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Significance of Bioconvective and Thermally Dissipation Flow of Viscoelastic Nanoparticles with Activation Energy Features: Novel Biofuels Significance

Hassan Waqas ¹ , Sami Ullah Khan ², Iskander Tlili ^{3,4,*}, Muhammad Awais ⁵ and Mostafa S. Shadloo ⁶

¹ Department of Mathematics, Government College University, Faisalabad 38000, Pakistan; syedhasanwaqas@hotmail.com

² Department of Mathematics, COMSATS University Islamabad, Sahiwal 57000, Pakistan; sk_iuu@yahoo.com

³ Department for Management of Science and Technology Development, Ton Duc Thang University, Ho Chi Minh City 758307, Vietnam

⁴ Faculty of Applied Sciences, Ton Duc Thang University, Ho Chi Minh City 758307, Vietnam

⁵ Department of Biosciences, COMSATS University Islamabad, Sahiwal 57000, Pakistan; muhammad.awais@cuisahiwal.edu.pk

⁶ CORIA-CNRS(UMR6614), Normandie University, INSA of Rouen, 76000 Rouen, France; msshadloo@coria.fr

* Correspondence: iskander.tlili@tdtu.edu.vn

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Abstract: The analysis of bioconvection flow nanofluids is the topic of concern in recent decades as it involves a variety of physical significance in biotechnology. Bioconvection has many applications in the interdisciplinary field of sciences such as in biomedical science, biofuel biotechnology, and enzyme-based biosensors, among others. The aim of the current work is to analyze the bioconvection phenomenon in the two-dimensional steady flow of viscoelastic nanofluid over a vertical surface. Here, the effects of activation energy, second-order slip, and nanoparticles zero mass flux conditions are considered to investigate the flow problem. Based on dimensionless variables, the governing partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs) which are further solved numerically by using a built-in BVP4C approach in MATLAB software. Various controlling parameters like Hartman number, viscoelastic parameter, first and second-order slip factors, buoyancy ratio parameter, thermophoresis parameter, Brownian motion constant, bioconvection Lewis number and Peclet number are graphically illustrated for the distributions of velocity, temperature, concentration, and motile microorganism. Moreover, the variation of local Nusselt number, local Sherwood number, and motile density number are numerically investigated for the involved parameters.

Keywords: bioconvection; second grade fluid; activation energy; microorganism; Wu's slip features; shooting technique

1. Introduction

In recent years, researchers have shown a deep interest in nanofluid flow applications. Based on their results, it is observed that the working fluids in different engineering and biomedical areas encountered resilient properties of transferring heat, mass, and density within a solution. Although various base liquids have a certain capacity of heat transfer, usually they are not preferred for heat transport processes due to their poor thermal efficiency. To overcome this issue, it is experimentally

shown that the thermophysical properties of such traditional materials can be enhanced by applying some nanoparticle additives.

The suspension of base fluids with tinny-sized nanoparticles (1–100 nm) is considered as the most effective solution for enhancing the heat transfer which attained variety of applications in various heating and cooling systems. With this idea, the ordinary liquids with lower thermal performance have been replaced with nanofluids which have the ability to transfer more heat into the system. Compared to the base liquids, the nanofluids are usually more stable and potent for energy and mass transfer. Such nanoparticles can be achieved by adding various metals like gold, silver, copper, iron, carbides, oxides and nitrides with specific cautions. A variety of nanofluid applications include nano-medicine, cancer therapy, bio-fuels, light-based sensors, biological sensors, enzyme biotechnology, and extrusion systems.

The pioneering work on nanoparticles was suggested by Choi [1] which was further extended by many researchers in the last decade. Among these researches, the important slip mechanisms of nanofluid thermophoresis parameter and Brownian movement were discussed by Buongiorno [2]. Sundar et al. [3] investigated the features of thermal conductivity coefficients and viscosity by using a cobalt oxide-nano-diamond solution. Sheikholeslami and Bhatti [4] highlighted the effects of shapes of fluid particles in the presence of a constant magnetic field for the thermal drift. Hsiao [5] evaluated the magnetohydrodynamic (MHD) flow of Carreau nanofluid by using parameter control methods and claimed that the reported results can be useful for the enhancement of various thermal extrusion systems. Another study regarding mixed convection flow of nanoparticles in a micro-porous-channel influenced by strong magnetic force was conducted by Basant and Aina [6]. The flow of nanofluid in a rotating frame in the presence of a heat source and heat sink was explored numerically by Mahantesh et al. [7]. Siddidui and Turkyilmazoglu [8] followed a theoretical approach for wall transpiration in the cavity flow of iron-based liquids. Ahmad and Khan [9] worked on the flow of Sisko-magneto-fluid with additional features of activation energy over a porous curved surface. Another investigation reported by Khan et al. [10], is based on the flow of Prandtl-Eyring nanofluid with interesting features of entropy generation with Arrhenius activation energy. Turkyilmazoglu [11] exploited the study of nanofluid by using Buongiorno's model in an asymmetric channel. Khan and Shehzad [12] explored the thermophoresis and Brownian motion features in nanoparticle flow over an accelerated moving configuration. Malik et al. [13] analyzed the mixed convection flow of electrically conducting Eyring-Powell fluid over a stretched surface. Some recent studies that reported the thermal physical aspects of nanoparticles can be seen in references [14–20].

The phenomenon of bioconvection is associated with mixed microbes or biological solutions. This practice is usually observed in dilute fluids where the microbes tend to move in an upward direction in a suspended liquid which results in the formation of instability due to the density stratification. In microbial suspensions, the convection is driven by unicellular microorganisms, however, it cannot be well defined with colonial microbes. Thus, cell to cell interactions are neglected during the calculations. Most often the bioconvection phenomena are explained by Rayleigh–Benard convection that also presents in the overturning instability of nanofluids due to the random movement of nanoparticles. It is commonly observed that the movement, interaction, mass transport and heat transfer in the nano-liquids at a molecular level is prominent on a larger scale.

The main difference between particles and biological suspensions is that the nanoparticles are not self-propagated and follow the Brownian movements due to the hydrodynamic instability. The addition of microorganisms in nanofluid is considered to pose a huge shift in convection patterns of fluid. Recently, the researches tend towards the investigation of mixed nanofluids which captured the interesting significance in enzyme technology, bioremediation, nano-biotechnology, drug delivery, nonmaterial processing, biosensors, and biofuels. The microbial movements in a solution can be classified as taxis and, depending on the incident stimuli, they can be further categorized into chemotaxis, phototaxis, gravitaxis, oxytaxis, and gyrotaxis.

Moreover, to achieve a substantial bioconvection, the microbial convection can be regulated by various stimuli of chemicals, light, gravity, oxygen, and gravitational torques that otherwise are difficult to achieve in nanofluids. Practically, such microscopic movement in solutions can attribute to the purification of microbial cultures, cellular aggregation, and separation of various strains. In the experimental conditions, biofuel system plants that operate with algae are strictly reliable on the algae producing bioreactors. Algal production is further dependent on the optical gateway in reactors, specifically light penetration. The algae being positively phototactic moves gyrotactically in the bioreactor resulting in a density stratification and variable light exposure in different reactor areas.

The early efforts to evaluate the concept of bioconvection were made by Kuznetsov [21] where he explained the onset of bioconvection in the suspension of nanoparticles. Kuznetsov [22] further investigated the water-based mixed nanofluids with oxyntic microorganisms. Moreover, the bioconvection patterns in non-Newtonian fluids were studied by considering microbial transport in a free stream by Beg et al. [23]. Numerical treatment for the anisotropic slip flow of nanofluid was determined by Lu et al. [24]. Uddin et al. [25] focused on the slip flow of nanofluid over the wavy surface with the relevant significance of nano-biofuel cells. The investigations regarding the bioconvection flow of nanoparticles saturated by a non-Darcian medium were performed by Sarkar et al. [26]. The flow of Maxwell fluid containing gyrotactic microorganisms in the presence of applied magnetic force was discussed by Khan et al. [27]. Rashad and Hossam [28] investigated the mixed convection flow of nanofluid due to a stretching cylinder encountered the convective boundary conditions. Saini and Sharma [29] implemented a novel numerical algorithm for problems regarding the thermo-bioconvection flow of nanofluid in the existence of a porous medium. The impact of temperature-dependent viscosity in the flow of nanofluid over a rotating system was investigated by Xun et al. [30]. Dhanai et al. [31] discussed some multiple solutions regarding the flow of nanoparticles containing gyrotactic microorganisms. The investigations conducted by Mutuku and Makinde [32] were based on the hydromagnetic bioconvection flow of nanofluid over a vertical plate. Recently, Waqas et al. [33] reported the Falkner-Skan bioconvection flow of nanofluid by using Buongiorno's nanofluid model over a stretched surface. Many other examples can be found in recent literature [34–37].

Following the above-mentioned research and also considering the great significance of bioconvection phenomenon in various biotechnologies, the aim of this work is to apply the flow of viscoelastic nanoparticles containing gyrotactic microorganisms in the presence of thermal radiation, viscous dissipation, and activation energy features. The proposed analyses are performed with the utilization of second-order slip (Wu's slip) constraints. The governing equations for the formulated flow model are solved numerically by using a built-in BVP4C procedure in MATLAB software.

2. Mathematical Modeling

In this work, we investigate the two-dimensional flow of viscoelastic nanofluid over a heated stretched configuration. For the applied Cartesian coordinate system, x -axis is considered along the stretched sheet while y -axis is taken perpendicular to it. We assume that the flow is induced in the vertical direction with the velocity of $u_w = cx$, where c defines the rate of stretching. The fluid is assumed to be electrically conducting and magnetic field effects are imposed in a perpendicular direction to the stretched sheet. The flow model is constituted upon the following assumptions:

- i. Second-grade fluid model is used to analyze the rheological features of non-Newtonian fluid.
- ii. The Buongiorno's nanofluid model is utilized to report the Brownian movement and thermophoresis prospective of nanofluids.
- iii. The energy equation can capture the effects thermal radiation, heat absorption/generation and viscous dissipation.
- iv. The activation energy consequences are considered by using famous Arrhenius theory.

The governing equations for the flow problem can be written as:

$$\nabla \cdot \mathbf{V} = 0, \quad (1)$$

$$\rho_f \frac{d\mathbf{V}}{dt} = \nabla \cdot \boldsymbol{\tau} + \mathbf{J} \times \mathbf{B}, \quad (2)$$

$$\mathbf{V} \cdot \nabla T = \alpha_f \nabla^2 T + \tau \left[D_B \nabla T \cdot \nabla C + \left(\frac{D_T}{T_\infty} \right) \nabla T \cdot \nabla T \right] - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y}, \quad (3)$$

$$(\mathbf{V} \cdot \nabla) C = D_B \nabla^2 C + \left(\frac{D_T}{T_\infty} \right) \nabla^2 T, \quad (4)$$

$$\mathbf{V} \cdot \mathbf{J}_1 = 0, \quad (5)$$

where $\mathbf{V}$, $\mathbf{J}$, ρ_f , $\mathbf{B}$, T , α_f , τ , D_B , C , D_T represent the velocity vector, current density, density of fluid, magnetic flux vector, temperature, thermal diffusivity, ratio between heat capacity of nanoparticles material to heat capacity of fluid, Brownian diffusion coefficient concentration, and thermophoretic diffusion coefficient, respectively. The microorganisms flux associated with macroscopic convection of fluid $\mathbf{J}_1$ is defined as:

$$\mathbf{J}_1 = n\mathbf{V} + \mathbf{n}\hat{\mathbf{V}} - D_n \nabla n, \quad (6)$$

where n , $\hat{\mathbf{V}} = (bW_c/\Delta C)\nabla C$, b , W_c and D_n are, respectively, microorganisms motile density, cell swimming velocity, chemotaxis constant, maximum cell swimming speed, and microorganisms diffusion coefficient.

For the second-grade fluid, we define the Cauchy stress tensor in following form

$$\boldsymbol{\tau} = -p\mathbf{I} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2, \quad (7)$$

where p , $\mathbf{I}$, μ , α_1 , α_2 symbolize the pressure, identity tensor, dynamic viscosity, and material moduli parameters, respectively. The Rivlin Ericksen tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^t, \quad \mathbf{L} = \nabla \mathbf{V}, \quad \mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1 \mathbf{L} + \mathbf{L}^t \mathbf{A}_1, \quad (8)$$

For the two-dimensional flow, the required velocity field is

$$\mathbf{V} = [u(x, y), v(x, y), 0], \quad T = T(x, y), \quad C = C(x, y), \quad n = n(x, y), \quad (9)$$

Following the flow assumptions, the constituted boundary layer equations for the current problem are expressed as [38–40]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (10)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho_f} \left(u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \nu \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right) - \frac{\sigma_e B_0^2}{\rho_f} u + \frac{1}{\rho_f} \left[(1 - C_f) \rho_f \beta^* g^* (T - T_\infty) - (\rho_p - \rho_f) g^* (C - C_\infty) - (N - N_\infty) g \gamma^{**} (\rho_m - \rho_f) \right], \quad (11)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\alpha_m + \frac{16\sigma^* T_\infty^3}{3k^*(\rho c)_f} \right) \frac{\partial^2 T}{\partial y^2} + \tau \left\{ D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\} + \frac{Q}{(\rho c)_f} (T - T_\infty) + \frac{\sigma B_0^2}{(\rho c)_f} u^2 + \frac{\mu}{(\rho c)_f} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\alpha_1}{(\rho c)_f} \left(\left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial v}{\partial y} \left(\frac{\partial u}{\partial y} \right)^2 + v \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right), \quad (12)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - Kr^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^2 \exp \left(\frac{-E_a}{kT} \right), \quad (13)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{bW_c}{(C_w - C_\infty)} \left[\frac{\partial}{\partial y} \left(N \frac{\partial C}{\partial y} \right) \right] = D_m \left(\frac{\partial^2 N}{\partial y^2} \right), \quad (14)$$

where α_1 is the material constant, σ_e being electric conductivity, B_0 is the magnetic field strength, g gravity, α_m notified the thermal diffusivity, ρ_p liquid density, ρ_m signify motile microorganism particles density, Q is the heat source coefficient, σ^* is the Stefan–Boltzmann constant while k^* represents the mean absorption coefficient, E_a is the activation energy, and Kr is the chemical reaction constant.

Following boundary assumptions are introduced for the current situation

$$u = u_w + u_{slip}, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_f (T_f - T), \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0, \quad N = N_w, \quad \text{at} \quad y = 0, \quad (15)$$

$$u \rightarrow U_\infty = 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N \rightarrow N_\infty \quad \text{at} \quad y \rightarrow \infty, \quad (16)$$

Let us introduce following slip boundary conditions [38–40]

$$u_{slip} = \frac{2}{3} \beta_1 \left(\frac{3 - \alpha^* p^2}{\alpha^*} - \frac{3(1 - p^2)}{2K^*} \right) \frac{\partial u}{\partial y} - \frac{1}{4} \beta_1^2 \left[p^4 + \frac{2}{K_n^2} (1 - p^2) \right] \frac{\partial^2 u}{\partial y^2}, \quad (17)$$

$$u_{slip} = C_1 \frac{\partial u}{\partial y} + C_2 \frac{\partial^2 u}{\partial y^2}, \quad (18)$$

where K^* , α^* , β_1 , C_1 and C_2 are respectively, Knudsen number, momentum coefficient, free path for molecular mean, and arbitrary constants.

In order to attain the dimensionless form of governing equations, we suggest the following variables [38–40]

$$\zeta = \sqrt{\frac{a}{v}} y, \quad u = cx f'(\zeta), \quad v = -\sqrt{cv} f(\zeta), \quad \theta(\zeta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\zeta) = \frac{C - C_\infty}{C_\infty}, \quad \chi(\zeta) = \frac{N - N_\infty}{N_w - N_\infty}. \quad (19)$$

The executing above variables in the governing equations yield the following set of dimensionless equations

$$f''' - (f')^2 + f f'' - M f' + \alpha (2 f' f''' - f''^2 - f f^{iv}) + \Lambda (\theta - Nr \phi - Nc \chi) = 0, \quad (20)$$

$$\left(1 + \frac{4}{3} Rd \right) \theta'' + Pr Ec [f''^2 + M f'^2 + \alpha f'' (f' f' - f f''')] + Pr \left(\frac{f \theta' + f' \theta + Nb \theta' \phi'}{+ Nt \theta'^2 + S \theta} \right) = 0, \quad (21)$$

$$\phi'' + \left(\frac{Nt}{Nb} \right) \theta'' + Sc f \phi' + Sc f' \phi - Pr Sc \omega (1 + \sigma \theta)^m \exp \left(\frac{-E}{1 + \sigma \theta} \right) \phi = 0, \quad (22)$$

$$\chi'' + Lb f \chi' - Pe (\phi'' (\chi + \delta_1) + \chi' \phi') = 0 \quad (23)$$

Similarly, the boundary conditions are reduced to following

$$f(0) = 0, \quad f'(0) = 1 + \lambda f''(0) + \Gamma f'''(0), \quad \theta'(0) = Bi(\theta(0) - 1), \quad Nb \theta'(0) + Nt \phi'(0) = 0, \quad \chi(0) = 1, \quad (24)$$

$$f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0, \quad \chi(\infty) \rightarrow 0,$$

where α is the viscoelastic parameter, M represents the Hartman number, Λ relates the mixed convection parameter, Nr is used to determine the Buoyancy number, Nc is the bioconvection Rayleigh number, S stands for the heat generation parameter, Pr reflects the Prandtl number, Nt is the thermophoresis parameter, Nb is used for the Brownian motion parameter, Ec is the Eckert number, Rd is the radiation parameter, Sc is the Schmidt number, σ is the chemical reaction parameter, E is the activation energy, Pe is the Peclet number, δ_1 represents the motile microorganism differences parameter, λ defines the

first order slip factor, Γ is the second order slip constant, and Bi is the Biot number. These parameters are mathematically related in following forms

$$\left. \begin{aligned} \alpha &= \frac{\alpha_1 c}{\mu}, M = \frac{\sigma B_0^2}{c \rho_f}, \Lambda = \left(\frac{(T_\infty) \beta_1 g_1 (1 - C_\infty)}{a(\rho c)_f} \right), Nc = \frac{\gamma_1 (\rho_m - \rho_f) (N_w - N_\infty)}{\rho_f (1 - C_\infty) T_\infty \beta_1}, \\ Nr &= \frac{(\rho_p - \rho_f) (\tilde{C}_w - \tilde{C}_\infty)}{\tilde{T}_\infty \beta_1 \rho_f (1 - \tilde{C}_\infty)}, S = \frac{Q}{c(\rho c)_f}, Pr = \frac{\nu}{\alpha}, Nt = \frac{\tau D_T (T_f - T_0)}{T_\infty \nu}, \\ Ec &= \frac{u_w^2}{C_f (T_f - T_0)}, Nb = \frac{\tau D_B (C_f - C_0)}{\nu}, Rd = \frac{4\sigma^* T_\infty^3}{kk^*}, Sc = \frac{\nu}{D_B}, \omega = \frac{Kr^2}{a}, \\ E &= \frac{E_a}{kT_\infty}, Pe = \frac{bW_c}{D_m}, Lb = \frac{\nu}{D_m}, \delta_1 = \frac{N_\infty}{N_w - N_\infty}, Bi = \left(\frac{h_f}{k} \right) \sqrt{\nu/c}, \end{aligned} \right\} \quad (25)$$

The mathematical forms for the local Nusselt number, the local Sherwood number and the motile density number are expressed as

$$\left. \begin{aligned} Nu &= \frac{xq_w}{k(T_f - T_\infty)}, q_w = -\left(k + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \left(\frac{\partial T}{\partial y} \right)_{y=0}, Sh = \frac{xq_m}{D_B (C_f - C_\infty)}, \\ q_m &= -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}, j_n = -D_m \left(\frac{\partial N}{\partial y} \right)_{y=0} \end{aligned} \right\} \quad (26)$$

Above quantities in the non-dimensional forms can be written as

$$NuRe_x^{-\frac{1}{2}} = -\left(1 + \frac{4}{3}R \right) \theta'(0), ShRe_x^{-\frac{1}{2}} = \phi'(0), NnRe_x^{-\frac{1}{2}} = -\chi'(0). \quad (27)$$

where Nu , Sh , Nn and Re_x are being the local Nusselt, the local Sherwood, the motile density and the Reynolds numbers, respectively.

3. Numerical Scheme

This section conveys the solution procedure for the differential equations system of Equations (20)–(23) subjected to the boundary conditions presented in Equation (24). Since these equations are highly nonlinear in nature and the exact solution is not possible, we employ the BVP4C numerical method in MATLAB in order to solve this system iteratively. The BVP4C is a limited difference code that executes the three-step Lobatto-IIIa formula. It is a collocation formula and the polynomial for the collocation provides us with a C_1 -continuous solution that is uniformly precise up to the fourth-order in each interval. For better convenience, we have chosen the step-size of $\Delta\zeta = 0.01$. For iterative process, the accuracy of 10^{-6} is achieved for current problems. Following the solving procedure, the finite value of $\eta_{\max} = 14$ is been adjusted carefully. To start the simulations, the current boundary value problems (BVP) are transmuted into a first-order initial value problems (IVP) as follows:

$$\left. \begin{aligned} f &= p_1, \frac{df}{d\eta} = p_2, \frac{d^2f}{d\eta^2} = p_3, \frac{d^3f}{d\eta^3} = p_4, \frac{d^4f}{d\eta^4} = p_4', \theta = p_5, \frac{d\theta}{d\eta} = p_6, \frac{d^2\theta}{d\eta^2} = p_6', \\ \phi &= p_7, \frac{d\phi}{d\eta} = p_8, \frac{d^2\phi}{d\eta^2} = p_8', \chi = p_9, \frac{d\chi}{d\eta} = p_{10}, \frac{d^2\chi}{d\eta^2} = p_{10}', \\ p_4' &= \frac{1}{(\alpha p_1)} \left(-p_2^2 + p_1 p_3 + M^2 p_2 + p_4 - 2\alpha p_2 p_4 - \alpha p_3^2 + \Lambda(p_5 - Nr p_7 - Nc p_9) \right), \\ p_6' &= -\frac{Pr}{(1 + Rd)} \left[\left(Ec(p_3^2) + M^2 p_2^2 + \alpha p_3(p_3 p_2 - p_1 p_4) \right) \right. \\ &\quad \left. + (p_1 p_6 + p_2 p_5 + Nb p_6 p_8 + Nc p_6^2 + Sp_5) \right], \\ p_8' &= -Sc p_1 p_8 + Pr Sc \omega (1 + \sigma p_5)^m \exp\left(\frac{-E}{1 + \sigma p_5}\right) p_7 - \frac{Nt}{Nb} p_6', \\ p_{10}' &= -Lb p_1 p_{10} + Pe(p_8'(p_9 + \delta_1) + p_{10} p_8) \end{aligned} \right\} \quad (28)$$

On the similar way, the boundary assumptions are transformed to the following forms

$$\left. \begin{aligned} p_1(\zeta) &= 0, p_2(\zeta) - (1 + \lambda p_3(\zeta) + \Gamma p_4(\zeta)), p_6(\zeta) - Bi(p_5(\zeta) - 1) = 0, \\ Nbp_6(\zeta) + Ntp_8(\zeta) &= 0, p_{10}(\zeta) = 1, \text{ as } \zeta = 0, \end{aligned} \right\} \quad (29)$$

$$p_2(\zeta) \rightarrow 0, p_5(\zeta) \rightarrow 0, p_7(\zeta) \rightarrow 0, p_9(\zeta) \rightarrow 0, \text{ as } \zeta \rightarrow \infty \quad (30)$$

4. Validation of Results

In order to assure the correct physical explanation, the accuracy of the computed solution is quite essential. In order to validate the reported numerical computations, we compared our results with available studies presented by Ibrahim [38] (see in Table 1). As can be seen, our numerical solution shows a convincing accuracy with the aforementioned literature.

Table 1. Comparison of solution for $f''(0)$ with various values of M when $\alpha = Nr = Nc = Rc = \Gamma = \lambda = 0$.

M	Wubshet Ibrahim [38]	Present Results
0.0	1.0000	1.0000
1.0	1.4142	1.4142
5.0	2.4495	2.4496

5. Physical Consequences of Results

This section interpolates the physical consequences of various flow parameters graphically for profiles of velocity f' , temperature θ , concentration ϕ and microorganism χ . For this purpose each parameter has been varied while remaining constant attained. Unless stated otherwise, fixed values are $\alpha = 0.2$, $M = 0.5$, $\Lambda = 0.3$, $Nc = 0.1$, $S = 0.1$, $Nr = 0.2$, $Pr = 0.7$, $N_t = 0.2$, $Ec = 0.2$, $Rd = 0.4$, $N_b = 0.3$, $Sc = 0.3$, $\omega = 0.2$, $E = 0.5$, $Lb = 0.2$, $Pe = 0.3$, $\delta_1 = 0.3$ and $Bi = 0.2$.

Figure 1 explains the physical feature of viscoelastic parameter α and mixed convection parameter Λ on velocity distribution f' . It is witnessed that distribution of velocity increases as we uplift both parameters gradually. Physically, viscoelastic parameter α is associated with the effective fluid viscosity due to which the velocity distribution gets maximum range. Similarly, the variation in the mixed convection constant Λ also improves the alteration in the velocity f' due to the involvement of the Grashoff number. It is further emphasized that increment in f' is more dominant with the change of α .

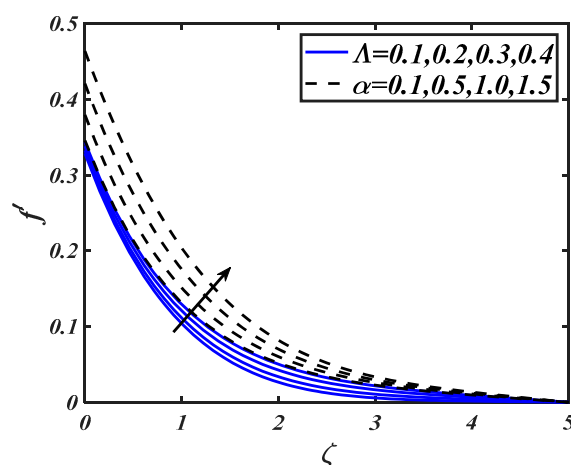


Figure 1. Profile f' for Λ and α .

Figure 2 is prepared to disclose the physical impact of the first-order slip parameter λ and the second-order slip parameter Γ on the velocity profile f' . Firstly, it is remarked that the first-order and the second-order slip factors are the characteristic of medium which is associated between pathway between fluid flow. The velocity magnitude reduces as we vary both slip parameters. A reduction in the thickness of boundary layer is also noted which is more privileged for first-order slip factor.

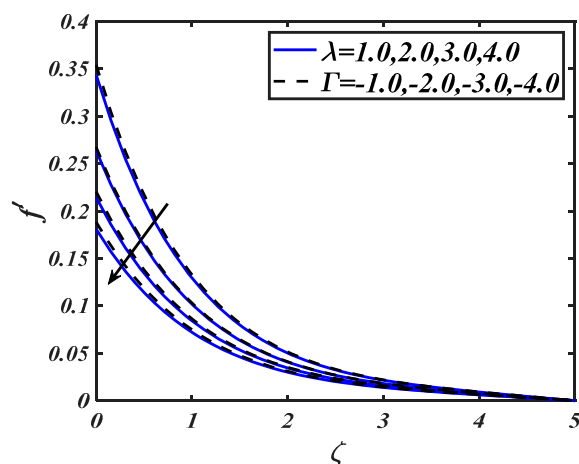


Figure 2. Profile f' for λ and Γ .

Figure 3 expresses the behavior of buoyancy ratio parameter Nc and the bioconvection Rayleigh number Nr on velocity field. This plot conveyed that an increment in either the buoyancy ratio parameter or the bioconvection Rayleigh number results in a declining velocity distribution f' . The reduction in f' is due to utilization of buoyancy forces which resist against the fluid particles' movement in the whole flow system and subsequently velocity decreases.

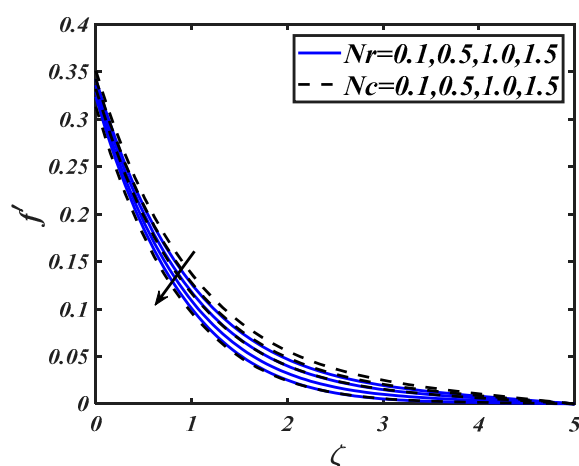


Figure 3. Profile f' for Nc and Nr .

To envision the impact of the first-order slip factor λ and the second-order slip parameter Γ on the temperature coefficient θ , Figure 4 is drawn. It is notified that both parameters are responsible for increase in the temperature distribution of nanoparticles θ . Therefore, it is concluded that slip effects may play a vital role to improve the nanoparticles temperate.

Figure 5 aims to depict the influence of the Prandtl number Pr and the radiation parameter Rd on the temperature coefficient θ . Physical Prandtl number is inversely associated with the thermal diffusivity of fluid, subsequently, a lower temperature distribution can be resulted in for leading values of Pr . Therefore, the proper evaluation of Prandtl number can control the thermal boundary layer growth. On the contrary, the larger values of radiation constant Rd boosts up the temperature distribution θ . The presence of thermal radiation is considered as an external source of energy which improves the temperature distribution effectively.

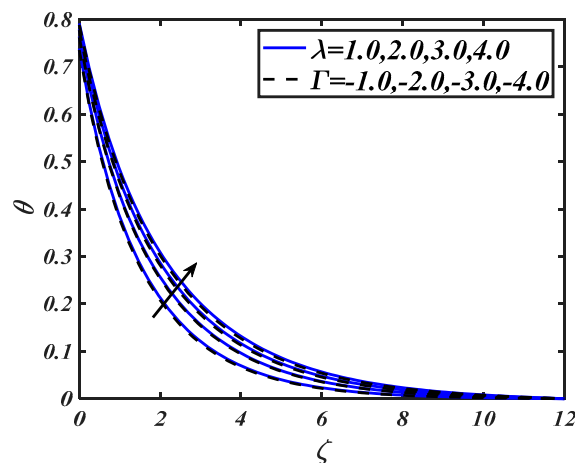


Figure 4. Profile θ for λ and Γ .

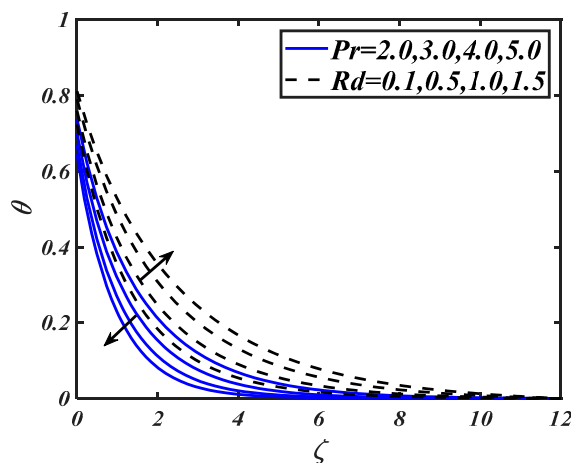


Figure 5. Profile θ for Pr and Rd .

The impact of Hartman number M and the viscoelastic parameter α on the temperature coefficient θ is illustrated in Figure 6. With variation in the Hartmann number M , an enhanced temperature distribution θ is noted. Since the interaction of the magnetic force is confronting the effects of Lorentz force, thus, it tends to improve the temperature distribution. In the same graph, the variation in α is presented which shows the opposite observations, i.e., the temperature distribution sufficiently increases with α . Physical aspects of such trend may be attributed to the involvement of the effective viscosity associated with the second-grade fluid which effectively enhanced the temperature distribution.

The result conveyed in Figure 7 deals with the variation of the thermophoresis constant Nt and the Biot number Bi . Thermophoresis is the phenomenon of moving particles towards the low-temperature region due to temperature gradient. Therefore, and as expected, an improvement in the thermophoresis constant enhances the temperature distribution θ . Similar graphical observations are noted for Biot number as Bi is related to the heat transfer coefficient. An increment in Bi results a larger heat coefficient which in return enlarges the temperature distribution θ .

The graphical observations for the concentration distribution ϕ against the first-order slip factor λ and the second slip factor Γ are presented in Figure 8. The concentration distribution ϕ is slightly increases with both parameters. However, changes in the variation of the concentration distribution ϕ is comparatively higher for the interaction of second-order slip coefficient Γ . Therefore, it is concluded that the interaction of the second-order slip is more convenient compared to the utilization of partial slip features. Further, the development in the boundary layer thickness is more stable by varying these parameters.

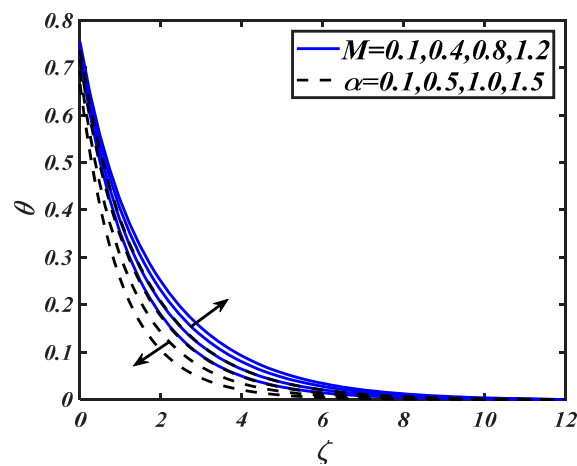
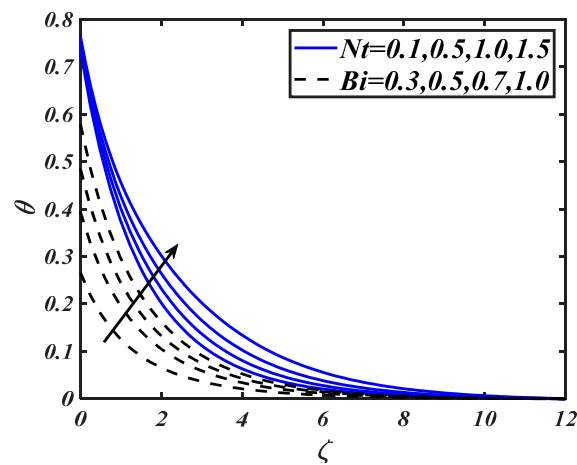
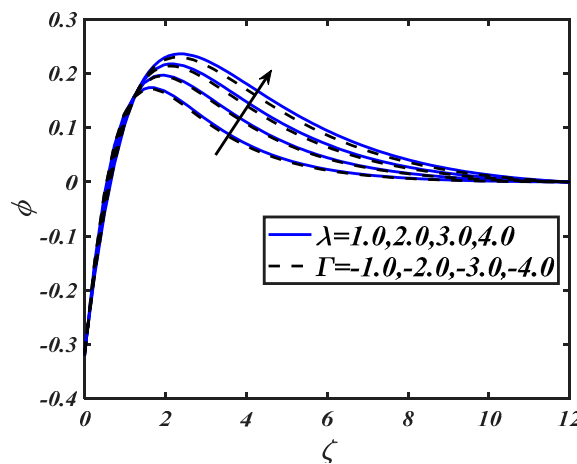
Figure 6. Profile θ for α and M .Figure 7. Profile θ for Nt and Nb .Figure 8. Profile ϕ for λ and Γ .

Figure 9 shows the variation of Lewis number Le and Prandtl number Pr on ϕ . The Lewis number Le is associated with the mass diffusion which decreases with an increment in Le . This results in a declining distribution of ϕ . Similarly, a declining nanoparticles concentration ϕ is inspected for higher Prandtl number Pr .

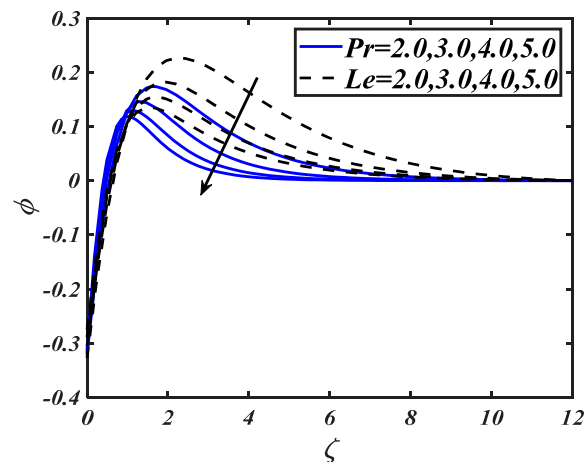


Figure 9. Profile ϕ for Pr and Le .

In Figure 10, the influence of the thermophoresis constant Nt and the Brownian motion parameter Nb are visualized against ϕ . The variation in the concentration distribution is maximum for Nt while the impact of Nb is quite opposite and controls the concentration distribution efficiently.

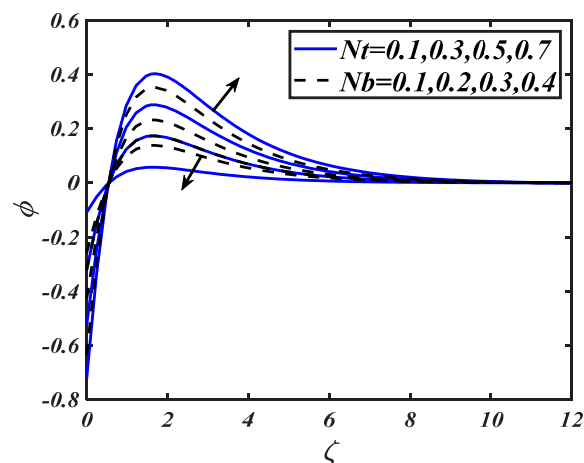


Figure 10. Profile ϕ for Nt and Nb .

The essential effects of the Hartmann number M and the viscoelastic fluid parameter α on concentration profile ϕ are presented in Figure 11. The viscoelastic parameter is physically related to the features of effective viscosity which altered with a variation of viscoelastic fluid parameter α . This results in a decreasing concentration distribution ϕ . While examining the effects of Hartmann number M on ϕ , it is seen that the nanoparticles concentration increases with M due to association of the Lorentz forces.

Figure 12 depicts the variation in the concentration field ϕ as results of changes in the activation energy E and the chemical reaction parameter ω . A contrasting behavior is noticed for a higher estimation of both parameters. Activation energy parameter is responsible for the incrimination of the concentration field ϕ while the dimensionless chemical reaction parameter caused a fall in the concentration distribution. The activation energy is the minimum energy amount to initiate the reaction process. Therefore, utilization of the activation energy enhances the reaction process and subsequently, the concentration distribution increases.

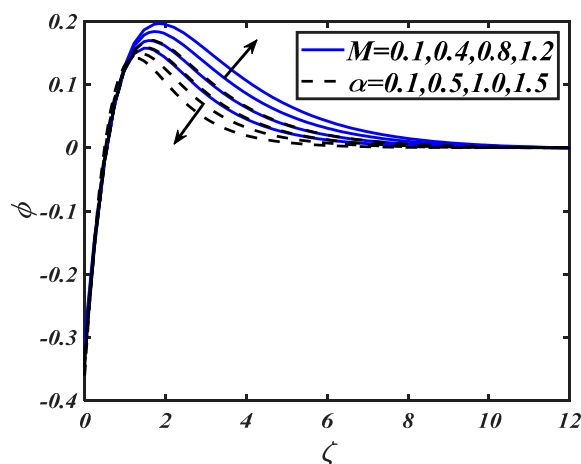
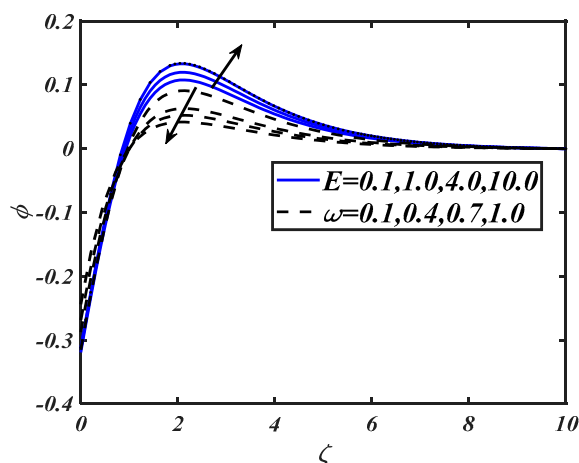
Figure 11. Profile ϕ for M and α .Figure 12. Profile ϕ for E and ω .

Figure 13 investigates the impact of the Hartman number M and the viscoelastic parameter α on the motile microorganism distribution χ . The motile microorganism distribution enhanced with M while the viscoelastic parameter depresses the profile of the motile microorganisms. The salient features of the first-order slip parameter λ and the second-order slip parameter Γ against the motile microorganism distribution χ is reported in Figure 14. It is noticed that by varying either slip parameter, the motility profile of nanoparticles is uplifted.

In order to visualize the effects of Peclet number Pe and bioconvection Lewis number Lb on the motile microorganism distribution χ , Figure 15 is plotted. The Peclet number has reverse relation with the motile diffusivity which means that the larger variation in Pe can cause a reduced motile microorganism profile. Similarly, the variation in Lewis number Lb also shows declining effects on χ and reduces the motile microorganism boundary layer.

Table 2 presents the variation in $-f''(0)$ for various values of M , α , Nr , Nc , λ and Γ . It is noted that numerical iteration in $-f''(0)$ increases with M , Nr and Nc . A decreasing rate of change in $-f''(0)$ is observed with the slip parameters λ and Γ . In Table 3, the variation in $-\theta'(0)$ for various values of M , Nr , Nc , α , λ , Γ , Pr , Nb , Nt and Bi are depicted which shows the numerical values of $-\theta'(0)$ for α , Pr and Bi in contrast to all other variables.

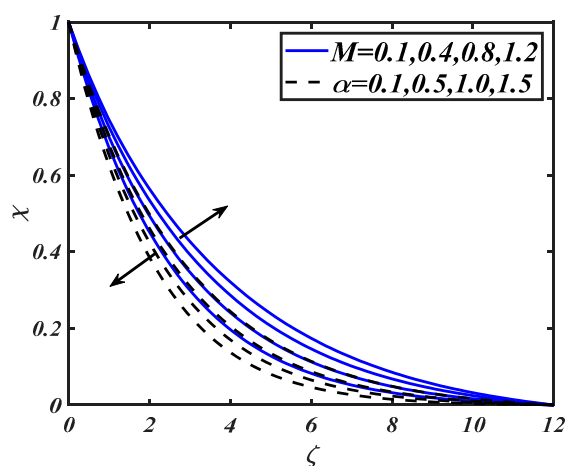
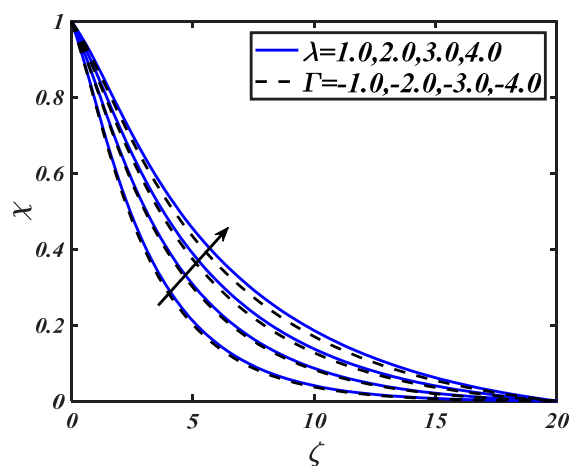
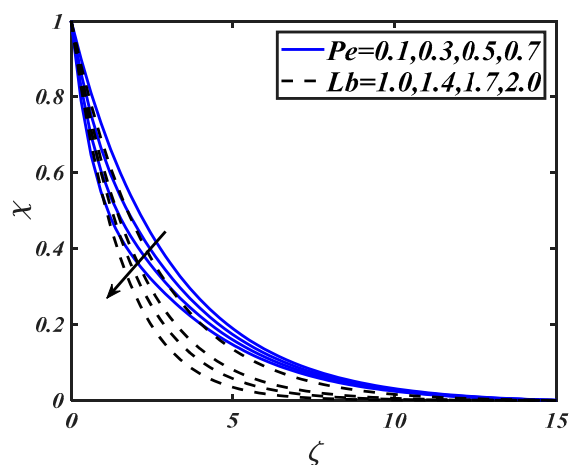
Figure 13. Profile χ for M and α .Figure 14. Profile χ for λ and Γ .Figure 15. Profile χ for Pe and Lb .

Table 2. Variation in $-f''(0)$ for $M, Nr, Nc, \alpha, \lambda$ and Γ .

M	Nr	Nc	α	λ	Γ	$-f''(0)$
0.1						0.3355
0.2	0.5	0.5	0.1	1.0	-1.0	0.3359
0.3						0.3361
	0.1					0.3349
	0.7					0.3366
	1.2					0.3381
		0.1				0.3328
		0.7				0.3377
		1.4				0.3434
			0.2			0.3346
			0.4			0.3339
			0.8			0.3335
				2.0		0.2507
				3.0		0.2005
				4.0		0.167
					-2.0	0.2552
					-3.0	0.2077
					-4.0	0.1756

Table 3. Variation in $-\theta'(0)$ for $M, Nr, Nc, \alpha, \lambda, \Gamma, Pr, Nb, Nt$ and Bi .

[illegible]

Table 4 shows the numerical values of $-\phi'(0)$ for different parameters which shows that it decreases for higher values of $\lambda, \Gamma, M, \Lambda$ and Nt . Finally, from Table 5, it is seen that the motile density number increases by increasing Pe, Lb and α .

Table 4. Variation in $-\phi'(0)$ for $M, \Lambda, Nr, Nc, \alpha, \lambda, \Gamma, Nt, Le$ and Bi .

M	Λ	Nr	Nc	α	λ	Γ	Nt	Le	$-\phi'(0)$
0.1									0.8271
0.2	0.1	0.5	0.5	0.1	1.0	−1.0	0.3	2.0	0.8150
0.3									0.8039
	0.1								0.6654
	0.2								0.5969
	0.3								0.5767
		0.1							0.7880
		0.7							0.7827
		1.2							0.7781
			0.1						0.7964
			0.7						0.7784
			1.4						0.7555
				0.2					0.7888
				0.4					0.7917
				0.8					0.7950
					2				0.7139
					3				0.6653
					4				0.6311
						−2			0.7181
						−3			0.6725
						−4			0.6401
							0.1		0.2659
							0.4		1.0458
							0.7		1.7996
								1.0	0.7915
								1.4	0.7903
								1.8	0.7892

Table 5. Of $-\chi'(0)$ for $\alpha, M, \Lambda, Nc, Nr, \lambda, \Gamma, Pe$ and Lb .

M	Λ	Nr	Nc	α	λ	Γ	Pe	Lb	$-\chi'(0)$
0.1									0.3904
0.2	0.1	0.5	0.5	0.1	1.0	−1.0	0.1	1.0	0.3777
0.3									0.3665
	0.1								0.2170
	0.2								0.1795
	0.3								0.1708
		0.1							0.3510
		0.7							0.3453
		1.2							0.3403
			0.1						0.3602
			0.7						0.3404
			1.4						0.3146

Table 5. Cont.

M	Λ	Nr	Nc	α	λ	Γ	Pe	Lb	$-\chi'(0)$
				0.2					0.3518
				0.4					0.3542
				0.8					0.3556
					2.0				0.3000
					3.0				0.2669
					4.0				0.2441
						−2.0			0.3029
						−3.0			0.2717
						−4.0			0.2500
							0.2		0.4310
							0.4		0.5902
							0.8		0.9119
								1.2	0.4070
								1.4	0.4603
								1.8	0.5611

6. Conclusions

In this research, the bioconvection flow for the thermally developed flow of the second-grade nanofluid over a vertical surface is examined. The additional features of viscous dissipation, activation energy, and the second-order slip factors are also considered. Some noteworthy observations from the current analysis are as follows:

- ❖ The utilization of second-order slip features controls the movement of fluid particles more effectively.
- ❖ An upsurges distribution of temperature has been noted for Hartmann number, slip parameters, thermophoresis constant, and radiation parameter.
- ❖ A lower solute distribution is noted for Schmidt number and viscoelastic parameter.
- ❖ The presence of viscoelastic parameter, Peclet number and bioconvection parameter decline the gyrotactic microorganism distribution.
- ❖ The gyrotactic microorganism distribution enhanced with the presence of slip factors.

Author Contributions: All authors contributed equally. All authors have read and agreed to the published version of the manuscript.

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