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The Effect of Wall Shear Stress on Two Phase Fluctuating Flow of Dusty Fluids by Using Light Hill Technique

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Citation: Khan, D.; Rahman, A.u.; Ali, G.; Kumam, P.; Kaewkhao, A.; Khan, I. The Effect of Wall Shear Stress on Two Phase Fluctuating Flow of Dusty Fluids by Using Light Hill Technique. *Water* **2021**, *13*, 1587. <https://doi.org/10.3390/w13111587>

Academic Editors:
Maksim Pakhomov and
Pavel Lobanov

Received: 22 April 2021
Accepted: 30 May 2021
Published: 4 June 2021

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Abstract: Due to the importance of wall shear stress effect and dust fluid in daily life fluid problems. This paper aims to discover the influence of wall shear stress on dust fluids of fluctuating flow. The flow is considered between two parallel plates that are non-conducting. Due to the transformation of heat, the fluid flow is generated. We consider every dust particle having spherical uniformly disperse in the base fluid. The perturb solution is obtained by applying the Poincare-Lighthill perturbation technique (PLPT). The fluid velocity and shear stress are discussed for the different parameters like Grashof number, magnetic parameter, radiation parameter, and dusty fluid parameter. Graphical results for fluid and dust particles are plotted through Mathcad-15. The behavior of base fluid and dusty fluid is matching for different embedded parameters.

Keywords: wall shear stress; oscillating two-phase fluctuation flow; heat transfer; magnetohydrodynamic (MHD); dust particles

1. Introduction

Fluid flow inserted with similar non-miscible inert solid particles is admitted as the two-phase structure of fluid. The fields of technology and engineering accommodate various uses of the flow of the gas-particles mixture. Nuclear reactors with gas-solid feeds, cooling of nuclear reactors, solid rocket exhaust nozzles, electrostatic drizzle, ablation cooling, polymer technology, the moving blast waves over the Earth's surface, the distillation of crude oil, environmental poison, the industry of petroleum, fluidized beds, transmit of powdered materials, physiological flows and various other fields of technologies are the practical examples where the dusty fluid are mostly applicable [1,2]. Different types of multiphase flows can be seen in the literature. However, the most familiar type of such flows, which are multiphase is two-phase flows. In these multiphase flows, liquid and liquid flow, gas and liquid flow, liquid and solid flow, solid and gas flow are frequently discussed by researchers [3,4]. In turbulent flows, the liquid droplets vaporization is a crucial multiphase flow area due to an extensive measure of energy used for heating. Electrical and impulse power generation is borrowed against liquid fuels that have been transformed into atomized sprays. The cooperation of the discharged vapors of droplets

with the turbulent flow formed immensely complicated problems in such situations. A comprehensive explanation of the fundamental flow processes and droplets involved in these situations can be initiated [5–7].

The researchers are very keen to study the dusty multiphase flows over the last few decades because of their numerous applications. In this regard, various researchers investigate preliminary and theoretical modeling of particle phase viscosity in a multiphase dusty fluid [8–11], but keep in mind Soo [12] is the first who presented the fundamental theory of multiphase flows. Other useful applications consist of dust particles in nuclear processing, in boundary layers contains soil emancipation which is occurred by natural winds, in aerodynamic refusal of plastic sheets, landing vehicle in a cloud designed during a nuclear detonation, and lunar surface extinction by the dust entrainment are investigated by many researchers in the literature [13–16]. Zhou et al. [17] discuss the fluid particles in translational motion and study the converging and diverging behavior in a microchannel. In another paper, Zhau et al. [18] established a model in which he examines the deformable interaction of particles, and he studies the contact of dielectrophoresis (DEP) thoroughly in the presence of an electric field generated by the alternating current. Recently, Ali et al. [19] present an article on fluctuating flow of absorbing heat viscoelastic dusty fluid with free convection and MHD effect past in a horizontal channel. In this reported study, the researcher discussed the consequence of different parameters on fluid velocity and particles. They investigate the solution for the fluid velocity and the dust particle's velocity by applying the Poincare-Light Hill technique. Similarly, in another paper, Ali et al. [20] established the study about the dusty viscoelastic fluid of two-phase fluctuating flow with heat transfer between rigid plates which are non-conducting and examine the connected effect of the heat transfer and magnetic field on the dusty viscoelastic fluid which is conducting electrically with the help of Light-Hill technique.

Furthermore, Attia et al. [21] work out the fallout of different substantial parameters on the steady flow of MHD incompressible non-Newtonian Oldroyd dusty fluid in a circular pipe and also study the consequence of Hall current. In this particular article, the author investigates the characteristics of the particle phase viscosity and non-Newtonian fluid, flow rates in terms of volume, and the coefficient of skin fraction for both the particle phase and fluid. Keeping in mind Ali et al. [22] present an article about second-grade fluid in which they present closed-form solutions of free convection unsteady flow. In this article, the author investigates the influence of different parameters on the velocity profile of second-grade fluid-like Grashof number, Prandtl number, and viscoelastic parameter.

Multiphase flow regimes have significant consideration because of their useful applications [23]. There is two specific access that has been used commonly. According to this particular access, if we study the first way, we can observe that the continuous phase's motion is not significantly affected by the dispersed phase. This phase's motion is so minute. This approach is commonly admitted as the "alter phased approach," which is also familiar with the Lagrangian approach. This approach is applied broadly in situations where the particles deal with dispersed phase like in sprays, atomization, and droplets [23].

On the other hand, the second approach is related to the two phases combined so that each phase precisely manipulates the altitude and motion of the other stage [23]. The second approach is called the method of dense phase. It is also known as the Eulerian approach. This approach is beneficial in solid-gas flows [24], aerial transmitting [25], in fluidization and is defined for a variety of uses and applications in suspensions [26,27].

The Light-Hill method was introduced by M. J. Light-Hill in 1949 [28]. He obtained uniformly reasonable approximate solutions for various classes of partial and ordinary differential equations. After that, this method was used by many researchers successfully in the research field and solve ordinary and partial differential Equations [29]. As far as wall-shear stress is concerned, many researchers present different problems in discussing wall-shear stress and interpret other behaviors of the various parameters on the fluid velocity. In this regard, Grobe et al. [30] study the boundary layer wall shear-stress of the turbulent wind tunnel with a high Reynolds number. In a similar way, Amili et al. [31]

represent wall shear-stress distribution in a turbulent flow in the channel and discuss the different influence of parameters on the fluid’s velocity. Keeping in mind, Orlu et al. [32] described the study on the fluctuating wall shear stress in zero pressure-gradient turbulent boundary layers flow. Recently, Mob et al. [33] present a paper in which they describe the study of the impact of wall-shear stress on the evolution and execution of electrochemically alive biofilm. The author referred to some recent contributions in the field [34–37].

In the literature mentioned above, the authors have considered different types of dusty fluids. Some of the dusty fluids are electrically conducting, and some are investigated with heat transfer and energy equations. According to our utmost knowledge, no investigation has been disclosed about the effect of wall shear stress on two phases of the fluctuating flow of dusty fluid by solving the Light-Hill technique. Therefore, we have to examine and study the different behavior of velocity of the problem theoretically and graphically in the present work. Hence, the purpose of this study is to interrogate the flow of fluid ingrained with dust particles along with transfer heat over the bounded plates.

2. Formulation of the Problem

The incompressible, unidirectional, and one-dimensional electrically conducting the unsteady flow of dusty fluid along the x-axis has been considered between two parallel plates. The magnetic field B_0 is transversely applied to the fluid, and due to the small size of the induced magnetic field, the emission is so small; therefore, the electric field inside is ignored. Also, flow origination is induced by heat transfer, the upper plate temperature, and the lower plate temperature T_w . The wall shear stress is applied to the lower plate, where the upper plate fluctuates with free stream velocity $U(t) = u_0(1 + \frac{\epsilon}{2}(e^{i\omega t} + e^{-i\omega t}))$. $u(y, t)$ and $v(y, t)$ denoted the velocities of fluid and dust particles, respectively. As shown in Figure 1. The effect of the equation of energy radiation is also getting hold of into description. By apply the assumptions of Bossiness resemblance and in sequence to escape similarities, the equation of momentum and energy is.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{K_0 N_0}{\rho} (v - u) - \frac{\sigma B_0^2 u}{\rho} + g\beta_T (T - T_\infty) \tag{1}$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

where $-\frac{\partial q_r}{\partial y} = 4\alpha_0(T - T_\infty)$.

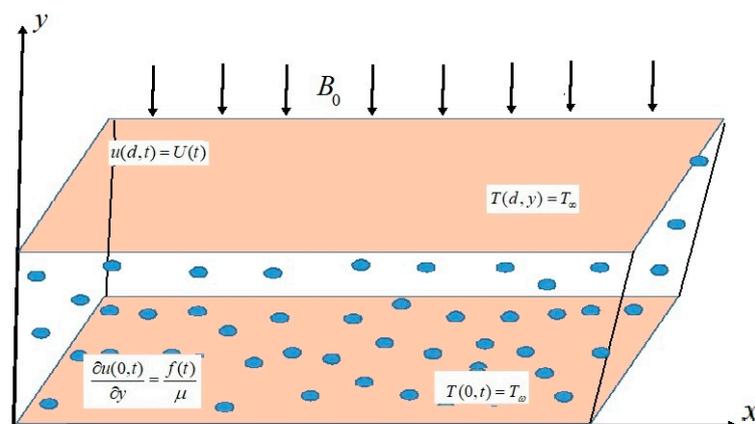


Figure 1. Geometry of the problem.

Equations (1) and (2), which are constantly spreader in the viscoelastic fluid, $q_r, \nu, c_p, k, g, \rho, \alpha_0, \sigma, B_0, N_0, \beta_T$ and K_0 are show the radiation heat flux, kinematic viscosity, specific heat capacity, thermal conductivity, gravitational acceleration, fluid density, mean radiation absorption coefficient, electrical conductivity, magnetic field, number of density of the dust

particle which is supposed to be constant, coefficient of thermal expansion, and stock’s resistance coefficient respectively.

This can be represented by Newton’s law of motion as:

$$m \frac{\partial v}{\partial t} = K_0(u - v) \tag{3}$$

where m represents the average mass of dust particles.

The physical boundary conditions are:

$$\left. \begin{aligned} \frac{\partial u(0,t)}{\partial y} &= \frac{f(t)}{\mu} \quad t > 0, \quad u(d,t) = U(t) \\ T(0,t) &= T_\omega, \quad T(d,t) = T_\infty \end{aligned} \right\} \tag{4}$$

where $U(t) = u_0(1 + \frac{\epsilon}{2}(e^{i\omega t} + e^{-i\omega t}))$.

To find the velocity of the dust particle let assume the following solution, obtained through Poincare-Light Hill Technique [28]:

$$v(y,t) = v_0(y)e^{i\omega t} \tag{5}$$

By using Equation (5) in Equation (3) velocity of the dust particles will be.

$$v(y,t) = \left(\frac{K_0}{mi\omega + K_0}\right)u(y,t) \tag{6}$$

Put Equation (6) in Equation (1). So, Equation (1) becomes,

$$\frac{\partial u}{\partial t} = v \frac{\partial^2 u}{\partial y^2} + \frac{K_0 N_0}{\rho} \left\{ \left(\frac{K_0}{mi\omega + K_0}\right)u - u \right\} - \frac{\sigma B_0^2 u}{\rho} + g\beta_T(T - T_\infty) \tag{7}$$

By use of the following dimensionless variables.

$$u^* = \frac{u}{u_0}, y^* = \frac{y}{d}, t^* = \frac{u_0 t}{d}, \theta^* = \frac{T - T_\infty}{T_\omega - T_\infty} \tag{8}$$

For simplicity (*) sign has been ignored. So Equation (2), and Equation (7) becomes:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (M + K_1 - K_2)u + Gr\theta \tag{9}$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \tag{10}$$

with dimensionless physical conditions are:

$$\left. \begin{aligned} \frac{\partial u(0,t)}{\partial y} &= f(t) \quad , \quad u(1,t) = U(t) = 1 + \frac{\epsilon}{2}(e^{i\omega t} + e^{-i\omega t}) \\ \theta(0,t) &= 1 \quad , \quad \theta(1,t) = 0 \end{aligned} \right\} \tag{11}$$

where,

$$M = \frac{\sigma B_0^2 d^2}{\rho v}, Gr = \frac{g\beta_T d^2 (T_\omega - T_\infty)}{u_0 v}, K_2 = \frac{K_0^2 N_0 d^2}{\rho v(mi\omega + K_0)}, K_1 = \frac{K_0 N_0 d^2}{\rho v}, Pe = \frac{\rho c_p u_0 d}{k}, N^2 = \frac{4\alpha_0^2 d^2}{k}.$$

where h_s is a heat transfer coefficient, M is a magnetic variable, Gr is Grashof number, K_1 dusty fluid variable, K_2 Dust particles parameter, Pe Peclet number, N radiation variable.

Consider the following assume periodic solutions for energy equation obtained through Poincare-Light Hill Technique in [28];

$$\theta(y,t) = \theta_0(y) + \theta_1(y)e^{i\omega t} \tag{12}$$

By solving energy equation with the help of above assumed periodic solution we get:

$$\theta(y, t) = \frac{\sin(N - Ny)}{\sin(N)} \tag{13}$$

By putting Equation (13) in Equation (9), we get:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (M + K_1 - K_2)u + Gr \left\{ \frac{\sin(N - Ny)}{\sin(N)} \right\} \tag{14}$$

3. The Solution to the Problem

Basic idea of Poincare-Light Hill Technique and model framework [28]:

The original model of lighthill is firstly introduced by Lighthill in 1949 [28]

$$(x + \epsilon y) \frac{dy}{dx} + q(x)y = r(x), \quad 0 \leq x \leq 1 \tag{15}$$

$$y(1) = b > 1, \tag{16}$$

With condition

$$q(0) \neq 0. \tag{17}$$

This last condition is one to which we shall return later. If one tries to find y as a series in ϵ ,

$$y \sim y_0(x) + \epsilon y_1(x) + \epsilon^2 y_2(x) + \dots \tag{18}$$

$$x \frac{dy_0}{dx} + q(x)y_0 = r(x) \tag{19}$$

whose homogeneous part has a regular singular point at the origin. The original equation, as a simple phase plane analysis shows, does not have a singularity at the origin, and thus the perturbation series has a false singularity.

Lighthill’s idea was to move the singularity back out of the domain of interest by introducing a new, slightly stretched, independent variable z by the implicit equation

$$x = z + \epsilon x_1(z) + \epsilon^2 x_2(z) + \dots \tag{20}$$

And then to look for a solution y in the form

$$y = y_0(x) + \epsilon y_1(z) + \epsilon^2 y_2(z) + \dots \tag{21}$$

On the light of Equation (21) the following assume periodic solutions for Equation (14) obtained through Poincare-Light Hill Technique in [28] is considered as follow.

$$u(y, t) = F_0(y) + \frac{\epsilon}{2} \left(F_1(y)e^{i\omega t} + F_2(y)e^{-i\omega t} \right) \tag{22}$$

To find out the results for $F_0(y)$, $F_1(y)$, and $F_2(y)$, incorporate Equation (22) in (14) we get:

$$F_0(y) = \left(1 - \frac{(f(t) + H)\sinh(\sqrt{m_1})}{\sqrt{m_1} \cdot \cosh(\sqrt{m_1})} \right) \cosh(y\sqrt{m_1}) + \left(\frac{f(t) + H}{\sqrt{m_1}} \right) \sinh(y\sqrt{m_1}) + A \left\{ \frac{\sin(N - Ny)}{\sin(N)} \right\} \tag{23}$$

where $H = \frac{A \cdot N \cdot \cos(N)}{\sin(N)}$, and $m_1 = M + K_1 - K_2$, $A = \frac{Gr}{m_1}$.

$$F_1(y) = \frac{\cosh(y\sqrt{m_2})}{\cosh(\sqrt{m_2})} \tag{24}$$

where $m_2 = m_1 + i\omega$

$$F_2(y) = \frac{\cosh(y\sqrt{m_3})}{\cosh(\sqrt{m_3})} \tag{25}$$

where $m_3 = m_1 - i\omega$.

In last, we put the results in Equation (22) from Equations (23)–(25), we obtained the following output:

$$u(y, t) = \left\{ \begin{aligned} &\left(1 - \frac{(f(t)+H)\sinh(\sqrt{m_1})}{\sqrt{m_1} \cdot \cosh(\sqrt{m_1})} \right) \cosh(y\sqrt{m_1}) + \left(\frac{f(t)+H}{\sqrt{m_1}} \right) \sinh(y\sqrt{m_1}) \\ &+ A \left\{ \frac{\sin(N-Ny)}{\sin(N)} \right\} + \frac{\varepsilon}{2} \left(\frac{\cosh(y\sqrt{m_2})}{\cosh(\sqrt{m_2})} \right) e^{i\omega t} + \frac{\varepsilon}{2} \left(\frac{\cosh(y\sqrt{m_3})}{\cosh(\sqrt{m_3})} \right) e^{-i\omega t} \end{aligned} \right\}. \tag{26}$$

4. Graphical Results and Discussion

In this work, the unsteady motion because of an infinite plate that addresses wall-shear stress to a two-phase fluctuating flow of dusty fluids is considered utilizing the light hill technique. The solutions that have been achieved satisfy all the introduced initial and boundary conditions. To find out some specific and vital information about the repercussions of different flow parameters on dusty magnetic particles and fluid velocities. Certain numerical simultaneous results have been made with Mathcad-15 software. These influences of various physical parameters like magnetic parameter M , dusty fluid parameter K_2 , Grashof number Gr and radiation variable N are graphically shown in this section. According to these graphs, we get different results for the profile of fluid velocities and the velocities of dusty particles, briefly discussed in the following paragraph. The estimated values of parameters for all figures are taken from Table 1.

Table 1. Description of variables and parameters for graphical results [19].

Parameter	Description	Assumed Values
t	Time	0.5
M	Magnetic variable	2
K_1	Dust particles parameter	5
K_2	Dusty fluid variable	1
Gr	Grashof number	2
Pe	Peclet number	1
N	Radiation variable	2
ε	—————	0.001
ω	—————	$\frac{\pi}{2}$

The obtained results are shown in Figures 2 and 3, respectively, which reflect the Grashof number’s behavior on the velocity profile of fluid and dusty particles. We observed the direct variation between the base fluid’s velocities and dust particles’ velocity in these graphs. By increasing the Grashof number, the rate of fluid and dust particles also increases. According to Grashof number physics, we know that it is the ratio of buoyancy forces and drag forces. Therefore, by increasing this number, the buoyancy forces increase, and the viscosity decreases, which is why the velocities of fluid and dust particles increase. Figures 4 and 5 displayed the fluid and dust particles velocities against the dusty parameter K_2 these graphs show that the fluid and dusty particles’ velocity also increases by increasing the dusty parameter. To illustrate the effect of radiation on both the velocities of dusty particles and base fluid, for this behavior, one can observe Figures 6 and 7, respectively. These figures show that when the radiation increases, the velocity of fluid and dust particles also increases. It is evident from the physics of radiation that by increasing the radiation, the fluid temperature and dust particles’ kinetic energy are also increased because of this

increase in temperature. The corresponding Figures 8 and 9 are plotted respectively for the investigation of the influence of magnetic parameter M on the profile of fluid velocity and also on the velocity of dusty particles. It is noted from these figures that the velocity of the fluid shortens monotonically due to the rise in magnetic parameter M . This reduction in fluid velocity is actually the implementation of magnetic force against the direction of fluid flow. These figures also clear that velocity profiles for the velocity of fluid are much larger than those for dusty particles' velocity. The affiliation of the radiation and the temperature also check out in this article. Figure 10 tells about the relation of radiation variable parameter N and temperature. It is clear from this specific graph that an increase in radiation variable parameter N occurs to a rise in the fluid temperature. Figures 11 and 12 highlight the velocity of the fluid and dust particle in 3D. It shows the better result for the t . Figure 13 is plotted for the comparison of our problem with Narahari and Pendyala [38] which shows a strong agreement. Both solutions are exactly overlapped which shows the correctness and validity of our solutions.

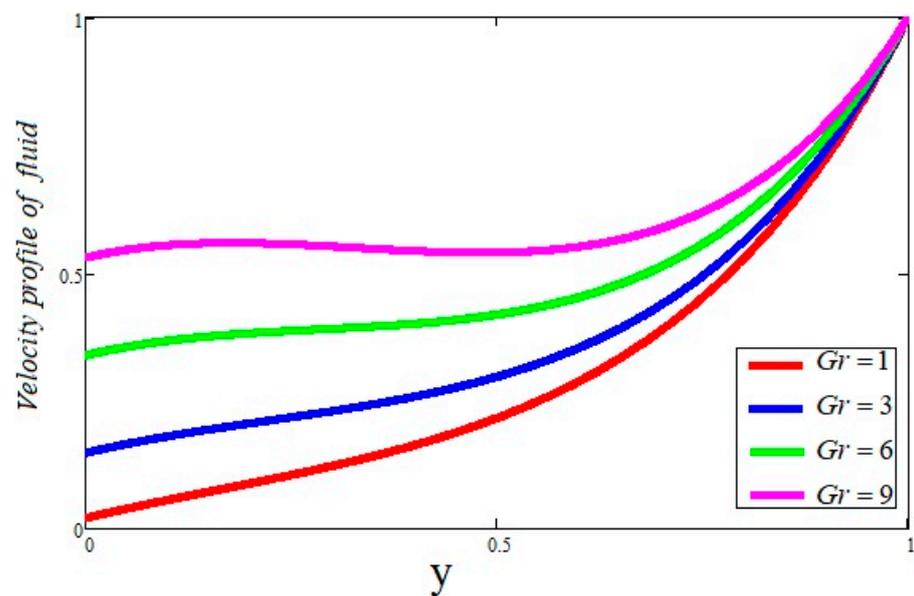


Figure 2. Impact of Gr on the profile of fluid velocity.

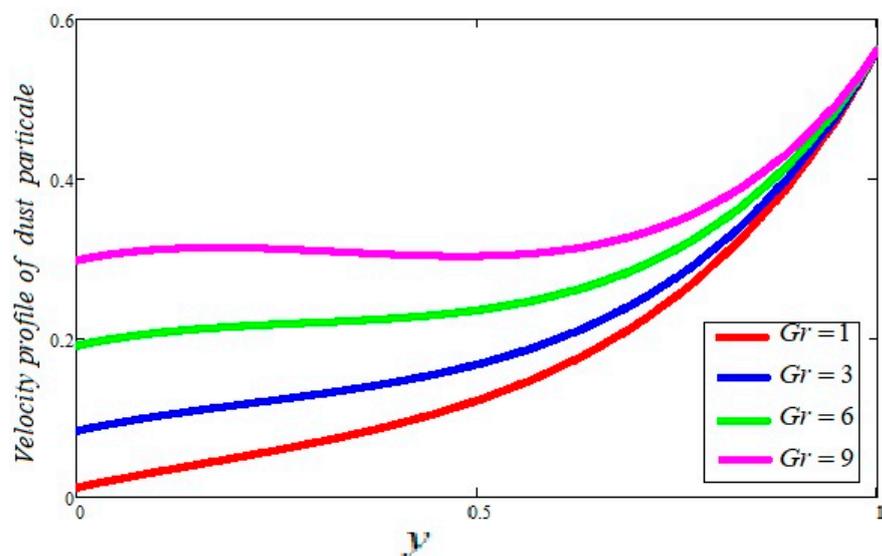


Figure 3. Impact of Gr on the profile of dust particles velocity.

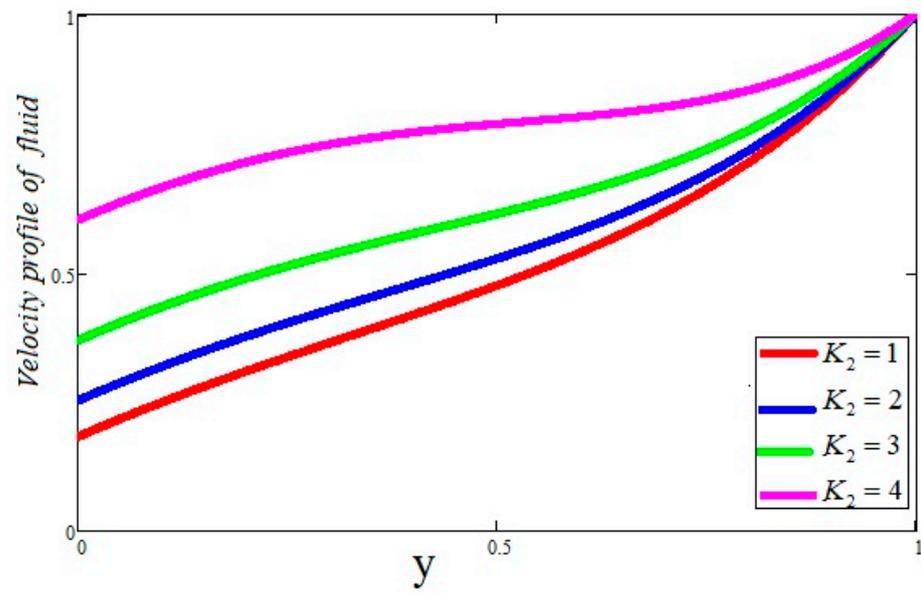


Figure 4. Impact of K_2 on the profile of fluid velocity.

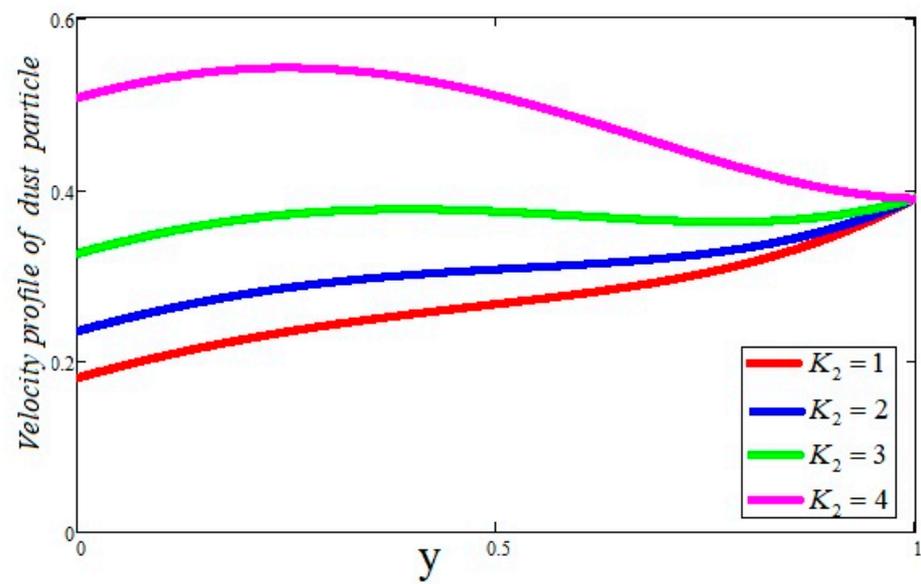


Figure 5. Impact of K_2 on the profile of dust particles velocity.

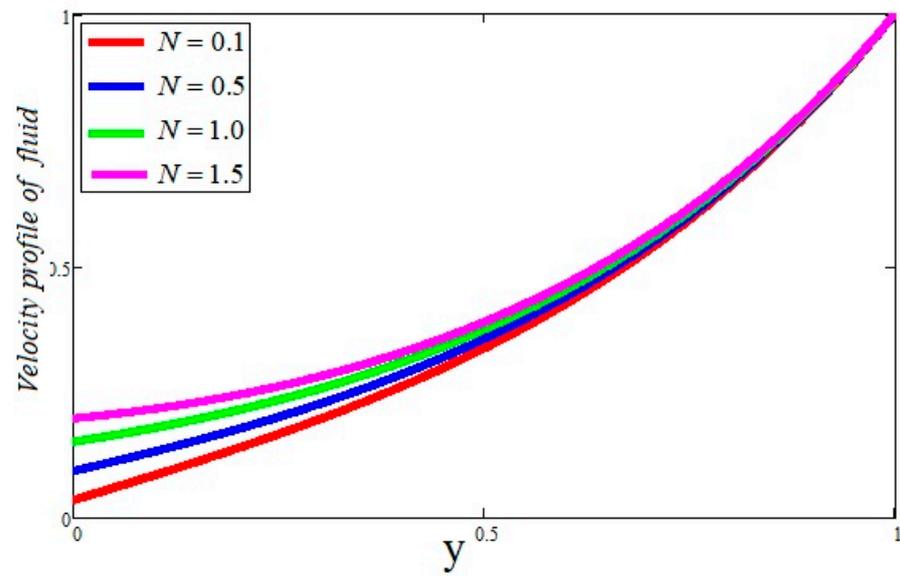


Figure 6. The behavior of N on velocity profile (fluid).

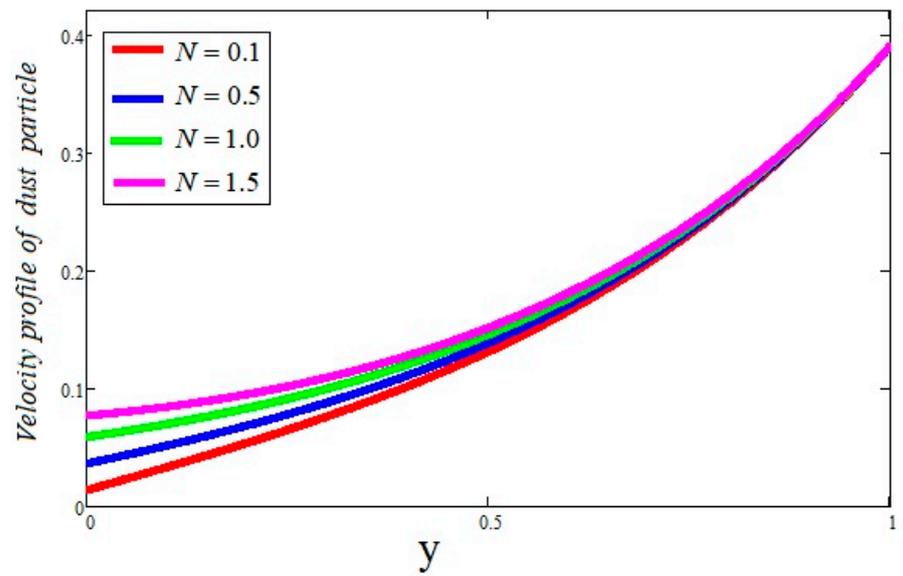


Figure 7. Impact of N on the profile of dust particle velocity.

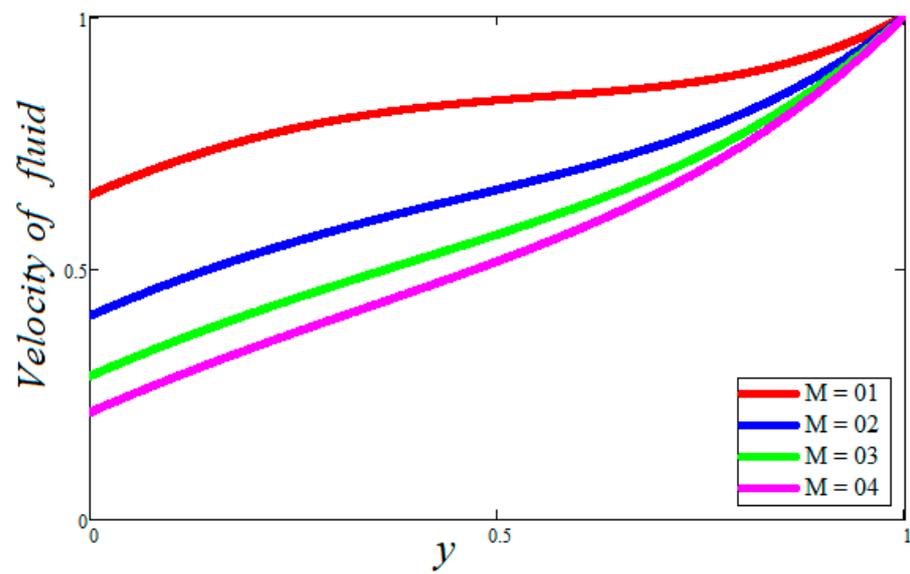


Figure 8. Impact of M on the profile of fluid velocity.

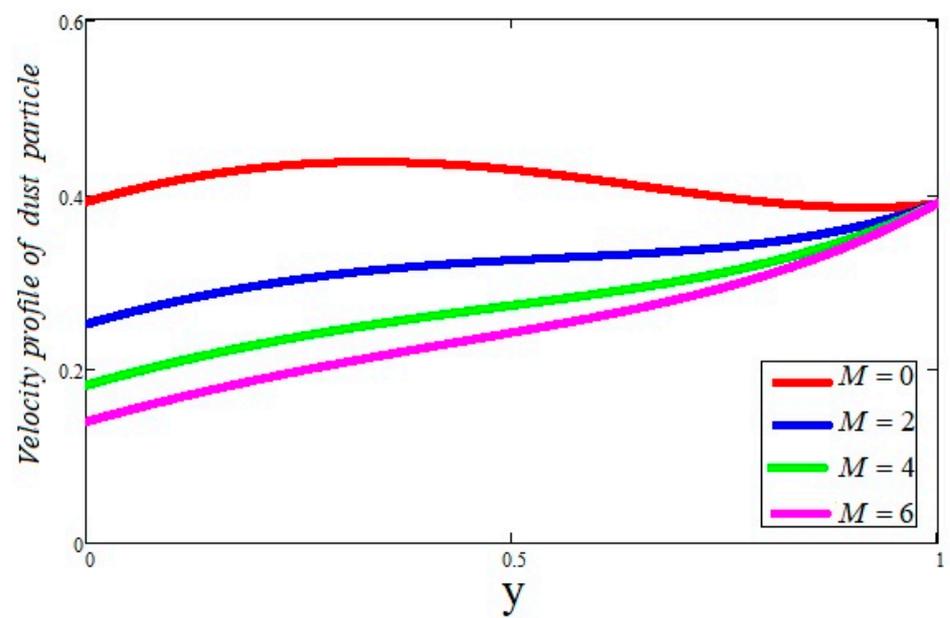


Figure 9. The impact of M on the profile velocity of dust particles.

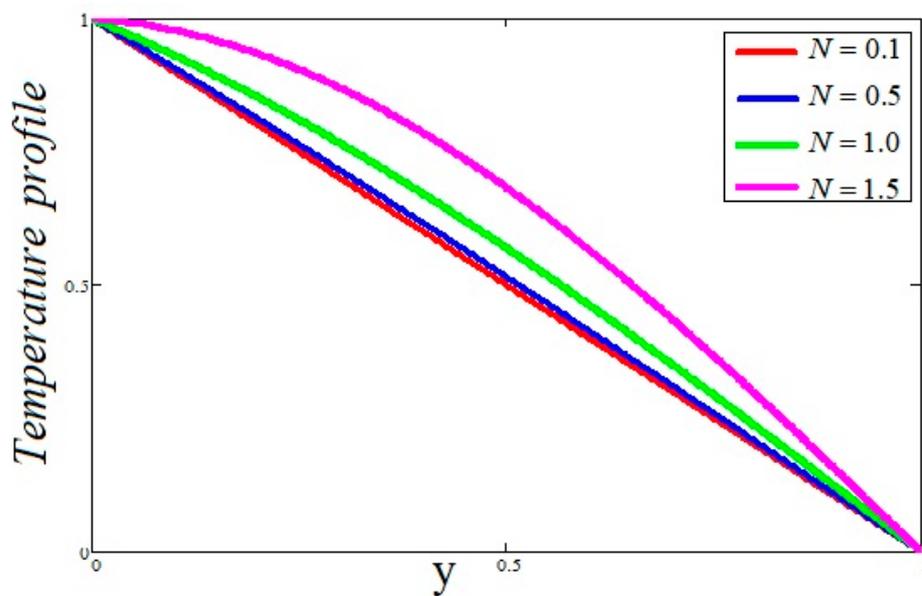


Figure 10. Impact of N on temperature.

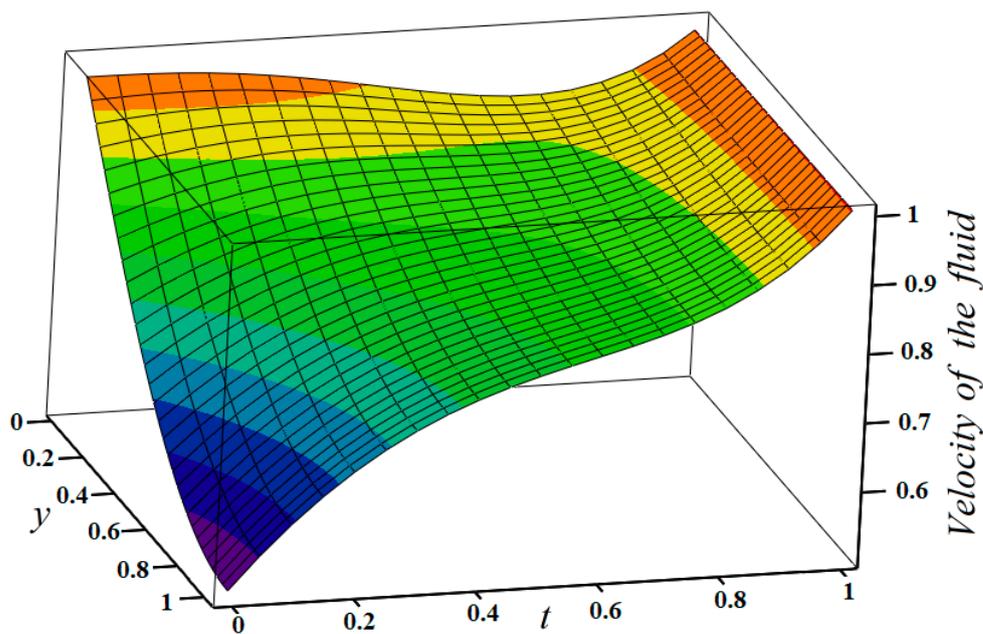


Figure 11. 3D plot of fluid velocity.

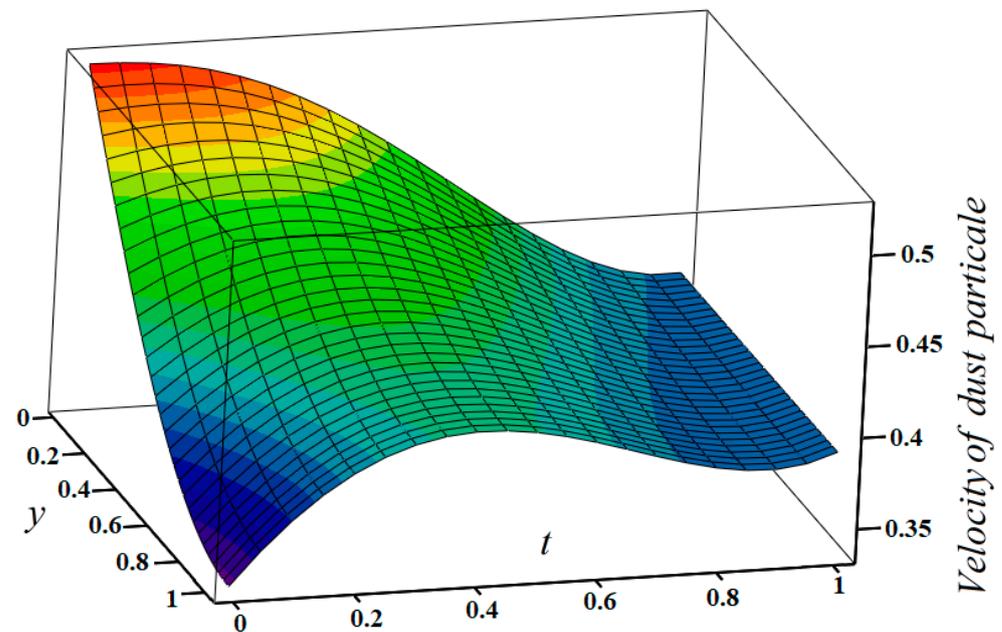


Figure 12. 3D plot of dust particle velocity.

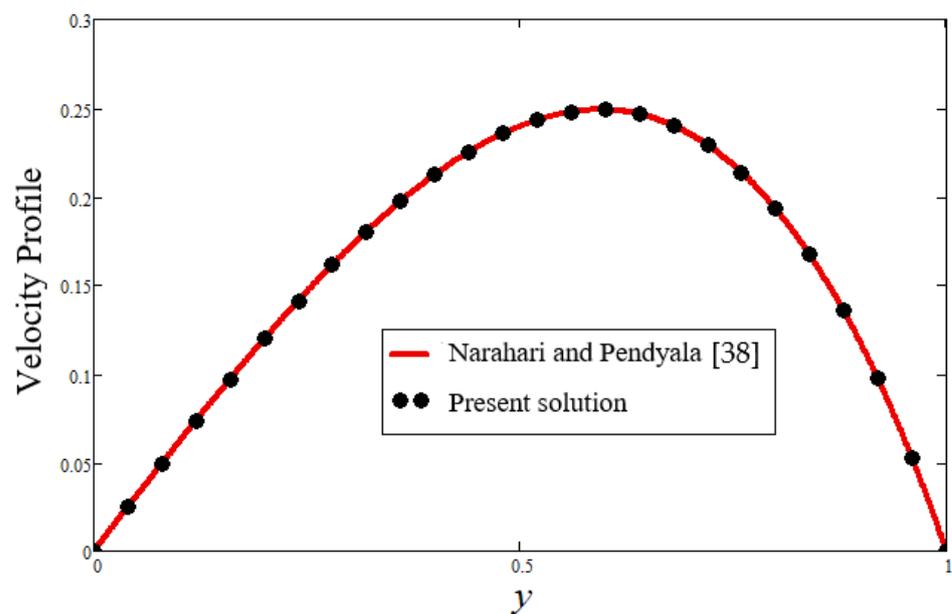


Figure 13. Comparison of the present solutions obtained in Equation (26) when $M = 0, K_1 = 0, u(1, t) = 0$ and $K_2 = 0$ with Narahari and Pendyala Equation (11).

5. Conclusions

A novel and theoretical analysis of wall share stress on the consequence of the various physical parameters on the MHD flow of two phases fluctuating flow of dusty fluid is examined along x-direction between two parallel plates, by Poincare-Light Hill Technique. It is pretended that the fluid flow is unidirectional, one dimensional, incompressible, conducting electrically, and heat convection with heat transfer is also appropriated in this problem. The ingrained dust particles are even pretended to be conducting and homogeneously dispersed in the fluid. The parametric consequence of the physical parameters on the profile of fluid velocity, dusty particles and temperature are examined comprehensively. It is observed that increase in Grashof number Gr , dusty parameter K_2 , radiation variable parameter N occurs an increase in both the velocities of fluid and dusty particles. The

increase in magnetic parameter M occurs a decrease in the velocities of fluid and dusty particles. The relation of radiation and temperature are also discussed graphically in this article and according to this relation the increase in radiation cause the increase in temperature. All the parameters show the identical behavior on fluid and dust particle velocity. While the wall share stress give the verity of velocity profile on the lower plate.

Author Contributions: D.K. model the problem. D.K and G.A. solved the modeled problem analytically. D.K. and G.A. draw the graphs. Results and discussions have reviewed by A.K. and P.K. reviewed the whole manuscript. Methodology, writing—original draft by A.u.R. Proof reading has performed by A.K, I.K. and P.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Thailand Science Research and Innovation (TSRI) Basic Research Fund: Fiscal year 2021 under project number 64A306000005 and Chiang Mai University” and “The APC was funded by Research Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors wish to thank the anonymous referees for their comments and suggestions. The authors acknowledge the financial support provided by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT. This research project is supported by Thailand Science Research and Innovation (TSRI) Basic Research Fund: Fiscal year 2021 under project number 64A306000005. Moreover, this research was supported by Chiang Mai University.

Conflicts of Interest: The authors declare no conflict of interest.

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