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A Novel Approach to Decision Making Based on Interval-Valued Fuzzy Soft Set

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Abstract: Interval-valued fuzzy soft set theory is a powerful tool that can provide the uncertain data processing capacity in an imprecise environment. The two existing methods for decision making based on this model were proposed. However, when there are some extreme values or outliers on the datasets based on interval-valued fuzzy soft set for making decisions, the existing methods are not reasonable and efficient, which may ignore some excellent candidates. In order to solve this problem, we give a novel approach to decision making based on interval-valued fuzzy soft set by means of the contrast table. Here, the contrast table has symmetry between the objects. Our proposed algorithm makes decisions based on the number of superior parameter values rather than score values, which is a new perspective to make decisions. The comparison results of three methods on two real-life cases show that, the proposed algorithm has superiority to the existing algorithms for the feasibility and efficiency when we face up to the extreme values of the uncertain datasets. Our proposed algorithm can also examine some extreme or unbalanced values for decision making if we regard this method as supplement of the existing algorithms.

Keywords: interval-valued fuzzy soft set; extreme value; decision making

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1. Introduction

There exists uncertain and fuzzy data when we face up to practical and complicated problems in a lot of domains as diverse as social science, medical science, economics, engineering [1,2], and so on. Many classical approaches such as probability theory, fuzzy sets and rough sets [3], and so on, have been developed to deal with vagueness. However, these solutions have inherent difficulties which were figured out in [4]. Molodtsov initiated the concept of soft set theory [4] which is an effective mathematical tool in handling uncertainty. There are rich variety of applications of soft set theory in many fields as diverse as game theory [4], operations research, decision making [5–8], data mining [9,10] Screening alternatives [11], resource discovery [12] and data filling [13] for incomplete datasets [14], and so on. In addition to the soft set theory, recently, scholars have developed and studied plenty of combination models of the soft set theory with other mathematical models such as fuzzy soft set [15–17], intuitionistic fuzzy soft set [18,19], belief interval-valued soft set [20], interval-valued intuitionistic fuzzy soft sets [21], hesitant N-soft sets [22], confidence soft sets [23], fault-tolerant enhanced bijective soft set [24], trapezoidal interval type-2 fuzzy soft sets [25], soft rough set [26], Z-soft fuzzy rough set [27], Z-soft rough fuzzy set [28], and so on.

In a number of fuzzy applications, the related membership functions are extremely individual, and so, giving an interval-valued data to express degree of membership is more reasonable and rational. As a result, the interval-valued fuzzy soft set was created [29], which is constructed by integrating interval valued fuzzy set and soft set. Interval-valued fuzzy soft set (IVFSS) is one of the most successful extended models of soft set. Because of inheriting the merits of the above two models, this model is robust when we are in the face of difficulties of handling fuzzy and unclearly defined datasets. At present, the main applications of this model involve the field of data filling, parameter reduction evaluation system and decision making. There is some incomplete information in the datasets described by IVFSS, which can be handled by the method of [30]. For the purpose of ignoring the needless parameters, Ma et al. [31] depicted and analyzed demerits and merits four parameter reduction ideas in the process of decision making. The research of [32] further displayed a complete evaluation system framework constructed on this model. Decision making is an important application of this model. The paper of [33] expressed one adjustable algorithms of decision making created on the definition of level soft sets. However, this method transformed interval-valued data into binary data, which lost the original superiority of interval-valued description for IVFSS. The combined weights for the parameters in IVFSS were considered in [34,35] when we have to make stochastic multi-criteria decision and emergency decision. However, the two methods were complicated and not easy to implement. In [29], one decision making method based on scores for IVFSS was proposed and an efficient decision making approach which considered the added objects was given in [36] in 2021. however, when there are some extreme values or outliers on the datasets for making decisions, the methods proposed in [29,36] were not reasonable and efficient. That is, according to the methods proposed in [29,36], the object with the maximum score value will be selected as the best choice. However, if the objects have some outliers or extreme value for some specific parameters, the two existing methods are likely to ignore some excellent candidates. And in some situation, we need to find some extreme value or some objects which have outstanding performances on some specific parameters. In order to solve this problem, in this paper, we give a novel approach to decision making based on IVFSS. Our contributions are as follows:

(1) We propose a novel approach to decision making based on IVFSS by means of the contrast table, which considers the extreme values or outliers.
(2) Our proposed algorithm can find and examine some extreme or unbalanced values for decision making when the two existing approaches have the different decision making results with our algorithm; if the three methods have the same results, we can verify that the datasets have no extreme data.
(3) The comparison results of three methods on one example and two real-life applications including 5-star Sydney Hotel Rating Systems and Scenic Spots Weather Condition Evaluation Systems show that, the proposed algorithm has superiority to the two existing algorithms when we face up to extreme or outlier values in the processing of decision making.

The remainder of this paper is organized as follows. The basic related terms are introduced in Section 2. Section 3 recalls the two existing algorithms of [29,36] and points to the weakness of the two methods. Section 4 proposes a novel approach to decision making based on IVFSS by means of the contrast table. Section 5 depicts the comparison results among three methods on two real-life cases. Finally, Section 6 shows the conclusion from our research.

2. Basic Notions

In this section, we briefly recall some definitions with regard to soft sets and IVFSS.

Definition 1. ([4]). *Let U be a non-empty initial universe of objects, E be a set of parameters in relation to objects in U, $P(U)$ be the power set of U, and A be a subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 2. ([29]). *Let an interval-valued fuzzy set X on an universe U be a mapping such that $\hat{\mathrm{X}} : \mathrm{U} \to \mathrm{Int}([0,1])$ where the set of all closed sub-intervals is denoted by Int([0,1]). Suppose that $\hat{\mathrm{X}} \in \widetilde{\psi}(U)$, where $\widetilde{\psi}(U)$ represents the set of all interval-valued fuzzy sets on U. $\mu^-_{\hat{\mathrm{X}}}(x)$ and $\mu^+_{\hat{\mathrm{X}}}(x)$ represents the lower and upper degrees of membership x to $\hat{\mathrm{X}}$ $(0 \le \mu^-_{\hat{\mathrm{X}}}(x) \le \mu^+_{\hat{\mathrm{X}}}(x) \le 1)$. For every $x \in \mathrm{U}$, $\mu_{\hat{\mathrm{X}}}(x) = [\mu^-_{\hat{\mathrm{X}}}(x), \mu^+_{\hat{\mathrm{X}}}(x)]$ is denoted as the degree of membership an element x to*

$\hat{X}$. *Suppose further that E is a set of parameters in relation to objects in U. A pair* $(\widetilde{\omega}, E)$ *is defined as an interval-valued fuzzy soft set over* $\widetilde{\psi}(U)$*, where* $\widetilde{\omega}$ *is a mapping given by* $\widetilde{\omega} : E \rightarrow \widetilde{\psi}(U)$.

We give the following example for illustration of IVFSS.

Example 1. *One movie critic wants to express the feeling of four movies from four dimensionalities. We can apply the model of IVFSS to describe the fuzzy expression. Note that the universe U represents the set of the four different movies and U = {h_1,h_2,h_3,h_4}. A represents the set of four parameters and A = {e_1,e_2,e_3,e_4} = {deliciously clever story, tight and witty plots, transporting visual beauty, high-profile releases}. Then IVFSS (F, A) on U is described by the following Table 1. The lower and upper approximations of such an assessment for four movies from four aspects are displayed in Table 1. For instance, a movie h_1 has at least tight and witty plots on the level of 0.3 and it has at most tight and witty plots on the level of 0.7.*

Table 1. IVFSS (F, A) for Examples 1 and 2.

U	e_1	e_2	e_3	e_4	c_i	r_i
h_1	[0.2, 0.4]	[0.3, 0.7]	[0.4, 0.6]	[0.5, 0.9]	[1.4, 2.6]	−1
h_2	[0.3, 0.6]	[0.4, 1.0]	[0.5, 0.9]	[0.1, 0.3]	[1.3, 2.8]	−0.6
h_3	[0.7, 0.8]	[0.5, 0.7]	[0.3, 0.5]	[0.6, 0.9]	[2.1, 2.9]	2.1
h_4	[0.5, 0.8]	[0.0, 0.4]	[0.3, 0.8]	[0.5, 0.6]	[1.3, 2.6]	−1.4

3. Related Work

Though there are some decision making methods based on the models such as soft sets [37] and fuzzy soft sets [38], the two mathematical models and interval-valued fuzzy soft set are different which have the different characteristics. The entries of soft set are 0 and 1, while the data of fuzzy soft set lies between 0 and 1. Interval-valued fuzzy soft sets have the lower and upper degrees of membership which are between 0 and 1. Different models need the different decision making approaches. Here we only focus on the decision making methods based on this model of interval-valued fuzzy soft sets. In this section, we briefly introduce the two existing Algorithms 1 and 2 to solve fuzzy decision making problems based on IVFSS such as score based decision making approach (SBDM) [29] and decision making method considering the added objects (CAODM) in [36] as follows.

Algorithm 1: SBDM [29]

Input: an IVFSS $(\widetilde{Z}, P)$.
Output: the optimal object.
Step 1: figure out the choice value c_i for each object hi by the equation of
$c_i = [c_i^-, c_i^+] = [\sum_{p \in P} \mu^-_{\widetilde{Z}(P)}(h_i), \sum_{p \in P} \mu^+_{\widetilde{Z}(P)}(h_i)]$.
Step 2: find the score value r_i of h_i by the equation of $r_i = \sum_{h_i \in U} ((c_i^- - c_j^-) + (c_i^+ - c_j^+))$.
Step 3: find the maximum of the score value and the corresponding object is referred to as the best outcome.

Here, we apply Example 1. to illustrate the performance of SBDM.

Example 2. *According to the dataset of Example 1, that is, this movie critic wants to assess these movies with its tabular form given by Table 1 and find the best one. According to SBDM, we compute choice value c_i and the score r_i for all the objects. The corresponding results are shown in Table 1, from which we see that h_3 is the most desirable movie, since it has the maximum score $r_3 = max_{hi \in U}\{r_i\} = 2.1$. If we sort the movies according to the scores in descending order, we are able to get $h_3 > h_2 > h_1 > h_4$. In this dataset, the algorithm of SBDM works well. Similarly, CAODM get the same outcome as $h_3 > h_2 > h_1 > h_4$.*

However, if we face up to some extreme situation, the two algorithms seem unreasonable, which is likely to ignore some good candidates. Let us look at this following Example 3.

Algorithm 2: CAODM [36]

Input: an IVFSS $\left(\tilde{S}, E\right)$.
Output: the optimal object.
Step 1: Compute the choice value c_i for each object h_i by the equation as
$$c_i = [c_i^-, c_i^+] = \left[\sum_{p\in P} \mu^-_{\tilde{Z}(P)}(h_i), \sum_{p\in P} \mu^+_{\tilde{Z}(P)}(h_i)\right],$$
For every $h_i \in U$, where c_i^- and c_i^+ are termed as the upper choice value and lower choice value, respectively. Here $\mu^-_{\tilde{Z}(e_j)}(h_i), \mu^+_{\tilde{Z}(e_j)}(h_i)$ are the upper and lower degrees for object h_i and parameter e_j, respectively.
Step 2: Figure out the overall choice value $C_i^{overall}$ of h_i by the following equation as
$$C_i^{overall} = c_i^- + c_i^+,$$
For every $h_i \in U$.
Step 3: Find $C_k^{overall} = \text{Max}\left\{C_i^{overall}\right\}$. That is, find the optimal object which has the maximum overall choice value.

Example 3. *One family is planning to buy an apartment building for living. There are five alternative apartment candidates from five different property developers. This family hesitates about which to buy. We are able to evaluate the alternatives from four aspects: "reasonable price", "excellent geographical location", "perfect facilities", "cozy environment". We choose the model of IVFSS to describe the customer's feeling for the five candidates from four aspects. Hence, suppose that the universe U represents the set of the five different alternative apartment candidates and* $U = \{h_1, h_2, h_3, h_4, h_5\}$. *A represents the set of four parameters and* $A = \{e_1, e_2, e_3, e_4\}$ *= {reasonable price, excellent geographical location, perfect facilities, cozy environment}. Then IVFSS (F, A) on U is described by Table 2.*

Table 2. IVFSS (F, A) for Example 3.

U	e_1	e_2	e_3	e_4	c_i	r_i
h_1	[0.3, 0.5]	[0.6, 0.7]	[0.2, 0.4]	[0.4, 0.5]	[1.5, 2.1]	−0.4
h_2	[0.3, 0.4]	[0.4, 0.5]	[0.6, 0.7]	[0.1, 0.3]	[1.4, 1.9]	−1.9
h_3	[0.5, 0.6]	[1.0, 1.0]	[0.2, 0.3]	[0.2, 0.4]	[1.9, 2.3]	2.6
h_4	[0.5, 0.7]	[0.0, 0.1]	[0.7, 0.8]	[0.6, 0.7]	[1.8, 2.3]	2.1
h_5	[0.3, 0.6]	[0.3, 0.4]	[0.4, 0.7]	[0.2, 0.3]	[1.2, 2.0]	−2.4

This family wants to assess these apartments with its tabular form given by Table 2 and finds the best one. According to SBDM, we compute choice values and the scores for all the objects. The corresponding results are shown in Table 2, from which we see that h_3 seems the most choice, since it has the maximum score 2.6. If we sort the apartments according to the scores in descending order, we are able to obtain $h_3 > h_4 > h_1 > h_2 > h_5$. CAODM considers the new added objects and reduces the computational complexity. However, about the outcome of decision making, the two methods are equivalent. Hence, we can get $h_3 > h_4 > h_1 > h_2 > h_5$ by CAODM. However, when we look through this dataset, it seems that h_3 is not the best choice. h_3 has the highest level of excellent geographical location, but it has the poorer performance at the level of another three parameters such as "reasonable price", "perfect facilities" and "cozy environment" compared with h_4. That is, h_4 has the reasonable price, perfect facilities, and cozy environment, only it has poor geographical location. Compared with h_3, object h_4 outperforms from three aspects. h_4 has the maximum score because of the extreme value or outlier such as [1.0, 1.0] with regard to $e_{2..}$ h_3 has the lower score due to the extreme value or outlier such as [0.0, 0.1] with regard to $e_{2..}$ It is clear that h_4 is likely to be the best choice rather than h_3 for this family. In this situation, the two algorithms of SBDM and CAODM are likely to ignore the excellent candidates. In order to solve this problem, we propose a new method to decision making based on the model of IVFSS.

4. A New Approach to Decision Making Based on Interval-Valued Fuzzy Soft Set

In this section, firstly, we give some related definitions as diverse as average degree of membership and contrast table, and so on. And then we describe a novel approach to decision making based on IVFSS.

Definition 3. *For interval-valued fuzzy soft set* $\left(\widetilde{S}, E\right)$, $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, $\mu_{\widetilde{S}(e_j)}(h_i) = [\mu^-_{\widetilde{S}(e_j)}(h_i), \mu^+_{\widetilde{S}(e_j)}(h_i)]$ *is the degree of membership an element* h_i *to* $\widetilde{S}(e_j)$. *We define* $\overline{\mu}_{\widetilde{S}(e_j)}(h_i)$ *as average degree of membership for every entry, where it is computed by the formula as*

$$\overline{\mu}_{\widetilde{S}(e_j)}(h_i) = \frac{\mu^-_{\widetilde{S}(e_j)}(h_i) + \mu^+_{\widetilde{S}(e_j)}(h_i)}{2} \tag{1}$$

We create a table in which both of rows and columns are the corresponding objects of the interval-valued fuzzy soft set $\left(\widetilde{S}, E\right)$, and the entries M_{ij} are the number of parameters for which the average degree of membership value of object h_i goes over or equal to the average degree of membership value of object h_j. We term this table as the contrast table of IVFSS $(\widetilde{S}, E)$.

It is clear that $0 \leq M_{ij} \leq m$ and where m is the number of parameters in IVFSS $\left(\widetilde{S}, E\right)$. The number of diagonals of this contrast table is m, that is, $M_{ii} = m$. M_{ij} implies object h_i dominates object h_j in M_{ij} number of parameters based on the average degree of membership value.

Definition 4. *For IVFSS* $\left(\widetilde{S}, E\right) U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, *we have a contrast table and the entries* M_{ij} *are the number of parameters for which the average degree of membership value of object* h_i *goes over or equal to the average degree of membership value of object* h_j. *We define* R_i *as the row dominant sum of an object* h_i *and* T_j *as the column dominant sum of an object* h_j, *which is calculated by the Formulas (2) and (3) as*

$$R_i = \sum_{j=1}^{n} M_{ij} \tag{2}$$

$$T_j = \sum_{i=1}^{n} M_{ij} \tag{3}$$

From above Definition 4, we find that the row dominant sum of an object displays the total number of parameters in which this object dominates all the other objects of U. Likewise, the integer T_j indicates the total number of parameters in which h_j is dominated by all the other objects of U.

Definition 5. *For IVFSS* $\left(\widetilde{S}, E\right) U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$; *for the corresponding contrast table,* R_i *and* T_i *are the row dominant sum and column dominant sum of an object* h_i, *respectively.*

We define S_i as the overall dominant score, which is obtained by the formula as,

$$S_i = R_i - T_i \tag{4}$$

Based on the above given definitions, we describe our proposed algorithm as follows Algorithm 3:

Algorithm 3: Our proposed algorithm

Step 1: Input an interval-valued fuzzy soft set $\left(\tilde{S}, E\right)$, $U = \{h_1, h_2, h_3, \cdots h_n\}, \cdots$
$E = \{e_1, e_2, \cdots e_m\}$.
Step 2: Calculate average degree of membership for every entry by the formula as
$\overline{\mu}_{\tilde{S}(e_j)}(h_i) = \frac{\mu^-_{\tilde{S}(e_j)}(h_i)+\mu^+_{\tilde{S}(e_j)}(h_i)}{2}$,
Step 3: Create the contrast table for this IVFSS.
Step 4: Compute the row dominant sum and column dominant sum for every object, respectively.
Step 5: Calculate the overall dominant score for every object.
Step 6: Get the maximum of the overall dominant score for all of objects. Then the corresponding object is the optimal choice.

In order to display our proposed algorithm, let us come back to Example 3. According to our proposed algorithm, firstly, we should get the average degree of membership for every entry, which is shown in Table 3. Secondly, we should construct the contrast table for this interval-valued fuzzy soft set, which is given in Table 4. Here, it is clear that the contrast table has symmetry between the objects.And then, we should calculate the row dominant sum, column dominant sum and the overall dominant score for every object, which are illustrated in Table 5. Finally, we find the object h_4 has the maximum of the overall dominant score for all of objects as 10, which is the best choice for this family. The sequence of these candidates is $h_4 > h_3 > h_5 > h_1 > h_2$, which is different with the results by SBDM and CAODM. This is because that h_3 has the highest level of excellent geographical location, but it has the poorer performance at the level of another three parameters such as "reasonable price", "perfect facilities" and "cozy environment" compared with h_4. That is, h_4 has the reasonable price, perfect facilities and cozy environment, only it has poor geographical location. Hence, we think that there are some extreme values such as the high value of "excellent geographical location" and low value of "perfect facilities" for object h_3, which lead to the highest score value by SBDM and lower overall dominant score by our proposed algorithm. In this situation, our proposed algorithm is more reasonable and feasible.

Table 3. Average degree of membership for Example 3.

U	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	$\mathbf{e}_4$
h_1	[0.40]	[0.65]	[0.30]	[0.45]
h_2	[0.35]	[0.45]	[0.65]	[0.20]
h_3	[0.55]	[1.00]	[0.25]	[0.30]
h_4	[0.60]	[0.05]	[0.75]	[0.65]
h_5	[0.45]	[0.35]	[0.55]	[0.25]

Table 4. The contrast table for Example 3.

	$\mathbf{h}_1$	$\mathbf{h}_2$	$\mathbf{h}_3$	$\mathbf{h}_4$	$\mathbf{h}_5$
h_1	5	3	2	1	2
h_2	2	5	1	1	2
h_3	3	4	5	1	3
h_4	4	4	4	5	3
h_5	3	3	2	2	5

Table 5. The row dominant sum, column dominant sum and overall dominant score for Example 3.

	Row Dominant Sum (R_i)	Column Dominant Sum (T_i)	Overall Dominant Score (S_i)
h_1	13	17	−4
h_2	11	19	−8
h_3	16	14	2
h_4	20	10	10
h_5	15	15	0

5. Comparison Results on Real-Life Cases

In this part, we compare the proposed algorithm with the two existing algorithms such as SBDM [29] and CAODM [36] on two real-life applications.

Case 1: 5-star Sydney Hotel Rating Systems

A traveler is going to arrive in Sydney to have a splendid trip, who is looking for 5-star accommodation. We browse the website of www.agoda.com (accessed on 1 September 2021) to obtain evaluation data. All guests who checked in this hotel give scores to this hotel from these aspects such as "Cleanliness", "Location", "Service", "Facilities", "Room comfort and quality" and "Value for money". All guests comprise the traveler for business, couple traveler, solo traveler, family with young children, family with older children and group travelers. Every guest category gave the average scores to this hotel. We find the maximum and minimum score value based on the evaluation scores from six guest categories as lower and upper degrees of membership, which are normalized and described by the model of IVFSS (F, A). Here we have 21 candidate hotels $U = \{h_1, h_2, \ldots, h_{21}\}$ = { Amora Jamison Hotel, Establishment Hotel, Fraser Suites Sydney, Hilton Sydney, Swissotel Sydney, Zara Tower-Luxury Suites and Apartments, Sofitel Sydney Wentworth Hotel, Radisson Blu Hotel Sydney, Ovolo Woolloomooloo Hotel, QT Sydney, Sheraton Grand Sydney Hyde Park, The Old Clare Hotel, Meriton Suites World Tower, Sydney Central YHA, The Langham Sydney Hotel, Primus Hotel Sydney, Hyatt Regency Sydney, ParkRoyal Darling Harbour Hotel, Shangri-la Hotel, The Darling at The Star, Four Seasons Hotel Sydney} and the set of six parameters as $A = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ = {"Cleanliness", "Location", "Service", "Facilities", "Room comfort and quality", "Value for money"}. These collected data are presented in Table 6.

5.1. Decision Making by SBDM

According to the algorithm of SBDM, we can follow the related steps to solve this problem.

Step 1: Input an IVFSS (F, A) as shown in Table 6.

Step 2: figure out the choice value for each object as given in Table 6.

Step 3: find the score value of every object as shown in Table 6.

Step 4: find the maximum of the score value and the corresponding object is referred to as the best outcome.

Finally, we discover that object h_{15} has the maximum of the score value as 12.94 among all of objects. Hence, h_{15}, that is, The Langham Sydney Hotel is the optimal choice. The sequence of objects is $h_{15} > h_{20} > h_{11} > h_9 > h_8 > h_{10} = h_{12} > h_{13} > h_{16} > h_{21} > h_5 > h_1 > h_{19} > h_7 > h_3 > h_4 > h_6 > h_{18} > h_2 > h_{17} > h_{14}$.

5.2. Decision Making by CAODM

Compared with SBDM, CAODM considers the new added objects and reduces the computational complexity. However, about the outcome of decision making, the two methods are equivalent. According to the algorithm of CAODM, we compute the choice value for each object, and then figure out the overall choice value for every object. As a result, we discover that object h_{15} has the maximum overall choice value among all of objects. Hence, h_{15}, that is, The Langham Sydney Hotel is the optimal choice. The sequence of objects is $h_{15} > h_{20} > h_{11} > h_9 > h_8 > h_{10} = h_{12} > h_{13} > h_{16} > h_{21} > h_5 > h_1 > h_{19} > h_7 > h_3 > h_4 > h_6 > h_{18} > h_2 > h_{17} > h_{14}$.

Table 6. Interval-valued fuzzy soft set (F, A) for Case 1.

U	e_1	e_2	e_3	e_4	e_5	e_6	c_i	r_i
h_1	[0.89, 0.92]	[0.84, 0.87]	[0.90, 0.93]	[0.86, 0.90]	[0.87, 0.90]	[0.82, 0.86]	[5.18, 5.38]	−0.92
h_2	[0.80, 0.94]	[0.87, 1.00]	[0.91, 0.95]	[0.87, 0.94]	[0.50, 0.95]	[0.67, 0.85]	[4.62, 5.63]	−7.43
h_3	[0.88, 0.90]	[0.83, 0.89]	[0.92, 0.94]	[0.86, 0.89]	[0.85, 0.89]	[0.79, 0.84]	[5.13, 5.35]	−2.6
h_4	[0.88, 0.92]	[0.83, 0.86]	[0.92, 0.96]	[0.83, 0.87]	[0.84, 0.92]	[0.77, 0.82]	[5.07, 5.35]	−3.86
h_5	[0.90, 0.91]	[0.85, 0.88]	[0.94, 0.96]	[0.86, 0.89]	[0.83, 0.90]	[0.82, 0.84]	[5.20, 5.38]	−0.5
h_6	[0.90, 0.92]	[0.84, 0.87]	[0.86, 0.87]	[0.84, 0.90]	[0.81, 0.94]	[0.80, 0.86]	[5.05, 5.36]	−4.07
h_7	[0.86, 0.98]	[0.81, 0.88]	[0.89, 0.90]	[0.84, 0.96]	[0.80, 1.00]	[0.76, 0.82]	[4.96, 5.54]	−2.18
h_8	[0.92, 0.96]	[0.87, 0.92]	[0.92, 0.96]	[0.90, 0.93]	[0.85, 0.95]	[0.83, 0.86]	[5.29, 5.58]	5.59
h_9	[0.88, 0.98]	[0.87, 0.97]	[0.88, 0.90]	[0.86, 0.97]	[0.85, 1.00]	[0.85, 0.92]	[5.19, 5.74]	6.85
h_{10}	[0.91, 0.96]	[0.86, 0.90]	[0.94, 0.98]	[0.89, 0.93]	[0.88, 0.96]	[0.78, 0.87]	[5.26, 5.60]	5.38
h_{11}	[0.91, 0.93]	[0.87, 0.91]	[0.93, 0.99]	[0.89, 0.90]	[0.90, 0.93]	[0.83, 0.90]	[5.43, 5.56]	8.11
h_{12}	[0.93, 1.00]	[0.80, 0.93]	[0.90, 1.00]	[0.88, 1.00]	[0.80, 1.00]	[0.78, 0.84]	[5.09, 5.77]	5.38
h_{13}	[0.87, 0.93]	[0.84, 0.95]	[0.93, 0.96]	[0.87, 0.95]	[0.85, 0.91]	[0.84, 0.88]	[5.20, 5.58]	3.7
h_{14}	[0.78, 0.79]	[0.75, 0.80]	[0.92, 0.93]	[0.78, 0.81]	[0.76, 0.84]	[0.79, 0.82]	[4.78, 4.99]	−17.51
h_{15}	[0.95, 1.00]	[0.91, 0.96]	[0.85, 0.92]	[0.95, 0.99]	[0.95, 1.00]	[0.86, 0.88]	[5.47, 5.75]	12.94
h_{16}	[0.92, 0.94]	[0.86, 0.90]	[0.92, 0.97]	[0.85, 0.94]	[0.81, 0.90]	[0.86, 0.90]	[5.22, 5.55]	3.49
h_{17}	[0.84, 0.88]	[0.77, 0.81]	[0.88, 0.92]	[0.81, 0.85]	[0.81, 0.83]	[0.74, 0.80]	[4.85, 5.09]	−13.94
h_{18}	[0.84, 0.88]	[0.81, 0.88]	[0.90, 0.91]	[0.84, 0.87]	[0.84, 0.86]	[0.80, 0.85]	[5.03, 5.25]	-6.8
h_{19}	[0.89, 0.93]	[0.85, 0.91]	[0.89, 0.90]	[0.85, 0.89]	[0.89, 0.92]	[0.81, 0.82]	[5.18, 5.37]	−1.13
h_{20}	[0.91, 0.96]	[0.88, 0.96]	[0.89, 0.95]	[0.90, 0.96]	[0.91, 1.00]	[0.83, 0.89]	[5.32, 5.72]	9.16
h_{21}	[0.89, 0.91]	[0.82, 0.87]	[0.94, 0.96]	[0.89, 0.90]	[0.88, 0.92]	[0.80, 0.84]	[5.22, 5.40]	0.34

5.3. Decision Making by Our Proposed Algorithm

According to our proposed algorithm, we can follow the related steps.

Step 1: Input an IVFSS (F, A) as shown in Table 6.

Step 2: Calculate average degree of membership for every entry as shown in Table 7.

Step 3: Create the contrast table for this IVFSS as shown in Table 8.

Step 4: Compute the row dominant sum and column dominant sum for every object, respectively. These data are presented in Table 9.

Step 5: Calculate the overall dominant score for every object as shown in Table 9.

Step 6: Get the maximum of the overall dominant score for all of 21 objects. Then the corresponding object is the optimal choice.

As a result, we find that h_{15} is the best choice. That is, The Langham Sydney Hotel is the best choice for this traveler. The sequence of the candidate hotels is $h_{15} > h_{20} > h_{11} > h_{10} > h_8 = h_9 > h_{12} > h_{16} > h_{13} > h_5 > h_{21} > h_1 > h_{19} > h_2 > h_7 > h_3 > h_6 > h_4 > h_{18} > h_{14} > h_{17}$.

Table 7. Average degree of membership of IVFSS (F, A) for Case 1.

U	e_1	e_2	e_3	e_4	e_5	e_6
h_1	0.91	0.86	0.92	0.88	0.89	0.84
h_2	0.87	0.94	0.93	0.91	0.73	0.76
h_3	0.89	0.86	0.93	0.88	0.87	0.82
h_4	0.90	0.85	0.94	0.85	0.88	0.80
h_5	0.91	0.87	0.95	0.88	0.87	0.83
h_6	0.91	0.86	0.87	0.87	0.88	0.83
h_7	0.92	0.85	0.90	0.90	0.90	0.79
h_8	0.94	0.90	0.94	0.92	0.90	0.85
h_9	0.93	0.92	0.89	0.92	0.93	0.89
h_{10}	0.94	0.88	0.96	0.91	0.92	0.83
h_{11}	0.92	0.89	0.96	0.90	0.92	0.87
h_{12}	0.97	0.87	0.95	0.94	0.90	0.81
h_{13}	0.90	0.90	0.95	0.91	0.88	0.86
h_{14}	0.79	0.78	0.93	0.80	0.80	0.81
h_{15}	0.98	0.94	0.89	0.97	0.98	0.87
h_{16}	0.93	0.88	0.95	0.90	0.86	0.88
h_{17}	0.86	0.79	0.90	0.83	0.82	0.77
h_{18}	0.86	0.85	0.91	0.86	0.85	0.83
h_{19}	0.91	0.88	0.90	0.87	0.91	0.82
h_{20}	0.94	0.92	0.92	0.93	0.96	0.86
h_{21}	0.90	0.85	0.95	0.90	0.90	0.82

Table 8. The contrast table for Case 1.

U	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}	h_{16}	h_{17}	h_{18}	h_{19}	h_{20}	h_{21}
h_1	6	3	5	5	4	6	3	0	1	1	0	1	2	5	1	1	6	6	4	1	3
h_2	3	6	3	2	2	3	3	1	2	2	2	1	2	4	2	2	4	4	3	2	2
h_3	3	4	6	3	2	3	3	0	1	0	0	1	0	6	1	1	6	5	3	1	2
h_4	1	4	3	6	1	2	3	1	1	0	0	0	2	5	1	1	6	5	1	1	2
h_5	4	4	6	5	6	6	3	1	1	1	0	3	2	6	1	2	6	6	4	1	4
h_6	2	3	4	5	3	6	2	0	0	1	0	1	1	5	0	1	5	5	3	0	3
h_7	3	3	3	4	3	4	6	1	1	0	2	1	2	4	1	2	6	4	3	0	4
h_8	6	5	6	6	5	6	6	6	3	4	3	3	4	6	1	4	6	6	5	2	5
h_9	5	4	5	5	5	6	5	4	6	4	5	3	5	5	2	5	5	5	5	2	5
h_{10}	5	5	6	6	6	6	6	3	2	6	4	4	4	6	1	5	6	6	6	2	6
h_{11}	6	4	6	6	6	6	6	6	3	4	6	4	4	6	2	4	6	6	6	2	6
h_{12}	5	5	5	6	5	5	6	4	3	3	2	6	4	6	1	4	6	5	3	3	5
h_{13}	4	5	6	6	5	5	4	3	1	3	2	3	6	6	1	4	6	6	4	2	5
h_{14}	1	3	1	1	0	1	2	0	1	0	0	1	0	6	1	0	2	1	1	1	0
h_{15}	5	5	5	5	5	6	5	5	5	5	5	5	5	5	6	4	5	5	5	5	5
h_{16}	5	4	5	5	5	5	5	2	3	2	3	3	3	6	2	6	6	6	5	2	5
h_{17}	0	2	0	0	0	1	1	0	1	0	0	0	0	4	1	0	6	1	1	0	0
h_{18}	0	2	1	3	1	2	3	0	1	1	0	1	0	4	1	0	6	6	2	0	2
h_{19}	3	3	4	5	4	5	4	1	1	1	0	3	2	5	1	2	6	4	6	0	4
h_{20}	6	4	5	5	5	6	6	5	5	5	4	3	5	5	1	4	6	6	6	6	5
h_{21}	3	4	5	6	3	3	5	2	1	0	1	3	3	6	1	3	6	5	3	1	6

We find that the sequence by our proposed algorithm is a little different with SBDM and CAODM, although three methods conduct the same top three. We can analyze the sequence of h_{10}, h_8 and h_9. By means of our proposed algorithm, h_{10} is better than h_8 and h_9. However, h_8 and h_9 are better than h_{10} by the method of SBDM and CAODM. This is because h_{10} has the comparative extreme values such as the performance of h_{10} from the aspect of value for money, which leads to the lower score value than h_8 and h_9. Our proposed method can also examine some extreme or unbalanced values for decision making. We find that the two methods have the similar decision results, which shows that there is no too much extreme value or outliers in this case.

As a result, we illustrate the comparison results among three methods on this case shown in Table 10.

Table 9. The row dominant sum, column dominant sum and overall dominant score for Case 1.

U	Row Dominant Sum	Column Dominant Sum	Overall Dominant Score
h_1	64	76	−12
h_2	55	82	−27
h_3	51	90	−39
h_4	46	59	−49
h_5	72	76	−4
h_6	50	93	−43
h_7	57	87	−30
h_8	98	45	53
h_9	96	43	53
h_{10}	101	43	58
h_{11}	105	39	66
h_{12}	92	50	42
h_{13}	87	56	31
h_{14}	23	111	−88
h_{15}	106	29	77
h_{16}	88	55	33
h_{17}	18	117	−99
h_{18}	36	103	−67
h_{19}	64	79	−15
h_{20}	103	34	69
h_{21}	70	79	−9

Table 10. Comparison results about three methods on case 1.

Algorithm	Considering The Extreme Data	Whether Examining Extreme Data	Decision Making Results
Our method	Yes	Yes(h_{10} has extreme value)	$h_{15} > h_{20} > h_{11} > h_{10} > h_8 = h_9 > h_{12} > h_{16} > h_{13} > h_5 > h_{21} > h_1 > h_{19} > h_2 > h_7 > h_3 > h_6 > h_4 > h_{18} > h_{14} > h_{17}$
SBDM	No	No	$h_{15} > h_{20} > h_{11} > h_9 > h_8 > h_{10} = h_{12} > h_{13} > h_{16} > h_{21} > h_5 > h_1 > h_{19} > h_7 > h_3 > h_4 > h_6 > h_{18} > h_2 > h_{17} > h_{14}$
CAODM	No	No	$h_{15} > h_{20} > h_{11} > h_9 > h_8 > h_{10} = h_{12} > h_{13} > h_{16} > h_{21} > h_5 > h_1 > h_{19} > h_7 > h_3 > h_4 > h_6 > h_{18} > h_2 > h_{17} > h_{14}$

Case 2: Scenic Spots Weather Condition Evaluation Systems

A person has 7 days off for his annual holiday. He desires to spend and enjoy his holiday at one scenic spot. He goes to visit the web site of www.weather.com.cn (accessed on 1 September 2021), which displays the weather forecast for sixteen destination scenic spots. The data of weather forecast are described from four aspects such as "temperature", "relative humidity", "air quality index", "wind speed". We find the maximum and minimum values for every parameter as the upper limits and lower limits of the interval. Here we apply the model of IVFSS to describe this Scenic Spots Weather Condition Evaluation Systems. There are sixteen scenic spots in China as the candidates. Propose that the universe = {Forbidden City, The Bund, Bangchui Island, West Lake, Five Avenue, Ciqikou, Confucius Temple, Yellow Crane Tower, Mount Tai, Jiuzhai Valley, Zhangjiajie, Gulangyu Islet, The ancient City of Ping Yao, Terra Cotta Warriors, Mogao Grottoes, Erhai Lake} and the parameter set.

It is necessary to normalize the original data into IVFSS. We transform maximum value and minimum value into sub-intervals of [0, 1], which is normalized as upper and lower degree of membership. After normalization, we get IVFSS for Scenic Spots Weather Condition Evaluation Systems, which is illustrated in Table 10.

5.4. Decision Making by SBDM

According to the algorithm of SBDM, we can follow the related steps to solve this problem.

Step 1: Input an IVFSS (F, A) as shown in Table 10.

Step 2: Compute the choice value for each object as shown in Table 10.

Step 3: Obtain the related score value of every object as presented in Table 10.

Step 4: Sort the score values and find the maximum of the score value and the corresponding object is referred to as the best outcome.

Finally, we discover that object h_{16} has the maximum of the score value among all of objects. Hence, h_{16}, that is, Erhai Lake is the optimal choice. The sequence of Scenic Spots is $h_{16} > h_1 > h_6 > h_{11} > h_{12} > h_2 > h_5 > h_7 > h_3 > h_8 > h_{15} > h_{10} > h_4 > h_{13} > h_9 > h_{14}$ according to the respective weather condition.

5.5. Decision Making by CAODM

According to the algorithm of CAODM, similarly, object h_{16} has the maximum of the overall choice value among all of objects. Hence, h_{16}, that is, Erhai Lake is the optimal choice. The sequence of Scenic Spots is $h_{16} > h_1 > h_6 > h_{11} > h_{12} > h_2 > h_5 > h_7 > h_3 > h_8 > h_{15} > h_{10} > h_4 > h_{13} > h_9 > h_{14}$ according to the respective weather condition.

5.6. Decision Making by Our Proposed Algorithm

According to our proposed algorithm, we can follow the related steps.

Step 1: Input an IVFSS (F, A) as shown in Table 11.

Step 2: Calculate average degree of membership for every entry as shown in Table 12.

Step 3: Construct the contrast table for this interval-valued fuzzy soft set as shown in Table 13.

Step 4: Compute the row dominant sum and column dominant sum for every object, which are depicted in Table 14.

Step 5: Calculate the overall dominant score for every object as shown in Table 14.

Step 6: Rank the overall dominant score and get the maximum of them for all of 16 objects. Then the corresponding object is the optimal choice.

As a result, we find that h_{16} is the best choice. That is, Erhai Lake is the best choice for this traveler. The sequence of the candidate scenic spots is $h_{16} > h_6 > h_1 > h_{12} > h_{11} > h_2 > h_8 > h_7 > h_3 = h_5 > h_{15} = h_{10} > h_{13} > h_4 > h_{14} > h_9$.

Table 11. IVFSS (F, A) for Case 2.

U	e_1	e_2	e_3	e_4	c_i	r_i
h_1	[0.13, 0.52]	[0.60, 0.98]	[0.27, 0.95]	[0.50, 1.00]	[1.50, 3.45]	13.01
h_2	[0.35, 0.71]	[0.06, 0.49]	[0.62, 0.89]	[0.25, 1.00]	[1.28, 3.09]	3.73
h_3	[0.23, 0.52]	[0.36, 0.64]	[0.25, 0.81]	[0.50, 0.75]	[1.34, 2.72]	−1.23
h_4	[0.39, 0.74]	[0.06, 0.37]	[0.46, 0.74]	[0.25, 0.75]	[1.16, 2.60]	−6.03
h_5	[0.19, 0.55]	[0.50, 1.00]	[0.04, 0.76]	[0.50, 0.75]	[1.23, 3.06]	2.45
h_6	[0.45, 0.81]	[0.06, 0.20]	[0.73, 0.85]	[0.50, 1.00]	[1.74, 2.86]	7.4
h_7	[0.32, 0.71]	[0.00, 0.51]	[0.57, 0.73]	[0.50, 0.75]	[1.39, 2.70]	−0.75
h_8	[0.39, 0.77]	[0.01, 0.10]	[0.57, 0.96]	[0.50, 0.75]	[1.47, 2.58]	−1.39
h_9	[0.03, 0.42]	[0.33, 0.89]	[0.06, 0.76]	[0.00, 0.75]	[0.42, 2.82]	−14.35
h_{10}	[0.06, 0.42]	[0.20, 0.48]	[0.37, 0.67]	[0.75, 1.00]	[1.38, 2.57]	−2.99
h_{11}	[0.42, 0.81]	[0.01, 0.16]	[0.91, 1.00]	[0.50, 0.75]	[1.84, 2.72]	6.77
h_{12}	[0.65, 1.00]	[0.32, 0.44]	[0.66, 0.91]	[0.00, 0.50]	[1.63, 2.85]	5.49
h_{13}	[0.13, 0.58]	[0.19, 0.90]	[0.04, 0.55]	[0.25, 1.00]	[0.61, 3.03]	−7.95
h_{14}	[0.19, 0.68]	[0.15, 0.34]	[0.00, 0.36]	[0.50, 0.75]	[0.84, 2.13]	−18.67
h_{15}	[0.00, 0.48]	[0.32, 0.91]	[0.29, 0.78]	[0.50, 0.75]	[1.11, 2.92]	−1.71
h_{16}	[0.42, 0.87]	[0.20, 0.78]	[0.46, 0.92]	[0.50, 1.00]	[1.58, 3.57]	16.21

It is clear that h_{16}, that is, Erhai Lake has high-level performance associated with four weather condition aspects. As a result, Erhai Lake is the best choice by both two methods. However, for object h_6 and object h_1, which one is better? By the method of SBDM, object h_1 is better than object h_6. There exists a contradiction that object h_6 has the better performance than object h_1 by our proposed algorithm. The reason for this contradiction: the extreme values. Let us come back to Table 11. It is clear that object h_6 has the much better performance from "temperature", "air quality index" and "wind speed" than object h_1 except for "relative humidity". It is clear that object h_6 is the better choice. However, object h_6 has the very low relative humidity, which results in object h_6 has the lower score value than object h_1 by the methods of SBDM and CAODM. We think that h_6 is the better choice which is more reasonable. Our proposed method can also examine some extreme or unbalanced values for decision making. We find that the three methods have the different decision results, which shows that there is some extreme value or outliers in this case. Therefore, we come back to this dataset. We can find that h_6 has the extreme data such as [0.06, 0.20] with regard to relative humidity.

Table 12. Average degree of membership of IVFSS (F, A) for Case 2.

U	e_1	e_2	e_3	e_4
h_1	0.33	0.79	0.61	0.75
h_2	0.53	0.28	0.76	0.63
h_3	0.38	0.50	0.53	0.63
h_4	0.57	0.22	0.60	0.50
h_5	0.37	0.75	0.40	0.63
h_6	0.63	0.13	0.79	0.75
h_7	0.52	0.26	0.65	0.63
h_8	0.58	0.06	0.77	0.63
h_9	0.23	0.61	0.41	0.38
h_{10}	0.24	0.34	0.52	0.88
h_{11}	0.62	0.09	0.96	0.63
h_{12}	0.83	0.38	0.79	0.25
h_{13}	0.36	0.55	0.30	0.63
h_{14}	0.44	0.25	0.18	0.63
h_{15}	0.24	0.62	0.54	0.63
h_{16}	0.65	0.49	0.69	0.75

Table 13. The contrast table for Case 2.

U	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}	h_{16}
h_1	4	2	3	3	3	2	2	2	4	3	2	2	3	3	4	2
h_2	2	4	3	3	3	1	4	2	3	2	2	1	3	4	3	1
h_3	1	2	4	2	3	1	2	2	3	3	2	2	3	3	2	1
h_4	1	1	2	4	2	1	1	1	3	2	1	1	2	2	2	0
h_5	1	2	2	2	4	1	2	2	3	2	2	2	4	3	3	1
h_6	3	3	3	3	3	4	3	4	3	2	3	2	3	3	3	2
h_7	2	1	3	3	3	1	4	2	3	2	2	1	3	4	3	0
h_8	2	3	3	3	3	0	3	4	3	2	1	1	3	3	3	1
h_9	0	1	1	1	1	1	1	1	4	1	1	2	2	2	0	1
h_{10}	1	2	1	2	2	2	2	2	3	4	2	1	2	3	2	1
h_{11}	2	3	3	3	3	1	2	4	3	2	4	2	3	3	3	1
h_{12}	2	3	2	3	2	3	3	3	2	3	2	4	2	3	2	2
h_{13}	1	2	2	2	1	1	2	2	2	2	2	2	4	3	2	1
h_{14}	1	1	2	2	2	1	1	2	2	1	2	1	2	4	2	0
h_{15}	0	2	3	2	2	1	2	2	4	3	2	2	3	3	4	1
h_{16}	3	3	3	4	3	3	4	3	3	3	3	2	3	4	3	4

Consequently, we present the comparison results among three methods on case 2 shown in Table 15.

From above two down-to-earth applications, we can draw the conclusion that our proposed algorithm is more reasonable and feasible when there exists some extreme values or outliers in the datasets for making decisions than the two existing algorithms of SBDM and CAODM. Our proposed algorithm makes decision based on the number of superior parameter values, which is a new perspective to make decision. Our proposed algorithm can examine some extreme or unbalanced values for decision making if we regard this method as supplement of the existing algorithms of SBDM and CAODM. Finally, we demonstrate the characteristics of three methods as the following Table 16.

Table 14. The row dominant sum, column dominant sum and overall dominant score for Case 2.

U	Row Dominant Sum	Column Dominant Sum	Overall Dominant Score
h_1	44	26	18
h_2	41	35	6
h_3	36	40	−4
h_4	26	42	−16
h_5	36	40	−4
h_6	47	24	23
h_7	37	39	−2
h_8	38	38	0
h_9	20	48	−28
h_{10}	32	37	−5
h_{11}	43	33	10
h_{12}	41	28	13
h_{13}	31	45	−14
h_{14}	26	50	−24
h_{15}	36	51	−5
h_{16}	51	19	32

Table 15. Comparison results about three methods on case 2.

Algorithm	Considering the Extreme Data	Whether Examining Extreme Data	Decision Making Results
Our method	Yes	Yes (h_6 has extreme value)	$h_{16} > h_6 > h_1 > h_{12} > h_{11} > h_2 > h_8 > h_7 > h_3 = h_5 > h_{15} = h_{10} > h_{13} > h_4 > h_{14} > h_9$
SBDM	No	No	$h_{16} > h_1 > h_6 > h_{11} > h_{12} > h_2 > h_5 > h_7 > h_3 > h_8 > h_{15} > h_{10} > h_4 > h_{13} > h_9 > h_{14}$
CAODM	No	No	$h_{16} > h_1 > h_6 > h_{11} > h_{12} > h_2 > h_5 > h_7 > h_3 > h_8 > h_{15} > h_{10} > h_4 > h_{13} > h_9 > h_{14}$

Table 16. Comparison results about three methods.

Algorithm	Considering the Extreme Data	Whether Examining Extreme Data	Considering the Added Objects
Our method	Yes	Yes	No
SBDM	No	No	No
CAODM	No	No	Yes

6. Conclusions

This paper analyzes the score-based decision making approach (SBDM) [29] and CAODM [36], and then points out the weakness and irrationality when there are some extreme values or outliers in the datasets based on IVFSS for making decisions. In order to overcome this shortcoming, a novel approach to decision making based on IVFSS by means of the contrast table is proposed. Comparison results on two real-life application cases such as five-star Sydney Hotel Rating Systems and Scenic Spots Weather Condition Evaluation Systems between two methods provide the testing and verification for the feasibility and efficiency when we face up to the extreme values or outliers of the uncertain datasets. Our proposed algorithm makes decision based on the number of superior parameter values, which is a new perspective to make decision. Our proposed algorithm can also examine if there are some extreme or unbalanced values for decision making if we regard this method as supplement of the existing algorithm of SBDM and CAODM. Future scope of this research work might be to apply the decision making methods into more practical applications such as evaluation systems, recommender system, conflict handling, and so on, and give the complete solution.

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