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Hidden Dynamics and Hybrid Synchronization of Fractional-Order Memristive Systems

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Abstract: A fractional-order memristive system without equilibrium is addressed. Hidden attractors in the proposed system are discussed and the coexistence of a hidden attractor is found. Via theoretical analysis, the hybrid synchronization of the proposed system with partial controllers is investigated using fractional stability theory. Numerical simulation verifies the validity of the hybrid synchronization scheme.

Keywords: hidden attractor; hybrid synchronization; fractional-order memristive system

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1. Introduction

In the past few decades, chaotic systems have received significant attention and were applied in many fields [1–4] because of their complex behavior and sensitivity to the initial value [5–7]. Then, many chaotic or hyperchaotic systems were investigated, such as chaotic Jerk circuit capabilities and their application in communication [8], a novel chaotic system deduced from different 3D five-term chaotic flows and its realization in electronic circuit [9], robust chaos in an exponential chaotic model [10], etc.

With further research on chaotic systems, fractional calculus [11] was introduced and many fractional-order systems were proposed [12–14]. Chaotic behavior of a fractional-order Liu system with time delay can be controlled to an appointed point via designing only one controller [15]. Effects of system parameters on the dynamics of a fractional-order system were revealed and dynamic behavior transition was given [16]. A fractional-order system with negative parameters is proposed and its complex dynamics were analyzed [17]. Recently, as a vital electronic component with complex dynamics, a memeristor was introduced into the circuit system and corresponding dynamics were investigated. For example, a delayed fractional-order system with a memristor was presented and the system's stability interval was deduced [18]; an active fractional-order memristor model was addressed and coexisting bifurcations as well as coexisting attractors were found [19]. Hidden attractors of Chua's circuit coupled with the memristor were found [20]. Existing results suggest that introducing a memristor into a chaotic system can make the system appear more complex in its dynamics. Simultaneously, it can be found that the mentioned memristor is complicated and has some difficulty in application. Therefore, a simple memristor should be explored.

As a vital collective behavior, synchronization fractional-order systems have been focused on. Various controlling methods were proposed to realize different kinds of synchronization. Function projective synchronization and generalized synchronization of fractional-order systems were achieved using tracking control [21] and the pole-placement technique with one controller [22], respectively. Synchronization between multidrive systems and one response system was obtained by designing suitable controllers [23]. By designing fractional-order proportion integral sliding mode surface, synchronization of fractional-order systems was realized [24]. Complete synchronization between fractional-order systems with external disturbance was realized by designing a feedback controller [25].

Finite time synchronization of fractional-order networks with discontinuous activation was discussed via discontinuous controller [26]. Global synchronization of delayed fractional-order networks was discussed and the upper bound of the setting time for synchronization was given [27]. Dual synchronization between fractional-order systems with uncertain parameters was explored with adaptive controllers and adaptive laws were depicted [28].

Based on existing results, hybrid synchronization of a fractional-order memristive system without an equilibrium point is to be considered. Other parts of this paper are arranged as follows: Section 2 describes preliminaries to be used in the research. In Section 3, the system to be investigated is introduced and its hidden attractors are discussed. In Section 4, a hybrid synchronization scheme is given and the result is verified via theoretical analysis as well as numerical simulations. In Section 5, some conclusions are drawn.

2. Preliminaries

A fractional-order differential operator can be regarded as an extended concept of an integer-order differential operator and can be written as

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}}, & \alpha > 0 \\ 1, & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0 \end{cases} \tag{1} $$

where α is the fractional order. With the development of a fractional-order derivative, several definitions were given, including Riemann–Liouville, Grünwald–Letnikov, and Caputo definition [28]. Much attention has been paid to the Caputo definition because its Laplace transformation formula has the same form as that of an integer-order derivative. Thus, the Caputo definition will be utilized in the next discussion, which is given as Definition 1 [29].

Definition 1. *Suppose $f(t)$ is a continuous function, Caputo fractional-order derivative of $f(t)$ with order α $(0 < \alpha \le 1)$ is denoted as*

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^m(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1 < \alpha < m, \\ \frac{d^m}{dt^m} f(t), & \alpha = m, \end{cases} \tag{2} $$

where $m = [\alpha]$ suggests the least integer no less than α. $\Gamma(\cdot)$ means Gamma function.

Specifically, for $0 < \alpha < 1$, Equation (2) can be simplified as

$$ {}_{t_0}^{c}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau \tag{3} $$

For simplicity, in the following discussion, ${}_{t_0}^{c}D_t^{\alpha} f(t)$ is denoted as $D^{\alpha} f(t)$.

3. System Description and Its Dynamical Behaviors

In this paper, to investigate the dynamics of the circuit system deeply, a kind of fractional-order system is constructed, refereed to the system in [30] and depicted as

$$ \begin{cases} D^{\alpha}x = y \\ D^{\alpha}y = (z + x^2 - \beta x^4)y - \omega_0^2 W(\omega)x \\ D^{\alpha}z = \mu - x^2 \\ D^{\alpha}w = x - w \end{cases}, \alpha \in (0,1) \tag{4} $$

where $W(w) = a + 2bw$ is memductance, a and b are positive parameters for controlling the behavior of memductance. Obviously, μ is a key parameter determining the solutions of system (4), which has no equilibrium for $\mu \neq 0$ and a single equilibrium point can be obtained for $\mu = 0$.

Compared to the system in [30], the novelty of system (4) lies in the linear property of the memristor, which is simple in construction and implication.

According to the description about the attractor [31–34], it can be known that continuous chaotic systems can be classified into two categories: self-excited attractors and hidden attractors. When the basin of attraction of attractors intersects with any arbitrary neighborhood of an unstable equilibrium point, it is called self-excited attractors; otherwise, it goes by the name of hidden attractors. Therefore, hidden attractors involve two cases. In one case, the basin of attraction of attractors does not intersect with any arbitrary neighborhood of an unstable equilibrium point. In another case, the continuous chaotic system itself has no equilibrium point but with attractors. In this manuscript, the case when the system has no equilibrium point is taken into account. In the following study, suppose $\mu \neq 0$, but system (4) can also generate various attractors with the change in the order of the system and other parameters, including periodic attractor, quasi-periodic attractor, and chaotic attractor. These attractors are hidden attractors. Additionally, coexisting attractors can be detected. These results are discussed as follows via numerical simulations.

3.1. Hidden Attractors in the Proposed System

Choose a = 1, b = 0.1, μ = 0.5, ω_0 = 2.01, β = 0.75 and initial value (0, 2, 0, 0), with α changing from 0.21 to 0.98, system (4) can show different hidden attractors, such as chaotic attractor (Figure 1), periodic attractor (Figure 2), and quasi-periodic attractor (Figure 3), which is confirmed in Figure 4. Furthermore, for a chaotic attractor, Lyapunov exponents are calcuted and depicted in Figure 5, which shows that when $\alpha = 0.28$, the largest Lyapunov exponent in system (4) is positive. It verifies the result in Figure 1.

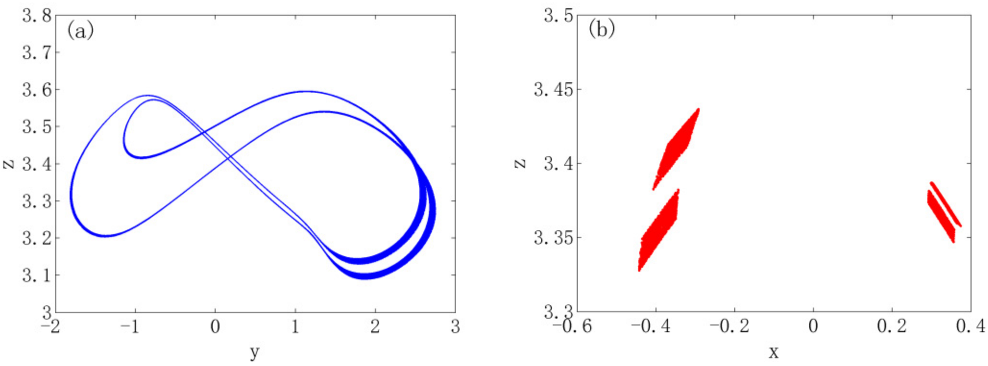

Figure 1. Hidden chaotic attractor of system (4) when $\alpha = 0.28$. (**a**) Phase trajectory diagram in yz plane; (**b**) Poincare map in xz plane.

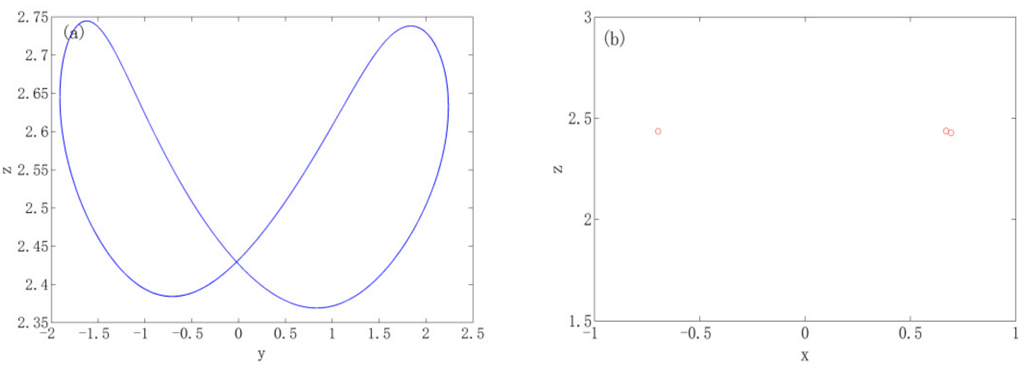

Figure 2. Hidden periodic attractor of system (4) when $\alpha = 0.5$. (**a**) Phase trajectory diagram in yz plane; (**b**) Poincare map in xz plane.

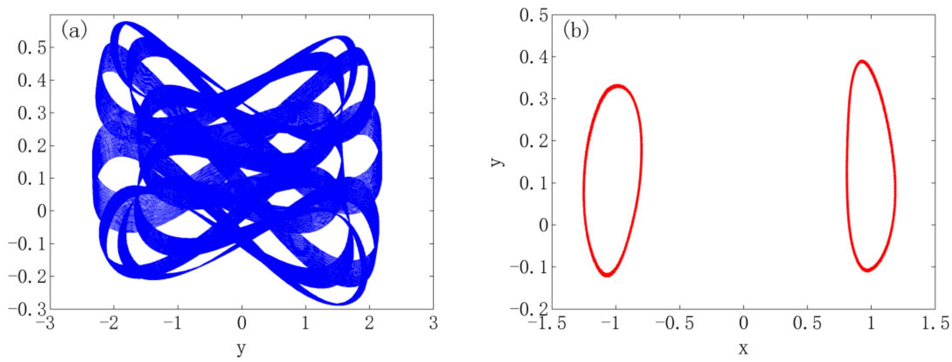

Figure 3. Hidden quasi-periodic attractor of system (4) when $\alpha = 0.95$. (**a**) Phase trajectory diagram in yz plane; (**b**) Poincare map in xz plane.

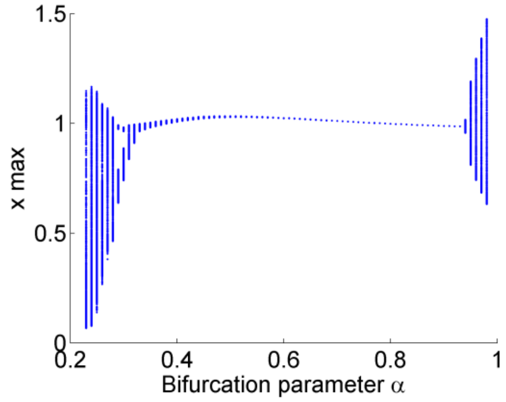

Figure 4. Bifurcation of x in system (4) with order α changing from 0.21 to 0.98.

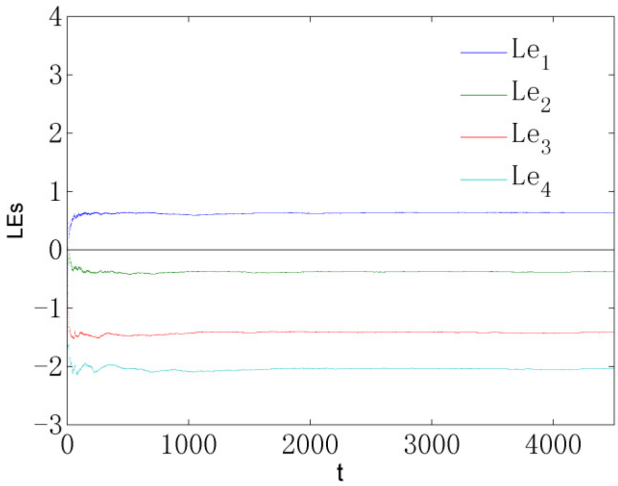

Figure 5. Lyapunov exponents of system (4) when $\alpha = 0.28$.

Select $a = 1$, $b = 0.1$, $\mu = 0.5$, $\omega_0 = 2.01$, initial value (0, 2, 0, 0), fixed $\alpha = 0.95$, numerical simulations suggest that system (4) can also appear with different hidden attractors with β changing from 0.1 to 0.9 (Figures 6–8). It can be verified in the bifurcation diagram in Figure 9. Then, Lyapunov exponents in system (4) are calculated and given in Figure 10, which indicates that, for $\beta = 0.3$, system (4) demonstrates a hidden chaotic attractor.

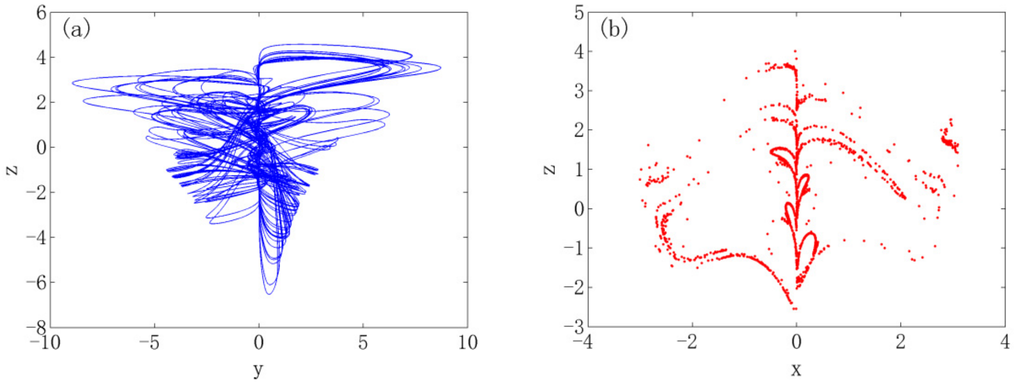

Figure 6. Hidden chaotic attractor of system (4) when $\beta = 0.30$. (**a**) Phase trajectory diagram in *yz* plane; (**b**) Poincare map in *xz* plane.

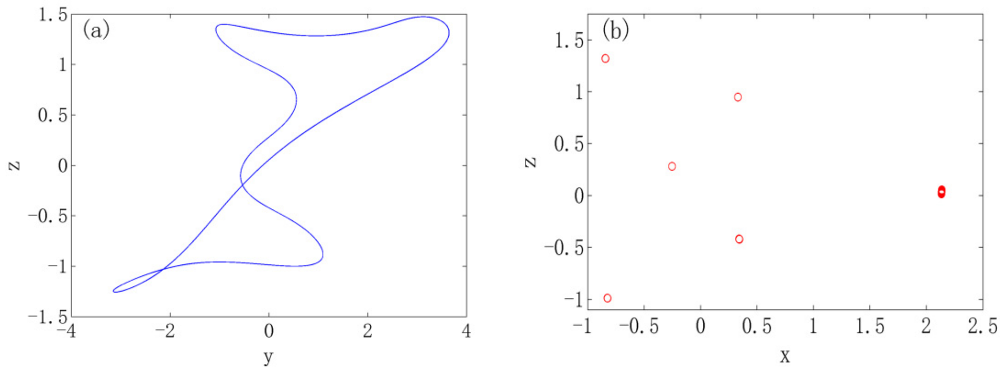

Figure 7. Hidden periodic attractor of system (4) when $\beta = 0.4$. (**a**) Phase trajectory diagram in *yz* plane; (**b**) Poincare map in *xz* plane.

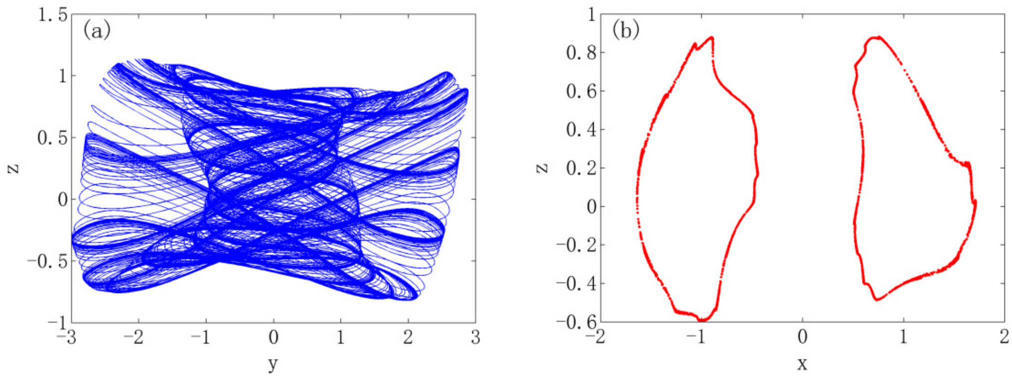

Figure 8. Hidden quasi-periodic attractor of system (4) when $\beta = 0.58$. (**a**) Phase trajectory diagram in *yz* plane; (**b**) Poincare map in *xz* plane.

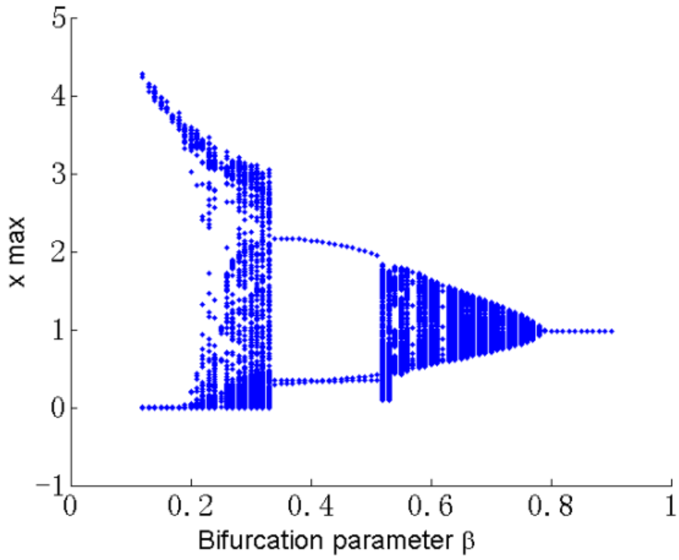

Figure 9. Bifurcation diagram of x in system (4) with change of β from 0.1 to 0.8.

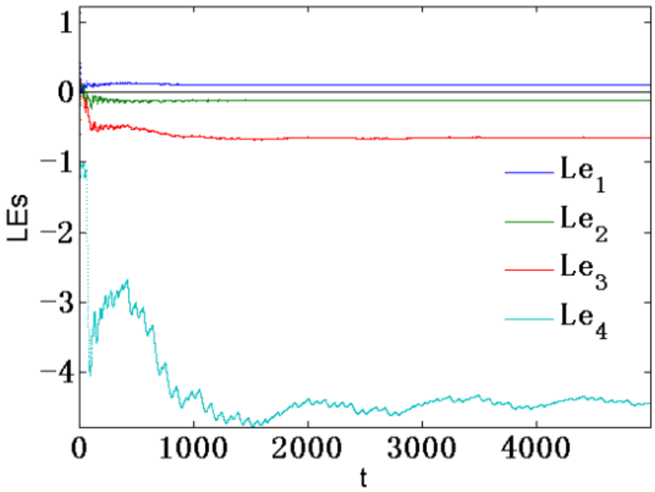

Figure 10. Lyapunov exponents in system (4) when $\beta = 0.3$.

3.2. Coexistence of Different Hidden Attractors

When system parameters and the order are all taken as fixed constants, the proposed system (4) can exhibit various hidden attractors according to different initial values, which means that the coexistence of different hidden attractors can be found in fractional-order system (4) (see Figures 11 and 12). Figure 11 depicts the phase trajectory and the Poincare map of the coexistences of hidden chaotic attractors in system (4) when $\alpha = 0.28$, $\beta = 0.8$. Phase trajectory and the Poincare map of the coexistences of the hidden chaotic attractor and 2D torus are shown in Figure 12, where $\alpha = 0.95$, $\beta = 0.58$. In all simulations, the other values of parameters are chosen as above.

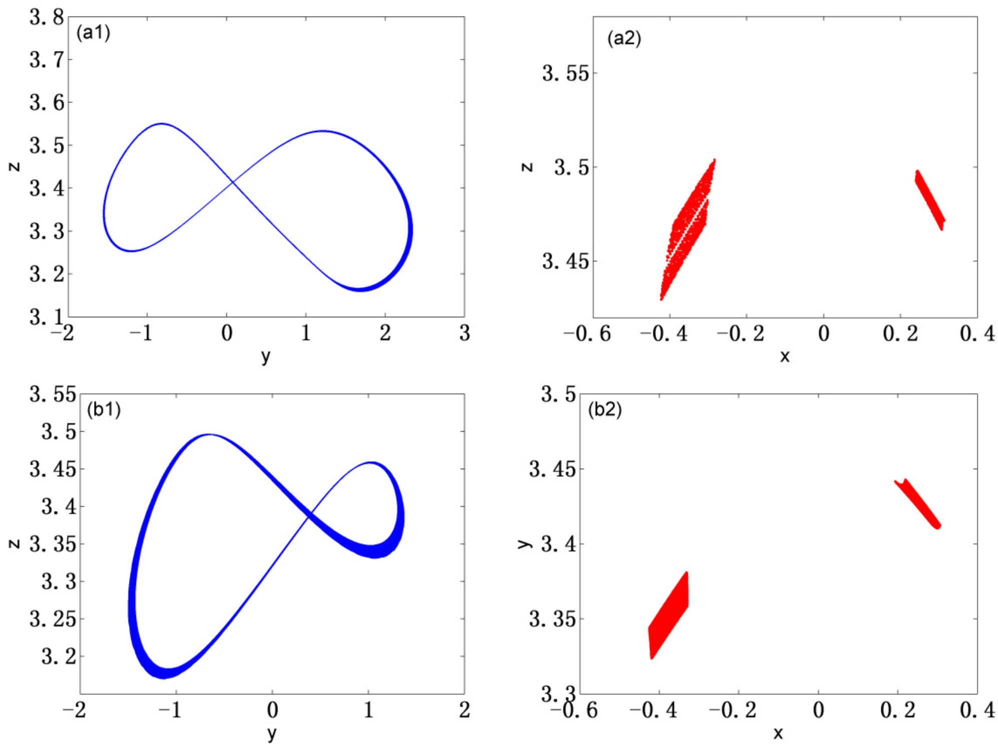

Figure 11. Coexistence of hidden chaotic attractors with symmetrical structure for different initial values. (**a1**,**a2**) are phase trajectory and Poincare maps with initial value (0, 2, 0, 0), respectively. (**b1**,**b2**) are phase trajectory and Poincare maps with initial value (1, −2, 0, 1), respectively.

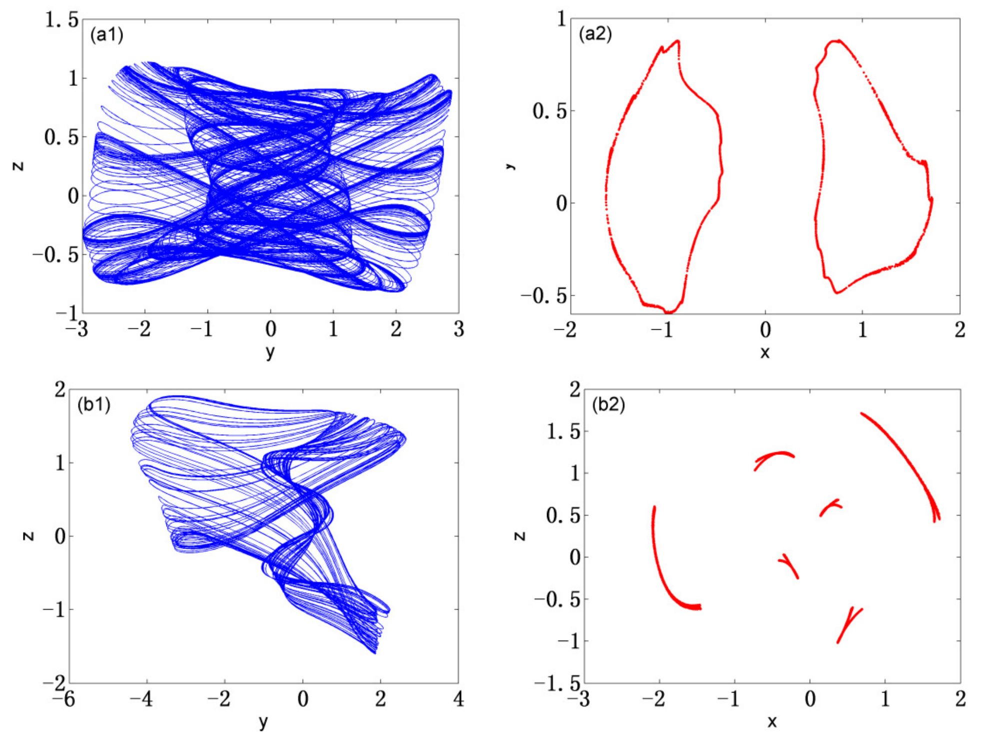

Figure 12. Coexistence of hidden quasi-periodic and chaotic attractor for different initial values. (**a1**,**a2**) are phase trajectory and Poincare maps with initial value (0, 2, 0, 0), respectively. (**b1**,**b2**) are phase trajectory and Poincare maps with initial value (1, −2, 0, 0), respectively.

4. Hybrid Synchronization Scheme

A scheme to realize the hybrid synchronization for system (4) is investigated utilizing fractional-order stability theory. For this purpose, the master system is taken as

$$\begin{cases} D^{\alpha}x_1 = y_1 \\ D^{\alpha}y_1 = (z_1 + x_1^2 - \beta x_1^4)y_1 - \omega_0^2 W(w_1)x_1 \\ D^{\alpha}z_1 = \mu - x_1^2 \\ D^{\alpha}w_1 = x_1 - w_1 \end{cases} \tag{5}$$

and corresponding slave system with controllers is written as

$$\begin{cases} D^{\alpha}x_2 = y_2 + u_1 \\ D^{\alpha}y_2 = (z_2 + x_2^2 - \beta x_2^4)y_2 - \omega_0^2 W(w_2)x_2 + u_2 \\ D^{\alpha}z_2 = \mu - x_2^2 + u_3 \\ D^{\alpha}w_2 = x_2 - w_2 \end{cases} \tag{6}$$

where $U_i (i = 1, 2, 3)$ are controllers to be determined.

To deliberate hybrid synchronization between systems (5) and (6), some definitions and lemmas are given as follows.

Lemma 1 ([35]). *Considering system*

$$D^{\alpha}X = A(X)X \tag{7}$$

where $X = (x_1, x_2, \cdots, x_n) \in R^n$ is state variable with $0 < \alpha < 1$. It is said that system (7) converges to be stable when every eigenvalue λ of $A(X)$ in (7) satisfies $|arg(\lambda)| \geq \alpha\pi/2$.

Lemma 2 ([36]). *The fractional-order system (7) will be stable if there is positive definite matrix P, for any variable X, $X^T P D^{\alpha} X \leq 0$ holds.*

Theorem 1. *If the controllers are chosen as*

$$\begin{cases} u_1 = -k_1(x_2 + x_1) \\ u_2 = -k_2(y_2 + y_1) \\ u_3 = -k_3(z_2 - z_1) \end{cases} \tag{8}$$

with k_i $(i = 1, 2, 3)$ being the feedback gains, then hybrid synchronization between systems (5) and (6) can be realized.

Proof. Let $e_1 = x_2 + x_1$, $e_2 = y_2 + y_1$, $e_3 = z_2 - z_1$ and $e_4 = w_2 + w_1$, and error system between systems (6) and (5) can be achieved as □

$$\begin{cases} D^{\alpha}e_1 = e_2 + u_1 \\ D^{\alpha}e_2 = -\omega_2^0 a e_1 + f_1 e_1 + f_2 e_2 + f_3 e_3 + f + u_2 \\ D^{\alpha}e_3 = -(x_2 - x_1)e_1 + u_3 \\ D^{\alpha}e_4 = e_1 - e_4 \end{cases} \tag{9}$$

where

$$f_1 = \beta(x_2^2 + x_1^2)(x_2 - x_1)y_1 - (x_2 - x_1)y_1,$$
$$f_2 = z_2 + x_2^2 - \beta x_2^4,$$
$$f_3 = -y_1,$$
$$f = -2b\omega_0^2(w_2 x_2 + w_1 x_1)$$

The f_1, f_2, f_3, f mentioned below are the same as here. Substitute (8) into (9) and we can obtain

$$\begin{cases} D^\alpha e_1 = e_2 - k_1(x_2 + x_1) \\ D^\alpha e_2 = -\omega_0^2 a e_1 + f_1 e_1 + f_2 e_2 + f_3 e_3 + f - k_2(y_2 + y_1) \\ D^\alpha e_3 = -(x_2 - x_1)e_1 - k_3(z_2 - z_1) \\ D^\alpha e_4 = e_1 - e_4 \end{cases} \tag{10}$$

Namely,

$$\begin{cases} D^\alpha e_1 = e_2 - k_1 e_1 \\ D^\alpha e_2 = -\omega_0^2 a e_1 + f_1 e_1 + f_2 e_2 + f_3 e_3 + f - k_2 e_2 \\ D^\alpha e_3 = -(x_2 - x_1)e_1 - k_3 e_3 \\ D^\alpha e_4 = e_1 - e_4 \end{cases} \tag{11}$$

Furthermore,

$$\begin{gathered} e_1 D^\alpha e_1 + e_2 D^\alpha e_2 + e_3 D^\alpha e_3 + e_4 D^\alpha e_4 = e_1(e_2 - k_1 e_1) + \\ e_2\left(-\omega_0^2 a e_1 + f_1 e_1 + f_2 e_2 + f_3 e_3 + f - k_2 e_2\right) \\ + e_3(-(x_2 - x_1)e_1 - k_3 e_3) + e_4(e_1 - e_4) \\ = e_1 e_2 - k_1 e_1^2 + z_2 e_2^2 - y_1 e_2 e_3 + x_2^2 e_2^2 - (x_2 - x_1) y_1 e_1 e_2 - \beta x_2^4 e_2^2 \\ + \beta\left(x_2^2 + x_1^2\right)(x_2 - x_1) y_1 e_1 e_2 - \omega_0^2 a e_1 e_2 - 2b\omega_0^2(w_2 x_2 + w_1 x_1) e_2 - k_2 e_2^2 - \\ (x_2 - x_1) e_1 e_3 - k_3 e_3^2 + e_1 e_4 - e_4^2 \end{gathered} \tag{12}$$

Due to the boundness of chaotic systems, there exist constant M, such that $|x_i| < M$, $|y_i| < M$, $|z_i| < M$, $|w_i| < M$ $(i = 1, 2)$. Therefore, Equation (12) can be calculated as

$$\begin{gathered} e_1 D^\alpha e_1 + e_2 D^\alpha e_2 + e_3 D^\alpha e_3 + e_4 D^\alpha e_4 \le \tfrac{1}{2}\left(e_1^2 + e_2^2\right) - k_1 e_1^2 + M e_2^2 + \tfrac{M}{2}\left(e_2^2 + e_3^2\right) + \\ M^2 e_2^2 + M^2\left(e_1^2 + e_2^2\right) + \beta M^2 e_2^2 + 2M^4 \beta\left(e_1^2 + e_2^2\right) + \tfrac{\omega_0^2 a}{2}\left(e_1^2 + e_2^2\right) - k_2 e_2^2 + \\ M\left(e_1^2 + e_3^2\right) - k_3 e_3^2 + \tfrac{1}{2}\left(e_1^2 + e_4^2\right) - e_4^2 \\ = \left(1 + M + M^2 + 2\beta M^4 + \tfrac{\omega_0^2 a}{2} - k_1\right) e_1^2 + \\ \left(\tfrac{1}{2} + \tfrac{3M}{2} + (2 + \beta) M^2 + 2\beta M^4 + \tfrac{\omega_0^2 a}{2} - k_2\right) e_2^2 + \\ \left(\tfrac{3M}{2} - k_3\right) e_3^2 - \tfrac{1}{2} e_4^2 \end{gathered} \tag{13}$$

Denote

$$\begin{gathered} L_1 = 1 + M + M^2 + 2\beta M^4 + \tfrac{\omega_0^2 a}{2} - k_1, \\ L_2 = \tfrac{1}{2} + \tfrac{3M}{2} + (2 + \beta) M^2 + 2\beta M^4 + \tfrac{\omega_0^2 a}{2} - k_2, \\ L_3 = \tfrac{3M}{2} - k_3, \end{gathered} \tag{14}$$

It is easy to know that, if k_1, k_2, k_3 are selected large enough, one can obtain $L_1 < 0$, $L_2 < 0$, $L_3 < 0$. Then, we can obtain that

$$e_1 D^\alpha e_1 + e_2 D^\alpha e_2 + e_3 D^\alpha e_3 + e_4 D^\alpha e_4 \le \ L_1 e_1^2 + L_2 e_2^2 + L_3 e_3^2 - \frac{1}{2} e_4^2 \le 0 \tag{15}$$

In line with Lemma 2, one can know that error system (10) or (11) will stabilize to zero under a feedback controller (8). That is to say, hybrid synchronization of systems (6) and (5) can be achieved with less controllers than the dimension of the system. Theorem 1 is proved.

To test the aforementioned result, the Adams–Bashforth–Moulton predictor-corrector algorithm in MATLAB program is used. In the following numerical simulations, system parameters are selected as $a = 1$, $b = 0.1$, $\mu = 0.5$, $\omega_0 = 2.01$, $\beta = 0.75$ and fractional order $\alpha = 0.98$. Initial values are chosen as $(x_1, y_1, z_1, w_1) = (0, 2, 0, 0)$, $(x_2, y_2, z_2, w_2) = (1,\ 0.2, -2, -10)$, respectively. Feedback gains in controllers are set as $k_1 = k_2 = k_3 = 2.1$. The time evolutions in the error system are given in Figure 13, from which it can be obtained that the error states converge to zero quickly, which indicates that hybrid synchronization between the master system (5) and slave system (6) can be achieved in a short time.

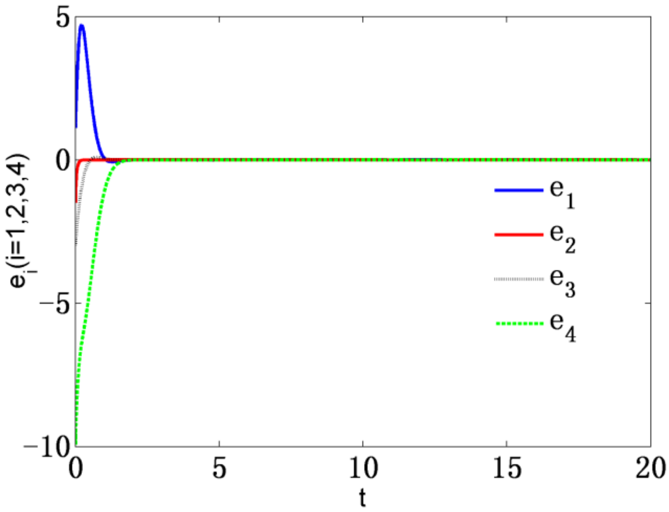

Figure 13. Curves of errors in system (9) with controllers (8).

5. Conclusions

In this paper, a novel fractional-order memristive system without equilibrium is presented via introducing a memristor into the considered system [3]. Some dynamics of the mentioned system are investigated. Some results are obtained as follows. (1) Various hidden attractors are found via altering the order of the system and value of system parameter. (2) Different coexistences of hidden attractors are detected via numerical simulations. (3) A hybrid synchronization scheme is put forward utilizing less controllers than the dimension of the system by fractional-order stability theory and effectiveness of the considered scheme is tested via numerical simulation.

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