Supplementary Material File S3:

The preference/trait model with natural selection on both sexes, recombination between the preference and trait loci, and migration between two demes

by

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1. Model and derivation of recursion relations

P2 is a preference allele for T2, with strength $\alpha 2$,
P1 is a preference allele for T1, with strength $\alpha 1$.
Migration into deme 1 occurs at rate $m1$, i.e., a fraction $m1$ of individuals in deme 1 is replaced by individuals from deme 2; migration into deme 2 occurs at rate $m2$.
T1 (T2) is favored by viability selection in both sexes in deme 1 (2) with selection coefficient $s1$ ($s2$).
The recombination rate between the P and T locus is $r \geq 0$.

There are the four genotypes P1T1, P1T2, P2T1, and P2T2. Their frequencies are denoted by
x1[1] = frequency of P1T1 in deme 1
x1[2] = frequency of P1T2 in deme 1
x1[3] = frequency of P2T1 in deme 1
x1[4] = frequency of P2T2 in deme 1
x2[1] = frequency of P1T1 in deme 2
x2[2] = frequency of P1T2 in deme 2
x2[3] = frequency of P2T1 in deme 2
x2[4] = frequency of P2T2 in deme 2

We use mainly allele frequencies and linkage disequilibria to describe the dynamics:

p12 = x1[3]+x1[4] = frequency of preference allele P2 in deme 1 (Note: the ordering of subscripts differs from that in the manuscript)
t12 = x1[2]+x1[4] = frequency of trait allele T2 in deme 1 (Note: the ordering of subscripts differs from that in the manuscript)
dd1 = x1[1]x1[4]-x1[2]x1[3] = linkage disequilibrium between P and T in deme 1
p22, t22, dd2 are the analogous quantities in deme 2

The phenotype-matching model is obtained as a special case if *r* = 0 and p12 = t12 and p22 = t22. As a consequence, linkage disequilibrium is maximal, i.e., dd1 = t12(1-t12) and dd2 = t22(1-t22).

1.1 Modeling the stages of the life cycle

1.1.1 Migration

Gamete frequencies after migration

```
x1m[1] = (1 - m1) x1[1] + m1 x2[1];
x1m[2] = (1 - m1) x1[2] + m1 x2[2];
x1m[3] = (1 - m1) x1[3] + m1 x2[3];
x1m[4] = (1 - m1) x1[4] + m1 x2[4];
```

```
x2m[1] = (1 - m2) x2[1] + m2 x1[1];
x2m[2] = (1 - m2) x2[2] + m2 x1[2];
x2m[3] = (1 - m2) x2[3] + m2 x1[3];
x2m[4] = (1 - m2) x2[4] + m2 x1[4];
```

1.1.2 Viability selection

Gamete frequencies after viability selection

pop 1

```
w1bar = 1 + s1 (x1m[1] + x1m[3]);
```

```
x1v[1] = x1m[1] (1 + s1) / w1bar;
x1v[2] = x1m[2] / w1bar;
x1v[3] = x1m[3] (1 + s1) / w1bar;
x1v[4] = x1m[4] / w1bar;
```

pop 2

```
w2bar = 1 + s2 (x2m[2] + x2m[4]);
```

```
x2v[1] = x2m[1] / w2bar;
x2v[2] = (1 + s2) x2m[2] / w2bar;
x2v[3] = x2m[3] / w2bar;
x2v[4] = (1 + s2) x2m[4] / w2bar;
```

1.1.3 Sexual selection

P1 prefers T1 (by a factor of 1+α1), and P2 prefers T2 (by a factor of 1+α2)

pop 1

```
Mate1 = Table[x1v[i] x1v[j], {i, 4}, {j, 4}];
Coef = Table[1, {i, 4}, {j, 4}];
Do[Coef[[i, 1]] = (1 + α1), {i, 1, 2}];
Do[Coef[[i, 3]] = (1 + α1), {i, 1, 2}];
Do[Coef[[i, 2]] = (1 + α2), {i, 3, 4}];
Do[Coef[[i, 4]] = (1 + α2), {i, 3, 4}];
G1 = Mate1 Coef;
```

```
Simplify[Coef] // TableForm
```

$1+\alpha1$	1	$1+\alpha1$	1
$1+\alpha1$	1	$1+\alpha1$	1
1	$1+\alpha2$	1	$1+\alpha2$
1	$1+\alpha2$	1	$1+\alpha2$

```
Do[z1[i] = Sum[x1v[j] Coef[[i, j]], {j, 4}], {i, 4}]
```

```
F1 = Table[0, {i, 4}, {j, 4}];
Do[F1[[i, j]] = G1[[i, j]] / z1[i], {i, 4}, {j, 4}];
```

```
TableForm[Simplify[F1]]
```

$$-\frac{}{(-1+s1\,((-1+m1)\,x1[1]+(-1+m1)\,x1[3]-m1\,(x2[1]+x2[3])))\,(-(-1+m1)\,(1+s1)\,(1+\alpha1)\,x1[1]+x1[2]-m1\,x1[2]+x1[3]-m1\,x1[3]+s1\,x1[3}$$

$$-\frac{}{(-1+s1\,((-1+m1)\,x1[1]+(-1+m1)\,x1[3]-m1\,(x2[1]+x2[3])))\,(-(-1+m1)\,(1+s1)\,(1+\alpha1)\,x1[1]+x1[2]-m1\,x1[2]+x1[3]-m1\,x1[3]+s1\,x1[3}$$

$$-\frac{}{(-1+s1\,((-1+m1)\,x1[1]+(-1+m1)\,x1[3]-m1\,(x2[1]+x2[3])))\,(-(-1+m1)\,(1+s1)\,x1[1]-(-1+m1)\,(1+\alpha2)\,x1[2]+x1[3]-m1\,x1[3]+s1\,x1[3]}$$

$$-\frac{}{(-1+s1\,((-1+m1)\,x1[1]+(-1+m1)\,x1[3]-m1\,(x2[1]+x2[3])))\,(-(-1+m1)\,(1+s1)\,x1[1]-(-1+m1)\,(1+\alpha2)\,x1[2]+x1[3]-m1\,x1[3]+s1\,x1[3]}$$

pop 2

```
Mate2 = Table[x2v[i] x2v[j], {i, 4}, {j, 4}];
G2 = Mate2 Coef;
```

```
Do[z2[i] = Sum[x2v[j] Coef[[i, j]], {j, 4}], {i, 4}]
```

```
F2 = Table[0, {i, 4}, {j, 4}];
Do[F2[[i, j]] = G2[[i, j]] / z2[i], {i, 4}, {j, 4}];
```

```
TableForm[F2]
```

$$\frac{(1+\alpha1)\,xnsf2[1]\,xnsm2[1]}{(1+\alpha1)\,xnsm2[1]+xnsm2[2]+(1+\alpha1)\,xnsm2[3]+xnsm2[4]} \qquad \frac{xnsf2[1]\,xnsm2[2]}{(1+\alpha1)\,xnsm2[1]+xnsm2[2]+(1+\alpha1)\,xnsm2[3]+xnsm2[4]} \qquad \frac{(1+\alpha1}{(1+\alpha1)\,xnsm2[1]+xn}$$

$$\frac{(1+\alpha1)\,xnsf2[2]\,xnsm2[1]}{(1+\alpha1)\,xnsm2[1]+xnsm2[2]+(1+\alpha1)\,xnsm2[3]+xnsm2[4]} \qquad \frac{xnsf2[2]\,xnsm2[2]}{(1+\alpha1)\,xnsm2[1]+xnsm2[2]+(1+\alpha1)\,xnsm2[3]+xnsm2[4]} \qquad \frac{(1+\alpha1}{(1+\alpha1)\,xnsm2[1]+xn}$$

$$\frac{xnsf2[3]\,xnsm2[1]}{xnsm2[1]+(1+\alpha2)\,xnsm2[2]+xnsm2[3]+(1+\alpha2)\,xnsm2[4]} \qquad \frac{(1+\alpha2)\,xnsf2[3]\,xnsm2[2]}{xnsm2[1]+(1+\alpha2)\,xnsm2[2]+xnsm2[3]+(1+\alpha2)\,xnsm2[4]} \qquad \frac{xn}{xnsm2[1]+(1+\alpha2)\,xn}$$

$$\frac{xnsf2[4]\,xnsm2[1]}{xnsm2[1]+(1+\alpha2)\,xnsm2[2]+xnsm2[3]+(1+\alpha2)\,xnsm2[4]} \qquad \frac{(1+\alpha2)\,xnsf2[4]\,xnsm2[2]}{xnsm2[1]+(1+\alpha2)\,xnsm2[2]+xnsm2[3]+(1+\alpha2)\,xnsm2[4]} \qquad \frac{xn}{xnsm2[1]+(1+\alpha2)\,xn}$$

1.1.4 Recombination

pop 1

```
x1t1[1] = Simplify[F1[[1, 1]] + (1 / 2) F1[[1, 2]] + (1 / 2) F1[[1, 3]] +
    (1 / 2) (1 - r) F1[[1, 4]] + (1 / 2) F1[[2, 1]] + (1 / 2) r F1[[2, 3]] +
    (1 / 2) F1[[3, 1]] + (1 / 2) r F1[[3, 2]] + (1 / 2) (1 - r) F1[[4, 1]]];

x1t1[2] = Simplify[(1/2) F1[[1,2]] + (1/2) r F1[[1,4]] +
    (1/2) F1[[2,1]] + F1[[2,2]] + (1/2)(1-r)F1[[2,3]] +
    (1/2) F1[[2,4]] + (1/2) (1-r) F1[[3,2]] +
    (1/2) r F1[[4,1]] + (1/2) F1[[4,2]]];

x1t1[3] = Simplify[(1/2) F1[[1,3]] + (1/2) r F1[[1,4]] +
    (1/2) (1-r) F1[[2,3]] + (1/2) F1[[3,1]] +
    (1/2) (1-r) F1[[3,2]] + F1[[3,3]] + (1/2) F1[[3,4]] +
    (1/2) r F1[[4,1]] + (1/2) F1[[4,3]]];

x1t1[4] = Simplify[(1/2) (1-r) F1[[1,4]] + (1/2) r F1[[2,3]] +
    (1/2) F1[[2,4]] + (1/2) r F1[[3,2]] + (1/2) F1[[3,4]] +
    (1/2) (1-r) F1[[4,1]] + (1/2) F1[[4,2]] + (1/2) F1[[4,3]] + F1[[4,4]]];
```

pop 2

```
x2t1[1] = Simplify[F2[[1, 1]] + (1 / 2) F2[[1, 2]] + (1 / 2) F2[[1, 3]] +
    (1 / 2) (1 - r) F2[[1, 4]] + (1 / 2) F2[[2, 1]] + (1 / 2) r F2[[2, 3]] +
    (1 / 2) F2[[3, 1]] + (1 / 2) r F2[[3, 2]] + (1 / 2) (1 - r) F2[[4, 1]]];

x2t1[2] = Simplify[(1/2) F2[[1,2]] + (1/2) r F2[[1,4]] +
    (1/2) F2[[2,1]] + F2[[2,2]] + (1/2)(1-r)F2[[2,3]] +
    (1/2) F2[[2,4]] + (1/2) (1-r) F2[[3,2]] +
    (1/2) r F2[[4,1]] + (1/2) F2[[4,2]]];

x2t1[3] = Simplify[(1/2) F2[[1,3]] + (1/2) r F2[[1,4]] +
    (1/2) (1-r) F2[[2,3]] + (1/2) F2[[3,1]] +
    (1/2) (1-r) F2[[3,2]] + F2[[3,3]] + (1/2) F2[[3,4]] +
    (1/2) r F2[[4,1]] + (1/2) F2[[4,3]]];

x2t1[4] = Simplify[(1/2) (1-r) F2[[1,4]] + (1/2) r F2[[2,3]] +
    (1/2) F2[[2,4]] + (1/2) r F2[[3,2]] + (1/2) F2[[3,4]] +
    (1/2) (1-r) F2[[4,1]] + (1/2) F2[[4,2]] + (1/2) F2[[4,3]] + F2[[4,4]]];
```

1.2 Recursion relation for allele frequencies and LD

The notation here is that the 1st subscript represents the population and the second represents the allele.

So all is done with respect to allele 2 in both populations.

1.2.1 Derivation of the recursion relations

```
x1[1] = (1 - p12) (1 - t12) + dd1;
x1[2] = (1 - p12) t12 - dd1;
x1[3] = p12 (1 - t12) - dd1;
x1[4] = p12 t12 + dd1;
x2[1] = (1 - p22) (1 - t22) + dd2;
x2[2] = (1 - p22) t22 - dd2;
x2[3] = p22 (1 - t22) - dd2;
x2[4] = p22 t22 + dd2;

p1t1 = Simplify[x1t1[3] + x1t1[4]]

t1t1 = Simplify[x1t1[2] + x1t1[4]]
```

```
dd1t1 = x1t1[1] x1t1[4] - x1t1[2] x1t1[3]

p2t1 = Simplify[x2t1[3] + x2t1[4]]

t2t1 = Simplify[x2t1[2] + x2t1[4]]

dd2t1 = x2t1[1] x2t1[4] - x2t1[2] x2t1[3]
```

1.2.2 The main recursion relation, itgen[s1, s2, α1, α2, m1, m2, r][{p12, t12, dd1, p22, t22, dd2}]

For given parameters {s1, s2, α1, α2, m1, m2, r}, itgen computes {p12 (t + 1), t12 (t + 1), dd1 (t + 1), p22 (t + 1), t22 (t + 1), dd2 (t + 1)} from {p12 (t), t12 (t), dd1 (t), p22 (t), t22 (t), dd2 (t)} .
Because computation of p1t1 etc (see above) is time consuming, we define itgen[s1, s2, α1, α2, m1, m2, r][{p12, t12, dd1, p22, t22, dd2}] explicitly by the right-hand side of {p1t1, t1t1, dd1t1, p2t1, t2t1, dd2t1}.

```
itgen[s1_, s2_, α1_, α2_, m1_, m2_, r_][{p12_, t12_, dd1_, p22_, t22_, dd2_}] :=
  {(-(-1 + m1)^2 m1 p12^2 (1 + s1) (-1 + s1 (-1 + t12)) (t12 - t22) (α1 + α2 + α1 α2) -
      dd1 (α1 + 2 s1^3 (-1 + t12)^2 (1 + α1) + t12 α1 α2 +
          s1 (2 + dd1 α2 + 2 t12 α2 + α1 (4 + dd1 - 3 t12 + dd1 α2 + 3 t12 α2 - 2 t12^2 α2)) +
          s1^2 (4 + dd1 α2 - 2 t12^2 (α1 (-1 + α2) + α2) +
              α1 (5 + dd1 + dd1 α2) + t12 (-4 - 7 α1 + 2 α2 + 2 α1 α2))) -
      m1^3 (t12 - t22) (-2 (dd1 - dd2) s1 (t12 - t22) (s1 + α1 + s1 α1) (s1 - α2) +
          p22^2 (1 + s1) (-1 + s1 (-1 + t22)) (α1 + α2 + α1 α2) +
          p22 (2 s1^3 (t12 - t22) (-1 + t22) (1 + α1) + (t12 - t22) α1 α2 +
              s1 (-dd1 (α1 + α2 + α1 α2) + dd2 (α1 + α2 + α1 α2) - (t12 - t22)
                  (α1 - 2 α2 - 3 α1 α2 + 2 t22 α1 α2)) + s1^2 (t22 (2 + α1 (3 - 2 α2) - 2 α2) + 2
                  t22^2 (α1 (-1 + α2) + α2) - (dd1 - dd2) (α1 + α2 + α1 α2) + t12
                  (-2 + 2 α2 - 2 t22 α2 + α1 (-3 + 2 t22 + 2 α2 - 2 t22 α2))))) +
      m1 (-dd2 (-α1 + 2 s1^2 (-1 + t12) (1 + α1) + s1 (-2 + (-3 + 2 t12) α1))
          (-1 + s1 (-1 + t12) - t12 α2) + 2 dd1^2 s1 (1 + s1) (α1 + α2 + α1 α2) +
          dd1 (2 s1^3 (-1 + t12) (-1 + 3 t12 - 2 t22) (1 + α1) + α1 (1 + 2 t12 α2 - t22 α2) +
              s1^2 (4 + 5 α1 - 2 dd2 α1 - 2 dd2 α2 - 2 dd2 α1 α2 - 6 t12^2
                  (α1 (-1 + α2) + α2) + t22 (4 + 7 α1 - 2 α2 - 2 α1 α2) + 2 t12
                  (α1 (-7 + 2 t22 (-1 + α2) + 2 α2) + 2 (-2 + α2 + t22 α2))) +
              s1 (2 - 2 dd2 α2 + 4 t12 α2 - 2 t22 α2 + α1 (4 + 3 t22 - 6 t12^2 α2 -
                    3 t22 α2 - 2 dd2 (1 + α2) + t12 (-6 + 6 α2 + 4 t22 α2)))) +
          p22 (2 - 2 s1^3 (-1 + t12)^2 (-1 + t22) (1 + α1) + dd1 α2 + 2 t12 α2 +
              α1 (dd1 (1 + α2) - (-2 + t12 + t22) (1 + t12 α2)) + s1^2 (6 + 6 α1 + dd1 α1 + dd1
                  α2 + dd1 α1 α2 + t12^2 (2 + α1 (3 + 2 t22 (-1 + α2) - 2 α2) + 2 (-1 + t22) α2) +
                t12 (-8 + t22 (4 + α1 (7 - 2 α2) - 2 α2) + 2 α2 + dd1 α2 + α1 (-9 + dd1 +
                      2 α2 + dd1 α2)) - t22 (4 + 2 dd1 α2 + α1 (5 + 2 dd1 (1 + α2)))) +
              s1 (t12^2 (α1 - 2 α2 - 3 α1 α2 + 2 t22 α1 α2) + t12 (-4 + (4 + dd1 - 2 t22) α2 +
                    α1 (-6 + dd1 + 3 t22 + 4 α2 + dd1 α2 - 3 t22 α2)) - 2 (-3 - dd1 α2 -
                    α1 (3 + dd1 + dd1 α2) + t22 (1 + dd1 α2 + α1 (2 + dd1 + dd1 α2)))))) +
      m1^2 (-dd1^2 s1 (1 + s1) (α1 + α2 + α1 α2) + p22^2 (1 + s1) (-1 + s1 (-1 + t22))
          (t12 - t22) (α1 + α2 + α1 α2) -
          dd2 (-4 s1^3 (-1 + t12) (t12 - t22) (1 + α1) + (-t12 + t22) α1 α2 +
```

```
    s1 (dd2 (α1 + α2 + α1 α2) + (t12 - t22) (-2 α2 + α1 (3 - 3 α2 + 4 t12 α2))) + s1^2
     (4 t12^2 (α1 (-1 + α2) + α2) + dd2 (α1 + α2 + α1 α2) + t22 (-4 - 7 α1 + 2 α2 + 2 α1
          α2) + t12 (α1 (7 + 4 t22 - 2 α2 - 4 t22 α2) - 2 (-2 + α2 + 2 t22 α2)))) +
  dd1 (-2 s1^3 (3 t12^2 + t22 (2 + t22) - 2 t12 (1 + 2 t22)) (1 + α1) +
    (-t12 + t22) α1 α2 + s1 (2 dd2 (α1 + α2 + α1 α2) + (t12 - t22)
        (-2 α2 + α1 (3 + (-3 + 6 t12 - 2 t22) α2))) +
    s1^2 (6 t12^2 (α1 (-1 + α2) + α2) + 2 t22^2 (α1 (-1 + α2) + α2) + 2 dd2
        (α1 + α2 + α1 α2) + t22 (-4 - 7 α1 + 2 α2 + 2 α1 α2) + t12
        (α1 (7 + 8 t22 - 2 α2 - 8 t22 α2) - 2 (-2 + α2 + 4 t22 α2)))) +
  p22 (-dd1 α1 + dd2 α1 + t12 α1 - t22 α1 + 4 s1^3 (-1 + t12) (t12 - t22)
      (-1 + t22) (1 + α1) - dd1 α2 + dd2 α2 - 2 t12 α2 + 2 t22 α2 - dd1 α1 α2 +
    dd2 α1 α2 - 2 t12 α1 α2 + 2 t12^2 α1 α2 + 2 t22 α1 α2 - t12 t22 α1 α2 -
    t22^2 α1 α2 + s1^2 (t22^2 (4 + α1 (7 - 2 α2) - 2 α2) - (dd1 - dd2) (α1 + α2 + α1 α2) +
      t12^2 (-4 - 4 (-1 + t22) α2 + α1 (-6 - 4 t22 (-1 + α2) + 4 α2)) + t12
       (8 + (-2 - 2 dd1 + dd2 - 2 t22 + 4 t22^2) α2 + α1 (9 + dd2 - t22 - 4 t22^2 - 2 α2 +
          dd2 α2 - 2 t22 α2 + 4 t22^2 α2 - 2 dd1 (1 + α2))) + t22 (-8 + 2 α2 +
        3 dd1 α2 - 2 dd2 α2 + α1 (-9 + 2 α2 + 3 dd1 (1 + α2) - 2 dd2 (1 + α2)))) +
    s1 (t22^2 (-3 α1 (-1 + α2) - 2 α2) - 2 (dd1 - dd2) (α1 + α2 + α1 α2) - 2
       t12^2 (α1 - 2 α2 - 3 α1 α2 + 2 t22 α1 α2) + t12
       (4 + (-4 - 2 dd1 + dd2 - 2 t22) α2 + α1 (6 + dd2 - t22 - 4 α2 + dd2 α2 -
          3 t22 α2 + 4 t22^2 α2 - 2 dd1 (1 + α2))) + t22 (-4 + (4 + 3 dd1 - 2 dd2)
        α2 + α1 (3 dd1 (1 + α2) - 2 (3 + dd2 - 2 α2 + dd2 α2)))))) +
 (-1 + m1) p12 (-2 - 2 α1 - dd1 α1 + dd1 m1 α1 - dd2 m1 α1 + 2 t12 α1 -
  m1 t12 α1 + m1 p22 t12 α1 - 2 m1^2 p22 t12 α1 +
  m1 t22 α1 - m1 p22 t22 α1 + 2 m1^2 p22 t22 α1 +
  2 s1^3 (-1 + t12) (-1 + t12 - m1 t12 + m1 t22)^2 (1 + α1) -
  dd1 α2 + dd1 m1 α2 - dd2 m1 α2 - 2 t12 α2 + 2 m1 t12 α2 +
  m1 p22 t12 α2 - 2 m1^2 p22 t12 α2 - 2 m1 t22 α2 - m1 p22 t22 α2 +
  2 m1^2 p22 t22 α2 - dd1 α1 α2 + dd1 m1 α1 α2 - dd2 m1 α1 α2 -
  2 t12 α1 α2 + 2 m1 t12 α1 α2 + m1 p22 t12 α1 α2 -
  2 m1^2 p22 t12 α1 α2 + 2 t12^2 α1 α2 - 3 m1 t12^2 α1 α2 + m1^2 t12^2 α1 α2 -
  2 m1 t22 α1 α2 - m1 p22 t22 α1 α2 + 2 m1^2 p22 t22 α1 α2 +
  3 m1 t12 t22 α1 α2 - 2 m1^2 t12 t22 α1 α2 + m1^2 t22^2 α1 α2 +
  s1 (-2 (-1 + m1)^2 t12^3 α1 α2 - 2 (3 + dd1 α2 + α1 (3 + dd1 + dd1 α2)) - m1^2 t22
      (-4 p22 α1 + t22 α1 + p22 t22 α1 - 4 p22 α2 - 2 t22 α2 + p22 t22 α2 - 4 p22 α1 α2 -
        3 t22 α1 α2 + p22 t22 α1 α2 - dd1 (α1 + α2 + α1 α2) + dd2 (α1 + α2 + α1 α2)) -
    m1 (-2 (dd1 - dd2) (α1 + α2 + α1 α2) + t22 (-4 + (4 + dd1 + 2 p22) α2 + α1
          (-6 + dd1 + 2 p22 + 4 α2 + dd1 α2 + 2 p22 α2))) + (-1 + m1) t12^2 ((-4 + m1
          (2 + p22)) α2 + α1 (4 - 6 α2 + m1 (-1 + p22 + 3 α2 + p22 α2 + 4 t22 α2))) +
    t12 (6 + 10 α1 + dd1 α1 - 4 α2 + dd1 α2 - 4 α1 α2 + dd1 α1 α2 - m1^2
        (4 p22 α1 - 2 t22 α1 + 4 p22 α2 + 4 t22 α2 + 4 p22 α1 α2 + 6 t22 α1 α2 +
          2 t22^2 α1 α2 + dd1 (α1 + α2 + α1 α2) - dd2 (α1 + α2 + α1 α2)) + m1
        (-4 + 4 α2 + dd2 α2 + 2 p22 α2 + 6 t22 α2 + p22 t22 α2 + α1
          (-6 - 5 t22 + 4 α2 + 9 t22 α2 + dd2 (1 + α2) + p22 (2 + t22) (1 + α2))))) -
    s1^2 (6 + 6 α1 + dd1 α1 + dd1 α2 + dd1 α1 α2 + 2 (-1 + m1)^2 t12^3 (α1 (-1 + α2) + α2) +
      m1^2 t22 (-(dd1 - dd2 + 2 p22) (α1 + α2 + α1 α2) + t22 (2 + (-2 + p22) α2 +
          α1 (3 + p22 - 2 α2 + p22 α2))) + m1 (-(dd1 - dd2) (α1 + α2 + α1 α2) + t22
        (-8 + (2 + dd1 + p22) α2 + α1 (-9 + dd1 + p22 + 2 α2 + dd1 α2 + p22 α2))) -
      (-1 + m1) t12^2 (6 + 10 α1 - 4 α2 - 4 α1 α2 + m1 (-2 + (2 + p22 + 4 t22) α2 +
          α1 (-3 + p22 - 4 t22 + 2 α2 + p22 α2 + 4 t22 α2))) + t12 (-12 - 14 α1 - dd1
```

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α1 + 2 α2 - dd1 α2 + 2 α1 α2 - dd1 α1 α2 + m1^2 (2 t22^2 (α1 (-1 + α2) + α2) +
      (dd1 - dd2 + 2 p22) (α1 + α2 + α1 α2) + t22 (-4 - 6 α1 + 4 α2 + 4 α1 α2)) -
   m1 (-8 + 2 α2 + dd2 α2 + p22 α2 + α1 (-9 + dd2 + p22 + 2 α2 + dd2 α2 + p22 α2) +
      t22 (-8 + (6 + p22) α2 + α1 (-13 + p22 + 6 α2 + p22 α2))))))) /
 (2 (1 + s1 (1 + (-1 + m1) t12 - m1 t22)) (1 + (1 + (-1 + m1) t12 - m1 t22) α1 +
    s1 (1 + (-1 + m1) t12 - m1 t22) (1 + α1))
  (1 + s1 (1 + (-1 + m1) t12 - m1 t22) + m1 t22 α2 + t12 (α2 - m1 α2))),
-((((-1 + m1) t12 - m1 t22) (2 + p12 α2 - m1 p12 α2 + m1 p22 α2 + 2 t12 α2 -
      2 m1 t12 α2 - p12 t12 α2 + 2 m1 p12 t12 α2 - m1^2 p12 t12 α2 -
      m1 p22 t12 α2 + m1^2 p22 t12 α2 + 2 m1 t22 α2 - m1 p12 t22 α2 +
      m1^2 p12 t22 α2 - m1^2 p22 t22 α2 - (1 + (-1 + m1) t12 - m1 t22) α1
       (-1 - t12 α2 + (-1 + m1) p12 (1 + α2) - m1 (p22 + p22 α2 - t12 α2 + t22 α2)) +
      s1 (1 + (-1 + m1) t12 - m1 t22) (4 - dd1 α2 + dd1 m1 α2 - dd2 m1 α2 + 2 p12 α2 -
        2 m1 p12 α2 + 2 m1 p22 α2 + 2 t12 α2 - 2 m1 t12 α2 - p12 t12 α2 + m1 p12 t12 α2 +
        2 m1 t22 α2 - m1 p22 t22 α2 + α1 (2 + 2 p12 - 2 m1 p12 + 2 m1 p22 - t12 + m1 t12 -
          p12 t12 + m1 p12 t12 - m1 t22 - m1 p22 t22 + 2 p12 α2 - 2 m1 p12 α2 +
          2 m1 p22 α2 + t12 α2 - m1 t12 α2 - p12 t12 α2 + m1 p12 t12 α2 +
          m1 t22 α2 - m1 p22 t22 α2 + dd1 (-1 + m1) (1 + α2) - dd2 m1 (1 + α2))) +
      s1^2 (1 + (-1 + m1) t12 - m1 t22) (2 + α1 - dd1 α1 + p12 α1 - dd1 α2 + p12 α2 -
        dd1 α1 α2 + p12 α1 α2 + (-1 + m1) t12 (2 + p12 α2 + α1 (1 + p12 + p12 α2)) -
        m1 (-(dd1 - dd2 - p12 + p22) (α1 + α2 + α1 α2) +
          t22 (2 + p22 α2 + α1 (1 + p22 + p22 α2)))))) /
 (2 (1 + s1 (1 + (-1 + m1) t12 - m1 t22)) (1 + (1 + (-1 + m1) t12 - m1 t22) α1 +
    s1 (1 + (-1 + m1) t12 - m1 t22) (1 + α1))
  (1 + s1 (1 + (-1 + m1) t12 - m1 t22) + m1 t22 α2 + t12 (α2 - m1 α2)))),
-(((1 + s1) (-1 + s1 ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) + (-1 + m1)
        (-dd1 + p12 (1 - t12)) - m1 ((1 - p22) (1 - t22) + p22 (1 - t22))))^2
   (-((r ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
        ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) / (-dd1 - m1 (-dd1 + p12
          (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) +
        p12 (1 - t12) + (1 - p12) t12 + p12 t12 - m1 (-dd1 + (1 - p12) t12) -
        m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
        m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) + m1 s1
         (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
        (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
        s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
        m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
        m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
        (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1))) -
     ((1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
       ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22))) (1 + α1)) /
      (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
        m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 - m1
         (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
        m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
        m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
        (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
        s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
        m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
        m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
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    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) +
((-1 + r) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 - m1
     (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
((1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
    ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12))) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
(2 (1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))^2) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12))) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
(r ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
    ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12))) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
(((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12))) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) +
((-1 + r) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α2)) /
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(-(-1+m1) (1+s1) (dd1+(1-p12) (1-t12))-m1 (-dd1+p12 (1-t12))+
  s1 (-dd1+p12 (1-t12))-m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+
  p12 t12-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
  m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
  m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
  (dd1+p12 t12) α2-m1 (dd1+p12 t12) α2+m1 (-dd2+(1-p22) t22) α2+
  m1 (dd2+p22 t22) α2-(-1+m1) (-dd1+(1-p12) t12) (1+α2))-
 (((-1+m1) (-dd1+p12 (1-t12))-m1 (-dd2+p22 (1-t22)))
  ((-1+m1) (dd1+p12 t12)-m1 (dd2+p22 t22)) (1+α2))/
 (-(-1+m1) (1+s1) (dd1+(1-p12) (1-t12))-m1 (-dd1+p12 (1-t12))+
  s1 (-dd1+p12 (1-t12))-m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+
  p12 t12-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
  m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
  m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
  (dd1+p12 t12) α2-m1 (dd1+p12 t12) α2+m1 (-dd2+(1-p22) t22) α2+
  m1 (dd2+p22 t22) α2-(-1+m1) (-dd1+(1-p12) t12) (1+α2)))
(-(((1+s1) ((-1+m1) (dd1+(1-p12) (1-t12))-m1 (dd2+
          (1-p22) (1-t22))) ((-1+m1) (-dd1+(1-p12) t12)-
       m1 (-dd2+(1-p22) t22)))/(-dd1-m1 (-dd1+p12 (1-t12))+
    s1 (-dd1+p12 (1-t12))-m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+
    (1-p12) t12+p12 t12-m1 (-dd1+(1-p12) t12)-m1 (dd1+p12 t12)+
    m1 (dd2+(1-p22) (1-t22))+m1 s1 (dd2+(1-p22) (1-t22))+
    m1 (-dd2+p22 (1-t22))+m1 s1 (-dd2+p22 (1-t22))+
    m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
    (-dd1+p12 (1-t12)) α1-m1 (-dd1+p12 (1-t12)) α1+
    s1 (-dd1+p12 (1-t12)) α1-m1 s1 (-dd1+p12 (1-t12)) α1+
    m1 (dd2+(1-p22) (1-t22)) α1+m1 s1 (dd2+(1-p22) (1-t22)) α1+
    m1 (-dd2+p22 (1-t22)) α1+m1 s1 (-dd2+p22 (1-t22)) α1-
    (-1+m1) (1+s1) (dd1+(1-p12) (1-t12)) (1+α1)))-
 (2 ((-1+m1) (-dd1+(1-p12) t12)-m1 (-dd2+(1-p22) t22))^2)/
 (-dd1-m1 (-dd1+p12 (1-t12))+s1 (-dd1+p12 (1-t12))-
    m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+(1-p12) t12+p12 t12-m1
     (-dd1+(1-p12) t12)-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
    m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
    m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
    (-dd1+p12 (1-t12)) α1-m1 (-dd1+p12 (1-t12)) α1+
    s1 (-dd1+p12 (1-t12)) α1-m1 s1 (-dd1+p12 (1-t12)) α1+
    m1 (dd2+(1-p22) (1-t22)) α1+m1 s1 (dd2+(1-p22) (1-t22)) α1+
    m1 (-dd2+p22 (1-t22)) α1+m1 s1 (-dd2+p22 (1-t22)) α1-
    (-1+m1) (1+s1) (dd1+(1-p12) (1-t12)) (1+α1))-
 (r (1+s1) ((-1+m1) (dd1+(1-p12) (1-t12))-m1 (dd2+(1-p22)
          (1-t22))) ((-1+m1) (dd1+p12 t12)-m1 (dd2+p22 t22)))/
 (-dd1-m1 (-dd1+p12 (1-t12))+s1 (-dd1+p12 (1-t12))-
    m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+(1-p12) t12+p12 t12-m1
     (-dd1+(1-p12) t12)-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
    m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
    m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
    (-dd1+p12 (1-t12)) α1-m1 (-dd1+p12 (1-t12)) α1+
    s1 (-dd1+p12 (1-t12)) α1-m1 s1 (-dd1+p12 (1-t12)) α1+
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    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
(((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22))
    ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 - m1
     (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
((1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 - m1
     (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) +
((-1 + r) (1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 - m1
     (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
(r (1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22)
          (1 - t22))) ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) +
((-1 + r) (1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α2)) /
```

```
(-(-1+m1) (1+s1) (dd1+(1-p12) (1-t12))-m1 (-dd1+p12 (1-t12))+
  s1 (-dd1+p12 (1-t12))-m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+
  p12 t12-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
  m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
  m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
  (dd1+p12 t12) α2-m1 (dd1+p12 t12) α2+m1 (-dd2+(1-p22) t22) α2+
  m1 (dd2+p22 t22) α2-(-1+m1) (-dd1+(1-p12) t12) (1+α2))-
 (((-1+m1) (-dd1+(1-p12) t12)-m1 (-dd2+(1-p22) t22))
  ((-1+m1) (dd1+p12 t12)-m1 (dd2+p22 t22)) (1+α2))/
 (-(-1+m1) (1+s1) (dd1+(1-p12) (1-t12))-m1 (-dd1+p12 (1-t12))+
  s1 (-dd1+p12 (1-t12))-m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+
  p12 t12-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
  m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
  m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
  (dd1+p12 t12) α2-m1 (dd1+p12 t12) α2+m1 (-dd2+(1-p22) t22) α2+
  m1 (dd2+p22 t22) α2-(-1+m1) (-dd1+(1-p12) t12) (1+α2)))/
(4 (1+s1 (-m1 (dd1+(1-p12) (1-t12))-m1 (-dd1+p12 (1-t12))+
    (1-p12) (1-t12)+p12 (1-t12)+
    m1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))))^4))+
((1+s1) (-1+s1 ((-1+m1) (dd1+(1-p12) (1-t12))+(-1+m1) (-dd1+
    p12 (1-t12))-m1 ((1-p22) (1-t22)+p22 (1-t22))))^2
 (-((((-1+m1) (dd1+(1-p12) (1-t12))-m1 (dd2+(1-p22) (1-t22)))
   ((-1+m1) (-dd1+(1-p12) t12)-m1 (-dd2+(1-p22) t22)))/
  (-dd1-m1 (-dd1+p12 (1-t12))+s1 (-dd1+p12 (1-t12))-
   m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+(1-p12) t12+p12 t12-m1
    (-dd1+(1-p12) t12)-m1 (dd1+p12 t12)+m1 (dd2+(1-p22) (1-t22))+
   m1 s1 (dd2+(1-p22) (1-t22))+m1 (-dd2+p22 (1-t22))+
   m1 s1 (-dd2+p22 (1-t22))+m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
   (-dd1+p12 (1-t12)) α1-m1 (-dd1+p12 (1-t12)) α1+
   s1 (-dd1+p12 (1-t12)) α1-m1 s1 (-dd1+p12 (1-t12)) α1+
   m1 (dd2+(1-p22) (1-t22)) α1+m1 s1 (dd2+(1-p22) (1-t22)) α1+
   m1 (-dd2+p22 (1-t22)) α1+m1 s1 (-dd2+p22 (1-t22)) α1-
   (-1+m1) (1+s1) (dd1+(1-p12) (1-t12)) (1+α1)))+
 ((-1+r) ((-1+m1) (dd1+(1-p12) (1-t12))-m1 (dd2+(1-p22) (1-t22)))
  ((-1+m1) (dd1+p12 t12)-m1 (dd2+p22 t22)))/
 (-dd1-m1 (-dd1+p12 (1-t12))+s1 (-dd1+p12 (1-t12))-
  m1 s1 (-dd1+p12 (1-t12))+p12 (1-t12)+(1-p12) t12+
  p12 t12-m1 (-dd1+(1-p12) t12)-m1 (dd1+p12 t12)+
  m1 (dd2+(1-p22) (1-t22))+m1 s1 (dd2+(1-p22) (1-t22))+
  m1 (-dd2+p22 (1-t22))+m1 s1 (-dd2+p22 (1-t22))+
  m1 (-dd2+(1-p22) t22)+m1 (dd2+p22 t22)+
  (-dd1+p12 (1-t12)) α1-m1 (-dd1+p12 (1-t12)) α1+
  s1 (-dd1+p12 (1-t12)) α1-m1 s1 (-dd1+p12 (1-t12)) α1+
  m1 (dd2+(1-p22) (1-t22)) α1+m1 s1 (dd2+(1-p22) (1-t22)) α1+
  m1 (-dd2+p22 (1-t22)) α1+m1 s1 (-dd2+p22 (1-t22)) α1-
  (-1+m1) (1+s1) (dd1+(1-p12) (1-t12)) (1+α1))-
 (2 (1+s1) ((-1+m1) (dd1+(1-p12) (1-t12))-m1 (dd2+(1-p22) (1-t22)))^2
  (1+α1))/(-dd1-m1 (-dd1+p12 (1-t12))+s1 (-dd1+p12 (1-t12))-
```

```
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 +
    p12 t12 - m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) +
    m1 (dd2 + (1 - p22) (1 - t22)) + m1 s1 (dd2 + (1 - p22) (1 - t22)) +
    m1 (-dd2 + p22 (1 - t22)) + m1 s1 (-dd2 + p22 (1 - t22)) +
    m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
((1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22))) -
    ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22))) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 +
    p12 t12 - m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) +
    m1 (dd2 + (1 - p22) (1 - t22)) + m1 s1 (dd2 + (1 - p22) (1 - t22)) +
    m1 (-dd2 + p22 (1 - t22)) + m1 s1 (-dd2 + p22 (1 - t22)) +
    m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
(((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22))) -
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 +
    p12 t12 - m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) +
    m1 (dd2 + (1 - p22) (1 - t22)) + m1 s1 (dd2 + (1 - p22) (1 - t22)) +
    m1 (-dd2 + p22 (1 - t22)) + m1 s1 (-dd2 + p22 (1 - t22)) +
    m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
(r ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22))) -
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 +
    p12 t12 - m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) +
    m1 (dd2 + (1 - p22) (1 - t22)) + m1 s1 (dd2 + (1 - p22) (1 - t22)) +
    m1 (-dd2 + p22 (1 - t22)) + m1 s1 (-dd2 + p22 (1 - t22)) +
    m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
((1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
```

```
      ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))) /
   (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
     s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
     p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
     m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
     m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
     (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
     m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) +
  ((-1 + r) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 + (1 - p22) (1 - t22)))
     ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
   (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
     s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
     p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
     m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
     m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
     (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
     m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
  (r ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
     ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α2)) /
   (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
     s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
     p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
     m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
     m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
     (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
     m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)))
((((-1 + r) (1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 +
          (1 - p22) (1 - t22))) ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
   (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
     m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 -
     m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
     m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
     m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
     (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
     s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
     m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
     m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
     (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
  (((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22))
     ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
   (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
     m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 -
     m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
     m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
     m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
     (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
     s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
     m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
     m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
     (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) -
```

```
(r (1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α1)) /
 (-dd1 - m1 (-dd1 + p12 (1 - t12)) + s1 (-dd1 + p12 (1 - t12)) -
    m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) + (1 - p12) t12 + p12 t12 -
    m1 (-dd1 + (1 - p12) t12) - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (-dd1 + p12 (1 - t12)) α1 - m1 (-dd1 + p12 (1 - t12)) α1 +
    s1 (-dd1 + p12 (1 - t12)) α1 - m1 s1 (-dd1 + p12 (1 - t12)) α1 +
    m1 (dd2 + (1 - p22) (1 - t22)) α1 + m1 s1 (dd2 + (1 - p22) (1 - t22)) α1 +
    m1 (-dd2 + p22 (1 - t22)) α1 + m1 s1 (-dd2 + p22 (1 - t22)) α1 -
    (-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) (1 + α1)) +
((-1 + r) (1 + s1) ((-1 + m1) (dd1 + (1 - p12) (1 - t12)) - m1 (dd2 +
         (1 - p22) (1 - t22))) ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
((1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
(r (1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22)) (1 + α2)) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
((1 + s1) ((-1 + m1) (-dd1 + p12 (1 - t12)) - m1 (-dd2 + p22 (1 - t22)))
    ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22)) (1 + α2)) /
 (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
    s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
    p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
    m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
    m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
    (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
    m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
(((-1 + m1) (-dd1 + (1 - p12) t12) - m1 (-dd2 + (1 - p22) t22))
    ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22)) (1 + α2)) /
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        (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
           s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
           p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
           m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
           m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
           (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
           m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)) -
        (2 ((-1 + m1) (dd1 + p12 t12) - m1 (dd2 + p22 t22))^2 (1 + α2))) /
       (-(-1 + m1) (1 + s1) (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
          s1 (-dd1 + p12 (1 - t12)) - m1 s1 (-dd1 + p12 (1 - t12)) + p12 (1 - t12) +
          p12 t12 - m1 (dd1 + p12 t12) + m1 (dd2 + (1 - p22) (1 - t22)) +
          m1 s1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22)) +
          m1 s1 (-dd2 + p22 (1 - t22)) + m1 (-dd2 + (1 - p22) t22) + m1 (dd2 + p22 t22) +
          (dd1 + p12 t12) α2 - m1 (dd1 + p12 t12) α2 + m1 (-dd2 + (1 - p22) t22) α2 +
          m1 (dd2 + p22 t22) α2 - (-1 + m1) (-dd1 + (1 - p12) t12) (1 + α2)))) /
     (4 (1 + s1 (-m1 (dd1 + (1 - p12) (1 - t12)) - m1 (-dd1 + p12 (1 - t12)) +
           (1 - p12) (1 - t12) + p12 (1 - t12) +
           m1 (dd2 + (1 - p22) (1 - t22)) + m1 (-dd2 + p22 (1 - t22))))^4),
   (p22 (1 + s2 t22) (2 + dd2 α2 + 2 t22 α2 + 2 s2^2 t22^2 (1 + α2) +
        α1 (dd2 (1 + α2) - 2 (-1 + t22) (1 + t22 α2)) +
        s2 (-2 t22^2 (α1 - α2 + α1 α2) + dd2 (α1 + α2 + α1 α2) + 2 t22 (2 + α1 + α2 + α1 α2))) +
     dd2 (2 s2^3 t22^2 (1 + α2) - α1 (1 + t22 α2) +
        s2 (2 + dd2 α2 + 2 t22 α2 + α1 (1 + dd2 - 3 t22 + dd2 α2 - 2 t22^2 α2)) +
        s2^2 (-2 t22^2 (α1 - α2 + α1 α2) + dd2 (α1 + α2 + α1 α2) + t22 (4 + α1 + 2 α2 + α1 α2))) -
     m2^3 (t12 - t22) (-2 (dd1 - dd2) s2 (t12 - t22) (s2 - α1) (s2 + α2 + s2 α2) +
        p12^2 (1 + s2) (1 + s2 t12) (α1 + α2 + α1 α2) +
        p22^2 (1 + s2) (1 + s2 t22) (α1 + α2 + α1 α2) -
        p12 (2 p22 α1 + 2 p22 α2 + 2 p22 α1 α2 - t12 α1 α2 + t22 α1 α2 +
           2 s2^3 t12 (t12 - t22) (1 + α2) + s2 (2 p22 α1 - t12 α1 + p22 t12 α1 + t22
                α1 + p22 t22 α1 + 2 p22 α2 + 2 t12 α2 + p22 t12 α2 - 2 t22 α2 + p22 t22
                α2 + 2 p22 α1 α2 + p22 t12 α1 α2 - 2 t12^2 α1 α2 + p22 t22 α1 α2 + 2
                t12 t22 α1 α2 - dd1 (α1 + α2 + α1 α2) + dd2 (α1 + α2 + α1 α2)) +
           s2^2 (-2 t12^2 (α1 - α2 + α1 α2) - (dd1 - dd2) (α1 + α2 + α1 α2) + t22
                (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2)) + t12
                (2 + (2 + p22 - 2 t22) α2 + (1 + p22 + 2 t22) α1 (1 + α2)))) +
        p22 ((-t12 + t22) α1 α2 + 2 s2^3 (t12 - t22) t22 (1 + α2) + s2 (-dd1 (α1 + α2 +
                α1 α2) + dd2 (α1 + α2 + α1 α2) - (t12 - t22) (α1 - 2 α2 + 2 t22 α1 α2)) +
           s2^2 (-t22 (2 + α1) (1 + α2) + 2 t22^2 (α1 - α2 + α1 α2) - (dd1 - dd2) (α1 +
                α2 + α1 α2) + t12 (-(-1 + 2 t22) α1 (1 + α2) + 2 (1 + α2 + t22 α2))))) +
     m2 (2 dd1 s2 - 2 dd2 s2 + 4 dd2 s2^2 t12 + 4 dd1 s2^2 t22 - 8 dd2 s2^2 t22 +
        4 dd2 s2^3 t12 t22 + 2 dd1 s2^3 t22^2 -
        6 dd2 s2^3 t22^2 - dd1 α1 + dd2 α1 +
        dd1 s2 α1 - dd2 s2 α1 + 2 dd1 dd2 s2 α1 -
        2 dd2^2 s2 α1 + 2 dd1 dd2 s2^2 α1 -
        2 dd2^2 s2^2 α1 - 3 dd2 s2 t12 α1 +
        dd2 s2^2 t12 α1 - 3 dd1 s2 t22 α1 +
        6 dd2 s2 t22 α1 + dd1 s2^2 t22 α1 -
        2 dd2 s2^2 t22 α1 - 4 dd2 s2^2 t12 t22 α1 -
        2 dd1 s2^2 t22^2 α1 + 6 dd2 s2^2 t22^2 α1 +
        2 dd1 dd2 s2 α2 - 2 dd2^2 s2 α2 +
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      2 dd1 dd2 s2^2 α2 - 2 dd2^2 s2^2 α2 +
      2 dd2 s2 t12 α2 + 2 dd2 s2^2 t12 α2 +
      2 dd1 s2 t22 α2 - 4 dd2 s2 t22 α2 +
      2 dd1 s2^2 t22 α2 - 4 dd2 s2^2 t22 α2 +
      4 dd2 s2^2 t12 t22 α2 + 4 dd2 s2^3 t12 t22 α2 +
      2 dd1 s2^2 t22^2 α2 - 6 dd2 s2^2 t22^2 α2 +
      2 dd1 s2^3 t22^2 α2 - 6 dd2 s2^3 t22^2 α2 +
      2 dd1 dd2 s2 α1 α2 - 2 dd2^2 s2 α1 α2 +
      2 dd1 dd2 s2^2 α1 α2 - 2 dd2^2 s2^2 α1 α2 -
      dd2 t12 α1 α2 + dd2 s2^2 t12 α1 α2 -
      dd1 t22 α1 α2 + 2 dd2 t22 α1 α2 +
      dd1 s2^2 t22 α1 α2 - 2 dd2 s2^2 t22 α1 α2 -
      4 dd2 s2 t12 t22 α1 α2 - 4 dd2 s2^2 t12 t22 α1 α2 -
      2 dd1 s2 t22^2 α1 α2 + 6 dd2 s2 t22^2 α1 α2 -
      2 dd1 s2^2 t22^2 α1 α2 + 6 dd2 s2^2 t22^2 α1 α2 -
      p22^2 (1 + s2) (t12 - t22) (1 + s2 t22) (α1 + α2 + α1 α2) +
      p22 (-2 - 2 α1 + dd1 α1 - 2 dd2 α1 - t12 α1 + 3 t22 α1 + dd1 α2 - 2 dd2 α2 +
          2 t12 α2 - 4 t22 α2 + dd1 α1 α2 - 2 dd2 α1 α2 + 2 t12 α1 α2 - 4 t22 α1 α2 -
          3 t12 t22 α1 α2 + 5 t22^2 α1 α2 + 2 s2^3 (2 t12 - 3 t22) t22^2 (1 + α2) -
          s2 (-6 t22^3 α1 α2 + t22^2 (-9 α1 + 10 α2) - (dd1 - 2 dd2)
                (α1 + α2 + α1 α2) + t12 (-4 + (-2 + dd2 - 6 t22) α2 +
                  α1 (-3 + dd2 + 5 t22 - 2 α2 + dd2 α2 + 4 t22^2 α2)) + t22
                (10 + (4 - dd1 + dd2) α2 + α1 (7 + dd2 + 4 α2 + dd2 α2 - dd1 (1 + α2)))) -
          s2^2 (t22 (-6 t22^2 (α1 - α2 + α1 α2) - (dd1 - dd2) (α1 + α2 + α1 α2) +
                  t22 (14 + 5 α1 + 10 α2 + 5 α1 α2)) + t12 (4 t22^2 (α1 - α2 + α1 α2) +
                  dd2 (α1 + α2 + α1 α2) - t22 (8 + 6 α2 + 3 α1 (1 + α2))))) +
      p12 (2 + 2 α1 + dd2 α1 - t12 α1 + p22 t12 α1 - t22 α1 - p22 t22 α1 + dd2 α2 +
          p22 t12 α2 + 2 t22 α2 - p22 t22 α2 + dd2 α1 α2 + p22 t12 α1 α2 + 2 t22 α1 α2 -
          p22 t22 α1 α2 - t12 t22 α1 α2 - t22^2 α1 α2 + 2 s2^3 t12 t22^2 (1 + α2) +
          s2 (dd2 (α1 + α2 + α1 α2) - t22^2 ((-2 + p22) α2 + α1 (1 + p22 + p22 α2)) - t22
                (-4 + (-2 + dd2 + p22) α2 + α1 (-3 + dd2 + p22 - 2 α2 + dd2 α2 + p22 α2)) +
              t12 (2 + 2 dd2 α2 + p22 α2 + 2 t22 α2 + p22 t22 α2 +
                  α1 (1 - 3 t22 - 2 t22^2 α2 + 2 dd2 (1 + α2) + p22 (1 + t22) (1 + α2)))) +
          s2^2 (-t22 (dd2 (α1 + α2 + α1 α2) + t22 (-2 + (-2 + p22) α2 + (-1 + p22)
                      α1 (1 + α2))) + t12 (-2 t22^2 (α1 - α2 + α1 α2) + 2 dd2
                    (α1 + α2 + α1 α2) + t22 (4 + (2 + p22) α2 + (1 + p22) α1 (1 + α2)))))) +
  m2^2 (4 dd1 s2^2 t12 - 4 dd2 s2^2 t12 + 2 dd2 s2^3 t12^2 - 4 dd1 s2^2 t22 +
      4 dd2 s2^2 t22 + 4 dd1 s2^3 t12 t22 -
      8 dd2 s2^3 t12 t22 - 4 dd1 s2^3 t22^2 +
      6 dd2 s2^3 t22^2 + dd1^2 s2 α1 - 2 dd1 dd2 s2 α1 +
      dd2^2 s2 α1 + dd1^2 s2^2 α1 - 2 dd1 dd2 s2^2 α1 +
      dd2^2 s2^2 α1 - 3 dd1 s2 t12 α1 + 3 dd2 s2 t12 α1 +
      dd1 s2^2 t12 α1 - dd2 s2^2 t12 α1 -
      2 dd2 s2^2 t12^2 α1 + 3 dd1 s2 t22 α1 -
      3 dd2 s2 t22 α1 - dd1 s2^2 t22 α1 +
      dd2 s2^2 t22 α1 - 4 dd1 s2^2 t12 t22 α1 +
      8 dd2 s2^2 t12 t22 α1 + 4 dd1 s2^2 t22^2 α1 -
      6 dd2 s2^2 t22^2 α1 + dd1^2 s2 α2 -
      2 dd1 dd2 s2 α2 + dd2^2 s2 α2 + dd1^2 s2^2 α2 -
      2 dd1 dd2 s2^2 α2 + dd2^2 s2^2 α2 + 2 dd1 s2 t12 α2 -
      2 dd2 s2 t12 α2 + 2 dd1 s2^2 t12 α2 -
      2 dd2 s2^2 t12 α2 + 2 dd2 s2^2 t12^2 α2 +
      2 dd2 s2^3 t12^2 α2 - 2 dd1 s2 t22 α2 +
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        2 dd2 s2 t22 α2 - 2 dd1 s2^2 t22 α2 +
        2 dd2 s2^2 t22 α2 + 4 dd1 s2^2 t12 t22 α2 -
        8 dd2 s2^2 t12 t22 α2 + 4 dd1 s2^3 t12 t22 α2 -
        8 dd2 s2^3 t12 t22 α2 - 4 dd1 s2^2 t22^2 α2 +
        6 dd2 s2^2 t22^2 α2 - 4 dd1 s2^3 t22^2 α2 +
        6 dd2 s2^3 t22^2 α2 + dd1^2 s2 α1 α2 -
        2 dd1 dd2 s2 α1 α2 + dd2^2 s2 α1 α2 +
        dd1^2 s2^2 α1 α2 - 2 dd1 dd2 s2^2 α1 α2 +
        dd2^2 s2^2 α1 α2 - dd1 t12 α1 α2 + dd2 t12 α1 α2 +
        dd1 s2^2 t12 α1 α2 - dd2 s2^2 t12 α1 α2 -
        2 dd2 s2 t12^2 α1 α2 - 2 dd2 s2^2 t12^2 α1 α2 +
        dd1 t22 α1 α2 - dd2 t22 α1 α2 -
        dd1 s2^2 t22 α1 α2 + dd2 s2^2 t22 α1 α2 -
        4 dd1 s2 t12 t22 α1 α2 + 8 dd2 s2 t12 t22 α1 α2 -
        4 dd1 s2^2 t12 t22 α1 α2 + 8 dd2 s2^2 t12 t22 α1 α2 +
        4 dd1 s2 t22^2 α1 α2 - 6 dd2 s2 t22^2 α1 α2 +
        4 dd1 s2^2 t22^2 α1 α2 - 6 dd2 s2^2 t22^2 α1 α2 +
        p12^2 (1 + s2) (1 + s2 t12) (t12 - t22) (α1 + α2 + α1 α2) +
        2 p22^2 (1 + s2) (t12 - t22) (1 + s2 t22) (α1 + α2 + α1 α2) +
        p22 (- dd1 α1 + dd2 α1 + t12 α1 - t22 α1 - dd1 α2 + dd2 α2 - 2 t12 α2 +
            2 t22 α2 - dd1 α1 α2 + dd2 α1 α2 - 2 t12 α1 α2 - t12^2 α1 α2 + 2 t22 α1 α2 +
            5 t12 t22 α1 α2 - 4 t22^2 α1 α2 + 2 s2^3 t22 (t12^2 - 4 t12 t22 + 3 t22^2) (1 + α2) -
            s2 (t22^2 (6 α1 - 8 α2) + 6 t22^3 α1 α2 + (dd1 - dd2) (α1 + α2 + α1 α2) +
                t12^2 (α1 - 2 α2 + 2 t22 α1 α2) + t22 (- 4 + (- 2 + dd2) α2 +
                    α1 (- 3 + dd2 - 2 α2 + dd2 α2)) + t12 (4 + (2 + dd1 - 2 dd2 + 10 t22) α2 +
                    α1 (3 - 7 t22 + 2 α2 - 8 t22^2 α2 + dd1 (1 + α2) - 2 dd2 (1 + α2)))) -
            s2^2 (t12^2 ((- 1 + 2 t22) α1 (1 + α2) - 2 (1 + α2 + t22 α2)) + t12
                  (- 8 t22^2 (α1 - α2 + α1 α2) + (dd1 - 2 dd2) (α1 + α2 + α1 α2) +
                    t22 (12 + 5 α1 + 10 α2 + 5 α1 α2)) + t22 (6 t22^2 (α1 - α2 + α1 α2) +
                    dd2 (α1 + α2 + α1 α2) - 2 t22 (5 + 4 α2 + 2 α1 (1 + α2))))) +
        p12 (dd1 α1 - dd2 α1 - t12 α1 - 3 p22 t12 α1 + t22 α1 + 3 p22 t22 α1 +
            dd1 α2 - dd2 α2 + 2 t12 α2 - 3 p22 t12 α2 - 2 t22 α2 + 3 p22 t22 α2 +
            dd1 α1 α2 - dd2 α1 α2 + 2 t12 α1 α2 - 3 p22 t12 α1 α2 - t12^2 α1 α2 -
            2 t22 α1 α2 + 3 p22 t22 α1 α2 - t12 t22 α1 α2 + 2 t22^2 α1 α2 +
            4 s2^3 t12 (t12 - t22) t22 (1 + α2) - s2 (- (dd1 - dd2) (α1 + α2 + α1 α2) - 2
                  t22^2 ((- 2 + p22) α2 + α1 (1 + p22 + p22 α2)) + t12^2 ((- 2 + p22) α2 +
                    α1 (3 + p22 + p22 α2 + 4 t22 α2)) + t22 (4 + (2 + dd1 - 2 dd2 - 3 p22) α2 +
                    α1 (3 - 3 p22 + 2 α2 - 3 p22 α2 + dd1 (1 + α2) - 2 dd2 (1 + α2))) +
                t12 (- 4 + (- 2 - 2 dd1 + 3 dd2 + 3 p22 - 2 t22 + p22 t22) α2 +
                    α1 (- 3 + 3 p22 - t22 + p22 t22 - 2 α2 + 3 p22 α2 + p22 t22 α2 -
                        4 t22^2 α2 - 2 dd1 (1 + α2) + 3 dd2 (1 + α2)))) -
            s2^2 (t12^2 (- 4 + (- 2 + p22 - 4 t22) α2 + (- 1 + p22 + 4 t22) α1 (1 + α2)) + t22
                  ((dd1 - 2 dd2) (α1 + α2 + α1 α2) - 2 t22 (- 2 + (- 2 + p22) α2 + (- 1 + p22) α1
                        (1 + α2))) + t12 (- 2 dd1 (α1 + α2 + α1 α2) + 3 dd2 (α1 + α2 + α1 α2) +
                    t22 ((- 2 + p22 + 4 t22) α2 + (- 1 + p22 - 4 t22) α1 (1 + α2)))))))) /
    (2 (1 + m2 s2 (t12 - t22) + s2 t22) (1 + s2 t22 + m2 (t12 - t22) (s2 - α1) +
        α1 - t22 α1)
      (1 + t22 α2 + s2 t22 (1 + α2) + m2 (t12 - t22) (s2 + α2 + s2 α2))),
  ((1 + s2) (m2 (t12 - t22) + t22)
      (2 + α1 + p22 α1 -
        t22 α1 - p22 t22 α1 +
        p22 α2 + 2 t22 α2 -
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        p22 t22 α2 +
        p22 α1 α2 + t22 α1 α2 -
        p22 t22 α1 α2 -
        t22^2 α1 α2 +
        2 s2^2 t22^2 (1 + α2) +
        s2 (dd2 (α1 + α2 + α1 α2) + t22 (4 + (2 - dd2 + p22) α2 - (-1 + dd2 - p22) α1 (1 + α2)) -
            t22^2 ((-2 + p22) α2 + (1 + p22) α1 (1 + α2))) +
        m2^2 (t12 - t22) (p22 α1 + p22 α2 + p22 α1 α2 - t12 α1 α2 + t22 α1 α2 +
            2 s2^2 (t12 - t22) (1 + α2) - p12 (α1 + α2 + α1 α2) +
            s2 (-t12 α1 - p12 t12 α1 + t22 α1 + p22 t22 α1 + 2 t12 α2 - p12 t12 α2 -
                2 t22 α2 + p22 t22 α2 - t12 α1 α2 - p12 t12 α1 α2 + t22 α1 α2 +
                p22 t22 α1 α2 - dd1 (α1 + α2 + α1 α2) + dd2 (α1 + α2 + α1 α2))) +
        m2 (p12 α1 - p22 α1 - t12 α1 - p22 t12 α1 + t22 α1 - p12 t22 α1 + 2 p22 t22 α1 +
            p12 α2 - p22 α2 + 2 t12 α2 - p22 t12 α2 - 2 t22 α2 -
            p12 t22 α2 + 2 p22 t22 α2 + p12 α1 α2 - p22 α1 α2 + t12 α1 α2 -
            p22 t12 α1 α2 - t22 α1 α2 - p12 t22 α1 α2 + 2 p22 t22 α1 α2 -
            2 t12 t22 α1 α2 + 2 t22^2 α1 α2 + 4 s2^2 (t12 - t22) t22 (1 + α2) - s2
              (-(dd1 - dd2) (α1 + α2 + α1 α2) - 2 t22^2 ((-2 + p22) α2 + (1 + p22) α1 (1 + α2)) +
                t22 (4 + (2 + dd1 - 2 dd2 + p22) α2 + (1 + dd1 - 2 dd2 + p22) α1 (1 + α2)) +
                t12 (-4 + (-2 + dd2 + p12 (-1 + t22) - 4 t22 + p22 t22) α2 +
                    (-1 + dd2 + p12 (-1 + t22) + 2 t22 + p22 t22) α1 (1 + α2))))))) /
    (2 (1 + m2 s2 (t12 - t22) + s2 t22) (1 + s2 t22 + m2 (t12 - t22) (s2 - α1) +
        α1 - t22 α1)
      (1 + t22 α2 + s2 t22 (1 + α2) + m2 (t12 - t22) (s2 + α2 + s2 α2))),
  -(((1 + s2) (1 + m2 s2 ((1 - p12) t12 + p12 t12 - (1 - p22) t22 - p22 t22) +
          s2 ((1 - p22) t22 + p22 t22))^2
        ((r (1 + s2) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
              (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
            (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
              s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22))
                (1 + α1) + m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 +
                  s2 (-dd1 + (1 - p12) t12) + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) -
                  p22 (1 - t22) - (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2
                    (dd2 + p22 t22) + (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22))
                    α1 - (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
          ((dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) + (1 - p22)
                (1 - t22)) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) +
                p22 (1 - t22)) (1 + α1)) / (-dd2 + p22 (1 - t22) + (1 - p22) t22 +
              p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
              (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
              m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
                  s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 -
                  p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
                  (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
                  (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) -
          ((-1 + r) (1 + s2) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) +
                p22 (1 - t22)) (-dd2 + (1 - p22) t22 +
                m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22)) (1 + α1)) /
            (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
              s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22))
```

```
(1 + α1) + m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 +
      s2 (-dd1 + (1 - p12) t12) + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) -
      p22 (1 - t22) - (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2
       (dd2 + p22 t22) + (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22))
       α1 - (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
 ((dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) + (1 - p22)
       (1 - t22)) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) +
      p22 (1 - t22))) / ((1 - p22) (1 - t22) + p22 (1 - t22) +
    (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
    (-dd2 + (1 - p22) t22) α2 + s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 +
    s2 (dd2 + p22 t22) α2 + m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) +
      p12 t12 + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) -
      (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
      (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
      s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
      s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 (2 (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))^2) /
  ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
      (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
      s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
      s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
      s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
      s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 (r (1 + s2) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
  ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
      (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
      s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
      s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
      s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
      s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 ((1 + s2) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) +
      p22 (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
  ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
      (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
      s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
      s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
      s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
      s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) -
 ((-1 + r) (1 + s2) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) +
      p22 (1 - t22)) (-dd2 + (1 - p22) t22 +
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        m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22)) (1 + α2)) /
   ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
     s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
     s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
     m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
         (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
         s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
         s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
         s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
         s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
   ((1 + s2) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
      (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22)) (1 + α2)) /
   ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
     s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
     s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
     m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
         (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
         s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
         s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
         s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
         s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))))
 (((dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
        (1 - p22) (1 - t22)) (-dd2 + (1 - p22) t22 +
       m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))) / (-dd2 + p22 (1 - t22) +
     (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
     (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
     m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
         s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 -
         p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
         (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
   (2 (1 + s2) (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 -
           (1 - p22) t22))^2) / (-dd2 + p22 (1 - t22) + (1 - p22) t22 +
     p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
     (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
     m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
         s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 -
         p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
         (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
   (r (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
        (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
    (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
     s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22))
      (1 + α1) + m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 +
       s2 (-dd1 + (1 - p12) t12) + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) -
       p22 (1 - t22) - (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2
        (dd2 + p22 t22) + (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22))
        α1 - (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
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((1 + s2) (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) / (-dd2 + p22 (1 - t22) +
    (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
    (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
   m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
       s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 -
       p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
       (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
       (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
((dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22)) (-dd2 + (1 - p22) t22 +
     m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22)) (1 + α1)) /
 (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
   s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22))
    (1 + α1) + m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 +
       s2 (-dd1 + (1 - p12) t12) + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) -
       p22 (1 - t22) - (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2
        (dd2 + p22 t22) + (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22))
        α1 - (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) -
((-1 + r) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
    (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (1 + α1)) / (-dd2 + p22 (1 - t22) + (1 - p22) t22 +
   p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
   (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
   m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
       s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 -
       p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
       (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
       (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
(r (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
 ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
   s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
   s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
   m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
       (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
       s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
       s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
       s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
       s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) -
((-1 + r) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
    (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (1 + α2)) / ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 +
   p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
   (-dd2 + (1 - p22) t22) α2 + s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 +
   s2 (dd2 + p22 t22) α2 + m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) +
       p12 t12 + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) -
       (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
       (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
       s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
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                s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
          ((1 + s2) (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
            (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22)) (1 + α2)) /
          ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
            s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
            s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
            m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2 (dd1 + p12 t12) -
                (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22) t22 - p22 t22 -
                s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) + (dd1 + p12 t12) α2 +
                s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22) α2 -
                s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 -
                s2 (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2)))))) /
     (4 (1 + s2 ((1 - p22) t22 + p22 t22 + m2 ((1 - p12) t12 + p12 t12 -
                (1 - p22) t22 - p22 t22)))^4) +
   ((1 + s2) (1 + m2 s2 ((1 - p12) t12 + p12 t12 - (1 - p22) t22 - p22 t22) +
       s2 ((1 - p22) t22 + p22 t22))^2
     (((1 + s2) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
           (1 - p22) (1 - t22))
         (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))) /
       (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
         s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 +
         (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
         m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
             s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
              t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
             (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
             (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) -
       ((-1 + r) (1 + s2) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
           (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
       (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
         s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 +
         (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
         m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
             s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
              t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
             (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
             (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
       (2 (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
           (1 - p22) (1 - t22))^2 (1 + α1)) /
       (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
         s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 +
         (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
         m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
             s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
              t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
             (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
             (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
       ((dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
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      (1 - p22) (1 - t22))
    (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
    (1 + α1)) / (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
    (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
    m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
        s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
         t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
        (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
        (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
((1 + s2) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22)) (-dd2 + (1 - p22) t22 +
     m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22)) (1 + α1)) /
 (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
    s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 +
    (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
    m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
        s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
         t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
        (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
        (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
(r (1 + s2) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
    (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (1 + α1)) / (-dd2 + p22 (1 - t22) + (1 - p22) t22 +
    p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
    (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
    m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
        s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
         t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
        (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
        (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
((dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22))
    (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))) /
 ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
         (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
         t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
        (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
         α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
         (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) -
((-1 + r) (1 + s2) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
 ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
         (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
```

```
t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
        (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
        α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
        (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
  (r (1 + s2) (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
    (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (1 + α2)) / ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
        (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
        t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
        (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
        α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
        (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))))
 (-(((-1 + r) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
        (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
    (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
      s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22))
      (1 + α1) + m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 +
        s2 (-dd1 + (1 - p12) t12) + s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) -
        p22 (1 - t22) - (1 - p22) t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2
        (dd2 + p22 t22) + (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22))
        α1 - (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1)))) +
  ((1 + s2) (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
   (-dd2 + p22 (1 - t22) + (1 - p22) t22 + p22 t22 + s2 (-dd2 + (1 - p22) t22) +
    s2 (dd2 + p22 t22) + (-dd2 + p22 (1 - t22)) α1 +
    (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
    m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
      s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
      t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
      (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
      (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) +
  (r (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22)) + p22 (1 - t22))
    (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
    (1 + α1)) / (-dd2 + p22 (1 - t22) + (1 - p22) t22 +
    p22 t22 + s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) +
    (-dd2 + p22 (1 - t22)) α1 + (dd2 + (1 - p22) (1 - t22)) (1 + α1) +
    m2 (-dd1 + p12 (1 - t12) + (1 - p12) t12 + p12 t12 + s2 (-dd1 + (1 - p12) t12) +
      s2 (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
      t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
      (-dd1 + p12 (1 - t12)) α1 - (dd2 + (1 - p22) (1 - t22)) α1 -
      (-dd2 + p22 (1 - t22)) α1 + (dd1 + (1 - p12) (1 - t12)) (1 + α1))) -
  ((-1 + r) (dd2 + m2 (dd1 - dd2 + (1 - p12) (1 - t12) - (1 - p22) (1 - t22)) +
      (1 - p22) (1 - t22)) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
   ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
```

```
          (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
          t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
          α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
          (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 ((-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22))) + p22 (1 - t22))
     (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))) /
  ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
          (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
          t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
          α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
          (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 (r (-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22))) + p22 (1 - t22))
     (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
     (1 + α2)) / ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
          (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
          t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
          α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
          (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 ((-dd2 + m2 (-dd1 + dd2 + p12 (1 - t12) - p22 (1 - t22))) + p22 (1 - t22))
     (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22)) (1 + α2)) /
  ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
          (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
          t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
          α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
          (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 ((1 + s2) (-dd2 + (1 - p22) t22 + m2 (-dd1 + dd2 + (1 - p12) t12 - (1 - p22) t22))
     (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22)) (1 + α2)) /
  ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
    s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
    s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
    m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
          (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
          t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
         (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
          α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
          (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))) +
 (2 (1 + s2) (dd2 + p22 t22 + m2 (dd1 - dd2 + p12 t12 - p22 t22))^2 (1 + α2)) /
```

```
        ((1 - p22) (1 - t22) + p22 (1 - t22) + (1 - p22) t22 + p22 t22 +
          s2 (-dd2 + (1 - p22) t22) + s2 (dd2 + p22 t22) + (-dd2 + (1 - p22) t22) α2 +
          s2 (-dd2 + (1 - p22) t22) α2 + (dd2 + p22 t22) α2 + s2 (dd2 + p22 t22) α2 +
          m2 (dd1 + (1 - p12) (1 - t12) + p12 (1 - t12) + p12 t12 + s2
                (dd1 + p12 t12) - (1 - p22) (1 - t22) - p22 (1 - t22) - (1 - p22)
                t22 - p22 t22 - s2 (-dd2 + (1 - p22) t22) - s2 (dd2 + p22 t22) +
               (dd1 + p12 t12) α2 + s2 (dd1 + p12 t12) α2 - (-dd2 + (1 - p22) t22)
                α2 - s2 (-dd2 + (1 - p22) t22) α2 - (dd2 + p22 t22) α2 - s2
                (dd2 + p22 t22) α2 + (1 + s2) (-dd1 + (1 - p12) t12) (1 + α2))))) /
    (4 (1 + s2 ((1 - p22) t22 + p22 t22 + m2 ((1 - p12) t12 + p12 t12 -
                (1 - p22) t22 - p22 t22)))^4)};
```

```
itgen[.1, .2, 3, 3.5, .1, .1, .5][{.8, .5, .05, .6, .9, .04}]
```

```
{0.790576, 0.602846, 0.0474188, 0.621487, 0.865297, 0.0358771}
```

1.2.3 Computing the equilibrium from itgen using FindRoot

```
Fritgen[s1_, s2_, α1_, α2_, m1_, m2_, r_][
  {fp12_, ft12_, fdd1_, fp22_, ft22_, fdd2_}] :=
 FindRoot[itgen[s1, s2, α1, α2, m1, m2, r][{p12, t12, dd1, p22, t22, dd2}] ==
   {p12, t12, dd1, p22, t22, dd2},
  {{p12, fp12}, {t12, ft12}, {dd1, fdd1}, {p22, fp22}, {t22, ft22}, {dd2, fdd2}}]
```

```
testdif[n_][x_, y_] := If[Max[Abs[x - y]] < 10.^(-n), False, True]
```

Because there exists a curve of equilibria, the above procedure is of limited use and sometimes has convergence problems

2. Analysis

Under the assumption of symmetry of the two populations, this models was analyzed in Servedio and Bürger (2014)

2.1 Equilibria with fixation of one allele

It is easy to show that if dd1 = dd2=0 and p12 = p22, then only t12 = t22 = 0 and t12 = t22 = 1 are possible equilibria

2.1.1 Determination of equilibria

Assume t12 = t22 = 0 (loss of trait allele T2)

```
Simplify[itgen[s1, s2, α1, α2, m1, m2, r][{p12, 0, 0, p22, 0, 0}] - {p12, 0, 0, p22, 0, 0}]
```

```
{m1 (-p12 + p22), 0, 0, m2 (p12 - p22), 0, 0}
```

If m1 > 0 or m2 > 0, then all equilibria on this edge satisfy p12 = p22 (and dd1=dd2=0).

Assume t12 = t22 = 1

```
Simplify[itgen[s1, s2, α1, α2, m1, m2, r][{p12, 1, 0, p22, 1, 0}] - {p12, 1, 0, p22, 1, 0}]
```

$$\{m1\,(-p12+p22),\ 0,\ 0,\ m2\,(p12-p22),\ 0,\ 0\}$$

If m1 > 0 or m2 > 0, then all equilibria on this edge satisfy p12 = p22 (and dd1=dd2=0).

Assume p12 = p22 = 0

```
Factor[itgen[s1, s2, α1, α2, m1, m2, r][{0, t12, 0, 0, t22, 0}] - {0, t12, 0, 0, t22, 0}]
```

$$\{0,\ -\big(\big(2\,m1\,t12+2\,s1\,t12+2\,m1\,s1\,t12+2\,s1^2\,t12-2\,s1\,t12^2+2\,m1^2\,s1\,t12^2-4\,s1^2\,t12^2+4\,m1\,s1^2\,t12^2+2\,s1^2\,t12^3-4\,m1\,s1^2\,t12^3+2\,m1^2\,s1^2\,t12^3-2\,m1\,t22-2\,m1\,s1\,t22-4\,m1^2\,s1\,t12\,t22-4\,m1\,s1^2\,t12\,t22+4\,m1\,s1^2\,t12^2\,t22-4\,m1^2\,s1^2\,t12^2\,t22+2\,m1^2\,s1\,t22^2+2\,m1^2\,s1^2\,t12\,t22^2+t12\,\alpha1+m1\,t12\,\alpha1+3\,s1\,t12\,\alpha1+m1\,s1\,t12\,\alpha1+2\,s1^2\,t12\,\alpha1-t12^2\,\alpha1+m1^2\,t12^2\,\alpha1-5\,s1\,t12^2\,\alpha1+4\,m1\,s1\,t12^2\,\alpha1+m1^2\,s1\,t12^2\,\alpha1-4\,s1^2\,t12^2\,\alpha1+4\,m1\,s1^2\,t12^2\,\alpha1+2\,s1\,t12^3\,\alpha1-4\,m1\,s1\,t12^3\,\alpha1+2\,m1^2\,s1\,t12^3\,\alpha1+2\,s1^2\,t12^3\,\alpha1-4\,m1\,s1^2\,t12^3\,\alpha1+2\,m1^2\,s1^2\,t12^3\,\alpha1-m1\,t22\,\alpha1-m1\,s1\,t22\,\alpha1-2\,m1^2\,t12\,t22\,\alpha1-4\,m1\,s1\,t12\,t22\,\alpha1-2\,m1^2\,s1\,t12\,t22\,\alpha1-4\,m1\,s1^2\,t12\,t22\,\alpha1+4\,m1\,s1\,t12^2\,t22\,\alpha1-4\,m1^2\,s1\,t12^2\,t22\,\alpha1+4\,m1\,s1^2\,t12^2\,t22\,\alpha1-4\,m1^2\,s1^2\,t12^2\,t22\,\alpha1+m1^2\,t22^2\,\alpha1+m1^2\,s1\,t22^2\,\alpha1+2\,m1^2\,s1\,t12\,t22^2\,\alpha1+2\,m1^2\,s1^2\,t12\,t22^2\,\alpha1\big)\ /\ \big(2\,\big(1+s1-s1\,t12+m1\,s1\,t12-m1\,s1\,t22\big)\,\big(1+s1-s1\,t12+m1\,s1\,t12-m1\,s1\,t22+\alpha1+s1\,\alpha1-t12\,\alpha1+m1\,t12\,\alpha1-s1\,t12\,\alpha1+m1\,s1\,t12\,\alpha1-m1\,t22\,\alpha1-m1\,s1\,t22\,\alpha1\big)\big)\big),$$
$$0,\ 0,\ -\big(\big(-2\,m2\,t12-2\,m2\,s2\,t12-2\,m2^2\,s2\,t12^2-2\,m2^2\,s2^2\,t12^2+2\,m2\,t22-2\,s2\,t22+2\,m2\,s2\,t22+4\,m2^2\,s2\,t12\,t22-4\,m2\,s2^2\,t12\,t22+4\,m2^2\,s2^2\,t12\,t22+2\,m2^2\,s2^2\,t12^2\,t22+2\,s2\,t22^2-2\,m2^2\,s2\,t22^2-2\,s2^2\,t22^2+4\,m2\,s2^2\,t22^2-2\,m2^2\,s2^2\,t22^2+4\,m2\,s2^2\,t12\,t22^2-4\,m2^2\,s2^2\,t12\,t22^2+2\,s2^2\,t22^3-4\,m2\,s2^2\,t22^3+2\,m2^2\,s2^2\,t22^3-m2\,t12\,\alpha1-m2\,s2\,t12\,\alpha1+m2^2\,t12^2\,\alpha1+m2^2\,s2\,t12^2\,\alpha1+t22\,\alpha1+m2\,t22\,\alpha1-s2\,t22\,\alpha1+m2\,s2\,t22\,\alpha1-2\,m2^2\,t12\,t22\,\alpha1+4\,m2\,s2\,t12\,t22\,\alpha1-2\,m2^2\,s2\,t12\,t22\,\alpha1-2\,m2^2\,s2\,t12^2\,t22\,\alpha1-t22^2\,\alpha1+m2^2\,t22^2\,\alpha1+3\,s2\,t22^2\,\alpha1-4\,m2\,s2\,t22^2\,\alpha1+m2^2\,s2\,t22^2\,\alpha1-4\,m2\,s2\,t12\,t22^2\,\alpha1+4\,m2^2\,s2\,t12\,t22^2\,\alpha1-2\,s2\,t22^3\,\alpha1+4\,m2\,s2\,t22^3\,\alpha1-2\,m2^2\,s2\,t22^3\,\alpha1\big)\ /\ \big(2\,\big(-1-m2\,s2\,t12-s2\,t22+m2\,s2\,t22\big)\,\big(-1-m2\,s2\,t12-s2\,t22+m2\,s2\,t22-\alpha1+m2\,t12\,\alpha1+t22\,\alpha1-m2\,t22\,\alpha1\big)\big)\big),\ 0\}$$

```
Simplify[Numerator[Factor[
    itgen[s1, s2, α1, α2, m1, m2, r][{0, t12, 0, 0, t22, 0}] - {0, t12, 0, 0, t22, 0}]]]
```

$$\{0,\ -m1^2\,(t12-t22)^2\,(\alpha1+2\,s1^2\,t12\,(1+\alpha1)+s1\,(2+\alpha1+2\,t12\,\alpha1))-(-1+t12)\,t12\,(-\alpha1+2\,s1^2\,(-1+t12)\,(1+\alpha1)+s1\,(-2+(-3+2\,t12)\,\alpha1))+m1\,(t12-t22)\,(-2-\alpha1+4\,s1^2\,(-1+t12)\,t12\,(1+\alpha1)+s1\,(-2+(-1-4\,t12+4\,t12^2)\,\alpha1)),$$
$$0,\ 0,\ -m2\,(t12-t22)\,\big(-2+4\,s2^2\,(-1+t22)\,t22-\alpha1-s2\,\big(2+(1-2\,t22)^2\,\alpha1\big)\big)-m2^2\,(t12-t22)^2\,(2\,s2^2\,(-1+t22)+\alpha1+s2\,(-2+\alpha1-2\,t22\,\alpha1))-(-1+t22)\,t22\,(2\,s2^2\,t22-\alpha1+s2\,(2+\alpha1-2\,t22\,\alpha1)),\ 0\}$$

This is independent of $\alpha 2$ (and of r, of course)!

The above may or may not have solutions. If $\alpha 1$ is small, then it usually has (close to migration-selection balance). For large $\alpha 1$, there is typically no solution.

```
NSolve[{-m1^2 (t12 - t22)^2 (α1 + 2 s1^2 t12 (1 + α1) + s1 (2 + α1 + 2 t12 α1)) -
     (-1 + t12) t12 (-α1 + 2 s1^2 (-1 + t12) (1 + α1) + s1 (-2 + (-3 + 2 t12) α1)) +
     m1 (t12 - t22) (-2 - α1 + 4 s1^2 (-1 + t12) t12 (1 + α1) + s1 (-2 + (-1 - 4 t12 + 4 t12^2) α1)),
    -m2 (t12 - t22) (-2 + 4 s2^2 (-1 + t22) t22 - α1 - s2 (2 + (1 - 2 t22)^2 α1)) -
     m2^2 (t12 - t22)^2 (2 s2^2 (-1 + t22) + α1 + s2 (-2 + α1 - 2 t22 α1)) -
     (-1 + t22) t22 (2 s2^2 t22 - α1 + s2 (2 + α1 - 2 t22 α1))} /.
  {s1 → 0.2, s2 → 0.2, m1 → 0.01, m2 → 0.01, α1 → 0.4}, {t12, t22}]
```

```
{{t12 → 4.29625, t22 → -0.67648}, {t12 → 4.27168 + 0.0098123 i, t22 → 1.30515 - 0.790794 i},
 {t12 → 4.27168 - 0.0098123 i, t22 → 1.30515 + 0.790794 i},
 {t12 → 1.04237, t22 → -0.293469}, {t12 → 0.0413652, t22 → 1.6079}, {t12 → 1., t22 → 1.},
 {t12 → 0.0101669, t22 → 0.411472}, {t12 → 0.989981, t22 → 1.29379}, {t12 → 0., t22 → 0.}}
```

```
NSolve[{-m1^2 (t12 - t22)^2 (α1 + 2 s1^2 t12 (1 + α1) + s1 (2 + α1 + 2 t12 α1)) -
     (-1 + t12) t12 (-α1 + 2 s1^2 (-1 + t12) (1 + α1) + s1 (-2 + (-3 + 2 t12) α1)) +
     m1 (t12 - t22) (-2 - α1 + 4 s1^2 (-1 + t12) t12 (1 + α1) + s1 (-2 + (-1 - 4 t12 + 4 t12^2) α1)),
    -m2 (t12 - t22) (-2 + 4 s2^2 (-1 + t22) t22 - α1 - s2 (2 + (1 - 2 t22)^2 α1)) -
     m2^2 (t12 - t22)^2 (2 s2^2 (-1 + t22) + α1 + s2 (-2 + α1 - 2 t22 α1)) -
     (-1 + t22) t22 (2 s2^2 t22 - α1 + s2 (2 + α1 - 2 t22 α1))} /.
  {s1 → 0.2, s2 → 0.2, m1 → 0.01, m2 → 0.01, α1 → 0.5}, {t12, t22}]
```

```
{{t12 → 4.18932, t22 → -0.872673},
 {t12 → 4.16661 + 0.00833685 i, t22 → 0.894128 - 0.648105 i},
 {t12 → 4.16661 - 0.00833685 i, t22 → 0.894128 + 0.648105 i},
 {t12 → 1.04553, t22 → -0.523105}, {t12 → 0.0277905, t22 → 1.20646},
 {t12 → 1.01521, t22 → 0.502458}, {t12 → 1., t22 → 1.},
 {t12 → -0.00456512, t22 → -0.206632}, {t12 → 0., t22 → 0.}}
```

```
NSolve[{-m1^2 (t12 - t22)^2 (α1 + 2 s1^2 t12 (1 + α1) + s1 (2 + α1 + 2 t12 α1)) -
     (-1 + t12) t12 (-α1 + 2 s1^2 (-1 + t12) (1 + α1) + s1 (-2 + (-3 + 2 t12) α1)) +
     m1 (t12 - t22) (-2 - α1 + 4 s1^2 (-1 + t12) t12 (1 + α1) + s1 (-2 + (-1 - 4 t12 + 4 t12^2) α1)),
    -m2 (t12 - t22) (-2 + 4 s2^2 (-1 + t22) t22 - α1 - s2 (2 + (1 - 2 t22)^2 α1)) -
     m2^2 (t12 - t22)^2 (2 s2^2 (-1 + t22) + α1 + s2 (-2 + α1 - 2 t22 α1)) -
     (-1 + t22) t22 (2 s2^2 t22 - α1 + s2 (2 + α1 - 2 t22 α1))} /.
  {s1 → 0.2, s2 → 0.2, m1 → 0.01, m2 → 0.01, α1 → 100}, {t12, t22}]
```

```
NSolve[{-m1^2 (t12 - t22)^2 (α1 + 2 s1^2 t12 (1 + α1) + s1 (2 + α1 + 2 t12 α1)) -
     (-1 + t12) t12 (-α1 + 2 s1^2 (-1 + t12) (1 + α1) + s1 (-2 + (-3 + 2 t12) α1)) +
     m1 (t12 - t22) (-2 - α1 + 4 s1^2 (-1 + t12) t12 (1 + α1) + s1 (-2 + (-1 - 4 t12 + 4 t12^2) α1)),
    -m2 (t12 - t22) (-2 + 4 s2^2 (-1 + t22) t22 - α1 - s2 (2 + (1 - 2 t22)^2 α1)) -
     m2^2 (t12 - t22)^2 (2 s2^2 (-1 + t22) + α1 + s2 (-2 + α1 - 2 t22 α1)) -
     (-1 + t22) t22 (2 s2^2 t22 - α1 + s2 (2 + α1 - 2 t22 α1))} /.
  {s1 → 0.2, s2 → 0.2, m1 → 0.01, m2 → 0.01, α1 → 10000}, {t12, t22}]
```

```
{{t12 → 3.59725, t22 → -2.13778}, {t12 → 3.55978, t22 → 0.0504928},
 {t12 → 3.54386, t22 → 0.974298}, {t12 → 1.03107, t22 → -2.07591},
 {t12 → -0.0144, t22 → -2.05009}, {t12 → 1.00995, t22 → 0.0147497},
 {t12 → 0.00719977, t22 → 1.01003}, {t12 → 1., t22 → 1.}, {t12 → 0., t22 → 0.}}
```

Assume p12 = p22 = 1

Analogous to the case p12 = p22 = 0

2.1.2 Jacobians and stability of equilibria, where one trait allele is fixed

Jacobian at edge t12 = t22 = 0

```
Jacobdt20 =
  Simplify[D[itgen[s1, s2, α1, α2, m1, m2, r][{p12, t12, dd1, p22, t22, dd2}], {{p12,
       t12, dd1, p22, t22, dd2}}] /. {t12 → 0, dd1 → 0, t22 → 0, dd2 → 0, p12 → p22}];
```

An equilibrium is stable if the modulus of eigenvalues of its Jacobian is < 1

```
TableForm[Jacobdt20]
```

$$\begin{array}{lll}
1-m1 & 0 & \frac{(-1+m1)\,(2\,s1\,(1+\alpha1)-p22\,\alpha2-\alpha1\,(-1+p22+p22\,\alpha2))}{2\,(1+s1)\,(1+\alpha1)} \\
0 & -\frac{(-1+m1)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))}{2\,(1+s1)\,(1+\alpha1)} & 0 \\
0 & \frac{(-1+m1)\,(-1+p22)\,p22\,r\,(\alpha1+\alpha2+\alpha1\,\alpha2)}{2\,(1+s1)\,(1+\alpha1)} & \frac{(-1+m1)\,(-1+r)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))}{2\,(1+s1)\,(1+\alpha1)} \\
m2 & 0 & \frac{m2\,(p22\,\alpha2+\alpha1\,(-1+p22+p22\,\alpha2)+s2\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2)))}{2\,(1+\alpha1)} \\
0 & \frac{m2\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))}{2\,(1+\alpha1)} & 0 \\
0 & -\frac{m2\,(-1+p22)\,p22\,r\,(1+s2)\,(\alpha1+\alpha2+\alpha1\,\alpha2)}{2\,(1+\alpha1)} & -\frac{m2\,(-1+r)\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))}{2\,(1+\alpha1)}
\end{array}$$

Characteristic polynomial at this equilibrium:

```
cpolgenT20 = Simplify[Det[Jacobdt20 - x IdentityMatrix[6]]]
```

$$\begin{aligned}
&\frac{1}{16\,(1+s1)^2\,(1+\alpha1)^4}\\
&(-1+x)\,(-1+m1+m2+x)\,\Big(-m1\,m2\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))^2+(-2-\alpha1-\\
&\quad p22\,\alpha1+2\,(1+s1)\,x\,(1+\alpha1)-p22\,\alpha2-p22\,\alpha1\,\alpha2+m1\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2)))\\
&\quad(-2+2\,x-\alpha1-p22\,\alpha1+2\,x\,\alpha1-p22\,\alpha2-p22\,\alpha1\,\alpha2-s2\,(2+\alpha1+p22\,\alpha1+p22\,\alpha2+p22\,\alpha1\,\alpha2)+\\
&\quad m2\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2)))\Big)\\
&\Big(-m1\,m2\,(-1+r)^2\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))^2+\\
&\quad(2-2\,x-2\,s1\,x+\alpha1+p22\,\alpha1-2\,x\,\alpha1-2\,s1\,x\,\alpha1+p22\,\alpha2+p22\,\alpha1\,\alpha2-\\
&\quad r\,(2+\alpha1+p22\,\alpha1+p22\,\alpha2+p22\,\alpha1\,\alpha2)+m1\,(-1+r)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2)))\\
&\quad(2+2\,s2-2\,x+\alpha1+p22\,\alpha1+s2\,\alpha1+p22\,s2\,\alpha1-2\,x\,\alpha1+p22\,\alpha2+p22\,s2\,\alpha2+\\
&\quad p22\,\alpha1\,\alpha2+p22\,s2\,\alpha1\,\alpha2+m2\,(-1+r)\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2))-\\
&\quad r\,(1+s2)\,(2+p22\,\alpha2+\alpha1\,(1+p22+p22\,\alpha2)))\Big)
\end{aligned}$$

There are four factors; x = 1 and x = 1 - m1 - m2 are always eigenvalues.
In addition, there is one factor without r, and one factor with r. Both factors are quadratic in x and given below:

```
cpolgenT20noR[x_, p22_] :=
  (-m1 m2 (1 + s2) (2 + p22 α2 + α1 (1 + p22 + p22 α2))^2 + (-2 - α1 - p22 α1 +
       2 (1 + s1) x (1 + α1) - p22 α2 - p22 α1 α2 + m1 (2 + p22 α2 + α1 (1 + p22 + p22 α2))) (-2 +
       2 x - α1 - p22 α1 + 2 x α1 - p22 α2 - p22 α1 α2 - s2 (2 + α1 + p22 α1 + p22 α2 + p22 α1 α2) +
       m2 (1 + s2) (2 + p22 α2 + α1 (1 + p22 + p22 α2))))
```

```
Simplify[Series[cpolgenT20noR[x, p22], {x, 0, 3}]]
```

$$-\left(-1+m1+m2\right)\left(1+s2\right)\left(2+p22\,\alpha2+\alpha1\left(1+p22+p22\,\alpha2\right)\right)^2+2\left(-2+m1-s1-s2-s1\,s2+m2\left(1+s1\right)\left(1+s2\right)\right)\left(1+\alpha1\right)\left(2+p22\,\alpha2+\alpha1\left(1+p22+p22\,\alpha2\right)\right)x+4\left(1+s1\right)\left(1+\alpha1\right)^2x^2+O[x]^4$$

```
cpolgenT20R[x_, p22_] := (-m1 m2 (-1 + r)^2 (1 + s2) (2 + p22 α2 + α1 (1 + p22 + p22 α2))^2 +
    (2 - 2 x - 2 s1 x + α1 + p22 α1 - 2 x α1 - 2 s1 x α1 + p22 α2 + p22 α1 α2 -
       r (2 + α1 + p22 α1 + p22 α2 + p22 α1 α2) + m1 (-1 + r) (2 + p22 α2 + α1 (1 + p22 + p22 α2)))
     (2 + 2 s2 - 2 x + α1 + p22 α1 + s2 α1 + p22 s2 α1 - 2 x α1 + p22 α2 + p22 s2 α2 +
       p22 α1 α2 + p22 s2 α1 α2 + m2 (-1 + r) (1 + s2) (2 + p22 α2 + α1 (1 + p22 + p22 α2)) -
       r (1 + s2) (2 + p22 α2 + α1 (1 + p22 + p22 α2))))
```

```
Simplify[Series[cpolgenT20R[x, p22], {x, 0, 3}]]
```

$$-\left(-1+m1+m2\right)\left(-1+r\right)^2\left(1+s2\right)\left(2+p22\,\alpha2+\alpha1\left(1+p22+p22\,\alpha2\right)\right)^2-2\left(\left(-1+r\right)\left(-2+m1-s1-s2-s1\,s2+m2\left(1+s1\right)\left(1+s2\right)\right)\left(1+\alpha1\right)\left(2+p22\,\alpha2+\alpha1\left(1+p22+p22\,\alpha2\right)\right)\right)x+4\left(1+s1\right)\left(1+\alpha1\right)^2x^2+O[x]^4$$

```
Simplify[cpolgenT20R[x (1 - r), p22] - (1 - r)^2 cpolgenT20noR[x, p22]]
```

```
0
```

Therefore, the two polynomials cpolgenT20noR and cpolgenT20R (in x) have the same leading term, and it is positive.

Therefore, cpolgenT20noR[x,p22] = 0 if and only if cpolgenT20R[(1-r)x,p22] = 0. Hence, x is an eigenvalue resulting from cpolgenT20noR[x,p22] = 0 if and only if (1-r)x is an eigenvalue resulting from cpolgenT20R[x,p22] = 0.

As a consequence, an equilibrium p22 (=p12) at this edge (t12=t22=0, dd1=dd2=0) is unstable if and only if there exists and eigenvalue x>1 with cpolgenT20noR[x,p22] = 0.
Thus, a change of stability at this edge occurs at a point p22 if cpolgenT20noR[1,p22] = 0.

The curve of equilibria that connects the edges t12=t22=0 and t12=t22=1 has its endpoint at the value p22, at which the stability at this edge changes, i.e., where cpolgenT20noR[1,p22] = 0.
This is easily computed numerically.

Now, we show: The value of both polynomials at x = 0 is positive (and proportional); their derivatives at x=0 have the same sign, and they are negative!

```
Factor[D[cpolgenT20noR[x, p22], {x, 1}] /. x → 0]
```

$$2\left(-2+m1+m2-s1+m2\,s1-s2+m2\,s2-s1\,s2+m2\,s1\,s2\right)\left(1+\alpha1\right)\left(2+\alpha1+p22\,\alpha1+p22\,\alpha2+p22\,\alpha1\,\alpha2\right)$$

```
Simplify[Reduce[(-2 + m1 + m2 - s1 + m2 s1 - s2 + m2 s2 - s1 s2 + m2 s1 s2) < 0 &&
    -(1 - m2) < m1/s1 - m2/s2 < 1 - m1 && s1 > 0 && s2 > 0 && 1 > m1 > 0 && 1 > m2 > 0],
 Assumptions → {s1 > 0, s2 > 0, 1 > m1 > 0, 1 > m2 > 0}]
```

$$\left(s2 < m2 + m2\, s2 \;\&\&\; \frac{m1\, s2}{m2 + s2 - m1\, s2} < s1 < \frac{m1\, s2}{m2 - s2 + m2\, s2}\right) \;||\; \left(s2 \geq m2 + m2\, s2 \;\&\&\; m2\, s1 > (m1 - s1 + m1\, s1)\, s2\right)$$

```
Simplify[Reduce[(-2 + m1 + m2 - s1 + m2 s1 - s2 + m2 s2 - s1 s2 + m2 s1 s2) > 0 &&
    -(1 - m2) < m1/s1 - m2/s2 < 1 - m1 && s1 > 0 && s2 > 0 && 1 > m1 > 0 && 1 > m2 > 0],
 Assumptions → {s1 > 0, s2 > 0, 1 > m1 > 0, 1 > m2 > 0}]
```

```
False
```

```
FindInstance[(-2 + m1 + m2 - s1 + m2 s1 - s2 + m2 s2 - s1 s2 + m2 s1 s2) < 0 &&
   -(1 - m2) < m1/s1 - m2/s2 < 1 - m1 && s1 > 0 && s2 > 0 && 1 > m1 > 0 && 1 > m2 > 0, {s1, s2, m1, m2}]
```

$$\left\{\left\{s1 \to \frac{5}{16},\ s2 \to \frac{1}{2},\ m1 \to \frac{1}{4},\ m2 \to \frac{1}{2}\right\}\right\}$$

Stability of equilibria at the edge t12 = t22 = 0

By the above, the equilibrium p22 at this edge is unstable if cpolgenT20noR[1, p22] < 0.

```
Simplify[Series[cpolgenT20noR[1, p22], {p22, 0, 3}]]
```

$$\left(-m1\, m2\, (1 + s2)\, (2 + \alpha 1)^2 + (\alpha 1 + 2\, s1\, (1 + \alpha 1) + m1\, (2 + \alpha 1))\, (\alpha 1 - s2\, (2 + \alpha 1) + m2\, (1 + s2)\, (2 + \alpha 1))\right) - 2\, ((m1 + s1 - s2 + s1\, s2 + \alpha 1 + s1\, \alpha 1 + s1\, s2\, \alpha 1 + m1\, s2\, (2 + \alpha 1) - m2\, (1 + s2)\, (-1 + s1 + s1\, \alpha 1))\, (\alpha 1 + \alpha 2 + \alpha 1\, \alpha 2))\, p22 - (-1 + m1 + m2)\, (1 + s2)\, (\alpha 1 + \alpha 2 + \alpha 1\, \alpha 2)^2\, p22^2 + O[p22]^4$$

Stability of p22 = 0:

```
FullSimplify[cpolgenT20noR[1, 0]]
```

$$m2\, (1 + s2)\, (2 + \alpha 1)\, (\alpha 1 + 2\, s1\, (1 + \alpha 1)) - (\alpha 1 + 2\, s1\, (1 + \alpha 1) + m1\, (2 + \alpha 1))\, (-\alpha 1 + s2\, (2 + \alpha 1))$$

This is positive if $\alpha 1 > 2\, s2/(1 - s2)$. Then

```
Simplify[D[cpolgenT20noR[x, 0], x] /. x → 1]
```

$$2\, (1 + \alpha 1)\, (\alpha 1 + 2\, s1\, (1 + \alpha 1) + m1\, (2 + \alpha 1) + (1 + s1)\, (\alpha 1 - s2\, (2 + \alpha 1) + m2\, (1 + s2)\, (2 + \alpha 1)))$$

is positive. Therefore, p22 = 0 is stable because there are no zeros > 1.

A similar analysis yields stability conditions of p22 = 1.

Complete symmetry of parameters and determination of the endpoints of the curve of equilibria for r>0

By the above, it is sufficient to study cpolgenT20noR[x, p22]

```
Simplify[Series[
  cpolgenT20noR[x, p22] /. {m2 → m, m1 → m, s1 → s, s2 → s, α1 → α, α2 → α}, {x, 0, 2}]]
```

$$-\left(-1+2\,m\right)\left(1+s\right)\left(2+\alpha\right)^2\left(1+p22\,\alpha\right)^2+2\left(-1+m\right)\left(2+2\,s+s^2\right)\left(1+p22\,\alpha\right)\left(2+3\,\alpha+\alpha^2\right)x+4\left(1+s\right)\left(1+\alpha\right)^2x^2+O[x]^3$$

```
cpolgenT20noRsym[p22_, x_, s_, α_, m_] := -(-1 + 2 m) (1 + s) (2 + α)^2 (1 + p22 α)^2 +
  2 (-1 + m) (2 + 2 s + s^2) (1 + p22 α) (2 + 3 α + α^2) x + 4 (1 + s) (1 + α)^2 x^2
```

Compute p22, where the eigenvalue is 1:

```
p22ev1noR = Simplify[Solve[cpolgenT20noRsym[p22, 1, s, α, m] == 0, p22],
  Assumptions → {0 < r < 1/2, α > 0, s > 0, m > s}]
```

$$\left\{\left\{p22\to-\left(\left(2\,m+2\,m\,s+s^2-\sqrt{\left(s^2\left(2+s\right)^2-2\,m\,s^2\left(2+s\right)^2+m^2\left(2+2\,s+s^2\right)^2\right)}+\alpha+s\,\alpha+s^2\,\alpha-\sqrt{\left(s^2\left(2+s\right)^2-2\,m\,s^2\left(2+s\right)^2+m^2\left(2+2\,s+s^2\right)^2\right)}\,\alpha-m\,s^2\left(1+\alpha\right)\right)\Big/\left(\left(-1+2\,m\right)\left(1+s\right)\alpha\left(2+\alpha\right)\right)\right)\right\},\right.$$
$$\left.\left\{p22\to-\left(\left(2\,m+2\,m\,s+s^2+\sqrt{\left(s^2\left(2+s\right)^2-2\,m\,s^2\left(2+s\right)^2+m^2\left(2+2\,s+s^2\right)^2\right)}+\alpha+s\,\alpha+s^2\,\alpha+\sqrt{\left(s^2\left(2+s\right)^2-2\,m\,s^2\left(2+s\right)^2+m^2\left(2+2\,s+s^2\right)^2\right)}\,\alpha-m\,s^2\left(1+\alpha\right)\right)\Big/\left(\left(-1+2\,m\right)\left(1+s\right)\alpha\left(2+\alpha\right)\right)\right)\right\}\right\}$$

```
N[{p22ev1noR /. {s → 0.2, α → 1, m → 0.01},
  p22ev1noR /. {s → 0.2, α → 10, m → 0.01}, p22ev1noR /. {s → 0.2, α → 100, m → 0.01}}]
```

```
{{{p22 → 0.122074}, {p22 → 0.616701}},
 {{p22 → 0.0542852}, {p22 → 0.122296}}, {{p22 → 0.00666611}, {p22 → 0.0140128}}}
```

Stability of p22 = 0:

```
FullSimplify[cpolgenT20noRsym[0, 1, s, α, m]]
```

$$2\left(-1+m\right)s^2\left(1+\alpha\right)\left(2+\alpha\right)+\alpha\left(\alpha+2\,m\left(2+\alpha\right)\right)+s\,\alpha\left(\alpha+2\,m\left(2+\alpha\right)\right)$$

This is always positive for m sufficiently close to 1

```
FullSimplify[cpolgenT20noRsym[0, 1, s, α, 0]]
```

$$-\left(\alpha+2\,s\left(1+\alpha\right)\right)\left(-\alpha+s\left(2+\alpha\right)\right)$$

This is positive if $\alpha > \frac{2\,s}{1-s}$ and negative otherwise!

```
FullSimplify[cpolgenT20noRsym[0, 1, s, α, m] /. α -> 2 s/(1 - s) (1 + z)]
```

$$\frac{1}{\left(-1+s\right)^2}4\,s\left(-\left(-1+s\right)s\,z\left(2+s+z+2\,s\,z\right)+m\left(1+s\,z\right)\left(2+3\,s+s^2+2\left(1+s+s^2\right)z\right)\right)$$

This is positive whenever $\alpha > \frac{2\,s}{1-s}$ (and $s < 1$)

```
FullSimplify[D[cpolgenT20noRsym[0, x, s, α, m], x] /. x → 1]
```

$$2\left(1+\alpha\right)\left(4\left(1+s\right)\left(1+\alpha\right)+\left(-1+m\right)\left(2+s\left(2+s\right)\right)\left(2+\alpha\right)\right)$$

```
FullSimplify[D[cpolgenT20noRsym[0, x, s, α, m], x] /. x → 1 /. α -> 2 s/(1 - s) (1 + z)]
```

$$\frac{1}{(-1+s)^2} 4\,(1+s+2\,s\,z)\,\left(m\,(2+s\,(2+s))\,(1+s\,z)+s\,(s-(-2+s)\,s\,z+2\,(1+z))\right)$$

This is positive whenever s < 1.

Therefore, p22 = 0 is stable if $\alpha > \frac{2\,s}{1-s}$

In a similar way it can be shown that p22 = 1 unstable, and stability changes exactly once between p22 = 0 and p22 = 1

The following computes the eigenvalue that determines stability of p22 :

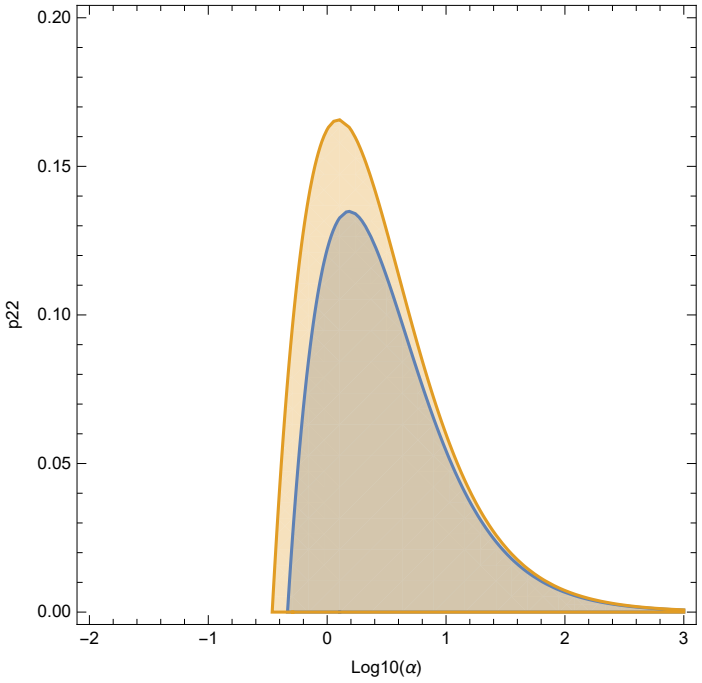
```
EVsgenT20noRsym[p22_, s_, α_, m_] := x /. NSolve[cpolgenT20noRsym[p22, x, s, α, m] == 0, x]
```

```
EVsgenT20noRsym[0.1, 0.2, 10^0, 0.01]
```

```
{0.680398, 0.980327}
```

Plot region in which the maximum eigenvalue is less than 1 in modulus (implying stability of the edge equilibrium t2 = 0, p22, D = 0). This region increases with increases m:

```
RegionPlot[{Abs[Max[EVsgenT20noRsym[p22, 0.2, 10^a, 0.01]]] < 1,
  Abs[Max[EVsgenT20noRsym[p22, 0.2, 10^a, 0.05]]] < 1},
 {a, -2, 3}, {p22, 0, 0.2}, FrameLabel → {"Log10(α)", "p22"}]
```




```
NSolve[cpolgenT20noRsym[p22, 1, 0.2, 1, 0.01] == 0, p22]
```

```
{{p22 → 0.122074}, {p22 → 0.616701}}
```

```
NSolve[cpolgenT20noRsym[p22, 1, 0.2, 10, 0.01] == 0, p22]
```

```
{{p22 → 0.0542852}, {p22 → 0.122296}}
```

```
NSolve[cpolgenT20noRsym[p22, 1, 0.2, 100, 0.01] == 0, p22]
```

```
{{p22 → 0.00666611}, {p22 → 0.0140128}}
```

Therefore, if {s -> 0.2, α -> 1, m -> 0.01, r > 0}, then the equilibria with p22 < 0.122074 are stable.
Therefore, if {s -> 0.2, α -> 10, m -> 0.01, r > 0}, then the equilibria with p22 < 0.0542852 are stable.
Therefore, if {s -> 0.2, α -> 100, m -> 0.01, r > 0}, then the equilibria with p22 < 0.00666611 are stable.

Jacobian at edge t12 = t22 = 1 and stability of edge equilibria

This analysis is analogous to that at t12 = t22 = 0

```
Jacobdt21 =
  Simplify[D[itgen[s1, s2, α1, α2, m1, m2, r][{p12, t12, dd1, p22, t22, dd2}], {{p12,
       t12, dd1, p22, t22, dd2}}] /. {t12 → 1, dd1 → 0, t22 → 1, dd2 → 0, p12 → p22}];
```

An equilibrium is stable if the modulus of eigenvalues of its Jacobian is < 1

```
TableForm[Jacobdt21]
```

$$\begin{array}{lll}
1-m1 & 0 & -\frac{(-1+m1)\,(p22\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)+s1\,(-2+(-2+p22)\,\alpha2+} {2\,(1+\alpha2)} \\
0 & \frac{(-1+m1)\,(1+s1)\,(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2))}{2\,(1+\alpha2)} & 0 \\
0 & -\frac{(-1+m1)\,(-1+p22)\,p22\,r\,(1+s1)\,(\alpha1+\alpha2+\alpha1\,\alpha2)}{2\,(1+\alpha2)} & -\frac{(-1+m1)\,(-1+r)\,(1+s1)\,(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha} {2\,(1+\alpha2)} \\
m2 & 0 & \frac{m2\,(p22\,\alpha2+2\,s2\,(1+\alpha2)+(-1+p22)\,\alpha1\,(1+\alpha2))}{2\,(1+s2)\,(1+\alpha2)} \\
0 & -\frac{m2\,(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2))}{2\,(1+s2)\,(1+\alpha2)} & 0 \\
0 & \frac{m2\,(-1+p22)\,p22\,r\,(\alpha1+\alpha2+\alpha1\,\alpha2)}{2\,(1+s2)\,(1+\alpha2)} & \frac{m2\,(-1+r)\,(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2))}{2\,(1+s2)\,(1+\alpha2)}
\end{array}$$

```
cpolgenT21 = Simplify[Det[Jacobdt21 - x IdentityMatrix[6]]]
```

$$\begin{aligned}
&\frac{1}{16\,(1+s2)^2\,(1+\alpha2)^4}\\
&(-1+x)\,(-1+m1+m2+x)\,\Big(-m1\,m2\,(1+s1)\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)^2+\\
&\quad\big(2+\alpha1-p22\,\alpha1+2\,\alpha2-p22\,\alpha2+\alpha1\,\alpha2-p22\,\alpha1\,\alpha2-2\,(1+s2)\,x\,(1+\alpha2)+\\
&\qquad m2\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)\big)\,\big(2-2\,x+\alpha1-p22\,\alpha1+2\,\alpha2-\\
&\qquad p22\,\alpha2-2\,x\,\alpha2+\alpha1\,\alpha2-p22\,\alpha1\,\alpha2+s1\,\big(2-(-2+p22)\,\alpha2-(-1+p22)\,\alpha1\,(1+\alpha2)\big)+\\
&\qquad m1\,(1+s1)\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)\big)\Big)\\
&\Big(-m1\,m2\,(-1+r)^2\,(1+s1)\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)^2+\\
&\quad\big(-2+2\,x+2\,s2\,x-\alpha1+p22\,\alpha1-2\,\alpha2+p22\,\alpha2+2\,x\,\alpha2+2\,s2\,x\,\alpha2-\\
&\qquad\alpha1\,\alpha2+p22\,\alpha1\,\alpha2+r\,\big(2-(-2+p22)\,\alpha2-(-1+p22)\,\alpha1\,(1+\alpha2)\big)+\\
&\qquad m2\,(-1+r)\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)\big)\\
&\quad\big(-2-2\,s1+2\,x-\alpha1+p22\,\alpha1-s1\,\alpha1+p22\,s1\,\alpha1-2\,\alpha2+p22\,\alpha2-2\,s1\,\alpha2+\\
&\qquad p22\,s1\,\alpha2+2\,x\,\alpha2-\alpha1\,\alpha2+p22\,\alpha1\,\alpha2-s1\,\alpha1\,\alpha2+p22\,s1\,\alpha1\,\alpha2+\\
&\qquad m1\,(-1+r)\,(1+s1)\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)-\\
&\qquad r\,(1+s1)\,\big(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)\big)\big)\Big)
\end{aligned}$$

There are four factors; x = 1 and x = 1 - m1 - m2 are always eigenvalues;
in addition, there is one factor without r, and one factor with r (each of them quadratic)

```
cpolgenT21noR[x_, p22_] := (-m1 m2 (1 + s1) (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2))^2 +
    (2 + α1 - p22 α1 + 2 α2 - p22 α2 + α1 α2 - p22 α1 α2 - 2 (1 + s2) x (1 + α2) +
       m2 (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2))) (2 - 2 x + α1 - p22 α1 + 2 α2 -
       p22 α2 - 2 x α2 + α1 α2 - p22 α1 α2 + s1 (2 - (-2 + p22) α2 - (-1 + p22) α1 (1 + α2)) +
       m1 (1 + s1) (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2))))
```

```
Simplify[Series[cpolgenT21noR[x, p22], {x, 0, 3}]]
```

$$-(-1+m1+m2)\,(1+s1)\,(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2))^2-2\,((-2+m2-s1-s2-s1\,s2+m1\,(1+s1)\,(1+s2))\,(1+\alpha2)\,(-2+(-2+p22)\,\alpha2+(-1+p22)\,\alpha1\,(1+\alpha2)))\,x+4\,(1+s2)\,(1+\alpha2)^2\,x^2+O[x]^4$$

```
cpolgenT21R[x_, p22_] :=
  (-m1 m2 (-1 + r)^2 (1 + s1) (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2))^2 +
    (-2 + 2 x + 2 s2 x - α1 + p22 α1 - 2 α2 + p22 α2 + 2 x α2 + 2 s2 x α2 -
       α1 α2 + p22 α1 α2 + r (2 - (-2 + p22) α2 - (-1 + p22) α1 (1 + α2)) +
       m2 (-1 + r) (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2)))
     (-2 - 2 s1 + 2 x - α1 + p22 α1 - s1 α1 + p22 s1 α1 - 2 α2 + p22 α2 - 2 s1 α2 +
       p22 s1 α2 + 2 x α2 - α1 α2 + p22 α1 α2 - s1 α1 α2 + p22 s1 α1 α2 +
       m1 (-1 + r) (1 + s1) (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2)) -
       r (1 + s1) (-2 + (-2 + p22) α2 + (-1 + p22) α1 (1 + α2)))))
```

```
Simplify[cpolgenT21R[x (1 - r), p22] - (1 - r)^2 cpolgenT21noR[x, p22]]
```

```
0
```

We have (1-r)^2 cpolgenT21noR[x, p22] = cpolgenT20R[x, p22] and an analysis as above shows that stability at this edge is changes at the value p22 at which cpolgenT21noR[1, p22]=0.

We have cpolgenT21noR[x, p22] = a0 + a1 x + a2 x^2, where a0 > 0, a1 < 0, a2 > 0.
We have cpolgenT21R[x, p22] = (1 - r)^2 a0 + (1 - r) a1 x + a2 x^2

Complete symmetry of parameters and determination of the endpoints of the curve of equilibria for r>0

```
Simplify[Series[
  cpolgenT21noR[x, p22] //. {s2 → s1, s1 → s, α2 → α1, α1 → α, m2 → m1, m1 → m}, {x, 0, 3}]]
```

$$-(-1+2\,m)\,(1+s)\,(2+\alpha)^2\,(-1+(-1+p22)\,\alpha)^2-2\,((-1+m)\,(2+2\,s+s^2)\,(-1+(-1+p22)\,\alpha)\,(2+3\,\alpha+\alpha^2))\,x+4\,(1+s)\,(1+\alpha)^2\,x^2+O[x]^4$$

```
cpolgenT21noRsym[p22_, x_, s_, α_, m_] := -(-1 + 2 m) (1 + s) (2 + α)^2 (-1 + (-1 + p22) α)^2 -
  2 ((-1 + m) (2 + 2 s + s^2) (-1 + (-1 + p22) α) (2 + 3 α + α^2)) x + 4 (1 + s) (1 + α)^2 x^2
```

An analogous analysis yields the conditions of stability at the edge t12 = t22 =1. In particular, we obtain:

Therefore, if {s -> 0.2, α -> 10, m -> 0.01, r > 0}, then the equilibria with p22 > 0.945715 are

stable.
If {s -> 0.2, α -> 1, m -> 0.01, r > 0}, then the equilibria with p22 > 0.877926 are stable.

2.2 Equilibria under random mating (α1 = α2 = 0)

```
condeq = FullSimplify[itgen[s1, s2, 0, 0, m1, m2, r][{p12, t12, d1, p22, t22, d2}] -
    {p12, t12, d1, p22, t22, d2}]
```

$$\Big\{\Big(-d1\,s1+m1\,\big(p22+d1\,s1-d2\,s1+p22\,s1+p12\,\big(-1+s1\,\big(-1+t22\big)\big)-p22\,s1\,t22\big)\Big)\Big/\big(1+s1\,\big(1+\big(-1+m1\big)\,t12-m1\,t22\big)\big),\ \frac{s1\,\big(-1+t12\big)\,t12-m1\,\big(1+s1\,t12\big)\,\big(t12-t22\big)}{1+s1\,\big(1+\big(-1+m1\big)\,t12-m1\,t22\big)},$$

$$-\frac{1}{\big(1+s1\,\big(1+\big(-1+m1\big)\,t12-m1\,t22\big)\big)^2}\Big(m1\,\big(-1+r\big)\,\big(1+s1\big)\,\big(d2-\big(-1+m1\big)\,\big(p12-p22\big)\,\big(t12-t22\big)\big)+d1\,\Big(r+s1+s1\,\Big(r+s1\,\big(-1+t12\big)^2-2\,t12\Big)+m1\,\big(1+s1-r\,\big(1+s1\big)-2\,s1\,\big(-1+s1\,\big(-1+t12\big)\big)\,\big(t12-t22\big)\big)+m1^2\,s1^2\,\big(t12-t22\big)^2\Big)\Big),$$

$$\big(d2\,s2+m2\,\big(p12+\big(d1-d2\big)\,s2+p12\,s2\,t12-p22\,\big(1+s2\,t12\big)\big)\big)\Big/\big(1+m2\,s2\,\big(t12-t22\big)+s2\,t22\big),$$

$$1+\frac{1}{s2}-t22-\frac{1+s2}{s2\,\big(1+m2\,s2\,\big(t12-t22\big)+s2\,t22\big)},$$

$$\frac{1}{\big(1+m2\,s2\,\big(t12-t22\big)+s2\,t22\big)^2}\Big(-d1\,m2\,\big(-1+r\big)\,\big(1+s2\big)+\big(-1+m2\big)\,m2\,\big(p12-p22\big)\,\big(-1+r\big)\,\big(1+s2\big)\,\big(t12-t22\big)-d2\,\Big(r-s2+m2^2\,s2^2\,\big(t12-t22\big)^2+m2\,\big(1+s2-r\,\big(1+s2\big)+2\,s2\,\big(t12-t22\big)\,\big(1+s2\,t22\big)\big)+s2\,\big(r+t22\,\big(2+s2\,t22\big)\big)\Big)\Big)\Big\}$$

```
Simplify[Solve[{s1 (-1 + t12) t12 - m1 (1 + s1 t12) (t12 - t22),
    1 + 1/s2 - t22 - (1 + s2)/(s2 (1 + m2 s2 (t12 - t22)) + s2 t22)} == 0,
  {t12, t22}], Assumptions → {s1 > 0, s2 > 0, m1 > 0, m2 > 0}]
{{t12 → 0, t22 → 0}, {t12 → 1, t22 → 1}, {t12 →
   -1/(2 (-1 + m1) (-1 + m1 + m2) s1 s2) (-m1 m2 s1 - 2 m1 s2 + 2 m1^2 s2 + m1 m2 s2 - s1 s2 + m1 s1 s2 + m2
       s1 s2 - m1 m2 s1 s2 + √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
         (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2)),
  t22 → (1 / (4 (-1 + m1) m1 (-1 + m2) (-1 + m1 + m2) s1 s2^2))
    (-m1 m2 s1 - 2 m1 s2 + 2 m1^2 s2 + m1 m2 s2 - s1 s2 + m1 s1 s2 + m2 s1 s2 - m1 m2 s1 s2 +
      √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
        (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2))
    (m1 m2 s1 - m1 m2 s2 + s1 s2 - m1 s1 s2 - m2 s1 s2 + m1 m2 s1 s2 +
      √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
        (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2))},
 {t12 → 1/(2 (-1 + m1) (-1 + m1 + m2) s1 s2) (m1 m2 s1 + 2 m1 s2 - 2 m1^2 s2 - m1 m2 s2 + s1 s2 - m1 s1 s2 -
      m2 s1 s2 + m1 m2 s1 s2 + √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
        (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2)),
  t22 → (1 / (4 (-1 + m1) m1 (-1 + m2) (-1 + m1 + m2) s1 s2^2))
    (-m1 m2 s1 + m1 m2 s2 - s1 s2 + m1 s1 s2 + m2 s1 s2 - m1 m2 s1 s2 +
      √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
        (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2))
    (m1 m2 s1 + 2 m1 s2 - 2 m1^2 s2 - m1 m2 s2 + s1 s2 - m1 s1 s2 - m2 s1 s2 + m1 m2 s1 s2 +
      √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
        (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2))}}
```

Comparison with the polymorphism from 1-locus model shows that the migration-selection equilibrium is

```
migselsol = {t12 → -1/(2 (-1 + m1) (-1 + m1 + m2) s1 s2)
      (-m1 m2 s1 - 2 m1 s2 + 2 m1^2 s2 + m1 m2 s2 - s1 s2 + m1 s1 s2 + m2 s1 s2 - m1 m2 s1 s2 +
        √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
            (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2)),
   t22 → (1 / (4 (-1 + m1) m1 (-1 + m2) (-1 + m1 + m2) s1 s2^2))
      (-m1 m2 s1 - 2 m1 s2 + 2 m1^2 s2 + m1 m2 s2 - s1 s2 + m1 s1 s2 + m2 s1 s2 - m1 m2 s1 s2 +
        √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
            (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2))
      (m1 m2 s1 - m1 m2 s2 + s1 s2 - m1 s1 s2 - m2 s1 s2 + m1 m2 s1 s2 +
        √(-4 (-1 + m1) m1 (-1 + m1 + m2) s2 ((m1 + s1) s2 - m2 s1 (1 + s2)) +
            (2 m1^2 s2 + (-1 + m2) s1 s2 + m1 ((-2 + s1) s2 - m2 (s1 - s2 + s1 s2)))^2))};
```

```
Simplify[{condeq[[2]], condeq[[5]]} /. migselsol,
 Assumptions → {s1 > 0, s2 > 0, m1 > 0, m2 > 0}]
```

```
{0, 0}
```

Determine other coordinates:

```
Simplify[Numerator[Factor[{condeq[[1]], condeq[[4]]} /. migselsol]],
 Assumptions → {s1 > 0, s2 > 0, m1 > 0, m2 > 0}]
```

```
{2 d1 (-1 + m2)^2 s1 s2 + m1^2 ((2 (d1 - d2) s1 - p12 (2 + s1) + p22 (2 + s1)) s2 +
      m2 (2 (-d1 + d2) s1 s2 + p12 (s1 (-1 + s2) + 3 s2) + p22 (s1 - 3 s2 - s1 s2))) +
  m1 (2 m2 p12 s1 - 2 m2^2 p12 s1 - 2 m2 p22 s1 + 2 m2^2 p22 s1 + 2 p12 s2 - 4 m2 p12 s2 + 2 m2^2 p12 s2 -
      2 p22 s2 + 4 m2 p22 s2 - 2 m2^2 p22 s2 - 4 d1 s1 s2 + 2 d2 s1 s2 + 6 d1 m2 s1 s2 - 4 d2 m2 s1 s2 -
      2 d1 m2^2 s1 s2 + 2 d2 m2^2 s1 s2 + p12 s1 s2 - m2 p12 s1 s2 - p22 s1 s2 + m2 p22 s1 s2 -
      p12 √((-1 + m2)^2 s1^2 s2^2 - 2 m1 (-1 + m2) s1 s2 (-s1 s2 + m2 (2 + s1 + s2 + s1 s2)) +
          m1^2 (s1^2 s2^2 + m2^2 (s1 + s2 + s1 s2)^2 - 2 m2 s1 s2 (2 + s1 + s2 + s1 s2))) +
      p22 √((-1 + m2)^2 s1^2 s2^2 - 2 m1 (-1 + m2) s1 s2 (-s1 s2 + m2 (2 + s1 + s2 + s1 s2)) +
          m1^2 (s1^2 s2^2 + m2^2 (s1 + s2 + s1 s2)^2 - 2 m2 s1 s2 (2 + s1 + s2 + s1 s2)))),
 -2 d2 (-1 + m1)^2 s1 s2 + m2^2 (-2 (d1 - d2) (-1 + m1) s1 s2 +
      p12 (m1 s2 + s1 (2 + s2 - m1 (3 + s2))) + p22 (-m1 s2 + s1 (-2 - s2 + m1 (3 + s2)))) +
  m2 (-2 p12 s1 + 4 m1 p12 s1 - 2 m1^2 p12 s1 + 2 p22 s1 - 4 m1 p22 s1 + 2 m1^2 p22 s1 - 2 m1 p12 s2 +
      2 m1^2 p12 s2 + 2 m1 p22 s2 - 2 m1^2 p22 s2 - 2 d1 s1 s2 + 4 d2 s1 s2 + 4 d1 m1 s1 s2 - 6 d2 m1 s1 s2 -
      2 d1 m1^2 s1 s2 + 2 d2 m1^2 s1 s2 - p12 s1 s2 + m1 p12 s1 s2 + p22 s1 s2 - m1 p22 s1 s2 +
      p12 √((-1 + m2)^2 s1^2 s2^2 - 2 m1 (-1 + m2) s1 s2 (-s1 s2 + m2 (2 + s1 + s2 + s1 s2)) +
          m1^2 (s1^2 s2^2 + m2^2 (s1 + s2 + s1 s2)^2 - 2 m2 s1 s2 (2 + s1 + s2 + s1 s2))) -
      p22 √((-1 + m2)^2 s1^2 s2^2 - 2 m1 (-1 + m2) s1 s2 (-s1 s2 + m2 (2 + s1 + s2 + s1 s2)) +
          m1^2 (s1^2 s2^2 + m2^2 (s1 + s2 + s1 s2)^2 - 2 m2 s1 s2 (2 + s1 + s2 + s1 s2))))}
```

```
Simplify[Solve[% == 0, {d1, d2}]]
```

$$\left\{\left\{d1 \to \frac{1}{2\,(-1+m1)\,(-1+m1+m2)\,s1\,s2}\,m1\,(p12-p22)\right.\right.$$
$$\left(-m1\,m2\,s1-2\,s2+2\,m1\,s2+2\,m2\,s2-m1\,m2\,s2-s1\,s2+m1\,s1\,s2+m2\,s1\,s2-m1\,m2\,s1\,s2+\right.$$
$$\sqrt{\left((-1+m2)^2\,s1^2\,s2^2-2\,m1\,(-1+m2)\,s1\,s2\,(-s1\,s2+m2\,(2+s1+s2+s1\,s2))+\right.}$$
$$\left.\left.\left.m1^2\,(s1^2\,s2^2+m2^2\,(s1+s2+s1\,s2)^2-2\,m2\,s1\,s2\,(2+s1+s2+s1\,s2))\right)\right)\right),$$
$$d2 \to \frac{1}{2\,(-1+m2)\,(-1+m1+m2)\,s1\,s2}\,m2\,(p12-p22)\,(-2\,s1+2\,m1\,s1+2\,m2\,s1-$$
$$m1\,m2\,s1-m1\,m2\,s2-s1\,s2+m1\,s1\,s2+m2\,s1\,s2-m1\,m2\,s1\,s2+$$
$$\sqrt{\left((-1+m2)^2\,s1^2\,s2^2-2\,m1\,(-1+m2)\,s1\,s2\,(-s1\,s2+m2\,(2+s1+s2+s1\,s2))+\right.}$$
$$\left.\left.\left.m1^2\,(s1^2\,s2^2+m2^2\,(s1+s2+s1\,s2)^2-2\,m2\,s1\,s2\,(2+s1+s2+s1\,s2))\right)\right)\right\}\right\}$$

```
d1d2alpha0 = {d1 → 1/(2 (-1 + m1) (-1 + m1 + m2) s1 s2) m1 (p12 - p22)
    (-m1 m2 s1 - 2 s2 + 2 m1 s2 + 2 m2 s2 - m1 m2 s2 - s1 s2 + m1 s1 s2 + m2 s1 s2 - m1 m2 s1 s2 +
      Sqrt((-1 + m2)^2 s1^2 s2^2 - 2 m1 (-1 + m2) s1 s2 (-s1 s2 + m2 (2 + s1 + s2 + s1 s2)) +
        m1^2 (s1^2 s2^2 + m2^2 (s1 + s2 + s1 s2)^2 - 2 m2 s1 s2 (2 + s1 + s2 + s1 s2)))),
  d2 → 1/(2 (-1 + m2) (-1 + m1 + m2) s1 s2) m2 (p12 - p22) (-2 s1 + 2 m1 s1 + 2 m2 s1 -
    m1 m2 s1 - m1 m2 s2 - s1 s2 + m1 s1 s2 + m2 s1 s2 - m1 m2 s1 s2 +
    Sqrt((-1 + m2)^2 s1^2 s2^2 - 2 m1 (-1 + m2) s1 s2 (-s1 s2 + m2 (2 + s1 + s2 + s1 s2)) +
      m1^2 (s1^2 s2^2 + m2^2 (s1 + s2 + s1 s2)^2 - 2 m2 s1 s2 (2 + s1 + s2 + s1 s2))))};
```

We have p12 = p22 if and only if d1 = d2 = 0!!!

```
Simplify[condeq /. migselsol /. d1d2alpha0]
```

$$\left\{0,\ 0,\ -\left((2\,m1\,(-1+m2)\,(p12-p22)\,r\,(1+s1)\,s2)\Big/\right.\right.$$
$$\left(s1\,\left(m1\,m2\,s1+2\,s2-2\,m1\,s2-2\,m2\,s2+m1\,m2\,s2+s1\,s2-m1\,s1\,s2-m2\,s1\,s2+\right.\right.$$
$$\left.\left.m1\,m2\,s1\,s2+\sqrt{\left(-4\,(-1+m1)\,m1\,(-1+m1+m2)\,s2\,((m1+s1)\,s2-m2\,s1\,(1+s2))+\right.\right.\right.$$
$$\left.\left.\left.\left.(2\,m1^2\,s2+(-1+m2)\,s1\,s2+m1\,((-2+s1)\,s2-m2\,(s1-s2+s1\,s2)))^2\right)\right)\right)\right),$$
$$0,\ 0,\ -\left((2\,(-1+m1)\,m2\,(p12-p22)\,r\,s1\,(1+s2))\Big/\right.$$
$$\left(s2\,\left(2\,s1-2\,m1\,s1-2\,m2\,s1+m1\,m2\,s1+m1\,m2\,s2+s1\,s2-m1\,s1\,s2-m2\,s1\,s2+\right.\right.$$
$$\left.\left.m1\,m2\,s1\,s2+\sqrt{\left(-4\,(-1+m1)\,m1\,(-1+m1+m2)\,s2\,((m1+s1)\,s2-m2\,s1\,(1+s2))+\right.\right.\right.$$
$$\left.\left.\left.\left.(2\,m1^2\,s2+(-1+m2)\,s1\,s2+m1\,((-2+s1)\,s2-m2\,(s1-s2+s1\,s2)))^2\right)\right)\right)\right)\right\}$$

```
Simplify[Solve[% == 0, {p12, p22}]]
```

Solve: Equations may not give solutions for all "solve" variables.

$$\{\{p22 \to p12\}\}$$

Therefore: If $\alpha 1 = \alpha 2 = 0$, the equilibria are given by
p12 = p22, d1 = d2 = 0, and {t12, t22} are the allele frequencies at migration-selection balance (migselsol).
Therefore, as expected, there is a line of equilibria.

The condition for existence of the polymorphic migration – selection equilibrium is $-(1-m2) < \frac{m1}{s1} - \frac{m2}{s2} < 1 - m1$

2.3 Derivation of manifold/curve of equilibria for r=0/r>0 (assuming symmetric parameters)

2.3.1 Equilibrium conditions and their structure

Solving deltagensym = 0 yields the potential equilibria (conditional on admissibility)

```
deltagensym = Simplify[itgen[s, s, α, α, m, m, r][{p12, t12, dd1, p22, t22, dd2}] -
    {p12, t12, dd1, p22, t22, dd2}]
```

```
deltagensym =
  {-p12 + (-(-1 + m)^2 m p12^2 (1 + s) (-1 + s (-1 + t12)) (t12 - t22) α (2 + α) - dd1 (α + t12 α^2 +
          2 s^3 (-1 + t12)^2 (1 + α) + s (2 + (4 + 2 dd1 - t12) α + (dd1 + (3 - 2 t12) t12) α^2) +
          s^2 (4 + (5 + 2 dd1) α + dd1 α^2 - 2 t12^2 α^2 + t12 (-4 - 5 α + 2 α^2))) -
      m^3 (t12 - t22) (p22^2 (1 + s) (-1 + s (-1 + t22)) α (2 + α) - 2 (dd1 - dd2) s (t12 - t22)
        (s - α) (s + α + s α) + p22 ((t12 - t22) α^2 + 2 s^3 (t12 - t22) (-1 + t22) (1 + α) -
          s α (dd1 (2 + α) - dd2 (2 + α) + (t12 - t22) (-1 - 3 α + 2 t22 α)) + s^2 (2 t22^2 α^2 -
            (dd1 - dd2) α (2 + α) + t22 (2 + α - 2 α^2) - t12 (2 + α + 2 (-1 + t22) α^2)))) +
      m^2 (-dd1^2 s (1 + s) α (2 + α) + p22^2 (1 + s) (-1 + s (-1 + t22)) (t12 - t22)
        α (2 + α) - dd2 ((-t12 + t22) α^2 - 4 s^3 (-1 + t12) (t12 - t22) (1 + α) +
          s (dd2 α (2 + α) + (t12 - t22) α (1 + (-3 + 4 t12) α)) + s^2 (4 t12^2 α^2 +
            dd2 α (2 + α) + t22 (-4 - 5 α + 2 α^2) + t12 (4 + 5 α - 2 (1 + 2 t22) α^2))) +
        dd1 ((-t12 + t22) α^2 - 2 s^3 (3 t12^2 + t22 (2 + t22) - 2 t12 (1 + 2 t22)) (1 + α) +
          s (2 dd2 α (2 + α) + (t12 - t22) α (1 + (-3 + 6 t12 - 2 t22) α)) +
          s^2 (6 t12^2 α^2 + 2 t22^2 α^2 + 2 dd2 α (2 + α) +
            t22 (-4 - 5 α + 2 α^2) + t12 (4 + 5 α - 2 (1 + 4 t22) α^2))) +
        p22 (-2 dd1 α + 2 dd2 α - t12 α + t22 α - dd1 α^2 + dd2 α^2 - 2 t12 α^2 + 2 t12^2 α^2 +
          2 t22 α^2 - t12 t22 α^2 - t22^2 α^2 + 4 s^3 (-1 + t12) (t12 - t22) (-1 + t22) (1 + α) +
          s (-2 (dd1 - dd2) α (2 + α) + 2 t12^2 α (1 + (3 - 2 t22) α) + t22^2 (α - 3 α^2) +
            t22 (-4 + (-2 + 6 dd1 - 4 dd2) α + (4 + 3 dd1 - 2 dd2) α^2) + t12
              (4 + (2 - 4 dd1 + 2 dd2 - 3 t22) α + (-4 - 2 dd1 + dd2 - 3 t22 + 4 t22^2) α^2)) +
          s^2 (-(dd1 - dd2) α (2 + α) + t22^2 (4 + 5 α - 2 α^2) + t22 (-8 + (-7 + 6 dd1 - 4 dd2)
                α + (2 + 3 dd1 - 2 dd2) α^2) - 2 t12^2 (2 + α + 2 (-1 + t22) α^2) + t12 (8 +
              (7 - 4 dd1 + 2 dd2 - 3 t22) α + (-2 - 2 dd1 + dd2 - 2 t22 + 4 t22^2) α^2)))) +
      m (2 dd1^2 s (1 + s) α (2 + α) - dd2 (-1 + s (-1 + t12) - t12 α)
        (-α + 2 s^2 (-1 + t12) (1 + α) + s (-2 + (-3 + 2 t12) α)) +
        dd1 (2 s^3 (-1 + t12) (-1 + 3 t12 - 2 t22) (1 + α) + α (1 + 2 t12 α - t22 α) +
          s (2 + (4 - 4 dd2 - 2 t12 + t22) α + (-2 dd2 + 6 t12 - 6 t12^2 - 3 t22 + 4 t12 t22)
              α^2) + s^2 (4 + 5 α - 4 dd2 α - 2 dd2 α^2 - 6 t12^2 α^2 +
            t22 (4 + 5 α - 2 α^2) + 2 t12 (-4 - 5 α + 2 (1 + t22) α^2))) +
        p22 (2 + dd1 α + 2 t12 α - 2 s^3 (-1 + t12)^2 (-1 + t22) (1 + α) +
          α (dd1 (1 + α) - (-2 + t12 + t22) (1 + t12 α)) + s^2 (6 + 6 α + 2 dd1 α + dd1 α^2 -
            t22 (4 + (5 + 4 dd1) α + 2 dd1 α^2) + t12^2 (2 + α + 2 (-1 + t22) α^2) + t12
```

$$
\begin{aligned}
&\left(-8+\left(-7+2\,\text{dd1}\right)\alpha+\left(2+\text{dd1}\right)\alpha^2+\text{t22}\left(4+5\,\alpha-2\,\alpha^2\right)\right)+\text{s}\left(\text{t12}^2\,\alpha\left(-1+\left(-3+2\,\text{t22}\right)\alpha\right)+\text{t12}\left(-4+\left(-2+2\,\text{dd1}+\text{t22}\right)\alpha+\left(4+\text{dd1}-3\,\text{t22}\right)\alpha^2\right)-2\left(-3-\text{dd1}\,\alpha-\alpha\left(3+\text{dd1}+\text{dd1}\,\alpha\right)+\text{t22}\left(1+2\left(1+\text{dd1}\right)\alpha+\text{dd1}\,\alpha^2\right)\right)\right)\right)+\\
&\left(-1+\text{m}\right)\text{p12}\left(-2-2\,\alpha-2\,\text{dd1}\,\alpha+2\,\text{dd1}\,\text{m}\,\alpha-2\,\text{dd2}\,\text{m}\,\alpha+\text{m}\,\text{t12}\,\alpha+2\,\text{m}\,\text{p22}\,\text{t12}\,\alpha-4\,\text{m}^2\,\text{p22}\,\text{t12}\,\alpha-\text{m}\,\text{t22}\,\alpha-2\,\text{m}\,\text{p22}\,\text{t22}\,\alpha+4\,\text{m}^2\,\text{p22}\,\text{t22}\,\alpha-\text{dd1}\,\alpha^2+\text{dd1}\,\text{m}\,\alpha^2-\text{dd2}\,\text{m}\,\alpha^2-2\,\text{t12}\,\alpha^2+2\,\text{m}\,\text{t12}\,\alpha^2+\text{m}\,\text{p22}\,\text{t12}\,\alpha^2-2\,\text{m}^2\,\text{p22}\,\text{t12}\,\alpha^2+2\,\text{t12}^2\,\alpha^2-3\,\text{m}\,\text{t12}^2\,\alpha^2+\text{m}^2\,\text{t12}^2\,\alpha^2-2\,\text{m}\,\text{t22}\,\alpha^2-\text{m}\,\text{p22}\,\text{t22}\,\alpha^2+2\,\text{m}^2\,\text{p22}\,\text{t22}\,\alpha^2+3\,\text{m}\,\text{t12}\,\text{t22}\,\alpha^2-2\,\text{m}^2\,\text{t12}\,\text{t22}\,\alpha^2+\text{m}^2\,\text{t22}^2\,\alpha^2+2\,\text{s}^3\left(-1+\text{t12}\right)\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)^2\left(1+\alpha\right)-\text{s}^2\left(6+6\,\alpha+2\,\text{dd1}\,\alpha+\text{dd1}\,\alpha^2+2\left(-1+\text{m}\right)^2\,\text{t12}^3\,\alpha^2+\text{m}^2\,\text{t22}\left(\left(-\text{dd1}+\text{dd2}-2\,\text{p22}\right)\alpha\left(2+\alpha\right)+\text{t22}\left(2+\alpha+2\,\text{p22}\,\alpha+\left(-2+\text{p22}\right)\alpha^2\right)\right)+\text{m}\left(\left(-\text{dd1}+\text{dd2}\right)\alpha\left(2+\alpha\right)+\text{t22}\left(-8+\left(-7+2\,\text{dd1}+2\,\text{p22}\right)\alpha+\left(2+\text{dd1}+\text{p22}\right)\alpha^2\right)\right)-\left(-1+\text{m}\right)\text{t12}^2\left(6+6\,\alpha-4\,\alpha^2+\text{m}\left(-2+\left(-1+2\,\text{p22}\right)\alpha+\left(2+\text{p22}+4\,\text{t22}\right)\alpha^2\right)\right)+\text{t12}\left(-12-12\,\alpha-2\,\text{dd1}\,\alpha+2\,\alpha^2-\text{dd1}\,\alpha^2+\text{m}^2\left(2\,\text{t22}^2\,\alpha^2+\left(\text{dd1}-\text{dd2}+2\,\text{p22}\right)\alpha\left(2+\alpha\right)+\text{t22}\left(-4-2\,\alpha+4\,\alpha^2\right)\right)-\text{m}\left(-8+\left(-7+2\,\text{dd2}+2\,\text{p22}\right)\alpha+\left(2+\text{dd2}+\text{p22}\right)\alpha^2+\text{t22}\left(-8+\left(-7+2\,\text{p22}\right)\alpha+\left(6+\text{p22}\right)\alpha^2\right)\right)\right)\right)+\\
&\text{s}\left(-2\left(-1+\text{m}\right)^2\,\text{t12}^3\,\alpha^2+\text{m}^2\,\text{t22}\,\alpha\left(8\,\text{p22}+\text{t22}-2\,\text{p22}\,\text{t22}+4\,\text{p22}\,\alpha+3\,\text{t22}\,\alpha-\text{p22}\,\text{t22}\,\alpha+\text{dd1}\left(2+\alpha\right)-\text{dd2}\left(2+\alpha\right)\right)-2\left(3+\text{dd1}\,\alpha+\alpha\left(3+\text{dd1}+\text{dd1}\,\alpha\right)\right)-\text{m}\left(-2\left(\text{dd1}-\text{dd2}\right)\alpha\left(2+\alpha\right)+\text{t22}\left(-4+2\left(-1+\text{dd1}+2\,\text{p22}\right)\alpha+\left(4+\text{dd1}+2\,\text{p22}\right)\alpha^2\right)\right)+\left(-1+\text{m}\right)\text{t12}^2\,\alpha\left(-6\,\alpha+\text{m}\left(1+3\,\alpha+4\,\text{t22}\,\alpha+\text{p22}\left(2+\alpha\right)\right)\right)+\text{t12}\left(6+6\,\alpha+2\,\text{dd1}\,\alpha-4\,\alpha^2+\text{dd1}\,\alpha^2+\text{m}\left(-4+\left(-2+2\,\text{dd2}+\text{t22}+2\,\text{p22}\left(2+\text{t22}\right)\right)\alpha+\left(4+\text{dd2}+9\,\text{t22}+\text{p22}\left(2+\text{t22}\right)\right)\alpha^2\right)-\text{m}^2\,\alpha\left(\text{dd1}\left(2+\alpha\right)-\text{dd2}\left(2+\alpha\right)+2\left(2\,\text{p22}\left(2+\alpha\right)+\text{t22}\left(1+\left(3+\text{t22}\right)\alpha\right)\right)\right)\right)\right)\Big)\Big/\\
&\left(2\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\right)\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)+\text{m}\,\text{t22}\,\alpha+\text{t12}\left(\alpha-\text{m}\,\alpha\right)\right)\right),\\
&-\text{t12}-\left(\left(\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(2+\text{p12}\,\alpha-\text{m}\,\text{p12}\,\alpha+\text{m}\,\text{p22}\,\alpha+2\,\text{t12}\,\alpha-2\,\text{m}\,\text{t12}\,\alpha-\text{p12}\,\text{t12}\,\alpha+2\,\text{m}\,\text{p12}\,\text{t12}\,\alpha-\text{m}^2\,\text{p12}\,\text{t12}\,\alpha-\text{m}\,\text{p22}\,\text{t12}\,\alpha+\text{m}^2\,\text{p22}\,\text{t12}\,\alpha+2\,\text{m}\,\text{t22}\,\alpha-\text{m}\,\text{p12}\,\text{t22}\,\alpha+\text{m}^2\,\text{p12}\,\text{t22}\,\alpha-\text{m}^2\,\text{p22}\,\text{t22}\,\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(4+\left(2+2\,\text{dd1}\left(-1+\text{m}\right)-2\,\text{dd2}\,\text{m}+4\,\text{p12}-4\,\text{m}\,\text{p12}+4\,\text{m}\,\text{p22}+\text{t12}-\text{m}\,\text{t12}-2\,\text{p12}\,\text{t12}+2\,\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-2\,\text{m}\,\text{p22}\,\text{t22}\right)\alpha+\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+2\,\text{p12}-2\,\text{m}\,\text{p12}+2\,\text{m}\,\text{p22}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\alpha^2\right)-\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha\left(-1-\text{t12}\,\alpha+\left(-1+\text{m}\right)\text{p12}\left(1+\alpha\right)-\text{m}\left(\text{p22}+\text{p22}\,\alpha-\text{t12}\,\alpha+\text{t22}\,\alpha\right)\right)+\text{s}^2\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(2+\alpha\right)\left(1-\text{dd1}\,\alpha+\text{p12}\,\alpha+\left(-1+\text{m}\right)\text{t12}\left(1+\text{p12}\,\alpha\right)-\text{m}\left(\text{t22}+\left(-\text{dd1}+\text{dd2}+\text{p12}-\text{p22}\right)\alpha+\text{p22}\,\text{t22}\,\alpha\right)\right)\right)\Big)\Big/\\
&\left(2\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\right)\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)+\text{m}\,\text{t22}\,\alpha+\text{t12}\left(\alpha-\text{m}\,\alpha\right)\right)\right),\\
&\frac{1}{4}\left(-4\,\text{dd1}-\frac{1}{\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\right)^4}\left(1+\text{s}\right)\left(-1+\text{s}\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)\right)^2\left(-\left(\left(\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\left(1+\alpha\right)\right)\Big/\left(-1+\text{s}\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)+\left(-1+\text{m}\right)\text{t12}\,\alpha-\text{m}\,\text{t22}\,\alpha\right)\right)-\left(\left(-1+\text{r}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.
\end{aligned}
$$

```
        (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)
        (1 + α)) / (-1 + s (-1 + t12 - m t12 + m t22) + (-1 + m) t12 α - m t22 α) -
     (r (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22) (-1 + dd1 (-1 + m) - dd2 m +
          p12 - m p12 + m p22 + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
     ((-1 + r) (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)
        (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22) (1 +
          α)) / (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
     ((1 + s) (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)
        (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
          m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22) (1 + α)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
     ((dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
        (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)) /
      (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)) -
     (r (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22) (-1 + dd1 (-1 + m) - dd2 m +
          p12 - m p12 + m p22 + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
      (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)) +
     ((1 + s) (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)
        (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
          m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
      (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)) - (2 (1 + s)
        (dd1 - dd1 m + dd2 m - p12 + m p12 - m p22 + p12 t12 - m p12 t12 + m p22 t22)^2) /
      (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)))
   (-(((dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22) (dd1 (-1 + m) -
            dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22) (1 + α)) /
        (-1 + s (-1 + t12 - m t12 + m t22) + (-1 + m) t12 α - m t22 α)) +
     ((dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
        (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) -
     (r (1 + s) (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
        (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
          m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
     ((1 + s) (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)
        (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
          m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) -
     (2 (dd1 - dd1 m + dd2 m - t12 + m t12 + p12 t12 - m p12 t12 - m t22 + m p22 t22)^2) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
     ((-1 + r) (1 + s) (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 +
          m p12 t12 - m p22 t22) (dd1 (-1 + m) - dd2 m + t12 -
          m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22) (1 + α)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
     ((1 + s) (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)
        (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
          m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22) (1 + α)) /
      (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) -
     (r (1 + s) (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
```

$$
\begin{aligned}
&\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\Big)\Big/\\
&\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)+\text{m}\,\text{t22}\,\alpha+\text{t12}\,\left(\alpha-\text{m}\,\alpha\right)\right)+\left(\left(-1+\text{r}\right)\left(1+\text{s}\right)\right.\\
&\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\\
&\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\\
&\left.\left(1+\alpha\right)\right)\Big/\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)+\text{m}\,\text{t22}\,\alpha+\text{t12}\,\left(\alpha-\text{m}\,\alpha\right)\right)\Big)+\\
&\frac{1}{\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\right)^4}\left(1+\text{s}\right)\left(-1+\text{s}\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)\right)^2\\
&\Big(\Big(\text{r}\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\\
&\quad\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\\
&\quad\left.\left(1+\alpha\right)\right)\Big/\left(-1+\text{s}\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)+\left(-1+\text{m}\right)\text{t12}\,\alpha-\text{m}\,\text{t22}\,\alpha\right)+\\
&\left(\left(-1+\text{r}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right)\Big/\\
&\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)+\\
&\left(\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right)\Big/\\
&\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)-\\
&\left(\text{r}\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\left(1+\right.\\
&\quad\left.\left.\alpha\right)\right)\Big/\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)+\\
&\left(\left(1+\text{s}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\left(1+\alpha\right)\right)\Big/\\
&\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)+\\
&\left(\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\left(1+\alpha\right)\right)\Big/\\
&\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)-\\
&\left(2\left(1+\text{s}\right)\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)^2\left(1+\alpha\right)\right)\Big/\\
&\left(1+\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\alpha+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)\left(1+\alpha\right)\right)+\\
&\left(\left(-1+\text{r}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right)\Big/\\
&\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)+\text{m}\,\text{t22}\,\alpha+\text{t12}\,\left(\alpha-\text{m}\,\alpha\right)\right)+\\
&\left(\left(1+\text{s}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left(-1+\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{p12}-\text{m}\,\text{p12}+\text{m}\,\text{p22}+\text{t12}-\right.\\
&\quad\left.\left.\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right)\Big/\\
&\left.\left(1+\text{s}\left(1+\left(-1+\text{m}\right)\text{t12}-\text{m}\,\text{t22}\right)+\text{m}\,\text{t22}\,\alpha+\text{t12}\,\left(\alpha-\text{m}\,\alpha\right)\right)\right)\\
&\Big(-\left(\left(\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\left(\text{dd1}\left(-1+\text{m}\right)-\right.\right.\right.\\
&\quad\left.\left.\left.\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\left(1+\alpha\right)\right)\Big/\\
&\left(-1+\text{s}\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)+\left(-1+\text{m}\right)\text{t12}\,\alpha-\text{m}\,\text{t22}\,\alpha\right)+\\
&\left(2\left(\text{dd1}-\text{dd1}\,\text{m}+\text{dd2}\,\text{m}+\text{p12}\,\text{t12}-\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{p22}\,\text{t22}\right)^2\left(1+\alpha\right)\right)\Big/\\
&\left(-1+\text{s}\left(-1+\text{t12}-\text{m}\,\text{t12}+\text{m}\,\text{t22}\right)+\left(-1+\text{m}\right)\text{t12}\,\alpha-\text{m}\,\text{t22}\,\alpha\right)+\\
&\left(\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}-\text{m}\,\text{p22}\,\text{t22}\right)\right.\\
&\quad\left.\left(\text{dd1}\left(-1+\text{m}\right)-\text{dd2}\,\text{m}+\text{t12}-\text{m}\,\text{t12}-\text{p12}\,\text{t12}+\text{m}\,\text{p12}\,\text{t12}+\text{m}\,\text{t22}-\text{m}\,\text{p22}\,\text{t22}\right)\right)\Big/
\end{aligned}
$$

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        (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
       ((-1 + r) (1 + s) (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
          (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
            m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
        (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) -
       (r (1 + s) (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)
          (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22) (1 +
            α)) / (1 + (1 + (-1 + m) t12 - m t22) α + s (1 + (-1 + m) t12 - m t22) (1 + α)) +
       ((1 + s) (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
          (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)) /
        (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)) +
       ((-1 + r) (1 + s) (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
          (-1 + dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 + t12 -
            m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)) /
        (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)) +
       ((1 + s) (dd1 (-1 + m) - dd2 m - p12 t12 + m p12 t12 - m p22 t22)
          (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)
          (1 + α)) / (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)) -
       (r (1 + s) (dd1 (-1 + m) - dd2 m + p12 - m p12 + m p22 - p12 t12 + m p12 t12 - m p22 t22)
          (dd1 (-1 + m) - dd2 m + t12 - m t12 - p12 t12 + m p12 t12 + m t22 - m p22 t22)
          (1 + α)) / (1 + s (1 + (-1 + m) t12 - m t22) + m t22 α + t12 (α - m α)))),
  -p22 + (p22 (1 + s t22) (2 + 2 (1 + dd2) α + (dd2 - 2 (-1 + t22) t22) α^2 +
        2 s^2 t22^2 (1 + α) + s (-2 t22^2 α^2 + dd2 α (2 + α) + 2 t22 (2 + 2 α + α^2))) +
     dd2 (2 s^3 t22^2 (1 + α) - α (1 + t22 α) + s (2 + (1 + 2 dd2 - t22) α + (dd2 - 2 t22^2) α^2) +
        s^2 (-2 t22^2 α^2 + dd2 α (2 + α) + t22 (4 + 3 α + α^2))) -
     m^3 (t12 - t22) (p12^2 (1 + s) (1 + s t12) α (2 + α) + p22^2 (1 + s) (1 + s t22) α (2 + α) -
        2 (dd1 - dd2) s (t12 - t22) (s - α) (s + α + s α) +
        p22 ((-t12 + t22) α^2 + 2 s^3 (t12 - t22) t22 (1 + α) -
           s α (dd1 (2 + α) - dd2 (2 + α) + (t12 - t22) (-1 + 2 t22 α)) + s^2 (2 t22^2 α^2 -
              (dd1 - dd2) α (2 + α) - t22 (1 + α) (2 + α) + t12 (2 + 3 α + (1 - 2 t22) α^2))) -
        p12 (2 s^3 t12 (t12 - t22) (1 + α) + s α (4 p22 + t12 + 2 p22 t12 - t22 +
              2 p22 t22 + 2 p22 α + p22 t12 α - 2 t12^2 α + p22 t22 α + 2 t12 t22 α -
              dd1 (2 + α) + dd2 (2 + α)) + α ((-t12 + t22) α + 2 p22 (2 + α)) +
           s^2 (-2 t12^2 α^2 + t12 (2 + (3 + 2 p22) α + (1 + p22 + 2 t22) α^2) +
              (2 + α) ((-dd1 + dd2) α + t22 (-1 + (-1 + p22) α))))) +
     m (2 dd1 s - 2 dd2 s + 4 dd2 s^2 t12 + 4 dd1 s^2 t22 - 8 dd2 s^2 t22 + 4 dd2 s^3 t12 t22 +
        2 dd1 s^3 t22^2 - 6 dd2 s^3 t22^2 - dd1 α + dd2 α + dd1 s α - dd2 s α +
        4 dd1 dd2 s α - 4 dd2^2 s α + 4 dd1 dd2 s^2 α - 4 dd2^2 s^2 α - dd2 s t12 α +
        3 dd2 s^2 t12 α - dd1 s t22 α + 2 dd2 s t22 α + 3 dd1 s^2 t22 α - 6 dd2 s^2 t22 α +
        4 dd2 s^3 t12 t22 α + 2 dd1 s^3 t22^2 α - 6 dd2 s^3 t22^2 α + 2 dd1 dd2 s α^2 -
        2 dd2^2 s α^2 + 2 dd1 dd2 s^2 α^2 - 2 dd2^2 s^2 α^2 - dd2 t12 α^2 + dd2 s^2 t12 α^2 -
        dd1 t22 α^2 + 2 dd2 t22 α^2 + dd1 s^2 t22 α^2 - 2 dd2 s^2 t22 α^2 - 4 dd2 s t12 t22 α^2 -
        4 dd2 s^2 t12 t22 α^2 - 2 dd1 s t22^2 α^2 + 6 dd2 s t22^2 α^2 - 2 dd1 s^2 t22^2 α^2 +
        6 dd2 s^2 t22^2 α^2 - p22^2 (1 + s) (t12 - t22) (1 + s t22) α (2 + α) +
        p22 (-2 - 2 α + 2 dd1 α - 4 dd2 α + t12 α - t22 α + dd1 α^2 - 2 dd2 α^2 + 2 t12 α^2 -
           4 t22 α^2 - 3 t12 t22 α^2 + 5 t22^2 α^2 + 2 s^3 (2 t12 - 3 t22) t22^2 (1 + α) -
           s (t22^2 α - 6 t22^3 α^2 - (dd1 - 2 dd2) α (2 + α) +
              t22 (10 + (11 - 2 dd1 + 2 dd2) α + (4 - dd1 + dd2) α^2) +
              t12 (-4 + (-5 + 2 dd2 - t22) α + (-2 + dd2 + 4 t22^2) α^2)) -
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          s^2 (t12 (4 t22^2 α^2 + dd2 α (2 + α) - t22 (8 + 9 α + 3 α^2)) +
              t22 (-6 t22^2 α^2 - (dd1 - dd2) α (2 + α) + t22 (14 + 15 α + 5 α^2)))) +
       p12 (2 + 2 α + 2 dd2 α - t12 α + 2 p22 t12 α + t22 α - 2 p22 t22 α + dd2 α^2 +
          p22 t12 α^2 + 2 t22 α^2 - p22 t22 α^2 - t12 t22 α^2 - t22^2 α^2 + 2 s^3 t12 t22^2 (1 + α) +
          s (dd2 α (2 + α) - t22 (-4 + (-5 + 2 dd2 + 2 p22) α + (-2 + dd2 + p22) α^2) +
              t12 (2 + (1 + 4 dd2 - t22 + 2 p22 (1 + t22)) α +
                  (2 dd2 + p22 + p22 t22 - 2 t22^2) α^2) - t22^2 α (-1 + p22 (2 + α))) +
          s^2 (-t22 (2 + α) (dd2 α + t22 (-1 + (-1 + p22) α)) + t12 (-2 t22^2 α^2 +
                2 dd2 α (2 + α) + t22 (4 + (2 + p22) α + (1 + p22) α (1 + α)))))) +
    m^2 (4 dd1 s^2 t12 - 4 dd2 s^2 t12 + 2 dd2 s^3 t12^2 - 4 dd1 s^2 t22 + 4 dd2 s^2 t22 +
       4 dd1 s^3 t12 t22 - 8 dd2 s^3 t12 t22 - 4 dd1 s^3 t22^2 + 6 dd2 s^3 t22^2 +
       2 dd1^2 s α - 4 dd1 dd2 s α + 2 dd2^2 s α + 2 dd1^2 s^2 α - 4 dd1 dd2 s^2 α +
       2 dd2^2 s^2 α - dd1 s t12 α + dd2 s t12 α + 3 dd1 s^2 t12 α - 3 dd2 s^2 t12 α +
       2 dd2 s^3 t12^2 α + dd1 s t22 α - dd2 s t22 α - 3 dd1 s^2 t22 α +
       3 dd2 s^2 t22 α + 4 dd1 s^3 t12 t22 α - 8 dd2 s^3 t12 t22 α -
       4 dd1 s^3 t22^2 α + 6 dd2 s^3 t22^2 α + dd1^2 s α^2 - 2 dd1 dd2 s α^2 +
       dd2^2 s α^2 + dd1^2 s^2 α^2 - 2 dd1 dd2 s^2 α^2 + dd2^2 s^2 α^2 - dd1 t12 α^2 +
       dd2 t12 α^2 + dd1 s^2 t12 α^2 - dd2 s^2 t12 α^2 - 2 dd2 s t12^2 α^2 -
       2 dd2 s^2 t12^2 α^2 + dd1 t22 α^2 - dd2 t22 α^2 - dd1 s^2 t22 α^2 + dd2 s^2 t22 α^2 -
       4 dd1 s t12 t22 α^2 + 8 dd2 s t12 t22 α^2 - 4 dd1 s^2 t12 t22 α^2 +
       8 dd2 s^2 t12 t22 α^2 + 4 dd1 s t22^2 α^2 - 6 dd2 s t22^2 α^2 + 4 dd1 s^2 t22^2 α^2 -
       6 dd2 s^2 t22^2 α^2 + p12^2 (1 + s) (1 + s t12) (t12 - t22) α (2 + α) +
       2 p22^2 (1 + s) (t12 - t22) (1 + s t22) α (2 + α) +
       p22 (2 s^3 t22 (t12^2 - 4 t12 t22 + 3 t22^2) (1 + α) -
          α (dd1 (2 + α) - dd2 (2 + α) + (t12 - t22) (1 + (2 + t12 - 4 t22) α)) -
          s (-2 t22^2 α + 6 t22^3 α^2 + (dd1 - dd2) α (2 + α) + t12^2 α (-1 + 2 t22 α) + t22
                (-4 + (-5 + 2 dd2) α + (-2 + dd2) α^2) + t12 (4 + (5 + 2 dd1 - 4 dd2 + 3 t22)
                  α + (2 + dd1 - 2 dd2 - 8 t22^2) α^2)) + s^2 (t12^2 (2 + 3 α + (1 - 2 t22) α^2) +
              t22 (-6 t22^2 α^2 - dd2 α (2 + α) + 2 t22 (5 + 6 α + 2 α^2)) +
              t12 (8 t22^2 α^2 - (dd1 - 2 dd2) α (2 + α) - t22 (12 + 15 α + 5 α^2))))) +
       p12 (2 dd1 α - 2 dd2 α + t12 α - 6 p22 t12 α - t22 α + 6 p22 t22 α + dd1 α^2 -
          dd2 α^2 + 2 t12 α^2 - 3 p22 t12 α^2 - t12^2 α^2 - 2 t22 α^2 + 3 p22 t22 α^2 - t12 t22 α^2 +
          2 t22^2 α^2 + 4 s^3 t12 (t12 - t22) t22 (1 + α) - s (-(dd1 - dd2) α (2 + α) +
              t22 (4 + (5 + 2 dd1 - 4 dd2 - 6 p22) α + (2 + dd1 - 2 dd2 - 3 p22) α^2) +
              t12 (-4 + (-5 - 4 dd1 + 6 dd2 + 6 p22 - 3 t22 + 2 p22 t22) α +
                  (-2 - 2 dd1 + 3 dd2 + 3 p22 + p22 t22 - 4 t22^2) α^2) -
              2 t22^2 α (-1 + p22 (2 + α)) + t12^2 α (1 + 4 t22 α + p22 (2 + α))) -
          s^2 (t12^2 (-4 + (-3 + 2 p22) α + (-1 + p22 + 4 t22) α^2) +
              t22 (2 + α) ((dd1 - 2 dd2) α + t22 (2 - 2 (-1 + p22) α)) - t12 α
                (2 dd1 (2 + α) - 3 dd2 (2 + α) + t22 (3 + α + 4 t22 α - p22 (2 + α)))))))) /
  (2 (1 + m s (t12 - t22) + s t22) (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α)
    (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α))),
-t22 + ((1 + s) (m (t12 - t22) + t22) (2 + α + 2 p22 α + t22 α - 2 p22 t22 α +
       p22 α^2 + t22 α^2 - p22 t22 α^2 - t22^2 α^2 + 2 s^2 t22^2 (1 + α) +
       s (dd2 α (2 + α) + t22 (4 + (3 - 2 dd2 + 2 p22) α + (1 - dd2 + p22) α^2) -
          t22^2 α (-1 + α + p22 (2 + α))) + m^2 (t12 - t22) (2 s^2 (t12 - t22) (1 + α) +
          s α (t12 - 2 p12 t12 - t22 + 2 p22 t22 - t12 α - p12 t12 α + t22 α + p22 t22 α - dd1
                (2 + α) + dd2 (2 + α)) + α ((-t12 + t22) α - p12 (2 + α) + p22 (2 + α))) +
       m (4 s^2 (t12 - t22) t22 (1 + α) - α (p22 (1 + t12 - 2 t22) (2 + α) + p12
                (-1 + t22) (2 + α) + (t12 - t22) (-1 - α + 2 t22 α)) -
          s (-(dd1 - dd2) α (2 + α) + t22 (4 + (3 + 2 dd1 - 4 dd2 + 2 p22) α +
```

```
                (1 + dd1 - 2 dd2 + p22) α^2) + t12 (-4 + (-3 + 2 dd2 + 2 p12 (-1 + t22) -
                  2 t22 + 2 p22 t22) α + (-1 + dd2 + p12 (-1 + t22) + 2 t22 +
                  p22 t22) α^2) - 2 t22^2 α (-1 + α + p22 (2 + α))))))) /
   (2 (1 + m s (t12 - t22) + s t22) (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α)
     (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α))),
 1/4 (-4 dd2 - 1/(1 + m s (t12 - t22) + s t22)^2 (1 + s)
     (((2 (1 + s) (dd2 + dd1 m - dd2 m - m t12 + m p12 t12 - t22 + m t22 + p22 t22 -
           m p22 t22))^2) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
       ((1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
         (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)) /
        (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) +
       (r (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
         (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
           p22 t22 + m p22 t22)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
       ((dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
         (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
           p22 t22 + m p22 t22)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
       ((-1 + r) (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
         (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
         (1 + α)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
       (1 / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α))
        (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
         (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 -
           m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22) (1 + α) +
       (r (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22) (-1 + dd2 (-1 + m) - dd1 m +
           m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)) /
        (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
       ((1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
         (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
         (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
       ((-1 + r) (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
         (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
         (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)))
     ((r (1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
         (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 -
           m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)) /
        (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) - ((-1 + r) (1 + s)
         (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
         (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
         (1 + α)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
       (1 / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α))
        (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
         (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 -
           m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22) (1 + α) +
       (2 (dd2 + dd1 m - dd2 m - m p12 - p22 + m p22 + m p12 t12 + p22 t22 - m p22 t22))^2) /
        (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
       ((1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
```

```
      (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)) /
    (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) +
   (r (1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
      (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
        p22 t22 + m p22 t22)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
   ((dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
      (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
        p22 t22 + m p22 t22)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
   ((1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22) (dd2 (-1 + m) -
        dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22) (1 + α)) /
    (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) - ((-1 + r) (1 + s)
      (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
      (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
      (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α))) +
  1/(1 + m s (t12 - t22) + s t22)^2 (1 + s) (-(((1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 -
          p22 t22 + m p22 t22) (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 -
          p22 t22 + m p22 t22)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α)) -
   ((-1 + r) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
      (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
        p22 t22 + m p22 t22)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) +
   (r (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
      (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
      (1 + α)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
   ((dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
      (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)) /
    (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
   ((-1 + r) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
      (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
        p22 t22 + m p22 t22)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) +
   (2 (1 + s) (dd2 + dd1 m - dd2 m + m p12 t12 + p22 t22 - m p22 t22)^2 (1 + α)) /
    (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
   ((dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
      (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
      (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
   ((1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
      (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
      (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) +
   (r (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
      (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
      (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α))) +
  (-(((-1 + r) (1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
        (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
          p22 t22 + m p22 t22)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α)) -
   (1 / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α)) (1 + s)
    (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
    (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 +
      m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22) +
   (r (1 + s) (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
```

```
        (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
        (1 + α)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
     (1 / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α))
      (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
       (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 -
         m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22) (1 + α) -
     (1 / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α)) (1 + s)
       (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
       (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 -
         m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22) (1 + α) +
     (2 (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 +
           t22 - m t22 - p22 t22 + m p22 t22)^2
        (1 + α)) / (1 + s t22 + m (t12 - t22) (s - α) + α - t22 α) -
     ((-1 + r) (1 + s) (dd2 (-1 + m) - dd1 m - m p12 t12 - p22 t22 + m p22 t22)
        (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
          p22 t22 + m p22 t22)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) -
     ((dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
        (-1 + dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 + m t12 - m p12 t12 + t22 - m t22 -
          p22 t22 + m p22 t22)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)) +
     (r (1 + s) (dd2 (-1 + m) - dd1 m + m p12 + p22 - m p22 - m p12 t12 - p22 t22 + m p22 t22)
        (dd2 (-1 + m) - dd1 m + m t12 - m p12 t12 + t22 - m t22 - p22 t22 + m p22 t22)
        (1 + α)) / (1 + t22 α + s t22 (1 + α) + m (t12 - t22) (s + α + s α)))};
```

```
Coefficient[deltagensym[[1]], r]
```

```
0
```

```
Coefficient[deltagensym[[2]], r]
```

```
0
```

```
Exponent[deltagensym[[3]], r]
```

```
1
```

```
Coefficient[deltagensym[[3]], r]
```

The following gives the structure of the (numerator of the) components of deltagensym

```
Exponent[Numerator[Factor[deltagensym[[1]]]], {p12, t12, dd1, p22, t22, dd2}]
```

```
{2, 2, 2, 2, 3, 2}
```

```
Exponent[Numerator[Factor[deltagensym[[2]]]], {p12, t12, dd1, p22, t22, dd2}]
```

```
{1, 4, 1, 1, 3, 1}
```

```
Exponent[Numerator[Factor[deltagensym[[3]]]], {p12, t12, dd1, p22, t22, dd2}]
```

```
{3, 6, 3, 3, 6, 3}
```

```
Exponent[Numerator[Factor[deltagensym[[4]]]], {p12, t12, dd1, p22, t22, dd2}]
```

```
{2, 3, 2, 2, 2, 2}
```

```
Exponent[Numerator[Factor[deltagensym[[5]]]], {p12, t12, dd1, p22, t22, dd2}]
```

```
{1, 3, 1, 1, 4, 1}
```

```
Exponent[Numerator[Factor[deltagensym[[6]]]], {p12, t12, dd1, p22, t22, dd2}]
```

```
{3, 6, 3, 3, 6, 3}
```

Therefore, deltagensym[[2]] and deltagensym[[5]] can be used to compute p12 and p22 (at equilibrium)

```
Exponent[deltagensym[[3]], r]
```

```
Exponent[deltagensym[[6]], r]
```

```
1
```

There is symmetry between deme 1 and deme 2, as it should be :

```
Simplify[(deltagensym[[1]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x2, p22 → x1, t11 → y2, t22 → y1, dd1 → z2, dd2 → z1}) +
   (deltagensym[[4]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x1, p22 → x2, t11 → y1, t22 → y2, dd1 → z1, dd2 → z2})]
```

```
Simplify[(deltagensym[[2]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x2, p22 → x1, t11 → y2, t22 → y1, dd1 → z2, dd2 → z1}) +
   (deltagensym[[5]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x1, p22 → x2, t11 → y1, t22 → y2, dd1 → z1, dd2 → z2})]
```

```
Simplify[(deltagensym[[3]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x2, p22 → x1, t11 → y2, t22 → y1, dd1 → z2, dd2 → z1}) -
   (deltagensym[[6]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x1, p22 → x2, t11 → y1, t22 → y2, dd1 → z1, dd2 → z2})]
```

```
0
```

```
Simplify[(deltagensym[[3]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x1, p22 → x2, t11 → y1, t22 → y2, dd1 → z1, dd2 → z2}) -
   (deltagensym[[6]] /. {p12 → 1 - p11, t12 → 1 - t11} /.
     {p11 → x2, p22 → x1, t11 → y2, t22 → y1, dd1 → z2, dd2 → z1})]
```

```
0
```

2.3.2 Computing the equilibrium values of (p12,p22) and (dd1,dd2) as functions of (t12,t22) for arbitrary r

The structure of deltagensym (see above) suggests that we can obtain (p12, dd1, p22, dd2) as functions of (t12, t22) by solving {deltagensym[[2]], deltagensym[[5]]} = 0 and ,subsequently, {deltagensym[[1]], deltagensym[[4]]} = 0

```
Simplify[Solve[{deltagensym[[2]], deltagensym[[5]]} == 0, {p12, p22}]]
```

The following is (p12, p22) as a function of (t12, t22, dd1, dd2); it is independent of r:

```
p12p22gentdd =
  {p12 → -((((-1 + t12) t12 (-1 + t22) t22 (1 + s t22) (2 s^3 (-1 + t12)^2 (1 + α) + α (1 + t12 α) +
            s (2 + (4 + 2 dd1 - t12) α + (dd1 + (3 - 2 t12) t12) α^2) +
```

$$
\begin{aligned}
&s^2 \left(4 + 5\,\alpha + 2\,dd1\,\alpha + dd1\,\alpha^2 - 2\,t12^2\,\alpha^2 + t12\,\left(-4 - 5\,\alpha + 2\,\alpha^2\right)\right)\Big) + \\
&m^6\,s\,\left(t12 - t22\right)^4\,\Big(2\,s^3\,\Big(t12^2\,t22 + \left(-1 + t22\right)^2\,t22 + t12\,\left(-1 + 2\,t22 - 2\,t22^2\right)\Big)\,\left(1 + \alpha\right) - \\
&\quad 2\,\alpha\,\left(-dd1\,\left(2 + \alpha\right) + dd2\,\left(2 + \alpha\right) + \left(t12 - t22\right)\,\left(1 + \left(-1 + t12 + 2\,t22\right)\,\alpha\right)\right) + \\
&\quad s\,\alpha\,\left(3\,dd1\,\left(2 + \alpha\right) - 3\,dd2\,\left(2 + \alpha\right) - \left(t12 - t22\right)\right. \\
&\qquad \left.\left(3 + \left(-3 + 7\,t22 - 2\,t22^2 + 2\,t12\,\left(1 + t22\right)\right)\,\alpha\right)\right) + \\
&\quad s^2\,\left(-2\,t22^3\,\alpha^2 + \left(dd1 - dd2\right)\,\alpha\,\left(2 + \alpha\right) + t22\,\left(4 + 5\,\alpha - \alpha^2\right) + 3\,t22^2\,\left(-2 - 2\,\alpha + \alpha^2\right) + \right. \\
&\qquad \left.t12^2\,\left(2 + 2\,\alpha - 2\,t22\,\alpha^2\right) + t12\,\left(-4 - 5\,\alpha + \alpha^2 + 4\,t22^2\,\alpha^2 + t22\,\left(4 + 4\,\alpha - 3\,\alpha^2\right)\right)\right)\Big) - \\
&m^5\,\left(t12 - t22\right)^4\,\Big(2\,s^4\,\Big(4\,t12^2\,t22 + \left(-1 + t22\right)^2\,\left(1 + 3\,t22\right) + t12\,\left(-2 + 2\,t22 - 5\,t22^2\right)\Big) \\
&\qquad \left(1 + \alpha\right) + 2\,\alpha\,\left(1 + t22\,\alpha\right) - 2\,s\,\alpha\,\left(-2 + 3\,t12 - 3\,t22 - 3\,t12\,\alpha + 4\,t12^2\right. \\
&\qquad \left.\alpha + t22\,\alpha + t12\,t22\,\alpha - 5\,t22^2\,\alpha - 3\,dd1\,\left(2 + \alpha\right) + 3\,dd2\,\left(2 + \alpha\right)\right) - \\
&\quad s^2\,\left(-8 - 10\,\alpha - 16\,dd1\,\alpha + 18\,dd2\,\alpha - 8\,dd1\,\alpha^2 + 9\,dd2\,\alpha^2 - 19\,t22^2\,\alpha^2 + 6\,t22^3\right. \\
&\qquad \left.\alpha^2 + 8\,t12^2\,\left(1 + t22\right)\,\alpha^2 + t22\,\left(14 - 4\,\left(-1 + dd1\right)\,\alpha + \left(7 - 2\,dd1\right)\,\alpha^2\right) + 2\right. \\
&\qquad \left.t12\,\left(1 + \left(5 + t22\right)\,\alpha + \left(-4 + 3\,t22 - 5\,t22^2\right)\,\alpha^2\right)\right) + \\
&\quad s^3\,\left(8 + 8\,\alpha + 4\,dd1\,\alpha - 6\,dd2\,\alpha + 2\,dd1\,\alpha^2 - 3\,dd2\,\alpha^2 - 6\,t22^3\,\alpha^2 + 3\,t22^2\right. \\
&\qquad \left.\left(-4 - 4\,\alpha + 3\,\alpha^2\right) + t12^2\,\left(8 + 8\,\alpha - 8\,t22\,\alpha^2\right) + t22\,\left(-4 - 3\,\alpha^2 + 2\,dd1\,\alpha\,\left(2 + \alpha\right)\right) + 2\right. \\
&\qquad \left.t12\,\left(-6 - 7\,\alpha + \alpha^2 + 5\,t22^2\,\alpha^2 + t22\,\left(2 + \alpha - 2\,\alpha^2\right)\right)\right)\Big) + m^4\,\left(t12 - t22\right)^2 \\
&\quad \Big(2\,s^4\,\Big(6\,t12^4\,t22 + 3\,\left(-1 + t22\right)^2\,t22^2\,\left(1 + t22\right) - t12^3\,\left(1 + 3\,t22 + 18\,t22^2\right) - \\
&\qquad t12\,t22\,\left(4 + t22 - 7\,t22^2 + 12\,t22^3\right) + t12^2\,\left(1 + 5\,t22 - t22^2 + 21\,t22^3\right)\Big)\,\left(1 + \alpha\right) + \\
&\qquad \left(t12 - t22\right)^2\,\alpha\,\left(5 + t12\,\alpha + 4\,t22\,\alpha\right) + s\,\left(-12\,t12^4\,\alpha^2 + 8\,t22^4\,\alpha^2 - 2\right. \\
&\qquad \left.\left(dd1 - dd2\right)\,\alpha\,\left(2 + \alpha\right) + t22^3\,\alpha\,\left(7 + 6\,\alpha\right) + t12^3\,\alpha\,\left(-7 + \left(7 + 31\,t22\right)\,\alpha\right) + \right. \\
&\qquad \left.t22^2\,\left(-14 + 2\,\left(-1 + 7\,dd1 - 6\,dd2\right)\,\alpha + \left(-6 + 7\,dd1 - 6\,dd2\right)\,\alpha^2\right) + 2\right. \\
&\qquad \left.t22\,\left(4 + \left(3 + 2\,dd1 - 2\,dd2\right)\,\alpha + \left(1 + dd1 - dd2\right)\,\alpha^2\right) + t12^2\right. \\
&\qquad \left.\left(2 + \left(10 + 14\,dd1 - 12\,dd2 + 25\,t22\right)\,\alpha + \right.\right. \\
&\qquad \left.\left.\left(4 + 7\,dd1 - 6\,dd2 - 24\,t22 - 6\,t22^2\right)\,\alpha^2\right) - t12\,\left(t22^2\,\left(25 - 11\,\alpha\right)\,\alpha + \right. \\
&\qquad \left.21\,t22^3\,\alpha^2 + 2\,t22\,\left(-6 + 2\,\left(2 + 9\,dd1 - 8\,dd2\right)\,\alpha + \left(-1 + 9\,dd1 - 8\,dd2\right)\,\alpha^2\right) + \right. \\
&\qquad \left.2\,\left(4 + \left(3 - 2\,dd1 + 2\,dd2\right)\,\alpha + \left(1 - dd1 + dd2\right)\,\alpha^2\right)\right) + \\
&\quad s^3\,\left(-6\,t22^5\,\alpha^2 - \left(dd1 - dd2\right)\,\alpha\,\left(2 + \alpha\right) + t22^4\,\left(-6 - 6\,\alpha + 7\,\alpha^2\right) + t22^3\right. \\
&\qquad \left.\left(-22 + 2\,\left(-8 + 5\,dd1\right)\,\alpha + \left(2 + 5\,dd1\right)\,\alpha^2\right) + t22^2\right. \\
&\qquad \left.\left(12 + \left(13 + 2\,dd1 - 6\,dd2\right)\,\alpha + \left(-4 + dd1 - 3\,dd2\right)\,\alpha^2\right) + t22\right. \\
&\qquad \left.\left(4 + \left(3 + 2\,dd1 - 2\,dd2\right)\,\alpha + \left(1 + dd1 - dd2\right)\,\alpha^2\right) - 12\,t12^4\,\left(-1 - \alpha + t22\,\alpha^2\right) + \right. \\
&\qquad \left.t12^3\,\left(-14 - 15\,\alpha + \alpha^2 + 36\,t22^2\,\alpha^2 + t22\,\left(-36 - 41\,\alpha + \alpha^2\right)\right) + t12^2\right. \\
&\qquad \left.\left(12 + \left(11 + 2\,dd1 - 6\,dd2\right)\,\alpha + \left(1 + dd1 - 3\,dd2\right)\,\alpha^2 - 42\,t22^3\,\alpha^2 + \right.\right. \\
&\qquad \left.\left.t22^2\,\left(14 + 24\,\alpha + 11\,\alpha^2\right) + t22\,\left(30 + 10\,\left(4 + dd1\right)\,\alpha + \left(-8 + 5\,dd1\right)\,\alpha^2\right)\right) + \right. \\
&\qquad \left.t12\,\left(-4 + \left(-3 + 2\,dd1 - 2\,dd2\right)\,\alpha + \left(-1 + dd1 - dd2\right)\,\alpha^2 + 24\,t22^4\,\alpha^2 + \right.\right. \\
&\qquad \left.\left.t22^3\,\left(16 + 11\,\alpha - 19\,\alpha^2\right) + t22^2\,\left(6 - \left(9 + 20\,dd1\right)\,\alpha + \left(5 - 10\,dd1\right)\,\alpha^2\right) + \right.\right. \\
&\qquad \left.\left.t22\,\left(-24 - 8\,\left(3 + dd1 - 2\,dd2\right)\,\alpha + \left(3 - 4\,dd1 + 8\,dd2\right)\,\alpha^2\right)\right)\right) + \\
&\quad s^2\,\left(15\,t22^4\,\alpha^2 - 6\,t22^5\,\alpha^2 - 12\,t12^4\,\left(1 + t22\right)\,\alpha^2 - 3\,\left(dd1 - dd2\right)\,\alpha\right. \\
&\qquad \left.\left(2 + \alpha\right) + t22^3\,\left(-26 + \left(-13 + 10\,dd1\right)\,\alpha + \left(4 + 5\,dd1\right)\,\alpha^2\right) + t22^2\right. \\
&\qquad \left.\left(-4 + 2\,\left(2 + 8\,dd1 - 9\,dd2\right)\,\alpha + \left(-10 + 8\,dd1 - 9\,dd2\right)\,\alpha^2\right) + 3\right. \\
&\qquad \left.t22\,\left(4 + \left(3 + 2\,dd1 - 2\,dd2\right)\,\alpha + \left(1 + dd1 - dd2\right)\,\alpha^2\right) + t12^3\right. \\
&\qquad \left.\left(-8 - \left(16 + 5\,t22\right)\,\alpha + \left(7 + 32\,t22 + 36\,t22^2\right)\,\alpha^2\right) + t12^2\right. \\
&\qquad \left.\left(20 + 2\,\left(11 + 8\,dd1 - 9\,dd2\right)\,\alpha + \left(5 + 8\,dd1 - 9\,dd2\right)\,\alpha^2 - 42\,t22^3\,\alpha^2 + \right.\right. \\
&\qquad \left.\left.5\,t22^2\,\alpha\,\left(2 + \alpha\right) + t22\,\left(-10 + 5\,\left(5 + 2\,dd1\right)\,\alpha + \left(-34 + 5\,dd1\right)\,\alpha^2\right)\right) + t12\right. \\
&\qquad \left.\left(24\,t22^4\,\alpha^2 - 5\,t22^3\,\alpha\,\left(1 + 8\,\alpha\right) + t22^2\,\left(44 + \left(4 - 20\,dd1\right)\,\alpha + \left(23 - 10\,dd1\right)\,\alpha^2\right) + \right.\right. \\
&\qquad \left.\left.3\,\left(-4 + \left(-3 + 2\,dd1 - 2\,dd2\right)\,\alpha + \left(-1 + dd1 - dd2\right)\,\alpha^2\right) + \right.\right. \\
&\qquad \left.\left.t22\,\left(-16 + \left(-26 - 44\,dd1 + 48\,dd2\right)\,\alpha + \left(5 - 22\,dd1 + 24\,dd2\right)\,\alpha^2\right)\right)\right)\Big) - \\
&m^3\,\left(t12 - t22\right)^2\,\Big(2\,s^4\,t22\,\Big(4\,t12^4 + \left(-1 + t22\right)^2\,t22\,\left(3 + t22\right) - 3\,t12^3\right. \\
&\qquad \left.\left(1 + 5\,t22\right) + t12^2\,t22\,\left(11 + 13\,t22\right) + t12\,\left(-1 - 3\,t22 + t22^2 - 7\,t22^3\right)\Big)\,\left(1 + \alpha\right) +
\end{aligned}
$$

$$
\begin{aligned}
&2\,\big(2+\alpha+t12^3\,\alpha^2+t22^3\,\alpha^2+t22^2\,\alpha\,(2+\alpha)+t12^2\,\alpha\,(2-t22\,\alpha)-t22\\
&\quad(3+2\,\alpha+\alpha^2)+t12\,(-1+t22\,(-6+\alpha)\,\alpha-3\,t22^2\,\alpha^2)\big)+\\
&s^2\,\big(t22^4\,\alpha^2-2\,t22^5\,\alpha^2-8\,t12^4\,(1+t22)\,\alpha^2-(2+\alpha)\,(-2+5\,dd1\,\alpha-6\,dd2\,\alpha)+\\
&\quad t22^3\,(-16+8\,(-1+dd1)\,\alpha+(15+4\,dd1)\,\alpha^2)+t22\\
&\quad(16+(11+6\,dd1-12\,dd2)\,\alpha+(4+3\,dd1-6\,dd2)\,\alpha^2)+t22^2\\
&\quad(-22+(-11+12\,dd1-6\,dd2)\,\alpha+(-16+6\,dd1-3\,dd2)\,\alpha^2)+2\\
&\quad t12^3\,(-5-(7+2\,t22)\,\alpha+t22\,(16+15\,t22)\,\alpha^2)+t12^2\\
&\quad(14+(13+8\,dd1-6\,dd2)\,\alpha+(11+4\,dd1-3\,dd2)\,\alpha^2-26\,t22^3\,\alpha^2-\\
&\quad\quad 3\,t22^2\,\alpha\,(-4+7\,\alpha)+2\,t22\,(7+(19+4\,dd1)\,\alpha+2\,(-6+dd1)\,\alpha^2))+t12\\
&\quad(-14+(-7+10\,dd1-12\,dd2)\,\alpha+(-5+5\,dd1-6\,dd2)\,\alpha^2+14\,t22^4\,\alpha^2-\\
&\quad\quad 2\,t22^3\,\alpha\,(2+11\,\alpha)+t22^2\,(32-4\,(1+6\,dd1)\,\alpha-3\,(-7+4\,dd1)\,\alpha^2)-\\
&\quad\quad 2\,t22\,(11+(15+16\,dd1-18\,dd2)\,\alpha+(-1+8\,dd1-9\,dd2)\,\alpha^2)))-\\
&2\,s\,\big(4\,t12^4\,\alpha^2-t22^4\,\alpha^2-2\,(dd1-dd2)\,t22\,\alpha\,(2+\alpha)-2\,t22^3\,\alpha\,(1+2\,\alpha)+\\
&\quad 2\,(2+\alpha)\,(-1+dd1\,\alpha-dd2\,\alpha)-t12^3\,\alpha\,(-2+\alpha+16\,t22\,\alpha)+t22^2\\
&\quad(10+(4-4\,dd1+2\,dd2)\,\alpha+(3-2\,dd1+dd2)\,\alpha^2)+t12^2\,(-2-\\
&\quad\quad 2\,(2+2\,dd1-dd2+5\,t22)\,\alpha+(-5-2\,dd1+dd2+14\,t22+7\,t22^2)\,\alpha^2)+t12\\
&\quad(-5\,t22^2\,(-2+\alpha)\,\alpha+6\,t22^3\,\alpha^2+2\,t22\,(-4+(2+8\,dd1-6\,dd2)\,\alpha+(-1+4\,dd1-\\
&\quad\quad 3\,dd2)\,\alpha^2)+2\,(4+(2-2\,dd1+2\,dd2)\,\alpha+(1-dd1+dd2)\,\alpha^2)))-\\
&s^3\,\big(t22^4\,\alpha^2+2\,t22^5\,\alpha^2+(dd1-2\,dd2)\,\alpha\,(2+\alpha)+t22^3\,(16+(12-8\,dd1)\,\alpha-\\
&\quad\quad(9+4\,dd1)\,\alpha^2)+t22^2\,(2+(-1-4\,dd1+2\,dd2)\,\alpha+(8-2\,dd1+dd2)\,\alpha^2)+\\
&\quad t22\,(-10+(-7+2\,dd1+4\,dd2)\,\alpha+(-2+dd1+2\,dd2)\,\alpha^2)+8\,t12^4\\
&\quad(-1-\alpha+t22\,\alpha^2)+t12^3\,(6+40\,t22+6\,\alpha+44\,t22\,\alpha-30\,t22^2\,\alpha^2)+t12^2\\
&\quad(2+(3+2\,dd2)\,\alpha+(-1+dd2)\,\alpha^2+26\,t22^3\,\alpha^2+t22^2\,(-32-44\,\alpha+7\,\alpha^2)-\\
&\quad\quad 2\,t22\,(21+(23+4\,dd1)\,\alpha+(1+2\,dd1)\,\alpha^2))+t12\\
&\quad(-14\,t22^4\,\alpha^2+2\,t22^3\,\alpha\,(2+5\,\alpha)+t22\,(26+(22-12\,dd2)\,\alpha-6\,dd2\,\alpha^2)+t22^2\\
&\quad\quad\alpha\,(16-5\,\alpha+12\,dd1\,(2+\alpha))+\alpha\,(-1+\alpha-dd1\,(2+\alpha)+2\,dd2\,(2+\alpha))))\big)+\\
&m^2\,\big(2\,s\,t12^6\,(1+s\,t22)\,(-\alpha^2-s\,\alpha^2+s^2\,(1+\alpha))-t12^5\,(1+s\,t22)\\
&\quad(-\alpha^2+2\,s^3\,(-1+10\,t22)\,(1+\alpha)+s\,\alpha\\
&\quad\quad(1+(3-20\,t22)\,\alpha)+s^2\,(4+5\,\alpha+(4-20\,t22)\,\alpha^2))+\\
&\quad t12^4\,(1+s\,t22)\,(2\,s^3\,(-6+11\,t22+21\,t22^2)\,(1+\alpha)+\alpha\,(1+\alpha-6\,t22\,\alpha)+\\
&\quad\quad s\,(2+(1+2\,dd1+10\,t22)\,\alpha+(11+dd1-12\,t22-42\,t22^2)\,\alpha^2)+s^2\\
&\quad\quad(-2+2\,(-2+dd1)\,\alpha+(10+dd1)\,\alpha^2-42\,t22^2\,\alpha^2+t22\,(28+38\,\alpha-6\,\alpha^2)))+\\
&\quad t22^2\,\big(4+\alpha+2\,s^4\,(-1+t22)^2\,t22^2\,(1+\alpha)+t22^2\,\alpha\,(1+2\,\alpha)-t22\,(6+3\,\alpha+2\,\alpha^2)+\\
&\quad\quad s^2\,(-2\,t22^4\,\alpha^2-3\,(2+\alpha)\,(-1+dd1\,\alpha-dd2\,\alpha)+t22^3\,(-4+2\,(-1+dd1)\,\alpha+\\
&\quad\quad\quad(7+dd1)\,\alpha^2)+t22^2\,(-14+(-7+8\,dd1)\,\alpha+(-6+4\,dd1)\,\alpha^2)+\\
&\quad\quad\quad t22\,(6+(3-6\,dd2)\,\alpha+(1-3\,dd2)\,\alpha^2))+s\,(10+(4-6\,dd1+4\,dd2)\,\alpha+\\
&\quad\quad\quad(-3\,dd1+2\,dd2)\,\alpha^2+t22^3\,\alpha\,(1+2\,\alpha)+t22^2\,(2+\alpha)\,(-5+dd1\,\alpha)+\\
&\quad\quad\quad t22\,(-6+(-3+6\,dd1-4\,dd2)\,\alpha+(-2+3\,dd1-2\,dd2)\,\alpha^2))+s^3\\
&\quad\quad\quad(-2\,t22^4\,\alpha^2+dd2\,\alpha\,(2+\alpha)+t22^3\,(-2+(-1+2\,dd1)\,\alpha+(5+dd1)\,\alpha^2)+\\
&\quad\quad\quad t22^2\,(-8+(-5+6\,dd1)\,\alpha+(-4+3\,dd1)\,\alpha^2)+\\
&\quad\quad\quad t22\,(8+(5-6\,dd1-2\,dd2)\,\alpha-(-1+3\,dd1+dd2)\,\alpha^2))\big)-t12^3\\
&\quad(2-\alpha+10\,t22\,\alpha+\alpha^2-t22\,\alpha^2-6\,t22^2\,\alpha^2+2\,s^4\,t22\,(-5-3\,t22+24\,t22^2+18\,t22^3)\\
&\quad\quad(1+\alpha)+s^2\,(-6-6\,(2+dd1-dd2)\,\alpha+3\,t22^3\,(6-17\,\alpha)\,\alpha+(4-3\,dd1+3\,dd2)\\
&\quad\quad\quad\alpha^2-36\,t22^4\,\alpha^2+2\,t22^2\,(19+2\,(19+5\,dd1)\,\alpha+(-14+5\,dd1)\,\alpha^2)+\\
&\quad\quad\quad t22\,(28+(23+14\,dd1-12\,dd2)\,\alpha+(29+7\,dd1-6\,dd2)\,\alpha^2))+s\\
&\quad\quad\quad(6+(-3-6\,dd1+4\,dd2)\,\alpha+15\,t22^2\,(2-3\,\alpha)\,\alpha+(5-3\,dd1+2\,dd2)\,\alpha^2-\\
&\quad\quad\quad 32\,t22^3\,\alpha^2+2\,t22\,(2+2\,(4+5\,dd1-2\,dd2)\,\alpha+(11+5\,dd1-2\,dd2)\,\alpha^2))+s^3\\
&\quad\quad\quad(-10+2\,(-5+dd2)\,\alpha+dd2\,\alpha^2-36\,t22^4\,\alpha^2+t22^3\,(72+90\,\alpha-19\,\alpha^2)+\\
&\quad\quad\quad t22^2\,(70+(78+20\,dd1)\,\alpha+(11+10\,dd1)\,\alpha^2)-
\end{aligned}
$$

```
      t22 (12 + (19 + 6 dd1 + 4 dd2) α + (-6 + 3 dd1 + 2 dd2) α^2))) +
  t12^2 (4 - 2 t22 + α - 5 t22 α + 18 t22^2 α - 3 t22^2 α^2 + 2 t22^3 α^2 + 2 s^4
    t22 (-1 - 5 t22 + 5 t22^2 + 14 t22^3 + 8 t22^4) (1 + α) + s^3
    (-2 + 2 (-1 + dd2) α + dd2 α^2 - 16 t22^5 α^2 + t22^4 (28 + 38 α - 9 α^2) +
      2 t22^3 (25 + 18 (2 + dd1) α + (2 + 9 dd1) α^2) +
      t22^2 (16 + (7 - 4 dd1 - 10 dd2) α + (3 - 2 dd1 - 5 dd2) α^2) -
      2 t22 (5 + (7 + 4 dd1 - 2 dd2) α + (-1 + 2 dd1 - dd2) α^2)) + s
    (6 + 4 dd2 α + t22^3 (30 - 29 α) α + 2 dd2 α^2 - 3 dd1 α (2 + α) +
      t22^2 (-16 + 20 (1 + 2 dd1 - dd2) α + (1 + 20 dd1 - 10 dd2) α^2) +
      t22 (18 + (1 - 10 dd1 + 8 dd2) α + (10 - 5 dd1 + 4 dd2) α^2)) + s^2 (t22^4
       (10 - 9 α) α - 16 t22^5 α^2 + t22^3 (-2 + (50 + 36 dd1) α + 9 (-3 + 2 dd1) α^2) +
      t22^2 (36 + (45 + 36 dd1 - 30 dd2) α + (7 + 18 dd1 - 15 dd2) α^2) -
      3 t22 (-4 + (2 + 6 dd1 - 4 dd2) α + (3 dd1 - 2 (2 + dd2)) α^2) -
      3 α (1 + dd1 (2 + α) - dd2 (2 + α)))) -
  t12 t22 (8 - 10 t22 + 2 α - 7 t22 α + 10 t22^2 α - 3 t22 α^2 + t22^2 α^2 + 3 t22^3
    α^2 + 2 s^4 t22 (-1 + t22 + 3 t22^3 + 2 t22^4) (1 + α) + s^2 (6 + 18 dd2 α + 9 dd2 α^2 -
      4 t22^5 α^2 + t22^4 α (1 + α) - 9 dd1 α (2 + α) + t22^3 (-18 + (6 + 20 dd1) α +
         (9 + 10 dd1) α^2) + t22 (24 + (9 - 12 dd2) α + (9 - 6 dd2) α^2) +
      t22^2 (-8 + (11 + 26 dd1 - 12 dd2) α + (13 dd1 - 6 (3 + dd2)) α^2)) +
    s^3 (-2 - 2 (1 + dd1 - 3 dd2) α - (dd1 - 3 dd2) α^2 - 4 t22^5 α^2 + t22^4
       (4 + 5 α - 2 α^2) + t22 (8 + (1 - 10 dd1 - 4 dd2) α + (3 - 5 dd1 - 2 dd2) α^2) +
      t22^2 (8 + (9 + 6 dd1 - 4 dd2) α + (-7 + 3 dd1 - 2 dd2) α^2) +
      t22^3 α (13 + 8 α + 10 dd1 (2 + α))) + s (3 t22^4 α^2 + t22^3 α (11 + 4 α) +
      t22 (6 + (1 + 10 dd1 - 8 dd2) α + (3 + 5 dd1 - 4 dd2) α^2) +
      2 (8 + (2 - 8 dd1 + 6 dd2) α + (-4 dd1 + 3 dd2) α^2) +
      2 t22^2 (-14 - 5 α^2 + 5 dd1 α (2 + α) - 2 dd2 α (2 + α))))) +
m (2 t12 t22 - 2 t22^2 - 2 t12 t22^2 + 2 t22^3 + t12^2 α - t12^3 α - 2 t12 t22 α +
  3 t12^2 t22 α + 2 t12^3 t22 α - t22^2 α + t12 t22^2 α -
  6 t12^2 t22^2 α + t22^3 α + 2 t12 t22^3 α + t12^3 α^2 - t12^4 α^2 -
  2 t12^2 t22 α^2 + t12^3 t22 α^2 + 2 t12^4 t22 α^2 - t12 t22^2 α^2 +
  3 t12^2 t22^2 α^2 - 4 t12^3 t22^2 α^2 + t12 t22^3 α^2 + 2 s^4 t12 t22
    (t12^4 (-1 + 2 t22) - t12^3 (-3 + t22 + 6 t22^2) + t22 (-2 + t22 + t22^2 + t22^3) + 3 t12^2
       (-1 - 2 t22 + 4 t22^2 + t22^3) - t12 (-1 - 7 t22 + 7 t22^2 + 4 t22^3 + t22^4)) (1 + α) +
  s^3 (-(-1 + t22) t22^3 (2 + α) (-1 + dd1 α) - 2 t12^5 (-1 + 2 t22) (-1 - α + t22 α^2) +
    t12^4 (12 t22^3 α^2 + 6 (1 + α) + t22 (2 + 3 α - 4 α^2) - 2 t22^2 (10 + 11 α + α^2)) + t12^3
      (-6 t22^4 α^2 - 6 (1 + α) + t22^3 (24 + 30 α - 14 α^2) - t22 (20 + 2 (11 + dd1) α +
         (-2 + dd1) α^2) + t22^2 (26 + (25 + 4 dd1) α + 2 (6 + dd1) α^2)) + t12^2
      (2 t22^5 α^2 + 2 (1 + α) + t22^4 (-8 - 10 α + 3 α^2) - t22^3 (28 + (37 + 12 dd1) α +
         (-4 + 6 dd1) α^2) + t22 (18 + (19 + 2 dd1 - 2 dd2) α + (dd1 - dd2) α^2) +
      t22^2 (-4 + (3 + 6 dd1 + 2 dd2) α + (-7 + 3 dd1 + dd2) α^2)) + t12
    t22 (-4 + 2 (-2 + dd2) α + dd2 α^2 - 2 t22^4 α^2 + t22^3 (2 + α)
       (3 + (3 + 2 dd1) α) + t22^2 (6 + (8 + 6 dd1) α + (-2 + 3 dd1) α^2) -
      t22 (2 + 2 (3 + 4 dd1 + dd2) α + (-1 + 4 dd1 + dd2) α^2))) +
  s (2 t12^5 (1 - 2 t22) α^2 + (-1 + t22) t22^2 (2 + α) (2 + t22 - dd1 α) + t12^4 α
    (1 - 5 α + 14 t22^2 α - t22 (2 + α)) + t12^3 (-2 - (5 + 2 dd1) α + t22^2 (8 - 17 α) α -
       (-3 + dd1) α^2 - 10 t22^3 α^2 + 2 t22 (2 + 2 (1 + dd1) α + (7 + dd1) α^2)) + t12
    t22 (2 + (-6 - 8 dd1 + 4 dd2) α + (-4 dd1 + 2 dd2) α^2 + t22^3 α (2 + α) +
      2 t22^2 (-2 + 2 (1 + dd1) α + (-1 + dd1) α^2) + t22 (4 + (5 + 6 dd1 - 4 dd2) α +
         (1 + 3 dd1 - 2 dd2) α^2)) + t12^2 (2 + 2 (2 + dd1) α + dd1 α^2 + t22^3 α
       (-8 + 11 α) + t22 (-4 + (7 + 6 dd1 - 4 dd2) α + (-8 + 3 dd1 - 2 dd2) α^2) +
```

$$
\begin{aligned}
&\mathrm{t22}^2 \left(-2-2 \left(7+6\,\mathrm{dd1}-2\,\mathrm{dd2}\right) \alpha+\left(1-6\,\mathrm{dd1}+2\,\mathrm{dd2}\right) \alpha^2\right)\Big)- \\
&\mathrm{s}^2 \Big(2\,\mathrm{t12}^5 \left(-1+\mathrm{t22}+2\,\mathrm{t22}^2\right) \alpha^2+\left(-1+\mathrm{t22}\right) \mathrm{t22}^2 \left(2+\alpha\right) \left(-1+\mathrm{dd1}\,\alpha+ \\
&\mathrm{t22} \left(-2+\mathrm{dd1}\,\alpha\right)\right)+\mathrm{t12}^4 \left(-4-5\,\alpha+2\,\mathrm{t22}^2 \left(1-6\,\alpha\right) \alpha+4\,\alpha^2-12\,\mathrm{t22}^3\,\alpha^2+ \\
&\mathrm{t22} \left(8+9\,\alpha+7\,\alpha^2\right)\right)+\mathrm{t12}^2 \left(-4-\left(5+2\,\mathrm{dd1}\right) \alpha+\mathrm{t22}^4 \left(2-3\,\alpha\right) \alpha- \\
&\mathrm{dd1}\,\alpha^2-2\,\mathrm{t22}^5\,\alpha^2+\mathrm{t22}^3 \left(8+\left(25+12\,\mathrm{dd1}\right) \alpha+3 \left(-5+2\,\mathrm{dd1}\right) \alpha^2\right)+ \\
&\mathrm{t22}^2 \left(24+\left(23+6\,\mathrm{dd1}-6\,\mathrm{dd2}\right) \alpha+3 \left(3+\mathrm{dd1}-\mathrm{dd2}\right) \alpha^2\right)+ \\
&\mathrm{t22} \left(-12+\left(-21-8\,\mathrm{dd1}+6\,\mathrm{dd2}\right) \alpha+\left(6-4\,\mathrm{dd1}+3\,\mathrm{dd2}\right) \alpha^2\right)\Big)+\mathrm{t12}^3 \\
&\left(8+2 \left(5+\mathrm{dd1}\right) \alpha+\left(-2+\mathrm{dd1}\right) \alpha^2+6\,\mathrm{t22}^4\,\alpha^2+6\,\mathrm{t22}^3\,\alpha \left(-1+4\,\alpha\right)+\mathrm{t22}^2\right. \\
&\left.\left(-22-\left(29+4\,\mathrm{dd1}\right) \alpha+\left(1-2\,\mathrm{dd1}\right) \alpha^2\right)-\mathrm{t22}\,\alpha \left(-4+15\,\alpha+\mathrm{dd1} \left(2+\alpha\right)\right)\right)+ \\
&\mathrm{t12}\,\mathrm{t22} \left(4+8\,\alpha+8\,\mathrm{dd1}\,\alpha-6\,\mathrm{dd2}\,\alpha+4\,\mathrm{dd1}\,\alpha^2-3\,\mathrm{dd2}\,\alpha^2+2\,\mathrm{t22}^4\,\alpha^2-\right. \\
&\mathrm{t22}^2 \left(6+2 \left(7+5\,\mathrm{dd1}\right) \alpha+5 \left(-1+\mathrm{dd1}\right) \alpha^2\right)-\mathrm{t22}^3 \left(-2+\left(3+4\,\mathrm{dd1}\right) \alpha+ \\
&\left.\left.\left.\left.2 \left(2+\mathrm{dd1}\right) \alpha^2\right)+\mathrm{t22} \left(\mathrm{dd1}\,\alpha \left(2+\alpha\right)+3\,\mathrm{dd2}\,\alpha \left(2+\alpha\right)-3 \left(2+\alpha^2\right)\right)\right)\right)\right)\Big)\Big/ \\
&\Big(\left(1+\mathrm{s}\right) \left(\mathrm{m}^2\,\mathrm{s} \left(2+\mathrm{s}\right) \left(\mathrm{t12}-\mathrm{t22}\right)+\left(-1+\mathrm{s} \left(-1+\mathrm{t12}\right)\right) \left(1+\mathrm{s}\,\mathrm{t22}\right)- \\
&2\,\mathrm{m} \left(-1+\mathrm{s} \left(-1+\mathrm{t12}\right)\right) \left(1+\mathrm{s}\,\mathrm{t22}\right)\Big) \\
&\Big(-2\,\mathrm{m}^3 \left(\mathrm{t12}-\mathrm{t22}\right)^4+\mathrm{m}^4 \left(\mathrm{t12}-\mathrm{t22}\right)^4+\left(-1+\mathrm{t12}\right) \mathrm{t12} \left(-1+\mathrm{t22}\right) \mathrm{t22}+ \\
&\mathrm{m}^2 \left(\mathrm{t12}-\mathrm{t22}\right)^2 \left(-1+\mathrm{t12}+\mathrm{t12}^2+\mathrm{t22}-4\,\mathrm{t12}\,\mathrm{t22}+\mathrm{t22}^2\right)+ \\
&\mathrm{m} \left(\mathrm{t12}-\mathrm{t22}\right)^2 \left(1-\mathrm{t22}+\mathrm{t12} \left(-1+2\,\mathrm{t22}\right)\right)\Big) \alpha \left(2+\alpha\right)\Big)\Big), \\
&\mathrm{p22} \to \Big(\left(-1+\mathrm{t22}\right) \mathrm{t22} \left(2\,\mathrm{s}^3\,\mathrm{t22}^2 \left(1+\alpha\right)-\alpha \left(1+\mathrm{t22}\,\alpha\right)+ \\
&\mathrm{s} \left(2+\left(1+2\,\mathrm{dd2}-\mathrm{t22}\right) \alpha+\left(\mathrm{dd2}-2\,\mathrm{t22}^2\right) \alpha^2\right)+ \\
&\mathrm{s}^2 \left(-2\,\mathrm{t22}^2\,\alpha^2+\mathrm{dd2}\,\alpha \left(2+\alpha\right)+\mathrm{t22} \left(4+3\,\alpha+\alpha^2\right)\right)\Big)+ \\
&\mathrm{m}^3 \left(\mathrm{t12}-\mathrm{t22}\right)^2 \left(\left(\mathrm{t12}-\mathrm{t22}\right) \alpha^2+2\,\mathrm{s}^3 \left(\mathrm{t12}-\mathrm{t22}\right) \left(-1+\mathrm{t22}\right) \left(1+\alpha\right)+ \\
&\mathrm{s}\,\alpha \left(\mathrm{dd1} \left(2+\alpha\right)-\mathrm{dd2} \left(2+\alpha\right)-\left(\mathrm{t12}-\mathrm{t22}\right) \left(1+2 \left(-1+\mathrm{t22}\right) \alpha\right)\right)+ \\
&\mathrm{s}^2 \left(2\,\mathrm{t22}^2\,\alpha^2+\left(\mathrm{dd1}-\mathrm{dd2}\right) \alpha \left(2+\alpha\right)+ \\
&\mathrm{t22} \left(2+3\,\alpha-\alpha^2\right)+\mathrm{t12} \left(-2-3\,\alpha+\left(1-2\,\mathrm{t22}\right) \alpha^2\right)\right)+ \\
&\mathrm{m}^2 \left(\mathrm{t12}-\mathrm{t22}\right) \left(6\,\mathrm{s}^3 \left(\mathrm{t12}-\mathrm{t22}\right) \left(-1+\mathrm{t22}\right) \mathrm{t22} \left(1+\alpha\right)+\right. \\
&\left(\mathrm{t12}-\mathrm{t22}\right) \alpha \left(-1+\left(-1+\mathrm{t22}\right) \alpha\right)+ \\
&\mathrm{s}^2 \left(\mathrm{t22}^2 \left(3-5\,\alpha\right) \alpha+6\,\mathrm{t22}^3\,\alpha^2-\left(\mathrm{dd1}-\mathrm{dd2}\right) \alpha \left(2+\alpha\right)+\right. \\
&\mathrm{t22} \left(4+\left(3+4\,\mathrm{dd1}-6\,\mathrm{dd2}\right) \alpha+\left(1+2\,\mathrm{dd1}-3\,\mathrm{dd2}\right) \alpha^2\right)+ \\
&\left.\mathrm{t12} \left(-4+\left(-3+2\,\mathrm{dd2}-3\,\mathrm{t22}\right) \alpha+\left(-1+\mathrm{dd2}+5\,\mathrm{t22}-6\,\mathrm{t22}^2\right) \alpha^2\right)\right)+ \\
&\mathrm{s} \left(3\,\mathrm{t22}^2 \left(1-2\,\alpha\right) \alpha+6\,\mathrm{t22}^3\,\alpha^2-\left(\mathrm{dd1}-\mathrm{dd2}\right) \alpha \left(2+\alpha\right)+\right. \\
&\mathrm{t22} \left(4+\left(4+4\,\mathrm{dd1}-6\,\mathrm{dd2}\right) \alpha+\left(2+2\,\mathrm{dd1}-3\,\mathrm{dd2}\right) \alpha^2\right)+ \\
&\left.\left.\mathrm{t12} \left(-4+\left(-4+2\,\mathrm{dd2}-3\,\mathrm{t22}\right) \alpha+\left(-2+\mathrm{dd2}+6\,\mathrm{t22}-6\,\mathrm{t22}^2\right) \alpha^2\right)\right)+ \\
&\Big(\mathrm{m} \left(1+\mathrm{s}\,\mathrm{t12}\right) \Big(-\left(-1+\mathrm{m}\right) \left(-1+\mathrm{m} \left(\mathrm{t12}-\mathrm{t22}\right)+\mathrm{t22}\right) \left(\mathrm{m} \left(\mathrm{t12}-\mathrm{t22}\right)+\mathrm{t22}\right) \\
&\left(1+\mathrm{s}\,\mathrm{t22}\right) \Big(2 \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right)+4\,\mathrm{s} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \\
&\left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right)+2\,\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha-2\,\mathrm{m}\,\mathrm{t12} \\
&\left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha+\left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \\
&\alpha+2\,\mathrm{s} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha+2 \\
&\mathrm{dd1} \left(-1+\mathrm{m}\right) \mathrm{s} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \\
&\alpha-2\,\mathrm{dd2}\,\mathrm{m}\,\mathrm{s} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \\
&\alpha+\mathrm{s}\,\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha-\mathrm{m}\,\mathrm{s} \\
&\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha-2\,\mathrm{m}\,\mathrm{t22} \\
&\left(\mathrm{t12}-\mathrm{m}\,\mathrm{t12}+\mathrm{m}\,\mathrm{t22}\right) \alpha+\mathrm{m}\,\mathrm{s}\,\mathrm{t22} \left(-1+\mathrm{t12}-\mathrm{m}\,\mathrm{t12}+\mathrm{m}\,\mathrm{t22}\right) \left(\mathrm{t12}-\mathrm{m}\,\mathrm{t12}+\mathrm{m}\,\mathrm{t22}\right) \\
&\alpha+\mathrm{dd1} \left(-1+\mathrm{m}\right) \mathrm{s} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \\
&\alpha^2-\mathrm{dd2}\,\mathrm{m}\,\mathrm{s} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \\
&\alpha^2+\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha^2-\mathrm{m} \\
&\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha^2+\mathrm{s} \\
&\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha^2-\mathrm{m} \\
&\mathrm{s}\,\mathrm{t12} \left(\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \left(1+\left(-1+\mathrm{m}\right) \mathrm{t12}-\mathrm{m}\,\mathrm{t22}\right) \alpha^2+\mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
&t22\,(-1+t12-m\,t12+m\,t22)\,(t12-m\,t12+m\,t22)\,\alpha^2+m\,s\,t22\\
&(-1+t12-m\,t12+m\,t22)\,(t12-m\,t12+m\,t22)\,\alpha^2+s^2\,((-1+m)\,t12-m\,t22)\\
&(1+(-1+m)\,t12-m\,t22)\,(2+\alpha)+(-1+m)\,s^2\,t12\,((-1+m)\,t12-m\,t22)\\
&(1+(-1+m)\,t12-m\,t22)\,(2+\alpha)-m\,s^2\,t22\,(-1+t12-m\,t12+m\,t22)\\
&(t12-m\,t12+m\,t22)\,(2+\alpha)-dd1\,s^2\,((-1+m)\,t12-m\,t22)\\
&(1+(-1+m)\,t12-m\,t22)\,\alpha\,(2+\alpha)+dd1\,m\,s^2\,((-1+m)\,t12-m\,t22)\\
&(1+(-1+m)\,t12-m\,t22)\,\alpha\,(2+\alpha)-dd2\,m\,s^2\,((-1+m)\,t12-m\,t22)\\
&(1+(-1+m)\,t12-m\,t22)\,\alpha\,(2+\alpha)+2\,t12\,(1+s\,(1+(-1+m)\,t12-m\,t22))\\
&(1+(1+(-1+m)\,t12-m\,t22)\,\alpha+s\,(1+(-1+m)\,t12-m\,t22)\,(1+\alpha))\\
&(1+s\,(1+(-1+m)\,t12-m\,t22)+m\,t22\,\alpha+t12\,(\alpha-m\,\alpha)))+\\
&m\,(-1+s\,(-1+t22))\,((-1+m)\,t12-m\,t22)\,(-1+t12-m\,t12+m\,t22)\\
&(2\,(1+s)\,(m\,(t12-t22)+t22)+4\,m\,s\,(1+s)\,t12\,(m\,(t12-t22)+t22)+4\,s\\
&(1+s)\,t22\,(m\,(t12-t22)+t22)-4\,m\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)+\\
&(1+s)\,(m\,(t12-t22)+t22)\,\alpha+3\,m\,s\,(1+s)\,t12\,(m\,(t12-t22)+t22)\\
&\alpha-2\,dd2\,m\,s\,(1+s)\,t12\,(m\,(t12-t22)+t22)\,\alpha+m^2\,s\,(1+s)\,t12\,(t12-t22)\\
&(m\,(t12-t22)+t22)\,\alpha+(1+s)\,t22\,(m\,(t12-t22)+t22)\,\alpha+3\,s\,(1+s)\\
&t22\,(m\,(t12-t22)+t22)\,\alpha-2\,dd2\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)\\
&\alpha-3\,m\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)\,\alpha-2\,dd1\,m\,s\,(1+s)\,t22\\
&(m\,(t12-t22)+t22)\,\alpha+4\,dd2\,m\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)\,\alpha+2\\
&m\,s\,(1+s)\,t12\,t22\,(m\,(t12-t22)+t22)\,\alpha-m^2\,s\,(1+s)\,(t12-t22)\,t22\\
&(m\,(t12-t22)+t22)\,\alpha+s\,(1+s)\,t22^2\,(m\,(t12-t22)+t22)\,\alpha-2\,m\,s\\
&(1+s)\,t22^2\,(m\,(t12-t22)+t22)\,\alpha+m\,s\,(1+s)\,t12\,(m\,(t12-t22)+t22)\\
&\alpha^2-dd2\,m\,s\,(1+s)\,t12\,(m\,(t12-t22)+t22)\,\alpha^2-m^2\,s\,(1+s)\,t12\\
&(t12-t22)\,(m\,(t12-t22)+t22)\,\alpha^2-m^2\,(1+s)\,(t12-t22)^2\\
&(m\,(t12-t22)+t22)\,\alpha^2+(1+s)\,t22\,(m\,(t12-t22)+t22)\,\alpha^2+s\,(1+s)\\
&t22\,(m\,(t12-t22)+t22)\,\alpha^2-dd2\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)\\
&\alpha^2-m\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)\,\alpha^2-dd1\,m\,s\,(1+s)\,t22\\
&(m\,(t12-t22)+t22)\,\alpha^2+2\,dd2\,m\,s\,(1+s)\,t22\,(m\,(t12-t22)+t22)\,\alpha^2-2\\
&m\,s\,(1+s)\,t12\,t22\,(m\,(t12-t22)+t22)\,\alpha^2+m^2\,s\,(1+s)\,(t12-t22)\,t22\\
&(m\,(t12-t22)+t22)\,\alpha^2-(1+s)\,t22^2\,(m\,(t12-t22)+t22)\,\alpha^2-s\,(1+s)\\
&t22^2\,(m\,(t12-t22)+t22)\,\alpha^2+2\,m\,s\,(1+s)\,t22^2\,(m\,(t12-t22)+t22)\\
&\alpha^2+2\,m^2\,s^2\,(1+s)\,(t12-t22)^2\,(m\,(t12-t22)+t22)\,(1+\alpha)+4\,m\,s^2\\
&(1+s)\,(t12-t22)\,t22\,(m\,(t12-t22)+t22)\,(1+\alpha)+2\,s^2\,(1+s)\,t22^2\\
&(m\,(t12-t22)+t22)\,(1+\alpha)+dd2\,s\,(1+s)\,(m\,(t12-t22)+t22)\,\alpha\\
&(2+\alpha)+(dd1-dd2)\,m\,s\,(1+s)\,(m\,(t12-t22)+t22)\,\alpha\,(2+\alpha)-dd1\\
&m^2\,s\,(1+s)\,(t12-t22)\,(m\,(t12-t22)+t22)\,\alpha\,(2+\alpha)+dd2\,m^2\\
&s\,(1+s)\,(t12-t22)\,(m\,(t12-t22)+t22)\,\alpha\,(2+\alpha)+m\,(1+s)\\
&(t12-t22)\,(m\,(t12-t22)+t22)\,\alpha\,(1+\alpha-2\,t22\,\alpha)-2\,t22\\
&(1+m\,s\,(t12-t22)+s\,t22)\,(1+s\,t22+m\,(t12-t22)\,(s-\alpha)+\alpha-t22\,\alpha)\\
&(1+t22\,\alpha+s\,t22\,(1+\alpha)+m\,(t12-t22)\,(s+\alpha+s\,\alpha))))\Big)\Big/\\
&\Big(\big((-1+m)^2\,t12^2+m\,t22\,(-1+m\,t22)-(-1+m)\,t12\,(-1+2\,m\,t22)\big)\\
&(m^2\,s\,(2+s)\,(t12-t22)+(-1+s\,(-1+t12))\,(1+s\,t22)-\\
&2\,m\,(-1+s\,(-1+t12))\,(1+s\,t22))\Big)+\\
&m\,(t22\,(2+\alpha+2\,t22\,\alpha+t22^2\,\alpha^2-6\,s^3\,(-1+t22)\,t22^2\,(1+\alpha)+\\
&s\,(3\,t22^2\,(1-2\,\alpha)\,\alpha+6\,t22^3\,\alpha^2-(2+\alpha)\,(-1+dd1\,\alpha-2\,dd2\,\alpha)+\\
&t22\,(2+2\,(1+dd1-3\,dd2)\,\alpha+(2+dd1-3\,dd2)\,\alpha^2))+\\
&s^2\,(6\,t22^3\,\alpha^2-(dd1-2\,dd2)\,\alpha\,(2+\alpha)-t22^2\,(6+3\,\alpha+7\,\alpha^2)+\\
&t22\,(8+2\,(3+dd1-3\,dd2)\,\alpha+(2+dd1-3\,dd2)\,\alpha^2)))+\\
&t12\,(-2-\alpha-2\,t22\,\alpha-t22^2\,\alpha^2+6\,s^3\,(-1+t22)\,t22^2\,(1+\alpha)-
\end{aligned}
$$

```
        s^2 (6 t22^3 α^2 + dd2 α (2 + α) - t22^2 (6 + 3 α + 7 α^2) +
            t22 (8 + (6 - 4 dd2) α - 2 (-1 + dd2) α^2)) - s (3 t22^2 (1 - 2 α) α +
            6 t22^3 α^2 + (2 + α) (1 + dd2 α) + 2 t22 (1 + α - 2 dd2 α + α^2 - dd2 α^2)))))) /
    ((-1 + m) (1 + s) (1 + s t22) (m^2 (t12 - t22)^2 + (-1 + t22) t22 +
        m (t12 - t22) (-1 + 2 t22)) α (2 +
        α))};
```

```
del1 = Factor[deltagensym[[1]] /. p12p22gentdd];
```

```
Exponent[Numerator[del1], {t12, t22, dd1, dd2}]
```

```
{9, 9, 1, 1}
```

```
del4 = Factor[deltagensym[[4]] /. p12p22gentdd];
```

```
Exponent[Numerator[del4], {t12, t22, dd1, dd2}]
```

```
{9, 9, 1, 1}
```

Therefore, we can indeed solve {deltagensym[[1]], deltagensym[[4]]} = 0 for (dd1, dd2):

```
Simplify[{dd1, dd2} /. Solve[{del1, del4} == 0, {dd1, dd2}]]
```

The following is (dd1, dd2) as a function of (t12, t22); this is independent of r:

```
dd12gent12 =
  {dd1 → -(((-1 + t12) t12 (m^6 s (t12 - t22)^6 (-2 α^2 + s (-3 - 2 t12 + 2 t22) α^2 + 2 s^3 (1 + t12 -
                  t22) (1 + α) + s^2 (4 + 4 α + (-1 - 2 t12 + 2 t22) α^2)) - m^5 (t12 - t22)^5
            (2 α^2 + 2 s (2 - t12 + t22) α^2 + 2 s^4 (-2 + t12 + 3 t12^2 - t22 - 6 t12 t22 + 3 t22^2)
              (1 + α) - 3 s^2 (4 + 4 α + (-1 + t12 + 2 t12^2 - t22 - 4 t12 t22 + 2 t22^2) α^2) +
            s^3 (-12 - 12 α + α^2 - 6 t12^2 α^2 - 6 t22^2 α^2 + t22 (-4 - 4 α + α^2) +
              t12 (4 + 4 α + (-1 + 12 t22) α^2))) +
          m^4 (t12 - t22)^4 (3 (t12 - t22) α^2 + 2 s^4 (1 + 3 t12^3 + 5 t22 - 12 t12^2 t22 -
              3 t22^2 - 3 t22^3 + 2 t12 (-2 + t22 + 6 t22^2)) (1 + α) +
            s (12 + 12 α + (2 + 4 t12^2 + t12 (3 - 2 t22) - 9 t22 + 4 t22^2) α^2) -
            3 s^2 (-6 - 6 α - α^2 + 2 t12^3 α^2 - 3 t22^2 α^2 - 2 t22^3 α^2 - t12^2 (1 + 8 t22) α^2 +
              t22 (-6 - 6 α + 4 α^2) + t12 (6 + 6 α + (-1 + t22 + 8 t22^2) α^2)) +
            s^3 (10 + 10 α + α^2 - 6 t12^3 α^2 + 6 t22^3 α^2 + t22 (20 + 20 α - 6 α^2) +
              t22^2 (-6 - 6 α + 5 α^2) + t12^2 (-6 - 6 α + (-1 + 24 t22) α^2) -
              t12 (16 + 16 α - 3 α^2 + 24 t22^2 α^2 + t22 (-8 - 8 α + α^2))))) -
        (-1 + t12) t12 (-1 + t22) t22 ((t12 - t22) α^2 + 2 s^4 (-1 + t12) (-1 + t12 - t22)
            t22 (1 + α) + s (4 + 4 α - (-3 t12 + 2 t12^2 + t22 - 2 t12 t22 + 2 t22^2) α^2) +
          s^2 (-4 t22^2 α^2 - 2 t12^2 (1 + t22) α^2 + 6 (1 + α) + t22 (6 + 6 α + α^2) +
            t12 (-6 - 6 α + (2 + 3 t22 + 2 t22^2) α^2)) +
          s^3 (2 (1 + α) + t22^2 (2 + 2 α - 2 α^2) + t22 (8 + 8 α + α^2) + t12^2
              (2 + 2 α - 2 t22 α^2) + t12 (2 t22^2 α^2 - 4 (1 + α) + t22 (-8 - 8 α + α^2)))) -
        m^3 (t12 - t22)^3 (2 s^4 (t12^4 + t12^3 (1 - 10 t22) + t12^2 (-5 + 5 t22 + 18 t22^2) +
            t12 (3 + 8 t22 - 11 t22^2 - 10 t22^3) + t22 (-3 - 3 t22 + 5 t22^2 + t22^3))
          (1 + α) - 2 (2 + 2 α + (1 - t22 + t12 (-1 + 2 t22)) α^2) +
          2 s (3 t12^3 α^2 + t22^2 α^2 - 3 t22^3 α^2 - t12^2 (1 + 7 t22) α^2 + 2 t22 (-4 - 4 α + α^2) -
            2 (2 + 2 α + α^2) + t12 (8 + 8 α + (2 - 4 t22 + 7 t22^2) α^2)) -
          s^2 (2 t12^4 α^2 + 15 t22^3 α^2 + 2 t22^4 α^2 - t12^3 (3 + 20 t22) α^2 +
            t22^2 (6 + 6 α - 9 α^2) - 3 t22 (-8 - 8 α + α^2) +
            3 (2 + 2 α + α^2) + 3 t12^2 (2 + 2 α + (-1 + 5 t22 + 12 t22^2) α^2) -
```

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t12 (27 t22^2 α^2 + 20 t22^3 α^2 + 3 (8 + 8 α + α^2) - 6 t22 (-2 - 2 α + 3 α^2))) -
s^3 (2 + 2 α + α^2 + 2 t12^4 α^2 + 2 t22^4 α^2 + t22^2 (2 + 2 α - 7 α^2) + t22 (20 + 20 α -
α^2) + t22^3 (-8 - 8 α + 9 α^2) + t12^3 (8 + 8 α + (3 - 20 t22) α^2) + t12^2
(36 t22^2 α^2 + t22 (-32 - 32 α + α^2) - 5 (-2 - 2 α + α^2)) - t12 (20 + 20 α +
α^2 + 20 t22^3 α^2 - 2 t22 (-6 - 6 α + 7 α^2) + t22^2 (-32 - 32 α + 13 α^2)))) -
m (t12 - t22) (2 s t12^4 (-1 - 2 (-1 + s) t22 + 3 s t22^2) (-α^2 - s α^2 + s^2 (1 + α)) +
(1 + s) (-1 + t22) t22 (-2 - 2 α - t22 α^2 + 2 s^3 t22^2 (1 + α) +
s^2 t22 (2 + 2 α + (1 - 2 t22) α^2) - 2 s t22 (1 + α + t22 α^2)) +
t12^3 ((-1 + 2 t22) α^2 - 6 s^4 t22 (-2 + t22 + 2 t22^2) (1 + α) +
s (2 + 2 α + (-5 + 2 t22 + 10 t22^2) α^2) +
s^3 (-3 + 4 t22) (3 t22^2 α^2 - 2 (1 + α) + 2 t22 (-3 - 3 α + α^2)) +
s^2 (6 + 6 α - 4 α^2 + 9 t22^2 α^2 + 12 t22^3 α^2 - 6 t22 (1 + α + α^2))) +
t12 (2 + 2 α + t22^2 (-1 + 2 t22) α^2 - 2 s^4 t22 (-2 - 3 t22 + 2 t22^2 + 4 t22^3)
(1 + α) + s (6 t22^3 α^2 + 4 t22^4 α^2 + 6 (1 + α) - t22^2 (2 + 2 α + 7 α^2)) +
s^3 (2 (1 + α) + t22^2 (2 + 2 α - 5 α^2) + 2 t22 (9 + 9 α + α^2) - 2 t22^3
(11 + 11 α + 2 α^2) + t22^4 (-4 - 4 α + 8 α^2)) + s^2 (12 t22^4 α^2 + 6 (1 + α) -
6 t22^3 (1 + α) + 2 t22 (9 + 9 α + α^2) - t22^2 (18 + 18 α + 11 α^2))) +
t12^2 (-2 - 2 α - (-1 + t22 + 2 t22^2) α^2 + 6 s^4 t22 (-2 - t22 + 3 t22^2 + t22^3)
(1 + α) + s (-8 - 8 α + 3 α^2 - 4 t22^2 α^2 - 10 t22^3 α^2 + t22 (2 + 2 α + 3 α^2)) +
s^2 (-21 t22^3 α^2 - 6 t22^4 α^2 + 2 (-6 - 6 α + α^2) + 4 t22 (-3 - 3 α + α^2) +
t22^2 (24 + 24 α + 11 α^2)) + s^3 (-6 t22^4 α^2 - 6 (1 + α) - 32 t22 (1 + α) +
t22^3 (24 + 24 α - 11 α^2) + t22^2 (24 + 24 α + 13 α^2)))) -
m^2 (t12 - t22)^2 (2 s t12^4 (1 + s (-1 + 3 t22)) (-α^2 - s α^2 + s^2 (1 + α)) +
t22 (-4 - 4 α + (-1 + t22 - t22^2) α^2 + 2 s^4 t22 (-3 + t22 + 2 t22^2) (1 + α) -
s^2 (6 + 6 α + 4 α^2 + 4 t22^2 α^2 + 6 t22^3 α^2 + t22 (6 + 6 α - 11 α^2)) -
s (4 + 4 α + 3 α^2 + 5 t22^2 α^2 + 2 t22^3 α^2 + t22 (8 + 8 α - 7 α^2)) + s^3 (8 t22^2 (1 +
α) + t22^3 (2 + 2 α - 4 α^2) + 5 t22 (-2 - 2 α + α^2) - 2 (2 + 2 α + α^2))) +
t12^2 (-(1 + t22) α^2 + 6 s^4 (-1 - 3 t22 + 4 t22^2 + 4 t22^3) (1 + α) -
s (8 + 8 α + (-3 + 5 t22 + 18 t22^2) α^2) +
s^3 (14 t22 α^2 - 24 t22^3 α^2 - 22 (1 + α) + t22^2 (48 + 48 α - 9 α^2)) +
s^2 (-27 t22^2 α^2 - 24 t22^3 α^2 + 2 (-9 - 9 α + 2 α^2) + 2 t22 (6 + 6 α + 5 α^2))) +
t12^3 (α^2 + s (-1 + 14 t22) α^2 + s^2 (-5 + 9 t22 + 24 t22^2) α^2 - 6 s^4 (-1 + 4 t22^2)
(1 + α) + s^3 (8 + 8 α - 3 α^2 + 24 t22^2 α^2 - t22 (24 + 24 α + 5 α^2))) +
t12 (4 + 4 α + (1 + t22^2) α^2 - 2 s^4 (-1 - 6 t22 - 3 t22^2 + 12 t22^3 + 3 t22^4)
(1 + α) + s (12 + 12 α + α^2 - t22^2 α^2 + 14 t22^3 α^2 + t22 (8 + 8 α - 2 α^2)) + s^3
(12 + 12 α + α^2 + 6 t22^4 α^2 + t22 (28 + 28 α - α^2) - t22^2 (24 + 24 α + 17 α^2) +
t22^3 (-24 - 24 α + 19 α^2)) + s^2 (18 + 18 α + α^2 + 33 t22^3 α^2 +
6 t22^4 α^2 - 3 t22 (-4 - 4 α + α^2) - t22^2 (12 + 12 α + 19 α^2)))))) /
((1 + s) (-1 + m (t12 - t22) + t22) (m (t12 - t22) + t22) ((-1 + m) t12 - m t22)
(1 + (-1 + m) t12 - m t22)
((-1 + m) m s (2 + s) t12^2 +
t12 (-1 - 2 s t22 - s^2 t22 - 2 m^2 s (2 + s) t22 + m (2 + 2 s + s^2 + 4 s t22 + 2 s^2 t22)) +
t22 ((1 + s)^2 + m^2 s (2 + s) t22 - m (2 + 2 s (1 + t22) + s^2 (1 + t22)))) α (2 + α)),
dd2 → ((-1 + t22) t22 (-m^6 s (t12 - t22)^6 (-2 α^2 + s (-3 - 2 t12 + 2 t22) α^2 +
2 s^3 (1 + t12 - t22) (1 + α) + s^2 (4 + 4 α + (-1 - 2 t12 + 2 t22) α^2)) +
m^5 (t12 - t22)^5 (2 α^2 + 2 s (2 - t12 + t22) α^2 +
2 s^4 (-2 + t12 + 3 t12^2 - t22 - 6 t12 t22 + 3 t22^2) (1 + α) -
3 s^2 (4 + 4 α + (-1 + t12 + 2 t12^2 - t22 - 4 t12 t22 + 2 t22^2) α^2) +
s^3 (-12 - 12 α + α^2 - 6 t12^2 α^2 - 6 t22^2 α^2 + t22
```

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        (-4 - 4 α + α^2) + t12 (4 + 4 α + (-1 + 12 t22) α^2))) -
m^4 (t12 - t22)^4 (3 (t12 - t22) α^2 + 2 s^4 (1 + 3 t12^3 + 5 t22 - 12 t12^2
        t22 - 3 t22^2 - 3 t22^3 + 2 t12 (-2 + t22 + 6 t22^2)) (1 + α) +
    s (12 + 12 α + (2 + 4 t12^2 + t12 (3 - 2 t22) - 9 t22 + 4 t22^2) α^2) -
    3 s^2 (-6 - 6 α - α^2 + 2 t12^3 α^2 - 3 t22^2 α^2 - 2 t22^3 α^2 - t12^2 (1 + 8 t22)
        α^2 + t22 (-6 - 6 α + 4 α^2) + t12 (6 + 6 α + (-1 + t22 + 8 t22^2) α^2)) +
    s^3 (10 + 10 α + α^2 - 6 t12^3 α^2 + 6 t22^3 α^2 + t22 (20 + 20 α - 6 α^2) + t22^2
        (-6 - 6 α + 5 α^2) + t12^2 (-6 - 6 α + (-1 + 24 t22) α^2) - t12
        (16 + 16 α - 3 α^2 + 24 t22^2 α^2 + t22 (-8 - 8 α + α^2)))) + (-1 + t12) t12
  (-1 + t22) t22 ((t12 - t22) α^2 + 2 s^4 (-1 + t12) (-1 + t12 - t22) t22 (1 + α) +
    s (4 + 4 α - (-3 t12 + 2 t12^2 + t22 - 2 t12 t22 + 2 t22^2) α^2) +
    s^2 (-4 t22^2 α^2 - 2 t12^2 (1 + t22) α^2 + 6 (1 + α) + t22
        (6 + 6 α + α^2) + t12 (-6 - 6 α + (2 + 3 t22 + 2 t22^2) α^2)) +
    s^3 (2 (1 + α) + t22^2 (2 + 2 α - 2 α^2) + t22 (8 + 8 α + α^2) + t12^2
        (2 + 2 α - 2 t22 α^2) + t12 (2 t22^2 α^2 - 4 (1 + α) + t22 (-8 - 8 α + α^2)))) +
m^3 (t12 - t22)^3 (2 s^4 (t12^4 + t12^3 (1 - 10 t22) + t12^2 (-5 + 5 t22 + 18 t22^2) +
      t12 (3 + 8 t22 - 11 t22^2 - 10 t22^3) + t22 (-3 - 3 t22 + 5 t22^2 + t22^3))
     (1 + α) - 2 (2 + 2 α + (1 - t22 + t12 (-1 + 2 t22)) α^2) +
    2 s (3 t12^3 α^2 + t22^2 α^2 - 3 t22^3 α^2 - t12^2 (1 + 7 t22) α^2 + 2 t22
        (-4 - 4 α + α^2) - 2 (2 + 2 α + α^2) + t12 (8 + 8 α + (2 - 4 t22 + 7 t22^2) α^2)) -
    s^2 (2 t12^4 α^2 + 15 t22^3 α^2 + 2 t22^4 α^2 - t12^3 (3 + 20 t22) α^2 + t22^2
        (6 + 6 α - 9 α^2) - 3 t22 (-8 - 8 α + α^2) + 3 (2 + 2 α + α^2) + 3
        t12^2 (2 + 2 α + (-1 + 5 t22 + 12 t22^2) α^2) - t12
        (27 t22^2 α^2 + 20 t22^3 α^2 + 3 (8 + 8 α + α^2) - 6 t22 (-2 - 2 α + 3 α^2))) -
    s^3 (2 + 2 α + α^2 + 2 t12^4 α^2 + 2 t22^4 α^2 + t22^2 (2 + 2 α - 7 α^2) + t22
        (20 + 20 α - α^2) + t22^3 (-8 - 8 α + 9 α^2) + t12^3 (8 + 8 α + (3 - 20 t22) α^2) +
      t12^2 (36 t22^2 α^2 + t22 (-32 - 32 α + α^2) - 5 (-2 - 2 α + α^2)) - t12 (20 + 20 α +
          α^2 + 20 t22^3 α^2 - 2 t22 (-6 - 6 α + 7 α^2) + t22^2 (-32 - 32 α + 13 α^2)))) +
m (t12 - t22) (2 s t12^4 (-1 - 2 (-1 + s) t22 + 3 s t22^2) (-α^2 - s α^2 + s^2 (1 + α)) +
    (1 + s) (-1 + t22) t22 (-2 - 2 α - t22 α^2 + 2 s^3 t22^2 (1 + α) + s^2
        t22 (2 + 2 α + (1 - 2 t22) α^2) - 2 s t22 (1 + α + t22 α^2)) +
    t12^3 ((-1 + 2 t22) α^2 - 6 s^4 t22 (-2 + t22 + 2 t22^2) (1 + α) + s
        (2 + 2 α + (-5 + 2 t22 + 10 t22^2) α^2) + s^3 (-3 + 4 t22)
        (3 t22^2 α^2 - 2 (1 + α) + 2 t22 (-3 - 3 α + α^2)) + s^2
        (6 + 6 α - 4 α^2 + 9 t22^2 α^2 + 12 t22^3 α^2 - 6 t22 (1 + α + α^2))) +
    t12 (2 + 2 α + t22^2 (-1 + 2 t22) α^2 - 2 s^4 t22 (-2 - 3 t22 + 2 t22^2 + 4 t22^3)
        (1 + α) + s (6 t22^3 α^2 + 4 t22^4 α^2 + 6 (1 + α) - t22^2 (2 + 2 α + 7 α^2)) + s^3
        (2 (1 + α) + t22^2 (2 + 2 α - 5 α^2) + 2 t22 (9 + 9 α + α^2) - 2 t22^3
            (11 + 11 α + 2 α^2) + t22^4 (-4 - 4 α + 8 α^2)) + s^2 (12 t22^4 α^2 + 6 (1 + α) -
          6 t22^3 (1 + α) + 2 t22 (9 + 9 α + α^2) - t22^2 (18 + 18 α + 11 α^2))) +
    t12^2 (-2 - 2 α - (-1 + t22 + 2 t22^2) α^2 + 6 s^4 t22 (-2 - t22 + 3 t22^2 + t22^3)
        (1 + α) + s (-8 - 8 α + 3 α^2 - 4 t22^2 α^2 - 10 t22^3 α^2 + t22 (2 + 2 α + 3 α^2)) + s^2
        (-21 t22^3 α^2 - 6 t22^4 α^2 + 2 (-6 - 6 α + α^2) + 4 t22 (-3 - 3 α + α^2) +
          t22^2 (24 + 24 α + 11 α^2)) + s^3 (-6 t22^4 α^2 - 6 (1 + α) - 32 t22 (1 + α) +
          t22^3 (24 + 24 α - 11 α^2) + t22^2 (24 + 24 α + 13 α^2)))) +
m^2 (t12 - t22)^2 (2 s t12^4 (1 + s (-1 + 3 t22)) (-α^2 - s α^2 + s^2 (1 + α)) +
    t22 (-4 - 4 α + (-1 + t22 - t22^2) α^2 + 2 s^4 t22 (-3 + t22 + 2 t22^2) (1 + α) - s^2
        (6 + 6 α + 4 α^2 + 4 t22^2 α^2 + 6 t22^3 α^2 + t22 (6 + 6 α - 11 α^2)) - s
        (4 + 4 α + 3 α^2 + 5 t22^2 α^2 + 2 t22^3 α^2 + t22 (8 + 8 α - 7 α^2)) + s^3 (8 t22^2
          (1 + α) + t22^3 (2 + 2 α - 4 α^2) + 5 t22 (-2 - 2 α + α^2) - 2 (2 + 2 α + α^2))) +
```

```
        t12^2 (-(1 + t22) α^2 + 6 s^4 (-1 - 3 t22 + 4 t22^2 + 4 t22^3) (1 + α) - s
              (8 + 8 α + (-3 + 5 t22 + 18 t22^2) α^2) + s^3
              (14 t22 α^2 - 24 t22^3 α^2 - 22 (1 + α) + t22^2 (48 + 48 α - 9 α^2)) + s^2
              (-27 t22^2 α^2 - 24 t22^3 α^2 + 2 (-9 - 9 α + 2 α^2) + 2 t22 (6 + 6 α + 5 α^2))) +
        t12^3 (α^2 + s (-1 + 14 t22) α^2 + s^2 (-5 + 9 t22 + 24 t22^2) α^2 - 6 s^4 (-1 + 4 t22^2)
              (1 + α) + s^3 (8 + 8 α - 3 α^2 + 24 t22^2 α^2 - t22 (24 + 24 α + 5 α^2))) +
        t12 (4 + 4 α + (1 + t22^2) α^2 - 2 s^4 (-1 - 6 t22 - 3 t22^2 + 12 t22^3 + 3 t22^4)
              (1 + α) + s (12 + 12 α + α^2 - t22^2 α^2 + 14 t22^3 α^2 + t22 (8 + 8 α - 2 α^2)) + s^3
              (12 + 12 α + α^2 + 6 t22^4 α^2 + t22 (28 + 28 α - α^2) - t22^2 (24 + 24 α + 17 α^2) +
                t22^3 (-24 - 24 α + 19 α^2)) + s^2 (18 + 18 α + α^2 + 33 t22^3 α^2 +
                6 t22^4 α^2 - 3 t22 (-4 - 4 α + α^2) - t22^2 (12 + 12 α + 19 α^2))))))) /
    ((1 + s) (-1 + m (t12 - t22) + t22) (m (t12 - t22) + t22)
      ((-1 + m) t12 - m t22)
      (1 + (-1 + m) t12 - m t22)
      ((-1 + m) m s (2 + s) t12^2 +
        t12 (-1 - 2 s t22 - s^2 t22 - 2 m^2 s (2 + s) t22 + m (2 + 2 s + s^2 + 4 s t22 + 2 s^2 t22)) +
        t22 ((1 + s)^2 + m^2 s (2 + s) t22 - m (2 + 2 s (1 + t22) + s^2 (1 + t22)))) α (2 +
        α))};
```

```
Exponent[Numerator[dd2] /. dd12gent12, {p12, t12, dd1, p22, t22, d22}]
```

```
{0, 7, 0, 0, 9, 0}
```

Now we can substitute dd12gent12 into p12p22gentdd to obtain (p12, p22) as functions of (t12, t22):

```
Simplify[p12p22gentdd /. dd12gent12]
```

```
p12p22gent12 =
  {p12 → -((-m^6 s (t12 - t22)^6 (2 s^3 t12 (t12 - t22) (1 + α) + 2 α (1 + t22 α) + s α
              (3 - 2 t12^2 α + 2 t22 α + t12 (α + 2 t22 α)) + s^2 (α - 2 t12^2 α^2 - 2 t22
                (1 + α) + t12 (2 + 2 α + (1 + 2 t22) α^2))) + (-1 + t12) t12 (-1 + t22) t22
            (2 s^4 (-1 + t12) t22 (1 + t12^2 - t12 (2 + t22)) (1 + α) + (t12 - t22) α (1 + t12 α) -
              s (1 + t12 α) (2 t12^2 α + t22 (2 + 5 α) - t12 (2 + (3 + 4 t22) α)) - s^2 (-1 + t12)
              (2 t12^2 (1 + t22) α^2 - 3 t22 (2 + 3 α) + t12 (6 + 6 α - t22 (5 + 2 t22) α^2)) -
            s^3 (-1 + t12) (-t22 (6 + 7 α) + 2 t12^2 (-1 - α + t22 α^2) + t12
                (-2 t22^2 α^2 + 2 (1 + α) + t22 (10 + 10 α - α^2)))) + m^5 (t12 - t22)^5
            (2 s^4 t12 (3 t12^2 - 2 t12 (1 + 3 t22) + t22 (2 + 3 t22)) (1 + α) - 2 α (1 + t22 α) -
              2 s α (2 - 2 t12^2 α + 3 t22^2 α + t22 (3 + α) + t12 (-3 + α - t22 α)) -
              3 s^2 (t22 (-2 + α) + α + 2 t12^3 α^2 + 2 t22^2 α^2 - t12^2
                (3 + 4 t22) α^2 + t12 (2 - α + (1 + t22 + 2 t22^2) α^2)) -
              s^3 (6 t12^3 α^2 + t12^2 (2 + 2 α - (5 + 12 t22) α^2) - (1 + 3 t22)
                (-α + 2 t22 (1 + α)) + t12 (2 - α + α^2 + 6 t22^2 α^2 + t22 (4 + 4 α + 5 α^2)))) -
          m^4 (t12 - t22)^4 (2 s^4 t12 (1 + 3 t12^3 - 2 t22 - 6 t22^2 - 3 t22^3 - 3 t12^2
                (1 + 4 t22) + t12 (-1 + 11 t22 + 12 t22^2)) (1 + α) -
              (t12 - t22) α (5 + t12 α + 4 t22 α) + s (10 t12^3 α^2 + 6 t22^3 α^2 + 6 t22^2
                α (1 + α) + 2 (2 + α) + t12^2 α (6 - 7 (1 + 2 t22) α) + t22
                (-10 + 3 α - 2 α^2) + t12 (2 - (5 + 18 t22) α + (7 - 8 t22) t22 α^2)) -
              s^2 (6 t12^4 α^2 - 6 t22^3 α^2 - 3 t12^3 (5 + 8 t22) α^2 - 3 (2 + α) + t22^2
                (12 + 3 α - 2 α^2) + 2 t22 (6 + α^2) + t12^2 (6 - 3 α + (5 + 33 t22 + 24 t22^2) α^2) +
                t12 (-3 t22^2 α^2 - 6 t22^3 α^2 + α (3 + α) - 3 t22 (6 - 3 α + 4 α^2))) +
```

```
    s^3 (2 + 2 t22 (-1 + α) + α - 6 t12^4 α^2 - 6 t22^3 (1 + α) - 3 t22^2
          (2 + α) + t12^3 (-12 - 12 α + (5 + 24 t22) α^2) + t12^2
          (2 + 5 α + α^2 - 24 t22^2 α^2 + t22 (20 + 20 α - 19 α^2)) + t12
          (2 + α - α^2 + 6 t22^3 α^2 + t22 (-8 - 17 α + 2 α^2) + t22^2 (6 + 6 α + 11 α^2)))) +
  m (t12 - t22) (2 t12 t22 - 2 t22^2 - 2 t12 t22^2 + 2 t22^3 + t12^2 α - t12^3 α -
    2 t12 t22 α + 3 t12^2 t22 α + 2 t12^3 t22 α - t22^2 α + t12 t22^2 α - 6 t12^2 t22^2 α +
    t22^3 α + 2 t12 t22^3 α + t12^3 α^2 - t12^4 α^2 - 2 t12^2 t22 α^2 + t12^3 t22 α^2 +
    2 t12^4 t22 α^2 - t12 t22^2 α^2 + 3 t12^2 t22^2 α^2 - 4 t12^3 t22^2 α^2 + t12 t22^3 α^2 +
    2 s^4 (-1 + t12) t12 t22 (2 - 2 t22^2 - t22^3 + t12^3 (-2 + 3 t22) - 3 t12^2
          (-2 + t22 + 2 t22^2) + 3 t12 (-2 + 2 t22^2 + t22^3)) (1 + α) - s^3 (-1 + t12)
        ((-1 + t22) t22^2 (2 + α) + 2 t12^4 (1 + α + 3 t22^2 α^2 - 2 t22 (1 + α + α^2)) + t12
            t22 (t22 (2 + α)^2 - 2 (6 + 7 α) - 2 t22^3 (1 + α + α^2) + t22^2 (16 + 19 α + α^2)) +
          t12^3 (-12 t22^3 α^2 - 4 (1 + α) + 6 t22 (-3 - 3 α + α^2) + t22^2 (30 + 30 α + α^2)) +
          t12^2 (6 t22^4 α^2 + 2 (1 + α) + t22 (34 + 36 α - 2 α^2) -
              3 t22^2 (8 + 9 α + 2 α^2) + t22^3 (-24 - 24 α + 5 α^2))) - s^2 (-1 + t12)
        (2 t12^4 (-1 + 3 t22^2) α^2 + 3 (-1 + t22) t22^2 (2 + α) + t12^2 (15 t22^3 α^2 +
            6 t22^4 α^2 + 6 (1 + α) + t22 (18 + 24 α - 10 α^2) - t22^2 (30 + 39 α + 4 α^2)) +
          t12 t22 (t22^2 (9 - 5 α) α - 6 (2 + 3 α) + t22 (18 + 18 α + 5 α^2)) + t12^3
            (-15 t22^2 α^2 - 12 t22^3 α^2 + 2 (-3 - 3 α + α^2) + 2 t22 (3 + 3 α + 7 α^2))) +
    s (2 t12^5 (1 - 2 t22) α^2 + 3 (-1 + t22) t22^2 (2 + α) + t12^4
          (2 + 2 α + (-5 - 2 t22 + 16 t22^2) α^2) + t12^2 (4 + 5 α - 2 t22^4 α^2 +
            2 t22^3 α (-3 + 8 α) + t22 (-4 + 7 α - 10 α^2) + t22^2 (-2 - 14 α + α^2)) + t12
          t22 (2 - 6 α + 2 t22^3 α^2 + t22^2 (-4 + 6 α - 5 α^2) + t22 (4 + 5 α + 3 α^2)) + t12^3
          (-6 - 7 α + 2 t22^2 (3 - 11 α) α + 3 α^2 - 10 t22^3 α^2 + t22 (4 + 2 α + 17 α^2)))) +
  m^3 (t12 - t22)^3 (2 s^4 t12 (-2 + t12^4 - 10 t12^3 t22 + 5 t22^2 + 6 t22^3 + t22^4 +
        t12^2 (-5 + 14 t22 + 18 t22^2) - 2 t12 (-3 + 2 t22 + 10 t22^2 + 5 t22^3)) (1 + α) -
    2 (2 + α + t12^3 α^2 + t22^3 α^2 + t22^2 α (2 + α) + t12^2 α (2 - t22 α) - t22
          (3 + 2 α + α^2) + t12 (-1 + t22 (-6 + α) α - 3 t22^2 α^2)) + 2 s (4 t12^4
          α^2 - t22^4 α^2 - 2 (2 + α) - t22^3 α (1 + 3 α) + t12^3 α (1 - (2 + 15 t22) α) + t22^2
          (7 + α^2) + t22 (3 + 3 α + α^2) + t12^2 (1 - 3 α^2 + 7 t22^2 α^2 + t22 α (-9 + 13 α)) +
        t12 (5 + α + t22^2 (9 - 4 α) α + α^2 + 5 t22^3 α^2 - 2 t22 (4 - 2 α + α^2))) -
    s^3 (2 + t22^2 (-4 + α) + α + 2 t12^5 α^2 - 2 t22^4 (1 + α) + t22 (2 + α) - t22^3 (6 + 5 α) +
        t12^4 (10 + 10 α + (1 - 20 t22) α^2) + t12^2 (-22 - 23 α + 5 α^2 - 20 t22^3 α^2 + t22^2
            (32 + 32 α - 31 α^2) + t22 (46 + 55 α + 2 α^2)) + t12 (6 + 9 α - α^2 + 2 t22^4 α^2 +
            t22^2 (-20 - 29 α + 3 α^2) - t22 (10 + 16 α + 3 α^2) + t22^3 (8 + 8 α + 11 α^2)) +
        t12^3 (4 + 3 α - 7 α^2 + 36 t22^2 α^2 + t22 (-48 - 48 α + 19 α^2))) +
    s^2 (-2 t12^5 α^2 - 2 t22^4 α^2 + t12^4 (7 + 20 t22) α^2 - 3 (2 + α) - 3 t22
          (2 + α) + t12^3 α (3 + (5 - 49 t22 - 36 t22^2) α) + t22^3
          (6 + 3 α - 4 α^2) + t22^2 (24 + 9 α + 4 α^2) + t12 (-t22^3 α^2 - 2 t22^4 α^2 +
            t22^2 (-24 + 3 α - 17 α^2) + 3 (2 - α + α^2) + t22 (6 + 24 α + α^2)) + t12^2
          (6 + 9 α - 11 α^2 + 45 t22^2 α^2 + 20 t22^3 α^2 + t22 (-6 - 33 α + 22 α^2)))) +
  m^2 (t12 - t22)^2 (2 s t12^5 (1 + s (-1 + 3 t22)) (-α^2 - s α^2 + s^2 (1 + α)) +
    t12^4 (α^2 + s (-3 + 20 t22) α^2 + 3 s^2 (-3 + 7 t22 + 8 t22^2)
          α^2 - 2 s^4 (-4 + 3 t22 + 12 t22^2) (1 + α) + s^3
          (24 t22^2 α^2 + t22 (-30 - 30 α + α^2) - 5 (-2 - 2 α + α^2))) +
    t22 (-4 - α - t22^2 α (1 + 2 α) + t22 (6 + 3 α + 2 α^2) + s^3
          (2 t22^2 + 2 t22^3 (1 + α) - 2 (2 + α) + t22 (2 + α)) + s^2
          (-2 t22^3 α^2 - 6 (2 + α) + 3 t22 (2 + α) + 2 t22^2 (6 + 3 α + α^2)) + s
          (-t22^2 (-4 + α) - 2 t22^3 α^2 - 7 (2 + α) + t22 (16 + 11 α + 2 α^2))) +
```

```
t12 (4 + α - t22^2 (-9 + α) α + 3 t22^3 α^2 + 2 s^4 (-1 - 3 t22 + 3 t22^2 + 4 t22^3 + 2 t22^4)
      (1 + α) - t22 (2 + α)^2 - s^3 (6 + 7 α + t22 (6 + 11 α - 2 α^2) +
        4 t22^4 (1 + α + α^2) - t22^2 (26 + 35 α + 3 α^2) + t22^3 (-8 - 11 α + 4 α^2)) + s
      (2 - 3 α - t22^3 (-6 + α) α + 4 t22^4 α^2 + t22^2 (-20 + 7 α - 9 α^2) +
        t22 (16 + 6 α + 5 α^2)) - s^2 (6 + 9 α + t22^2 α (-27 + 5 α) +
        t22^3 (12 + 3 α + 8 α^2) - t22 (24 + 9 α + 8 α^2))) +
    t12^3 (6 s^4 (-2 - 2 t22 + 7 t22^2 + 4 t22^3) (1 + α) + α (1 + α - 5 t22 α) - s
      (4 + (5 - 6 t22) α + (-11 + 17 t22 + 30 t22^2) α^2) + s^2 (-18 - 21 α + 14 α^2 -
        57 t22^2 α^2 - 24 t22^3 α^2 + t22 (24 + 33 α + 2 α^2)) + s^3 (-32 - 33 α +
        4 α^2 - 24 t22^3 α^2 + t22^2 (60 + 60 α - 27 α^2) + t22 (24 + 27 α + 14 α^2))) -
    t12^2 (2 + (-1 + 9 t22) α - (-1 + 2 t22 + t22^2) α^2 + 2 s^4
      (-4 - 9 t22 + 12 t22^2 + 15 t22^3 + 3 t22^4) (1 + α) + s
      (-4 - 11 α + t22^2 (18 - 35 α) α + 5 α^2 - 2 t22^3 α^2 + t22 (4 + 13 α + 10 α^2)) + s^3
      (-26 - 28 α + α^2 - 6 t22^4 α^2 + t22^3 (12 + 12 α - 25 α^2) + t22^2 (84 + 93 α - α^2) +
        t22 (-16 - 16 α + 11 α^2)) + s^2 (-24 - 30 α + 5 α^2 - 27 t22^3 α^2 -
        6 t22^4 α^2 + t22^2 (12 + 39 α - 35 α^2) + t22 (30 + 30 α + 23 α^2))))) /
  ((1 + s) (-1 + m (t12 - t22) + t22) (m (t12 - t22) + t22) ((-1 + m) t12 - m t22)
    (1 + (-1 + m) t12 - m t22)
    ((-1 + m) m s (2 + s) t12^2 +
      t12 (-1 - 2 s t22 - s^2 t22 - 2 m^2 s (2 + s) t22 + m (2 + 2 s + s^2 + 4 s t22 + 2 s^2 t22)) +
      t22 ((1 + s)^2 + m^2 s (2 + s) t22 - m (2 + 2 s (1 + t22) + s^2 (1 + t22)))) α (2 + α))),
p22 → -((-m^6 s (t12 - t22)^6 (2 s^3 (t12 - t22) (-1 + t22) (1 + α) + 2 α (1 + t12 α) +
      s α (3 - 2 t12 (-2 + t22) α - t22 α + 2 t22^2 α) +
      s^2 (α + 2 t22^2 α^2 + t22 (2 + 2 α - α^2) - 2 t12 (1 + α + (-1 + t22) α^2))) +
    m^5 (t12 - t22)^5 (2 s^4 (-1 + t22) (3 t12^2 - 2 t12 (1 + 3 t22) + t22 (2 + 3 t22))
      (1 + α) - 2 α (1 + t12 α) +
      2 s α (-2 + t22 (-3 + α) + 3 t12^2 α - 2 t22^2 α - t12 (-3 + (3 + t22) α)) -
      3 s^2 (α + 2 t12^2 (-2 + t22) α^2 + t22^2 α^2 + 2 t22^3 α^2 + t22 (2 + 5 α - α^2) + t12
        (-2 - 5 α + (2 + 3 t22 - 4 t22^2) α^2)) + s^3 (-α - 6 t22^3 α^2 + t22
        (-10 - 13 α + α^2) + t22^2 (2 + 2 α + α^2) - 6 t12^2 (1 + α + (-1 + t22) α^2) +
        t12 (10 + 13 α - 2 α^2 + 12 t22^2 α^2 + t22 (4 + 4 α - 7 α^2)))) +
    (-1 + t12) t12 (-1 + t22) t22 (-2 s^4 t22 (1 - t12^2 (-1 + t22) - t22 -
        t22^2 + t12 (-2 + 2 t22 + t22^2)) (1 + α) + (t12 - t22) α (1 + t22 α) -
      s (1 + t22 α) (t12 (2 + α - 4 t22 α) + t22 (-2 + α + 2 t22 α)) +
      s^2 t22 (-3 α - 2 t12^2 (-1 + t22) α^2 - 4 t22^2 α^2 + t22
        (6 + 6 α + α^2) + t12 (-6 - 3 α + (-4 + 5 t22 + 2 t22^2) α^2)) +
      s^3 t22 (-4 - 5 α - 2 t12^2 (-1 + t22) α^2 + t22^2 (2 + 2 α - 2 α^2) + t22
        (8 + 8 α + α^2) + t12 (4 + 5 α - 2 α^2 + 2 t22^2 α^2 + t22 (-10 - 10 α + α^2)))) -
    m^4 (t12 - t22)^4 (-2 s^4 (-1 + t22) (-3 t12^3 + 3 t12^2 (1 + 4 t22) - t12
        t22 (11 + 12 t22) + t22 (1 + 6 t22 + 3 t22^2)) (1 + α) -
      (t12 - t22) α (5 + 4 t12 α + t22 α) + s (-3 t22^2 (-2 + α) α + 6 t12^3 α^2 +
        10 t22^3 α^2 + 2 (2 + α) + t22 (2 + 15 α) - 2 t12^2 α (-3 + (5 + 4 t22) α) -
        t12 (10 + (17 + 18 t22) α + (2 - 19 t22 + 14 t22^2) α^2)) +
      s^3 (2 + α + 6 t22^4 α^2 + t22^2 (14 + 17 α - 6 α^2) + t22 (8 + 12 α + α^2) + t22^3
        (-12 - 12 α + 5 α^2) - 6 t12^3 (1 + α + (-1 + t22) α^2) + t12^2
        (18 + 21 α - 2 α^2 + 24 t22^2 α^2 + t22 (6 + 6 α - 19 α^2)) - t12
        (8 + 9 α + 2 α^2 + 24 t22^3 α^2 - 5 t22^2 (2 + α)^2 + t22 (44 + 53 α - 11 α^2))) +
```

```
    s^2 (-6 t12^3 (-2 + t22) α^2 + 15 t22^3 α^2 + 6 t22^4 α^2 + 3 (2 + α) + t22^2 (6 + 15 α -
          10 α^2) + t22 (6 + 18 α + α^2) + t12^2 (12 + 21 α + (-8 - 27 t22 + 24 t22^2) α^2) -
        t12 (18 + 21 α + 4 α^2 + 9 t22^2 α^2 + 24 t22^3 α^2 - 9 t22 (-2 - 5 α + 3 α^2)))) +
  m^3 (t12 - t22)^3 (2 s^4 (-1 + t22) (t12^4 - 10 t12^3 t22 + t22^2 (3 + 6 t22 + t22^2) +
        t12^2 (-3 + 14 t22 + 18 t22^2) - 2 t12 (-1 + 2 t22 + 10 t22^2 + 5 t22^3)) (1 + α) -
      2 (2 - t22 + α + 2 t22^2 α + t12^3 α^2 + t22^3 α^2 + t12^2 α (2 + α - 3 t22 α) -
        t12 (3 + (2 + 6 t22) α + (1 - t22 + t22^2) α^2)) +
      2 s (t12^4 α^2 - 4 t22^4 α^2 - 2 (2 + α) - t22^3 α (1 + 2 α) - t12^3 α
          (-1 + α + 5 t22 α) - t22 (1 + α + α^2) + t22^2 (-1 - 8 α + 3 α^2) - t12^2
          (7 + (8 + 9 t22) α + (5 - 16 t22 + 7 t22^2) α^2) + t12
          (9 + 5 α - 9 t22^2 (-1 + α) α + 3 α^2 + 15 t22^3 α^2 + t22 (8 + 20 α - 2 α^2))) +
      s^2 (-2 t12^4 (-2 + t22) α^2 - 17 t22^4 α^2 - 2 t22^5 α^2 - 3 (2 + α) + t22^3
          α (-3 + 5 α) + t22^2 α (-15 + 7 α) - 3 t22 (4 + 3 α + α^2) + t12^3
          (6 + 9 α + (2 - 31 t22 + 20 t22^2) α^2) - t12^2 (18 + 15 α + 14 α^2 - 3 t22^2 α^2 +
            36 t22^3 α^2 + t22 (24 + 51 α - 43 α^2)) + t12 (41 t22^3 α^2 + 20 t22^4 α^2 +
            t22^2 (-6 + 21 α - 44 α^2) + t22 (54 + 72 α + α^2) + 3 (4 + α + 2 α^2))) +
      s^3 (-2 - α - 2 t22^5 α^2 + t22^4 (10 + 10 α - 9 α^2) + t22^2 (-16 - 21 α + α^2) -
        t22 (4 + 3 α + α^2) + t22^3 (4 + 3 α + 7 α^2) - 2 t12^4 (1 + α + (-1 + t22) α^2) +
        t12^3 (6 + 7 α + 2 α^2 + 20 t22^2 α^2 + t22 (8 + 8 α - 21 α^2)) + t12
          (-4 - 7 α + 2 α^2 + 20 t22^4 α^2 + t22^2 (34 + 43 α - 24 α^2) + t22 (50 + 56 α + 3 α^2) +
            t22^3 (-48 - 48 α + 11 α^2)) + t12^2 (2 + 3 α - 6 α^2 - 36 t22^3 α^2 +
            t22 (-68 - 77 α + 17 α^2) + t22^2 (32 + 32 α + 17 α^2)))) + m^2 (t12 - t22)^2
    (2 s t12^4 (1 + s - 2 t22 - 4 s t22 + 3 s t22^2) (-α^2 - s α^2 + s^2 (1 + α)) +
      t12^2 (-6 - 3 (1 + 3 t22) α + (-2 + t22 - t22^2) α^2 + 6 s^4
          (1 - 8 t22^2 + 3 t22^3 + 4 t22^4) (1 + α) - s^2 (33 t22^3 α^2 + 24 t22^4 α^2 +
            t22^2 (-12 + 15 α - 64 α^2) + 6 (-1 - 2 α + α^2) + 4 t22 (15 + 15 α + 4 α^2)) - s^3
          (24 t22^4 α^2 + t22^2 (60 + 69 α - 32 α^2) + 3 t22^3 (-20 - 20 α + α^2) +
            2 (-7 - 8 α + α^2) + 2 t22 (17 + 17 α + 5 α^2)) - s
          (8 + α + t22^2 (18 - 31 α) α + 6 α^2 + 30 t22^3 α^2 + t22 (20 + 29 α + 5 α^2))) +
      t22 (-4 - α - t22^3 α^2 + 2 s^4 t22^2 (-3 + t22 + 2 t22^2) (1 + α) - t22^2 α (1 + α) +
        t22 (2 - α + α^2) - s (7 t22^3 α^2 + 2 t22^4 α^2 + 7 (2 + α) + t22^2 (4 + 9 α - 7 α^2) +
            t22 (-12 - 7 α + α^2)) - s^3 (-10 t22^3 (1 + α) + 2 (2 + α) +
            t22^2 (8 + 10 α - 5 α^2) + t22 (2 + 3 α + 2 α^2) + t22^4 (-2 - 2 α + 4 α^2)) - s^2
          (6 t22^3 α^2 + 6 t22^4 α^2 + 6 (2 + α) + t22 α (3 + 4 α) - t22^2 (6 + 13 α^2))) +
      t12^3 (-6 s^4 (1 - 2 t22 - 3 t22^2 + 4 t22^3) (1 + α) + α (1 + (2 - 3 t22) α) +
        s (4 + (3 + 6 t22) α + (8 - 13 t22 + 2 t22^2) α^2) + s^2 (-21 t22^2 α^2 +
            24 t22^3 α^2 + α (-3 + 10 α) + t22 (12 + 21 α - 13 α^2)) + s^3 (-10 - 11 α +
            4 α^2 + 24 t22^3 α^2 + t22 (32 + 35 α - 3 α^2) - t22^2 (12 + 12 α + 23 α^2))) +
      t12 (4 + α + t22^2 (9 - 2 α) α + 5 t22^3 α^2 - 2 s^4 (1 + 2 t22 - 12 t22^2 -
            6 t22^3 + 12 t22^4 + 3 t22^5) (1 + α) + t22 (2 + α)^2 + s (2 - 3 α + 20 t22^4 α^2 +
            3 t22^3 α (2 + α) + t22^2 (-4 + 23 α - 18 α^2) + t22 (32 + 22 α + 9 α^2)) + s^3
          (-6 - 7 α + 6 t22^5 α^2 + t22^2 (64 + 73 α - 3 α^2) + 5 t22 (2 + α + α^2) -
            3 t22^3 (8 + 7 α + 7 α^2) + t22^4 (-30 - 30 α + 19 α^2)) + s^2
          (-6 - 9 α + 39 t22^4 α^2 + 6 t22^5 α^2 + t22^2 (18 + 45 α - 19 α^2) +
            t22 (48 + 33 α + 13 α^2) - t22^3 (24 + 15 α + 23 α^2)))) + m (t12 - t22)
    (2 s t12^4 (-1 + t22) t22 (-1 + s (-2 + 3 t22)) (-α^2 - s α^2 + s^2 (1 + α)) +
      (1 + s) (-1 + t22) t22^2 (2 s^3 t22^2 (1 + α) - α (1 + t22 α) - 2 s
          (-2 + t22 - α + t22 α + t22^2 α^2) + s^2 (2 + α - 2 t22^2 α^2 + t22 (2 + 2 α + α^2))) +
```

```
t12 t22 (2 + (-2 + 3 t22 + 2 t22^2) α + t22 (-2 + t22 + 2 t22^2) α^2 - 4 s^4
      (1 + t22)^2 (1 - 3 t22 + 2 t22^2) (1 + α) + s (6 - 2 α + 10 t22^3 α^2 +
        4 t22^4 α^2 + t22^2 (-4 + 6 α - 13 α^2) + t22 (4 + 5 α + 2 α^2)) + s^3
      (-2 (5 + 6 α) + t22^2 (16 + 20 α - 7 α^2) - 2 t22^3 (13 + 13 α + 2 α^2) + t22
          (26 + 25 α + 4 α^2) + t22^4 (-4 - 4 α + 8 α^2)) + s^2 (12 t22^4 α^2 - 6 (1 + 2 α) +
        t22^3 (-6 - 6 α + 4 α^2) - 3 t22^2 (8 + 4 α + 7 α^2) + t22 (42 + 39 α + 8 α^2)))) +
   t12^3 (-12 s^4 t22 (1 - 2 t22 + t22^3) (1 + α) + (2 + α) (1 + t22 α) + s^2
      t22 (6 + t22 (9 - 28 α) α + 16 α^2 + 3 t22^2 α^2 + 12 t22^3 α^2) + s^3 t22 (12 t22^3 α^2 +
        t22 (40 + 43 α - 12 α^2) + 2 (-7 - 8 α + 4 α^2) - t22^2 (24 + 24 α + 7 α^2)) + s
      (2 + α + 2 t22^2 (3 - 8 α) α + 10 t22^3 α^2 + t22 (4 + 2 α + 9 α^2))) +
   t12^2 (-2 - α + 3 t22^2 (-2 + α) α - 4 t22^3 α^2 + 6 s^4 t22 (2 - 3 t22 - 3 t22^2 + 3 t22^3 +
        t22^4) (1 + α) + t22 (-2 + α - α^2) + s^3 t22 (22 + 26 α - 4 α^2 - 6 t22^4 α^2 +
        t22^3 (30 + 30 α - 11 α^2) - t22 (62 + 65 α + 2 α^2) + 3 t22^2 (2 + α + 7 α^2)) - s
      (2 + α - 6 t22^3 (-1 + α) α + 16 t22^4 α^2 + t22^2 (-2 + 10 α - 11 α^2) +
        t22 (12 + α + 7 α^2)) + s^2 t22 (2 (6 - 5 α) α - 27 t22^3 α^2 -
        6 t22^4 α^2 + t22 (-42 - 51 α + 6 α^2) + t22^2 (30 + 21 α + 31 α^2))))) /
 ((1 + s) (-1 + m (t12 - t22) + t22) (m (t12 - t22) + t22) ((-1 + m) t12 - m t22)
   (1 + (-1 + m) t12 - m t22)
   ((-1 + m) m s (2 + s) t12^2 +
     t12 (-1 - 2 s t22 - s^2 t22 - 2 m^2 s (2 + s) t22 + m (2 + 2 s + s^2 + 4 s t22 + 2 s^2 t22)) +
     t22 ((1 + s)^2 + m^2 s (2 + s) t22 - m (2 + 2 s (1 + t22) + s^2 (1 + t22)))) α (2 +
     α)))};
```

We know from 2.3.1 that deltagensym[[3]] is linear in r. The following gives the constant term :

```
factdelta3r0 = Factor[Coefficient[deltagensym[[3]], r, 0]]
```

```
-((4 dd1 m - 4 dd2 m + 4 dd1 s + 20 dd1 m s - 20 dd2 m s + 20 dd1 s^2 + 40 dd1 m s^2 -
      40 dd2 m s^2 + ··· 37075 ··· + 48 dd1 m^5 s^3 t12 t22^5 α^4 - 48 dd1 m^6 s^3 t12 t22^5 α^4 +
      24 dd1 m^5 s^4 t12 t22^5 α^4 - 24 dd1 m^6 s^4 t12 t22^5 α^4 + 4 dd1 m^6 s^2 t22^6 α^4 +
      8 dd1 m^6 s^3 t22^6 α^4 + 4 dd1 m^6 s^4 t22^6 α^4) / (4 (1 + s - s t12 + m s t12 - m s t22)^2
      (1 + s - s t12 + m s t12 - m s t22 + t12 α - m t12 α + m t22 α)^2 (1 + s - s t12 + m s t12 -
        m s t22 + α + s α - t12 α + m t12 α - s t12 α + m s t12 α - m t22 α - m s t22 α)^2))
```

large output | show less | show more | show all | set size limit...

```
Denominator[factdelta3r0]
```

```
4 (1 + s - s t12 + m s t12 - m s t22)^2 (1 + s - s t12 + m s t12 - m s t22 + t12 α - m t12 α + m t22 α)^2
  (1 + s - s t12 + m s t12 - m s t22 + α + s α - t12 α + m t12 α - s t12 α + m s t12 α - m t22 α - m s t22 α)^2
```

```
Numerator[factdelta3r0]
```

```
-4 dd1 m + 4 dd2 m - 4 dd1 s - 20 dd1 m s + 20 dd2 m s - 20 dd1 s^2 - 40 dd1 m s^2 + 40 dd2 m s^2 -
 40 dd1 s^3 - 40 dd1 m s^3 + 40 dd2 m s^3 - 40 dd1 s^4 - 20 dd1 m s^4 + 20 dd2 m s^4 - 20 dd1 s^5 -
 4 dd1 m s^5 + ··· 37 286 ··· + 2 m^6 p22^2 s^4 t22^5 α^4 - m^5 p12 p22^2 s^4 t22^5 α^4 + m^6 p12 p22^2 s^4 t22^5 α^4 +
 m^5 p22^3 s^4 t22^5 α^4 - m^6 p22^3 s^4 t22^5 α^4 - 24 dd1 m^5 s^2 t12 t22^5 α^4 + 24 dd1 m^6 s^2 t12 t22^5 α^4 -
 48 dd1 m^5 s^3 t12 t22^5 α^4 + 48 dd1 m^6 s^3 t12 t22^5 α^4 - 24 dd1 m^5 s^4 t12 t22^5 α^4 +
 24 dd1 m^6 s^4 t12 t22^5 α^4 - 4 dd1 m^6 s^2 t22^6 α^4 - 8 dd1 m^6 s^3 t22^6 α^4 - 4 dd1 m^6 s^4 t22^6 α^4

large output | show less | show more | show all | set size limit...
```

Therefore, the constant term in deltagensym[[3]] is given by

```
Factor[factdelta3r0 /. p12p22gent12 /. dd12gent12]
```

```
0
```

This gives indeed 0, but takes > 12 CPU hours. An analogous fact holds for the constant term in deltagensym[[6]].

In addition, we find :

```
Factor[
 Numerator[Factor[Coefficient[deltagensym[[3]], r, 1] /. p12p22gent12 /. dd12gent12]] -
  Numerator[Factor[Coefficient[deltagensym[[6]], r, 1] /. p12p22gent12 /. dd12gent12]]]
0
```

Therefore, Coefficient[deltagensym[[3]], r, 1] = 0 is satisfied if and only if Coefficient[deltagensym[[6]], r, 1] = 0. Therefore, the conditions are dependent and the equilibria for r>0 are determined by solving Coefficient[deltagensym[[3]], r, 1] = 0. This implies that t22 is obtained as function of t12. Hence, there is a curve of equilibria!

Summary.

If r=0, then p12p22gent12 and dd12gent12 give the manifold of equilibria, i.e., for every (t12,t22) there is a potential equilibrium (not necessarily admissible).
For given (t12, t22), p12p22gent12 gives the values of p12 and p22, and dd12gent12 gives dd1and dd2.

Because the constant terms in deltagensym[[3]] and deltagensym[[6]] vanish, this also shows that if r>0, then the set of equilibria does not depend on r!
If r>0, it is sufficient to equate the coefficient of r in deltagensym[[3]] to zero to obtain the equilibria!
Because this coefficient is a polynomial of high degree in t12 and t22, an explicit analytical solution is impossible.
However, there exists a curve of equilibria (in each deme), and it is independent of r!

This curve was computed numerically by taking a large number of initial conditions, iterating until equilibrium was reached, and then using curve fitting through these points to display the curve (see Section 3.1.1 for an example).
The endpoints of this curve were computed numerically as outlined in Section 2.1.2 above.

One numerical check:

```
Simplify[deltagensym[[3]] /. p12p22gent12 /. dd12gent12 /. {t12 → 0.3, t22 → 0.64} /.
  {s → 0.2, α → 10, m → 0.01}]
```

$$-4.16334 \times 10^{-17} + 0.0205532\, r$$

2.4 One isolated population: Derivation of curve of equilibria

This is analogous to Kirkpatrick' s (1982) model, who assumed that the trait is expressed only in males. It is also a psecial case of the continent-island model with m=0 (Servedio and Bürger 2015).

2.4.1 Basics and import definitions

It is sufficient to focus on p2 = p22, t2 = t22, and d2 = dd2, and set m1 = m2 = 0 in itgen. In addition, s1 becomes irrelevant.

```
Simplify[
 (itgen[s1, s, α1, α2, 0, 0, r][{p12, t12, d1, p2, t2, d2}] - {p12, t12, d1, p2, t2, d2})[[
  4 ;; 6]]]
```

```
delitgenNoMig =
  {(d2 (p2 α2 + 2 s^3 t2^2 (1 + α2) + α1 (-1 + p2 + p2 α2 - t2 α2) + s (2 + d2 α2 + p2 α2 + 2 t2 α2 +
            p2 t2 α2 + α1 (1 - 3 t2 - 2 t2^2 α2 + d2 (1 + α2) + p2 (1 + t2) (1 + α2))) +
       s^2 (-2 t2^2 (α1 - α2 + α1 α2) + d2 (α1 + α2 + α1 α2) +
          t2 (4 + (2 + p2) α2 + (1 + p2) α1 (1 + α2))))) /
    (2 (1 + s t2) (1 + s t2 + α1 - t2 α1) (1 + t2 α2 + s t2 (1 + α2))),
   1/2 t2 (-2 + ((1 + s) (2 + p2 α2 + 2 t2 α2 - p2 t2 α2 + 2 s^2 t2^2 (1 + α2) - (-1 + t2) α1
           (1 + p2 + p2 α2 + t2 α2) + s (d2 (α1 + α2 + α1 α2) + t2 (4 + (2 - d2 + p2) α2 -
              (-1 + d2 - p2) α1 (1 + α2)) - t2^2 ((-2 + p2) α2 + (1 + p2) α1 (1 + α2))))) /
      ((1 + s t2) (1 + s t2 + α1 - t2 α1) (1 + t2 α2 + s t2 (1 + α2)))),
   (-4 d2 (1 + s t2)^2 (1 + s t2 + α1 - t2 α1)^2 (1 + t2 α2 + s t2 (1 + α2))^2 +
      (1 + s) (d2^2 (r + s - s t2 + r s t2) (α1 + α2 + α1 α2) +
         p2 t2 (2 + p2 α2 + r α2 - p2 r α2 + 2 t2 α2 - p2 t2 α2 - r t2 α2 + p2 r t2 α2 +
            2 s^2 t2^2 (1 + α2) - (-1 + t2) α1 (1 + r + r α2 + t2 α2 - p2 (-1 + r) (1 + α2)) +
            s t2 (4 + (2 + r + p2 (-1 + r) (-1 + t2) + 2 t2 - r t2) α2 + (-1 + p2 (-1 + r) - r)
               (-1 + t2) α1 (1 + α2))) + d2 (2 + α1 + p2 α1 - t2 α1 - p2 t2 α1 + p2 α2 +
            2 t2 α2 - p2 t2 α2 + p2 α1 α2 + t2 α1 α2 - p2 t2 α1 α2 - t2^2 α1 α2 + 2 s^2 t2^2 (1 + α2) +
            s t2 (4 + 2 (1 + p2 + t2 - p2 t2) α2 - (1 + 2 p2) (-1 + t2) α1 (1 + α2)) +
            r (-2 - p2 α2 - 2 t2 α2 + 2 p2 t2 α2 + α1 (-1 - t2 α2 + p2 (-1 + 2 t2) (1 + α2))) + s
               t2 (-2 - α2 - t2 (α1 + α2 + α1 α2) + p2 (-1 + 2 t2) (α1 + α2 + α1 α2))))
      (d2^2 (1 + s) (r - s t2 + r s t2) (α1 + α2 + α1 α2) + (-1 + p2) (-1 + t2)
         (2 + 2 α1 - t2 α1 - p2 t2 α1 + p2 r t2 α1 + 2 t2 α2 - p2 t2 α2 + p2 r t2 α2 +
           2 t2 α1 α2 - p2 t2 α1 α2 + p2 r t2 α1 α2 - t2^2 α1 α2 + s^2 t2^2 (2 + (2 + p2 (-1 + r))
               α2 + (1 + p2 (-1 + r)) α1 (1 + α2)) + s t2 (4 + (2 + p2 (-1 + r))
               (1 + t2) α2 + α1 (3 - t2 + 2 α2 + p2 (-1 + r) (1 + t2) (1 + α2)))) +
        d2 (2 + 2 α1 - t2 α1 - p2 t2 α1 + 2 t2 α2 - p2 t2 α2 + 2 t2 α1 α2 - p2 t2 α1 α2 -
```

```
t2^2 α1 α2 + s t2 (4 + α2 + 3 t2 α2 - 2 p2 t2 α2 + α1
      (2 + α2 + t2 α2 - 2 p2 t2 (1 + α2))) + s^2 t2 ((-1 + p2) (α1 + α2 + α1 α2) +
    t2 (2 + 3 α2 - 2 p2 α2 - 2 (-1 + p2) α1 (1 + α2))) + r (1 + s)
   (-2 - p2 α2 - 2 t2 α2 + 2 p2 t2 α2 + α1 (-1 - t2 α2 + p2 (-1 + 2 t2)) (1 + α2)) +
    s t2 (-2 - α2 - t2 (α1 + α2 + α1 α2) + p2 (-1 + 2 t2)) (α1 + α2 + α1 α2)))))) -
(1 + s) (d2^2 (r + s - s t2 + r s t2) (α1 + α2 + α1 α2) + (-1 + p2) t2
   (2 + p2 α2 - p2 r α2 + 2 t2 α2 - p2 t2 α2 + p2 r t2 α2 + 2 s^2 t2^2 (1 + α2) -
    (-1 + t2) α1 (1 + t2 α2 - p2 (-1 + r) (1 + α2)) + s t2 (4 + p2 (-1 + r) (-1 + t2)
      α2 + 2 (1 + t2) α2 + (-1 + p2 (-1 + r)) (-1 + t2) α1 (1 + α2))) +
  d2 (2 + α1 + p2 α1 - t2 α1 - p2 t2 α1 + p2 α2 + 2 t2 α2 - p2 t2 α2 + p2 α1 α2 +
    t2 α1 α2 - p2 t2 α1 α2 - t2^2 α1 α2 + 2 s^2 t2^2 (1 + α2) +
    s t2 (4 + α2 + 3 t2 α2 - 2 p2 (-1 + t2) (α1 + α2 + α1 α2)) +
    r (-2 - p2 α2 - 2 t2 α2 + 2 p2 t2 α2 + α1 (-1 - t2 α2 + p2 (-1 + 2 t2)) (1 + α2)) + s
      t2 (-2 - α2 - t2 (α1 + α2 + α1 α2) + p2 (-1 + 2 t2)) (α1 + α2 + α1 α2)))))
 (d2^2 (1 + s) (r - s t2 + r s t2) (α1 + α2 + α1 α2) + p2 (-1 + t2)
   (2 + 2 α1 - t2 α1 - p2 t2 α1 - r t2 α1 + p2 r t2 α1 + 2 t2 α2 - p2 t2 α2 - r t2 α2 +
    p2 r t2 α2 + 2 t2 α1 α2 - p2 t2 α1 α2 - r t2 α1 α2 + p2 r t2 α1 α2 - t2^2 α1 α2 +
    s^2 t2^2 (2 + (2 + p2 (-1 + r) - r) α2 + (-1 + p2) (-1 + r) α1 (1 + α2)) +
    s t2 (4 + (2 + p2 (-1 + r) - r) (1 + t2) α2 + α1
      (3 - t2 + 2 α2 + p2 (-1 + r) (1 + t2) (1 + α2) - r (1 + t2) (1 + α2)))) +
  d2 (2 + 2 α1 - t2 α1 - p2 t2 α1 + 2 t2 α2 - p2 t2 α2 + 2 t2 α1 α2 - p2 t2 α1 α2 -
    t2^2 α1 α2 + s t2 (4 + 2 (1 + t2 - p2 t2) α2 - α1 (-3 + t2 - 2 α2 + 2 p2 t2 (1 + α2))) +
    s^2 t2 (p2 (α1 + α2 + α1 α2) + t2 (2 + 2 α2 - 2 p2 α2 - (-1 + 2 p2) α1 (1 + α2))) +
    r (1 + s) (-2 - p2 α2 - 2 t2 α2 + 2 p2 t2 α2 + α1
      (-1 - t2 α2 + p2 (-1 + 2 t2)) (1 + α2)) + s t2
      (-2 - α2 - t2 (α1 + α2 + α1 α2) + p2 (-1 + 2 t2)) (α1 + α2 + α1 α2)))))) /
(4 (1 + s t2)^2 (1 + s t2 + α1 - t2 α1)^2 (1 + t2 α2 + s t2 (1 + α2))^2)};
```

As in 2.1.1, the edges t2 = 0 and t2 = 1 consist of equilibria.

2.4.2 Stability of edge equilibria

Jacobian at t2 = 0:

```
Simplify[D[delitgenNoMig, {{p2, t2, d2}}] //. {t2 → 0, d2 → 0}]
```

$$\left\{\left\{0,\ 0,\ \frac{1}{2\,(1+\alpha 1)}\left(p2\,\alpha 2+\alpha 1\,\left(-1+p2+p2\,\alpha 2\right)+s\,\left(2+p2\,\alpha 2+\alpha 1\,\left(1+p2+p2\,\alpha 2\right)\right)\right)\right\},\right.$$
$$\left\{0,\ -1+\frac{(1+s)\,\left(2+p2\,\alpha 2+\alpha 1\,\left(1+p2+p2\,\alpha 2\right)\right)}{2\,(1+\alpha 1)},\ 0\right\},$$
$$\left.\left\{0,\ -\frac{(-1+p2)\,p2\,r\,(1+s)\,(\alpha 1+\alpha 2+\alpha 1\,\alpha 2)}{2\,(1+\alpha 1)},\ \frac{1}{2\,(1+\alpha 1)}\,\left(-\alpha 1+p2\,\alpha 1+p2\,\alpha 2+p2\,\alpha 1\,\alpha 2+\right.\right.\right.$$
$$\left.\left.\left.s\,\left(2+\alpha 1+p2\,\alpha 1+p2\,\alpha 2+p2\,\alpha 1\,\alpha 2\right)-r\,(1+s)\,\left(2+p2\,\alpha 2+\alpha 1\,\left(1+p2+p2\,\alpha 2\right)\right)\right)\right\}\right\}$$

Eigenvalues:

```
eigenvt20 = Simplify[Eigenvalues[%]]
```

$$\left\{0,\ \frac{1}{2\left(1+\alpha 1\right)}\left(p2\,\alpha 2+\alpha 1\left(-1+p2+p2\,\alpha 2\right)+s\left(2+p2\,\alpha 2+\alpha 1\left(1+p2+p2\,\alpha 2\right)\right)\right),\right.$$
$$\left.\frac{1}{2\left(1+\alpha 1\right)}\left(-\alpha 1+p2\,\alpha 1+p2\,\alpha 2+p2\,\alpha 1\,\alpha 2+\right.\right.$$
$$\left.\left.s\left(2+\alpha 1+p2\,\alpha 1+p2\,\alpha 2+p2\,\alpha 1\,\alpha 2\right)-r\left(1+s\right)\left(2+p2\,\alpha 2+\alpha 1\left(1+p2+p2\,\alpha 2\right)\right)\right)\right\}$$

```
Simplify[eigenvt20[[3]] - eigenvt20[[2]]]
```

$$-\frac{r\left(1+s\right)\left(2+p2\,\alpha 2+\alpha 1\left(1+p2+p2\,\alpha 2\right)\right)}{2\left(1+\alpha 1\right)}$$

This is always negative! Thus, the second ev is always larger than the third! (This could be also inferred from the analysis of the characteristic polynomials cpolgenT20noR and cpolgenT20R in Section 2.1.2)
Hence, stability is determined by the second eigenvalue.

```
Simplify[Series[Numerator[eigenvt20[[2]]], {p2, 0, 2}]] // Normal
```

$$-\alpha 1+s\left(2+\alpha 1\right)+p2\left(1+s\right)\left(\alpha 1+\alpha 2+\alpha 1\,\alpha 2\right)$$

This is strictly monotone increasing in p2. Therefore, stability of equilibria on this edge changes at:

```
Simplify[Solve[-α1 + s (2 + α1) + p2 (1 + s) (α1 + α2 + α1 α2) == 0, p2]]
```

$$\left\{\left\{p2\to\frac{\alpha 1-s\left(2+\alpha 1\right)}{\left(1+s\right)\left(\alpha 1+\alpha 2+\alpha 1\,\alpha 2\right)}\right\}\right\}$$

Analogously, one obtains the value of p2 at t2 = 1, at which stability of equilibria changes (i.e., for smaller p2, equilibria are stable, for larger they are unstable).

Important points on the edges t2 = 0, t2 = 1 :

```
p2t0ast = (α1 - s (2 + α1))/((1 + s) (α1 + α2 + α1 α2));
p2t1ast = ((α1 - 2 s) (1 + α2))/(α1 + α2 + α1 α2);
```

We observe that :

$0 < \text{p2t0ast} < 1$ if and only if $\frac{\alpha 1}{2+\alpha 1} > s$;
$0 < \text{p2t1ast} < 1$ if and only if $\alpha 1 > 2\,s$.

Simple considerations show the following

Edge t2 = 0:

(a) If $\frac{\alpha 1}{2+\alpha 1} < s$, then p2t0ast ≤ 0 and all equilibria are unstable.
The condition $\frac{\alpha 1}{2+\alpha 1} < s$ is equivalent to $s \geq 1$ or $\left(s < 1 \text{ and } \alpha 1 < \frac{2\,s}{1-s}\right)$.

(b) If $\frac{\alpha 1}{2+\alpha 1} > s$, then 0 < p2t0ast < 1 and equilibria with p2 < p2t0ast are stable, and equilibria with p2 > p2t0ast are unstable.

The condition $\frac{\alpha 1}{2+\alpha 1} > s$ is equivalent to $s < 1$ and $\alpha 1 > \frac{2s}{1-s}$.

Edge t2 = 1:

(a) If $\frac{\alpha 1}{2} \leq s$, then p2t1ast ≤ 0 and all equilibria are stable.

(b) If $\frac{\alpha 1}{2} > s$, then 0 < p2t1ast < 1 and equilibria p2 < p2t1ast are unstable and equilibria p2 > p2t1ast are stable.

If $\frac{\alpha 1}{2+\alpha 1} < s < \frac{\alpha 1}{2}$, the equilibrium p2=0 and t2 = $\frac{\alpha 1 - s\,(2+\alpha 1)}{2\,s\,(s-\alpha 1)}$ exists. It is the endpoint of a curve and stable.

2.4.3 Determining the curve of polymorphic equilibria

The following shows that the equilibrium conditions for p2 and t2 are proportional (hence, there will be a curve of polymorphic equilibria):

```
Simplify[delitgenNoMig[[1]] / delitgenNoMig[[2]]]
```

$$\frac{d2}{t2 - t2^2}$$

```
Numerator[Factor[delitgenNoMig[[2]]]]
```

Compute d2 as a function of p2 and t2 delitgenNoMig[[2]] :

```
FullSimplify[Solve[delitgenNoMig[[2]] == 0, d2]]
```

Substitute this into delitgenNoMig[[3]] to compute p2

```
Simplify[Numerator[Factor[
   delitgenNoMig[[3]] /. d2 → -(((1 + s t2) ((-1 + p2) α1 + s (2 + 2 s t2 + (1 + p2 - 2 t2) α1)) +
         (1 + s) (-t2 α1 + p2 (1 + s t2) (1 + α1) + s t2 (2 + 2 s t2 + α1 - 2 t2 α1)) α2) / (s
         (1 + s) (α1 + α2 + α1 α2)))]]]
```

The above is quadratic in p2 (and r is just a factor). Solve for p2 :

```
Simplify[Solve[% == 0, p2], Assumptions → {s > 0, t2 ≥ 0, α1 > 0, α2 > 0, r > 0}]
```

A straightforward analysis shows that only the second solution is admissible.

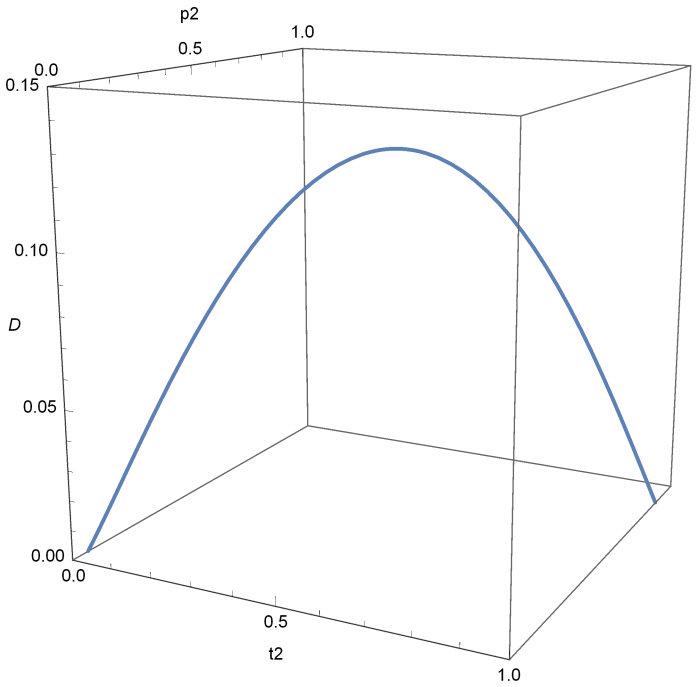
Therefore, we obtain the following curve of equilibria as a function of t2 (and the other parameters):

```
p2curve[t2_, s_, α1_, α2_] := -1/((1 + s)^2 (1 + s t2) (α1 + α2 + α1 α2))
  (3 s + 2 s^2 + 8 s^2 t2 + 5 s^3 t2 + 7 s^3 t2^2 + 4 s^4 t2^2 + 2 s^4 t2^3 + s^5 t2^3 - α1 + s α1 + s^2 α1 -
    4 s t2 α1 + s^2 t2 α1 + 2 s^3 t2 α1 - 5 s^2 t2^2 α1 - s^3 t2^2 α1 + s^4 t2^2 α1 - 2 s^3 t2^3 α1 -
    s^4 t2^3 α1 + 3 s t2 α2 + 5 s^2 t2 α2 + 2 s^3 t2 α2 + 5 s^2 t2^2 α2 + 8 s^3 t2^2 α2 + 3 s^4 t2^2 α2 +
    2 s^3 t2^3 α2 + 3 s^4 t2^3 α2 + s^5 t2^3 α2 - t2 α1 α2 + 2 s^2 t2 α1 α2 + s^3 t2 α1 α2 - 3 s t2^2 α1 α2 -
    3 s^2 t2^2 α1 α2 + s^3 t2^2 α1 α2 + s^4 t2^2 α1 α2 - 2 s^2 t2^3 α1 α2 - 3 s^3 t2^3 α1 α2 -
    s^4 t2^3 α1 α2 - s √((1 + s t2) (1 + s t2 + α1 - t2 α1) (1 + t2 α2 + s t2 (1 + α2))
        (1 + α1 - t2 α1 + t2 α2 + s^5 t2^3 (1 + α2) + s t2 (5 - 2 (-1 + t2) α1 + (3 + 2 t2) α2) +
          s^4 t2^2 (5 + (4 + t2) α2 - (-1 + t2) α1 (1 + α2)) +
          s^3 t2 (-t2^2 α1 α2 + (2 + α1) (2 + α2) + t2 (6 - 2 α1 + 8 α2)) +
          s^2 t2 (6 + 4 α2 + α1 (2 + α2) + t2 (4 + 6 α2 - α1 (2 + α2)))))));
d2curve[t2_, s_, α1_, α2_] := 1/((1 + s)^2 (α1 + α2 + α1 α2))
  (1 + 4 s t2 + s^2 t2 + 5 s^2 t2^2 + 2 s^3 t2^2 + 2 s^3 t2^3 + s^4 t2^3 + α1 - t2 α1 + 3 s t2 α1 + s^2 t2 α1 -
    3 s t2^2 α1 + s^2 t2^2 α1 + s^3 t2^2 α1 - 2 s^2 t2^3 α1 - s^3 t2^3 α1 + t2 α2 + s t2 α2 + 3 s t2^2 α2 +
    4 s^2 t2^2 α2 + s^3 t2^2 α2 + 2 s^2 t2^3 α2 + 3 s^3 t2^3 α2 + s^4 t2^3 α2 + t2 α1 α2 + s t2 α1 α2 -
    t2^2 α1 α2 + s t2^2 α1 α2 + 3 s^2 t2^2 α1 α2 + s^3 t2^2 α1 α2 - 2 s t2^3 α1 α2 - 3 s^2 t2^3 α1 α2 -
    s^3 t2^3 α1 α2 - √((1 + s t2) (1 + s t2 + α1 - t2 α1) (1 + t2 α2 + s t2 (1 + α2))
        (1 + α1 - t2 α1 + t2 α2 + s^5 t2^3 (1 + α2) + s t2 (5 - 2 (-1 + t2) α1 + (3 + 2 t2) α2) +
          s^4 t2^2 (5 + (4 + t2) α2 - (-1 + t2) α1 (1 + α2)) +
          s^3 t2 (-t2^2 α1 α2 + (2 + α1) (2 + α2) + t2 (6 - 2 α1 + 8 α2)) +
          s^2 t2 (6 + 4 α2 + α1 (2 + α2) + t2 (4 + 6 α2 - α1 (2 + α2)))))));
```

It indeed connects (p2t0ast, 0, 0) with (p2t1ast, 1, 0).

2.4.4 Graphs

```
ParametricPlot3D[{t2, p2curve[t2, 0.2, 10, 10], d2curve[t2, 0.2, 10, 10]}, {t2, 0, 1},
 PlotRange → {{0, 1}, {0, 1}, {0, 0.15}}, BoxRatios → 1, AxesLabel → {t2, p2, D}]
```



2.4.5. Properties of the curve (summary)

If $s < \frac{\alpha 1}{2}$, the curve exists and connects equilibria on the boundaryas follows :

1. If $\frac{\alpha 1}{2+\alpha 1} < s$ $\left(\text{or, equivalently, } \alpha 1 > \frac{2\,s}{1-s} \text{ and } s < 1\right)$,

then it connects $(p2, t2, d2) = (p2t0ast, 0, 0)$ with $(p2t1ast, 1, 0)$.

2. If $\frac{\alpha 1}{2+\alpha 1} < s < \frac{\alpha 1}{2}$,

the it connects $(p2, t2, d2) = \left(0, \frac{\alpha 1 - s\,(2+\alpha 1)}{2\,s\,(s-\alpha 1)}, 0\right)$ with $(p2t1ast, 1, 0)$.

Note that case 2 does not exist if selection acts only on males.

If $s > \frac{\alpha 1}{2}$, then the edge $t2 = 1$ is stable, i.e.,

trajectories from the interior converge to some point on this edge (or value p2).

It can also be shown that the two nonzero eigenvalues at the curve equilibria are negative. Hence, the curve is attracting.

3. Numerics

3.1 3D Graphs

3.1.1 Computing the curve of equilibria for r>0

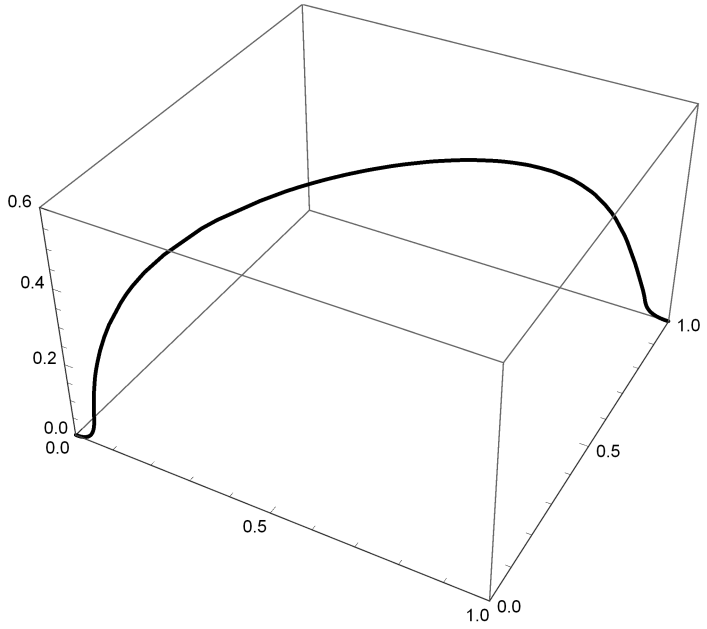
Here is a list of points at the equilibrium curve (in deme 2) generated by forward iteration from many initial conditions and adding the two endpoints of the curve (see Section 2.1.2). The parameters are s1=s2=0.2, α1=α2=10, m1=m2=0.01.

```
curvetestcor = {{0, 0, 0}, {0.0543`, 0, 0},
   {0.05660184549540463`, 0.0037979456900286836`, 0.08956592091594318`},
   {0.06293855717352347`, 0.01416124988810209`, 0.16830729752874554`},
   {0.06697210466039187`, 0.020735137757946442`, 0.20034661333909676`},
   {0.07187476326385878`, 0.028698339084111516`, 0.2312458954044843`},
   {0.07544512372411044`, 0.034477312711836634`, 0.2501032537113619`},
   {0.07717254954432423`, 0.03726690132983597`, 0.25839209179040895`},
   {0.08083017489183085`, 0.043159256993953836`, 0.27447405890270077`},
   {0.08633379352554969`, 0.05198773295467537`, 0.2956236194244254`},
   {0.08761521824010333`, 0.054036622403928584`, 0.300111656227197`},
   {0.08973001650999651`, 0.05741239751113015`, 0.3072066941250179`},
   {0.09814867464065638`, 0.07078034924694529`, 0.3321888747519035`},
   {0.10864419728816559`, 0.08728515952065873`, 0.3577159585557959`},
   {0.10870477365339459`, 0.08737989463453665`, 0.3578488302527233`},
   {0.11127287951672861`, 0.09139058129873132`, 0.3633491761294169`},
```

```
{0.11812426799323449`, 0.10203735791085268`, 0.37686452761090644`},
{0.11901022085398592`, 0.10340843611575098`, 0.37850044234757785`},
{0.1209895230416802`, 0.10646687804899065`, 0.3820705626920394 3`},
{0.12917967091488097`, 0.11905387276980248`, 0.3957093973091313`},
{0.13817711838388322`, 0.1327550965517995`, 0.4088825748528965`},
{0.14470515205365492`, 0.1426138843371581`, 0.41744699919639994`},
{0.15706596239702128`, 0.16109541730424295`, 0.4317699225248873`},
{0.1896641867757147`, 0.20870873113438498`, 0.4607171431040528`},
{0.20759819201782306`, 0.23424062261727313`, 0.4727135883051294`},
{0.2134783491466003`, 0.2425143551970263`, 0.4761834705190131`},
{0.21779411537695048`, 0.24855696602625846`, 0.4786008647940324`},
{0.23519344478351012`, 0.27266725572017686`, 0.4873474570893123`},
{0.24511873060981698`, 0.28624545557885095`, 0.4916974110617419`},
{0.2922048377543788`, 0.34903103154930987`, 0.5073368199586237`},
{0.3149159753823795`, 0.37841264105294037`, 0.5125151968552388`},
{0.3174900231636321`, 0.3817077853226969`, 0.5130211497579203`},
{0.33129893831153834`, 0.39926742753133965`, 0.5154762025192541`},
{0.3359364468607152`, 0.4051207696824208`, 0.5162065531815315`},
{0.34534059458250055`, 0.41692429037741024`, 0.5175494289931301`},
{0.3667408164820048`, 0.4434631850094184`, 0.519954015721805`},
{0.3784466413933181`, 0.45779694161704004`, 0.5209094747718551`},
{0.38287673965708674`, 0.4631887979796312`, 0.5212078563674708`},
{0.4076699312356923`, 0.49304216287247343`, 0.5222645091159591`},
{0.40888996243457`, 0.4944973931147247`, 0.52229039235307`},
{0.40915658900192814`, 0.49481525118937486`, 0.5222957297700026`},
{0.4339569087470273`, 0.5241207307124023`, 0.5223001359510501`},
{0.4446090855901148`, 0.5365539498479851`, 0.5220086853028184`},
{0.4699251377135113`, 0.5657472827711086`, 0.5206226004516094`},
{0.47848775667480375`, 0.5755115234140892`, 0.5199338701986029`},
{0.47851970199604676`, 0.57554785025021`, 0.5199310922645882`},
{0.4792329740687634`, 0.5763587571519981`, 0.5198686627458622`},
{0.48643317738660835`, 0.5845237330803713`, 0.5191950770289251`},
{0.4949159898914635`, 0.5940951213831318`, 0.5182997888217864`},
{0.5159926309422291`, 0.6176568952074447`, 0.515592548504511`},
{0.5249854564755814`, 0.6276171547980913`, 0.5142237933026124`},
{0.5371679561106322`, 0.6410239976729145`, 0.512160036073494`},
{0.5404446474354232`, 0.6446133260559913`, 0.5115630154593267`},
{0.5413684583424265`, 0.6456240160053848`, 0.5113914369495139`},
{0.5420332714962874`, 0.6463510079923273`, 0.5112670711865185`},
{0.5439349817158053`, 0.6484290003160936`, 0.5109071912816989`},
{0.571784036425194`, 0.6785959990488386`, 0.5049139310684305`},
{0.5742766392404987`, 0.6812724667109248`, 0.5043091051332322`},
{0.576837201336487`, 0.6840179458670907`, 0.5036756170201854`},
{0.5894437181868252`, 0.6974769183145952`, 0.5003725862976591`},
{0.5946632739946408`, 0.7030215332474032`, 0.49891281837638485`},
{0.5979969238483621`, 0.7065543430834157`, 0.4979513536919065`},
{0.60491162289776`, 0.7138613077245021`, 0.4958829286847526`},
{0.6074027448992102`, 0.7164869042863247`, 0.4951127345087458`},
{0.6295483657040672`, 0.7396709598141 92`, 0.48765154270459543`},
{0.6382138932160628`, 0.7486670930088165`, 0.4844128844618195`},
{0.6415082123927798`, 0.7520760713559929`, 0.4831314693576045`},
{0.6433063614338183`, 0.7539342551416135`, 0.4824200421399131`},
{0.6474924239835468`, 0.7582531226112944`, 0.48073042924068976`},
{0.6517098416131714`, 0.7625945304106434`, 0.47897986266862896`},
{0.656967634858931`, 0.7679931634915529`, 0.4767275694047644`},
```

```
    {0.6635058538773997`, 0.7746853362417472`, 0.47381473882984537`},
    {0.673442257398772`, 0.7848109264476019`, 0.4691382168575423`},
    {0.6864834319878824`, 0.7980186716833192`, 0.46251015574925536`},
    {0.6874402827624676`, 0.7989840923306586`, 0.46200061403011555`},
    {0.6882878535959853`, 0.7998388369551203`, 0.46154651291756243`},
    {0.6890406903600791`, 0.800597716812932`, 0.4611409840507769`},
    {0.6922989018062435`, 0.8038785126162025`, 0.4593619090714377`},
    {0.6961184813158338`, 0.8077171736643235`, 0.4572256728998312`},
    {0.6969205193530782`, 0.8085222030191773`, 0.4567700152578396`},
    {0.698793776837788`, 0.8104010720140574`, 0.4556960186491584`},
    {0.7203909710548612`, 0.8319225862253015`, 0.44225752147076`},
    {0.7317880631593816`, 0.8431734688989039`, 0.4342985217686242`},
    {0.7334223430227556`, 0.8447805921595001`, 0.43310278562944093`},
    {0.7382808652967848`, 0.8495490260243432`, 0.4294626610617528`},
    {0.7384769545256583`, 0.8497411828467468`, 0.4293130062291318`},
    {0.74160938747052`, 0.8528076438223302`, 0.42689273729456756`},
    {0.7619357973570297`, 0.8725558572268646`, 0.40971888713657334`},
    {0.7695145612414444`, 0.8798480733153262`, 0.4025889141564653`},
    {0.7723627821992717`, 0.8825778492241741`, 0.39979540339410563`},
    {0.7759494703646802`, 0.886006663591593`, 0.39618424353839315`},
    {0.7768751778771739`, 0.8868900082206609`, 0.39523483717156965`},
    {0.7810743847121827`, 0.8908884646848251`, 0.3908354763542134`},
    {0.7984358528426314`, 0.9072564679952063`, 0.3708803298969047`},
    {0.8015210591546166`, 0.9101343693730436`, 0.36700490240665773`},
    {0.8108252528857616`, 0.9187478521743578`, 0.3546357642396313`},
    {0.811331533473403`, 0.9192135034258367`, 0.35393164651384595`},
    {0.8143056601135491`, 0.9219420875253119`, 0.34972690940189294`},
    {0.8165385396788698`, 0.923982611809436`, 0.3464913131256027`},
    {0.8263023333386098`, 0.9328156197867329`, 0.33149425270135274`},
    {0.8418273198029743`, 0.9464871298307743`, 0.3044335414653078`},
    {0.844379779046111`, 0.9486796476216544`, 0.2995595442428612`},
    {0.8482777314932839`, 0.951991435568306`, 0.2918637209630855`},
    {0.8519604417927931`, 0.9550757621266964`, 0.28430192523648307`},
    {0.8532439822946108`, 0.9561396687169569`, 0.2815979587194742`},
    {0.8614468864597258`, 0.9627848979299587`, 0.26344929769419717`},
    {0.8761036718543275`, 0.9738113661703788`, 0.2271314418759672`},
    {0.8769685155710457`, 0.9744195610745443`, 0.2248323831818406`},
    {0.877275490470237`, 0.9746341306033633`, 0.2240122861274224`},
    {0.8855047036089843`, 0.9801096353904881`, 0.20127326166811996`},
    {0.8909469842157165`, 0.9834051663463637`, 0.18551729997724517`},
    {0.9031038766252986`, 0.9896823612150077`, 0.14889336604266454`},
    {0.9076458532821965`, 0.9916160065182898`, 0.13494426712407512`},
    {0.9114143286570854`, 0.9930481602067847`, 0.12335228482418856`},
    {0.9280481225312094`, 0.9976275374149793`, 0.07263886559671419`},
    {0.934476877938098`, 0.9987362821293031`, 0.052907308944191794`},
    {0.945715`, 1, 0}, {1.`, 1.`, 0}};
```

```
f10cor = BSplineFunction[curvetestcor];
plotcurveequilcor10 = ParametricPlot3D[f10cor[x], {x, 0, 1},
  PlotRange → {{0, 1}, {0, 1}, {0, 0.6}}, BoxRatios → {1, 1, 0.6}, PlotStyle → Black]
```



3.1.2 Generating Figure 3b (see main text for details)

```
datsymfix1[p20_, t20_, dd0_, s_, α_, m_, r_, tend_] :=
 datsymfix1[p20, t20, dd0, s, α, m, r, tend] = NestList[itgen[s, s, α, α, m, m, r],
    {0, 0, 0, p2, t2, dd} /. {p2 → p20, t2 → t20, dd → dd0}, tend]
```

```
initphm = {{0.99, 0.99, 0.99 * 0.01}, {0.95, 0.95, 0.05 * 0.95}, {0.9, 0.9, 0.1 * 0.9},
    {0.8, 0.8, 0.2 * 0.8}, {0.7, 0.7, 0.3 * 0.7}, {0.6, 0.6, 0.4 * 0.6}};
```

```
tabdatphmfix1[ss_, αα_, mm_, r_, tend_][nvals_] :=
 Table[{n, Transpose[Table[datsymfix1[initphm[[i, 1]], initphm[[i, 2]],
         initphm[[i, 3]], ss, αα, mm, r, tend][[n]] // Chop, {i, 1, 6}]]}, {n, nvals}]
```

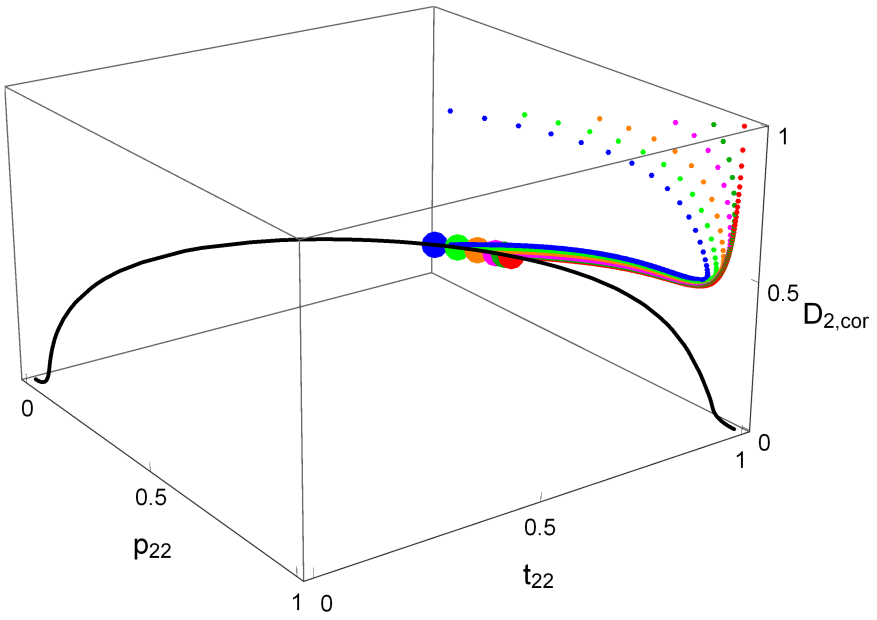
Data for specific generations ({1, 10, 50, 100, 250, 500, 1000}) :

```
Round[10^4 * tabdatphmfix1[0.2, 10, 0.01, 0.1, 2000][
     {1, 10, 50, 100, 250, 500, 1000, 2000}]] / 10^4. // TableForm
```

1.	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.
	0.	0.	0.	0.	0.	0.
	0.99	0.95	0.9	0.8	0.7	0.6
	0.99	0.95	0.9	0.8	0.7	0.6
	0.0099	0.0475	0.09	0.16	0.21	0.24
10.	0.0157	0.0156	0.0155	0.015	0.0143	0.0132
	0.0113	0.0112	0.0111	0.0108	0.0103	0.0095
	0.0095	0.0094	0.0093	0.009	0.0085	0.0078
	0.9834	0.9797	0.9746	0.961	0.938	0.8983
	0.9885	0.9876	0.9857	0.9777	0.9589	0.922
	0.0096	0.0103	0.0117	0.0177	0.0319	0.0596
50.	0.0541	0.0538	0.0534	0.0525	0.0515	0.0502
	0.0191	0.019	0.0189	0.0186	0.0182	0.0178
	0.0149	0.0148	0.0146	0.0143	0.0139	0.0133
	0.9448	0.9403	0.9346	0.9222	0.9073	0.8877
	0.9806	0.9794	0.9778	0.9738	0.9682	0.9594
	0.0152	0.016	0.0172	0.0199	0.0237	0.0293
100.	0.117	0.1158	0.1142	0.111	0.1075	0.1035
	0.0425	0.0418	0.0409	0.039	0.0369	0.0344
	0.0289	0.0284	0.0277	0.0262	0.0245	0.0225
	0.8809	0.8737	0.8642	0.8427	0.8158	0.7804
	0.9564	0.9525	0.9471	0.9334	0.9139	0.8854
	0.0296	0.0319	0.0351	0.0428	0.0531	0.0669
250.	0.3275	0.3197	0.3101	0.2913	0.2718	0.251
	0.2164	0.2085	0.1989	0.1801	0.161	0.1408
	0.0938	0.0911	0.0878	0.081	0.0737	0.0655
	0.6678	0.6573	0.6443	0.6174	0.588	0.5546
	0.7789	0.7683	0.755	0.7272	0.6963	0.6605
	0.0953	0.0984	0.1021	0.1092	0.1159	0.1221
500.	0.3694	0.3601	0.3489	0.3267	0.3039	0.2795
	0.2592	0.2496	0.238	0.2153	0.1923	0.168
	0.1031	0.1006	0.0973	0.0905	0.0829	0.0742
	0.6258	0.6163	0.6045	0.58	0.5533	0.5226
	0.7358	0.7259	0.7134	0.6875	0.6586	0.625
	0.1044	0.1068	0.1097	0.1151	0.12	0.1245
1000.	0.3702	0.3609	0.3496	0.3273	0.3045	0.28
	0.26	0.2504	0.2387	0.216	0.1929	0.1685
	0.1033	0.1007	0.0975	0.0907	0.0831	0.0744
	0.6251	0.6156	0.6038	0.5794	0.5526	0.522
	0.735	0.7251	0.7127	0.6867	0.6579	0.6243
	0.1045	0.107	0.1098	0.1152	0.1201	0.1245
2000.	0.3702	0.3609	0.3496	0.3273	0.3045	0.28
	0.26	0.2504	0.2387	0.216	0.1929	0.1685
	0.1033	0.1007	0.0975	0.0907	0.0831	0.0744
	0.6251	0.6156	0.6038	0.5794	0.5526	0.522
	0.735	0.7251	0.7127	0.6867	0.6579	0.6243
	0.1045	0.107	0.1098	0.1152	0.1201	0.1245

This figure was produced as follows:

```
plotcor2br01endp =
 ListPointPlot3D[Table[ddat = datsymfix1[initphm[[i, 1]], initphm[[i, 2]],
      initphm[[i, 3]], 0.2, 10, 0.01, 0.1, 2000][[1000, 4 ;; 6]];
   {{ddat[[1]], ddat[[2]], ddat[[3]] / (√(ddat[[1]] (1 - ddat[[1]]) ddat[[2]] (1 -
          ddat[[2]]))))}}, {i, 1, 6}], PlotStyle → {Directive[Red, PointSize[0.035]],
    Directive[Darker[Green], PointSize[0.035]], Directive[Magenta, PointSize[0.035]],
    Directive[Orange, PointSize[0.035]], Directive[Green, PointSize[0.035]],
    Directive[Blue, PointSize[0.035]]}, AxesLabel → {p2, t2, D}];
plotcor2br01all = ListPointPlot3D[Table[Table[ddat = datsymfix1[initphm[[i, 1]],
      initphm[[i, 2]], initphm[[i, 3]], 0.2, 10, 0.01, 0.1, 2000][[n, 4 ;; 6]];
    {ddat[[1]], ddat[[2]], ddat[[3]] / (√(ddat[[1]] (1 - ddat[[1]])
        ddat[[2]] (1 - ddat[[2]]))))}, {n, 1, 1000}], {i, 1, 6}],
  PlotStyle → {Red, Darker[Green], Magenta, Orange, Green, Blue}];
plotcor2br01 = Show[plotcor2br01endp, plotcor2br01all, plotcurveequilcor10,
  ImageSize → 450, PlotRange → {{0, 1}, {0, 1}, {-0.005, 1.005}},
  AxesLabel → {Text[Style[p22, Black, 15]],
    Text[Style[t22, Black, 15]], Text[Style[D2,cor, Black, 15]]},
  LabelStyle → Directive[Black, FontFamily → "Helvetica", 12], BoxRatios → {1, 1, 0.6},
  ViewPoint → {3.4, -2.4, 1.6}, AxesEdge → {{0, 0}, {1, 0}, {1, 1}},
  Ticks → {{0, 0.5, 1}, {{0, "    0"}, 0.5, 1}, {0, 0.5, 1}}]
```



3.1.3 Generating Figures 3c, 3d (see main text for details)

```
initvalD0 = {{0.99, 0.99, 0}, {0.95, 0.95, 0},
   {0.9, 0.9, 0}, {0.8, 0.8, 0}, {0.7, 0.7, 0}, {0.6, 0.6, 0}};
```

```
tabdatfixD0[ss_, αα_, mm_, r_, tend_][nvals_] := Table[
  {n, Transpose[Table[datsymfix1[initvalD0[[i, 1]], initvalD0[[i, 2]], initvalD0[[i,
        3]], ss, αα, mm, r, tend][[n, 4 ;; 6]] // Chop, {i, 1, 6}]]}, {n, nvals}]
```

Data for specific generations ({1,10,50,100,500,1000,5000,10000,20000,50000}) :

Figure 3c

```
Round[10^4 * tabdatfixD0[0.2, 10, 0.01, 0.001, 50000][
     {1, 10, 50, 100, 500, 1000, 5000, 10000, 20000, 50000}]] / 10^4. // TableForm
```

	0.99	0.95	0.9	0.8	0.7	0.6
1.	0.99	0.95	0.9	0.8	0.7	0.6
	0.	0.	0.	0.	0.	0.
	0.9775	0.9278	0.8635	0.7491	0.6489	0.5536
10.	0.9873	0.9749	0.9452	0.8547	0.7528	0.6475
	0.0123	0.0216	0.0345	0.0438	0.039	0.0308
	0.9769	0.9241	0.8255	0.591	0.4797	0.4019
50.	0.9871	0.9732	0.9173	0.7061	0.5818	0.4856
	0.0125	0.0247	0.0676	0.1474	0.1291	0.098
	0.9764	0.9232	0.8213	0.5138	0.379	0.3113
100.	0.987	0.9729	0.9138	0.623	0.4655	0.3758
	0.0126	0.0249	0.0702	0.177	0.1498	0.1103
	0.9721	0.9155	0.7929	0.4247	0.2928	0.2502
500.	0.9862	0.97	0.8914	0.5217	0.3554	0.2943
	0.0133	0.0273	0.0849	0.1844	0.1391	0.1032
	0.9665	0.9049	0.7483	0.3526	0.2704	0.2473
1000.	0.9851	0.9655	0.8534	0.4351	0.3251	0.2902
	0.0142	0.0308	0.1073	0.1733	0.1279	0.1011
	0.9086	0.6953	0.28	0.1162	0.2063	0.2379
5000.	0.9674	0.8046	0.3401	0.1004	0.2333	0.2765
	0.028	0.1259	0.1419	0.0406	0.0848	0.0939
	0.7237	0.3679	0.1612	0.1071	0.1975	0.2362
10000.	0.831	0.4479	0.168	0.085	0.22	0.2741
	0.1025	0.1443	0.0629	0.0307	0.0773	0.0925
	0.5545	0.3089	0.1548	0.1068	0.1966	0.236
20000.	0.66	0.3708	0.1577	0.0844	0.2186	0.2738
	0.1212	0.1145	0.0566	0.0304	0.0765	0.0923
	0.5506	0.308	0.1548	0.1068	0.1966	0.236
50000.	0.6557	0.3695	0.1577	0.0844	0.2186	0.2738
	0.1204	0.1139	0.0566	0.0304	0.0765	0.0923

Computing divergence between subpopulations (p22 - p12, t22 - t12):

```
divergefixD0[ss_, αα_, mm_, r_, tend_][nvals_] :=
 Table[{n, Transpose[Table[{datsymfix1[initvalD0[[i, 1]], initvalD0[[i, 2]],
           initvalD0[[i, 3]], ss, αα, mm, r, tend][[n]][[4]] -
        datsymfix1[initvalD0[[i, 1]], initvalD0[[i, 2]], initvalD0[[i, 3]],
           ss, αα, mm, r, tend][[n]][[1]], datsymfix1[initvalD0[[i, 1]],
           initvalD0[[i, 2]], initvalD0[[i, 3]], ss, αα, mm, r, tend][[n]][[5]] -
        datsymfix1[initvalD0[[i, 1]], initvalD0[[i, 2]], initvalD0[[i, 3]],
           ss, αα, mm, r, tend][[n]][[2]]} // Chop, {i, 1, 6}]]}, {n, nvals}]
```

```
Round[10^4 * divergefixD0[0.2, 10, 0.01, 0.001, 50000][
     {1, 10, 50, 100, 500, 1000, 5000, 10000, 20000, 50000}]] / 10^4. // TableForm
```

1.	0.99	0.95	0.9	0.8	0.7	0.6
	0.99	0.95	0.9	0.8	0.7	0.6
10.	0.9666	0.9163	0.8507	0.7324	0.6291	0.5319
	0.9767	0.9643	0.9348	0.8449	0.7439	0.6397
50.	0.9656	0.9122	0.8113	0.5591	0.425	0.3314
	0.9764	0.9625	0.9069	0.6957	0.5697	0.4725
100.	0.9646	0.9108	0.8066	0.4765	0.3068	0.2134
	0.9762	0.9621	0.9034	0.6131	0.4522	0.3579
500.	0.9565	0.8995	0.7745	0.3807	0.2088	0.1357
	0.9748	0.9587	0.8807	0.5125	0.3422	0.2727
1000.	0.946	0.8842	0.7249	0.3025	0.1826	0.1318
	0.9728	0.9535	0.8424	0.4265	0.3119	0.2682
5000.	0.8408	0.6315	0.21	0.0439	0.1041	0.119
	0.944	0.7858	0.3304	0.0971	0.2193	0.2535
10000.	0.552	0.2426	0.0722	0.0324	0.0926	0.1166
	0.7539	0.4146	0.1601	0.082	0.2056	0.2508
20000.	0.2553	0.1592	0.0639	0.032	0.0914	0.1164
	0.4723	0.3243	0.1499	0.0814	0.2042	0.2505
50000.	0.2478	0.1578	0.0638	0.032	0.0914	0.1163
	0.4645	0.3228	0.1499	0.0814	0.2042	0.2505

```
plotcor2cr0001endp =
 ListPointPlot3D[Table[ddat = datsymfix1[initvalD0[[i, 1]], initvalD0[[i, 2]],
      initvalD0[[i, 3]], 0.2, 10, 0.01, 0.001, 50000][[50000, 4 ;; 6]];
   {{ddat[[1]], ddat[[2]], ddat[[3]] / (√(ddat[[1]] (1 - ddat[[1]]) ddat[[2]] (1 -
           ddat[[2]])))}}, {i, 1, 6}], PlotStyle → {Directive[Red, PointSize[0.035]],
    Directive[Darker[Green], PointSize[0.035]], Directive[Magenta, PointSize[0.035]],
    Directive[Orange, PointSize[0.035]], Directive[Green, PointSize[0.035]],
    Directive[Blue, PointSize[0.035]]}, AxesLabel → {p2, t2, D}];
plotcor2cr0001all = ListPointPlot3D[Table[Table[ddat = datsymfix1[initvalD0[[i, 1]],
       initvalD0[[i, 2]], initvalD0[[i, 3]], 0.2, 10, 0.01, 0.001, 50000][[n, 4 ;; 6]];
    {ddat[[1]], ddat[[2]], ddat[[3]] /
      (√(ddat[[1]] (1 - ddat[[1]]) ddat[[2]] (1 - ddat[[2]])))},
    {n, Flatten[{Join[Table[j, {j, 1, 1000}], Table[1000 + 10 j, {j, 1, 4000}]]}]}],
   {i, 1, 6}], PlotStyle → {Red, Darker[Green], Magenta, Orange, Green, Blue}];
plotcor2cr0001 = Show[plotcor2cr0001all, plotcor2cr0001endp, plotcurveequilcor10,
  ImageSize → 450, PlotRange → {{0, 1}, {0, 1}, {-0.005, 1.005}},
  AxesLabel → {Text[Style[p22, Black, 15]],
    Text[Style[t22, Black, 15]], Text[Style[D2,cor, Black, 15]]},
  LabelStyle → Directive[Black, FontFamily → "Helvetica", 12], BoxRatios → {1, 1, 0.6},
  ViewPoint → {3.4, -2.4, 1.6}, AxesEdge → {{0, 0}, {1, 0}, {1, 1}},
  Ticks → {{0, 0.5, 1}, {{0, "    0"}, 0.5, 1}, {0, 0.5, 1}}]
```

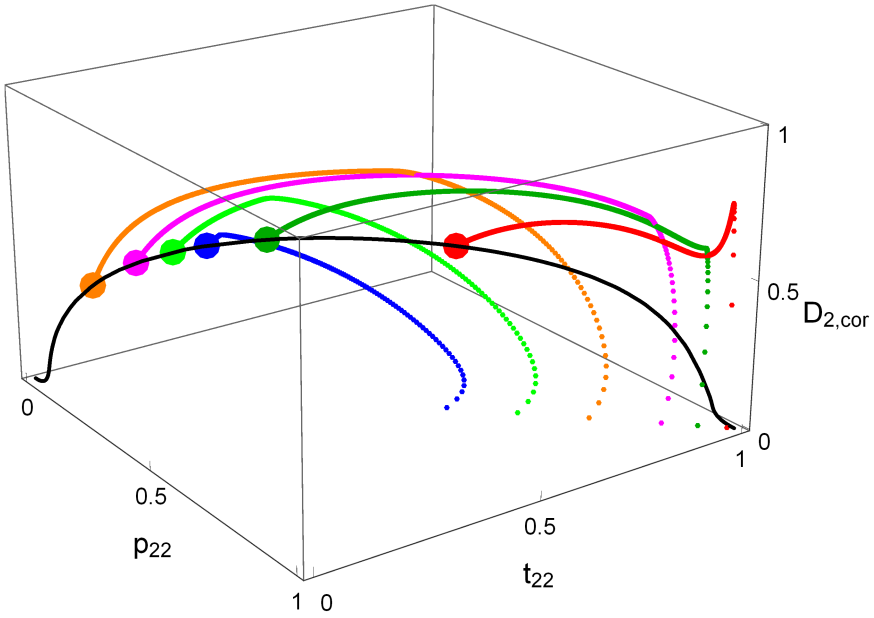


Figure 3d

```
Round[10^4 * tabdatfixD0[0.2, 10, 0.01, 0.1, 5000][{1, 10, 50, 100, 500, 1000, 5000}]] /
  10^4. // TableForm
```

	0.99	0.95	0.9	0.8	0.7	0.6
1.	0.99	0.95	0.9	0.8	0.7	0.6
	0.	0.	0.	0.	0.	0.
	0.9726	0.9255	0.867	0.7586	0.6591	0.5631
10.	0.9867	0.9745	0.9469	0.8622	0.7645	0.6616
	0.0111	0.0192	0.0325	0.0561	0.0702	0.0767
	0.9305	0.859	0.7527	0.6053	0.5134	0.4346
50.	0.9765	0.9437	0.8624	0.7196	0.6209	0.5311
	0.018	0.0389	0.08	0.1244	0.1368	0.1371
	0.8573	0.7337	0.6043	0.4762	0.4043	0.3425
100.	0.9428	0.844	0.7167	0.5774	0.4935	0.4178
	0.0375	0.0845	0.1223	0.1378	0.1353	0.1264
	0.5969	0.4917	0.4089	0.3347	0.2936	0.2539
500.	0.7054	0.5905	0.4945	0.4036	0.3508	0.2981
	0.1115	0.1275	0.1284	0.1195	0.1103	0.0986
	0.5962	0.4912	0.4085	0.3345	0.2934	0.2537
1000.	0.7046	0.5899	0.494	0.4033	0.3506	0.2979
	0.1116	0.1275	0.1284	0.1194	0.1103	0.0985
	0.5962	0.4912	0.4085	0.3345	0.2934	0.2537
5000.	0.7046	0.5899	0.494	0.4033	0.3506	0.2979
	0.1116	0.1275	0.1284	0.1194	0.1103	0.0985

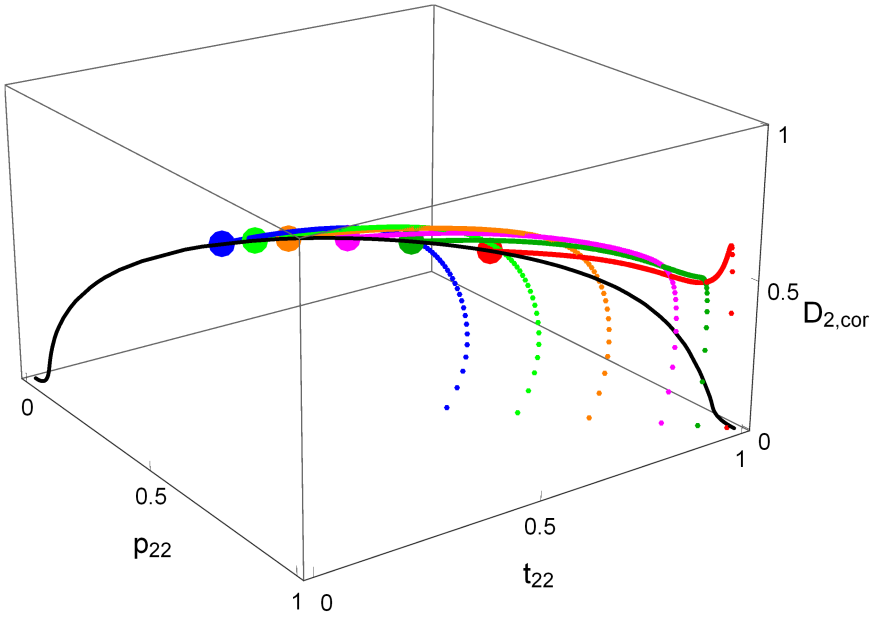
Divergence:

```
Round[10^4 * divergefixD0[0.2, 10, 0.01, 0.1, 5000][{1, 10, 50, 100, 500, 1000, 5000}]] /
  10^4. // TableForm
```

1.	0.99	0.95	0.9	0.8	0.7	0.6
	0.99	0.95	0.9	0.8	0.7	0.6
10.	0.9568	0.9095	0.8501	0.7394	0.6383	0.5416
	0.9755	0.9634	0.936	0.8521	0.7554	0.6538
50.	0.8771	0.8076	0.7018	0.5498	0.4535	0.3731
	0.9576	0.926	0.8461	0.7049	0.6071	0.5188
100.	0.7436	0.6312	0.5079	0.3812	0.3099	0.2516
	0.9023	0.8111	0.6896	0.5546	0.4733	0.4009
500.	0.2551	0.235	0.2059	0.1719	0.1501	0.1273
	0.4746	0.4449	0.4004	0.3461	0.3097	0.27
1000.	0.2537	0.234	0.2053	0.1715	0.1498	0.1271
	0.4732	0.4439	0.3997	0.3456	0.3093	0.2698
5000.	0.2537	0.234	0.2053	0.1715	0.1498	0.1271
	0.4732	0.4439	0.3997	0.3456	0.3093	0.2698

Note that the divergence here, both for p and t, is higher than for the smaller recombination rate in Figure 3c!

```
plotcor2dr01endp =
 ListPointPlot3D[Table[ddat = datsymfix1[initvalD0[[i, 1]], initvalD0[[i, 2]],
      initvalD0[[i, 3]], 0.2, 10, 0.01, 0.1, 5000][[5000, 4 ;; 6]];
   {{ddat[[1]], ddat[[2]], ddat[[3]] / (√(ddat[[1]] (1 - ddat[[1]]) ddat[[2]] (1 -
           ddat[[2]])))}}, {i, 1, 6}], PlotStyle → {Directive[Red, PointSize[0.035]],
    Directive[Darker[Green], PointSize[0.035]], Directive[Magenta, PointSize[0.035]],
    Directive[Orange, PointSize[0.035]], Directive[Green, PointSize[0.035]],
    Directive[Blue, PointSize[0.035]]}, AxesLabel → {p2, t2, D}];
plotcor2dr01all = ListPointPlot3D[Table[Table[ddat = datsymfix1[initvalD0[[i, 1]],
       initvalD0[[i, 2]], initvalD0[[i, 3]], 0.2, 10, 0.01, 0.1, 5000][[n, 4 ;; 6]];
    {ddat[[1]], ddat[[2]], ddat[[3]] /
      (√(ddat[[1]] (1 - ddat[[1]]) ddat[[2]] (1 - ddat[[2]])))},
    {n, Flatten[{Join[Table[j, {j, 1, 1000}], Table[1000 + 10 j, {j, 1, 400}]]}]}],
   {i, 1, 6}], PlotStyle → {Red, Darker[Green], Magenta, Orange, Green, Blue}];
plotcor2dr01 = Show[plotcor2dr01all, plotcor2dr01endp, plotcurveequilcor10,
  ImageSize → 450, PlotRange → {{0, 1}, {0, 1}, {-0.005, 1.005}},
  AxesLabel → {Text[Style[p22, Black, 15]],
    Text[Style[t22, Black, 15]], Text[Style[D2,cor, Black, 15]]},
  LabelStyle → Directive[Black, FontFamily → "Helvetica", 12], BoxRatios → {1, 1, 0.6},
  ViewPoint → {3.4, -2.4, 1.6}, AxesEdge → {{0, 0}, {1, 0}, {1, 1}},
  Ticks → {{0, 0.5, 1}, {{0, "    0"}, 0.5, 1}, {0, 0.5, 1}}]
```



Other 3D figures were produced analogously

3.2 Generate initial data with nearly symmetric initial conditions

```
datsymsymnear[p20_, t20_, dd0_, s_, α_, m_, r_, tend_] :=
 datsymsymnear[p20, t20, dd0, s, α, m, r, tend] =
  NestList[itgen[s, s, α, α, m, m, r], {(1 - p2) RandomReal[{0.95, 1.05}],
     (1 - t2) RandomReal[{0.95, 1.05}], 0.98 * Min[dd * RandomReal[{0.95, 1.05}],
       Min[(1 - p2) RandomReal[{0.95, 1.05}] * (1 - (1 - t2) RandomReal[{0.95, 1.05}]),
        (1 - (1 - p2) RandomReal[{0.95, 1.05}]) * (1 - t2) RandomReal[{0.95, 1.05}]]],
     p2, t2, dd} /. {p2 → p20, t2 → t20, dd → dd0}, tend]
```

```
tabdatsymnear[ss_, αα_, mm_, r_, tend_][nvals_] :=
 Table[{n, Transpose[Table[datsymsymnear[initvals[[i, 1]], initvals[[i, 2]],
        initvals[[i, 3]], ss, αα, mm, r, tend][[n]] // Chop, {i, 1, 8}]]}, {n, nvals}]
```

The following provides an example for the fact that slight deviations from symmetric initial conditions (in this case by up to 5%) lead to equilibria that are also close to the symmetric equilibrium.

```
SeedRandom[5]
```

```
tabdatsymnear[0.2, 10, 0.01, 0.01, 10000][{1, 10, 100, 1000, 5000, 10000}] // TableForm
```

1	0.047504	0.104707	0.308284	0.298176	0.0502728	0.0957219	0.285646	0.2940
	0.0478251	0.0972991	0.0502547	0.289269	0.0511082	0.0960357	0.0495603	0.3107
	0	0	0	0	0.0382716	0.0681139	0.0291374	0.1971
	0.95	0.9	0.7	0.7	0.95	0.9	0.7	0.7
	0.95	0.9	0.95	0.7	0.95	0.9	0.95	0.7
	0	0	0	0	0.04	0.07	0.03	0.2
10	0.0690184	0.139359	0.354977	0.327689	0.0238609	0.0382095	0.3946	0.0395
	0.0237022	0.0556002	0.239333	0.225387	0.0133083	0.0173711	0.255147	0.0305
	0.0192383	0.0300719	0.0362455	0.0210997	0.012598	0.0157453	0.105109	0.0274
	0.927858	0.866856	0.654343	0.670703	0.978517	0.959254	0.573493	0.9459
	0.975257	0.947898	0.770308	0.772628	0.987152	0.981808	0.716413	0.9618
	0.0199225	0.0286655	0.035643	0.0210803	0.0121895	0.0164315	0.114012	0.0345
100	0.0852246	0.195301	0.426903	0.419742	0.0330694	0.0472057	0.374527	0.033
	0.0296503	0.0972022	0.321911	0.314236	0.0147077	0.0178431	0.264674	0.014
	0.0248472	0.0652225	0.0918839	0.0896788	0.0137572	0.0161617	0.0999837	0.013
	0.910631	0.817269	0.583205	0.578504	0.969535	0.950209	0.5885	0.957
	0.968628	0.913016	0.689071	0.683857	0.985836	0.981488	0.696225	0.983
	0.0262359	0.0590644	0.0903139	0.0899407	0.0132568	0.0167547	0.108813	0.015
1000	0.317089	0.372389	0.379522	0.373623	0.226052	0.256262	0.355183	0.204557
	0.206161	0.262297	0.269806	0.263644	0.12265	0.148691	0.244507	0.103139
	0.0977929	0.106811	0.105158	0.103616	0.0769609	0.0844003	0.098942	0.0643932
	0.662475	0.648281	0.630555	0.624618	0.791993	0.725626	0.608514	0.728574
	0.773332	0.759058	0.740709	0.734521	0.893078	0.834256	0.717651	0.836449
	0.104753	0.100949	0.102515	0.104077	0.0683369	0.092177	0.108531	0.095076
5000	0.362008	0.383018	0.377593	0.371665	0.385235	0.362092	0.354615	0.330943
	0.251563	0.27345	0.267775	0.261592	0.275774	0.25165	0.24392	0.219651
	0.101032	0.106611	0.105224	0.103665	0.107168	0.101056	0.0989397	0.091804
	0.616773	0.637788	0.632473	0.626577	0.639939	0.616859	0.609094	0.583438
	0.726331	0.748226	0.742712	0.736577	0.750453	0.726421	0.718267	0.691077
	0.106664	0.101089	0.102551	0.104132	0.100487	0.106642	0.108562	0.114345
10000	0.362008	0.383018	0.377593	0.371665	0.385235	0.362092	0.354615	0.330943
	0.251563	0.27345	0.267775	0.261592	0.275775	0.25165	0.24392	0.219651
	0.101032	0.106611	0.105224	0.103665	0.107168	0.101056	0.0989397	0.091804
	0.616773	0.637788	0.632473	0.626577	0.639939	0.616859	0.609094	0.583438
	0.726331	0.748226	0.742712	0.736577	0.750453	0.726421	0.718267	0.691077
	0.106664	0.101089	0.102551	0.104132	0.100487	0.106642	0.108562	0.114345