

Supplementary Material

Supplementary Material contains a detailed proof of the model-altered part of this paper. The unaltered part is only presented, not proved.

S1. Households

Agents in each location order consumption baskets according to Cobb-Douglas preferences.

$$U(C_n) = \prod_{j=1}^J (c_n^j)^{s^j} \quad (S1)$$

with shares s^j , over their consumption of final domestic goods c_n^j , bought at prices P_n^j , in all sectors. $\sum_{j=1}^J s^j = 1$. where I_n is income earned by agents residing in region n , $P_n =$

$\prod_{j=1}^J (P_n^j / s^j)^{s^j}$ is the ideal price index in region n , and U is determined in equilibrium. Hence,

$$U = \frac{I_n}{P_n}, n \in \{1, \dots, N\} \quad (S2)$$

S2. Firms

S2.1 Intermediate Goods

$$q_n^j(z_n^j) = z_n^j \left([T_n^j [h_n^j(z_n^j)]]^{\beta_n} [l_n^j(z_n^j)]^{(1-\beta_n)} \gamma_n^j \prod_{k=1}^J [M_n^{jk}(z_n^j)]^{\gamma_n^{jk}} \right)^{1-\alpha_n^j} [e_n^j(z_n^j)]^{\alpha_n^j} \quad (S3)$$

where the productivity level, z_n^j , follows the Fréchet distribution, T_n^j is the fundamental productivity, $h_n^j(\cdot)$ and $l_n^j(\cdot)$ denote the demand for land and labor respectively, $M_n^{jk}(\cdot)$ is the demand for final material inputs by firms in sector j from sector k , $\gamma_n^{jk} \geq 0$ is the share of sector j goods spent on materials from sector k , and $\gamma_n^j \geq 0$ is the share of value added in gross output. We assume that the production function has constant returns to scale, namely that $\gamma_n^j + \sum_{k=1}^J \gamma_n^{jk} = 1$. α_n^j is the emission elasticity of industry j in region n .

**Proof of the production function of intermediate goods (S3):*

the production of potential output is

$$y_n^j(z_n^j) = z_n^j [T_n^j [h_n^j(z_n^j)]]^{\beta_n} [l_n^j(z_n^j)]^{(1-\beta_n)} \gamma_n^j \prod_{k=1}^J [M_n^{jk}(z_n^j)]^{\gamma_n^{jk}} \quad (S4)$$

Based on potential output, a producer can allocate fraction ϵ_n^j of $y_n^j(z_n^j)$ to emission-reduction activities to reduce payments. The remaining $1 - \epsilon_n^j$ fraction is the net output. We denote the net production after abatement investment using $q_n^j(z_n^j)$, wherein the production function is

$$q_n^j(z_n^j) = (1 - \epsilon_n^j) y_n^j(z_n^j) \quad (S5)$$

We assume that the emissions are also affected by z_n^j . Advanced production technologies can improve resource utilization efficiency, leading to positive externalities in emission reduction. Research on the technology list for China's response to climate change supports this assumption. Under this assumption, emissions from output at different technological levels differ. Then, the relationship between emissions and potential output can be expressed as:

$$e_n^j(z_n^j) = \left(\frac{1}{z_n^j} \right) (1 - \epsilon_n^j)^{\frac{1}{\alpha_n^j}} y_n^j(z_n^j) \quad (S6)$$

Putting Eq. (S5) and Eq. (S6) into Eq. (S4), the net production of intermediate goods is obtained, where

z_n^j includes the technical level of potential output $z_n^{j1-\alpha_n^j}$ and the technical level of emission reduction

$z_n^{j\alpha_n^j}$

Q.E.D.

The intermediate good producer's cost minimization problem can be written as

$$\begin{aligned} \min_{\{l_n^j, h_n^j, M_n^{jk}, e_n^j\}} \quad & w_n l_n^j(z_n^j) + r_n h_n^j(z_n^j) + \sum_{k=1}^J P_n^k M_n^{jk}(z_n^j) + t_n e_n^j(z_n^j) \\ \text{s. t.} \quad & z_n^j \left([T_n^j [h_n^j(z_n^j)]^{\beta_n} [l_n^j(z_n^j)]^{(1-\beta_n)}] \gamma_n^j \prod_{k=1}^J [M_n^{jk}(z_n^j)]^{\gamma_n^{jk}} \right)^{1-\alpha_n^j} [e_n^j(z_n^j)]^{\alpha_n^j} = 1 \end{aligned} \quad (\text{S7})$$

The government controls the emissions of sector j in region n , denoted as e_n^j , through emission reduction policies, denoted as t_n (including environmental taxes and emission penalties),

where $e_n = \sum_{j=1}^J e_n^j$.

We use x_n^j to denote the cost of the input bundle needed to produce intermediate good

varieties. Let $B_{1,n}^j = (\alpha_n^j)^{-\alpha_n^j} (1 - \alpha_n^j)^{\alpha_n^j - 1}$, $B_{2,n}^j = [\gamma_n^j (1 - \beta_n)^{(1-\beta_n)} \beta_n^{\beta_n}]^{-\gamma_n^j} \prod_{k=1}^J (\gamma_n^{jk})^{-\gamma_n^{jk}}$. ζ_n^j

is the unit cost of input bundles without pollution emissions. Then:

$$\begin{aligned} x_n^j &= B_{1,n}^j (t_n)^{\alpha_n^j} (\zeta_n^j)^{1-\alpha_n^j} \\ \zeta_n^j &= B_{2,n}^j (r_n^{\beta_n} w_n^{1-\beta_n})^{\gamma_n^j} \prod_{k=1}^J (P_n^k)^{\gamma_n^{jk}} \end{aligned} \quad (\text{S8})$$

The unit cost of an intermediate good with idiosyncratic draw z_n^j is then given by

$$\frac{x_n^j}{z_n^j (T_n^j)^{(1-\alpha_n^j)\gamma_n^j}} \quad (\text{S9})$$

S2.2 Final Goods

The production of final goods is given by

$$Q_n^j = \left[\int \tilde{q}_n^j(z^j)^{1-\frac{1}{\eta_n^j}} \phi^j(z^j) dz^j \right]^{\frac{\eta_n^j}{(\eta_n^j-1)}} \quad (\text{S10})$$

where $\phi^j(z^j) = \exp\{-\sum_{n=1}^N (z_n^j)^{-\theta^j}\}$ denotes the joint density function for the vector z^j , with

marginal densities given by $\phi_n^j(z_n^j) = \exp\{-(z_n^j)^{-\theta^j}\}$.

S3. Prices

The price of final good j in region n is given by

$$P_n^j = B_3^j \left[\sum_{i=1}^N (x_i^j \kappa_{ni}^j)^{-\theta^j} (T_i^j)^{(1-\alpha_i^j)\gamma_i^j \theta^j} \right]^{\frac{1}{\theta^j}} \quad (\text{S11})$$

where κ_{ni}^j is the transport costs of transporting goods from region i to region n , $B_3^j = \left[\Gamma \left(1 + \frac{1-\eta_n^j}{\theta^j} \right) \right]^{\frac{1}{1-\eta_n^j}}$.

*Proof of the final product prices (S11):

The price paid for a particular variety $p_n^j(z^j)$, is given by the minimum of the unit costs across locations,

$$p_n^j(z^j) = \min_i \left\{ \frac{\kappa_{ni}^j x_i^j}{z_i^j (T_i^j)^{(1-\alpha_i^j) \gamma_i^j}} \right\} \quad (S12)$$

We follow Eaton and Kortum (2002) in solving for the distribution of prices. Given the distribution of prices, when sector j is tradeable, the price of final good j in region n solves

$$P_n^{j1-\eta_n^j} = \int p_n^j(z^j)^{1-\eta_n^j} \phi^j(z^j) dz^j \quad (S13)$$

Given the assumptions on the distribution of z_i^j , and the unit cost of producing and shipping goods, we have that

$$\Pr[p_{ni}^j \leq p] = \Pr \left[\frac{\kappa_{ni}^j x_i^j}{z_i^j (T_i^j)^{(1-\alpha_i^j) \gamma_i^j}} \leq p \right] = \Pr \left[z_i^j \geq \frac{\kappa_{ni}^j x_i^j}{p (T_i^j)^{(1-\alpha_i^j) \gamma_i^j}} \right]$$

or

$$\Pr[p_{ni}^j \leq p] = 1 - \exp \left\{ -\lambda_{ni}^j p^{\theta^j} \right\}$$

where $\lambda_{ni}^j = \left[\kappa_{ni}^j x_i^j (T_i^j)^{-(1-\alpha_i^j) \gamma_i^j} \right]^{-\theta^j}$, then

$$\Pr[p_n^j \leq p] = 1 - \exp \left\{ -\Phi_n^j p^{\theta^j} \right\}$$

where $\Phi_n^j = \sum_{i=1}^N \lambda_{ni}^j = \sum_{i=1}^N (\kappa_{ni}^j x_i^j)^{-\theta^j} (T_i^j)^{(1-\alpha_i^j) \gamma_i^j \theta^j}$, the price of final good j in region n can be expressed as

$$P_n^j = \left[\Gamma \left(1 + \frac{1-\eta_n^j}{\theta^j} \right) \right]^{\frac{1}{1-\eta_n^j}} (\Phi_n^j)^{-\frac{1}{\theta^j}}$$

$$= \left[\Gamma \left(1 + \frac{1-\eta_n^j}{\theta^j} \right) \right]^{\frac{1}{1-\eta_n^j}} \left[\sum_{i=1}^N (x_i^j \kappa_{ni}^j)^{-\theta^j} (T_i^j)^{(1-\alpha_i^j) \gamma_i^j \theta^j} \right]^{\frac{1}{\theta^j}}$$

Q.E.D.

S4. Trade Shares

Let π_{ni}^j denote the share of region n 's expenditures on sector j composite goods purchased from region i ,

$$\pi_{ni}^j = \frac{(x_i^j \kappa_{ni}^j)^{-\theta^j} T_i^{j(1-\alpha_i^j) \theta^j \gamma_i^j}}{\sum_{m=1}^N (x_m^j \kappa_{nm}^j)^{-\theta^j} T_m^{j(1-\alpha_i^j) \theta^j \gamma_m^j}} \quad (S14)$$

*Proof of the trade share (S14):

Let X_n^j denote total expenditures on final good j in region n (or total revenue), $X_n^j = P_n^j Q_n^j$, then

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j}$$

and observe that

$$X_{ni}^j = Pr \left[p_{ni}^j(z^j) \leq \min_{m \neq i} \{p_{nm}^j(z^j)\} \right] X_n^j$$

From the properties of the Frechet distribution, it follows that

$$\begin{aligned} \pi_{ni}^j &= Pr \left[p_{ni}^j(z^j) \leq \min_{m \neq i} \{p_{nm}^j(z^j)\} \right] \\ &= Pr \left[p_{ni}^j(z^j)^{\theta^j} \leq \min_{m \neq i} \{p_{nm}^j(z^j)\}^{\theta^j} \right] \\ &= \frac{\lambda_{ni}^j}{\Phi_n^j} \\ &= \frac{\left[\kappa_{ni}^j x_i^j (T_i^j)^{-(1-\alpha_i^j) \nu_i^j} \right]^{-\theta^j}}{\sum_{i=1}^N (\kappa_{ni}^j x_i^j)^{-\theta^j} (T_i^j)^{(1-\alpha_i^j) \nu_i^j \theta^j}} \end{aligned}$$

Q.E.D.

S5. Asset Holdings and Regional Deficits

We used F_n^j to denote the revenue from emission penalties or carbon taxes received by region n from sector j through environmental regulations; that is, $F_n^j = t_n e_n^j$. The total environmental regulation revenue in region n is $F_n = \sum_{j=1}^J F_n^j$. The income of an agent residing in region n is

$$I_n = w_n + \chi + (1 - \iota_n) r_n H_n / L_n + F_n / L_n \quad (S15)$$

The term χ represents the return per person from the national portfolio of land and structures in all regions.

$$\chi = \frac{\sum_{i=1}^N \iota_i r_i H_i}{\sum_{i=1}^N L_i}$$

Hence, the difference between the remittances and the income in region n , generates imbalances given by

$$Y_n \equiv \iota_n r_n H_n - \chi L_n \quad (S16)$$

S6. Labour Mobility and Market Clearing

Regional labour market clearing requires that

$$\sum_{j=1}^J L_n^j = \sum_{j=1}^J \int_0^\infty l_n^j(z) \phi_n^j(z) dz = L_n \quad (S17)$$

where $\sum_{n=1}^N L_n = L$. Market clearing for land and structures in each region imply that

$$\sum_{j=1}^J H_n^j = \sum_{j=1}^J \int_0^\infty h_n^j(z) \phi_n^j(z) dz = H_n \quad (S18)$$

Profit maximization by intermediate goods producers, together with these equilibrium conditions, implies that $r_n H_n (1 - \beta_n) = \beta_n w_n L_n$. Then, defining $\omega_n \equiv [r_n / \beta_n]^{\beta_n} [w_n / (1 - \beta_n)]^{(1-\beta_n)}$, free mobility gives us

$$L_n = H_n \left(\frac{\omega_n}{P_n U + u_n} \right)^{\frac{1}{\beta_n}}$$

where $u_n \equiv Y_n/L_n = \iota_n r_n H_n/L_n - \chi$ denotes the trade surplus per capita in n . Combining these conditions with the labour market clearing condition, yields an expression for labour input in region n ,

$$L_n = \frac{H_n \left(\frac{\omega_n}{P_n U + u_n} \right)^{\frac{1}{\beta_n}}}{\sum_{i=1}^N H_i \left(\frac{\omega_i}{P_i U + u_i} \right)^{\frac{1}{\beta_i}}} L \quad (S19)$$

Regional market clearing in final goods is given by

$$L_n c_n^j + \sum_{k=1}^J M_n^{kj} = L_n c_n^j + \sum_{k=1}^J \int_0^\infty M_n^{kj}(z) \phi_n^k(z) dz = Q_n^j \quad (S20)$$

Then, regional market clearing in final goods implies that

$$X_n^j = \sum_{k=1}^J (1 - \alpha_n^k) \gamma_n^{kj} \sum_{i=1}^N \pi_{in}^k X_i^k + s^j L_n L_n \quad (S21)$$

In equilibrium, in any region n , total expenditures on intermediates purchased from other regions must equal total revenue from intermediates sold to other regions, formally,

$$\sum_{j=1}^J \sum_{i=1}^N \pi_{ni}^j X_n^j + Y_n = \sum_{j=1}^J \sum_{i=1}^N \pi_{in}^j X_i^j \quad (S22)$$

The optimization conditions for consumers and intermediate and final goods producers hold, all markets clear—equations (S8), (S11), (S14), (S16), (S17), (S18), (S20), (S21) hold—aggregate trade is balanced—(S22) holds—and utility is equalized across regions—(S19) holds.

S7. Equilibrium Conditions in Relative Terms

For any variable x , we denote the change in x following a change in the economic environment as $\hat{x} = x'/x$.

In changes, the cost of the input bundle becomes (JN equations),

$$\hat{x}_n^j = (\hat{t}_n)^{\alpha_n^j} \left((\hat{\omega}_n)^{\gamma_n^j} \prod_{k=1}^J (\hat{p}_n^k)^{\gamma_n^{jk}} \right)^{1 - \alpha_n^j} \quad (S23)$$

prices take the form (JN equations),

$$\hat{p}_n^j = \left[\sum_{i=1}^N \pi_{ni}^j (\hat{\kappa}_{ni}^j \hat{x}_i^j)^{-\theta^j} (\hat{T}_i^j)^{(1 - \alpha_i^j) \gamma_i^j \theta^j} \right]^{-\frac{1}{\theta^j}} \quad (S24)$$

Trade share after an exogenous change (JN^2 equations),

$$\pi_{ni}^{j'} = \pi_{ni}^j \left(\frac{\hat{x}_i^j \hat{\kappa}_{ni}^j}{\hat{p}_n^j} \right)^{-\theta^j} (\hat{T}_i^j)^{(1 - \alpha_i^j) \gamma_i^j \theta^j} \quad (S25)$$

Labour mobility condition (N equations), where $\varphi = 1 / \left(1 + \frac{u_n}{P_n U} \right)$,

$$\hat{L}_n^j = \frac{\left(\frac{\hat{\omega}_n}{\varphi_n \hat{P}_n \hat{U} + (1 - \varphi_n) \hat{u}_n} \right)^{\frac{1}{\beta_n}}}{\sum_{i=1}^N \left(\frac{\hat{\omega}_n}{\varphi_i \hat{P}_i \hat{U} + (1 - \varphi_i) \hat{u}_i} \right)^{\frac{1}{\beta_i}}} \quad (S26)$$

Regional market clearing in final goods (JN equations),

$$X_n^{j'} = \sum_{k=1}^J (1 - \alpha_n^k) \gamma_n^{kj} \sum_{i=1}^N \pi_{in}^{k'} X_i^{k'} + s^j I_n' L_n' \quad (S27)$$

Trade balance condition (N equations),

$$\hat{\omega}_n (\hat{L}_n)^{1-\beta_n} \omega_n (H_n)^{\beta_n} (L_n)^{1-\beta_n} = \sum_{j=1}^J (1 - \alpha_n^j) \gamma_n^j \sum_{i=1}^N \pi_{in}^{j'} X_i^{j'} \quad (S28)$$

The total number of unknowns is: $\hat{\omega}_n$ (N), \hat{L}_n (N), $X_n^{j'}$ (JN), \hat{P}_n^j (JN), $\pi_{ni}^{j'}$ ($J \times N \times N$), \hat{x}_n^j (JN). In the model, t_n , T_n^j and $\hat{\kappa}_{ni}^j$ are exogenous variables. Thus a total of $2N + 3JN + JN^2$ equations and unknowns.

S8. Changes in Green TFP and GDP

The green total factor productivity (Green TFP) of industry j in region n is

$$A_n^j = \frac{x_n^j}{P_n^j} \quad (S29)$$

Since $\hat{\pi}_{nn}^j = \left(\frac{\hat{x}_n^j}{\hat{P}_n^j}\right)^{-\theta^j} (\hat{T}_i^j)^{(1-\alpha_i^j)\gamma_i^j \theta^j}$, changes in green TFP is

$$\hat{A}_n^j = \frac{\hat{x}_n^j}{\hat{P}_n^j} = \frac{(\hat{T}_i^j)^{(1-\alpha_i^j)\gamma_i^j}}{(\hat{\pi}_{nn}^j)^{\frac{1}{\theta^j}}} \quad (S30)$$

*Proof of the green TFP (S29) and its changes (S30):

Measured green TFP in a region-sector pair (n, j) is commonly calculated as

$$\ln A_n^j = \ln \frac{Y_n^j}{P_n^j} - (1 - \beta_n) \gamma_n^j (1 - \alpha_n^j) \ln L_n^j - \beta_n \gamma_n^j (1 - \alpha_n^j) \ln H_n^j - \sum_{k=1}^J \gamma_n^{jk} (1 - \alpha_n^j) \ln M_n^{jk} - \alpha_n^j E_n^j \quad (S31)$$

The first term is gross output revenue over price, while the last four terms denote the log of the aggregate input bundle. The equilibrium factor demands of the intermediate good producers imply that

$$Y_n^j = w_n L_n^j + r_n H_n^j + \sum_{k=1}^J P_n^k M_n^{jk} + t_n E_n^j = \frac{w_n L_n^j}{\gamma_n^j (1 - \beta_n) (1 - \alpha_n^j)}$$

Zero profits imply that total production of intermediates is exactly offset by factor payments

$$w_n L_n^j + r_n H_n^j + \sum_{k=1}^J P_n^k M_n^{jk} + t_n E_n^j = \int p_n^j(z_n^j) q_n^j(z_n^j) \phi_n^j(z_n^j) dz_n^j = Y_n^j$$

From firms' optimality conditions, we have that

$$\frac{w_n l_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)} = \gamma_n^j (1 - \beta_n) (1 - \alpha_n^j)$$

or

$$w_n L_n^j = \gamma_n^j (1 - \beta_n) (1 - \alpha_n^j) \int p_n^j(z_n^j) q_n^j(z_n^j) \phi_n^j(z_n^j) dz_n^j$$

so that

$$\frac{w_n L_n^j}{Y_n^j} = \gamma_n^j (1 - \beta_n) (1 - \alpha_n^j) = \frac{w_n l_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)}$$

Similarly, it follows that

$$\begin{aligned}\frac{r_n H_n^j}{Y_n^j} &= \gamma_n^j \beta_n (1 - \alpha_n^j) = \frac{r_n h_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)} \\ \frac{P_n^k M_n^{jk}}{Y_n^j} &= \gamma_n^{jk} (1 - \alpha_n^j) = \frac{P_n^k M_n^{jk}(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)} \\ \frac{t_n e_n^j}{Y_n^j} &= \alpha_n^j = \frac{t_n e_n^j(z_n^j)}{p_n^j(z_n^j) q_n^j(z_n^j)}\end{aligned}$$

Therefore,

$$\begin{aligned}l_n^j(z_n^j) &= \frac{p_n^j(z_n^j) q_n^j(z_n^j) L_n^j}{Y_n^j}, & h_n^j(z_n^j) &= \frac{p_n^j(z_n^j) q_n^j(z_n^j) H_n^j}{Y_n^j}, \\ M_n^{jk}(z_n^j) &= \frac{p_n^j(z_n^j) q_n^j(z_n^j) M_n^{jk}(z_n^j)}{Y_n^j}, & e_n^j(z_n^j) &= \frac{p_n^j(z_n^j) q_n^j(z_n^j) e_n^j}{Y_n^j}\end{aligned}$$

Substituting these expressions in the production of intermediate goods (3) gives

$$q_n^j(z_n^j) = \left[\frac{p_n^j(z_n^j) q_n^j(z_n^j)}{Y_n^j} \right] z_n^j \left(\left[T_n^j (H_n^j)^{\beta_n} (L_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod_{k=1}^{k=1} (M_n^{jk})^{\gamma_n^{jk}} \right)^{1-\alpha_n^j} e_n^{j\alpha_n^j}$$

which, given that $p_n^j(z_n^j) = x_n^j / z_n^j (T_n^j)^{(1-\alpha_n^j)\gamma_n^j}$, implies

$$q_n^j(z_n^j) = \frac{x_n^j q_n^j(z_n^j)}{Y_n^j} \left(\left[(H_n^j)^{\beta_n} (L_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod_{k=1}^{k=1} (M_n^{jk})^{\gamma_n^{jk}} \right)^{1-\alpha_n^j} e_n^{j\alpha_n^j} \quad (S32)$$

From equation (31), it follows that

$$\frac{Y_n^j}{P_n^j} = A_n^j \left(\left[(H_n^j)^{\beta_n} (L_n^j)^{(1-\beta_n)} \right]^{\gamma_n^j} \prod_{k=1}^{k=1} (M_n^{jk})^{\gamma_n^{jk}} \right)^{1-\alpha_n^j} e_n^{j\alpha_n^j} \quad (S33)$$

Therefore, combining equations (S32) and (S33) yields the Green TFP for industry j in region n as $A_n^j = \frac{x_n^j}{P_n^j}$. Its changes can be expressed as $\hat{A}_n^j = \frac{\hat{x}_n^j}{\hat{P}_n^j}$. From equation (S25), we know that

$$\hat{\pi}_{nn}^j = \left(\frac{\hat{x}_n^j}{\hat{P}_n^j} \right)^{-\theta^j} (\hat{T}_n^j)^{(1-\alpha_n^j)\gamma_n^j \theta^j}$$

it follows that Green TFP changes in a given region-sector pair may also be expressed as

$$\hat{A}_n^j = \frac{(\hat{T}_n^j)^{(1-\alpha_n^j)\gamma_n^j}}{(\hat{\pi}_{nn}^j)^{\frac{1}{\theta^j}}}$$

Q.E.D.

Actual GDP changes in sector j in region n can be expressed as

$$\widehat{GDP}_n^j = \hat{A}_n^j \hat{L}_n^j \left(\frac{\hat{W}_n}{\hat{x}_n^j} \right) \quad (S34)$$

Sectors are categorized according to their emission characteristics into carbon-intensive sector j and other low-emission sector k , and ψ_n^j and ψ_n^k are used to represent their respective shares of GDP in the region. Thus, changes in regional GDP can be expressed as:

$$\widehat{GDP}_n = \psi_n^j \widehat{GDP}_n^j + \psi_n^k \widehat{GDP}_n^k \quad (S35)$$