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Fixed Effect Meta-Analytic Structural Equation Modeling (MASEM) Estimation Using Generalized Method of Moments (GMM)

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Abstract: The fixed effect meta-analytic structural equation modeling (MASEM) model assumes that the population effect is homogeneous across studies. It was first developed analytically using Generalized Least Squares (GLS) and computationally using Weighted Least Square (WLS) methods. The MASEM fixed effect was not estimated analytically using the estimation method based on moment. One of the classic estimation methods based on moment is the Generalized Method of Moments (GMM), whereas GMM can possibly estimate the data whose studies has parameter uncertainty problems, it also has a high accuracy on data heterogeneity. Therefore, this study estimates the fixed effect MASEM model using GMM. The symmetry of this research is based on the proof goodness of the estimator and the performance that it is analytical and numerical. The estimation results were proven to be the goodness of the estimator, unbiased and consistent. To show the performance of the obtained estimator, a comparison was carried out on the same data as the MASEM using GLS. The results show that the estimation of MASEM using GMM yields the SE value in each coefficient is smaller than the estimation of MASEM using GLS. Interactive GMM for the determination of the optimal weight on GMM in this study gave better results and therefore needs to be developed in order to obtain a Random Model MASEM estimator using GMM that is much more reliable and accurate in performance.

Keywords: GMM; fixed effect model; meta-analysis; MASEM



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1. Introduction

Currently, there are many scientific studies whose discussions have the same topic but with various characteristics and results. Those studies are conducted from time to time and space to space on several global problems with the same goals. This condition demands researchers to review those scientific studies known as literature review. The main reason of this is to strengthen the overall conclusion from the findings in those studies [1]. Statistical methods that are capable of synthesizing studies, integrating studies, and gathering evidence of the studies, for the purpose of integrating findings of those studies, are called a meta-analysis [2]. A meta-analysis is defined as statistical analysis used to integrate a large collection of the analytical results from previous studies. One of the requirements needed in a meta-analysis is an assessment of the results of similar studies by finding the effect size value.

The effect size meta-analysis value consists of four types, namely effect size based on mean, proportion, odd ratio and correlation. The effect size based on correlation is obtained from the correlation of measured models such as Structural Equation Modeling (SEM). Currently, SEM is a widely used method in quantitative research. Many studies apply SEM model in various fields, for example in the fields of economic and social [3,4], education and health [5,6], engineering [7,8], and many others. The previous studies using Structural Equation Modeling (SEM) technique were integrated with the meta-analysis

based on the effect size correlation. The combination of SEM and meta-analysis technique is called Meta-Analytic Structural Equation Modeling (MASEM). MASEM has two steps of analysis, the first one is synthesizing the correlation or covariance matrix from several studies, and the second one is applying the SEM technique to explain the relationship between variables using combined correlation or covariance matrix [9].

MASEM is a correlated-based meta-analysis such that the type of MASEM data is a correlation matrix of the previous studies. Effect size correlation is divided into univariate and multivariate. Univariate effect size correlation is based on the bivariate correlation of two studies, and multivariate effect size correlation is based on the multivariate correlation of multiple studies. MASEM belongs to multivariate effect size correlation [10]. In general, according to the homogeneity of the MASEM models, as well as meta-analysis models, MASEM models are divided into fixed effect MASEM model and random effect MASEM model [11,12]. The difference between those two models is on the fixed effect model in an assumption that the population effect is homogeneous across studies such that the variance between studies component is ignored, and there is only variance within studies component in it. The random effect model is heterogeneous or there is variation on the effect of the studies population that involves both variance components between studies and within studies. In addition, the applied research of the MASEM method in Indonesia is similar to the state of health in East Java [13–15].

The fixed effect MASEM model was first developed analytically using Generalized Least Squares (GLS) method [16]. Further development was carried out computationally using Weighted Least Square (WLS) method or also known as Two Stage SEM (TSSEM) [17]. The development of the fixed effect MASEM model is limited, although there are many estimation methods that can be used to obtain more detailed study of the MASEM model. This is contrast to the development of the random effect MASEM model which has been widely studied, such as the DerSimonian and Laird's method [18], the moment method [19], maximum likelihood method [20], and etc.

In this study, the focus is on the fixed effect model MASEM as the first step in the further development of MASEM. The MASEM fixed effect was not estimated analytically using the estimation method based on moment. One of the classic estimation methods based on moment is the Generalized Method of Moments (GMM). GMM was first developed by Las Peter Hansen in 1982. GMM is an extension of the moment method. Basically, the GMM method compares the population moments with the sample moments then making it possible to estimate data studies having uncertainty problem parameter. GMM also has high level accuracy on data heterogeneity.

The MASEM model is sensitive to heterogeneity and the objective of MASEM is to obtain the homogeneity of the data. Based on the description above, fixed effect MASEM model needs to be developed analytically in order to obtain a good estimator and considering that MASEM is a method that has a problem with heterogeneity, and there is currently no study developing the fixed effect MASEM model using GMM estimation technique. Therefore, the researchers, in this study, propose a new method for estimating the fixed effect MASEM model using GMM, which is estimated analytically.

The symmetry of this research is based on the proof goodness of the estimator and the performance carried out by analytical and numerical. Estimation results for goodness of estimator proven by analytical were unbiased and consistent, while for estimator performance test, the estimation results were tested in sample question [16] and then compared with the performance of the estimation using GLS technique. The estimation results obtained are intended to be a new benchmark for the MASEM model, which has more reliable performance such that these results become a solution by integrating research with the same subject often found on social, economic, educational and other problems.

2. Literature Review

This section describes the definition of the Fixed Effect Meta-Analysis Model, Meta-Analytic Structural Equation Modeling (MASEM) using Generalized Least Squares (GLS) Approach and Generalized Method of Moments (GMM).

2.1. Fixed Effect Meta-Analysis Model

The meta-analysis based on the homogeneity of the effect size model is divided into two types, namely fixed effect meta-analysis and random effect meta-analysis. The fixed effect meta-analysis assumes that the effect size value that is similar to or homogeneous among studies and the variance between studies is ignored in this model. There is only variance in within study. The random effect meta-analysis assumes that it is heterogeneous with there is variance between studies and within studies [11].

Fixed effect model is written as follows:

$$y_i = \beta_F + e_i \quad (1)$$

where

y = studies effect size vector;
 β = combined effect size parameter;
 e = sampling error;
 $i = 1, 2, \dots, k$ studies unit;
 F = fixed effect case.

If e_i assumes normally distributed with mean 0 and variance σ_i^2 . Weighted mean of effect size estimation \bar{y} in the fixed effect model is [4]

$$\bar{y} = \frac{\sum_{i=1}^k w_i y_i}{\sum_{i=1}^k w_i} \quad (2)$$

where

y_i = i -th studies effect size;
 $w_i = \frac{1}{\sigma_i^2}$ (i -th studies weight);
 k = number of studies.

Sample variance S_F^2 with w_i as weighting factor can be calculated using

$$S_F^2 = \frac{1}{\sum_{i=1}^k w_i} \quad (3)$$

2.2. Meta-Analytic Structural Equation Modeling (MASEM) Using Generalized Least Squares (GLS) Approach

Becker's initial model in [16] is defined as regression model with k a collection of correlation matrices and p predictor variables. Thus, the multiple correlation is $p^* = \frac{p(p+1)}{2}$ will be estimated. This model is a multivariate meta-analysis model because the effect size value in MASEM is based on multivariate correlation. The meta-analysis model, namely Equation (1) is a univariate meta-analysis model and if the equation is applied to a

multivariate model consisting of k studies and p^* correlations, then it can be written in the following form:

$$\begin{aligned}
 \mathbf{y}_i &= \boldsymbol{\beta}_F + \mathbf{e}_i \\
 \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1p^*} \\ y_{21} \\ y_{22} \\ \vdots \\ y_{2p^*} \\ \vdots \\ y_{k1} \\ y_{k2} \\ \vdots \\ y_{kp^*} \end{bmatrix}_{kp^* \times 1} &= \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{p^*} \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{p^*} \\ \vdots \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{p^*} \end{bmatrix}_{kp^* \times 1} + \begin{bmatrix} e_{11} \\ e_{12} \\ \vdots \\ e_{1p^*} \\ e_{21} \\ e_{22} \\ \vdots \\ e_{2p^*} \\ \vdots \\ e_{k1} \\ e_{k2} \\ \vdots \\ e_{kp^*} \end{bmatrix}_{kp^* \times 1} \\
 \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1p^*} \\ y_{21} \\ y_{22} \\ \vdots \\ y_{2p^*} \\ \vdots \\ y_{k1} \\ y_{k2} \\ \vdots \\ y_{kp^*} \end{bmatrix}_{kp^* \times 1} &= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & & 0 & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & & 0 & 1 \\ & & \vdots & \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & & 0 & 1 \end{bmatrix}_{kp^* \times kp^*} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{p^*} \end{bmatrix}_{p^* \times 1} + \begin{bmatrix} e_{11} \\ e_{12} \\ \vdots \\ e_{1p^*} \\ e_{21} \\ e_{22} \\ \vdots \\ e_{2p^*} \\ \vdots \\ e_{k1} \\ e_{k2} \\ \vdots \\ e_{kp^*} \end{bmatrix}_{kp^* \times 1}
 \end{aligned}$$

Then, the Becker model, in which \mathbf{y} equal \mathbf{r} and $\boldsymbol{\beta}$ equal $\boldsymbol{\rho}$, is:

$$\mathbf{r} = \mathbf{X}\boldsymbol{\rho} + \mathbf{e} \quad (4)$$

where

\mathbf{r} = correlation vector with $kp^* \times 1$ size;

\mathbf{X} = identity matrix with $kp^* \times p^*$ size;

$\boldsymbol{\rho}$ = combined correlation vector $p^* \times 1$ size;

\mathbf{e} = error vector (unexplained Variance) $kp^* \times 1$ which is approximated by $\boldsymbol{\Sigma}$ variance-covariance matrix with $kp^* \times kp^*$ size.

If the GLS estimation steps have been carried out, it obtains

$$\hat{\boldsymbol{\rho}} = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{r} \quad (5)$$

$$\text{var}(\hat{\boldsymbol{\rho}}) = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1} \quad (6)$$

There are two models in the meta-analysis, which are fixed effect and random effect. In the two models, there are differences in the estimator value, which lies in the value of the variance-covariance matrices. Here is the difference between the two models:

1. First effect model: Fixed effect meta-analysis model is $r_i = \rho + e$ where $r \sim N(0, \Sigma)$ then the correlation matrix estimator is

$$\hat{\rho} = (X' \Sigma^{-1} X)^{-1} X' \Sigma^{-1} r. \quad (7)$$

$$\text{var}(\hat{\rho}) = (X' \Sigma^{-1} X)^{-1} \quad (8)$$

2. Random effect model: Random effect meta-analysis model is $r_i = \rho_i + e$ and $\rho_i = \rho + u_i$ where $r \sim N(0, \Sigma + T)$ or equal to $r \sim N(0, \Psi)$ then the correlation matrix estimator is

$$\hat{\rho} = (X' \Psi^{-1} X)^{-1} X' \Psi^{-1} r. \quad (9)$$

$$\text{var}(\hat{\rho}) = (X' \Psi^{-1} X)^{-1} \quad (10)$$

2.3. Generalized Method of Moments (GMM)

There are many basic estimation methods, such as maximum likelihood estimation (MLE) and ordinary least square (OLS). In addition to the basic method there is also another basic estimation method which is Method of Moment (MM). MM can also be used as an alternative method which can be used to find a consistent estimator of a parameter. The basic pattern of this method of moment is to equate the population moments with the corresponding sample moments.

The weakness of MM can only be applied in the case where the number of moment conditions is equal to the number of estimated parameters, thus in 1982 The Generalized Method of Moments (GMM) was introduced. GMM is one of the methods to estimating parameters and an extension of Method of Moments (MM). The GMM estimator is used on parameters that are overidentified by moment conditions. If we have more moment conditions than the unknown parameters, it will result in overidentification estimates. However, GMM can be used when the number of moment conditions is not the same as the number of estimated parameters. This also allows GMM to work in cases of high heterogeneity of data. The definition of GMM based on [21] is supposed there is an observed sample $(x_k : k = 1, 2, \dots, K)$ to estimate the θ parameter which has size $t \times 1$ to the true value of θ_0 . Supposed $E = [f(x_k, \theta_0)] = 0$ is the set of q conditional moments and corresponding $f_K(\theta)$ sample moments. The criteria function is defined as follows:

$$Q_k(\theta) = f_K(\theta)^T W f_K(\theta) \quad (11)$$

where W is a positive definite matrix, such that $\hat{\theta} = \arg\min_{\theta} Q_k(\theta)$.

3. Methodology

This section describes the Meta-Analytic Structural Equation Modeling (MASEM) estimation method proposed using the method of Generalized Method of Moments (GMM).

3.1. Parameter Estimation of Meta-Analytic Structural Equation Modeling (MASEM)

The ordinary Structural Equation Modeling (SEM) model can be said to have v number of exogenous latent variables and one endogenous latent variable. Based on that model, if there are p number of indicators of exogeneous latent v and endogenous latent q indicators $(y_1, y_2, \dots, y_q, x_1, x_2, \dots, x_p)$, then there is $t = p + q$ random variable that can be written as (a_1, a_2, \dots, a_t) in each study having n_i samples with total of k study. The correlation among random variables of a_e and a_s where $a_e, a_s \in (a_1, a_2, \dots, a_t)$ and $e \neq s$ is $c^* = \frac{t(t-1)}{2}$ in the i study and is written as $\eta = [\eta_{i12}, \eta_{i13}, \dots, \eta_{i1t}, \eta_{i23}, \dots, \eta_{i(t-1)t}]'$ which η is the sample correlation vector whereas the population correlation is written as $\gamma = [\gamma_{i12}, \gamma_{i13}, \dots, \gamma_{i1t}, \gamma_{i23}, \dots, \gamma_{i(t-1)t}]'$.

In this study the researchers also use several assumptions including:

1. $\mathbf{a} = [a_1, a_2, \dots, a_t]' \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$;
2. asymptotic normal distribution $\sqrt{n_i}(\boldsymbol{\eta}_i - \boldsymbol{\gamma}_i) \sim N(0, \boldsymbol{\Sigma})$
3. error vector $\boldsymbol{\zeta} \sim N(0, \boldsymbol{\Sigma})$ such that the value of $\boldsymbol{\Sigma}_i$ according to [22] can be defined as follows:

$$\text{Var}(\eta_{ipq}) = \frac{(1 - \gamma_{ipq})^2}{n_i} \quad (12)$$

and

$$\text{Cov}(\eta_{ist}, \eta_{iuv}) = [0.5\gamma_{ist}\gamma_{iuv}(\gamma_{isu}^2 + \gamma_{isv}^2 + \gamma_{itu}^2 + \gamma_{itv}^2) + \gamma_{isu}\gamma_{itv} + \gamma_{isv}\gamma_{itu} - (\gamma_{ist}\gamma_{isu}\gamma_{isv} + \gamma_{its}\gamma_{itu}\gamma_{itv} + \gamma_{ius}\gamma_{iut}\gamma_{iuv} + \gamma_{ivs}\gamma_{ivt}\gamma_{ivu})]/n_i \quad (13)$$

3.2. Fixed Effect Generalized Method of Moment (GMM) Estimation

The fixed effect meta-analysis model assumes that the effect size value is similar for each study and variance between study component is ignored. To estimate the correlation vector in general with c^* model length is as follows:

$$\boldsymbol{\eta}_{kc^* \times 1} = \boldsymbol{\Xi}_{kc^* \times c^*} \boldsymbol{\gamma}_{c^* \times 1} + \boldsymbol{\zeta}_{kc^* \times 1} \quad (14)$$

where

$\boldsymbol{\eta}$ = correlation vector between indicators in each study with $kc^* \times 1$ size;

$\boldsymbol{\gamma}$ = combined correlation vector with $c^* \times 1$ size;

$\boldsymbol{\Xi}$ = matrix created by stacking k identity matrix with $kc^* \times c^*$ size;

$\boldsymbol{\zeta}$ = error vector (unexplained variance) with $kc^* \times 1$ size.

The steps for estimating the parameters from Equation (14) are as follows:

1. Determining the population moment condition [11]:

$$\mathbf{f}(\boldsymbol{\gamma}) = \boldsymbol{\Xi}'(\boldsymbol{\eta} - \boldsymbol{\Xi}\boldsymbol{\gamma}) \quad (15)$$

2. Determining the sample moment condition:

$$\mathbf{f}_K(\boldsymbol{\gamma}) = \frac{1}{k} \sum_{i=1}^k (\boldsymbol{\Xi}_i'(\boldsymbol{\eta}_i - \boldsymbol{\Xi}_i\boldsymbol{\gamma})) \quad (16)$$

3. Constructing GMM function which is quadratic function of the sample moment:

$$\mathbf{Q}_K(\boldsymbol{\gamma}) = \mathbf{f}_K(\boldsymbol{\gamma})' \mathbf{W} \mathbf{f}_K(\boldsymbol{\gamma}) \quad (17)$$

where \mathbf{W} is the weight estimator;

4. Minimizing function $\mathbf{Q}_K(\boldsymbol{\gamma})$:

$$\frac{\partial \mathbf{Q}_K(\boldsymbol{\gamma})}{\partial \boldsymbol{\gamma}} = 0 \quad (18)$$

$$\frac{\partial \mathbf{f}_K(\boldsymbol{\gamma})' \mathbf{W} \mathbf{f}_K(\boldsymbol{\gamma})}{\partial \boldsymbol{\gamma}} = 0 \quad (19)$$

5. After substituting Equation (16) to Equation (19), we obtain the parameter estimation of GMM below:

$$\hat{\boldsymbol{\gamma}}_{GMM} = (\boldsymbol{\Xi}' \boldsymbol{\Xi} \mathbf{W} \boldsymbol{\Xi}' \boldsymbol{\Xi})^{-1} \boldsymbol{\Xi}' \boldsymbol{\Xi} \mathbf{W} \boldsymbol{\Xi}' \boldsymbol{\eta} \quad (20)$$

$$\text{var}(\hat{\boldsymbol{\gamma}}_{GMM}) = (\boldsymbol{\Xi}' \boldsymbol{\Xi} \mathbf{W} \boldsymbol{\Xi}' \boldsymbol{\Xi})^{-1} \boldsymbol{\Xi}' \boldsymbol{\Xi} \mathbf{W} \boldsymbol{\Xi}' \boldsymbol{\Sigma} \boldsymbol{\Xi} \mathbf{W} \boldsymbol{\Xi}' \boldsymbol{\Xi} (\boldsymbol{\Xi}' \boldsymbol{\Xi} \mathbf{W} \boldsymbol{\Xi}' \boldsymbol{\Xi})^{-1} \quad (21)$$

3.2.1. The Unbiased Properties of the GMM Estimator

One of the properties of a good estimator is unbiased, which means that the expected value of the estimator is the same as the value of the parameter or it can be written as

$E(\hat{\gamma}_{GMM}) = \gamma$. The unbiased properties of the GMM estimator satisfies Lemma 1 which can be proved analytically by the proof found in Appendix A.

Lemma 1. *If there is a Meta-Analytic Structural Equation Modeling (MASEM) model with a population correlation vector $\eta \sim N_p(\mu, \Sigma)$, a sample correlation vector (γ) and an error vector $\zeta \sim N(0, \Sigma)$ estimated by the generalized method of moment (GMM), the value of $E(\hat{\gamma}_{GMM}) = \gamma$ is unbiased.*

3.2.2. The Consistent Properties of the GMM Estimator

The properties of good estimator are not only unbiased but also consistent. The unbiased estimator $\hat{\gamma}_{GMM}$ is consistent if $\lim_{k \rightarrow \infty} Q_K(\hat{\gamma}_k) = 0$ [23]. The consistent properties of the GMM estimator satisfies Lemma 2 in which the proof is shown in Appendix B.

Lemma 2. *If there is a Meta-Analytic Structural Equation Modeling (MASEM) model with a population correlation vector $\eta \sim N_p(\mu, \Sigma)$, a sample correlation vector (γ) and an error vector $\zeta \sim N(0, \Sigma)$ estimated by the generalized method of moment (GMM), the unbiased estimator $\hat{\gamma}_{GMM}$ is consistent that satisfies the value of $\lim_{k \rightarrow \infty} Q_K(\hat{\gamma}_k) = 0$.*

3.2.3. Determination of W Weight

Parameter estimation using GMM involves weighting W , which is a positive definite weight matrix, and the optimal value of W weight can produce the efficient estimator. The W weight is defined as follows [24]:

$$\lim_{k \rightarrow \infty} \text{Var}(\sqrt{k}f_K(\gamma)) = W(\gamma) \quad (22)$$

and

$$W_{opt} = [W(\gamma)]^{-1} \quad (23)$$

The value of $\sqrt{k}f_K(\gamma)$ if approached by the Central Limit Theorem yields

$$\sqrt{k}f_K(\gamma) \rightarrow N(0, S) \quad (24)$$

where S is the asymptotic moment variance of $\sqrt{k}f_K(\gamma)$. This implies that Equation (23) can also be written as

$$W_{opt} = S^{-1} \quad (25)$$

The S matrix is defined as follows as:

$$\begin{aligned} S &= \text{Var}(\sqrt{k}f_K(\gamma)) \\ &= k \text{Var}\left(\frac{1}{k} \sum_{i=1}^k (\Xi_i'(\eta_i - \Xi_i\gamma))\right) \\ &= \frac{1}{k} \sum_{i=1}^k [\Xi_i'(\eta_i - \Xi_i\gamma)] [(\eta_i - \Xi_i\gamma)' \Xi_i] \\ &= \frac{1}{k} \sum_{i=1}^k [\Xi_i' \zeta_i] [\zeta_i' \Xi_i] \end{aligned} \quad (26)$$

Because the Equation (26) still contains γ component, numerical iteration is carried out to find the solution. The numerical iteration steps are:

Step 1 Initialize $W^{(i)} = I$ which is substituted to Equation (20) such that the initial value of $\hat{\gamma}_{GMM}^{(i)}$ is obtained.

Step 2 The estimated result of $\hat{\gamma}_{GMM}^{(i)}$ is substituted to Equation (26) such that the value of $S^{(i)}$ is obtained.

Step 3 Update the weights by calculating $W^{(i+1)} = (S^{(i+1)})^{-1}$.

Step 4 Update the $\hat{\gamma}_{\text{GMM}}^{(i+1)}$ value from Equation (20).

Step 5 Returns to step 2 and Calculate iteration until convergence.

4. Results and Discussion

In this section, the estimation results of Meta-Analytical Structural Equation Modeling (MASEM) using the Generalized Method of Moments (GMM) are tested on simulation data to show the performance of the GMM method and the results are compared to the GLS method.

4.1. Results

The estimation of Meta-Analytical Structural Equation Modeling (MASEM) with the GMM method was applied to the case real data in the article [15] with size of sample Table 1. It aims to show the performance of the estimator. The example case in this article concerns the correlation between 4 variables, poverty, economy, human resource (HR) and health variables. The coefficient β_1 shows the correlation between economy and poverty, The coefficient β_2 shows the correlation between health and poverty and the coefficient β_3 shows the correlation between HR and poverty. There are six studies as data sources for the application of the sample questions. They are as follows:

Table 1. Data of Sample for Implementation.

No	Data of Sample	Size of Sample
1	East Java	$n1 = 38$
2	Central Java	$n2 = 35$
3	Yogyakarta	$n3 = 5$
4	West Java	$n4 = 27$
5	DKI Jakarta	$n5 = 6$
6	Banten	$n6 = 8$

There are six correlations from each data source (economy-poverty; health-poverty; HR-poverty; economy-health; economy-HR; health-HR) and arranged in $r_{36 \times 1}$ or $\eta_{36 \times 1}$ vector form,

$$r_{36 \times 1} = \begin{bmatrix} r_{16 \times 1}^T & r_{26 \times 1}^T & r_{36 \times 1}^T & r_{46 \times 1}^T & r_{56 \times 1}^T & r_{66 \times 1}^T \end{bmatrix}^T \text{ Where}$$

$$r_{16 \times 1}^T = \begin{bmatrix} 0.590 & -0.476 & -0.612 & -0.311 & -0.563 & 0.666 \end{bmatrix}^T$$

$$r_{26 \times 1}^T = \begin{bmatrix} 0.591 & -0.494 & 0.326 & -0.591 & 0.639 & -0.378 \end{bmatrix}^T$$

$$r_{36 \times 1}^T = \begin{bmatrix} 0.827 & -0.604 & -0.753 & -0.529 & -0.738 & 0.898 \end{bmatrix}^T$$

$$r_{46 \times 1}^T = \begin{bmatrix} 0.698 & -0.399 & -0.387 & -0.747 & -0.251 & 0.231 \end{bmatrix}^T$$

$$r_{56 \times 1}^T = \begin{bmatrix} 0.963 & -0.946 & -0.930 & -0.939 & -0.985 & 0.888 \end{bmatrix}^T$$

$$r_{66 \times 1}^T = \begin{bmatrix} 0.829 & -0.848 & -0.447 & -0.888 & -0.742 & 0.826 \end{bmatrix}^T$$

Or in GMM estimation

$$\eta_{36 \times 1} = \begin{bmatrix} \eta_{16 \times 1}^T & \eta_{26 \times 1}^T & \eta_{36 \times 1}^T & \eta_{46 \times 1}^T & \eta_{56 \times 1}^T & \eta_{66 \times 1}^T \end{bmatrix}^T \text{ with the same data}$$

The estimated value obtained consists of a pooled correlation matrix ($\hat{\rho}_{\text{GLS}}$ or $\hat{\gamma}_{\text{GMM}}$), the value of the coefficient between the latent variables and the fit of the model. The results of the GMM MASEM estimation are then compared to their performance with the results of the GLS MASEM estimator using initial weight $W^0 = I$ presented in Table 2.

Table 2. Result values.

Value	Method	
	GLS	GMM
pooled correlation vector	$\hat{r}_{GLS} = \begin{bmatrix} 0.906 \\ -0.976 \\ -0.832 \\ -0.956 \\ -0.978 \\ 0.910 \end{bmatrix}$	$\hat{\gamma}_{GMM} = \begin{bmatrix} 0.714 \\ -0.587 \\ -0.539 \\ -0.604 \\ -0.228 \\ 0.239 \end{bmatrix}$
Coefficient β_1	0.511	0.516
Coefficient β_2	−1.079	−0.185
Coefficient β_3	0.650	−0.378
The standard error of β_1	0.808	0.075
The standard error of β_2	0.309	0.078
The standard error of β_3	0.548	0.057

4.2. Discussion

The goodness of the estimator is unbiased and consistent. The MASEM estimator with GMM was proven to be unbiased and consistent; thus, it has the goodness of the estimator. To see the performance of the estimator, the estimation results of this study were compared with the estimations results of the MASEM GLS. The result of the model fit is the standard error of β_1 , β_2 and β_3 . This shows that the estimation using GMM produces the value better than the estimation using GLS because the SE value of the GMM method in each coefficient is smaller than the GLS method. Thus, the GMM performance using the interactive GMM has better performance than the GLS method for the case of poverty in the island of Java.

5. Conclusions

The reliability of the performance of GMM depends on its weight, thus there are many ways that can be used to obtain the optimal weight. In this study, using interactive GMM which turned out to have a better performance than GLS result. Of course, future studies are also needed to determine a more optimal weight for the random model MASEM estimator using GMM that is much more reliable and accurate in performance.

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Appendix A

The steps to prove that $\hat{\gamma}_{\text{GMM}}$ is an unbiased estimator is to simplify Equation (20) as follows as:

$$\hat{\gamma}_{\text{GMM}} = D\eta$$

where $D = \left(\Xi' \Xi W \Xi' \Xi \right)^{-1} \Xi' \Xi W \Xi'$, such that

$$\begin{aligned} E(\hat{\gamma}_{\text{GMM}}) &= E(D\eta) \\ &= DE(\Xi\gamma + \zeta) \text{ as, } E(\gamma) = 0 \\ &= \left(\Xi' \Xi W \Xi' \Xi \right)^{-1} \Xi' \Xi W \Xi' \Xi E(\Xi\gamma) \\ &= \left[\left(\Xi' \Xi W \Xi' \Xi \right)^{-1} \Xi' \Xi W \Xi' \Xi \right] \gamma \\ &= \gamma \end{aligned}$$

then, it is proven that $E(\hat{\gamma}_{\text{GMM}}) = \gamma$ and has the unbiased properties of estimator.

Appendix B

The GMM estimator is obtained by minimizing Equation (17); thus, it will be proved that

$$\begin{aligned} \lim_{k \rightarrow \infty} Q_K(\hat{\gamma}_k) &= 0 \\ &= \lim_{k \rightarrow \infty} \left(\frac{1}{k} \Xi' \eta - \frac{1}{k} \Xi' \Xi \hat{\gamma}_k \right)' W \left(\frac{1}{k} \Xi' \eta - \frac{1}{k} \Xi' \Xi \hat{\gamma}_k \right) = 0 \\ &= \lim_{k \rightarrow \infty} \frac{1}{k^2} \left[\left(\Xi' \eta - \Xi' \Xi \hat{\gamma}_k \right)' W \left(\Xi' \eta - \Xi' \Xi \hat{\gamma}_k \right) \right] = 0 \end{aligned}$$

It is true that the unbiased estimator $\hat{\gamma}_{\text{GMM}}$ is consistent with satisfies the value $\lim_{k \rightarrow \infty} Q_K(\hat{\gamma}_k) = 0$.

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