atmosphere

Article

Generation of Internal Gravity Waves in the Thermosphere during Operation of the SURA Facility under Parametric Resonance Conditions

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Abstract: The problem of excitation of internal gravity waves (IGWs) in the upper atmosphere by an external source of a limited duration of operation is investigated. An isothermal atmosphere was chosen as the propagation environment of IGWs in the presence of a uniform wind that changes over time according to the harmonic law. For the vertical component of the displacement of an environment, the Mathieu equation with zero initial conditions was solved with the right part simulating the effect of a powerful heating facility on the ionosphere. In the case of a small amplitude of the variable component of the wind, the time dependence of the vertical displacement under parametric resonance conditions using the perturbation method is obtained. The obtained dependence of the solution of the differential equation on the parameters allows us to perform a numerical analysis of the problem in the case of variable wind of arbitrary amplitude. For practical estimations of the obtained values, data on the operating modes of the SURA heating facility (56.15° N, 46.11° E) with periodic (15–30 min) switching on during of 2–3 h for ionosphere impact were used.

Keywords: internal gravity waves; Mathieu equation; parametric resonance; perturbation method; vertical displacement; powerful heating facility

1. Introduction

Internal gravity waves are one of the main components of the ionosphere dynamics. IGWs are studied using many methods including either measuring of parameters of the neutral or the ionized component of the atmosphere. Lidar observation, airglow imaging, measurements of the zonal and meridional wind by meteor radar, satellite observation allows to study IGWs and determine their vertical and horizontal parameters from the lower atmosphere to the thermosphere [1–8].

The operation of powerful radio transmitting devices has made the task of analyzing the artificial impact on the processes occurring in the Earth's atmosphere. It seems plausible that the operation of powerful facilities can lead to a noticeable and controlled impact on the environment, the results of which are recorded by modern techniques. This problem is discussed in more detail in the reviews [9,10]. They provide a detailed description of the equipment and technical capabilities when using it for experiments. Among the diagnostic and monitoring devices, we note: the automated digital ionosonde, the installation of partial reflections for a study the ionosphere using artificial periodic inhomogeneities, equipment for receiving the optical glow of the atmosphere, the devices for radio and electroacoustic sounding of the atmosphere, a complex of receiving radio-astronomical equipment for recording signals from onboard artificial earth satellites transmitters, a receiving complex for measuring electromagnetic signals in the range of 0.01 Hz–100 kHz, a receiving complex for measuring the characteristics of HF radio emission, including stimulated electromagnetic, etc.

Using the SURA facility (56.15° N, 46.11° E) which worked in special modes with the specified equipment, important problems for the physics of the ionosphere were solved. Let us note only some of them, which are close to the topic of the present paper. The generation of low-frequency radio emission in the kilohertz frequency range is realized when the lower ionosphere in the dynamo region is exposed to transmitters with amplitude modulation of power. Signals from the traveling wave antenna were recorded, formed during the operation of two groups of transmitters operating at close frequencies without power modulation. The SURA facility has been used for the study of natural IGW for many years. IGW parameters are determined on the basis of measuring the altitude-temporal variations of the electron concentration, temperature and density of the neutral atmosphere, and the velocity of vertical plasma motion by the method of creating artificial periodic irregularities [9,10]. As a result, we conclude that powerful radio transmitters can be used to create disturbances in the required frequency range, such as IGWs. It is necessary to set the operating mode of the transmitters, in which the spectrum of the radiated energy contain the required frequencies.

For qualified assessing the degree of exposure to powerful heating facility requires a detailed analysis of power flows in the atmosphere. Mechanisms of transformation of this power into heat and the part of it that is carried away from the impact area by various physical processes also need to be investigated.

2. Problem Statement

The purpose of this study is to analyze the possibility of enhancing the effect of powerful radio transmitters on the upper atmosphere under conditions of parametric resonance upon an excitation or an amplification of internal gravitational waves. In this work, we solve the problem of the excitation of low-frequency internal gravity waves (IGWs) during periodic operation of an external source. When solving the problem, information about the time regime of the heating facility is only used. As a source, we consider the energy supplied to the upper layers of the atmosphere by a powerful transmitter and changing over time with a frequency lower than the Brunt-Väisälä frequency (i.e., in the IGW frequency range). That is to say, we are considering periodic heating with the same heating and pause duration (the transmitter turning on and turning off). Here, we do not consider the mechanism of transformation of high-frequency energy into heat release and force action on the environment. Partially these tasks are solved in [11,12]. Dokuchaev and Troitsky [11] estimated the heating of atmospheric gas in the troposphere by electromagnetic radiation from high-power transmitters. Using the parameters of the SURA facility, the authors showed that the temperature change of atmospheric gas can reach more than 10 K. Another mechanism of influence of high-frequency heating of electrons in the dynamo region of the E-region of the ionosphere is considered in the work [12]. When the electrons are heated, the conductivity of the ionosphere and the current in the dynamo region are changed. In this case, an external Ampere force is aroused, under the influence of which IGWs can be generated. We consider the periodically switched on source as given and acting at the heights of the thermosphere.

As a model of the environment we choose an isothermal atmosphere in the presence of a uniform horizontal wind with a time harmonic component

$$V = V_1 + V_2 \sin \Omega t, \tag{1}$$

where V_1 and V_2 are amplitudes of velocity, and frequency Ω are constants. For this model of the atmosphere in [13] the differential equation in the linear approximation was obtained for the time dependence of the vertical component of the displacement, given in the form of a plane wave in space $\varsigma \approx \exp(ik_x x - ik_z z + k_0 z)$.

Reduced to canonical form in dimensionless variables it has the simple shape

$$\ddot{\varsigma} + (\delta + \varepsilon \cos 2\tau)\varsigma = 0. \tag{2}$$

The points at the top of this equation Mathieu means differentiation in dimensionless time. In the Equation (2) the following designations are introduced as $\tau = \Omega t/2$, $\varepsilon = k_x V_2/k_z H\Omega$, $\delta = \frac{4\omega_g^2 k_x^2}{\Omega^2\left(k_x^2 + k_z^2 + 1/4H^2\right)}$.

H is the height of a homogeneous atmosphere. Since the Mathieu Equation (2) describes the mechanism of parametric resonance [14], we will answer the question whether it is possible to increase the efficiency of the impact on the environment for the accepted model at a fixed power of the transmitter. For this purpose, it is required to find a solution to the Mathieu equation, the right part of which is determined by the time dependence of external sources

$$\ddot{\varsigma} + (\delta + \varepsilon \cos 2\tau)\varsigma = f(\tau). \tag{3}$$

In the next section we provide solutions for several specified sources.

3. Problem Solution

Since Equation (3) has second order in time, two initial conditions must be set. We choose as initial condition the equality to zero of displacement and velocity at the initial time $\tau = 0$

$$\varsigma = 0, \dot{\varsigma} = 0. \tag{4}$$

The Mathieu equation has a resonant character when the condition $\delta = n^2$ is fulfilled, where n = 1, 2, 3 is the order (number) of the resonance, the first of which is the strongest. We will consider it further. To find an approximate solution of the formulated problem for $\delta = 1 + \varepsilon\delta_1$, which is valid for small parameter values ε and small frequency offset $\varepsilon\delta_1$, we use the fundamental system of solutions of the homogeneous Equation (2), given with precision $0(\varepsilon)$ in [15,16]

$$\varsigma_1 = \exp(\Gamma\tau)\sin(\tau - \sigma)/\sin\sigma, \varsigma_2 = \exp(-\Gamma\tau)\sin(\tau + \sigma)/\sin\sigma. \tag{5}$$

Here and above, the following designations are introduced: $\tau = \frac{1}{2}\Omega t$ is dimensionless time, $2\delta_1 = \cos 2\sigma$ (using σ instead δ_1 makes it easier to convert formulas), $\Gamma = (\varepsilon/2)\cdot\sqrt{2^{-2} - \delta_1^2} = (\varepsilon/4)\sin 2\sigma$ is increment of instability under $\delta_1 \le 1/2$. For values ε comparable to one, we can use Mathieu functions instead of system (5).

The solution of the inhomogeneous Equation (3) using the system (5) is written at form [17,18]

$$\varsigma(\tau) = \int_0^\tau f(\xi)[\varsigma_1(\xi)\varsigma_2(\tau) - \varsigma_2(\xi)\varsigma_1(\tau)]W^{-1}(\xi)d\xi, \tag{6}$$

where $W(\xi) = \varsigma_1(\xi)\dot{\varsigma}_2(\xi) - \varsigma_2(\xi)\dot{\varsigma}_1(\xi)$ is the Vronsky determinant of the system (5). We note that the approximation can be taken for further calculations that this determinant equals to 1. As an example, we select several functions $f(\tau)$ below:

1. exponentially decreasing cosine $f_1(\tau)$,
2. a segment of the cosine $f_2(\tau)$ with frequency ω_1 and duration τ_1,
3. a periodic sequence of N rectangular pulses. For certainty, we assume that their amplitudes are the same and equal to 1, and the periods coincide.

The easiest way to assess the impact on the environment, to take the Dirac function $f(\tau) = \delta(\tau - \tau_0)$, the spectrum of which is constant over the entire frequency range. By integrating into the formula (6) with such a source, for $\tau > \tau_0$ we have

$$\varsigma = \exp(-\Gamma\tau + \Gamma\tau_0)\frac{\sin(\tau+\sigma)}{\sin\sigma}\sin(\tau_0 - \sigma) - \exp(\Gamma\tau - \Gamma\tau_0)\frac{\sin(\tau-\sigma)}{\sin\sigma}\sin(\tau_0 + \sigma). \tag{7}$$

It follows from (7) that in this case both proper modes are excited equally effectively: increasing and decreasing with time. Moreover, this solution can be regarded as an approximate Green function of Equation (3).

As the next example, consider the source in the form of an exponentially decreasing cosine over time

$$f_1(t) = \exp(-\alpha t) \cdot \cos(\omega_1 t + \varphi) \cdot \theta(t). \tag{8}$$

The duration of this source is limited by a parameter α, the beginning of the source action is determined by the Heaviside step function $\theta(t) = 1$ for $t > 0$, $\theta(t) = 0$ for $t < 0$ and the source phase φ has a strong influence on the solution. Reduced to a dimensionless form, Formula (8) contains new parameters $\alpha_1 = 2\alpha/\Omega, \chi = 2\omega_1/\Omega$ and is transformed to

$$f_1(\tau) = \exp(-\alpha_1\tau) \cdot \cos(\chi\tau + \varphi)\theta(\tau). \tag{9}$$

In contrast to the previous example the spectrum of this source depends on the frequency and is given by the formula

$$S(\omega) = \frac{(\alpha + i\omega)\cos\varphi - \omega_1\sin\varphi}{\omega_0^2\left[1 - \omega^2/\omega_0^2 + id\omega/\omega_0\right]}, \tag{10}$$

where $\omega_0^2 = \alpha^2 + \omega_1^2$, $d = 2\alpha/\omega_0$. For $d \leq 1$ the spectrum module $\left|S(\omega)\right|$ has a maximum near $\omega = \omega_0$ [19]. Substituting the source (9) into Formula (6), we get a linear approximation of the displacement $\varsigma(\tau)$. Calculations, although quite cumbersome, do not cause difficulties; here we only give the part of the calculation result that contains an increase in time

$$\varsigma_1 \approx \exp(\Gamma\tau)\sin(\tau - \sigma)\frac{a\sin(\varphi-\sigma) + b\cos(\varphi-\sigma)}{(a^2+b^2)\sin\sigma}, \tag{11}$$

where $a = \alpha_1 + \Gamma, b = (2\omega_1/\Omega) - 1$. The following conclusions can be drawn from the above formula. For a given power source, the growth of wave disturbances in time is possible if certain conditions are met. If the fraction in (11) has a maximum, this increase will be the largest. If the fraction in formula (11) turns to zero, then there is no a growth disturbance. The dependence of the function $y(\varphi)$

$$y(\varphi) = \frac{a\sin(\varphi-\sigma) + b\cos(\varphi-\sigma)}{(a^2+b^2)\sin\sigma} \tag{12}$$

is presented in Figure 1. For constructing this figure, the parameters were selected in accordance with the numerical estimates in Section 4 for the operating conditions of the SURA facility at which generation of IGWs is possible.

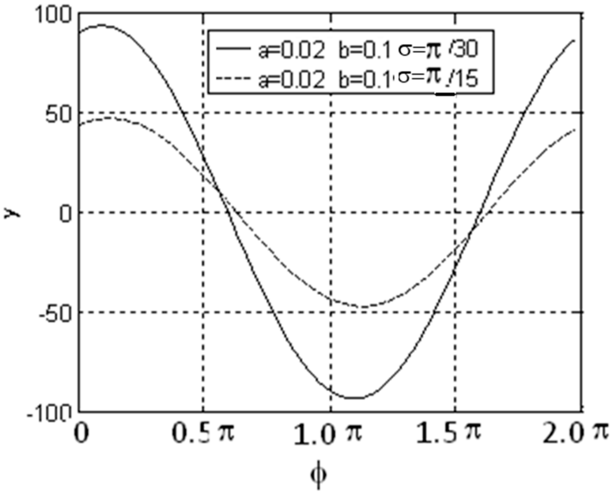

Figure 1. Dependence of the amplitude displacement on phase φ under $\alpha = 10^{-4}c^{-1}$, $\omega_1 = 5 \cdot 10^{-3}c^{-1}$, $d = 4 \cdot 10^{-2}$, $\varepsilon = 4 \cdot 10^{-2}$, $\sigma_1 = \pi/30$, $\sigma_2 = \pi/15$.

In practice high power heating facilities are often used in operation mode, in which their pulse activation during τ_0 is repeated for a time after a pause of duration τ_p with a period equal to T_1. We can model sources to this mode of operation for $0 < \tau < \tau_1$

$$f_2(t) = \cos(\chi\tau + \varphi), \tag{13}$$

$$f_3(\tau) = 2[\theta(\cos(\chi\tau + \varphi)) - 1/2]. \tag{14}$$

In this case, the solutions are almost identical for the same time of operation of the sources $0 \le \tau \le \tau_1$. Increasing in time the part of the solution under exact resonance conditions $\delta_1 = 0$, $\sigma = \pi/4$ is represented as

$$\varsigma(\tau) \simeq \frac{2\sqrt{2}}{\pi\Gamma}[\exp(\Gamma\tau)\sin(\tau - \pi/4)\sin(\varphi - \pi/4)]. \tag{15}$$

As follows from this formula, under resonant conditions and $\varphi = 3\pi/4$ the amplitude of perturbations increases, the stronger than the longer the operating time of the source.

The Figure 2 shows the dependence of the displacement on time for a source of N pulses with a repetition period $T_1 = 2\pi$, calculated using the full formula taking into account the part that is decreasing in time (thick line). For comparison the same figure shows a similar dependence on the source in the form of a segment of the cosine (thin line) with the same period and the same operating time $\tau_1 > 30$.

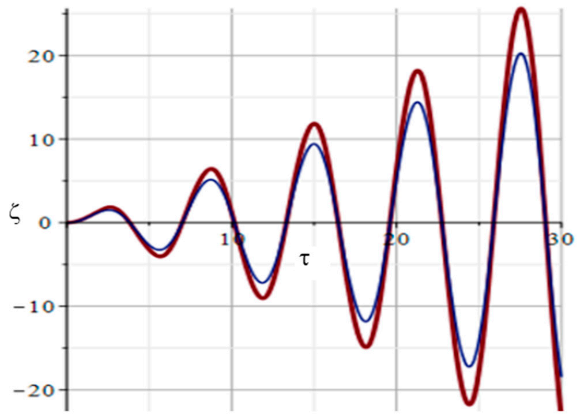

Figure 2. Time dependence of displacement for pulse sources (14) and a segment of the cosine (13) at $N = \tau_1/T_1$,$\tau_0 = \tau_p$. There is a slight difference of 4/π in the oscillation amplitudes for different sources.

In experiments on the IGW generation using the SURA heating facility, one of the modes of operation of powerful transmitters was a pulsed one with a period of 15–30 min, including switching on-switching off mode (at regular intervals) in sessions with a total duration of 2–3 h. In this mode, 4–12 heating pulses were emitted into the ionosphere.

4. Numerical Estimation

We now give numerical estimates of the effect of parametric resonance on the excitation of the IGWs by an external source. We apply the simplified dispersion equation of internal gravity waves in the Boussinesq approximation $\omega = \omega_g \frac{k_x}{k_z}$ [20], where ω_g is the Brunt–Väisälä frequency, k_x, k_z are the components of the wave vector along the x and z axes. Let's take into account the necessary ratios $\Omega = 2\omega$, $\varepsilon = \frac{k_x V_2}{\Omega k_z H} = \frac{V_2}{2\omega_g H}$, $\Gamma = \frac{\varepsilon}{4}\sin 2\sigma$. Let's set the parameters typical for the atmosphere and waves: $V_2 = 10\ m/s$, $\omega_g = 2{\cdot}10^{-2} s^{-1}$, $H = 6{\cdot}10^3\ m$, $\omega = 5{\cdot}10^{-3}\ \mathrm{s}^{-1}$, $2\sigma = \frac{\pi}{4}$. With the indicated values of the parameters, we obtain $\varepsilon = 4\cdot 10^{-2}$, $\Gamma = 7{\cdot}10^{-3}$. The size value of the increment is approximate $\Gamma = 3{,}6{\cdot}10^{-5} s^{-1}$. A not very significant value of the increment can be partially corrected by an increase of more than an order of magnitude at resonance from $y(\varphi)$ (see Figure 1). Note that the role of the variable component of the wind velocity can be played by IGWs of larger scales [21]. The condition of the parametric resonance considered in the article is the equality $\Omega = 2\omega$ between the frequencies of the generated ω and background field Ω. This condition makes it possible to distinguish between wave natural wave disturbances from generated by the heating facility.

5. Conclusions

The problem of excitation of internal gravity waves (IGWs) in the upper atmosphere by an external source of a limited duration of operation was investigated. An isothermal atmosphere was chosen as a model for the propagation environment of IGWs in the presence of a uniform wind that changes over time according to the harmonic law. For the vertical component of the displacement of an environment, the Mathieu equation with zero initial conditions was solved with the right part simulating the effect of a powerful heating facility on the ionosphere. In the case of small amplitude of the variable component of the wind the time dependence of the vertical displacement under parametric resonance conditions was obtained. The numerical value of increment for IGWs was estimated. Thus, under conditions of parametric resonance in the presence of a variable component of the wind, the excitation of internal gravity waves in the upper atmosphere during the operation of powerful radio transmitters can be realized. In support of this conclusion, we quote references [22–25], in which the authors report on the experimental observation of disturbances with IGW parameters during the periods of operation of the powerful SURA heating facility.

When the performed calculations compare with experimental data, the following circumstances should be borne in mind. Parametric resonance during IGW generation due to the operation of powerful radio transmitters is realized only in the presence of a variable wind component in the atmosphere. In this case, it is necessary to fulfill the resonance conditions $\omega = 2\Omega$ and certain phase relationships between the wind variations and the operation of the transmitters.

For a more convincing conclusion about the possibility of generating IGWs in the operation of high-power high-frequency radio transmitters, additional research is required. In theory, it is necessary to solve a number of problems:

1. To develop a physical model of a force source and an energy source (possibly significant energy to the upper atmosphere.
2. To find the relationship between the amplitude of atmospheric disturbances and the effective power of radio transmitters based on solving the nonlinear problem of waves in the atmosphere with specified localized sources. It is necessary to use a model of the medium with parameters as close as possible to those actually measured at the location of the transmitters (taking into

account the wind movements of the medium that change over time and space). Such calculations probably require the use of a supercomputer.

3. To analyze the possibility of solving the inverse problem: to restore the parameters of the atmosphere (possibly sources) from the measured space-time characteristics of disturbances.

In the experimental plan, it is necessary to analyze the available data, supplemented with new ones, in order to determine the current sources of their origin associated with high-power radio transmitters. It is necessary to take into account the anisotropic nature of IGW propagation in the presence of wind in the atmosphere. In observation points located to windward of localized sources, disturbances may not be detected if the group velocity of the waves is less than the wind velocity. The specified atmospheric instability with increment $\Gamma = (\varepsilon/4)\sin 2\sigma$ can also enhance natural IGVs when they pass through the disturbed heating region in the upper atmosphere.

In conclusion, we point out that the tasks of the propagation of IGWs in environments with periodically changing of spatial parameters are related to the problem considered. Complete information about these tasks can be found in reviews [26,27] and many monographs, for example [28,29].

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