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Cosine Similarity Measures of (m, n)-Rung Orthopair Fuzzy Sets and Their Applications in Plant Leaf Disease Classification

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Abstract: A fuzzy set is a powerful tool to handle uncertainty and ambiguity, and generally, the notions of symmetry and similarity are also exhibited in the fuzzy set theory. The class of (m, n)-rung orthopair fuzzy sets through two universes are more flexible and efficient than the q-rung orthopair fuzzy sets when discussing the symmetry and similarity between multiple objects. This research article comprehensively investigates ten similarity measures that employ cosine and cotangent functions for comparing (m, n)-rung orthopair fuzzy sets, which are a superclass of q-rung orthopair fuzzy sets. Moreover, the proposed weighted similarity measures are applied to real-world problems in building material analysis. A comparative analysis is conducted between the proposed measures and the existing cosine and cotangent measures of q-rung orthopair fuzzy sets, showing that the proposed measures are more efficient than existing ones. Additionally, a numerical example demonstrates the practical and scientific applications of these similarity measures in classifying plant leaf diseases. The sensitivity analysis shows that the existing measures cannot be applied to (m, n)-fuzzy data for distinct values of m and n. The results are supported by graphical interpretations, further illustrating the efficacy of the proposed measures.

Keywords: (m, n)-rung orthopair fuzzy sets; cosine similarity measure; pattern recognition; plant leaf disease

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1. Introduction

The concept of symmetry holds tremendous significance in science and engineering and is widely observed in nature, fine arts, and various human creative pursuits. Its foundations are rooted in mathematics, while its artistic expression and communication medium can be traced back to early human endeavors. Symmetry can be defined as an object's property that identifies two or more parts as identical concerning a point, line, or plane. In geometry, symmetry is formally defined as the invariance of a configuration of elements under a group of automorphic transformations [1–3]. Recent research has emphasized the versatility of symmetry as a tool for establishing connections across a wide range of disciplines, encompassing mathematics, physics, chemistry, biology, archaeology, geology, and pattern recognition. By integrating the deep theoretical foundations of symmetry and similarity with its practical applications in various domains, this research aims to advance our understanding and utilization of this fundamental concept. This interdisciplinary exploration will contribute to the comprehension of symmetry and its implications for diverse fields, opening the path to novel discoveries and practical advancements.

The fusion of technology and generalized forms of classical sets is instrumental in solving many real-world complex problems, which involve incomplete and uncertain information. A classical set is defined by its characteristic function from a universe of discourse to two point sets {0, 1}. The classical set theory falls short when dealing with intricate issues that encompass vague and uncertain information. To handle the vagueness and impreciseness in complex problems, fuzzy sets (FSs) were created by Zadeh [4] as a generalization of classical sets. The application of the Fuzzy set theory extends to multiple fields, including control theory, artificial intelligence, pattern recognition, database system, and medical diagnosis. In 1986, Atanassov [5] created intuitionistic fuzzy sets (IFSs), a superclass of FSs. After the occurrence of the Atanassov [5] paper, several generalizations of IFSs have appeared in the literature. In 2013, Yager [6] presented a superclass of IFSs called Pythagorean fuzzy sets (PFSs). PFSs are more extensive than the IFSs and can describe more imprecise and vague information in the decision-making process. In 2017, Yager introduced the q-rung ortho-pair fuzzy sets (q-ROFSs) [7]. This innovative approach offers a highly effective and powerful tool to manage imprecise and uncertain information across various real-world applications and problems. In 2019, Senapati and Yager [8] developed and introduced the concept of Fermatean fuzzy sets (FFSs), a specific instance of q-ROFSs when q = 3. Recently, Ibrahim and Alshamari [9] initiated the study of (m, n)-rung orthopair fuzzy sets ((m, n)-ROFSs) as a superclass of q-ROFSs. They discuss the applications of these fuzzy sets in the context of multi-criteria decision-making methods. This concept is also independently investigated by Jun and hur [10] and AI-Shami [11] with the name of (m, n)-fuzzy sets. The (m, n)-ROFSs are more flexible and effective as compared to q-ROFSs in handling the uncertainty and vagueness in MADM and MCDM.

Symmetry plays a fundamental role in shape analysis and object recognition, as it is considered a pre-attentive feature that enhances object recognition and reconstruction. After the invention of fuzzy sets, several authors have proposed methods and algorithms based on fuzzy measures of symmetry and similarity for pattern recognition and classification of fuzzy objects. Helgason and Jobe [12] investigated fuzzy measures of symmetry breaking, similarity, and comparison within the context of non-statistical information pertaining to a single patient. This research may be relevant to healthcare and medical decision-making. Zainuddin and Pauline [13] presented an effective fuzzy C-means algorithm based on a symmetry–similarity approach. The algorithm aims to improve the performance of fuzzy C-means clustering in handling symmetric patterns in data. Miranda and Grabisch [14] created p-symmetric fuzzy measures and explored the properties and applications of these measures in uncertainty, fuzziness, and knowledge-based systems. Saha and Bandyopadhyay [15] presented a new point symmetry-based fuzzy genetic clustering technique for automatically evolving clusters. This technique combines symmetry principles and genetic algorithms to improve the clustering process. Colliota and Tuzikovb [16] studied approximate reflectional symmetries of fuzzy objects and their application in model-based object recognition. This research focused on developing fuzzy models to recognize and analyze objects based on their approximate reflectional symmetries and similarities.

The similarity measure is a crucial metric that can evaluate the degree of similarity between two objects, making it an essential tool for distinguishing diverse patterns in practical applications. Adlassnig [17], Zwick et al. [18], Pappis and Karacapilidis [19], Chen et al. [20], Zeng and Li [21], Mitchel [22], and others, have extensively studied the similarity measures between fuzzy sets. Their research explored the potential of fuzzy sets to facilitate the development of corresponding applications in areas such as image processing, medical diagnosis, pattern recognition, and decision-making. Since the emergence of interval type-2 fuzzy sets (IFSs), several similarity measures between IFSs have appeared in this literature [23–36]. Some researchers have investigated and studied these similarity measures between IFSs based on cosine functions, including Ye [37–39], Shi and Ye [40], Zhau et al. [41], and Liu et al. [42,43]. Tian [44] and Rajarajeswari and Uma [45] proposed similarity measures between IFSs based on cotangent functions and demonstrated their applications in medical diagnoses. Recently, Garg et al. [46] proposed Choquet integral-based cosine similarity measures for interval-valued IFSs and presented their applications in pattern recognition.

PFS is a powerful tool for depicting vagueness and impreciseness in MADM and MCDM. Recently, many researchers, such as Garg [47], Zeng, Li, and Yin [48], Peng, Yuan, and Yang [49], Husain, and Yang [50], Wei and Wei [51], Ejeiwa et al. [52–54], and others, presented different similarity measures between PFSs for solving MADM problems. Recently, different similarity measures between FFSs and their applications in MADM and MCDM have appeared in the literature [55–60]. q-ROFSs are powerful mathematical tools for handling uncertain, imprecise, and vague information in real-world problems, surpassing PFS and FFS regarding capability. In 2019, Peng and Dai [61] introduced a similarity measure between q-ROFSs that assessed the quality of classroom teaching. Jan et al. [62] considered the generalized dice similarity between ROFSs. Farhadinia et al. explored a range of similarity measures for q-ROFSs. Peng and Liu [63] investigated information measures for q-ROFSs. Liu, Chen, and Peng [64], as well as Wang et al. [65], introduced several similarity measures between q-ROFSs based on cosine and cotangent functions and explored their properties and applications.

The existing generalizations of PFSs, FSSs, and q-ROFSs of IFSs exhibit symmetry between the powers of MD and NMD of an attribute within the universe of discourse. In decision-making, it is not flexible in prioritizing different powers of the MD or NMD of an attribute in these extensions of IFSs. The motivations for writing this research are to consider a new class of IFSs, called (m, n)-round orthopair fuzzy sets((m, n)-ROFSs) for creating cosine and cotangent similarity measures, which helps to expand the MD and NMD more than all types of q-ROFSs. Classes with distinct powers enable us to evaluate the input data with different levels of significance for MD and NMD, which is appropriate in MADM problems. This matter does not apply to the other generalizations of IFSs because they give equal significance to MD and NMD viz 1 in IFSs, 2 in PFSs, 3 in FFSs, and q in q-ROFSs. In (m, n)-orthopair fuzzy sets, different power function scales are utilized to widen the scope of the decision-making problems.(m, n)-rung orthopair fuzzy sets can be applied to more diverse scenarios than FFSs, PFSs, and IFSs sets, due to their wider range in depicting membership grades. The main advantage of(m, n)-rung orthopair fuzzy sets is that they can describe more uncertainties than q-ROFSs, which can be applied to many decision-making problems. The (m, n)-ROFSs through double universes are more flexible and efficient than m-ROFS and n-ROFS when discussing the similarity between multiple objects. In other words, (m, n)-ROFSs can more effectively address MADM problems, including all q-ROFS decision-making problems as a special case.

The structure of this article is as follows. Section two presents a review of generalized fuzzy structures along with their cosine and cotangent similarity measures. Section three establishes similarity and weighted similarity measures between (m, n)-ROFSs based on cosine and cotangent functions. Section four compares the newly established similarity measures for (m, n)-ROFSs with existing q-ROFSs, PFSs, and IFSs based on cosine and cotangent functions. The comparison is made by considering pattern recognition, medical diagnosis, and building material problems discussed in the literature. In section five, the established similarity measures are utilized to classify plant leaf disease, and the effectiveness and reasonableness of the proposed measures are demonstrated. Finally, section six concludes the article with some closing remarks.

All the abbreviations and their description used in the paper are presented in Table 1.

Table 1. Abbreviations and their description.

Abbreviation	Description
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PFS	Pythagorean fuzzy set
FFS	Fermatean fuzzy set
q-ROFS	q-rung orthopair fuzzy set
(m, n)-ROFS	(m, n)-rung orthopair fuzzy set
$(m,n)-ROFS(\mathbb{P})$	The family of all (m, n)-ROFSs defined over $\mathbb{P}$
MD	Membership degree
NMD	Non-membership degree
IMD	Indeterminacy membership degree
$\vartheta_{\mathbb{E}}(p)$	MD of p to $\mathbb{E}$
$\varsigma_{\mathbb{E}}(p)$	NMD of p to $\mathbb{E}$
$\pi_{\mathbb{E}}(p)$	IMD of p to $\mathbb{E}$

2. Preliminaries

In this section, we will examine different types of generalized fuzzy structures and the cosine and cotangent similarity measures currently used for these structures. In the remainder of the paper, we will assume that $\mathbb{P}$ is a finite, discrete, and non-empty discourse set consisting of r elements, denoted as $p_1, p_2, \ldots, p_r$.

2.1. Generalized Fuzzy Structures

Definition 1. *Let us consider that $\mathbb{P}$ is a fixed set. A structure $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$, where $\vartheta_{\mathbb{E}} : \mathbb{P} \to [0,1]$ and $\varsigma_{\mathbb{E}} : \mathbb{P} \to [0,1]$ denotes the membership and non-membership functions of $\mathbb{E}$, called:*

(a) *Intuitionistic fuzzy set [5] in $\mathbb{P}$ if $0 \le \vartheta_{\mathbb{E}}(p) + \varsigma_{\mathbb{E}}(p) \le 1, \ \forall \ p \in \mathbb{P}$.*
(b) *Pythagorean fuzzy set [6] in $\mathbb{P}$ if $0 \le (\vartheta_{\mathbb{E}}(p))^2 + (\varsigma_{\mathbb{E}}(p))^2 \le 1, \ \forall \ p \in \mathbb{P}$.*
(c) *Fermatean fuzzy set [8] in $\mathbb{P}$ if $0 \le (\vartheta_{\mathbb{E}}(p))^3 + (\varsigma_{\mathbb{E}}(p))^3 \le 1, \ \forall \ p \in \mathbb{P}$.*
(d) *q-rung orthopair fuzzy set [7] in $\mathbb{P}$ if $0 \le (\vartheta_{\mathbb{E}}(p))^q + (\varsigma_{\mathbb{E}}(p))^q \le 1, \ \forall \ p \in \mathbb{P}$ and $q \in \mathrm{N}$.*

Definition 2 ([9–11])**.** *A(m, n)-rung orthopair fuzzy set $\mathbb{E}$ in a universe of discourse $\mathbb{P}$ is a structure defined as follows:*

$$\mathbb{E} = \{\langle p, (\vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p))\rangle : p \in \mathbb{P}\}$$

where $\vartheta_{\mathbb{E}} : \mathbb{P} \to [0,1]$ and $\varsigma_{\mathbb{E}} : \mathbb{P} \to [0,1]$ denote the membership and non-membership functions of $\mathbb{E}$, which satisfies the following condition:

$$0 \le (\vartheta_{\mathbb{E}}(p))^m + (\varsigma_{\mathbb{E}}(p))^n \le 1$$

$\forall$ *p*$\in \mathbb{P}$ *and m, n* $\in$ N.

Remark 1 ([9–11])**.** *A (m, n)-ROFS $\mathbb{E}$ in $\mathbb{P}$ coincides with IFS (resp., PFS, FFS, q-ROFS) if m = n = 1 (resp., m = n = 2, m = n = 3, m = n = q).*

Proposition 1 ([11])**.** *For any universe of discourse $\mathbb{P}$:*

(a) *Every IFS is an (m, n)-ROFS.*
(b) *If m ≥ 2 and n ≥ 2 then a PFS is an (m, n)-ROFS.*
(c) *If m ≥ 3 and n ≥ 3 then a FFS is an (m, n)-ROFS.*
(d) *If m $\ge$ q and n $\ge$ q then a q-ROFS is an (m, n)-ROFS.*

Remark 2. *The converse of the relationships presented in Proposition 1 is false.*

Definition 3 ([9–11])**.** *For any (m, n)-ROFSs* $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$, $\mathbb{E}_1 = \{\langle p, \vartheta_{\mathbb{E}_1}(p), \varsigma_{\mathbb{E}_1}(p)\rangle : p \in \mathbb{P}\}$, *and* $\mathbb{E}_2 = \{\langle p, \vartheta_{\mathbb{E}_2}(p), \varsigma_{\mathbb{E}_2}(p)\rangle : p \in \mathbb{P}\}$ *in* $\mathbb{P}$.

The subset, equality, union, intersection, and complement operations over $(m, n) - ROFS(\mathbb{P})$ *are defined as follows:*

(a) $\mathbb{E}_1 \Subset \mathbb{E}_2 \Leftrightarrow \vartheta_{\mathbb{E}_1}(p) \leq \vartheta_{\mathbb{E}_2}(p)$ *and* $\varsigma_{\mathbb{E}_1}(p) \geq \varsigma_{\mathbb{E}_2}(p)\ \forall\ p \in \mathbb{P}$.
(b) $\mathbb{E}_1 = \mathbb{E}_2 \Leftrightarrow \vartheta_{\mathbb{E}_1} = \vartheta_{\mathbb{E}_2}$ *and* $\varsigma_{\mathbb{E}_1} = \varsigma_{\mathbb{E}_2}$.
(c) $\mathbb{E}_1 \Cup \mathbb{E}_2 = \left\{\begin{matrix} \langle p, \vartheta_{E_1}(p) \bigvee \vartheta_{\mathbb{E}_2}(p), \\ \varsigma_{\mathbb{E}_1}(p) \bigwedge \varsigma_{\mathbb{E}_2}(p)\rangle : p \in \mathbb{P} \end{matrix}\right\}$.
(d) $\mathbb{E}_1 \Cap \mathbb{E}_2 = \left\{\begin{matrix} \langle p, \vartheta_{\mathbb{E}_1}(p) \bigwedge \vartheta_{\mathbb{E}_2}(p), \\ \varsigma_{\mathbb{E}_1}(p) \bigvee \varsigma_{\mathbb{E}_2}(p)\rangle : p \in \mathbb{P} \end{matrix}\right\}$.
(e) $\mathbb{E}^c = \{\langle p, (\varsigma_{\mathbb{E}}(p))^{\frac{n}{m}}, (\vartheta_{\mathbb{E}}(p))^{\frac{m}{n}} : p \in \mathbb{P}\}$.

Proposition 2 ([9–11])**.** *Let* $\mathbb{E}_1 = \{\langle p, \vartheta_{\mathbb{E}_1}(p), \varsigma_{\mathbb{E}_1}(p)\rangle : p \in \mathbb{P}\}$ *and* $\mathbb{E}_2 = \{\langle p, \vartheta_{\mathbb{E}_2}(p), \varsigma_{\mathbb{E}_2}(p)\rangle : p \in \mathbb{P}\}$ *be two (m, n)-ROFS on* $\mathbb{P}$. *Then,*

(a) $\mathbb{E}_1 \Cup \mathbb{E}_2 = \mathbb{E}_2 \Cup \mathbb{E}_1$.
(b) $\mathbb{E}_1 \Cap \mathbb{E}_2 = \mathbb{E}_2 \Cap \mathbb{E}_1$.
(c) $(\mathbb{E}_1^c)^c = \mathbb{E}_1$.
(d) $(\mathbb{E}_1 \Cup \mathbb{E}_2)^c = \mathbb{E}_1^c \Cap \mathbb{E}_2^c$.
(e) $(P_1 \Cap \mathbb{E}_2)^c = \mathbb{E}_1^c \Cup \mathbb{E}_2^c$.

Definition 4 ([5–8])**.** *Let* $\mathbb{P}$ *be a universe of discourse. Then, for any*

(a) *IFS* $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$ *in* $\mathbb{P}$, *the expression*

$$\pi_{\mathbb{E}}(p) = (1 - \vartheta_{\mathbb{E}}(p) + \varsigma_{\mathbb{E}}(p))$$

is called the IMD of $p \in \mathbb{P}$.

(b) *PFS* $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$, *in* $\mathbb{P}$, *the expression*

$$\pi_{\mathbb{E}}(p) = \sqrt{(1 - \vartheta_{\mathbb{E}}(p))^2 + (\varsigma_{\mathbb{E}}(p))^2}$$

is called the IMD of $p \in \mathbb{P}$.

(c) *FFS* $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$, *in* $\mathbb{P}$, *the expression*

$$\pi_{\mathbb{E}}(p) = \sqrt[3]{(1 - \vartheta_{\mathbb{E}}(p))^3 + (\varsigma_{\mathbb{E}}(p))^3}$$

is called the IMD of $p \in \mathbb{P}$.

(d) *q-ROFS* $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$ *in* $\mathbb{P}$, *the expression*

$$\pi_{\mathbb{E}}(p) = \sqrt[q]{(1 - \vartheta_{\mathbb{E}}(p))^q + (\varsigma_{\mathbb{E}}(p))^q}$$

is called the IMD of $p \in \mathbb{P}$.

Definition 5. *For any (m, n)-ROFS* $\mathbb{E} = \{\langle p, \vartheta_{\mathbb{E}}(p), \varsigma_{\mathbb{E}}(p)\rangle : p \in \mathbb{P}\}$ *over* $\mathbb{P}$, *the expression*

$$\pi_{\mathbb{E}}(p) = \sqrt[\frac{m+n}{2}]{(1 - \vartheta_{\mathbb{E}}(p))^m + (\varsigma_{\mathbb{E}}(p))^n}$$

is called the IMD of $p \in \mathbb{P}$.

Remark 3. *The IMD* $\pi_{\mathbb{E}}(p)$ *of* $p \in \mathbb{P}$ *to IFS (resp., PFS, FFS, q-ROFS)* $\mathbb{E}$ *is a special case of IMD* $\pi_{\mathbb{E}}(p)$ *of p to (m, n)-ROFS* $\mathbb{E}$ *for m = n = 1 (resp., m = n = 2 m = n = 3, m = n = q).*

Remark 4. *Clearly, for each (m, n)-ROFS $\mathbb{E}$ in $\mathbb{P}$,*

$$\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p) + (\vartheta_{\mathbb{E}}(p))^m + (\varsigma_{\mathbb{E}}(p))^n = 1,$$

$\forall\ p \in \mathbb{P}$.

2.2. Cosine and Cotangent Similarity Measures for Generalized Fuzzy Structures

The cosine and cotangent similarity measures have been applied in numerous MADM and MCDM methods to calculate the degree of proximity between any two objects. The study of the cosine measure between two IFSs $\mathbb{E}$ and $\mathbb{F}$ was initiated by Ye [37] in 2011. In 2013, Shi and Ye [40] created new cosine similarity measures for IFSs, which extend the measures proposed by Ye [37] and are applied to the fault diagnosis of turbines. In 2016, Ye [38] created two cosine similarity measures for IFSs based on cosine functions and applied them to MADM. The cotangent function-based similarity measures for IFSs were created by Tian [44] and Rajeshwari and Uma [45] in 2013. The weighted cosine and cotangent similarity measures between IFSs were defined by Ye [37], Shi and Ye [40], Tian [44], and Rajarajeshwari and Uma [45]. In 2018, Wei and Wei [51] proposed cosine and cotangent measures and weighted cosine and cotangent measures between PFSs. In 2019, Wang et al. [65] extended cosine and cotangent measures of similarity for q-ROFSs, which are shown in Tables 2 and 3. The cosine and cotangent similarity and weighted similarity measures for IFSs and PFSs are exceptional cases of corresponding similarity measures defined by Wang et al. [65]. Recently, Kirisci [58] created some cosine similarity measures for Fermatean fuzzy sets, which is a particular case of q-ROFS for q = 3.

Table 2. Cosine and cotangent-based measures of similarity between q-ROFSs [51].

S. N.	Similarity Measure
1	$q-ROFC^1(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} \frac{\vartheta_{\mathbb{E}}^q(p_j)\vartheta_{\mathbb{F}}^q(p_j) + \varsigma_{\mathbb{E}}^q(p_j)\varsigma_{\mathbb{F}}^q(p_j)}{\sqrt{(\vartheta_{\mathbb{E}}^q(p_j))^2 + (\varsigma_{\mathbb{E}}^q(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^q(p_j))^2 + (\varsigma_{\mathbb{F}}^q(p_j))^2}}$
2	$q-ROFC^2(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r}[\frac{(\vartheta_{\mathbb{E}}^q(p_j)\vartheta_{\mathbb{F}}^q(p_j) + \varsigma_{\mathbb{E}}^q(p_j)\varsigma_{\mathbb{F}}^q(p_j) + \pi_{\mathbb{E}}^q(p_j)\pi_{\mathbb{F}}^q(p_j))}{\sqrt{(\vartheta_{\mathbb{E}}^q(p_j))^2 + (\varsigma_{\mathbb{E}}^q(p_j))^2 + (\pi_{\mathbb{E}}^q(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^q(p_j))^2 + (\varsigma_{\mathbb{F}}^q(p_j))^2 + (\pi_{\mathbb{F}}^q(p_j))^2}}]$
3	$q-ROFCS^1(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cos[\frac{\pi}{2}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert, \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert))]$
4	$q-ROFCS^2(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cos[\frac{\pi}{4}(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert + \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert)]$
5	$q-ROFCS^3(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cos[\frac{\pi}{2}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert, \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert, \lvert\pi_{\mathbb{E}}^q(p_j) - \pi_{\mathbb{F}}^q(p_j)\rvert))]$
6	$q-ROFCS^4(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cos[\frac{\pi}{4}(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert + \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert + \lvert\pi_{\mathbb{E}}^q(p_j) - \pi_{\mathbb{F}}^q(p_j)\rvert)]$
7	$q-ROFCot^1(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cot[\frac{\pi}{4} + \frac{\pi}{4}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert, \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert))]$
8	$q-ROFCot^2(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cot[\frac{\pi}{4} + \frac{\pi}{8}(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert + \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert)]$
9	$q-ROFCot^3(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cot[\frac{\pi}{4} + \frac{\pi}{4}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert, \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert, \lvert\pi_{\mathbb{E}}^q(p_j) - \pi_{\mathbb{F}}^q(p_j)\rvert))]$
10	$q-ROFCot^4(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r} cot[\frac{\pi}{4} + \frac{\pi}{8}(\lvert\vartheta_{\mathbb{E}}^q(p_j) - \vartheta_{\mathbb{F}}^q(p_j)\rvert + \lvert\varsigma_{\mathbb{E}}^q(p_j) - \varsigma_{\mathbb{F}}^q(p_j)\rvert + \lvert\pi_{\mathbb{E}}^q(p_j) - \pi_{\mathbb{F}}^q(p_j)\rvert))]$

Table 3. Cosine and cotangent-based weighted similarity measures between q-ROFSs [65].

S. N.	Similarity Measure
1	$q-ROFWC^1(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j[\frac{(\vartheta_{\mathbb{E}}^q(p_j)\vartheta_{\mathbb{F}}^q(p_j)+\varsigma_{\mathbb{E}}^q(p_j)\varsigma_{\mathbb{F}}^q(p_j))}{\sqrt{(\vartheta_{\mathbb{E}}^q(p_j))^2+(\varsigma_{\mathbb{E}}^q(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^q(p_j))^2+(\varsigma_{\mathbb{F}}^q(p_j))^2}}]$
2	$q-ROFWC^2(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j[\frac{(\vartheta_{\mathbb{E}}^q(p_j)\vartheta_{\mathbb{F}}^q(p_j)+\varsigma_{\mathbb{E}}^q(p_j)\varsigma_{\mathbb{F}}^q(p_j)+\pi_{\mathbb{E}}^q(p_j)\pi_{\mathbb{F}}^q(p_j))}{\sqrt{(\vartheta_{\mathbb{E}}^q(p_j))^2+(\varsigma_{\mathbb{E}}^q(p_j))^2+(\pi_{\mathbb{E}}^q(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^q(p_j))^2+(\varsigma_{\mathbb{F}}^q(p_j))^2+(\pi_{\mathbb{F}}^q(p_j))^2}}]$
3	$q-ROFWCS^1(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cos[\frac{\pi}{2}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert,\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert))]$
4	$q-ROFWCS^2(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cos[\frac{\pi}{4}(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert+\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert)]$
5	$q-ROFWCS^3(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cos[\frac{\pi}{2}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert,\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert,\lvert\pi_{\mathbb{E}}^q(p_j)-\pi_{\mathbb{F}}^q(p_j)\rvert))]$
6	$q-ROFWCS^4(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cos[\frac{\pi}{4}(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert+\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert+\lvert\pi_{\mathbb{E}}^q(p_j)-\pi_{\mathbb{F}}^q(p_j)\rvert)]$
7	$q-ROFWCot^1(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cot[\frac{\pi}{4}+\frac{\pi}{4}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert,\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert))]$
8	$q-ROFWCot^2(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cot[\frac{\pi}{4}+\frac{\pi}{8}(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert+\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert)]$
9	$q-ROFWCot^3(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cot[\frac{\pi}{4}+\frac{\pi}{4}(max(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert,\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert,\lvert\pi_{\mathbb{E}}^q(p_j)-\pi_{\mathbb{F}}^q(p_j)\rvert))]$
10	$q-ROFWCot^4(\mathbb{E},\mathbb{F}) = \sum_{j=1}^{r}\omega_j cot[\frac{\pi}{4}+\frac{\pi}{8}(\lvert\vartheta_{\mathbb{E}}^q(p_j)-\vartheta_{\mathbb{F}}^q(p_j)\rvert+\lvert\varsigma_{\mathbb{E}}^q(p_j)-\varsigma_{\mathbb{F}}^q(p_j)\rvert+\lvert\pi_{\mathbb{E}}^q(p_j)-\pi_{\mathbb{F}}^q(p_j)\rvert)]$

3. Cosine and Cotangent Similarity Measures for (m, n)-ROFSs

The (m, n)-ROFSs described by the degrees of membership and non-membership, for which the sum of the n-th power of the membership degree and the n-th power of the non-membership degree lies between 0 and 1, are more general than the IFSs, PFSs, and q-ROFSs, and can describe more vague and imprecise information. In other words, the (m, n)-ROFSs can deal with the MADM and MCDM problems, which IFSs, PFSs, and q-ROFSs cannot, and it is clear that IFSs, PFSs, and q-ROFSs are the special (m, n)-ROFSs, which indicates that (m, n)-ROFSs can be a more effective and powerful tool to deal with the vagueness and impreciseness involved in MADM and MCDM problems. In this section, we shall propose the(m, n)-rung ortho-pair fuzzy cosine similarity measures and (m, n)-rung orthopair fuzzy cotangent similarity measures under the (m, n)-ROFSs environment, which are new extensions of the similarity measures of IFSs, PFSs, and q-ROFSs.

3.1. Cosine Similarity Measures for (m, n)-ROFSs

This section introduces a cosine similarity measure and a weighted cosine similarity measure using (m, n)-ROFSs information in a manner analogous to the cosine similarity measure and weighted cosine similarity measure for IFSs, PFSs, and q-ROFSs.

Definition 6. *Let $\mathbb{P} = \{p_1, p_2, \dots, p_r\}$ be a fixed set. Assume that $\mathbb{E} = \{\langle p_j, \vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j)\rangle \mid p_j \in \mathbb{P}\}$ and $\mathbb{F} = \{\langle p_j, \vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j)\rangle \mid p_j \in \mathbb{P}\}$ are two (m, n)-ROFSs of $\mathbb{P}$, then the (m, n)-ROFSs cosine measure $(m,n)-ROFC^1$ between $\mathbb{E}$ and $\mathbb{F}$ is defined as follows:*

$$(m,n)-ROFC^1(\mathbb{E},\mathbb{F}) = \frac{1}{r}\sum_{j=1}^{r}\frac{\vartheta_{\mathbb{E}}^m(p_j)\vartheta_{\mathbb{F}}^m(p_j)+\varsigma_{\mathbb{E}}^n(p_j)\varsigma_{\mathbb{F}}^n(p_j)}{\sqrt{(\vartheta_{\mathbb{E}}^m(p_j))^2+(\varsigma_{\mathbb{E}}^n(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^m(p_j))^2+(\varsigma_{\mathbb{F}}^n(p_j))^2}} \quad (1)$$

Remark 5. *The cosine measures $IFC^1(\mathbb{E},\mathbb{F})$ (resp., $PFC^1(\mathbb{E},\mathbb{F})$, $q-ROFC^1(\mathbb{E},\mathbb{F})$) for IFSs (resp., PFSs, q-ROFSs) are special cases of cosine similarity measures of $(m,n)-ROFC^1(\mathbb{E},\mathbb{F})$ of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

Proposition 3. *Let $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$ and $\mathbb{E}, \mathbb{F} \in (m,n) - ROFS(\mathbb{P})$, then the cosine similarity measure of $(m,n) - ROFC^1(\mathbb{E}, \mathbb{F})$ satisfies the following conditions:*

(i) $0 \leq (m,n) - ROFC^1(\mathbb{E}, \mathbb{F}) \leq 1$.
(ii) $(m,n) - ROFC^1(\mathbb{E}, \mathbb{F}) = (m,n) - ROFC^1(\mathbb{F}, \mathbb{E})$.
(iii) $\mathbb{E} = \mathbb{F} \Rightarrow (m,n) - ROFC^1(\mathbb{E}, \mathbb{F}) = 1$.

Proof.

(i) It is true because of the cosine values within the closed interval [0,1].
(ii) It follows, noting that:

$$\begin{aligned}(m,n) - ROFC^1(\mathbb{E},\mathbb{F}) &= \frac{1}{r}\sum_{j=1}^{r}\frac{\vartheta_{\mathbb{E}}^m(p_j)\vartheta_{\mathbb{F}}^m(p_j) + \varsigma_{\mathbb{E}}^n(p_j)\varsigma_{\mathbb{F}}^n(p_j)}{\sqrt{(\vartheta_{\mathbb{E}}^m(p_j))^2 + (\varsigma_{\mathbb{E}}^n(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^m(p_j))^2 + (\varsigma_{\mathbb{F}}^n(p_j))^2}}\\ &= \frac{1}{r}\sum_{j=1}^{r}\frac{\vartheta_{\mathbb{F}}^m(p_j)\vartheta_{\mathbb{E}}^m(p_j) + \varsigma_{\mathbb{F}}^n(p_j)\varsigma_{\mathbb{E}}^n(p_j)}{\sqrt{(\vartheta_{\mathbb{F}}^m(p_j))^2 + (\varsigma_{\mathbb{F}}^n(p_j))^2}\sqrt{(\vartheta_{\mathbb{E}}^m(p_j))^2 + (\varsigma_{\mathbb{E}}^n(p_j))^2}}\\ &= (m,n) - ROFC^1(\mathbb{F},\mathbb{E}).\end{aligned}$$

(iii) If $\mathbb{E} = \mathbb{F}$, then $\vartheta_{\mathbb{E}}(p_j) = \vartheta_{\mathbb{F}}(p_j)$ and $\varsigma_{\mathbb{E}}(p_j) = \varsigma_{\mathbb{F}}(p_j)$ for j = 1, 2, ..., n. Thus, from Equation (1), we have the following: $(m,n) - ROFC^1(\mathbb{E}, \mathbb{F})$ = 1

□

Proposition 4. *Let $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$ and $\mathbb{E}, \mathbb{F} \in (m,n) - ROFS(\mathbb{P})$. The distance measure of angle is defined as follows:*

$$d(\mathbb{E}, \mathbb{F}) = arccos((m,n) - ROFC^1(\mathbb{E}, \mathbb{F}))$$

meeting the specified conditions:

(i) $0 \leq d(\mathbb{E}, \mathbb{F}) \leq 1$.
(ii) $\mathbb{E} = \mathbb{F} \Rightarrow d(\mathbb{E}, \mathbb{F}) = 0$.
(iii) $d(\mathbb{E}, \mathbb{F}) = d(\mathbb{F}, \mathbb{E})$.
(iv) $d(\mathbb{E}, \mathbb{G}) \leq d(\mathbb{E}, \mathbb{F}) + d(\mathbb{F}, \mathbb{G})$ *if* $\mathbb{E} \Subset \mathbb{F} \Subset \mathbb{G}$ *for any* $\mathbb{G} \in (m,n) - ROFS(\mathbb{P})$.

Proof. Proof of conditions (i), (ii), and (iii) follows from Proposition 3.

(iv) Suppose that $\mathbb{E} \subseteq \mathbb{F} \subseteq \mathbb{G}$ for any (m, n)-ROFS $\mathbb{G} = \{\langle p_j, \vartheta_T(p_j), \varsigma_T(p_j)\rangle | p_j \in \mathbb{P}\}$ over $\mathbb{P}$. Since Equation (1) is a sum of terms, it is appropriate to examine the distance measures based on the angle between the vectors:
$d_j(\mathbb{E}(p_j), \mathbb{F}(p_j)) = arccos((m,n) - ROFC_j^1(\mathbb{E}(p_j), \mathbb{F}(p_j)))$,
$d_j(\mathbb{E}(p_j), \mathbb{G}(p_j)) = arccos((m,n) - ROFC_j^1(\mathbb{E}(p_j), \mathbb{G}(p_j)))$,
$d_j(\mathbb{F}(p_j), \mathbb{G}(p_j)) = arccos((m,n) - ROFC_j^1(\mathbb{F}(p_j), \mathbb{G}(p_j)))$,
$(j = 1, 2, \ldots r)$, where,

$$(m,n) - ROFC_j^1(\mathbb{E}(p_j), \mathbb{F}(p_j) = \frac{\vartheta_{\mathbb{E}}^m(p_j)\vartheta_{\mathbb{F}}^m(p_j) + \varsigma_{\mathbb{E}}^n(p_j)\varsigma_{\mathbb{F}}^n(p_j)}{\sqrt{(\vartheta_{\mathbb{E}}^m(p_j))^2 + (\varsigma_{\mathbb{E}}^n(p_j))^2}\sqrt{(\vartheta_{\mathbb{F}}^m(p_j))^2 + (\varsigma_{\mathbb{F}}^n(p_j))^2}}$$

$$(m,n) - ROFC_j^1(\mathbb{E}(p_j), \mathbb{G}(p_j) = \frac{\vartheta_{\mathbb{E}}^m(p_j)\vartheta_{\mathbb{G}}^m(p_j) + \varsigma_{\mathbb{E}}^n(p_j)\varsigma_{\mathbb{G}}^n(p_j)}{\sqrt{(\vartheta_{\mathbb{E}}^m(p_j))^2 + (\varsigma_{\mathbb{E}}^n(p_j))^2}\sqrt{(\vartheta_{\mathbb{G}}^m(p_j))^2 + (\varsigma_{\mathbb{G}}^n(p_j))^2}}$$

$$(m,n) - ROFC_j^1(\mathbb{F}(p_j), \mathbb{G}(p_j) = \frac{\vartheta_{\mathbb{F}}^m(p_j)\vartheta_{\mathbb{G}}^m(p_j) + \varsigma_{\mathbb{F}}^n(p_j)\varsigma_{\mathbb{G}}^n(p_j)}{\sqrt{(\vartheta_{\mathbb{F}}^m(p_j))^2 + (\varsigma_{\mathbb{F}}^n(p_j))^2}\sqrt{(\vartheta_{\mathbb{G}}^m(p_j))^2 + (\varsigma_{\mathbb{G}}^n(p_j))^2}}$$

For three vectors, $\mathbb{E}(p_j) = \langle \vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j)\rangle, \mathbb{F}(p_j) = \langle \vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j)\rangle, \mathbb{G}(p_j) = \langle \vartheta_{\mathbb{G}}(p_j), \varsigma_{\mathbb{G}}(p_j)\rangle$ in one plane, if the $\mathbb{E}(p_j) \subseteq \mathbb{F}(p_j) \subseteq \mathbb{G}(p_j)$, $j = 1, 2, \ldots$, r, then by the triangle inequality, we have the following: $d_j(\mathbb{E}(p_j), \mathbb{G}(p_j)) \leq d_j(\mathbb{E}(p_j), \mathbb{F}(p_j)) + d_j(\mathbb{F}(p_j), \mathbb{G}(p_j))$. Combining the inequality with Equation (1), we can obtain $d(\mathbb{E}, \mathbb{G}) \leq d(\mathbb{E}, \mathbb{F}) + d(\mathbb{F}, \mathbb{G})$. Hence, the distance measure of angle d($\mathbb{E}, \mathbb{F}$) satisfies the property (iv). □

Now, we define the (m, n)-ROFS cosine measure by considering three terms, i.e.; MD, NMD, and IMD of (m, n)-ROFSs.

Definition 7. *Let* $\mathbb{E} = \{\langle p_j, \vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j)\rangle | p_j \in \mathbb{P}\}$, *and* $\mathbb{F} = \{\langle p_j, \vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j)\rangle | p_j \in \mathbb{P}\}$ *be two (m, n)-ROFSs in* $\mathbb{P}$, *then the (m, n)-rung orthopair fuzzy cosine measure (*$(m,n) - ROFC^2$*) between* $\mathbb{E}$ *and* $\mathbb{F}$ *can be expressed as follows:*

$(m,n) - ROFC^2(\mathbb{E}, \mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r}\frac{\left(\vartheta_{\mathbb{E}}^{m}(p_j)\vartheta_{\mathbb{F}}^{m}(p_j) + \varsigma_{\mathbb{E}}^{n}(p_j)\varsigma_{\mathbb{F}}^{n}(p_j) + \pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j)\pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)\right)}{\left((\sqrt{(\vartheta_{\mathbb{E}}^{m}(p_j))^2 + (\varsigma_{\mathbb{E}}^{n}(p_j))^2 + (\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j))^2})(\sqrt{(\vartheta_{\mathbb{F}}^{m}(p_j))^2 + (\varsigma_{\mathbb{F}}^{n}(p_j))^2 + (\pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j))^2})\right)} \quad (2)$$

Proposition 5. *Consider two (m, n)-ROFSs, denoted by* $\mathbb{E}$ *and* $\mathbb{F}$*, defined over* $\mathbb{P}$*, the cosine similarity measure* $(m,n) - ROFC^2(\mathbb{E}, \mathbb{F})$ *satisfies the following conditions:*

(i) $0 \leq (m,n) - ROFC^2(\mathbb{E}, \mathbb{F}) \leq 1$.
(ii) $(m,n) - ROFC^2(\mathbb{E}, \mathbb{F}) = (m,n) - ROFC^2(\mathbb{F}, \mathbb{E})$.
(iii) $\mathbb{E} = \mathbb{F} \Rightarrow (m,n) - ROFC^2(\mathbb{E}, \mathbb{F}) = 1$.

Remark 6. *The cosine measures* $IFC^2(\mathbb{E}, \mathbb{F})$ *(resp.,* $PFC^2(\mathbb{E}, \mathbb{F})$*,* $q - ROFC^2(\mathbb{E}, \mathbb{F})$*) for IFSs (resp., PFSs, q-ROFSs) are special cases of cosine similarity measures* $(m,n) - ROFC^2(\mathbb{E}, \mathbb{F})$ *of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

Now, we define the (m, n)-ROFS-weighted cosine measures between two (m, n)-ROFSs, $\mathbb{E}$ and $\mathbb{F}$, by considering the weighting vector of the elements in (m, n)-ROFSs.

Definition 8. *Let* $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$ *be a fixed set and* $\mathbb{E}, \mathbb{F} \in (m,n) - ROFS(\mathbb{P})$*. Assume* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ *is the weighting vector of the elements* p_j *(j = 1, 2, ..., r), satisfying the condition* $\sum_{j=1}^{r}\omega_j = 1$, $\forall\ \omega_j \in$ *[0, 1] and j = 1, 2, ..., r. Then, the (m, n)-rung orthopair fuzzy-weighted cosine measures, i.e.,* $(m,n) - ROFWC^1$ *and* $(m,n) - ROFWC^2$*, between* $\mathbb{E}$ *and* $\mathbb{F}$*, can be expressed as follows:*

$(m,n) - ROFWC^1(\mathbb{E}, \mathbb{F})$=

$$\sum_{j=1}^{r}\omega_j\frac{\left(\vartheta_{\mathbb{E}}^{m}(p_j)\vartheta_{\mathbb{F}}^{m}(p_j) + \varsigma_{\mathbb{E}}^{n}(p_j)\varsigma_{\mathbb{F}}^{n}(p_j)\right)}{\left((\sqrt{(\vartheta_{\mathbb{E}}^{m}(p_j))^2 + (\varsigma_{\mathbb{E}}^{n}(p_j))^2})(\sqrt{(\vartheta_{\mathbb{F}}^{m}(p_j))^2 + (\varsigma_{\mathbb{F}}^{n}(p_j))^2})\right)} \quad (3)$$

$(m,n) - ROFWC^2(\mathbb{E}, \mathbb{F})$ =

$$\sum_{j=1}^{r}\omega_j\frac{\left(\vartheta_{\mathbb{E}}^{m}(p_j)\vartheta_{\mathbb{F}}^{m}(p_j) + \varsigma_{\mathbb{E}}^{n}(p_j)\varsigma_{\mathbb{F}}^{n}(p_j) + \pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j)\pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)\right)}{\left((\sqrt{(\vartheta_{\mathbb{E}}^{m}(p_j))^2 + (\varsigma_{\mathbb{E}}^{n}(p_j))^2 + (\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j))^2})(\sqrt{(\vartheta_{\mathbb{F}}^{m}(p_j))^2 + (\varsigma_{\mathbb{F}}^{n}(p_j))^2 + (\pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j))^2})\right)} \quad (4)$$

When we take the weighting vector $\omega = (\frac{1}{r}, \frac{1}{r}, \ldots, \frac{1}{r})^T$, then the weighted cosine similarity measures $(m,n) - ROFWC^1(\mathbb{E}, \mathbb{F})$, and $(m,n) - ROFWC^2(\mathbb{E}, \mathbb{F})$ will reduce to cosine similarity measures $(m,n) - ROFC^1(\mathbb{E}, \mathbb{F})$ and $(m,n) - ROFC^2(\mathbb{E}, \mathbb{F})$, respectively.

Remark 7. *The weighted cosine similarity measures* $WIFC^k(\mathbb{E}, \mathbb{F})$ *(resp.,* $WPFC^k(\mathbb{E}, \mathbb{F})$*,* $q - ROFWC^k(\mathbb{E}, \mathbb{F})$*) for IFSs (resp., PFSs, q-ROFSs) are special cases of the weighted cosine similarity measures* $(m,n) - ROFWC^k(\mathbb{E}, \mathbb{F})$ *(k = 1, 2) of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

Example 1. *Let* $\mathbb{P} = \{p_1, p_2, p_3\}$ *and*

$$\mathbb{E} = \{(p_1, 0.5, 0.8), (p_2, 0.6, 0.4), (p_3, 0.8, 0.3)\}$$
$$\mathbb{F} = \{(p_1, 0.7, 0.6), (p_2, 0.8, 0.2), (p_3, 0.4, 0.3)\}$$

be two (m, n)-ROFSs over $\mathbb{P}$. *Assuming m = 4, n = 3, and* $\omega = (0.20, 0.45, 0.35)^T$ *is a weighting vector of the elements* p_1, p_2, p_3, *then,* $(m,n) - ROFWC^1(\mathbb{E},\mathbb{F})$ *= 0.8150 and* $(m,n) - ROFWC^2(\mathbb{E},\mathbb{F})$ *= 0.8610.*

Proposition 6. *Let* $\mathbb{E}$ *and* $\mathbb{F}$ *be two (m, n)-ROFSs over a fixed set* $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$. *Assuming* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ *is the weighting vector of the elements* p_j *(j = 1, 2, ..., r), satisfying the conditions* $\sum_{j=1}^{r} \omega_j = 1$, $\forall\ \omega_j \in [0, 1]$ *and j = 1, 2, ..., r, then the weighted cosine similarity measures* $(m,n) - ROFWC^k(\mathbb{E},\mathbb{F})$ *(k = 1, 2) meet the following conditions:*

(a) $0 \leq (m,n) - ROFWC^k(\mathbb{E},\mathbb{F}) \leq 1$.
(b) $(m,n) - ROFWC^k(\mathbb{E},\mathbb{F}) = (m,n) - ROFWC^k(\mathbb{F},\mathbb{E})$.
(c) $\mathbb{E} = \mathbb{F} \Rightarrow (m,n) - ROFWC^k(\mathbb{E},\mathbb{F}) = 1$.

3.2. Similarity Measures of (m, n)-ROFSs Based on the Cosine Function

This section introduces several (m, n)-ROFS cosine similarity measures between (m, n)-ROFSs, which are based on the cosine function, and examines their properties.

Definition 9. *Let* $\mathbb{P} = \{p_j : j = 1, 2, \ldots, r\}$ *and*

$$\mathbb{E} = \{\langle p_j, (\vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j))\rangle | p_j \in \mathbb{P}\}$$
$$\mathbb{F} = \{\langle p_j, (\vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j))\rangle | p_j \in \mathbb{P}\}$$

be two (m, n)-ROFSs over $\mathbb{P}$, *then two (m, n)-ROFS cosine similarity measures* $(m,n) - ROFCS^k$ *(k = 1, 2) between* $\mathbb{E}$ *and* $\mathbb{F}$ *can be expressed as follows:*

$(m,n) - ROFCS^1(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r} cos\left[\frac{\pi}{2}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| \vee |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)|\right)\right] \tag{5}$$

$(m,n) - ROFCS^2(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r} cos\left[\frac{\pi}{4}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| + |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)|\right)\right] \tag{6}$$

Proposition 7. *Let* $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$ *and* $\mathbb{E}, \mathbb{F} \in (m,n) - ROFS(\mathbb{P})$, *then the (m, n)-rung orthopair fuzzy cosine similarity measures* $(m,n) - ROFCS^k(\mathbb{E},\mathbb{F})$ *(k = 1, 2) meet the following properties:*

(a) $0 \leq (m,n) - ROFCS^k(\mathbb{E},\mathbb{F}) \leq 1$.
(b) $(m,n) - ROFCS^k(\mathbb{E},\mathbb{F}) = 1 \Leftrightarrow \mathbb{E} = \mathbb{F}$.
(c) $(m,n) - ROFCS^k(\mathbb{E},\mathbb{F}) = (m,n) - ROFCS^k(\mathbb{F},\mathbb{E})$.
(d) *If* $\mathbb{E} \subseteq \mathbb{F} \subseteq \mathbb{G}$, $\forall\ \mathbb{G} \in (m,n) - ROFS(\mathbb{P})$. *Then,* $(m,n) - ROFCS^k(\mathbb{E},\mathbb{G}) \leq (m,n) - ROFCS^k(\mathbb{E},\mathbb{F})$ *and* $(m,n) - ROFCS^k(\mathbb{E},\mathbb{G}) \leq (m,n) - ROFCS^k(\mathbb{F},\mathbb{G})$.

Proof.

(a) The values of cosine functions lie between 0 and 1, which makes it evident.
(b) If $\mathbb{E} = \mathbb{F}$ for any two (m, n)-ROFSs $\mathbb{E}$ and $\mathbb{F}$ in $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$, then for each j = 1, 2, ..., r, $\vartheta^m_{\mathbb{E}}(p_j) = \vartheta^m_{\mathbb{F}}(p_j)$ and $\varsigma^n_{\mathbb{E}}(p_j) = \varsigma^n_{\mathbb{F}}(p_j)$. It implies that $|\ \vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)\ |$ = 0 and $|\ \varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)\ |$ = 0. Hence, $(m,n) - ROFCS^k(\mathbb{E},\mathbb{F})$ = 1 for k = 1, 2. Suppose that

$(m,n) - ROFCS^k(\mathbb{E},\mathbb{F})$ = 1, k = 1, 2, then $\mid \vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j) \mid$ = 0 and $\mid \varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j) \mid$ = 0, for all j= 1, 2, ..., r. Since cos(0) = 1, there are $\vartheta^m_{\mathbb{E}}(p_j) = \vartheta^m_{\mathbb{F}}(p_j)$,$\varsigma^n_{\mathbb{E}}(p_j) = \varsigma^n_{\mathbb{F}}(p_j)$ (j = 1, 2, ..., r). Hence, $\mathbb{E} = \mathbb{F}$.

(c) Obvious.

(d) If $\mathbb{E}(p_j) \subseteq \mathbb{F}(p_j) \subseteq \mathbb{G}(p_j)$, $\forall$ j = 1, 2, ..., r, then $\vartheta_{\mathbb{E}}(p_j) \leq \vartheta_{\mathbb{F}}(p_j) \leq \vartheta_{\mathbb{G}}(p_j)$ and $\varsigma_{\mathbb{E}}(p_j) \geq \varsigma_{\mathbb{F}}(p_j) \geq \varsigma_{\mathbb{G}}(p_j)$, j = 1, 2, ..., r. It follows that $\vartheta^m_{\mathbb{E}}(p_j) \leq \vartheta^m_{\mathbb{F}}(p_j) \leq \vartheta^m_{\mathbb{G}}(p_j)$ and $\varsigma^n_{\mathbb{E}}(p_j) \geq \varsigma^n_{\mathbb{F}}(p_j) \geq \varsigma^n_{\mathbb{G}}(p_j)$. Thus, we obtain the following:
$\mid \vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j) \mid \leq \mid \vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{G}}(p_j) \mid$,
$\mid \vartheta^m_{\mathbb{F}}(p_j) - \vartheta^m_{\mathbb{G}}(p_j) \mid \leq \mid \vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{G}}(p_j) \mid$,
$\mid \varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j) \mid \leq \mid \varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{G}}(p_j) \mid$,
$\mid \varsigma^n_{\mathbb{F}}(p_j) - \varsigma^n_{\mathbb{G}}(p_j) \mid \leq \mid \varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{G}}(p_j) \mid$.
□

The cosine function is a decreasing function with the interval [0, $\frac{\pi}{2}$]; therefore, we obtain that $(m,n) - ROFCS^k(\mathbb{E},\mathbb{G}) \leq (m,n) - ROFCS^k(\mathbb{E},\mathbb{F})$, $(m,n) - ROFCS^k(\mathbb{E},\mathbb{G}) \leq (m,n) - ROFCS^k(\mathbb{F},\mathbb{G})$ for k = 1, 2.

Next, we introduce (m, n)-rung orthopair fuzzy cosine measures based on the cosine function. These measures are obtained by considering MD, NMD, and IMD for two (m, n)-ROFSs, i.e., $\mathbb{E}$ and $\mathbb{F}$ of $\mathbb{P}$.

Definition 10. *Let* $\mathbb{P} = \{p_j : j = 1, 2, \ldots, r\}$ *and*

$$\mathbb{E} = \{\langle p_j, (\vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j))\rangle | p_j \in \mathbb{P}\}$$
$$\mathbb{F} = \{\langle p_j, (\vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j))\rangle | p_j \in \mathbb{P}\}$$

be the two (m, n)-ROFSs over $\mathbb{P}$*, then two (m, n)-ROFS cosine similarity measures, i.e.,* $(m,n) - ROFCS^k$*) (k = 3, 4) between* $\mathbb{E}$ *and* $\mathbb{F}$*, by considering MD, NMD, and IMD, can be expressed as follows:*

$(m,n) - ROFCS^3(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r} cos\left[\frac{\pi}{2}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)|, \vee|\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)|, \vee|\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - |\pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \quad (7)$$

$(m,n) - ROFCS^4(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r} cos\left[\frac{\pi}{4}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| + |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)| + |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - |\pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \quad (8)$$

Remark 8. *The cosine measures* $IFCS^k(\mathbb{E},\mathbb{F})$ *(resp.,* $PFCS^k(\mathbb{E},\mathbb{F})$*,* $q - ROFWCS^k(\mathbb{E},\mathbb{F})$*) for IFSs (resp., PFSs, q-ROFSs) are special cases of cosine measures* $(m,n) - ROFCS^k(\mathbb{E},\mathbb{F})$ *(k = 1, 2, 3, 4) of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

We will now introduce the (m, n)-ROFS weighted cosine measures between two (m, n)-ROFSs, which are based on cosine functions $\mathbb{E}$ and $\mathbb{F}$, by taking into account the weighting vector associated with the elements in (m, n)-ROFSs.

Definition 11. *Let* $\mathbb{E}$ = $\{\langle p_j, \vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j)\rangle | p_j \in \mathbb{P}\}$ *and* $\mathbb{F}$ = $\{\langle p_j, \vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j)\rangle | p_j \in \mathbb{P}\}$ *be two (m, n)-ROFSs in* $\mathbb{P}$ *and let* $\omega = (\omega_1, \omega_2, \ldots, \omega_r)^T$ *be the weighting vector of the elements* p_j *(j = 1, 2, ..., r) that satisfies the condition* $\sum_{j=1}^{r} \omega = 1$, $\forall$ $\omega_j \in$ *[0, 1] and j = 1, 2, ..., r. The (m, n)-ROFS weighted cosine measures* $(m,n) - ROFWCS^k$*, (k = 1, 2, 3, 4) between* $\mathbb{E}$ *and* $\mathbb{F}$ *on the bases of cosine functions can be presented as follows:*

$(m,n) - ROFWCS^1(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r}\omega_j cos\left[\frac{\pi}{2}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| \vee |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)|\right)\right] \tag{9}$$

$(m,n) - ROFWCS^2(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r}\omega_j cos\left[\frac{\pi}{4}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| + |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)|\right)\right] \tag{10}$$

$(m,n) - -ROFWCS^3$ $(\mathbb{E},\mathbb{F})$ =

$$\frac{1}{r}\sum_{j=1}^{r}\omega_j cos\left[\frac{\pi}{2}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| \vee |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)| \vee |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - \pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \tag{11}$$

$(m,n) - ROFWCS^4(\mathbb{E},\mathbb{F})$=

$$\frac{1}{r}\sum_{j=1}^{r}\omega_j cos\left[\frac{\pi}{4}\left(|\vartheta^m_{\mathbb{E}}(p_j) - \vartheta^m_{\mathbb{F}}(p_j)| + |\varsigma^n_{\mathbb{E}}(p_j) - \varsigma^n_{\mathbb{F}}(p_j)| + |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - \pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \tag{12}$$

When the weighting vector $\omega_j = \frac{1}{r}, \quad j = 1,2,\ldots r$, then for k = 1, 2, 3, 4, we have (m, n)-ROFWCS $^k(\mathbb{E},\mathbb{F})$ = (m, n)-ROFCS$^k(\mathbb{E},\mathbb{F})$.

Remark 9. *The weighted cosine similarity measures $WIFCS^k(\mathbb{E},\mathbb{F})$ (resp., $WPFCS^k(\mathbb{E},\mathbb{F})$, $q - ROFWCS^k(\mathbb{E},\mathbb{F})$) for IFSs (resp., PFSs, q-ROFSs) are special cases of weighted cosine measures $(m,n) - ROFWCS^k(\mathbb{E},\mathbb{F})$(k = 1, 2, 3, 4) of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

Example 2. *Let $\mathbb{P} = \{p_1, p_2, p_3\}$ and*

$$\mathbb{E} = \{(p_1, 0.8, 0.5), (p_2, 0.4, 0.6), (p_3, 0.3, 0.8)\}$$
$$\mathbb{F} = \{(p_1, 0.6, 0.7), (p_2, 0.2, 0.8), (p_3, 0.4, 0.3)\}$$

be two (m, n)-ROFSs over $\mathbb{P}$. Assuming m = 4, n = 6 and $\omega = (0.25, 0.55, 0.20)^T$ are the weights for the elements p_1, p_2, p_3, then:

$$(m,n) - ROFWCS^1(\mathbb{E},\mathbb{F}) = 0.9283.$$
$$(m,n) - ROFWCS^2(\mathbb{E},\mathbb{F}) = 0.9743.$$
$$(m,n) - ROFWCS^3(\mathbb{E},\mathbb{F}) = 0.9283.$$
$$(m,n) - ROFWCS^4(\mathbb{E},\mathbb{F}) = 0.9283.$$

Proposition 8. *Assuming that there are any two (m, n)-ROFSs, $\mathbb{E}$ and $\mathbb{F}$ in $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$, the (m, n)-ROFS weighted cosine similarity measures (m, n)-ROFWCS^k $(\mathbb{E},\mathbb{F})$(k = 1, 2, 3, 4) should satisfy properties (a)–(b):*

(a) $0 \leq (m,n) - ROFWCS^k(\mathbb{E},\mathbb{F}) \leq 1.$

(b) $(m,n) - ROFWCS^k(\mathbb{E},\mathbb{F}) = 1 \Leftrightarrow \mathbb{E} = \mathbb{F}.$

(c) $(m,n) - ROFWCS^k(\mathbb{E},\mathbb{F})$
$= (m,n) - ROFWCS^k(\mathbb{F},\mathbb{E}).$

(d) *If $\mathbb{E} \subseteq \mathbb{F} \subseteq \mathbb{G}$, $\forall$ $\mathbb{G} \in (m,n) - ROFS(\mathbb{P})$. Then (m, n)-ROFWCS^k $(\mathbb{E},\mathbb{G})$ $\leq$(m, n)-ROFWCS^k $(\mathbb{E},\mathbb{F})$, (m, n)-ROFWCS^k $(\mathbb{E},\mathbb{G})$ $\leq$ (m, n)-ROFWCS^k $(\mathbb{F},\mathbb{G})$.*

3.3. Cotangent-Based Similarity Measures for (m, n)-ROFSs

Definition 12. *Let* $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$ *and*
$\mathbb{E}= \{\langle p_j, (\vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j))\rangle | p_j \in \mathbb{P}\}$,
Let $\mathbb{F}= \{\langle p_j, (\vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j))\rangle | p_j \in \mathbb{P}\}$ *be two (m, n)-ROFSs; the (m, n)-ROFS cotangent measures* $(m,n) - ROFCot^1$ *and* $(m,n) - ROFCot^2$ *between* $\mathbb{E}$ *and* $\mathbb{F}$ *are defined as follows:*
$(m,n) - ROFCot^1(\mathbb{E},\mathbb{F})=$

$$\frac{1}{r}\sum_{j=1}^{r} cot\left[\frac{\pi}{4} + \frac{\pi}{4}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| \vee |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)|\right)\right] \tag{13}$$

$(m,n) - ROFCot^2(\mathbb{E},\mathbb{F})=$

$$\frac{1}{r}\sum_{j=1}^{r} cot\left[\frac{\pi}{4} + \frac{\pi}{8}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| + |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)|\right)\right] \tag{14}$$

We will now incorporate MD, NMD, and IMD, all of which are components of (m, n)-ROFSs, to define two additional cotangent similarity measures between two (m, n)-ROFSs.

Definition 13. *Let* $\mathbb{P} = \{p_j : j = 1, 2, \ldots, r\}$ *and*

$$\mathbb{E} = \{\langle p_j, (\vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j)\rangle : p_j \in \mathbb{P}\}$$
$$\mathbb{F} = \{\langle p_j, (\vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j))\rangle : p_j \in \mathbb{P}\}.$$

be two (m, n)-ROFSs in $\mathbb{P}$*, then the (m, n)-ROFSs cotangent similarity measures* $(m,n) - ROFCot^3$ *and* $(m,n) - ROFCot^4$ *between* $\mathbb{E}$ *and* $\mathbb{F}$ *can be expressed as follows:*
$(m,n) - ROFCot^3(\mathbb{E},\mathbb{F}) =$

$$\frac{1}{r}\sum_{j=1}^{r} cot\left[\frac{\pi}{4} + \frac{\pi}{4}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| \vee |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)| \vee |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - \pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \tag{15}$$

$(m,n) - ROFCot^4$ $(\mathbb{E},\mathbb{F}) =$

$$\frac{1}{r}\sum_{j=1}^{r} cot\left[\frac{\pi}{4} + \frac{\pi}{8}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| + |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)| + |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - \pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \tag{16}$$

Remark 10. *The cotangent measures* $IFCT^k(\mathbb{E},\mathbb{F})$ *(resp.,* $PFCT^k(\mathbb{E},\mathbb{F})$*,* $q - ROFCot^k(\mathbb{E},\mathbb{F})$*) for IFSs (resp., PFSs, q-ROFSs) are special cases of cotangent measures* $(m,n) - ROFCot^k(\mathbb{E},\mathbb{F})$ *(k = 1, 2, 3, 4) of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

We will now introduce the (m, n)-ROFS weighted cotangent measures between two (m, n)-ROFSs, $\mathbb{E}$ and $\mathbb{F}$, by taking into account the weighting vector associated with the elements in (m, n)-ROFSs.

Definition 14. *Let* $\mathbb{P} = \{p_1, p_2, \ldots, p_r\}$ *be a fixed set and* $\mathbb{E} = \{\langle p_j, \vartheta_{\mathbb{E}}(p_j), \varsigma_{\mathbb{E}}(p_j)\rangle | p_j \in \mathbb{P}\}$, $\mathbb{F}$ $= \{\langle p_j, \vartheta_{\mathbb{F}}(p_j), \varsigma_{\mathbb{F}}(p_j)\rangle | p_j \in \mathbb{P}\}$ *be two (m, n)-ROFSs in* $\mathbb{P}$*. Assume that* $\omega = (\omega_1, \omega_2, \ldots, \omega_r)^T$ *is the weighting vector of the elements* p_j *(j = 1, 2, . . . , r) that satisfies the condition* $\sum_{j=1}^{r} \omega_j = 1$, $\forall\ \omega_j \in [0,1]$ *and j = 1, 2, . . . , r.*

The (m, n)-ROFS weighted cotangent measures $(m,n) - ROFWCot^k$ *(k = 1, 2, 3, 4) between* $\mathbb{E}$ *and* $\mathbb{F}$ *are expressed as follows:*
$(m,n) - ROFWCot^1(\mathbb{E},\mathbb{F}) =$

$$\sum_{j=1}^{r} \omega_j cot\left[\frac{\pi}{4} + \frac{\pi}{4}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| \vee |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)|\right)\right] \tag{17}$$

$(m,n) - ROFWCot^2(\mathbb{E},\mathbb{F}) =$

$$\sum_{j=1}^{r} \omega_j cot\left[\frac{\pi}{4} + \frac{\pi}{8}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| + |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)|\right)\right] \quad (18)$$

$(m,n) - ROFWCot^3(\mathbb{E},\mathbb{F}) =$

$$\sum_{j=1}^{r} \omega_j cot\left[\frac{\pi}{4} + \frac{\pi}{4}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| \vee |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)| \vee |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - \pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \quad (19)$$

$(m,n) - ROFWCot^4(\mathbb{E},\mathbb{F}) =$

$$\sum_{j=1}^{r} \omega_j cot\left[\frac{\pi}{4} + \frac{\pi}{8}\left(|\vartheta_{\mathbb{E}}^{m}(p_j) - \vartheta_{\mathbb{F}}^{m}(p_j)| + |\varsigma_{\mathbb{E}}^{n}(p_j) - \varsigma_{\mathbb{F}}^{n}(p_j)| + |\pi_{\mathbb{E}}^{\frac{m+n}{2}}(p_j) - \pi_{\mathbb{F}}^{\frac{m+n}{2}}(p_j)|\right)\right] \quad (20)$$

When we let the weighting vector $\omega = (\frac{1}{r}, \frac{1}{r}, \ldots \frac{1}{r})^T$, then $(m,n) - ROFWCot^k(\mathbb{E},\mathbb{F})$ coincides with $(m,n) - ROFCot^k(\mathbb{E},\mathbb{F})$, for $k = 1, 2, 3, 4$.

Example 3. *Let* $X = \{p_1, p_2, p_3\}$ *and*

$$\mathbb{E} = \{(p_1, 0.4, 0.9), (p_2, 0.9, 0.3), (p_3, 0.9, 0.6)\}$$
$$\mathbb{F} = \{(p_1, 0.3, 0.7), (p_2, 0.8, 0.3), (p_3, 0.7, 0.4)\}$$

be two (m, n)-ROFSs over $\mathbb{P}$*. Assuming that m = 5, n = 7 and* $\omega_1 = 0.25$, $\omega_2 = 0.35$ *and* $\omega_1 = 0.40$ *are the weights for the elements* p_1, p_2, p_3*, then*

$$(m,n) - ROFWCot^1(\mathbb{E},\mathbb{F}) = 0.5290.$$
$$(m,n) - ROFWCot^2(\mathbb{E},\mathbb{F}) = 0.7206.$$
$$(m,n) - ROFWCot^3(\mathbb{E},\mathbb{F}) = 0.5253.$$
$$(m,n) - ROFWCot^4(\mathbb{E},\mathbb{F}) = 0.5253.$$

Remark 11. *The weighted cotangent similarity measures* $WIFCT^k(\mathbb{E},\mathbb{F})$ *(resp.,* $WPFCT^k(\mathbb{E},\mathbb{F})$, $q - ROFWCot^k(\mathbb{E},\mathbb{F})$ *for IFSs (resp., PFSs, q-ROFSs) are special cases of weighted cotangent measures* $(m,n) - ROFWCot^k(\mathbb{E},\mathbb{F})$ *(k = 1, 2, 3, 4) of (m, n)-ROFSs for m = n = 1 (resp., m = n = 2, m = n = q).*

4. Comparisons of Existing Similarity Measures and Proposed Similarity Measures

The following section presents a comparison between the cosine and cotangent similarity measures for q-ROFSs and the newly introduced cosine and cotangent similarity measures for (m, n)-ROFSs. The evaluation is based on the example of the building material classification by Wang et al. [65].

Example 4 ([65])**.** *Let us consider a scenario where there are five known building construction materials, represented by q-ROFSs* $\mathbb{Z}_i$ *(i = 1, 2, 3, 4, 5), in the feature space* $\mathbb{P} = \{p_1, p_2, p_3, p_4, p_5\}$*, as shown in Table 4. We also have an unknown building material* $\mathbb{Z}$ *that needs to be classified into one of the following classes:* $\mathbb{Z}_1$, $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_5$*. Assuming that the weights w = (0.15, 0.20, 0.25, 0.10, 0.30)*^{T}*, we aim to determine the degree of similarity between* $\mathbb{Z}_3$ *and* $\mathbb{Z}$*.*

According to the findings presented in Table 5, the degree of weighted similarity between $\mathbb{Z}_3$ and $\mathbb{Z}$ is the highest among all ten weighted similarity measures from Table 3 for most building materials, except for $q - ROFWC^1$ and $q - ROFWC^2$. As a result, based on the principle of maximum weighted q-ROFWS similarity, the unknown building material

$\mathbb{Z}$ can be classified as similar to the known building material $\mathbb{Z}_2$ using these ten similarity measures.

Table 4. q-Orthopair fuzzy data for the material pattern (for q = 3) [65].

Feature	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$	$\mathbb{Z}_5$	$\mathbb{Z}$
p_1	(0.5, 0.8)	(0.6, 0.7)	(0.3, 0.4)	(0.5, 0.3)	(0.4, 0.7)	(0.7, 0.6)
p_2	(0.6, 0.4)	(0.7, 0.3)	(0.7, 0.5)	(0.4, 0.4)	(0.2, 0.6)	(0.8, 0.2)
p_3	(0.8, 0.3)	(0.6, 0.2)	(0.9, 0.3)	(0.6, 0.2)	(0.5, 0.4)	(0.4, 0.3)
p_4	(0.6, 0.9)	(0.8, 0.6)	(0.4, 0.8)	(0.4, 0.7)	(0.5, 0.3)	(0.7, 0.8)
p_5	(0.1, 0.4)	(0.3, 0.5)	(0.2, 0.3)	(0.2, 0.6)	(0.4, 0.2)	(0.4, 0.2)

Table 5. q-ROF weighted similarity measures for the data presented in Table 4 for q = 3.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
$q - ROFWC^1$	0.6728	0.7515	**0.7553**	0.6584	0.7336	$\mathbb{Z}_3 > \mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_4$
$q - ROFWC^2$	0.8457	0.8901	**0.8937**	0.8406	0.8735	$\mathbb{Z}_3 > \mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_4$
$q - ROFWCS^1$	0.8962	**0.9673**	0.8398	0.9114	0.8976	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_3$
$q - ROFWCS^2$	0.9601	**0.9838**	0.9487	0.9621	0.9464	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_5$
$q - ROFWCS^3$	0.8962	**0.9673**	0.8299	0.8986	0.8910	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
$q - ROFWCS^4$	0.8961	**0.9693**	0.8830	0.8883	0.8830	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3 = \mathbb{Z}_3$
$q - ROFWCot^1$	0.6740	**0.7831**	0.6478	0.6735	0.7474	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3$
$q - ROFWCot^2$	0.7740	**0.8482**	0.7700	0.7733	0.8065	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3$
$q - ROFWCot^3$	0.6740	**0.7831**	0.6356	0.6522	0.7324	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3$
$q - ROFWCot^4$	0.6727	**0.7866**	0.6356	0.6389	0.7284	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3$

Table 6 shows the results obtained by the proposed weighted similarity measures $(m, n) - ROFWSs$ for $m = 4$ and $n = 3$. Based on these results, it is evident in Table 6 that all ten similarity measures allocate unknown building material $\mathbb{Z}$ to building material $\mathbb{Z}_2$, with the degree of weighted similarity between $\mathbb{Z}$ and $\mathbb{E}_2$ being the largest. This result suggests that the proposed weighted $(m, n) - ROFWSs$ method accurately allocates unknown building materials to known building materials.

Table 6. Weighted (m, n)-ROFSs for the data presented in Table 4 for m = 4, n = 3.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
$(m, n) - ROFWC^1$	0.7353	**0.7620**	0.7334	0.5957	0.6426	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_4$
$(m, n) - ROFWC^2$	0.9008	**0.9527**	0.8244	0.8947	0.8481	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3$
$(m, n) - ROFWCS^1$	0.9073	**0.9648**	0.8614	0.9203	0.8930	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
$(m, n) - ROFWCS^2$	0.9661	**0.9828**	0.9557	0.9669	0.9417	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_5$
$(m, n) - ROFWCS^3$	0.9073	**0.9648**	0.8503	0.8906	0.8590	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3$
$(m, n) - ROFWCS^4$	0.9073	**0.9648**	0.8503	0.8906	0.8590	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3$
$(m, n) - ROFWCot^1$	0.6726	**0.7819**	0.6717	0.6936	0.7215	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_3$
$(m, n) - ROFWCot^2$	0.7853	**0.8515**	0.7818	0.7895	0.7803	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_5$
$(m, n) - ROFWCot^3$	0.6497	**0.7664**	0.6277	0.6119	0.6500	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_4$
$(m, n) - ROFWCot^4$	0.6530	**0.7704**	0.6372	0.6183	0.6603	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_4$

By comparing the results presented in Tables 5 and 6, it is clear that the proposed weighted $(m, n) - ROFWSs$ method is more accurate than the method proposed by Wang et al. [65] for assigning unknown building materials to the consistent class ($\mathbb{Z}_2$, $\mathbb{Z}$). Therefore, our results are more reliable and accurate.

Figure 1 shows that for the weighted q-ROFWS values of all building materials, except for $q - ROFWC^1$ and $q - ROFWC^2$, the degree of weighted similarity between ($\mathbb{Z}_3$, $\mathbb{Z}$) is the largest among the ten weighted similarity measures, and the proposed weighted (m, n)-ROFWSs for $m = 4, n = 3$ show that $\mathbb{Z}_3$ has the most significant and consistent value. This result indicates that the unknown pattern $\mathbb{Z}$ is the most similar to $\mathbb{Z}_3$.

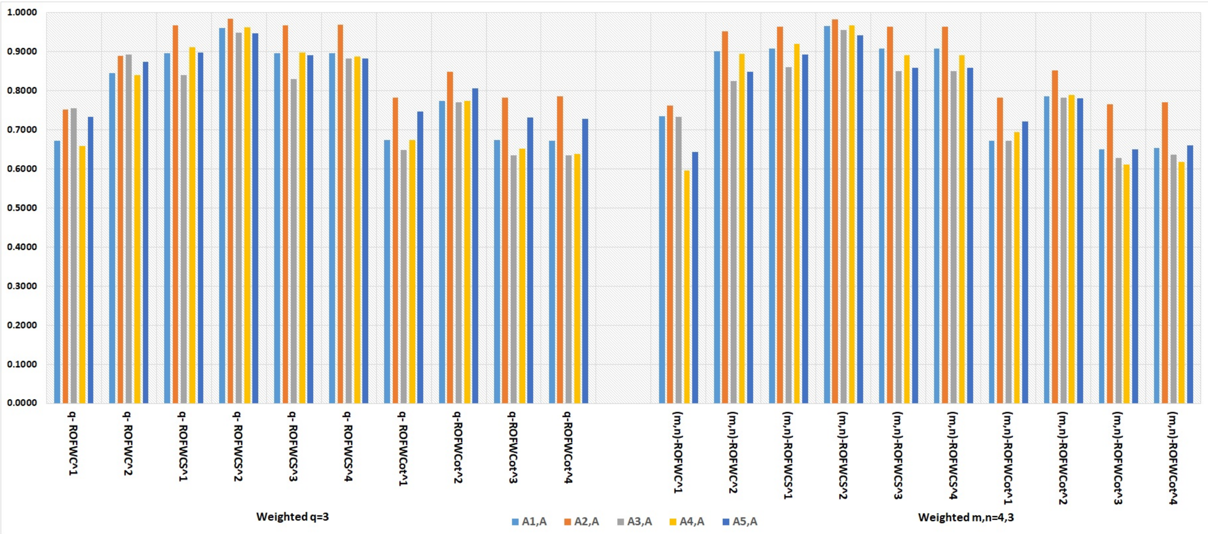

Figure 1. Comparison graph between weighted q-ROFWSs and the proposed weighted (m, n)-ROFWSs.

5. Applications of Proposed Similarity Measures in the Classification of Plant Leaf Disease

Plants are integral components of our ecosystem that provide us with vital resources such as oxygen and food. However, these crucial organisms are vulnerable to diseases that can significantly impact their growth and survival. One of the most prevalent problems plants face is leaf disease, which can considerably reduce crop yield and quality, significantly affecting farmers' livelihoods and the economy. This section aims to explore the issue of plant leaf disease and the measures that can be taken to prevent and manage it.

Several factors, such as bacteria, fungi, viruses, and other pathogens, can cause plant leaf diseases. Common types of leaf diseases include powdery mildew, downy mildew, leaf spot, and rust. These diseases can affect various parts of plants, including leaves, stems, and fruits, leading to discoloration, distortion, and wilting. The leaves may fall off in severe cases, leading to stunted growth and reduced yield. Several factors can facilitate the spread of plant leaf diseases, such as high humidity, poor air circulation, and contaminated soil or water. Additionally, using infected planting materials and inadequate crop management practices can contribute to the spreading of these diseases.

Plant leaf diseases pose significant problems that can adversely affect crop yield and quality, ultimately impacting farmers' livelihoods and the economy. Preventing and managing these diseases require a combination of preventive and curative measures, including disease-resistant plant varieties, good agricultural practices, and judicious chemical treatments. By taking these measures, we can ensure that our plants remain healthy and continue to provide us with the essential resources that we need for our survival.

Tomato plants are susceptible to various leaf diseases that can negatively impact their growth and yield. These diseases can be prevented by planting disease-resistant tomato varieties, keeping the soil well-drained, and avoiding overhead watering. Additionally, it is essential to remove any infected plant parts and keep the garden clean to prevent the spread of disease.

In the following illustrative example, we proposed a method to classify the plant leaf disease using the proposed cosine and cotangent similarity and weighted similarity measures for the study of the leaf disease classification that we generated Table 7 after carefully studying the dataset (the plant village dataset) [66] containing the different diseases and their symptoms.

Example 5. *Let us consider a set of five symptoms* $q = \{q_1, q_2, q_3, q_4, q_5\}$*, where* q_1 *= dark brown leaf,* q_2 *= brown leaf,* q_3 *= yellow leaf,* q_4 *= patches,* q_5 *= spots and five diagnoses* $\mathbb{Z}_i (i = 1, 2, 3, 4, 5)$*, which are presented by (m, n)-ROFSs,* $\mathbb{Z}_1$*(Gray leaf spot),* $\mathbb{Z}_2$ *(Bacterial Canker),* $\mathbb{Z}_3$ *(Bacterial Speck),* $\mathbb{Z}_4$ *(Bacterial Spot), and* $\mathbb{Z}_5$ *(Early Blight), defined in Table 7; let us also consider a sample pattern* $\mathbb{Z}$ *that will be recognized.*

Table 7. (m, n)-orthopair fuzzy data for the pattern of plant leaf diseases.

Symptom	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$	$\mathbb{Z}_5$	$\mathbb{Z}$
q_1	(0.45, 0.95)	(0.25, 0.75)	(0.95, 0.55)	(0.85, 0.45)	(0.15, 0.95)	(0.35, 0.70)
q_2	(0.95, 0.35)	(0.85, 0.25)	(0.35, 0.85)	(0.65, 0.45)	(0.25, 0.65)	(0.80, 0.30)
q_3	(0.95, 0.65)	(0.75, 0.35)	(0.95, 0.45)	(0.15, 0.95)	(0.95, 0.15)	(0.70, 0.40)
q_4	(0.45, 0.65)	(0.15, 0.95)	(0.85, 0.15)	(0.45, 0.75)	(0.95, 0.55)	(0.20, 0.90)
q_5	(0.55, 0.95)	(0.15, 0.85)	(0.55, 0.35)	(0.95, 0.15)	(0.55, 0.95)	(0.25, 0.80)

For the given plant leaf disease example, the proposed cosine and cotangent similarity measures for $(m, n) - ROFSs$ for the values m = 5, n = 7, and m = 6, n = 10, and m = 10, n = 100, and m = 100, n = 10 are shown in Tables 8–11. The results show consistent and accurate results, indicating that all ten proposed similarity measures show $\mathbb{Z}_2$ as having the largest value, suggesting that sample $\mathbb{Z}$ is the most similar to $\mathbb{Z}_2$.

For the given plant leaf disease example, the cosine and cotangent similarity measures $q - ROFSs$ for the values $q = 100$ and $q = 50$ are shown in Tables 12 and 13. The similarity measures for $q = 100$ and $q = 50$, respectively, show inconsistent results, indicating that the six $q - ROFSs$ similarity measures of Table 2, given by Wang et al [65], failed to classify.

Table 8. (m, n)-ROF similarity measures for the data presented in Table 7 for m = 7, n = 5.

Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-$ROFC^1$	0.9897	**0.9993**	0.2821	0.4250	0.6132	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3$
(m, n)-$ROFC^2$	0.5495	**0.9802**	0.5954	0.6602	0.5617	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFCS^1$	0.6733	**0.9836**	0.6536	0.7028	0.6630	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-$ROFCS^2$	0.9047	**0.9958**	0.8552	0.8728	0.8472	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5$
(m, n)-$ROFCS^3$	0.6400	**0.9836**	0.6515	0.7028	0.6598	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFCS^4$	0.6400	**0.9836**	0.6515	0.7028	0.6598	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFCot^1$	0.3917	**0.8477**	0.3889	0.4705	0.4105	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_3$
(m, n)-$ROFCot^2$	0.6402	**0.9194**	0.5816	0.6395	0.5958	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-$ROFCot^3$	0.3699	**0.8477**	0.3875	0.4705	0.4076	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFCot^4$	0.3699	**0.8477**	0.3875	0.4705	0.4076	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$

Table 9. (m, n)-ROF similarity measures for the data presented in Table 7 for m = 6, n = 10.

Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-$ROFC^1$	0.9698	**0.9996**	0.2162	0.4117	0.6037	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3$
(m, n)-$ROFC^2$	0.6335	**0.9703**	0.6921	0.7239	0.5816	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFCS^1$	0.7015	**0.9784**	0.7411	0.7358	0.6455	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFCS^2$	0.9185	**0.9946**	0.8969	0.9093	0.8682	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5$
(m, n)-$ROFCS^3$	0.6903	**0.9784**	0.7410	0.7358	0.6394	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFCS^4$	0.6903	**0.9784**	0.7410	0.7358	0.6394	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFCot^1$	0.4195	**0.8474**	0.5111	0.4871	0.3897	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFCot^2$	0.6648	**0.9193**	0.6545	0.6832	0.6141	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_5$
(m, n)-$ROFCot^3$	0.4110	**0.8474**	0.5111	0.4871	0.3847	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFCot^4$	0.4110	**0.8474**	0.5111	0.4871	0.3847	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$

Table 10. (m, n)-ROF similarity measures for the data presented in Table 7 for m = 10, n = 100.

Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-ROFC^1	0.4910	**0.8132**	0.8008	0.6008	0.4795	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFC^2	0.8465	**0.9984**	0.8203	0.9044	0.8260	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCS^1	0.8682	**0.9978**	0.8305	0.9060	0.8400	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCS^2	0.9655	**0.9995**	0.9555	0.9753	0.9578	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCS^3	0.8682	**0.9978**	0.8305	0.9060	0.8399	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCS^4	0.8682	**0.9978**	0.8305	0.9060	0.8399	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCot^1	0.7508	**0.9632**	0.6495	0.7750	0.7005	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCot^2	0.8580	**0.9811**	0.8031	0.8744	0.8300	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCot^3	0.7499	**0.9632**	0.6495	0.7750	0.6997	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFCot^4	0.7499	**0.9632**	0.6495	0.7750	0.6997	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$

Table 11. (m, n)-ROF similarity measures for the data presented in Table 7 for m = 100, n = 10.

Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-ROFC^1	0.7840	**0.9992**	0.5033	0.8000	0.6787	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFC^2	0.8241	**0.9750**	0.9691	0.8915	0.8231	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCS^1	0.8411	**0.9826**	0.9582	0.8942	0.8394	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCS^2	0.9586	**0.9956**	0.9894	0.9722	0.9579	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCS^3	0.8411	**0.9826**	0.9582	0.8942	0.8394	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCS^4	0.8411	**0.9826**	0.9582	0.8942	0.8394	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCot^1	0.6637	**0.8987**	0.8174	0.7496	0.6614	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCot^2	0.8097	**0.9462**	0.9007	0.8599	0.8085	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCot^3	0.6618	**0.8987**	0.8173	0.7496	0.6614	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFCot^4	0.6618	**0.8987**	0.8173	0.7496	0.6614	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$

Table 12. q-ROF similarity measures for the data presented in Table 7 for q = 100.

Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
q-ROFC^1	**1.0000**	**1.0000**	0.2000	0.4000	0.6000	cannot be classified
q-ROFC^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCS^1	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCS^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCS^3	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCS^4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCot^1	0.9926	**0.9982**	0.9963	0.9963	0.9926	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$
q-ROFCot^2	0.9963	**0.9991**	0.9981	0.9981	0.9963	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$
q-ROFCot^3	0.9926	**0.9982**	0.9963	0.9963	0.9926	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$
q-ROFCot^4	0.9926	**0.9982**	0.9963	0.9963	0.9926	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$

If we consider the weights of $w = (0.10, 0.30, 0.25, 0.15, 0.20)^T$, the proposed weighted cosine and cotangent similarity measures for (m, n)-ROFWSs with values of m = 5, n = 7, m = 6, n = 10, m = 10, n = 100, and m = 100, n = 10 are presented in Tables 14–17, respectively. The accurate and consistent results indicate that all ten proposed weighted similarity measures show $\mathbb{Z}_2$ as having the largest value, suggesting that the sample $\mathbb{Z}$ is the most similar to $\mathbb{Z}_2$.

Table 13. q-ROF similarity measures for the data presented in Table 7 for q = 50.

Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
q-ROFC^1	**1.0000**	**1.0000**	0.2000	0.4000	0.6000	cannot be classified
q-ROFC^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCS^1	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	cannot be classified
q-ROFCS^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFCS^3	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	cannot be classified
q-ROFCS^4	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	cannot be classified
q-ROFCot^1	0.9100	**0.9800**	0.9500	0.9500	0.9100	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$
q-ROFCot^2	0.9500	**0.9900**	0.9800	0.9800	0.9500	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$
q-ROFCot^3	0.9100	**0.9800**	0.9500	0.9500	0.9100	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$
q-ROFCot^4	0.9100	**0.9800**	0.9500	0.9500	0.9100	$\mathbb{Z}_2 > \mathbb{Z}_3 = \mathbb{Z}_4 > \mathbb{Z}_5 = \mathbb{Z}_1$

Table 14. Weighted (m, n)-ROFSs for the data presented in Table 7 for m = 7, n = 5.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-$ROFWC^1$	0.9897	**0.9991**	0.3256	0.4690	0.5595	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3$
(m, n)-$ROFWC^2$	0.5483	**0.9815**	0.6338	0.6444	0.6225	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCS^1$	0.6789	**0.9846**	0.6832	0.6876	0.7051	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCS^2$	0.9042	**0.9961**	0.8687	0.8699	0.8710	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3$
(m, n)-$ROFWCS^3$	0.6384	**0.9846**	0.6805	0.6876	0.7020	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCS^4$	0.6384	**0.9846**	0.6805	0.6876	0.7020	$\mathbb{Z}_2 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCot^1$	0.3958	**0.8507**	0.4105	0.4704	0.4534	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCot^2$	0.6401	**0.9209**	0.5972	0.6447	0.6266	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-$ROFWCot^3$	0.3696	**0.8507**	0.4088	0.4704	0.4506	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCot^4$	0.3696	**0.8507**	0.4088	0.4704	0.4506	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$

Table 15. Weighted (m, n)-ROF similarity measures for the data presented in Table 7 for m = 6, n = 10.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-$ROFWC^1$	0.9773	**0.9998**	0.2603	0.4557	0.5556	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4 > \mathbb{Z}_3$
(m, n)-$ROFWC^2$	0.6175	**0.9740**	0.7352	0.7171	0.6258	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCS^1$	0.6978	**0.9805**	0.7789	0.7331	0.6828	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFWCS^2$	0.9176	**0.9951**	0.9099	0.9069	0.8870	$\mathbb{Z}_2 > \mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5$
(m, n)-$ROFWCS^3$	0.6865	**0.9805**	0.7788	0.7331	0.6766	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFWCS^4$	0.6865	**0.9805**	0.7788	0.7331	0.6766	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFWCot^1$	0.4151	**0.8471**	0.5444	0.4931	0.4251	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCot^2$	0.6624	**0.9196**	0.6736	0.6852	0.6430	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-$ROFWCot^3$	0.4066	**0.8471**	0.5443	0.4931	0.4200	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCot^4$	0.4066	**0.8471**	0.5443	0.4931	0.4200	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$

Table 16. Weighted (m, n)-ROF similarity measures for the data presented in Table 7 for m = 10, n = 100.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-$ROFWC^1$	0.6350	**0.8506**	0.8132	0.6006	0.6293	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-$ROFWC^2$	0.7907	**0.9977**	0.8444	0.9067	0.8265	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCS^1$	0.8211	**0.9968**	0.8537	0.9096	0.8403	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCS^2$	0.9533	**0.9992**	0.9616	0.9762	0.9580	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCS^3$	0.8211	**0.9968**	0.8537	0.9096	0.8403	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCS^4$	0.8211	**0.9968**	0.8537	0.9096	0.8403	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3 > \mathbb{Z}_5 > \mathbb{Z}_1$
(m, n)-$ROFWCot^1$	0.6615	**0.9484**	0.6823	0.7860	0.6870	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCot^2$	0.8073	**0.9735**	0.8223	0.8803	0.8232	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCot^3$	0.6606	**0.9484**	0.6823	0.7860	0.6863	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$
(m, n)-$ROFWCot^4$	0.6606	**0.9484**	0.6823	0.7860	0.6863	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_3 > \mathbb{Z}_1$

Table 17. Weighted (m, n)-ROF similarity measures for the data presented in Table 7 for m = 100, n = 10.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
(m, n)-ROFWC^1	0.6803	**0.9987**	0.5636	0.8000	0.6590	$\mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_3$
(m, n)-ROFWC^2	0.8716	**0.9809**	0.9721	0.8739	0.8708	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_1 > \mathbb{Z}_5$
(m, n)-ROFWCS^1	0.8854	**0.9865**	0.9609	0.8789	0.8841	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCS^2	0.9702	**0.9966**	0.9901	0.9681	0.9697	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCS^3	0.8853	**0.9865**	0.9609	0.8789	0.8841	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCS^4	0.8853	**0.9865**	0.9609	0.8789	0.8841	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCot^1	0.7478	**0.9196**	0.8159	0.7391	0.7455	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCot^2	0.8573	**0.9573**	0.9007	0.8531	0.8565	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCot^3	0.7456	**0.9196**	0.8158	0.7391	0.7455	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$
(m, n)-ROFWCot^4	0.7456	**0.9196**	0.8158	0.7391	0.7455	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_1 > \mathbb{Z}_5 > \mathbb{Z}_4$

The $q - ROFWs$ weighted similarity measures in Table 3, given by Wang et al. [65], for the values q = 100 and q = 50 with same weighted values, and the results presented in Tables 18 and 19, shows that six weighted similarity measures failed to classify.

Table 18. Weighted (m, n)-ROF similarity measures for the data presented in Table 7 for q = 100.

Weighted Similarity Measures	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
q-ROFWC^1	**1.0000**	**1.0000**	0.2500	0.4500	0.5500	cannot be classified
q-ROFWC^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCS^1	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCS^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCS^3	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCS^4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCot^1	0.9921	**0.9986**	0.9968	0.9958	0.9935	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
q-ROFWCot^2	0.9961	**0.9993**	0.9984	0.9979	0.9967	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
q-ROFWCot^3	0.9921	**0.9986**	0.9968	0.9958	0.9935	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
q-ROFWCot^4	0.9921	**0.9986**	0.9968	0.9958	0.9935	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$

Table 19. Weighted (m, n)-ROF similarity measures for the data presented in Table 7 for q = 50.

Weighted Similarity Measure	$(\mathbb{Z}_1, \mathbb{Z})$	$(\mathbb{Z}_2, \mathbb{Z})$	$(\mathbb{Z}_3, \mathbb{Z})$	$(\mathbb{Z}_4, \mathbb{Z})$	$(\mathbb{Z}_5, \mathbb{Z})$	Ranking
q-ROFWC^1	**1.0000**	**1.0000**	0.2500	0.4500	0.5500	cannot be classified
q-ROFWC^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCS^1	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	cannot be classified
q-ROFWCS^2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	cannot be classified
q-ROFWCS^3	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	cannot be classified
q-ROFWCS^4	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	cannot be classified
q-ROFWCot^1	0.9000	**0.9800**	0.9600	0.9500	0.9200	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
q-ROFWCot^2	0.9500	**0.9900**	0.9800	0.9700	0.9600	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
q-ROFWCot^3	0.9000	**0.9800**	0.9600	0.9500	0.9200	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$
q-ROFWCot^4	0.9000	**0.9800**	0.9600	0.9500	0.9200	$\mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4 > \mathbb{Z}_5 > \mathbb{Z}_1$

6. Discussion

The existing cosine and cotangent similarity measures for IFS, PFS, and q-ROFs are special cases of the proposed (m, n)-ROFS similarity measure in the paper. The flexibility of the power of the MD and NMD attributes represents the uncertainty of attributes reflecting better performances for different values of m and n, compared to the other cases where the powers of the MD and NMD attributes are the same. The results of different values of m and n show consistent and accurate results for higher values of m and n in this paper. In q-ROFS, the similarity measure failed to classify the sets for the higher values of q, as shown in the paper.

After analyzing the data presented in several tables, it is clear that the most reliable classification of the plant leaf sample disease $\mathbb{Z}$ can be achieved using (m, n)-ROFSs and (m, n)-ROFWs. As a result, the plant leaf can be identified as disease $\mathbb{Z}_2$, a bacterial canker. However, other tables show that q-ROFSs and q-ROFWs are ineffective at classifying plant leaf diseases. These findings highlight the significance of the (m, n) condition for precise results and accurate disease classification. Overall, this analysis emphasizes the importance of selecting appropriate similarity measures for the specific type of fuzzy information to achieve accurate disease classification.

7. Conclusions

This research article demonstrates the effectiveness of (m, n)-ROFS, a generalized fuzzy structure, in addressing uncertainty and imprecision in decision-making problems. The (m, n)-ROFS framework surpasses other fuzzy structures, such as IFS, PFS, FFS, and q-ROFS (for $q > 3$), by accommodating a wider range of information. Our study introduces cosine, cotangent, and weighted similarity measures specifically designed for (m, n)-ROFSs, including measures for q-ROFSs information in special cases. We applied these similarity measures to evaluate their performances in building material problems. We compared the q-ROFSs cosine and cotangent measures with the existing ones. We also presented a numerical example to demonstrate the practical applications of these similarity measures in plant leaf disease classification.

The findings of our study indicate that the defined similarity measures are more suitable and applicable to real-world problems than existing measures. This research significantly contributes to decision-making under uncertainty and imprecision, providing improved tools for measuring similarity within the (m, n)-ROFSs framework. These findings have broad implications in domains such as medicine, pattern recognition, and material engineering, where robust decision-making techniques are essential. Future research can build upon this work by further exploring and expanding the (m, n)-ROFS framework and its similarity measures to address complex decision-making problems in diverse applications, by continually advancing the (m, n)-ROFS methodology, we can foster the innovation and improve the decision-making processes in various domains.

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