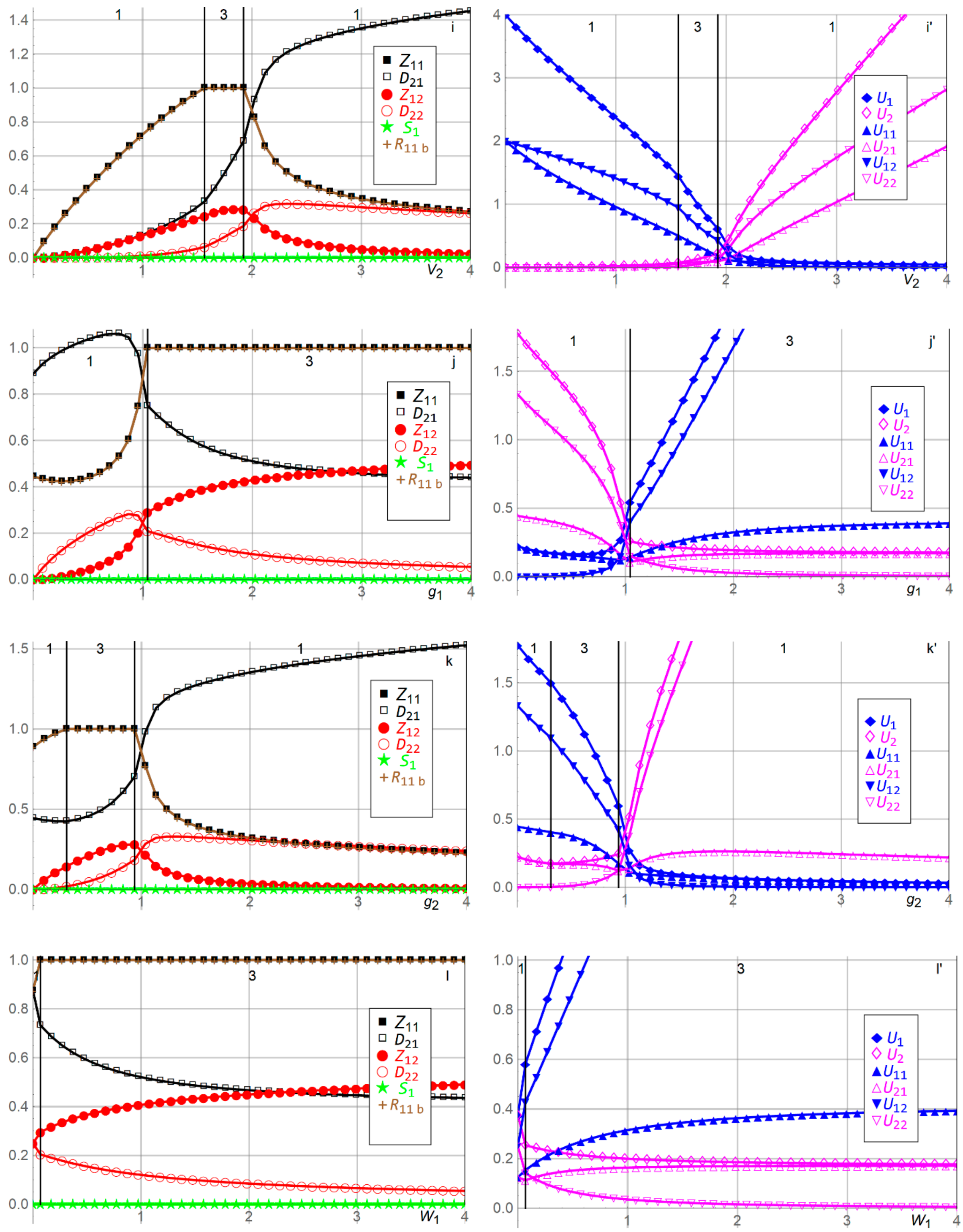

Figure 3. *Cont.*

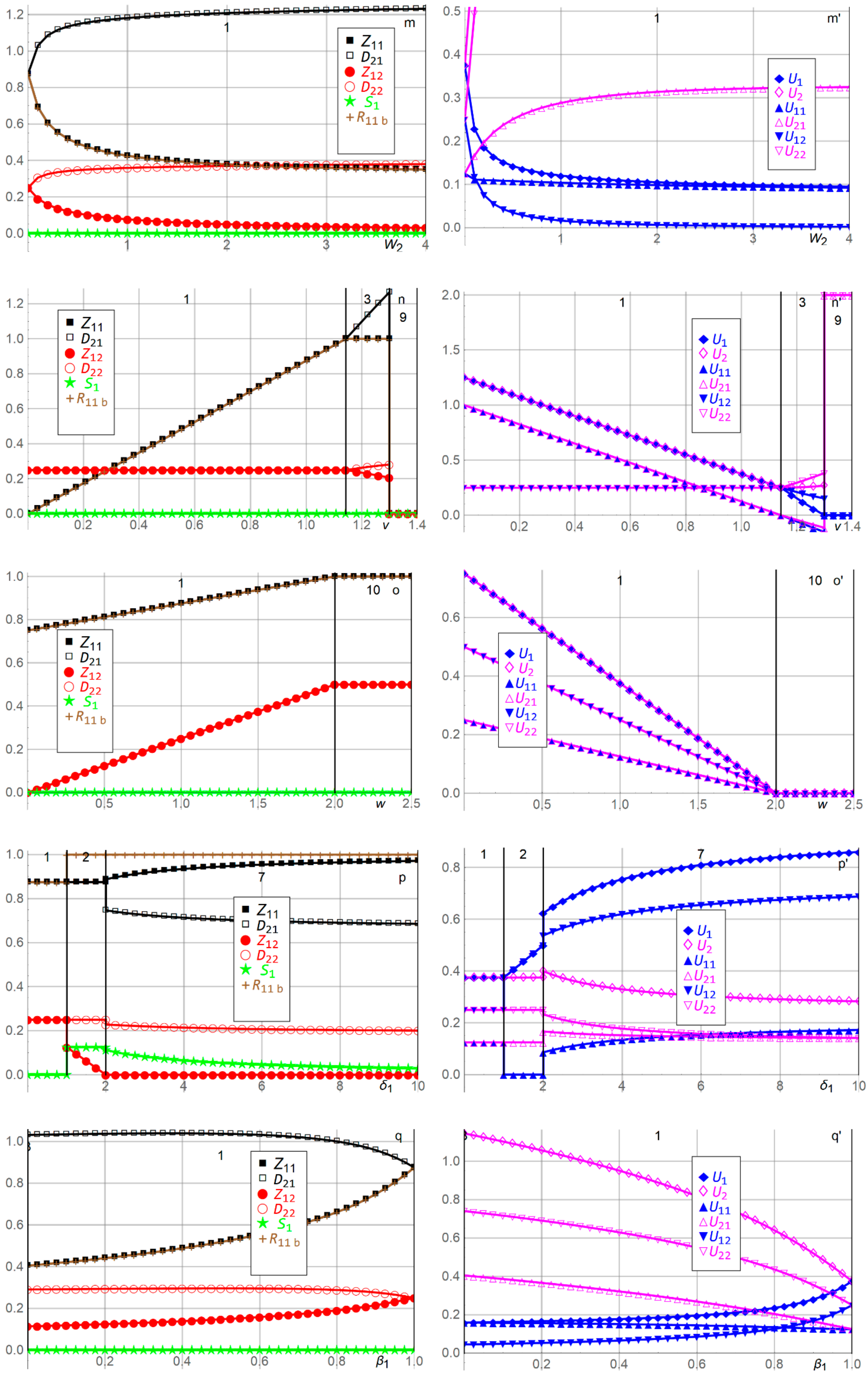

Figure 3. *Cont.*

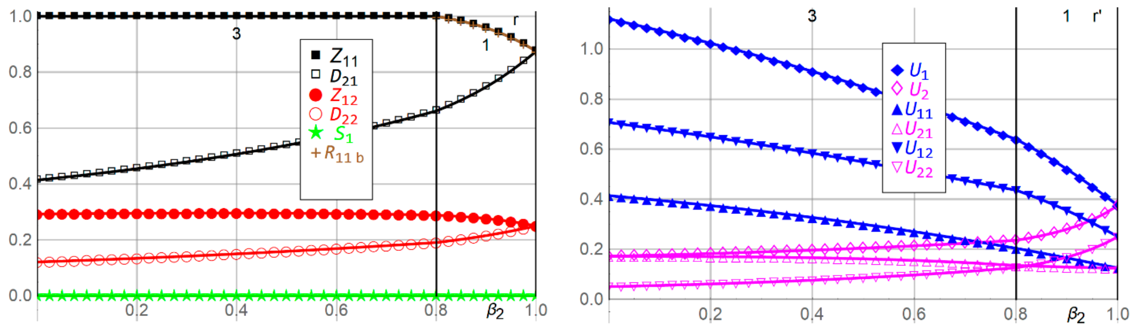

Figure 3. Efforts Z_{11}, D_{21}, Z_{12}, D_{22}, stockpiling S_1, used resources R_{11b}, and expected utilities U_1, U_2, U_{11},U_{21}, U_{12}, U_{22} for Players 1 and 2 as functions of R_{11}, $a_{2j}, b_{1j}, g_i, v, w, \delta_1, \beta_i$, V_i, W_i, $i, j = 1, 2$, relative to the benchmark parameter values $R_{11} = a_{2j} = b_{1j} = g_i = v = w = \delta_1 = \beta_i = 1$, $V_i = 2$, $W_i = 0$, $i, j = 1, 2$. See Table 2 and the text for an explanation of the 18 panels a,a′ to r,r′.

In Figure 3a,a′, when Player 1′s budget constraint R_{11} exceeds the amount R_{11b} of resources used at benchmark $R_{11b} = 0.875$, all variables remain at their benchmarks, as functions of R_{11}, since Player 1 is not constrained in any way. In contrast, as R_{11} decreases below $R_{11b} = 0.875$, Player 1 is constrained in its effort $Z_{11} = R_{11}/b_{11}$, which decreases linearly to $Z_{11} = 0$ as R_{11} decreases to $R_{11} = 0$. Player 2′s Period 1 defense effort D_{21} is inverse U-shaped in R_{11} since Player 1 first seeks to gain competitive advantage against Player 2 by competing more fiercely as R_{11} decreases below $R_{11b} = 0.875$. After D_{21} reaches a maximum, it decreases as Player 2 becomes more advantaged and succeeds with lower effort D_{21} due to Player 1′s decreasing budget R_{11}. Hence, as R_{11} decreases, Player 1′s expected utilities U_1,U_{11},U_{12} decrease and Player 2′s expected utilities U_2,U_{21},U_{22} increase.

In Figure 3b,b′, as Player 2′s unit effort cost a_{21} of defense in Period 1 increases above $a_{21} = 1$, the disadvantaged Player 2′s efforts D_{21} and D_{22} in both periods and its expected utilities U_2,U_{21},U_{22} decrease. Player 1′s efforts Z_{11} and Z_{12} in both periods are inverse U-shaped in a_{21}. Initially, as a_{21} increases above $a_{21} = 1$, Player 1 increases Z_{11} and Z_{12} to compete more successfully with Player 2. As a_{21} increases further, Player 1 decreases its efforts Z_{11} and Z_{12} due to strength and being advantaged, as Z_{11} and Z_{12} are less needed to compete successfully with Player 2. As a_{21} increases above $a_{21} = 1$, Player 1′s expected utilities U_1,U_{11},U_{12} thus increase. For the range $1.07 \leq a_{21} \leq 1.35$, Player 1 reaches its budget constraint $R_{11} = 1$ due to competing fiercely with Player 2 (and being neither strongly advantaged nor strongly disadvantaged), causing maximum Period 1 effort $Z_{11} = 1$, which depresses Player 1′s expected utility U_1 and increases Player 2′s expected utility U_2 slightly, relative to no budget constraint. In contrast, as a_{21} decreases below $a_{21} = 1$, the advantaged Player 2 increases its Period 1 defense effort D_{21}, while Player 1 decreases its efforts Z_{11} and Z_{12} in both periods. Player 2′s defense effort D_{22} in period 2 is inverse U-shaped for the same reason as above. As a_{21} approaches $a_{21} = 0$, less need exists for the advantaged Player 2 to exert effort D_{22} in Period 2, and the asset fought over is less valuable since most of the value was distributes in Period 1. Hence, as a_{21} decreases below $a_{21} = 1$, Player 2′s expected utilities U_2,U_{21},U_{22} increase, and Player 1′s expected utilities U_1,U_{11},U_{12} decrease. Player 1 does not stockpile $S_1 = 0$ since its efforts Z_{11} and Z_{12} are equally costly in both periods, its zero-day appreciation factor from Period 1 to Period 2 equals $\delta_1 = 1$, and its time discount factor equals $\beta_1 = 1$.

In Figure 3c,c′, Player 2′s unit defense costs are assumed equal $a_{21} = a_{22}$ in both periods. Player 1 is budget constrained when $1.04 \leq a_{21} \leq 1.28$. Panel c,c′ is qualitatively similar to Panel b,b′. The main differences are that Player 2 becomes more disadvantaged when $a_{21} = a_{22}$ increases above $a_{21} = a_{22} = 1$ and more advantaged when $a_{21} = a_{22}$ decreases below $a_{21} = a_{22} = 1$ compared with Panel b,b′, where only a_{21} varies. Hence, for example, when $a_{21} = a_{22} > 1$, the two inverse-U shapes for Z_{11} and Z_{12} are narrower in Panel c,c′ than in Panel d,d′.

In Figure 3d,d′, Player 2′s unit effort cost a_{22} of defense in Period 2 varies, causing results qualitatively similar to Panels b,b′ and c,c′. The main differences are that Player 2 prefers being disadvantaged in Period 2, with high a_{22} in Panel d,d′, rather than being disadvantaged in Period 1, with high a_{21} in Panel b,b′, and that Player 2 prefers being advantaged in Period 1 with low a_{21} in Panel b,b′ rather than being advantaged in Period 2 with high a_{22} in Panel b,b′. That is, Player 2 prefers to be advantaged in the important Period 1. If Player 2 is to be disadvantaged, it prefers to be so in the less important Period 2, where a less valuable asset is at stake. Player 1 is budget-constrained when $1.10 \leq a_{21} \leq 3.73$. The reason for the larger range of being budget-constrained (compared with Panels b,b′ and c,c′) is that when Player 1 is disadvantaged with a large unit effort cost $a_{21} \geq 1 = a_{11}$ in Period 2, which constrains its Period 2 effort Z_{12}, it becomes more important for Player 1 to compete as fiercely as possible with Player 2 in Period 1, utilizing the cheaper Period 1 effort Z_{11}.

In Figure 3e,e′, as Player 1′s unit effort cost b_{11} of developing zero-day capabilities in Period 1 increases above $b_{11} = 1$, stockpiling $S_1 = 0$ continues not to occur in Solution 1 and exerting effort Z_{12} in Period 2 at unit cost $b_{12} = 1$ is cheaper. Player 1′s efforts Z_{11} and Z_{12} in both periods decrease as b_{11} increases since Player 1 becomes more disadvantaged, cannot justify the costly efforts, and receives lower expected utilities U_1,U_{11},U_{12}. Player 2′s defense efforts D_{21} and D_{22} in the two periods are inverse U-shaped as b_{11} increases above $b_{11} = 1$, which is common in such situations. That is, for intermediate b_{11} above $b_{11} = 1$, the players are similarly advantaged and Player 2 exerts high efforts D_{21} and D_{22}. As b_{11} increases, Player 2 becomes more advantaged and decreases D_{21} and D_{22} due to strength since high expected utilities U_2,U_{21},U_{22} are obtained even with low efforts. As b_{11} decreases, Player 2 becomes more disadvantaged and decreases D_{21} and D_{22} due to weakness, earning lower expected utilities U_2,U_{21},U_{22}. In contrast, as b_{11} decreases below $b_{11} = 1$, Player 1 stockpiles $S_1 \geq 0$ when the budget R_{11} permits it and it is beneficial. More specifically, decreasing b_{11} marginally below $b_{11} = 1$ causes Player 1 to replace a maximum part of its Period 2 effort Z_{12} with stockpiling $S_1 \geq 0$ until its budget $R_{11} = 1$ is reached, causing Z_{12} and S_1 to be discontinuous through $b_{11} = 1$ and causing Solution 2. As b_{11} decreases below $b_{11} = 0.94$, Solution 3 emerges. Player 1′s unit efforts cost b_{11} is then so low that it chooses maximum Period 1 effort $Z_{11} = R_{11}/b_{11}$, as permitted by the budget $R_{11} = 1$, and zero stockpiling $S_1 = 0$. This continues with increasing expected utilities U_1,U_{11},U_{12} for Player 1 and decreasing expected utilities U_2,U_{21},U_{22} for Player 2, until $b_{11} = 0.74$, where Solution 2 again emerges. The reason is that for $b_{11} < 0.74$, Player 1 is sufficiently advantaged compared with Player 2, does not need to increase its Period 1 effort Z_{11} further, and prefers instead to stockpile to become more competitive in Period 2. Hence, as b_{11} decreases from $b_{11} = 0.74$ to $b_{11} = 0.63$, Player 1′s Period 2 effort Z_{12} decreases as it is cost effectively replaced with stockpiling $S_1 \geq 0$. As b_{11} decreases below $b_{11} = 0.63$, Solution 5 emerges, where, interestingly, Player 1 stockpiles sufficiently with $S_1 \geq 0$ in Period 1 to deter Player 2 from defending in Period 2, i.e., $D_{22} = 0$. Player 1 exerts no effort $Z_{12} = 0$ in Period 2 (at unit cost b_{12}) since stockpiling $S_1 \geq 0$ at unit cost $b_{11} < 0.63$ is more cost effective. To accomplish the substantial stockpiling $S_1 \geq 0$ required to deter Player 2 in Period 2, Player 1 must decrease its Period 1 effort $Z_{11} = \frac{R_{11} - b_{11}S_1}{b_{11}}$ substantially below its effort Z_{11} chosen when $b_{11} < 0.63$, as required by its budget constraint $R_{11} = 1$. As b_{11} decreases below $b_{11} = 0.63$, within Solution 5, Player 1 can gradually afford to increase its Period 1 effort Z_{11}, enabling more successful competition with Player 2 in Period 1, and thus less stockpiling $S_1 \geq 0$ is required to deter Player 2 in Period 2. This process continues until $b_{11} < 0.61$, where Solution 4 emerges. In Solution 4, Player 1 is so superior that it does not need to utilize its entire budget $R_{11} = 1$. Its low unit effort cost $b_{11} < 0.61$ in Period 1 enables it to stockpile $S_1 \geq 0$ sufficiently to deter Player 2 in Period 2 and to sufficiently avoid having to exert effort in Period 2, i.e., $Z_{12} = 0$. Furthermore, as b_{11} decreases below $b_{11} = 0.61$, Player 1 competes increasingly successfully through increasing effort Z_{11} with Player 2 in Period 1, which enables decreased stockpiling $S_1 \geq 0$, increased expected utilities U_1,U_{11},U_{12} for Player 1, and decreased expected utilities U_2,U_{21},U_{22} for Player 2.

In Figure 3f,f′, Player 1′s unit effort costs of developing zero-day capabilities are assumed to be equal $b_{11} = b_{12}$ in both periods. Since Player 1′s zero-days do not appreciate, $\delta_1 = 1$, and Player 1 does not discount time, $\beta_1 = 0$, Player 1 does not need to stockpile, i.e., $S_1 = 0$ throughout. As $b_{11} = b_{12}$ increases above $b_{11} = b_{12} = 1$, the players′ Period 1 efforts Z_{11} and D_{21} are qualitatively similar to

Panel e,e′, i.e., decreasing for Player 1 and inverse U-shaped for Player 2. In Period 2, Player 1 is more disadvantaged in Panel f,f′ than in Panel e,e′ since its unit effort cost b_{12} is higher (no longer $b_{12} = 1$). Thus Player 1′s Period 2 effort Z_{12} decreases more quickly towards zero than in Panel e,e′, enabling the advantaged Player 2 to also decrease its Period 2 defense effort D_{22} towards zero more quickly than in Panel e,e′. In contrast, as $b_{11} = b_{12}$ decreases below $b_{11} = b_{12} = 1$, Solution 2 with stockpiling does not arise as in Panel e,e′. Instead, Solution 1 continues to operate with increased Period 1 and Period 2 efforts Z_{11} and Z_{12} for Player 1 and decreased Period 1 and Period 2 efforts D_{21} and D_{22} for Player 2. This continues until $b_{11} = b_{12} = 0.96$, when Player 1 reaches its budget constraint $R_{11} = 1$ and Solution 3 emerges, as in Panel e,e′. Solution 3 is maintained, with increasing advantage for Player 1, until $b_{11} = b_{12} = 0.78$ when Player 1 is so advantaged that it does not need to utilize its entire budget $R_{11} = 1$. Instead, Solution 1 emerges for $b_{11} = b_{12} < 0.78$, where all the four efforts Z_{11}, Z_{12}, D_{21}, D_{22} are positive since stockpiling $S_1 \geq 0$ does not occur, which would deter Player 2 in Period 2, as in Panel e,e′. As $b_{11} = b_{12}$ decreases, Player 1′s Period 1 effort Z_{11} increases since the unit effort cost decreases, while Player 1′s Period 2 effort Z_{12} decreases due to Player 1′s advantage and less of Player 2′s asset left to compete in Period 2.

In Figure 3g,g′, as Player 1′s unit effort cost b_{12} of developing zero-day capabilities in Period 2 increases above $b_{12} = 1$, to the disadvantage of Player 1, stockpiling $S_1 \geq 0$ emerges in Solution 2 since Player 1′s Period 2 effort Z_{12} becomes increasingly expensive and reaches $Z_{12} = 0$ when $b_{12} > 1.05$. As b_{12} increases from $b_{12} = 1$ to $b_{12} = 1.05$, Player 1 accepts negative expected utility U_{11} in Period 1 in order to earn increasing positive expected utility U_{12} in Period 2. As b_{12} increases above $b_{12} = 1.05$, Player 1 exerts zero effort $Z_{12} = 0$ in Period 2, stockpiles optimally $S_1 \geq 0$, and chooses its Period 1 effort $Z_{11} = \frac{R_{11}-b_{11}S_1}{b_{11}}$ in Solution 6 to satisfy the budget constraint $R_{11} = 1$. Player 1 thus offsets its increasing unit effort cost $b_{12} > 1.05$ by stockpiling $S_1 \geq 0$ in Period 1. In contrast, as b_{12} decreases below $b_{12} = 1$, stockpiling $S_1 = 0$ continues not to occur in Solution 1 since exerting effort Z_{12} in Period 2 at unit cost $b_{12} = 1$ is cheaper. Player 1′s efforts Z_{11} and Z_{12} in both periods increase as b_{12} decreases since Player 1 becomes more advantaged and receives higher expected utilities U_1, U_{11},U_{12}. Player 2′s defense efforts D_{21} and D_{22} in the two periods decrease as b_{12} decreases below $b_{12} = 1$ since Player 2 becomes more disadvantaged and receives lower expected utilities U_2, U_{21}, U_{22}. This continues until $b_{12} = 0.91$, when Player 1′s Period 1 effort Z_{11} at unit cost $b_{11} = 1$ becomes too costly, Player 1 reaches its budget constraint $R_{11} = 1$, and Solution 3 emerges. Solution 3 is maintained as b_{12} decreases to $b_{12} = 0.27$, enabling Player 1 to increase its Period 2 effort Z_{12} and earn higher expected utilities U_1,U_{11},U_{12}. Player 2′s defense efforts D_{21} and D_{22} in the two periods decrease as b_{12} decreases below $b_{12} = 1$, earning lower expected utility U_2. As b_{12} decreases below $b_{12} = 0.27$, Player 1′s Period 2 effort Z_{12} becomes so high and cheap that Player 1 can rely on competing successfully with Player 2 in Period 2. Thus, Player 1 no longer needs to exert high Period 1 effort Z_{11} and no longer needs to apply its entire budget $R_{11} = 1$. Thus, Solution 1 re-emerges with higher expected utility U_1 to Player 1. Interestingly, Player 2 also receives higher expected utility U_2 as b_{12} decreases towards $b_{12} = 0$ since Player 1 still has the unit effort cost $b_{11} = 1$ of its Period 1 effort Z_{11}, and, thus, to some extent, Player 2 competes somewhat successfully with Player 1 in Period 1.

In Figure 3h,h′, when Player 1′s valuation V_1 of Player 2′s asset increases above the benchmark $V_1 = 2$, Player 1′s Period 1 effort Z_{11} increases rapidly from the benchmark $Z_{11} = 0.875$ and reaches the budget constraint $Z_{11} = R_{11} = 1$ when $V_1 > 2.06$. That causes a transition from Solution 1 to Solution 3. As V_1 increases, Player 2′s Period 1 effort D_{21} decreases, $\lim\limits_{V_1 \longrightarrow \infty} D_{21} = 0.41$, determined numerically. That is, although Player 1′s valuation V_1 increases arbitrarily, Player 2′s valuation remains at the benchmark $V_1 = 2$, causing Player 2 to compete to defend its asset in Period 1. In Period 2, this changes. As V_1 increases, Player 1 exerts increasing effort Z_{12}, $\lim\limits_{V_1 \longrightarrow \infty} Z_{12} = 0.59$, while Player 2 exerts decreasing effort D_{22}, $\lim\limits_{V_1 \longrightarrow \infty} D_{22} = 0$. As V_1 increases, Player 1 receives increasing expected utilities U_1,U_{11},U_{12}, $\lim\limits_{V_1 \longrightarrow \infty} U_1 = \lim\limits_{V_1 \longrightarrow \infty} U_{11} = \lim\limits_{V_1 \longrightarrow \infty} U_{12} = \infty$, while Player 2′s expected utility U_2 decreases, $\lim\limits_{V_1 \longrightarrow \infty} U_2 = \lim\limits_{V_1 \longrightarrow \infty} U_{21} = 0.17$, $\lim\limits_{V_1 \longrightarrow \infty} U_{22} = 0$. In contrast, as V_1 decreases below the benchmark $V_1 = 2$, the results are qualitatively similar to Player 1's budget R_{11}, decreasing below the benchmark $R_{11} = 0.785$ in Panel a,a'. That is, Player 1 exerts lower efforts Z_{11} and Z_{12} and receives lower expected utilities U_1,U_{11},U_{12}, while Player 2's efforts are inverse U-shaped and it receives increasing expected utilities U_2,U_{21},U_{22}.

In Figure 3i,i', when Player 2's valuation V_2 of its own asset increases above the benchmark $V_2 = 2$, Player 2 exerts concavely increasing Period 1 defense effort D_{21} for its more valuable asset, $\lim_{V_2 \longrightarrow \infty} D_{21} = 2.00$. Player 2's Period 2 defense effort D_{22} is inverse U-shaped, as it first competes more fiercely with Player 1 and eventually decreases D_{22} due to being advantaged $\lim_{V_2 \longrightarrow \infty} D_{22} = 0$. Player 2's expected utilities U_2,U_{21},U_{22} thus increase, $\lim_{V_2 \longrightarrow \infty} U_2 = \lim_{V_2 \longrightarrow \infty} U_{21} = \lim_{V_2 \longrightarrow \infty} U_{22} = \infty$. Player 1 responds by decreasing its efforts Z_{11} and Z_{12} in both periods, $\lim_{V_2 \longrightarrow \infty} Z_{11} = \lim_{V_2 \longrightarrow \infty} Z_{12} = 0$, receiving decreasing expected utilities U_1,U_{11},U_{12}, $\lim_{V_2 \longrightarrow \infty} U_1 = \lim_{V_2 \longrightarrow \infty} U_{11} = \lim_{V_2 \longrightarrow \infty} U_{12} = 0$. In contrast, as V_2 decreases below the benchmark $V_2 = 2$, Player 1's Period 1 effort Z_{11} increases rapidly from the benchmark $Z_{11} = 0.875$ and reaches the budget constraint $Z_{11} = R_{11} = 1$ when $V_2 < 1.92$. That causes a transition from Solution 1 to Solution 3, but in the opposite direction compared with Panel h,h'. As V_2 decreases, Player 2's Period 1 effort D_{21} decreases convexly until $V_2 < 1.57$, causing a transition back to Solution 1 since the advantaged Player 1 no longer needs to utilize its entire budget $R_{11} = 1$. Thus, Player 1's Period 1 effort Z_{11} decreases. As V_2 decreases below the benchmark $V_2 = 2$, Player 1's Period 2 effort Z_{11} is inverse U-shaped, causing increasing expected utilities U_1,U_{11},U_{12}, while both efforts D_{21} and D_{22} by Player 2 decrease, causing decreasing expected utilities U_2,U_{21},U_{22}.

In Figure 3j,j', when Player 1's growth factor g_1 of asset V_1 from Period 1 to Period 2 increases above the benchmark $g_1 = 1$, Player 1's Period 1 effort Z_{11} increases rapidly from the benchmark $Z_{11} = 0.875$, as in Panel h,h', and reaches the budget constraint $Z_{11} = R_{11} = 1$ when $g_1 > 1.04$. That causes a transition from Solution 1 to Solution 3. As g_1 increases, the results are qualitatively similar to V_1 increasing in Panel h,h', since Player 1's period 1 effort Z_{11} is locked to the budget constraint $Z_{11} = R_{11}/b_{11}$. The difference is that Player 1's Period 1 expected utility U_{11} does not approach infinity, since the growth factor g_1 is confined to Period 2, and, instead, approaches a constant concavely, $\lim_{g_1 \longrightarrow \infty} U_{11} = 0.41$. The other limit values are as in Panel h,h', i.e., $\lim_{g_1 \longrightarrow \infty} D_{21} = 0.41$, $\lim_{g_1 \longrightarrow \infty} Z_{12} = 0.59$, $\lim_{g_1 \longrightarrow \infty} D_{22} = 0$, $\lim_{g_1 \longrightarrow \infty} U_1 = \lim_{g_1 \longrightarrow \infty} U_{12} = \infty$, $\lim_{g_1 \longrightarrow \infty} U_2 = \lim_{g_1 \longrightarrow \infty} U_{21} = 0.17$, $\lim_{g_1 \longrightarrow \infty} U_{22} = 0$. In contrast, as g_1 decreases below the benchmark $g_1 = 1$, Player 1 decreases its Period 2 effort Z_{12} since the asset has less value in Period 2, receiving decreasing expected utility U_{12} in Period 2. Both efforts D_{21} and D_{22} by Player 2 are inverse U-shaped, as in Panel h,h', when the asset value V_1 decreases below the benchmark $V_1 = 2$. Player 1's Period 1 effort is slightly U-shaped since the asset still has value V_1 for Player 1 in Period 1. As g_1 decreases, Player 2's expected utilities U_2,U_{21},U_{22} increase, while Player 1's expected utilities U_1 and U_{11} are U-shaped. This latter remarkable result is caused by Player 1 focusing more explicitly on Period 1 when the growth factor g_1 is very low, while Player 2 focuses on both periods and strikes a balance between them.

In Figure 3k,k', when Player 2's growth factor g_2 of asset V_2 from Period 1 to Period 2 increases above the benchmark $g_2 = 1$, Player 2's Period 1 effort D_{21} increases rapidly from the benchmark $D_{21} = 0.875$, as in Panel i,i'. Although growth g_2 does not manifest until Period 2, Player 2 competes fiercely in Period 1, knowing that what it can protect in Period 1 grows in Period 2. Thus, Player 2 exerts concavely increasing Period 1 defense effort D_{21}, $\lim_{g_2 \longrightarrow \infty} D_{21} = 2.00$. As g_2 increases, the results are qualitatively similar to V_2 increasing in Panel i,i'. The difference is that Player 2's Period 1 expected utility U_{21} does not approach infinity, since the growth factor g_2 is confined to Period 2. Instead, it is inverse U-shaped and approaches zero, $\lim_{g_2 \longrightarrow \infty} U_{21} = 0$. The other limit values are as in Panel i,i', i.e., $\lim_{g_2 \longrightarrow \infty} U_2 = \lim_{g_2 \longrightarrow \infty} U_{22} = \infty$, $\lim_{g_2 \longrightarrow \infty} D_{22} = \lim_{g_2 \longrightarrow \infty} Z_{11} = \lim_{g_2 \longrightarrow \infty} Z_{12} = \lim_{g_2 \longrightarrow \infty} U_1 = \lim_{g_2 \longrightarrow \infty} U_{11} = \lim_{g_2 \longrightarrow \infty} U_{12} = 0$. In contrast, as g_2 decreases below the benchmark $g_2 = 1$, Player 2's Period 1 effort is slightly U-shaped since the asset still has value V_2 for Player 2 in Period 1. Solution 3 arises when

$0.31 \leq g_2 \leq 0.94$. Player 2 decreases its Period 2 effort D_{22} since the asset has less value in Period 2, receiving decreasing expected utility U_{22} in Period 2. Both efforts Z_{11} and Z_{12} by Player 1 are inverse U-shaped, as in Panel i,i′, when the asset value V_2 decreases below the benchmark $V_2 = 2$. As g_2 decreases, Player 1's expected utilities U_1,U_{11},U_{12} increase, while Player 2's expected utilities U_2 and U_{21} are U-shaped. This latter remarkable result is caused by Player 2 focusing more explicitly on Period 1, when the growth factor g_2 is very low, while Player 1 focuses on both periods and strikes a balance between them.

In Figure 3l,l′, when Player 1's valuation W_1 of Player 2's asset acquired in Period 2 increases above the benchmark $W_1 = 0$, Player 1's Period 1 effort Z_{11} quickly increases to its budget constraint $Z_{11} = R_{11}/b_{11}$, causing transition from Solution 1 to Solution 3 when $W_1 = 0.07$. Player 1's Period 1 expected utility U_{11} is thus constrained, increasing concavely to $\lim_{W_1 \longrightarrow \infty} U_{11} = 0.41$. Player 1's Period 2 effort Z_{12} increases concavely, $\lim_{W_1 \longrightarrow \infty} Z_{12} = 0.59$, and its expected utilities U_1 and U_{12} increase without bounds, $\lim_{W_1 \longrightarrow \infty} U_1 = \lim_{W_1 \longrightarrow \infty} U_{12} = \infty$. In contrast, Player 2's defense efforts D_{21} and D_{22} in the two periods and its expected utilities U_2 and U_{22} decrease convexly, $\lim_{W_1 \longrightarrow \infty} D_{21} = 0.41$, $\lim_{W_1 \longrightarrow \infty} D_{22} = 0$, $\lim_{W_1 \longrightarrow \infty} U_2 = 0.17$, $\lim_{W_1 \longrightarrow \infty} U_{22} = 0$. Player 2's Period 1 expected utility U_{21} increases concavely, $\lim_{W_1 \longrightarrow \infty} U_{21} = 0.17$, since Player 1 is budget-constrained in Period 1 and strongly focuses instead on Period 2 as W_1 increases.

In Figure 3m,m′, when Player 2's valuation W_2 of its own asset acquired in Period 2 increases above the benchmark $W_2 = 0$, Player 2's Period 1 defense effort D_{21} and expected utility U_{21} increase concavely, $\lim_{W_2 \longrightarrow \infty} D_{21} = 1.28$, $\lim_{W_2 \longrightarrow \infty} U_{21} = 0.32$. Player 1's Period 1 effort Z_{11} and expected utilities U_1 and U_{11} decrease concavely, $\lim_{W_2 \longrightarrow \infty} Z_{11} = 0.32$, $\lim_{W_2 \longrightarrow \infty} U_1 = \lim_{W_2 \longrightarrow \infty} U_{11} = 0.08$. Player 2's Period 2 defense effort D_{21} also increases concavely, $\lim_{W_2 \longrightarrow \infty} D_{22} = 0.4$, and Player 2's expected utilities U_2 and U_{22} increase without bounds, $\lim_{W_2 \longrightarrow \infty} U_2 = \lim_{W_2 \longrightarrow \infty} U_{22} = 0.08$. Player 1's Period 2 effort Z_{12} and expected utility U_{12} decrease convexly, $\lim_{W_2 \longrightarrow \infty} Z_{12} = \lim_{W_2 \longrightarrow \infty} U_{12} = 0$.

In Figure 3n,n′, when the contest intensity v in Period 1 increases above the benchmark $v = 1$, the players compete more fiercely with each other in Period 1, receiving decreasing expected utilities U_1,U_{11},U_2,U_{21} until Player 1 reaches its budget constraint $Z_{11} = R_{11}/b_{11} = 1$ when $v > 1.14$. When $v > 1.14$, which gives a transition from Solution 1 to Solution 3, Player 2 competes even more fiercely with increasing Period 1 defense effort D_{21} while accepting negative Period 1 expected utility U_2. Player 1's Period 1 expected utility U_{11} is even more negative. When $v > 1.14$, the advantaged Player 2 exerts slightly increasing Period 2 effort D_{22}, while Player 1 exerts decreasing effort Z_{12}. That continues until $v > 1.30$, when Player 1 starts to receive negative expected utility $U_1 < 0$ over the two periods, which is unacceptable for Player 1. Hence Solution 9 emerges, where Player 1 withdraws from both periods and receives zero expected utilities $Z_{11} = Z_{12} = U_1 = U_{11} = U_{12} = 0$. When $v > 1.30$, Player 2 exerts a arbitrarily small positive effort and keeps its asset, i.e., $D_{21} = D_{22} = \epsilon > 0$, where ϵ is arbitrarily small but positive, and receives expected utilities $U_2 = U_{21} = 2$, $U_2 = 4$. In contrast, as v decreases below the benchmark $v = 1$, both players exert lower Period 1 efforts Z_{11} and D_{21} and eventually zero effort $Z_{11} = D_{21} = 0$ at the limit for an egalitarian contest $v = 0$, where efforts do not matter. Concomitantly, both players' expected utilities U_1,U_{11},U_2,U_{21} increase. The players' Period 2 efforts and expected utilities are constant at $Z_{11} = D_{21} = U_{12} = U_{22} = 0.25$.

In Figure 3o,o′, when the contest intensity w in Period 2 increases from $w = 0$ (egalitarian contest) through to the benchmark $w = 1$ and to $w = 2$, the players' Period 2 efforts Z_{12} and D_{22} increase from $Z_{12} = D_{22} = 0$ through $Z_{12} = D_{22} = 0.25$, and to $Z_{12} = D_{22} = 0.5$. Simultaneously, the players' Period 1 efforts Z_{11} and D_{21} increase from $Z_{11} = D_{21} = 0.75$, when $w = 0$ (no egalitarian contest in Period 1), through the benchmark $Z_{11} = D_{21} = 0.875$, and to $Z_{11} = D_{21} = 1$ when $w = 2$. These increases in the efforts Z_{12}, D_{22}, Z_{11}, D_{21} depress the players' expected utilities U_1,U_{11},U_{12},U_2,U_{21},U_{22}, all of which decrease after reaching $U_1 = U_{11} = U_{12} = U_2 = U_{21} = U_{22} = 0$ when $w = 2$. When $w > 2$, causing transition from Solution 1 to Solution 10, we assume that the players choose the equilibrium, where they both exert the $w = 2$ efforts $Z_{12} = D_{22} = 0.5$ and $Z_{11} = D_{21} = 1$ and receive zero expected utilities $U_1 = U_{11} = U_{12} = U_2 = U_{21} = U_{22} = 0$. Increasing the Period 2 contest intensity w is quite costly for equally matched (equally advantaged) players.

In Figure 3p,p′, when Player 1′s zero-day appreciation factor δ_1 of stockpiled zero-day exploits S_1 from Period 1 to Period 2 increases above the benchmark $\delta_1 = 1$, causing transition from Solution 1 to Solution 2 in Table 1, Player 1 immediately utilizes its entire Period 1 budget $R_{11} = 1$, allocating $S_1 = \frac{R_{11}-b_{11}Z_{11}}{b_{11}} = 0.125$ to stockpiling, $Z_{11} = 0.875$ to the Period 1 attack, and $Z_{12} = 0.125$ to the Period 2 attack. Hence, Player 1 cuts the Period 2 attack in half, from the benchmark $Z_{12} = 0.25$ to $Z_{12} = 0.125$, utilizing stockpiling $S_1 = 0.125$ from Period 1 instead as δ_1 increases above $\delta_1 = 1$. As δ_1 increases above $\delta_1 = 1$, Player 1 keeps its stockpiling at $S_1 = 0.125$, as permitted by its budget constraint $R_{11} = 1$, but decreases its Period 2 attack Z_{12} linearly since stockpiling at S_1 gets multiplied with the increasing δ_1 (see $\delta_1 S_1$ in (5)). Player 1′s expected utilities U_1 and U_2 increase, while its Period 1 expected utility is zero, $U_{11} = 0$, since its stockpiling S_1 gives a cost in Period 1 and a benefit in Period 2. Player 2′s expected utilities U_2,U_{21},U_{22} remain at their benchmarks when $1 \leq \delta_1 \leq 2$ since Player 1′s allocation from Z_{12} to S_1 is all that happens when $1 \leq \delta_1 \leq 2$. As δ_1 increases above $\delta_1 = 2$, Player 1′s Period 2 attack Z_{12} decreases to $Z_{12} = 0$, as it gets entirely replaced by stockpiling S_1. That causes transition from Solution 2 to Solution 7 in Table 1. As δ_1 increases above $\delta_1 = 2$, Player 1 decreases its stockpiling S_1, $\lim_{\delta_1 \longrightarrow \infty} S_1 = 0$, which continues to impact Period 2 due to $\delta_1 S_1$ in (5). That enables Player 1 to increase its Period 1 attack Z_{11}, within its budget $R_{11} = 1$, $\lim_{\delta_1 \longrightarrow \infty} Z_{11} = 1$. Thus, Player 2 decreases its defense in both periods, $\lim_{\delta_1 \longrightarrow \infty} D_{21} = 0.66$, $\lim_{\delta_1 \longrightarrow \infty} D_{22} = 0.19$. Thus, Player 1′s expected utilities U_1,U_{11},U_{12} increase concavely, $\lim_{\delta_1 \longrightarrow \infty} U_1 = 0.948$, $\lim_{\delta_1 \longrightarrow \infty} U_{11} = 0.203$, $\lim_{\delta_1 \longrightarrow \infty} U_{12} = 0.745$, while Player 2′s expected utilities U_2,U_{21},U_{22} decrease convexly, $\lim_{\delta_1 \longrightarrow \infty} U_2 = 0.25$, $\lim_{\delta_1 \longrightarrow \infty} U_{21} = 0.13$, $\lim_{\delta_1 \longrightarrow \infty} U_{22} = 0.12$. In contrast, when δ_1 is less than 1, i.e., $0 \leq \delta_1 \leq 1$, which means depreciation, then Player 1 refrains from stockpiling, S_1. Hence, all variables are constant at their benchmark values as functions of δ_1 when $0 \leq \delta_1 \leq 1$.

In Figure 3q,q′, as Player 1′s time discount factor β_1 decreases below the benchmark $\beta_1 = 1$, so that Player 1 assigns less weight to the future Period 2, Player 1 exerts decreasing efforts Z_{11} and Z_{12} in both periods, receiving decreasing expected utilities U_1 and U_{12} but increasing expected utility U_{11} in Period 1, which is more important than Period 2 for Player 1, while Player 2 assigns equal importance to both periods. As β_1 decreases, Player 2 exerts increasing defense efforts D_{12} and D_{22} in both periods, which eventually decrease slightly, causing inverse U-shapes as β_1 approaches $\beta_1 = 0$. As β_1 decreases, Player 2 becomes more competitive due to weighing both periods equally and receiving increasing expected utilities U_2,U_{21},U_{22}. When $\beta_1 < 1$, Player 1 assigns less weight to Period 2 than Period 1, causing zero stockpiling $S_1 = 0$.

In Figure 3r,r′, as Player 2′s time discount factor β_2 decreases below the benchmark $\beta_2 = 1$, so that Player 2 assigns less weight to the future Period 2, Player 2 exerts decreasing defense efforts and D_{22} in both periods, receiving decreasing expected utilities U_2 and U_{22} but increasing expected utility U_{21} in Period 1, which is more important than Period 2 for Player 2, while Player 1 assigns equal importance to both periods. As β_2 decreases, Player 1 exerts increasing efforts Z_{11} and Z_{12} in both periods, becoming more competitive due to weighing both periods equally and receiving increasing expected utilities U_1,U_{11},U_{12}. As β_2 decreases below $\beta_2 = 0.80$, Player 1 reaches its budget constraint, which constricts its Period 1 effort $Z_{11} = R_{11}/b_{11} = 1$, causing a transition from Solution 1 to Solution 3.

5. Discussion

Table 2 presents the key findings from Section 4, including the three situations where Player 1 stockpiles in Panels e,e′, g,g′, and p,p′.

Table 2. Key findings from Section 4, including the three situations where Player 1 stockpiles in Panels e,e′, g,g′, and p,p′.

Panel	Parameter(s)	Key Findings
a,a′	R_{11}	As Player 1′s available resources R_{11} in Period 1 decrease, its efforts in both periods decrease, while Player 2′s efforts in both periods are inverse U-shaped. Player 2 transitions from being inferior when Player 1 is resourceful to being competitive when the players are equally matched and being superior when Player 1 lacks resources.
b,b′	a_{21}	As Player 2′s unit effort cost a_{21} of defense in Period 1 increases, its efforts decrease, while Player 1′s efforts are inverse U-shaped and resource-constrained. As a_{21} decreases, Player 2′s Period 1 effort increases, while its Period 2 effort is inverse U-shaped, and Player 1′s efforts decrease.
c,c′	$a_{21} = a_{22}$	As Player 2′s unit defense costs $a_{21} = a_{22}$ in both periods increase (decrease), Player 2 becomes more disadvantaged (advantaged) than when only its unit effort cost a_{21} of defense in Period 1 increases (decreases).
d,d′	a_{22}	If Player 2 can choose, it prefers being disadvantaged in Period 2 with high unit effort cost a_{22}, when a less valuable asset is at stake, rather than being disadvantaged in the more important Period 1 with high unit effort cost a_{21}. Similarly, Player 2 prefers being advantaged in the more important Period 1 with low unit effort cost a_{21}, rather than being advantaged in Period 2 with high a_{22}.
e,e′	b_{11}	Player 1 may stockpile when its unit effort cost b_{11} of developing zero-day capabilities in Period 1 decreases, through three phases, below that of Period 2. First, Player 1 stockpiles as permitted by the budget and cuts back on the Period 2 effort. Second, Player 1 utilizes its entire budget in Period 1 without stockpiling, to exploit its advantage competitively over Player 2. Third, Player 1 eventually does not need to utilize its entire budget, attacks optimally in Period 1, and stockpiles sufficiently in Period 1 to deter Player 2 from defending in Period 2.
f,f′	$b_{11} = b_{12}$	As Player 1′s unit effort costs $b_{11} = b_{12}$ of developing zero-day capabilities increase equally in both periods, Player 1 does not stockpile and becomes more disadvantaged than when only one unit effort cost increases. As $b_{11} = b_{12}$ decrease, Player 1 becomes more advantaged than when only one unit effort cost decreases.
g,g′	b_{12}	As Player 1′s unit effort cost b_{12} of developing zero-day capabilities in Period 2 increases above that of Period 1, Player 1 stockpiles more to exploit the advantage of the cheaper unit effort cost in Period 1, decreases the efforts in both periods, and accepts negative expected utility in Period 1 to ensure higher expected utility in Period 2. This continues until Player 1 can no longer afford to exert effort in Period 2. Player 1 instead focuses on Period 1 and stockpiles optimally for Period 2, as permitted by the budget constraint.
h,h′	V_1	As Player 1′s valuation V_1 of Player 2′s asset increases, Player 1 exerts higher efforts and eventually becomes resource-constrained, while Player 2 exerts lower efforts. As V_1 decreases, Player 1 exerts lower efforts and Player 2′s efforts are inverse U-shaped.
i,i′	V_2	As Player 2′s valuation V_2 of its own asset increases, Player 2 exerts concavely increasing Period 1 defense effort and inverse U-shaped Period 2 effort, while Player 1′s efforts decrease. As V_2 decreases, Player 2′s efforts decrease, while Player 1′s efforts are inverse U-shaped and resource-constrained.
j,j′	g_1	As Player 1′s growth factor g_1 of asset V_1 from Period 1 to Period 2 increases, Player 1′s efforts increase, subject to the resource constraint, while Player 2′s efforts decrease. As V_1 decreases, Player 1′s efforts decrease overall, while Player 2′s efforts are inverse U-shaped.
k,k′	g_2	As Player 2′s growth factor g_2 of asset V_2 from Period 1 to Period 2 increases, Player 2′s Period 1 effort increases, its Period 2 effort is inverse U-shaped, and Player 1′s efforts decrease. As V_2 decreases, Player 2′s efforts decrease overall, while Player 1′s efforts are inverse U-shaped and resource-constrained.
l,l′	W_1	As Player 1′s valuation W_1 of Player 2′s asset, acquired in Period 2, increases, Player 1′s efforts increase, subject to the budget constraint, while Player 2′s efforts decrease.
m,m′	W_2	As Player 2′s valuation W_2 of its own asset acquired in Period 2 increases, Player 2′s efforts increase concavely, while Player 1′s efforts decrease convexly.
n,n′	v	As the contest intensity v in Period 1 increases, both players′ Period 1 efforts increase due to more fierce competition, until Player 1 reaches its budget constraint, after which Player 2 benefits. As v decreases, both players′ Period 1 efforts decrease, causing higher expected utilities.
o,o′	w	As the contest intensity w in Period 2 increases, both players′ efforts in both periods increase until the fiercer competition causes zero expected utilities to both players, assuming they are equally matched.
p,p′	δ_1	As Player 1′s zero-day appreciation factor δ_1 of stockpiled zero-day exploits from Period 1 to Period 2 increases above one, Player 1 immediately utilizes its entire Period 1 budget to attack and stockpile, cutting back on its Period 2 attack. This continues until Player 1′s stockpiling is so large that the Period 2 attack is no longer cost effective. Thereafter, Player 1 decreases its stockpiling (due to its appreciation) and increases its Period 1 attack, while Player 2 decreases its defense in both periods.
q,q′	β_1	As Player 1′s time discount factor β_1 decreases, so that Player 1 assigns less weight to the future Period 2, Player 1′s efforts decrease, causing lower expected utilities, while Player 2′s efforts increase overall, causing higher expected utilities.
r,r′	β_2	As Player 2′s time discount factor β_2 decreases, so that Player 2 assigns less weight to the future Period 2, Player 2′s efforts decrease, causing lower expected utilities, while Player 1′s efforts increase, subject to the budget constraint, causing higher expected utilities.

6. Conclusions

The article presents a two-player two-period game between players producing zero-day exploits for immediate deployment in Period 1 or stockpiles for future deployment in Period 2. In Period 2, Player 1 produces zero-day exploits for immediate deployment, supplemented by stockpiled zero-day exploits from Period 1. Player 2 defends its asset against the attack in both periods. The analysis implies 11 solutions, where Player 1 may or may not stockpile, may or may not utilize its entire budget, may or may not attack in Period 2, and may or may not deter Player 2 from defending in Period 2. Relative to a benchmark solution with no stockpiling, 18 parameter values are altered to understand the nature of the zero-day phenomenon over two periods. Both players strike balances between how to exert efforts over the two periods, while Player 1 additionally decides whether to stockpile.

Player 1 may stockpile in three situations. First, as Player 1's unit effort cost of developing zero-day capabilities in Period 1 decreases below that of Period 2, it may exploit the Period 1 advantage for stockpiling and deployment in Period 2. Second, when Player 1's unit effort cost of developing zero-day capabilities in Period 2 increases above that of Period 1, it may similarly exploit the Period 1 advantage for stockpiling, potentially even accepting negative expected utility in Period 1 in order to benefit from subsequent deployment in Period 2. Third, when Player 1's zero-day appreciation factor of stockpiled zero-day exploits from Period 1 to Period 2 increases above one, it stockpiles for utilization in Period 2 until no additional Period 2 attack is required.

When the contest intensity in Period 1 increases, the players compete more fiercely with each other in Period 1, receiving decreasing expected utilities, until Player 1 reaches its budget constraint. Thereafter, Player 2 competes more fiercely, and both players receive negative Period 1 expected utilities. This continues until Player 1 receives negative expected utility over both periods, causing it to withdraw, while Player 2 keeps its asset. When the contest intensity in Period 2 increases, all efforts increase until both players receive zero expected utilities, assuming that they are equally advantaged.

If a player's time discount factor decreases, the player exerts lower efforts in both periods and receives lower expected utilities except in Period 1. The other player exerts higher efforts overall. The model confirms many intuitive results. For example, a player exerts more effort if it is cheaper, if it values the asset more, if the asset has a higher growth factor, and if the asset added in Period 2 is more valuable. If a player's unit effort costs increase (decrease) equally as much in both periods, the player becomes more disadvantaged (advantaged) than if the unit effort cost in only one period increases (decreases). The phenomenon of inversely U-shaped efforts is documented extensively. Typically, a player competes most fiercely when equally advantaged compared with the other player and decreases its efforts due to cost-effectiveness when too advantaged (due to superiority) or too disadvantaged (due to inferiority).

Future research should include more players, outside interference from governments and nongovernment bodies, regulation, and supervision and account for technological developments of the various aspects of zero-day exploits. The parameter values should be estimated by considering zero-day attacks that have occurred. Empirical support should be provided from contemporary and historical records. More complexity and more than two time periods may also be incorporated.

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Nomenclature

Parameters

R_{11}	Player 1's cyber resources in Period 1, $R_{11} \geq 0$
a_{2j}	Player 2's unit effort cost of defense in Period j, $j = 1, 2$, $a_{2j} \geq 0$
b_{1j}	Player 1's unit effort cost of developing zero-day capabilities in Period j, $j = 1, 2$, $b_{1j} \geq 0$
V_i	Player i's valuation of Player 2's asset, $V_i \geq 0$
g_i	Growth factor of asset V_i from Period 1 to Period 2, $g_i \geq 0$
W_i	Player i's valuation of Player 2's asset acquired in Period 2, $W_i \geq 0$
v	Contest intensity in Period 1, $v \geq 0$
w	Contest intensity in Period 2, $w \geq 0$

δ_1	Player 1's zero-day appreciation factor of stockpiled zero-day exploits S_1 from Period 1 to Period 2, $\delta_1 \geq 0$
β_i	Player i's time discount factor, $0 \leq \beta_i \leq 1$
Strategic Choice Variables	
Z_{11}	Player 1's effort to develop zero-day capabilities in Period 1, $Z_{11} \geq 0$
D_{21}	Player 2's defense effort in Period 1, $D_{21} \geq 0$
Z_{12}	Player 1's effort to develop zero-day capabilities in Period 2, $Z_{12} \geq 0$
D_{22}	Player 2's defense effort in Period 2, $D_{22} \geq 0$
Dependent Variables	
S_1	Player 1's stockpiling of zero-day exploits in Period 1 for use in Period 2, $S_1 \geq 0$
p_{ij}	Player i's expected contest success in Period j, $i, j = 1, 2$, $0 \leq p_{ij} \leq 1$
U_{ij}	Player i's expected utility in Period j, $i, j = 1, 2$
U_i	Player i's expected utility over both time periods, $i = 1, 2$
$R_{11b} = b_{11}Z_{11} + b_{11}S_1 \leq R_{11}$	The actual amount of resources used by Player 1 in Period 1

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