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Two-Stage Fuzzy Interactive Multi-Objective Approach under Interval Type-2 Fuzzy Environment with Application to the Remanufacture of Old Clothes

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Abstract: In this study, a two-stage approach is introduced to obtain a more interactive and flexible solution to deal with the multi-objective programming under interval type-2 fuzzy environment. In the first stage, the fuzzy multi-objective chance-constrained programming with regular symmetric triangular interval type-2 fuzzy set parameters is proposed and transferred into its crisp equivalent form. In the second stage, we use the fuzzy interactive approach to address the crisp multi-objective programming obtained in the first stage by introducing the trade-off rate, which helps the decision maker react via updating their reference member values to obtain a satisfactory solution. Finally, taking a remanufacture of old clothes problem as an example, the comparison of experimental results obtained using a non-interactive method and interactive method shows that the proposed approach is conducive to obtaining satisfactory solutions effectively and efficiently, which broadens the application scope of the multi-objective programming with regular symmetric triangular interval type-2 fuzzy set parameters for sustainable manufacturing.

Keywords: multi-objective programming; interval type-2 fuzzy set; fuzzy interactive approach; remanufacture of old clothes; sustainable manufacturing

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1. Introduction

Today's daily management work is not only faced with a single-objective decision optimization problem, but also involves multi-objective decision optimization problems. Enterprise management often encounters the problem of multi-objective decision making. When formulating production plans, enterprises must not only consider the total output value, but also consider profits, product quality and equipment utilization. Some objectives are often contradictory. For example, corporate profits may contradict environmental protection goals. How to take multi-objectives into account in an integrated manner and choose a reasonable plan is a very complicated issue. Multi-objective programming (MOP) has been applied in different areas widely for many years; there are a large number of application examples in economic planning [1], supplier selection [2], energy development [3] and financial decision making [4,5]. In real life, when settling multi-objective problems, there are often several uncertainties required to be taken into account. Therefore, based on the multi-objective environment and the authoritative theoretical basis of possibility theory [6], the fuzzy multi-objective programming (FMOP) has been widely researched. In accord with the way the decision maker (DM) specifies their preferences for the objectives, existing methods for solving MOP in various practical environments can be divided into two categories, namely non-interactive approaches and interactive approaches [7].

In relation to the non-interactive approaches, it is required to first clarify the preference information and personal wishes of DM, and then a solution process is performed in an attempt to find the Pareto optimal solution. Ahmadini et al. [8] considered an MOP that includes multiple products with back-ordered quantity and solved it with a LINGO optimization software. Khan et al. [9] discussed a production decision-making problem with multiple objectives, and figured out it by an intuitionistic and neutrosophic programming method. Two classical non-interactive approaches are the weighting method [10,11] and the ε-constraint method [12]. Firouzi and Jadidi [13] converted the multi-objective into single-objective in fuzzy programming by developing a weighting additive function, which could effectively take the preferences of DM into account and solve the problem. Pérez-Cañedo et al. [14] developed a scheme of fuzzy ε-constraint, which was used to work out the Pareto optimal fuzzy solutions of fuzzy multi-objective linear programming in LR fuzzy environment. Ehsani et al. [15] investigated an MOP in project management with fuzzy objectives, and a weighting maxmin model was applied for solving it. Ren et al. [16] put forward a FMOP method to improve the irrigation rate of the existing land under the fuzzy environment, and developed a minimum deviation method to solve it. Ali et al. [17] solved a FMOP about vendor selection using the aggregated weighted criterion method. The ε-constraint method and the weighting method both reflect to some extent that non-interactive approaches do not provide preference information well, which means the non-interactive approach cannot reflect the opinions of DMs in the decision-making process, so it can only solve a limited number of decision-making problems.

The interactive approach has been improved and developed accordingly to address the weaknesses of the non-interactive approach. In the process of interactive approaches, it is necessary to illustrate the preference of DM for each iteration for the purpose of making sure that the most preferred solution is obtained, rather than give the priori or posterior global preference structure that employed in the non-interactive approaches. Interactive approaches are generally classified into three categories: stand on the trade-off information, the classification-based approach, and reference points methods, respectively, all of which have difference in terms of technical elements and interaction style. Gupta et al. [18] established a comprehensive MOP method to solve the transportation problem of expansion capacitances, and a fuzzy interactive method was proposed to obtain preferred compromise transportation solutions. Shahbeig et al. [19] proposed an fuzzy interactive method to optimize the multi-objective problem of selecting the sub-optimal subset of genes from massive gene expression data. El Sayed et al. [20] worked out the two-level MOP about supply chain by means of interaction with decision-makers, and used the α-level methodology to defuzzify and change the fuzzy programming into a valent crisp one.

In real life, there are lots of phenomena that need to consider multiple uncertainties; thus, scholars have carried numerous works to enrich fuzzy set theory (e.g., fuzzy random sets [21,22], twofold fuzzy sets [23,24], bifuzzy sets [25], fuzzy soft sets [26]). Among them, type-2 fuzzy set (T2-FS) is a crucial embranchment that continues to draw attention because of its advantage in expressing the uncertainty of the membership function. Ahmad [27] investigated a multi-objective supplier decision making under type-2 fuzzy environment, and developed a novel interactive neutrosophic programming method to solve it. However, it is challenging to figure out the problems involving general T2-FS because of the complexity of its calculations. Compared with this, research into the interval type-2 fuzzy set (IT2-FS) has been developed rapidly in recent years. Gupta et al. [28] considered a transportation problem with interval type-2 fuzzy parameters and then converted it into the crisp form by using an expected value (EV) function approach. Calik [29] studied supplier decision making and order allocation in the context of sustainability, in which an interval type-2 fuzzy AHP approach was used to determine the weight of the selected standard. Kundu [30] considered a redundant allocation decision making under interval type-2 fuzzy environment, and applied NIMBUS method to obtain compromise solution. The literature related to FMOP mentioned above and the corresponding approaches for solving them are summarized in Table 1.

As shown in Table 1, most of the papers on FMOP related to our research considered non-interactive methods, and only a small number of the related works investigated interactive methods. Moreover, even fewer are associated with T2-FS (see, e.g., [27,30]), in which the presented methods involve complex operations and are difficult for solving realistic problems. Therefore, in order to better handle the type-2 fuzzy multi-objective programming (T2-FMOP) that often occurs in real life, we develop the two-stage fuzzy interactive multi-objective approach based on the operational law for T2-FS proposed recently by Li and Cai [31], which is easier to understand, saves time and better reflects the preference information of the DM compared with the current research. The procedure of this approach is summarized in Figure 1. In the first stage, we propose a type-2 fuzzy multi-objective chance constrained programming (CCP) and convert it into a crisp equivalent model. In the second stage, we update the reference membership function through a continuous interaction with the DM, and then convert the problem into a minmax model so as to solve the MOP models with regular symmetric triangular interval type-2 fuzzy set (RSTIT2-FS).

Table 1. Literature related to the fuzzy multi-objective programming.

Literature	Type of Fuzzy Parameters		Type of Programming		Type of Approach		Method
	Type-1	Type-2	Linear	Non-Linear	Interactive	Non-Interactive	
Ahmadini et al. [8]	✓			✓		✓	LINGO
Khan et al. [9]	✓		✓			✓	intuitionistic and neutrosophic programming approaches
Firouzi et al. [13]	✓		✓			✓	weighted additive function
Pérez-Cañedo et al. [14]	✓		✓			✓	ε-constraint method
Ehsani et al. [15]	✓			✓		✓	weighted max-min approach
Ren C et al. [16]	✓			✓		✓	minimum deviation method
Ali et al. [17]	✓			✓		✓	aggregated weighted criterion method
Gupta et al. [18]	✓		✓		✓		LINGO
Shahbeig et al. [19]	✓			✓	✓		min-max method
El Sayed et al. [20]	✓			✓	✓		ε-constraint method
Ahmad [27]		✓	✓		✓		interactive neutrosophic method
Gupta et al. [28]		✓	✓			✓	compromise criterion
Calik [29]		✓	✓			✓	generalized credibility measure
Kundu [30]		✓		✓	✓		NIMBUS method
Current paper		✓		✓	✓		fuzzy interactive approach

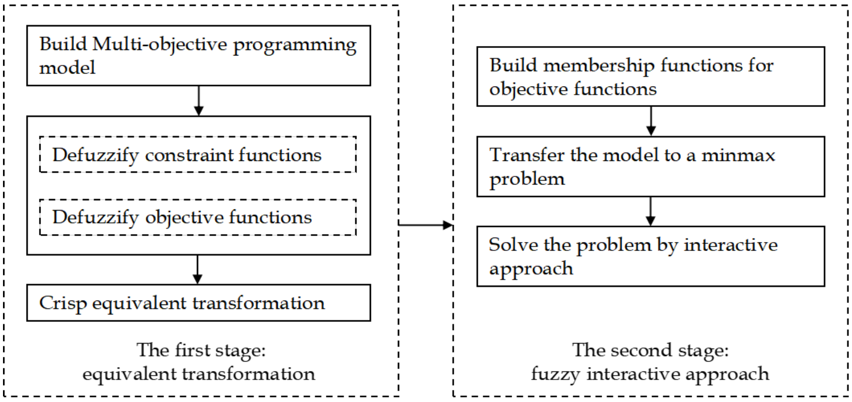

Figure 1. The procedure of the two-stage fuzzy interactive multi-objective approach.

Our contributions are mainly reflected in the following: First, we propose a two-stage fuzzy interactive multi-objective approach so as to more efficiently figure out multi-objective problems with interval type-2 fuzzy parameters. Second, we propose a type-2 fuzzy chance-constrained programming (CCP) to transfer the interval T2-FMOP into its crisp equivalent form. Third, we use the fuzzy interactive approach to solve the crisp model and apply the approach to a problem of remanufacturing old clothes.

The structure and main content of this paper are as follows. In Section 2, the related concepts about IT2-FS and RSTIT2-FS are briefly introduced. Then, the T2-FMOP model is presented and transferred into the crisp equivalent form in Section 3. In Section 4, the fuzzy interactive approach is provided to solve the crisp model. In Section 5, we apply the two-stage method to help remanufactured apparel companies in making decisions, and to achieve the win–win situation of sustainability development and profit maximization. Finally, this article is concluded in Section 6.

2. Preliminaries

For the sake of solving the T2-FMOP model, we should first transfer it to its crisp equivalent form. In this section, we introduce some basic concepts about IT2-FS and RSTIT2-FS in advance, which includes the definition, operational law and excepted value of it.

2.1. Interval Type-2 Fuzzy Set

Definition 1 (Zadeh [32])**.** *Let X be the universe of x, $\mu_N(x)$ the membership function of x. Then the type-1 fuzzy set N can be defined as*

$$N = \int_{x \in X} \frac{\mu_N(x)}{x}.$$

Definition 2 (Zadeh [32], Mendel and John [33])**.** *For a fuzzy variable $\widetilde{\Theta}$, if the membership of a given $x \in X$ is a type-1 fuzzy set, then $\widetilde{\Theta}$ can be called a T2-FS and can be defined as*

$$\widetilde{\Theta} = \left\{ \left((x, u), \mu_{\widetilde{\Theta}}(x, u) \right) \mid x \in X, u \in J_x \subseteq [0, 1] \right\},$$

where X is a infinite set, $0 \leq \mu_{\widetilde{\Theta}}(x, u) \leq 1$ is the secondary membership for u, $J_x \subseteq [0, 1]$ is the primary membership for x, and its range is the domain of $\mu_{\widetilde{\Theta}}$.

Definition 3 (Men et al. [34])**.** *For a T2-FS $\widetilde{\Theta}$, if $\mu_{\widetilde{\Theta}}(x, u)$ is identically equal to 1 for any $x \in X$ and $u \in J_x$, then $\widetilde{\Theta}$ can be called an IT2-FS.*

2.2. Regular Symmetric Triangular Interval Type-2 Fuzzy Set

Definition 4 (Li and Cai [31])**.** *An IT2-FS $\widetilde{\Theta}$ is called a RSTIT2-FS if its upper membership function (UMF) and lower membership function (LMF) are expressed as follows,*

$$\mathrm{UMF(x)} = \begin{cases} \dfrac{x - h + l_U}{l_U}, & x \in [h - l_U, h) \\ \dfrac{-x + h + l_U}{l_U}, & x \in [h, h + l_U] \\ 0, & otherwise, \end{cases}$$

and

$$\mathrm{LMF(x)} = \begin{cases} \dfrac{x - h + l_L}{l_L}, & x \in [h - l_L, h) \\ \dfrac{-x + h + l_L}{l_L}, & x \in [h, h + l_L] \\ 0, & otherwise, \end{cases}$$

and can be denoted as $\begin{pmatrix} h - l_U & h & h + l_U \\ h - l_L & h & h + l_L \end{pmatrix}$, where l_U and l_L are spreads of the UMF and LMF, satisfying $l_U > l_L$, and the peak of them, 1, is reached when x is equal to h.

An RSTIT2-FS is visualized in Figure 2.

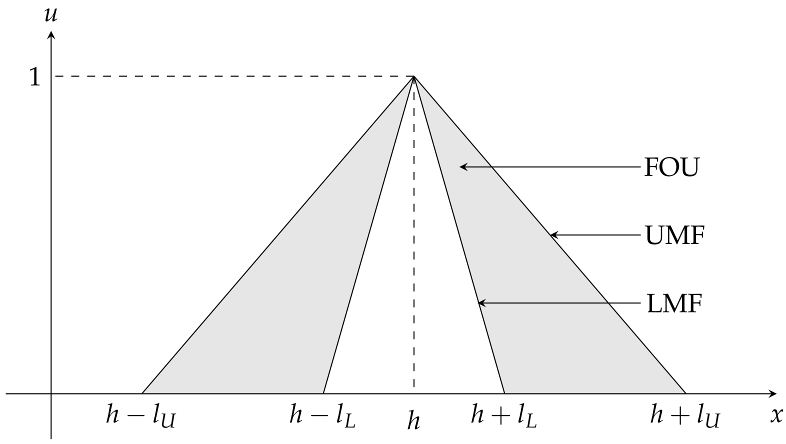

Figure 2. The plane visualization of an RSTIT2-FS.

Definition 5 (Li and Cai [31])**.** *Let $\widetilde{\Theta}$ be an RSTIT2-FS, λ a type-1 fuzzy set, and their membership functions satisfy*

$$\mu_\lambda(x) = \frac{1}{2}(UMF(x) + LMF(x)).$$

Then λ is called the medium of $\widetilde{\Theta}$, and μ_λ can be expressed as,

$$\mu_\lambda(x) = \begin{cases} \dfrac{x-h+l_U}{2l_U}, & x \in [h-l_U, h-l_L) \\ \dfrac{x-h+l_U}{2l_U} + \dfrac{x-h+l_L}{2l_L}, & x \in [h-l_L, h) \\ \dfrac{-x+h+l_U}{2l_U} + \dfrac{-x+h+l_L}{2l_L}, & x \in [h, h+l_L) \\ \dfrac{-x+h+l_U}{2l_U}, & x \in [h+l_L, h+l_U] \\ 0, & otherwise. \end{cases} \quad (1)$$

Definition 6 (Li and Cai [31])**.** *Let $\widetilde{\Theta}$ be an RSTIT2-FS, λ the medium of it, and E a fuzzy event from the universe. Then we can have the possibility, necessity, and credibility measures of E for $\widetilde{\Theta}$ are, respectively,*

$$\begin{aligned} &\mathrm{Pos}\{E\} = \mathrm{sup}_{x\in E}\mu_\lambda(x), \\ &\mathrm{Nec}\{E\} = 1 - \mathrm{sup}_{x\in E^c}\mu_\lambda(x), \\ &\mathrm{Cr}\{E\} = \frac{1}{2}(\mathrm{Pos}\{E\} + \mathrm{Nec}\{E\}). \end{aligned}$$

Definition 7 (Li and Cai [31])**.** *Assuming that $\widetilde{\Theta}$ is an RSTIT2-FS, λ is the medium of it. Then the credibility distribution (CD) of $\widetilde{\Theta}$ can be defined as $\Phi_{\widetilde{\Theta}} = \mathrm{Cr}\{\widetilde{\Theta} \le x\}$.*

It follows from Definition 6 and Equation (1) that we can lightly obtain that

$$\mathrm{Pos}\{\widetilde{\Theta} \le x\} = \begin{cases} 0, & x \in (-\infty, h - l_U] \\ \dfrac{x - h + l_U}{2l_U}, & x \in (h - l_U, h - l_L] \\ \dfrac{x - h + l_U}{2l_U} + \dfrac{x - h + l_L}{2l_L}, & x \in (h - l_L, h] \\ 1, & x \in (h, +\infty), \end{cases} \tag{2}$$

and

$$\mathrm{Nec}\{\widetilde{\Theta} \le x\} = \begin{cases} 0, & x \in (-\infty, h] \\ 1 + \dfrac{x - h - l_U}{2l_U} + \dfrac{x - h - l_L}{2l_L}, & x \in (h, h + l_L] \\ 1 + \dfrac{x - h - l_U}{2l_U}, & x \in (h + l_L, h + l_U] \\ 1, & x \in (h + l_U, +\infty). \end{cases} \tag{3}$$

Then the CD can be lightly calculated by making use of Definition 7 and Equations (2) and (3), which is as follows

$$\begin{aligned} \Phi_{\widetilde{\Theta}}(x) &= \mathrm{Cr}\{\widetilde{\Theta} \le x\} \\ &= \begin{cases} 0, & x \in (-\infty, h - l_U] \\ \dfrac{x - h + l_U}{4l_U} & x \in (h - l_U, h - l_L] \\ \dfrac{x - h + l_U}{4l_U} + \dfrac{x - h + l_L}{4l_L}, & x \in (h - l_L, h + l_L] \\ 1 + \dfrac{x - h - l_U}{4l_U}, & x \in (h + l_L, h + l_U] \\ 1, & x \in (h + l_U, +\infty). \end{cases} \end{aligned}$$

Definition 8 (Li and Cai [31])**.** *Let $\widetilde{\Theta}$ an RSTIT2-FS, and its inverse credibility distribution (ICD) is defined as the inverse function of $\Phi_{\widetilde{\Theta}}$ and can be calculated by*

$$\Phi^{-1}_{\widetilde{\Theta}}(\omega) = \begin{cases} 4l_U\omega + h - l_U, & \omega \in \left[0, \dfrac{l_U - l_L}{4l_U}\right) \\ \dfrac{4l_Ul_L\omega - 2l_Ul_L}{l_U + l_L} + h, & \omega \in \left[\dfrac{l_U - l_L}{4l_U}, 1 - \dfrac{l_U - l_L}{4l_U}\right) \\ 4l_U\omega + h - 3l_U, & \omega \in \left[1 - \dfrac{l_U - l_L}{4l_U}, 1\right]. \end{cases}$$

Definition 9 (Li and Cai [31])**.** *Supposing that $\widetilde{\Theta}_i, i = 1, 2, \cdots, n$ are RSTIT2-FVs with the mediums of λ_i, $i = 1, 2, \cdots, n$, respectively, and f is a function from $\mathbb{R}^n$ to $\mathbb{R}$, then the CD of $\widetilde{\Theta} = f(\widetilde{\Theta}_1, \widetilde{\Theta}_2, \cdots, \widetilde{\Theta}_n)$, $\Phi_{\widetilde{\Theta}}(x) = \mathrm{Cr}\{\widetilde{\Theta} \le x\}$ is defined as*

$$\Phi_{\widetilde{\Theta}}(x) = \Phi_\lambda(x),$$

where $\lambda = f(\lambda_1, \lambda_2, \cdots, \lambda_n)$ is the medium of $\widetilde{\Theta}$.

Remark 1. *According to Definitions 8 and 9, it can be easily deduced that* $\Phi^{-1}_{\widetilde{\Theta}}(\omega) = \Phi^{-1}_{\lambda}(\omega)$.

Theorem 1 (Li and Cai [31])**.** *Supposing that* $\widetilde{\Theta}_i, i = 1, 2, \cdots, n$*, are mutually independent RSTIT2-FVs with mediums of* λ_i*. If the function* $f(y_1, \cdots, y_\tau, y_{\tau+1}, \cdots, y_n)$ *is strictly increasing in regard to* $y_i, i = 1, 2, \cdots, \tau$ *and strictly decreasing in regard to* $y_i, i = \tau+1, \tau+2, \cdots, n$*, then*

$$\widetilde{\Theta} = f(\widetilde{\Theta}_1, \cdots, \widetilde{\Theta}_\tau, \widetilde{\Theta}_{\tau+1}, \cdots, \widetilde{\Theta}_n)$$

has the ICD of

$$\Phi^{-1}_{\widetilde{\Theta}}(\omega) = f\left(\Phi^{-1}_{\widetilde{\Theta}_1}(\omega), \cdots, \Phi^{-1}_{\widetilde{\Theta}_\tau}(\omega), \Phi^{-1}_{\widetilde{\Theta}_{\tau+1}}(1-\omega), \cdots, \Phi^{-1}_{\widetilde{\Theta}_n}(1-\omega)\right).$$

Definition 10 (Li and Cai [31])**.** *Let* $\widetilde{\Theta}$ *an RSTIT2-FS, and the EV of* $\widetilde{\Theta}$ *can be defined by*

$$E(\widetilde{\Theta}) = \int_0^{+\infty} \mathrm{Cr}\{\widetilde{\Theta} \geq x\}\mathrm{d}x - \int_{-\infty}^{0} \mathrm{Cr}\{\widetilde{\Theta} \leq x\}\mathrm{d}x.$$

Theorem 2 (Li and Cai [31])**.** *Given that* $\widetilde{\Theta}$ *is an RSTIT2-FS, its EV can be calculated by*

$$E[\widetilde{\Theta}] = \int_0^1 \Phi^{-1}_{\widetilde{\Theta}}(\omega)\mathrm{d}\omega.$$

Theorem 3 (Li and Cai [31])**.** *Let* $\widetilde{\Theta}_1$ *and* $\widetilde{\Theta}_2$ *two mutually independent RSTIT2-FSs. Then the linearity of the EV operator concerning the two RSTIT2-FSs can be expressed in the following form,*

$$E\left[k_1\widetilde{\Theta}_1 + k_2\widetilde{\Theta}_2\right] = k_1E[\widetilde{\Theta}_1] + k_2E[\widetilde{\Theta}_2].$$

3. Model Building and Equivalent Transformation

In the first stage, we introduce the MOP, and give the formulation of multi-objective CCP under type-2 fuzzy environment. Then, according to the operation law, we further transfer the type-2 fuzzy multi-objective CCP into its crisp equivalent form.

3.1. Multi-Objective Programming

MOPs have important applications in many areas of engineering and management [35]. Practically, a DM may want to optimize for more than one objectives under certain constraints, and these objectives may conflict with each other. So, we formulate a general MOP as follows,

$$\begin{cases} \min\limits_{x}(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})) \\ \text{subject to:} \\ \qquad z_j(\boldsymbol{x}) \leq 0, \quad j = 1, 2, \cdots, p, \end{cases}$$

where $\boldsymbol{x}$ is the decision vector, $f_i, i = 1, 2, \cdots, m$, are the objective functions, $z_j, j = 1, 2, \cdots, p$, are the constraint functions. In most cases, objective functions are mutually exclusive, namely, the optimization of one sub-objective may cause the degradation of the performance of another sub-objective or sub-objectives. Therefore, it is impossible for the DM to make each sub-objective completely optimal. For the purpose of solving this type of MOP, the only way is to find a balanced coordination among the sub-objectives, so that each sub-objective can be optimized under certain conditions.

3.2. Type-2 Fuzzy Multi-Objective Chance-Constrained Programming

Fuzzy CCP provides a method that allows DM to consider objectives and constraints based on the possibility of their acquirement, which is first proposed by Liu [36].

The following type-2 fuzzy CCP model is proposed in order to obtain the result of the decision to minimize the objective function under series of chance constraints. Assume that $\boldsymbol{x} = (x_1, x_2, \cdots, x_l)$ is an l-dimensional decision vector, $\widetilde{\boldsymbol{\sigma}} = (\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ is an n-dimensional type-2 fuzzy vector, $f(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ is the objective function, and $z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ are the constraint functions. Because the objective function $f(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ involves uncertain variable, it cannot be minimized immediately. Instead, the EV of the objective function can be minimized, i.e.,

$$\min_{\boldsymbol{x}} E[f(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})].$$

Due to the type-2 fuzzy constraints $z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}}) \leq 0$ cannot define a deterministic feasible set; we can provide a possibility β, called the confidence level, at which the type-2 fuzzy constraints are desired to hold. Thus the following chance constraints are proposed,

$$\mathrm{Cr}\{z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}}) \leq 0\} \geq \beta_j, \quad j = 1, 2, \cdots, p,$$

where β_j are the confidence levels.

Then, the following type-2 fuzzy single-objective CCP model is obtained,

$$\begin{cases} \min\limits_{\boldsymbol{x}} E[f(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})] \\ \text{subject to:} \\ \qquad \mathrm{Cr}\{z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}}) \leq 0\} \geq \beta_j, \quad j = 1, 2, \cdots, p, \end{cases} \tag{4}$$

where $\boldsymbol{x}$ is the decision vector, $\widetilde{\boldsymbol{\sigma}}$ is the type-2 fuzzy vector, $f(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ is the objective function, $z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ are the constraint functions, and β_j are the specified confidence levels.

In the following, we further propose a type-2 fuzzy multi-objective chance-constraint programming. Assume that $\boldsymbol{x} = (x_1, x_2, \cdots, x_l)$ is an l-dimensional decision vector, $\widetilde{\boldsymbol{\sigma}} = (\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ is an n-dimensional type-2 fuzzy vector, $f_i(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ are objective functions for $i = 1, 2, \cdots, m$, and $z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$ are constraint functions for $j = 1, 2, \cdots, p$. Similarly, since the objective functions $f_i(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})$, $i = 1, 2, \cdots, m$, also involve T2-FS, they cannot be minimized immediately. Instead, their EV can be minimized, i.e.,

$$E[f_1(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})], E[f_2(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})], \cdots, E[f_m(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})],$$

with the following chance constraints,

$$\mathrm{Cr}\{z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}}) \leq 0\} \geq \beta_j, \quad j = 1, 2, \cdots, p. \tag{5}$$

Then, we obtain the following T2-FMOP model,

$$\begin{cases} \min\limits_{\boldsymbol{x}} (E[f_1(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})], E[f_2(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})], \cdots, E[f_m(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})]) \\ \text{subject to:} \\ \qquad \mathrm{Cr}\{z_j(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}}) \leq 0\} \geq \beta_j, \quad j = 1, 2, \cdots, p. \end{cases} \tag{6}$$

Model (6) is to arrive at the result of a decision to minimize as many EVs $E[f_i(\boldsymbol{x}, \widetilde{\boldsymbol{\sigma}})]$ of objective functions as possible under a range of chance constraints. In general, either ideal or allowed choice are called solutions, as long as they are the specification of values for the decision vector $\boldsymbol{x}$.

3.3. Crisp Equivalent Model

We propose following theorems to transfer model 6 to its crisp equivalent form.

Theorem 4. *Assuming that the objective function $f_i(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ is strictly increasing in regard to $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_\xi$ and strictly decreasing in regard to $\widetilde{\sigma}_{\xi+1}, \widetilde{\sigma}_{\xi+2}, \cdots, \widetilde{\sigma}_n$. If $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_\xi$ are independent RSTIT2-FS, then the expected objective function in model (4) equals to*

$$\int_0^1 f(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\xi}^{-1}(r), \Phi_{\xi+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r))\mathrm{d}r,$$

where Φ_i^{-1} *is the ICD of* $\widetilde{\sigma}_i$ *for* $i = 1, 2, \cdots, n$.

Proof. According to Theorem 1, the ICD of $f_i(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ is

$$\Phi^{-1}(\boldsymbol{x}, r) = f(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\xi}^{-1}(r), \Phi_{\xi+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r)).$$

Additionally, making use of Theorem 2, we can have $E[f(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)] = \int_0^1 \Phi^{-1}(\boldsymbol{x}, r)\mathrm{d}r$. □

Theorem 5. *Assuming that the constraint function* $g_j(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ *is strictly increasing in regard to* $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_{\xi_j}$ *and strictly decreasing in regard to* $\widetilde{\sigma}_{\xi_j+1}, \widetilde{\sigma}_{\xi_j+2}, \cdots, \widetilde{\sigma}_n$*. If* $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_\xi$ *are independent RSTIT2-FS, then the chance constraint (5) holds if and only if*

$$z_j(\boldsymbol{x}, \Phi_1^{-1}(\beta_j), \cdots, \Phi_{\xi_j}^{-1}(\beta_j), \Phi_{\xi_j+1}^{-1}(1-\beta_j), \cdots, \Phi_n^{-1}(1-\beta_j)) \le 0, \tag{7}$$

where Φ_i^{-1} *is the ICD of* $\widetilde{\sigma}_i$ *for* $i = 1, 2, \cdots, n$.

Proof. According to Theorem 1, the ICD of $f_i(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ is

$$\Phi^{-1}(\boldsymbol{x}, r) = f(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\xi}^{-1}(r), \Phi_{\xi+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r)).$$

□

Furthermore, it is obvious that (5) holds if and only if $\Phi^{-1}(\boldsymbol{x}, r) \le 0$.

Let $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n$ in model (6) be independent RSTIT2-FS with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. We also assume that $f_i(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)$ is strictly increasing in regard to $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_{\eta_i}$ and strictly decreasing in regard to $\widetilde{\sigma}_{\eta_i+1}, \widetilde{\sigma}_{\eta_i+2}, \cdots, \widetilde{\sigma}_n$. Then in accordance with Theorem 4, each objective function in model (6) can be rewritten as

$$\begin{aligned} &E[f_i(\boldsymbol{x}, \widetilde{\sigma}_1, \widetilde{\sigma}_2, \cdots, \widetilde{\sigma}_n)] \\ &= \int_0^1 f_i(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\eta_i}^{-1}(r), \Phi_{\eta_i+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r))\mathrm{d}r. \end{aligned} \tag{8}$$

Thereby, combining Equations (7) and (8), the T2-FMOP model (6) can be transformed into its equivalent crisp model as follows,

$$\left\{\begin{array}{l} \min\limits_{\boldsymbol{x}} \Big(\int_0^1 f_1(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\eta_1}^{-1}(r), \Phi_{\eta_1+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r))\mathrm{d}r, \\ \qquad \int_0^1 f_2(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\eta_2}^{-1}(r), \Phi_{\eta_2+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r))\mathrm{d}r, \cdots, \\ \qquad \int_0^1 f_m(\boldsymbol{x}, \Phi_1^{-1}(r), \cdots, \Phi_{\eta_m}^{-1}(r), \Phi_{\eta_m+1}^{-1}(1-r), \cdots, \Phi_n^{-1}(1-r))\mathrm{d}r \Big) \\ \text{subject to:} \\ \qquad z_j(\boldsymbol{x}, \Phi_1^{-1}(\beta_j), \cdots, \Phi_{\xi_j}^{-1}(\beta_j), \Phi_{\xi_j+1}^{-1}(1-\beta_j), \cdots, \Phi_n^{-1}(1-\beta_j)) \le 0, \\ \qquad\qquad j = 1, 2, \cdots, p. \end{array}\right. \tag{9}$$

4. Model Solving Based on Fuzzy Interactive Approach

For the sake of a satisfactory result to the DM, we propose a fuzzy interactive approach. The core of the fuzzy interactive approach lies in the transformation of the model into the minmax problem and the continuous interaction with the DM to obtain new reference member values, and eventually achieve a satisfactory solution for the DM.

4.1. The Minmax Problem

For the interactive process, we first consider the inexactness of the DM's judgment of the importance of each objective $E[f_i(\boldsymbol{x},\widetilde{\sigma})]$ in model (9), so we construct a membership function for each objective. We denote $E[f_i(\boldsymbol{x},\widetilde{\sigma})]$ as $\bar{f}_i(\boldsymbol{x})$ and assume that $\bar{f}_i(\boldsymbol{x})$ should be not greater than a certain value is equivalent to fuzzy min, $\bar{f}_i(\boldsymbol{x})$ should be not less than a certain value is equivalent to fuzzy max. The crisp objective functions in model (9) can be indicated by the liner membership functions $\mu_i(\bar{f}_i(\boldsymbol{x})), i = 1, 2, \cdots, m$, defined by the following equations, as shown in Figures 3 and 4.

$$\text{fuzzy min} \qquad \mu_i(\bar{f}_i(\boldsymbol{x})) = \begin{cases} 0, & \bar{f}_i(\boldsymbol{x}) > \bar{f}_i^R \\ \dfrac{\bar{f}_i(\boldsymbol{x}) - \bar{f}_i^R}{\bar{f}_i^N - \bar{f}_i^R}, & \bar{f}_i^N \leq \bar{f}_i(\boldsymbol{x}) \leq \bar{f}_i^R \\ 1, & \bar{f}_i(\boldsymbol{x}) < \bar{f}_i^N, \end{cases}$$

$$\text{fuzzy max} \qquad \mu_i(\bar{f}_i(\boldsymbol{x})) = \begin{cases} 0, & \bar{f}_i(\boldsymbol{x}) < \bar{f}_i^L \\ \dfrac{\bar{f}_i(\boldsymbol{x}) - \bar{f}_i^L}{\bar{f}_i^M - \bar{f}_i^L}, & \bar{f}_i^L \leq \bar{f}_i(\boldsymbol{x}) \leq \bar{f}_i^M \\ 1, & \bar{f}_i(\boldsymbol{x}) > \bar{f}_i^M, \end{cases}$$

where $\bar{f}_i^N$ and $\bar{f}_i^L$ are the minimum values of the objective functions $\bar{f}_i(\boldsymbol{x})$ under the given constraints, $\bar{f}_i^R$ and $\bar{f}_i^M$ are the maximum values of the objective functions $\bar{f}_i(\boldsymbol{x})$ under the given constraints.

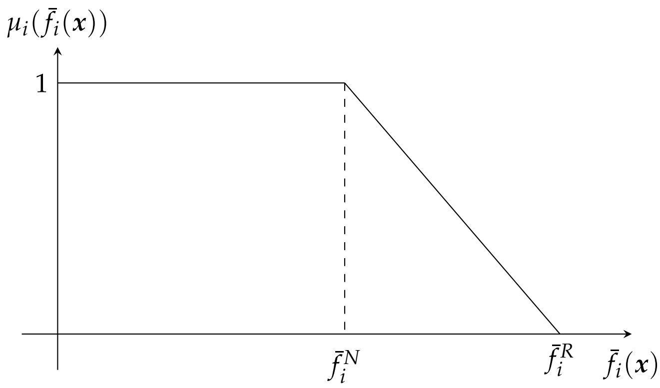

Figure 3. Image of fuzzy min membership function.

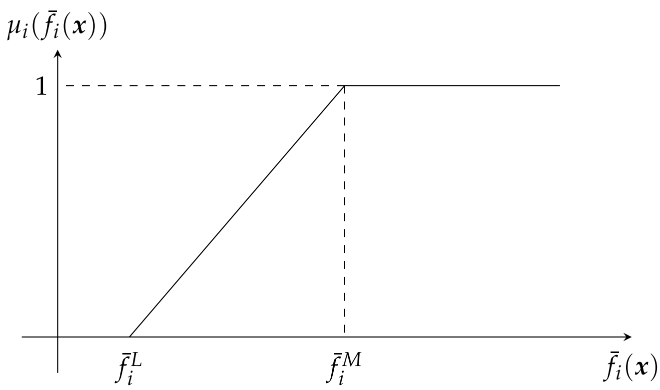

Figure 4. Image of fuzzy max membership function.

After eliciting the membership functions $\mu_i(\bar{f}_i(\boldsymbol{x})), i = 1, 2, \ldots, m$ from the DM, model (9) can be replaced by

$$\max_{1 \leq i \leq m} (\mu_1(\bar{f}_1(\boldsymbol{x})), \mu_2(\bar{f}_2(\boldsymbol{x})) \cdots, \mu_m(\bar{f}_m(\boldsymbol{x})). \tag{10}$$

In general, the objectives may conflict with each other. Therefore, we refer to a more flexible interactive method to obtain the DM's satisfactory solution founded on their preference degree for each goal through interaction. To explain clearly, we determine the preference information $\bar{\mu}_i, i = 1, 2, \cdots, m$ for each fuzzy objective from the DM, called the reference membership value. Here, the concept of reference member value is defined as an extended application of reference points proposed by Wierzbicki [37]. Subsequently, we express the difference between the actual value of the original objective function and the satisfactory value as the difference between the actual value of the member function and the reference member value, i.e., $\bar{\mu}_i - \mu_i(\bar{f}_i(\boldsymbol{x})), i = 1, 2, \cdots, m$, which means that the satisfactory solution of the DM can be approached by this iterative calculation. According to the approach, model (10) is interpreted as

$$\min_{x \in X} \max_{1 \leq i \leq m} \{\bar{\mu}_i - \mu_i(\bar{f}_i(\boldsymbol{x}))\}$$

or, equivalent to

$$\begin{cases} \min\limits_{v, \boldsymbol{x} \in X} v \\ \text{subject to:} \\ \qquad \bar{\mu}_1 - \mu_1(\bar{f}_1(\boldsymbol{x})) \leq v \\ \qquad \qquad \vdots \\ \qquad \bar{\mu}_m - \mu_m(\bar{f}_m(\boldsymbol{x})) \leq v. \end{cases} \tag{11}$$

4.2. Interactive Algorithm

The preliminary solution can be obtained by calculating model (11), but the result is not satisfactory for the DM. In order to obtain an approving result, the DM is obliged to renew the values of μ_i, by employing the trade-off rate (Sakawa et al. [38]). Denote the simplex multipliers (Haimes and Chankong [39]) associated with the constraints of mode (9) as $-{}^{\partial \bar{f}_i(x)}/_{\partial \bar{f}_1(x)}, i = 1, 2, \cdots$. Then the trade-off rate between $\bar{f}_i(\boldsymbol{x})$ and $\bar{f}_1(\boldsymbol{x})$ is easily obtained and can be represented by

$$-\frac{\partial \bar{f}_i(\boldsymbol{x})}{\partial \bar{f}_1(\boldsymbol{x})}, \quad i = 2, 3, \cdots, m.$$

The above equation means that under the condition of ensuring that the original objective function is unchanged, only sacrificing the ith objective function of one unit, $\bar{f}_i(\boldsymbol{x})$, can realize the improvement of one unit in $\bar{f}_1(\boldsymbol{x})$ of another objective function. If $-{}^{\partial \bar{f}_i(x)}/_{\partial \bar{f}_1(x)} = 1$, it means that neither of the two objective functions need to be changed. If $0 < -{}^{\partial \bar{f}_i(x)}/_{\partial \bar{f}_1(x)} < 1$, we will be willing to change $\bar{f}_1(\boldsymbol{x})$. If $-{}^{\partial \bar{f}_i(x)}/_{\partial \bar{f}_1(x)} > 1$, we will make more priority changes to $\bar{f}_i(\boldsymbol{x})$ for a better pursuit.

When the DM obtains the trade-off rate, they modify the reference membership values $\bar{\mu}_i$. Subsequently, the DM Changes the reference membership values $\bar{\mu}_i$ through a continuous interaction and solving model (11) to obtain a satisfactory solution.

In summary, when we obtain the crisp model, we have to choose the membership functions for all the objectives. Then we ask the DM to give a reference membership value for each objective function to solve model (11) by minmax method. At the same time, the DM can measure whether the result can achieve the expected effect. Then the DM modifies their reference membership value according to the trade-off rate, and can obtain a satisfactory solution. The fuzzy interactive algorithm can be summarized in Table 2.

Table 2. Fuzzy interactive algorithm.

Step 1	Let the DM give the satisficing levels α_i, $i = 1, 2, \cdots, m$ for each constraint in model (9).
Step 2	Calculate the individual minimum $\bar{f}_i^{min}$ and maximum $\bar{f}_i^{max}$ of each objective function $\bar{f}_i(x)$ under the chance-constrained conditions with satisfying levels. Elicit a membership function $\mu_i(\bar{f}_i(x))$, $i = 1, 2, \cdots, m$, from the DM for each objective function.
Step 3	Set up the initial reference membership level $\bar{\mu}_i$, $i = 1, 2, \cdots, m$, as required by the DM, and solve the corresponding minmax problem in model (11).
Step 4	Provide the DM with the corresponding trade-off rates $-{}^{\partial \bar{f}_i(x)}/_{\partial \bar{f}_1(x)}$, $i = 1, 2, \cdots, m$, between the objective functions. The calculation does not stop until it can obtain the satisfactory membership function value in the current stage. Otherwise, please update the reference member level $\bar{\mu}_i$ by the DM, by considering the trade-off rate to improve another objective function with higher priority at the expense of one objective function, and return to step 3.

5. Application to Remanufacture of Old Clothes Problem

To manifest the effectiveness of the fuzzy interactive approach, it will be applied to the remanufacture of old clothes problem, considering the sustainability development of remanufactured apparel. The overall process of the fuzzy interactive approach is illustrated clearly by the experiment, and the result is calculated to present whether this approach performs well.

5.1. Problem Description

Remanufactured apparel is designed to increase the usable value of textile and apparel materials. Pao [40] pointed out that the recycling of waste clothing is to recycle and remanufacture waste materials, so as to save raw materials and reduce the production of waste, which is of greater significance to the ecological protection of nature. From the perspective of sustainability, traditional clothing manufacturing uses a large number of cotton, hemp, polyester and other textiles for production. Riley [41] introduced that polyester is a high molecular compound, which is not easy to recycle. Secondly, Mason [42] introduced that due to people's pursuit of fashion, clothing has become a fast-moving consumer good, and clothes discarded by people have gradually imposed a burden on the environment. Therefore, in the process of sustainable manufacturing, the amount of recycled old clothes and renewable resources used in product production determines the social and economic benefits of products.

The traditional clothing manufacturing industry generally considers the production cost, product quality and profits in the manufacturing process. However, Manson [43] introduced that green sustainable manufacturing pursues the former goal, while considering resource interest rates and environmental impact. Based on this, this paper is based on the product manufacturing process, pondering on the affection of the manufacturing process on the economy, environment and society, and strives to use recycled old clothes and use environmentally friendly materials to make clothes to reduce emissions in production. By establishing a multi-objective planning model, we can weigh the impact of product manufacturing on the economy, environment and society, and obtain a sustainable production plan.

In this paper, we suppose that a factory DM will consider expanding production channels by utilizing uncertain customer demands in the future. They want to maximize the total profit and minimize the working hours of the machine under the control of the capital budget and stock limits. The decision variables x_i, $i = 1, 2$, are the number of new apparel and remanufactured apparel being produced. The process of remanufactured apparel production includes recycling old apparel, remanufacturing, storage, delivery and sale, as shown in Figure 5.

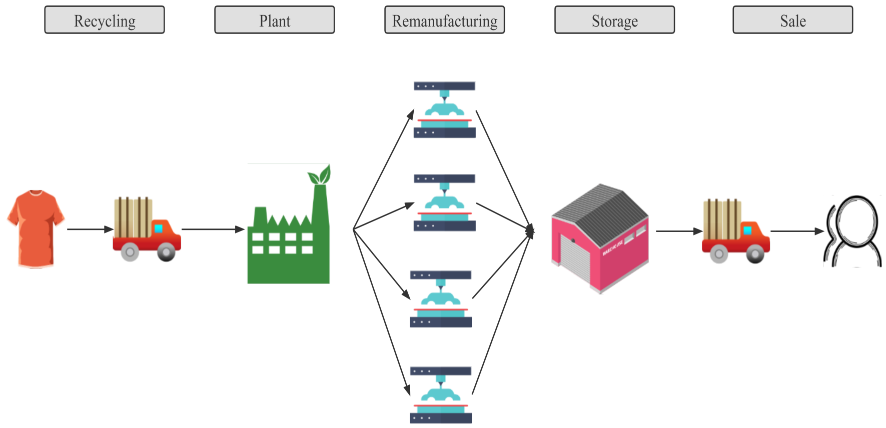

Figure 5. The flow of the remanufacture of old clothes problem.

We will demonstrate the parameters and decision variables applied in the model of this paper:

- $\boldsymbol{x} = (x_1, x_2)^T$: the decision vector of the number of new apparel and remanufactured apparel;
- α : the predetermined satisfying level of the DM, $0 \leq \alpha \leq 1$;
- a_i : the unit profit of the ith type of product, $i = 1, 2$;
- b_i : the unit machine hours to produce the ith type of product, $i = 1, 2$;
- c_i : the uncertain customer demands that the ith type of product covers, $i = 1, 2$.

Thus, the remanufacture of old clothes problem can be constructed by a T2-FMOP model as follows,

$$\begin{cases} \max\ a_1x_1 + a_2x_2 \\ \min\ b_1x_1 + b_2x_2 \\ \text{subject to:} \\ \qquad 30x_1 + 40x_2 \leq 50000 \\ \qquad x_1 + x_2 - c_1 - c_2 \leq 100 \\ \qquad x_1, x_2 \geq 0, \end{cases} \tag{12}$$

for example, assume that $a_1 = \begin{pmatrix} 10 & 50 & 90 \\ 20 & 50 & 80 \end{pmatrix}$, $a_2 = \begin{pmatrix} 10 & 40 & 70 \\ 20 & 40 & 60 \end{pmatrix}$, $b_1 = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 3 & 4 \end{pmatrix}$, $b_2 = \begin{pmatrix} 3 & 5 & 7 \\ 4 & 5 & 6 \end{pmatrix}$, $c_1 = \begin{pmatrix} 200 & 500 & 800 \\ 300 & 500 & 700 \end{pmatrix}$, $c_2 = \begin{pmatrix} 100 & 300 & 500 \\ 200 & 300 & 400 \end{pmatrix}$. The first objective function means to maximize the total profit, and the second function means to minimize the total machine hours. The first constraint limits the capital budget, and the second constraint limits the stock. According to the derivation result of model (6), model (12) of the remanufacture of old clothes problem can be transferred into

$$\begin{cases} \max\ E[a_1x_1 + a_2x_2] \\ \min\ E[b_1x_1 + b_2x_2] \\ \text{subject to:} \\ \qquad 30x_1 + 40x_2 \leq 50000 \\ \qquad \mathrm{Cr}\{x_1 + x_2 - 100 \leq T\} \geq \alpha \\ \qquad x_1, x_2 \geq 0, \end{cases} \tag{13}$$

where $T = c_1 + c_2$ for convenient expression. According to the DM, we set the satisfying level $\alpha = 0.9$.

According to Theorem 5, we can transfer the constraint functions, and model (13) can be written as

$$\begin{cases} \max\ E[a_1x_1 + a_2x_1] \\ \min\ E[b_1x_2 + b_2x_2] \\ \text{subject to:} \\ \qquad 30x_1 + 40x_2 \leq 50000 \\ \qquad x_1 + x_2 \geq \Phi_T^{-1}(\alpha) \\ \qquad x_1, x_2 \geq 0. \end{cases} \tag{14}$$

5.2. Problem Solving

Firstly, we can calculate the second constraint limiting the stock, where $\Phi^{-1}_{c_1}(\alpha)$ and $\Phi^{-1}_{c_2}(\alpha)$ are calculated as

$$\Phi^{-1}_{c_1}(\alpha) = \begin{cases} 1200\alpha + 200, & \alpha \in \left[0, \dfrac{1}{12}\right) \\ 480\alpha + 260, & \alpha \in \left[\dfrac{1}{12}, \dfrac{11}{12}\right) \\ 1200\alpha - 400, & \alpha \in \left[\dfrac{11}{12}, 1\right], \end{cases}$$

$$\Phi^{-1}_{c_2}(\alpha) = \begin{cases} 800\alpha + 100, & \alpha \in \left[0, \dfrac{1}{8}\right) \\ \dfrac{800}{3}\alpha + \dfrac{500}{3}, & \alpha \in \left[\dfrac{1}{8}, \dfrac{7}{8}\right) \\ 800\alpha - 300, & \alpha \in \left[\dfrac{7}{8}, 1\right]. \end{cases}$$

According to Theorem 1, we have

$$\Phi^{-1}_{T}(\alpha) = \Phi^{-1}_{c_1}(\alpha) + \Phi^{-1}_{c_2}(\alpha)$$

$$= \begin{cases} 2000\alpha + 300, & \alpha \in \left[0, \dfrac{1}{12}\right) \\ 1280\alpha + 360, & \alpha \in \left[\dfrac{1}{12}, \dfrac{1}{8}\right) \\ \dfrac{2240}{3}\alpha + \dfrac{1280}{3}, & \alpha \in \left[\dfrac{1}{8}, \dfrac{7}{8}\right) \\ 1280\alpha - 40, & \alpha \in \left[\dfrac{7}{8}, \dfrac{11}{12}\right) \\ 2000\alpha - 700, & \alpha \in \left[\dfrac{11}{12}, 1\right]. \end{cases}$$

Following that, we obtain the crisp expectations by Theorem 3, and model (14) can be described as

$$\begin{cases} \max\ 50x_1 + 40x_2 \\ \min\ 3x_1 + 5x_2 \\ \text{subject to:} \\ \qquad 30x_1 + 40x_2 \le 50000 \\ \qquad x_1 + x_2 \ge 1212 \\ \qquad x_1, x_2 \ge 0. \end{cases} \tag{15}$$

After obtaining the crisp model (15), we separately calculate the maximum $\bar{f}_i^{\max}$ and minimum $\bar{f}_i^{\min}$ of each objective function $\bar{f}_i(\boldsymbol{x})$ under the constraints. The results are derived as

$$\begin{cases} \bar{f}_1^{\min} = 48480 \\ \bar{f}_1^{\max} = 83333 \end{cases} \quad \begin{cases} \bar{f}_2^{\min} = 3636 \\ \bar{f}_2^{\max} = 6250. \end{cases}$$

Finally, based on the maximum $\bar{f}_i^{\max}$ and minimum $\bar{f}_i^{\min}$ values of each objective function, we can quantify the images of the two objective functions of maximizing total profit and minimizing machine working time (shown in Figures 6 and 7) and calculate the membership functions as follows,

$$\mu_1(\bar{f}_1(\boldsymbol{x})) = \begin{cases} 0, & \bar{f}_1(\boldsymbol{x}) < 48480 \\ \dfrac{\bar{f}_1(\boldsymbol{x}) - 48480}{83333 - 48480} & 48480 \le \bar{f}_1(\boldsymbol{x}) \le 83333 \\ 1, & \bar{f}_1(\boldsymbol{x}) > 83333, \end{cases}$$

and

$$\mu_2(\bar{f}_2(\boldsymbol{x})) = \begin{cases} 0, & \bar{f}_2(\boldsymbol{x}) > 6250 \\ \dfrac{\bar{f}_2(\boldsymbol{x}) - 6250}{3636 - 6250} & 3636 \le \bar{f}_2(\boldsymbol{x}) \le 6250 \\ 1, & \bar{f}_2(\boldsymbol{x}) < 3636. \end{cases}$$

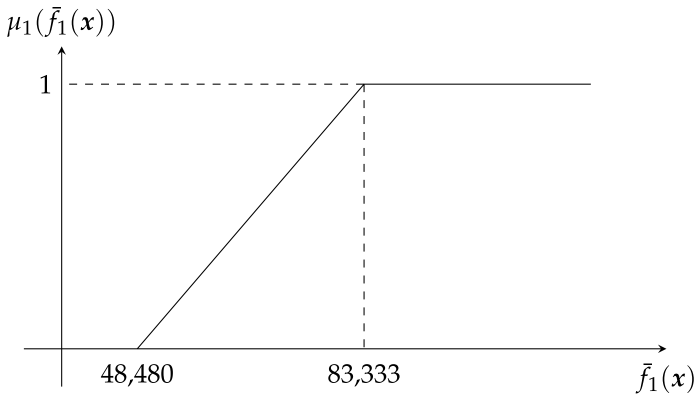

Figure 6. Image of fuzzy membership function $\bar{f}_1(\boldsymbol{x})$.

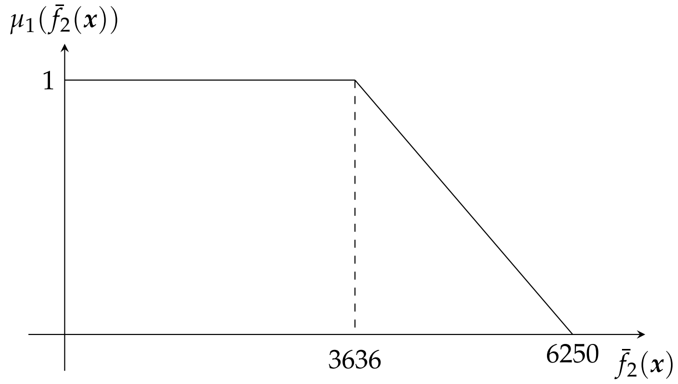

Figure 7. Image of fuzzy membership function $\bar{f}_2(x)$.

5.3. Interactive Process

When we obtain the membership functions for each objective function, we need the DM to give a reference membership value $\bar{\mu}_i$ for each membership function as a way to derive satisfactory results through an interactive approach. The process and results of the interaction are shown in Table 3.

Table 3. Solution of interaction.

Interaction	$(\bar{\mu}_1, \bar{\mu}_2)$	$(\mu_{\bar{f}_1}(x), \mu_{\bar{f}_2}(x))$	$-\frac{\partial f_i(x)}{\partial f_1(x)}$
Non-interactive	–	(0.3477, 1.0000)	–
First	(1.0000, 1.0000)	(0.7093, 0.7108)	0.1031
Second	(0.9500, 0.8000)	(0.7925, 0.6442)	0.1031
Third	(0.9500, 0.7000)	(0.8484, 0.5995)	0.1031
Fourth	(0.9000, 0.5500)	(0.8986, 0.5593)	0.1031

From the first row in Table 3, we can clearly observe that although the value of $\mu_{\bar{f}_1}(x)$ is too small, the value of $\mu_{\bar{f}_2}(x) = 1.0000$ (means that the DM can not make as much profit as possible, although the use time of the machine can be minimized), which cannot satisfy the requirements of the DM. Therefore, we need to interact with the DM to obtain a satisfactory solution. At the beginning, we set the initial reference membership values $\bar{\mu}_1 = 1.0000$, $\bar{\mu}_2 = 1.0000$. The result leaves the DM unsatisfied and the DM hopes to increase the total profit and reduce the working time of the machine. Then, the trade-off rate is figured out as $-{}^{\partial \bar{f}_i(x)}/_{\partial \bar{f}_1(x)} = 0.1031$, which signifies that in the second iteration, the second objective value of 0.1031 units needs to be sacrificed to improve the first objective value. Following that, they update the reference membership levels to $\bar{\mu}_1 = 0.9500$, $\bar{\mu}_2 = 0.8000$. We can see from the third column in Table 3, the value of $\mu_{\bar{f}_1}(x)$ increases and the value of $\mu_{\bar{f}_2}(x)$ decreases, which means that for the purpose of obtaining higher profits, it is supposed to properly extend the working time of the machine. After the DM completes the second interaction, they are still dissatisfied with the new solution. Then, the reference membership levels are updated to $\bar{\mu}_1 = 0.9500$, $\bar{\mu}_2 = 0.7000$, they remain dissatisfied with the results of $\mu_{\bar{f}_1}$ and $\mu_{\bar{f}_2}$, and therefore move on to the next interaction. In the fourth interaction, let the reference membership value be $\bar{\mu}_1 = 0.9000$, $\bar{\mu}_2 = 0.5500$, and the result shows that the DM obtains a production plan that can achieve both more total profit and less machine working time.

From Table 3, it can be seen that if we only interact with the DM once or not, it is impossible to acquire a pleased solution. Therefore, we can conclude that for MOP problem with RSTIT2-FS parameters, the results of interactive approach are usually more satisfactory for the DM than those without fuzzy interactive approach. It not only achieves satisfactory solutions for the DM, but also facilitates the application to other MOP problems under the uncertain environment in production or business activities due to its interactivity and flexibility.

In addition, we consider the sensitivity of uncontrollable parameters to the results. In order to present more obvious results, we assume that all fuzzy parameters increase and decrease by 10% and 20%. Table 4 shows the calculation results of uncontrollable parameter disturbance.

Table 4. Impact of item-specific data perturbations on production decisions.

NO.	Range	Interaction	$(\bar{\mu}_1, \bar{\mu}_2)$	$(\mu_{\tilde{f}_1}(x), \mu_{\tilde{f}_2}(x))$
1	10%	First	(1.0000, 1.0000)	(0.7203, 0.7238)
		Second	(0.9500, 0.8000)	(0.8020, 0.6584)
		Third	(0.9500, 0.7000)	(0.8588, 0.6130)
		Fourth	(0.9000, 0.5500)	(0.9005, 0.5797)
2	−10%	First	(1.0000, 1.0000)	(0.6877, 0.6877)
		Second	(0.9500, 0.8000)	(0.7705, 0.6215)
		Third	(0.9500, 0.7000)	(0.8270, 0.5763)
		Fourth	(0.9000, 0.5500)	(0.8821, 0.5322)
3	20%	First	(1.0000, 1.0000)	(0.7271, 0.7183)
		Second	(0.9500, 0.8000)	(0.8098, 0.6521)
		Third	(0.9500, 0.7000)	(0.8925, 0.5860)
		Fourth	(0.9000, 0.5500)	(0.9053, 0.5758)
4	−20%	First	(1.0000, 1.0000)	(0.7092, 0.7109)
		Second	(0.9500, 0.8000)	(0.7925, 0.6443)
		Third	(0.9500, 0.7000)	(0.8484, 0.5995)
		Fourth	(0.9000, 0.5500)	(0.8986, 0.5593)

As shown in Table 4, compared with the final result $(0.8986, 0.5593)$ in Table 3, the comparison of these four results $(0.9005, 0.5797)$, $(0.8821, 0.5322)$, $(0.9053, 0.5758)$, $(0.8986, 0.5593)$ shows that although the change of parameters makes the DM change the production decision of products to a certain extent, the DM can always obtain satisfactory results through the fuzzy interactive approach.

The results in Table 4 reveal two points: (1) By changing the value of uncontrollable factors such as the unit profit of the ith type of product, the time required to produce the product and the uncertainty of customer demand, it is found that their impact on the results is similar. (2) Considering the uncertainty of the DM to uncontrollable parameters, the fuzzy interactive approach can maximize the total profit and minimize the total production time under the constraints of capital budget and inventory constraints.

From the results of the example, it can be observed that the fuzzy interactive solution of the CCP model with RSTIT2-FS parameters can make the DM directly participate in the production decision making in practice. Meanwhile, as a garment manufacturing enterprise, it is a green and sustainable production activity to use the form of recycling old clothes, which can not only bring economic benefits but also environmental benefits. Consequently, in the pursuit of sustainable manufacturing, the DM needs to (1) pay attention to product quality to break consumers' prejudice against old products and (2) balance machine working time and material consumption.

Although in this numerical experiment, we use the example of remanufacturing old clothes to illustrate our proposed method, in fact, it can be extended to multi-objective programming problems in interval type-2 fuzzy environment where the objective functions and constraint conditions are strictly monotonically increasing or monotonically decreasing, such as logistics transportation programming, investment decisions and other programming problems.

6. Conclusions

This paper focused on the crisp transformation of the MOP problem with RSTIT2-FS parameters and the process of solving it by using the fuzzy interactive approach. After proposing the type-2 fuzzy CCP model, we obtained the crisp equivalent model through crisp transformation. Subsequently, we need to determine the membership function in the interactive approach in advance and calculate the EV by constantly updating the reference membership value, and finally obtain a satisfactory solution for the DM.

The main contributions of this paper are as follows: (1) A two-stage method was proposed to solve the CCP model with RSTIT2-FS parameters, which provides a useful theoretical framework for dealing with uncertain programming problems in practice. (2) A crisp transformation method was proposed to obtain the crisp equivalent model by defuzzifying objective and constraint functions. (3) The fuzzy interactive approach was used to solve the remanufacture of old clothes problem. Through a continuous interactive process, a satisfactory production decision scheme can be obtained for the DM. Meanwhile, it can also improve the participation of the DM in decision making, and provide an available measure for the disposal of programming problems in real life.

It is worth mentioning that there are still many limitations in this paper. Firstly, the remanufacture of old clothes problem assumed in Section 5 of this paper is a simple MOP problem. In addition, the objective function and constraint function are linear. Therefore, it is necessary to consider the nonlinear objective function and constraint function in future research. Secondly, the fuzzy interactive approach of MOP with RSTIT2-FS parameters studied in this paper can be extended to a stochastic and some mixed backgrounds of randomness and uncertainty. Meanwhile, it can be applied to logistics transportation, investment decision making and other issues. Finally, the research on the production problem of used clothes remanufacture proposed in this paper is in the preliminary application stage, and it is used to verify the applicability of the mentioned approach. In future work, we will further study how to establish a sustainable objective function that is more in line with the actual production decision and the various constraints in the corresponding model.

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Abbreviations

The following abbreviations are used in the manuscript:

DM	decision maker
MOP	multi-objective programming
FMOP	fuzzy multi-objective programming
T2-FS	type-2 fuzzy set
T2-FMOP	type-2 fuzzy multi-objective programming
IT2-FS	interval type-2 fuzzy set
RSTIT2-FS	regular symmetric triangular interval type-2 fuzzy set
UMF	upper membership function
LMF	lower membership function
CD	credibility distribution
ICD	inverse credibility distribution
CCP	chance-constrained programming
EV	expected value

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