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Copernican Paradigm beyond FLRW

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Abstract: We present the dipole cosmological principle, i.e., the notion that the Universe is a Copernican cosmology that agrees with the cosmic flow. It suits the most symmetric paradigm that generalizes the Friedmann–Lemaître–Robertson–Walker ansatz in the context of numerous suggestions that have appeared in the literature for non-kinematic components in the cosmic microwave background dipole. Field equations in our "dipole cosmology" are still ODEs, but we now have four instead of two Friedmann equations. The two extra functions can be regarded as additional scale factors that break the isotropy group from SO(3) to U(1) and a "tilt" that denotes the cosmic flow. The result is an axially isotropic Universe. We examined the dynamics of the expansion rate, anisotropic shear, and tilt in some cases. One important observation is that the cosmic flow (tilt) can grow while the anisotropy (shear) dies down.

Keywords: dipole; tilt; shear; acceleration; equations of state

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1. Introduction

The philosophy that we are non-privileged observers has been a powerful one in the history of science ever since Copernicus [1]. In modern cosmology, this idea in the context of both space and time is sometimes called the perfect cosmological principle. Any maximally symmetric spacetime (Minkowski or (anti-)de Sitter space) would satisfy this. It would be a candidate for the ultimate Copernican cosmology, but as the observations suggest—it is wrong. Hubble's discovery (regarding the expansion of our Universe and numerous shreds of evidence for the Big Bang paradigm) instructs that we do not have symmetry along the time direction. This eliminates the possibility of maximally symmetric spacetimes.

However, one could still pose the question: What is the *maximal* Copernican paradigm that agrees with an expanding Universe? The Friedmann–Lemaître–Robertson–Walker (FLRW) metric [2,3] provides an answer to this question. This spacetime allows for a time dependence but assumes maximal symmetry in the spatial slices (i.e., homogeneous and isotropic). This is the "Cosmological Principle". FLRW ansatz is just the start of modern cosmology, whereas the flat Lambda cold dark matter (ΛCDM) concordance model is a specific realization of it.

ΛCDM cosmology is a great success story in that it provides a concordant model of the Universe across datasets at different redshifts. However, in the last two decades, multiple worries have appeared in the cosmological standard model. The most striking among them is the sustained tension that has only worsened over the years in the measured value of the local Hubble constant, H_0 [4–7].

The inferred value of H_0 from early cosmology (e.g., as reported by the Planck mission, $\sim$67.4 $\pm$ 0.5 km/s/Mpc [8]) differs from those arising from local measurements ($\sim$72–73 km/s/Mpc; see, e.g., [7]). This discrepancy is often quoted at the 4–6 σ [7,9] level. The H_0 tension is one of the many tensions challenging the standard model of cosmology, giving rise to the "crisis in ΛCDM cosmology" [4]. Given this state of affairs, it seems worth exploring how one should modify models of cosmology if one were to take current data at face value. Clarification is needed as to whether the data and systematics involved are unquestionable; however, we were motivated to evaluate the weakest points in our current models.

If we want to work within the FLRW paradigm, taking the current data at face value, there is no way we can alleviate the H_0 and other cosmological tensions. This work explores some possibilities beyond the standard setting in cosmology. One reason that makes working with FLRW desirable is its simplicity. The background dynamical equations are ordinary differential equations (ODEs). Moreover, FLRW finds good support within inflationary settings where the lore is that accelerated expansion 'isotropizes' cosmology. This is based on Wald's cosmic no-hair theorem [10] and refers to shear anisotropies. While Wald's theorem is moderate during the inflationary epochs, the generic picture of suppressed anisotropies is accurate [11].

Moving significantly away from FLRW is problematic for several reasons. Primarily, the "microwave background radiation" is isotropic to a part in 10^5 [8]. (However, there are anomalies (most famously, high-ℓ–low-ℓ anomalies and quadrupole and octupole anomalies). Thus, at least at the time of the last scattering, the assumptions of homogeneity and isotropy on the spatial slices were quite suitable. However, there is scope for generalization. The cosmic microwave background (CMB) has an anisotropic dipole component, which is one part in 10^2 as opposed to 10^5. This bigger dipole moment is considered to have kinematic origins. The idea is that the CMB dipole is a result of the Earth's local motion and our host galaxy relative to the "CMB rest frame". If the CMB dipole was a result of our local peculiar motion, one would expect similar effects on the other distant homogeneously distributed sources, such as quasars and radio sources. We will see an analogous dipole moment in the sky distribution [12]. Thus, we expect to have isotropic distributions of such objects once we correct for the kinematic component.

This paper provides some exploratory steps in this setting. We identify the next most symmetric paradigm/ansatz that generalizes FLRW where cosmic flow is allowed. We present our ansatz with the metric, stress tensor, and equations of motion for these spacetimes and investigate these cosmologies for some equation of state (EOS) categories. We find that cosmologies do not have to isotropize as instructed by Wald's cosmic hair theorem, even if there is acceleration.

2. Background Metric and the Fluid

The most general metric compatible with the Copernican assumptions of the previous section (i.e., the dipole cosmological principle) is

$$ds^2 = -dt^2 + X^2(t)\, dz^2 + Y^2(t)\ \exp(-2A_0\, z)\big(dx^2 + dy^2\big) \tag{1}$$

The above metric has four killing vectors on space-like surfaces: $\partial_x, \partial_y, x\partial_y - y\partial_x$, and $K = A_0(x\partial_x + y\partial_y) + \partial_z$. A_0 being a positive constant can be set to unity by scaling the z coordinate, redefining $X(t)$.

The metric in Equation (1) falls into locally rotationally symmetric Bianchi V and VII$_h$ classes (e.g., chapter 18 [3]). A $t =$ constant slice of the metric describes a 3-dimensional hyperboloid with $SO(3,1)$ isometry. This means that for the cases where $X/Y =$ constant, the metric in Equation (1) describes the FLRW universe with $k = -1$. In the coordinates adapted here (for the $X(t)/Y(t) =$ const. case), we have an R^2 slicing of this H^3. Explicitly, the metric on a constant t_0 slice is

$$ds^2 = \frac{X(t_0)^2}{A_0^2 \tilde{z}^2}\left[d\tilde{z}^2 + d\tilde{x}^2 + d\tilde{y}^2\right]$$

where $\tilde{z} = \frac{e^{A_0 z}}{A_0}$, $\tilde{x} = \frac{Y(t_0)}{X(t_0)}x$, and $\tilde{y} = \frac{Y(t_0)}{X(t_0)}y$. The R^2 slicing is particularly suitable for our case, as it singles out one spatial direction (z-direction) along the cosmic flow.

In the (t, z, x, y) coordinates, the perfect fluid stress in the rest frame of the fluid with energy density ρ and pressure p with the flow (also referred to as "tilt" in various) takes the form [13]

$$T_{ab} = \begin{pmatrix} \rho + (\rho + p)\sinh^2\beta & -(\rho + p)X\sinh\beta\cosh\beta & 0 & 0 \\ -(\rho + p)X\sinh\beta\cosh\beta & \Big(p + (\rho + p)\sinh^2\beta\Big)X^2 & 0 & 0 \\ 0 & 0 & p\, e^{-2A_0 z}\, Y^2 & 0 \\ 0 & 0 & 0 & p\, e^{-2A_0 z}\, Y^2 \end{pmatrix} \tag{2}$$

or equivalently,

$$T^a{}_b = \mathrm{diag}(-\rho, p, p, p) + (\rho + p)\sinh\beta \begin{pmatrix} -\sinh\beta & X\cosh\beta & 0 & 0 \\ -X^{-1}\cosh\beta & \sinh\beta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

Though the fluid is perfect, this stress tensor has "imperfect" terms manifested by the off-diagonal elements in $T^a{}_b$. The parameter for the flow is $\beta(t)$. The independent functions of our system are $X(t)$, $Y(t)$, $\rho(t)$, $p(t)$ & $\beta(t)$. We have two extra functions of t than the FLRW setup, i.e., an extra scale factor that breaks the isotropy from $SO(3)$ to $U(1)$ and the flow velocity of the fluid $\beta(t)$.

For the specific cases with $p = -\rho$, the off-diagonal terms drop out along with the tilt parameter $\beta(t)$ from the stress tensor. This gives us the stress tensor for a perfect fluid, implying that the cosmological constant Λ is compatible with any flow or tilt. Instead of setting the EOS parameter $w = p/\rho$ to negative unity, we choose a generic function of t. Of all cases, functions with late time limits $w(t) = -1$ are of particular interest. We will emphasize the results where the flow velocities do not die down even when the Universe is accelerating.

Field Equations

The equations of motion from the metric and stress tensors mentioned in the previous sections are ordinary differential equations of t for FLRW. The number of equations has enhanced because of the two new functions of this system. Einstein's equations with explicit cosmological constants (along with the tilted–perfect fluid) are given by

$$G_{ab} + \Lambda\, g_{ab} = T_a \tag{4}$$

The two second-order equations are

$$\frac{\ddot{X}}{X} + 2\frac{\dot{X}}{X}\frac{\dot{Y}}{Y} - 2\frac{A_0^2}{X^2} = \frac{1}{2}(\rho - p) + (\rho + p)\sinh^2\beta + \Lambda \tag{5a}$$

$$\frac{\ddot{Y}}{Y} + \Big(\frac{\dot{Y}}{Y}\Big)^2 + \frac{\dot{X}}{X}\frac{\dot{Y}}{Y} - 2\frac{A_0^2}{X^2} = \frac{1}{2}(\rho - p) + \Lambda \tag{5b}$$

and the two first-order equations are

$$\frac{2A_0}{X}\Big(\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}\Big) = (\rho + p)\sinh\beta\cosh\beta \tag{6a}$$

$$2\frac{\dot{X}}{X}\frac{\dot{Y}}{Y} + \Big(\frac{\dot{Y}}{Y}\Big)^2 - \frac{3A_0^2}{X^2} = \rho + (\rho + p)\sinh^2\beta + \Lambda. \tag{6b}$$

These equations are generalizations for the Friedmann equations. Friedmann equations contain two equations—one second-order equation and one first integral. Hence, to incorporate two extra functions, two additional equations are necessary. The shear of the system is defined by

$$\sigma(t) = \Big(\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}\Big) \tag{7}$$

As in the case involving the usual Friedmann equations, here, it is also possible to replace the second-order equations with the conservation law for the stress tensor. The independent equations obtained from the covariant conservation of the stress tensor are:

$$\dot{\rho} + (\rho + p)\Big(\frac{d}{dt}\log(XY^2\cosh\beta) - \frac{2A_0}{X}\tanh\beta\Big) = 0 \tag{8a}$$

$$\dot{p} + (\rho + p)\frac{d}{dt}\log(X\sinh\beta) = 0 \tag{8b}$$

Similar to the case of FLRW, here, we can check that the conservation equations can be reproduced by taking one more derivative of the first-integral Equations (6a) and (6b), eliminating the double derivatives using the second-order Equations (5a) and (5b). Some features of our system are the following

- As manifested by Equation (6a), setting $\beta = 0$ either fixes A_0 to zero or sets X/Y to a constant ($\sigma = 0$). $A_0 = 0$ is a Bianchi-I model. Moreover, Bianchi-I universes cannot incorporate the flow. For zero-shear cases, the equations are reduced to FLRW equations for the open Universe, as we mentioned earlier.
- Here, the number of quantities is one more than the number of equations. The problem, such as FLRW, is solved by introducing the EOS $p(t) = w(t)\rho(t)$. The function $w(t)$ is either a constant or a function of our choice. Regarding the fixed EOS, the particular case is when $\rho + p = 0$, as discussed before. The tilt remains unspecified as β drops out from the equations.

3. Results

We will separate our results into two categories: First, we will look into the evolution where the EOS is constant. Secondly, we will choose a function $w(t)$, i.e., an *effective equation of state* of the tilted fluid. We solve our equations by defining the initial conditions at $t = 0.01$ Gyr. We define the variables at the initial times as X_{in}, Y_{in}, ρ_{in} & β_{in}. For phenomenological reasons, we take $X_{in} = Y_{in}$. In Section 3.3, the different curves correspond to different values of X_{in} & Y_{in}. Together, we encode this information as the effective initial scale factor $a_{in} = (X_{in}Y_{in}^2)^{1/3}$. The dynamical equations are FLRW-like; it will be interesting to see if the features evolve with time in a similar way.

3.1. Models with Constant EOSs

We take into account the constant cases for which $-1 < w \leq 1$. In these evolutions, the cosmological constant Λ is zero. The two cases $w = 0$ and $w = 1/3$ are of special interest as they describe pressureless matter and radiation, respectively. For $w = 0$ case, Equations (8a) and (8b) imply

$$X\sinh\beta = const., \qquad \dot{\beta}\coth\beta = -H - \frac{2}{3}\sigma \tag{9}$$

This shows that the cosmic tilt (β) monotonically decreases with time for pressureless matter. By simple manipulation, one can also find the shear as $\sigma = C\rho\cosh\beta$. Both the tilt and the shear become very small for this particular cosmology. At very late times, this model becomes an FLRW universe for dust. These features are evident in Figure 1.

$w = 1/3$, **dipole radiation model:** as the next example, we consider radiation $p = \rho/3$. For this case, Equations (8a) and (8b) imply

$$\dot{\beta}(3\coth\beta - \tanh\beta) = -\frac{2A_0\tanh\beta}{X} - 2\sigma, \qquad \rho X^4\sinh^4\beta = const. \tag{10}$$

The RHS of the first of the two equations is always negative, whereas the coefficient attached to the derivative of $\beta(t)$ is always positive. Therefore, $\dot{\beta} < 0$, and the tilt also decreases here. If we start from a significant value of β at initial times, the β continues to have a non-null value when the shear decreases to zero and the scale factor becomes large. Then only the system has a constant tilt even at late times. The numerically evolved system is plotted in Figure 1.

Generic constant w case. When $p = w\rho$, Equations (8a) and (8b) imply,

$$\dot{\beta}\big(\coth\beta - w\tanh\beta\big) = (3w-1)H - \frac{2}{3}\sigma - \frac{2wA_0\tanh\beta}{X}, \tag{11a}$$

$$\rho^{\frac{w}{1+w}} X\sinh\beta = C = const. \tag{11b}$$

Since the EOS parameter w lies within positive and negative unity, the quantity attached to $\dot{\beta}$ in Equation (11a) is always positive. The term with $\tanh\beta/X$ in the RHS becomes positive or negative depending on the sign of the w. However, this term and the shear vanish at late times. The first term in RHS is the only term that decides the feature of the evolution of tilt. For $w > 1/3$, tilt growth is possible depending on the other details of the evolution. The cosmic acceleration for this case is given by

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{\rho}{6}(1+3w) - \frac{2}{9}\sigma^2 - \frac{\rho}{3}(1+w)\sinh^2\beta, \tag{12}$$

where $a = (XY^2)^{1/3}$, the effective scale factor. The first term in the RHS is the usual FLRW term. We obtain accelerating Universe solutions for $w < -1/3$. The rest of the terms are negative-definite. The numerical analysis is presented in Figure 1. FLRW features dominate the dynamics of the late times. For the constant EOS, there is no case where the tilt growth and the acceleration overlap. Our results agree with the results presented in [14].

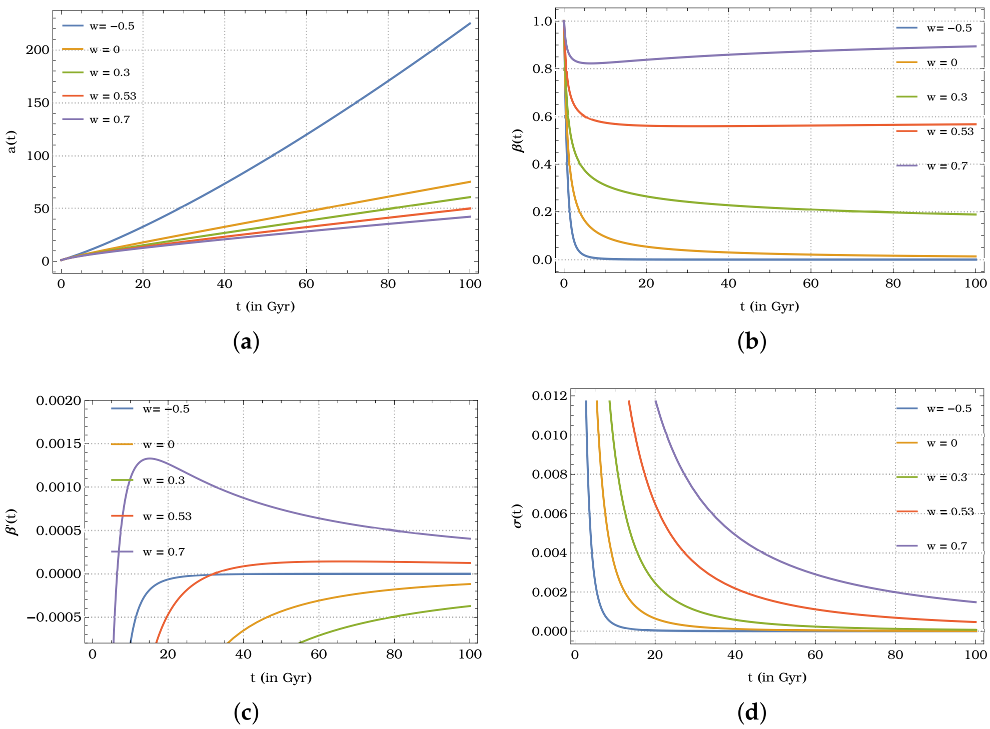
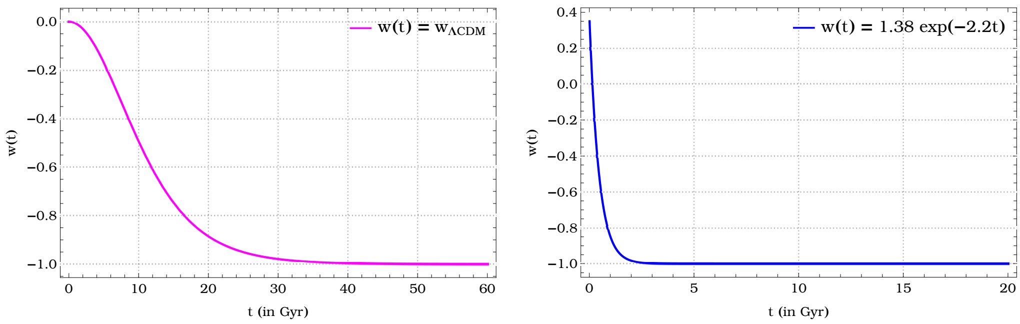


Figure 1. Evolution of the overall scale factor, tilt, the derivative of the tilt, and shear for a constant equation of the state within $-0.5 \leq w \leq 0.7$ for initial values $X_{in} = Y_{in} = 1.2$, $\rho_{in} = 0.6$, $\beta_{in} = 1$. We set our initial conditions at $t = 0.01$ Gyr. Plot (**a**): Scale factor evolution; Plot (**b**): Tilt evolution; Plot (**c**): Tilt derivative evolution; Plot (**d**): Shear Evolution.

3.2. Defining Models via $w(t)$

$w =$ constant cases are not realistic cases. Our Universe does not only consist of pressureless matter and radiation. Moreover, here, we do not consider the mixtures of different fluids with different tilts. Thus, a more phenomenologically interesting case would be to examine a Universe where all fluids move with an effective EOS $w(t)$ and a single tilt $\beta(t)$. We can think of this as an alternative to mixtures. One advantage of this effective $w(t)$ is that it fits directly into our system without further examination. In Figure 2a, we illustrate that the usual FLRW assumption of a multi-component fluid, each with a constant EOS, can be viewed as a specific choice of the total EOS. The function itself is the definition of our model. We present our analysis for the generic function in Figure 2b to explain how robust our results are.



(**a**) (**b**)

Figure 2. Time-varying effective EOS that we evolved our ansatz for. Both functions exhibit the evolution characteristics that we aimed for. By $w(t) = w_{\Lambda CDM}$, we mean (16). The detailed perturbative analysis is presented in [15]. The $w(t)$s are chosen such that the models are somewhat realistic (assuming that late-time cosmology is dominated by a matter with $p = -\rho$.) Plot (a): For w(t) = w$_\Lambda$cdm; plot (b): For a different EoS

Let us clarify this effective equation of the state in the context of the FLRW universe. We consider the fluid to be composed of "radiation", "pressureless matter", and the "cosmological constants" (perfect fluids with constant EOSs w_i). They all have constant EOSs. Thus, one may describe this model in terms of the effective EOS $w_{eff}(t)$ in the following manner:

$$w_{\text{eff}}(t) := \frac{\sum_i p_i}{\sum_i \rho_i} = \sum_i w_i \Omega_i, \qquad \Omega_i = \frac{\rho_i}{\sum_i \rho_i}, \quad \sum_i \Omega_i = 1. \tag{13}$$

For systems under our consideration, we satisfy the dominant energy condition $-1 \leq w_i \leq 1$, with respect to $0 \leq \Omega_i \leq 1$. Thus, at different cosmological epochs, the i^{th} component will dominate when $w_{eff} \simeq w_i$. In this setting, "radiation dominated", "matter dominated", and "dark energy dominated" epochs are presented in this way. Hereafter, we drop the subscript "eff" for brevity and write $w(t)$.

Late-time flat ΛCDM. Let us make things completely explicit and write down the form of $w(t)$ for flat ΛCDM after the radiation decouples. We will later use this specific form of $w(t)$ in one of our dipole cosmology toy examples (the dipole "ΛCDM" model), i.e., we will use it as a crude model to track quasi-realistic phenomenology in the dipole setting.

The Friedmann equation for this setting becomes

$$H(t) = \frac{\dot{a}}{a} = H_0 \sqrt{\frac{\Omega_{m0}}{a(t)^3} + \Omega_\Lambda}, \qquad \Omega_\Lambda = 1 - \Omega_{m0}. \tag{14}$$

One can then integrate the above equation and find

$$a(t) = \left(\frac{\Omega_{m0}}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2}\sqrt{\Omega_\Lambda H_0^2}\, t\right), \qquad H(t) = \sqrt{\Omega_\Lambda H_0^2} \coth\left(\frac{3}{2}\sqrt{\Omega_\Lambda H_0^2}\, t\right). \tag{15}$$

The integration constants are chosen, such that at present, epoch t_0, $H(t_0) = H_0$, i.e., $\sqrt{\Omega_\Lambda}\coth\left(\frac{3}{2}\sqrt{\Omega_\Lambda H_0^2}\, t_0\right) = 1$, and $a(t_0) = 1$. For H_0, Ω_{m0} may take the usual Planck value and find the effective EOS to be

$$w(t) = -\tanh^2\left(\frac{3}{2}\sqrt{\Omega_\Lambda H_0^2}\, t\right). \tag{16}$$

At very early times (i.e., when $H_0 t \ll 1$), as expected, $w(t) \to 0$, and we are dealing with the matter dominated Universe. Moreover, very late times, $H_0 \gg 1$ $w \sim -1$, imply that the DE dominates in this epoch. However, this is the effective equation of the state for the post-radiation era.

3.3. *Defining Models via $w(t)$: Dipole Cosmology*

We want to solve Equations (6a), (6b), (8a) and (8b) for the FLRW-like system Equation (16) and examine if the evolution is roughly similar to the Flat ΛCDM systems, i.e., where the "tilt" does not decrease. In Figures 3 and 4, we see that this can most certainly happen if the total EOSs at late times go to -1. Throughout our analysis, we chose our initial conditions on the scale factors as $X_{in} = Y_{in}$. This does not mean that the initial shear is zero, as the equality in the scale factors does not automatically ensure the initial derivatives are the same.

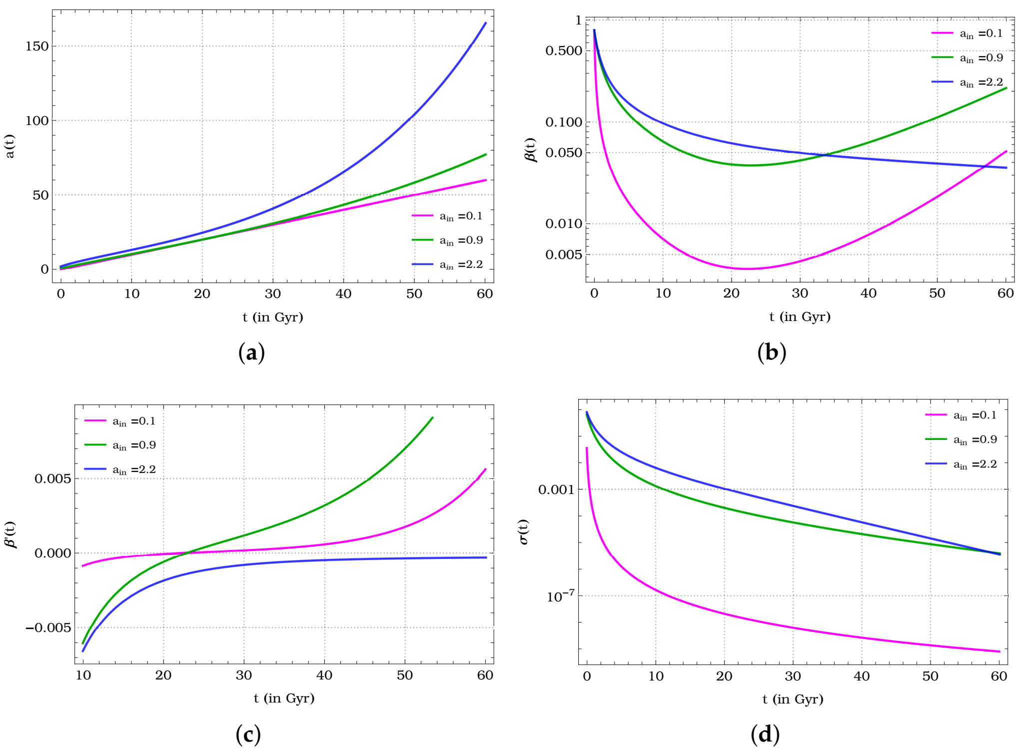


Figure 3. Time evolution of the dipole "ΛCDM" model. This model is defined by $w(t)$ in (Figure 2a). The initial conditions are $\rho_{in} = 0.6, \beta_{in} = 0.793, X_{in} = Y_{in}$, at $t = 0.01$ Gyr. While the shear σ dies off fast, the tilt β remains relatively large and, depending on the initial conditions, can also grow in time. We also check that, at late times, $\sigma \sim a^{-3}$. Plot (**a**): Scale factor evolution; Plot (**b**): Tilt evolution; Plot (**c**): Tilt derivative evolution; Plot (**d**): Shear Evolution.

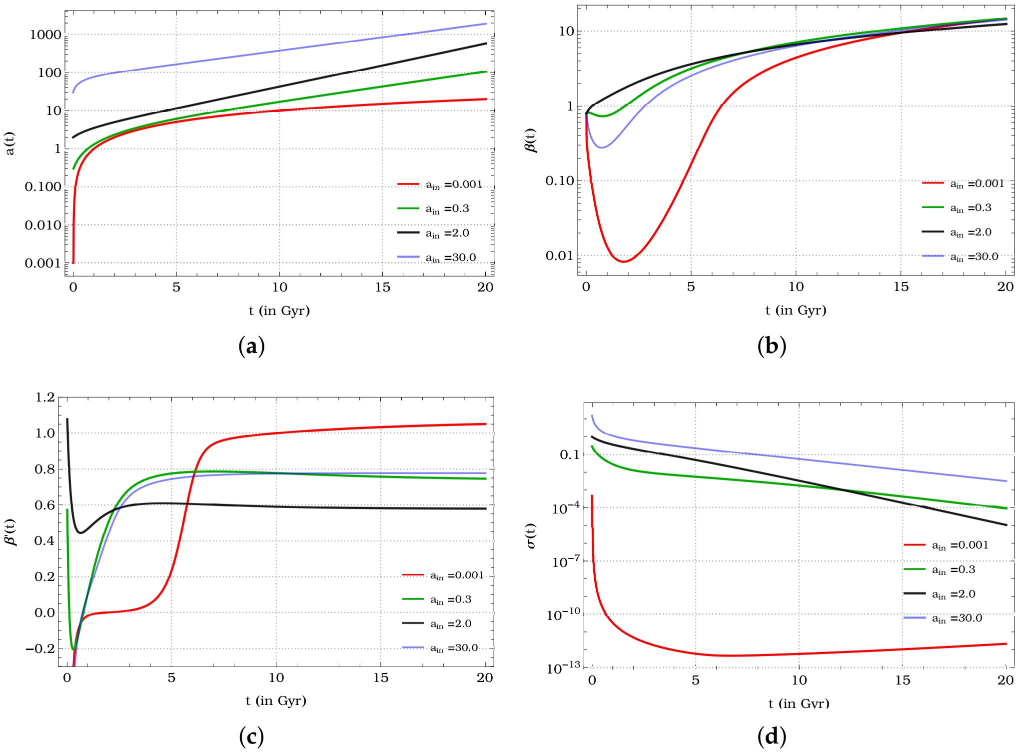


Figure 4. Time evolution of the model defined with $w(t)$ given in (Figure 2b). The initial conditions are the same as in the previous figure. While the anisotropic shear σ is a monotonically decreasing function, the tilt β can be increasing at late times, whereas the Universe is accelerating. We have also checked that, at late times, $\sigma \sim a^{-3}$. Plot (**a**): Scale factor evolution; Plot (**b**): Tilt evolution; Plot (**c**): Tilt derivative evolution; Plot (**d**): Shear Evolution.

3.4. Flows Can Grow Even in Accelerating Models

Apart from the dipole "ΛCDM" model, we consider an exponentially decreasing $w(t)$ that tends to -1. Figure 4 shows the system's evolution results. These observations are powerful but numerical. For a detailed analytical analysis, see [15]. We close this section with the following conclusions: We saw tilt growth in multiple accelerating dipole cosmologies. The evolutions did not contradict Wald's cosmic no-hair theorem. The main statement of the theorem focuses on the dying shear in the presence of a cosmological constant. It does not have any remark on the evolution of the dipole flow.

Λ-w models: Generalizing the above Λ-$w = 0$ expression to a system with the cosmological constant Λ plus a fluid of constant EOS w ($w \neq -1$):

$$H(t) = H_0\sqrt{\Omega_\Lambda + \frac{\Omega_w}{a(t)^{3(1+w)}}}, \qquad \Omega_\Lambda = 1 - \Omega_w, \tag{17a}$$

$$a(t) = \left(\frac{\Omega_w}{\Omega_\Lambda}\sinh^2(Kt)\right)^{\frac{1}{3(1+w)}}, \qquad H(t) = \sqrt{\Omega_\Lambda H_0^2}\coth(Kt) \tag{17b}$$

$$w(t) = -\tanh^2(Kt) + \frac{w}{\cosh^2(Kt)}, \qquad K = \frac{3}{2}(1+w)H_0\sqrt{\Omega_\Lambda}. \tag{17c}$$

Note that, in the far future, $Kt \gg 1$, $w(t) \simeq -1 + 4(1+w)e^{-2Kt}$, and $H(t) \simeq \sqrt{\Omega_\Lambda H_0^2}$. The fact that the rate at which the total EOS falls increases with the stiffness w will play a role in our discussions in a more general context of dipole cosmology.

3.5. One More Example with Late-Time Acceleration

There are other effective EOSs in which we can retrieve such cosmologies. Here, we present one such example that the evolution curves shown in Figure 3 are not confined to a specific model. In contrast, the features are generic. The perturbative analysis in support of this statement is well discussed in [15].

4. Concluding Remarks and Outlook

Therefore, our goal was to provide the most straightforward theoretical framework for the cosmological model building that accommodates such a cosmic dipole. To this end, we first phrased a "dipole cosmological principle", which led us directly to a specific class of tilted Bianchi cosmologies (see [3,16,17] for context). We worked out the field equations for dipole cosmologies and studied these equations analytically and numerically for some representative cosmic fluid classes.

We should also mention the various caveats in our results here. The symmetries that we assumed forced us to have only one independent component in the dipole flow. In other words, except for the possibility of a cosmological constant, all fluids in a dipole cosmology have to flow at the same velocity. This seems artificial since the fluids familiar with flat Λ CDM (ordinary matter, dark matter, and radiation) have decoupled from each other. We evaded this problem by considering the time-dependent total EOSs instead of component fluids with constant EOSs. This is useful for general lessons, but it will be necessary to go beyond this if we want to conduct detailed phenomenology. This may force us to drop the assumption of homogeneity, and we will have to deal with partial differential equations (PDE) instead of ODEs. Perhaps the perturbation theory around an FLRW background will be helpful at late times; it may be useful to track the non-linear evolutions of certain specific modes. Another observation that was forced on us by symmetry is that we worked with a universe whose natural isotropic limit was a $k = -1$ FLRW universe. It may be helpful to consider systems where the $k = 0$ limit is accessible while the dipole flow is non-vanishing. This can be done while retaining the homogeneity assumption; we will have more to say about issues of this kind in another paper [13].

It is desirable to construct more realistic dipole cosmology models. Such constructions may allow for a tunable non-kinematical CMB dipole (say, in the pre-recombination era) [18]. They may also help realize bulk flows that can yield realistic dipole anisotropy in various sources [19–22] or our local cluster [23].

Despite this, these ideas are worthy of exploration for the reasons we detailed throughout the paper. Of particular interest is the fact that direct evidence for the acceleration of the Universe comes from late-time observations, while there is a glaring $\sim 5\sigma$ observational difficulty in late-time cosmology in the form of H_0 and other tensions. Along with the challenges in constructing an accelerating Universe in UV-complete settings, such as string theory [24–27], it is evident that—whatever way the chips may fall—regarding late-time cosmology, and given the current status of cosmic tensions, we need a paradigm shift in cosmology. The β-growth we noticed in this paper is a possible loophole that can move us away from asymptotic homogeneity and isotropy in an accelerating Universe. Perhaps it is time to replace the cosmological principle with the dipole cosmological principle or something even weaker.

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