axioms

Article

Multi-Criteria Group Decision-Making Models in a Multi-Choice Environment

Qazi Shoeb Ahmad [1], Mohammad Faisal Khan [2,*] and Naeem Ahmad [3]

[1] Department of Statistics, Deanship of Educational Services, Qassim University, Buraydah 51452, Saudi Arabia
[2] Department of Basic Science, College of Science & Theoretical Studies, Saudi Electronic University, Riyadh 11673, Saudi Arabia
[3] Department of Civil Engineering, College of Engineering, Qassim University, Unaizah 56452, Saudi Arabia
* Correspondence: f.khan@seu.edu.sa

Abstract: The best–worst method (BWM) has recently demonstrated its applicability in addressing various decision-making problems in a practical setting. The traditional BWM method is based on deterministic information gathered from experts as pairwise comparisons of several criteria. The advantage of BWM is that it uses fewer calculations and analyses while maintaining good, acceptable consistency ratio values. A multi-choice best–worst method (MCBWM), which considers several options for pairwise comparison of preferences between the criteria, has recently been developed. The experts are given the option to select values from several comparison scales. The MCBWM technique has been shown to be better. Presenting the options for which an optimal solution has been found simplifies the calculation and establishes the ideal weight values. This study proposes two different mathematical programming models for solving multi-criteria decision-making problems having multiple decision-makers. The two methods are proposed considering the multi-choice uncertainty assumption in pairwise criteria comparisons. Additionally, it considers the best–worst method as the base model. The multi-choice uncertainty is applied to determine the best choice out of multiple choices. It gives a real-life scenario to the decision-making problems. Although there are many other forms of uncertainty, such as fuzzy, intuitionistic fuzzy, neutrosophic, probabilistic, etc., it focuses on choices instead of ambiguity in terms of the probabilistic or fuzzy nature of parameters. The parameter considered as multi-choice is the pairwise comparison. These parameters are handled by applying the Lagrange interpolating polynomial method. The proposed models are novel in terms of their mathematical structure and group decision-making approach. The models are formulated and further validated by solving numerical examples. It provides a framework for solving mcdm problems where the weightage to the decision-makers is also incorporated. The CR values for all the models of example 1 and 2, and the case study has been found acceptable.

Keywords: best–worst method; multi-choice best–worst method; multi-criteria decision-making; group decision-making; mathematical programming

Citation: Ahmad, Q.S.; Khan, M.F.; Ahmad, N. Multi-Criteria Group Decision-Making Models in a Multi-Choice Environment. *Axioms* **2022**, *11*, 659. https://doi.org/10.3390/axioms11110659

Academic Editors: Goran Ćirović and Dragan Pamučar

Received: 17 September 2022
Accepted: 15 November 2022
Published: 21 November 2022

1. Introduction

Multiple-criteria decision-making (MCDM) plays an important role in real-life decision-making problems. The MCDM approach determines the ranking of the options and selects the best option using the appropriate approach based on certain criteria. There are an enormous number of applications of MCDM methods in real-life problems. These methods include elimination and choice expressing reality (ELECTRE) [1,2], data envelopment analysis (DEA) [3,4], analytic hierarchy process (AHP) [5,6], preference ranking organization method for enrichment evaluations (PROMETHEE) [7,8], VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [9,10], decision-making trial and evaluation laboratory (DEMATEL), the technique for order of preference by similarity to ideal solution (TOPSIS) [7,11], analytic network process (ANP) [12,13], and best–worst method (BWM) [14,15].

The best suitable method to handle an MCDM problem is determined based on the structure of the problem of decision-making. In general, the process of MCDM consists of multiple steps from formulation to identification of criteria to decision metric and, finally, calculation of weights and rank of criteria.

Among the above-mentioned MCDM approaches, In 2015, Rezai [14] developed an MCDM method named as the best–worst method. As compared to the mostly applied MCDM approach, i.e., AHP, BWM has shown a more reliable approach as it takes fewer number pairwise comparisons leading to fewer calculations and, hence, low inconsistency of pairwise comparisons.

Let us consider for AHP [16], a matrix X having $x_{ij}; i, j = 1, 2, 3, \ldots, n$ as a pairwise importance comparison between the i-th criteria and the j-th criteria. from the n criteria $C_1, C_2, C_3, \ldots, C_n$, then the comparison matrix X will become as follows:

$$X = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix} \end{array} \tag{1}$$

where x_{ij} represents the degree of relative importance (or the relative preference) of criterion C_i over criterion C_j, the values of x_{ij} range between the $1/9$ to 9 scale. For an equal preference of criterion C_i over C_j, x_{ij} will take value 1. $x_{ij} \geq 1$ shows that criterion C_1 is relatively more important than criterion C_2, with $x_{ij} = 9$ exhibiting the extreme preference of criterion C_i over C_j. The relative importance of C_j over C_i is given by x_{ji}. It is essential that the relative importance of C_i over C_i is ($x_{ii} = 1$) and $x_{ij} = 1/x_{ji}$, $\forall i, j$. For X to be perfectly consistent $x_{ik} \times x_{kj} = x_{ij}, \forall i, j$.

In comparison to AHP, the BWM approach is based on pairwise comparisons between the best criterion to all other criteria and all others to the worst criterion. All the favorites are assigned to a number through a scale ranging from 1 to 9. Since the secondary comparisons are not performed, the BWM technique seems to be more convenient, very precise, and far less redundant [14]. Meanwhile, executing a pairwise comparison, a decision-maker expresses both direction and the strength of the preference of i over j. In most situations, the decision-maker has no problem in conveying the direction; however, representing the strength of the choice is a difficult task that is almost the primary source of inconsistency. Since the degree of preference relation is assigned with a numeric number, it may be possible that the provided value is insufficient to accommodate the uncertainty with the linguistic term. Based on the above discussion, in this paper, the pairwise reference comparisons are considered multi-choice comparisons [17,18]. While taking responses from decision-makers (DMs), uncertain information may come up in terms of options. When comparing criterion i over j pairwise, the DM may offer many responses.

Multi-choice programming problem [17,18] is a sort of mathematical programming in which the goal is to select the best alternative among multiple potential combinations to optimize an objective function under a number of constraints. A good review of multi-choice mathematical programming is presented in [19]. The situation of multiple choices for a parameter exists in many managerial decision-making problems. Choosing the optimum combination of parameter values from a variety of parameter values is aided by the multi-choice programming approach. Our goal is to better comprehend the supposed multi-choice parameter, which is multiple opinions in the pairwise comparison that forms the foundation of our proposed method. In the MCDM problem, the multi-choice parameters were essential when a specialist was unsure of the significance of a certain criterion. It is due to a lack of information, unclear criteria, unfavorable factors, and the decision-point makers of view and judgment. The reference comparison's reference parameters with several choices will increase the decision-making problems' flexibility. It prompts the decision-makers to use not only a single value but more than one benefit to compare criterion i over j. This approach of pairwise comparison results in inconsistency. The decision-makers are supported in their efforts to comprehend accurately and correctly with regard to their final decisions by the obtained value of inconsistency. This approach aids in examining the consistency of the assessments of the decision's importance. Because there are so many options for criteria, it is possible to take into account every viewpoint when comparing one criterion to another in pairs. The multi-choice best–worst method is a novel MCDM strategy that is introduced in this research to supplement the BWM for multi-choice comparisons.

The BWM method is a well-established, tested, and verified method. Since its inception, it has been applied to various kinds of real-life problems. A lot of extensions, including uncertainty and hybrid methods, have also been proposed in recent years. Additionally, we have seen that there is no extension of BWM with multi-choice parameters in a group decision-making scenario. This motivated us to carry out this proposed work. In recent years, some new methods have also been developed for determining the weights of the criteria, such as the fully consistency method (FUCOM) [20], level-based weight assessment (LBWA) [21], and defining interrelationships between ranked (DIBR) [22]. In literature, it has been shown that these methods are better than the AHP and BWM. The reason behind it is the less number of pairwise comparisons, higher level of consistency, requires an $(n-1)$ number of pairwise comparisons, reliability of results, and simple algorithm. As a limited study has only been carried out using them, so we have incorporated the multi-choice concept for group decision-making in the best–worst method. This work is not yet explored by anyone. The key reasons for the motivation behind this work are as follows:

- Numerous group decision-making models have been put out in the literature, but no one has ever taken into account pairwise comparisons as multi-choice parameters.
- Multiple options for a parameter, such as pairwise comparisons, are another type of uncertainty that can be managed in real-world problems utilizing the multi-choice mathematical programming approach in MCDM.
- The decision that determines whether a solution is optimal is revealed in the solution that results from solving the proposed models. The goal is to reduce inconsistency. Therefore, multiple models can be solved to find the option with the least amount of inconsistency.

Following are the main objectives of this study:

- This study's major goal is to provide group decision-making techniques that incorporate the freedom to select several options for pairwise comparisons.
- To validate the proposed model by applying it to experimental studies.

Rest of this paper is organized as follows: The next Section 2 is about the literature survey. Some preliminaries are presented in the next Section 3. The proposed models incorporating multi-choice in group decision-making are presented in Section 4. In Section 5, an experimental study conducted using the proposed approach is presented. In Section 6, a case study of the piping selection problem has been presented. Finally, the manuscript concludes in Section 7 and discusses the future directions of the work along with the limitations of the proposed work.

2. Literature Survey

When choosing, organizing, and prioritizing various actions, MCDM considers the decision-makers subjectivity. It also examines the acceptability of alternative options in light of the resources at hand. There are multiple kinds of MCDM methods for ranking and prioritization of criteria. In MCDM, the best–worst method [14] has played a key role in solving many kinds of real-life decision-making problems. Initially, it was applied to a mobile selection problem. Later, It was applied in linking supplier segmentation [23] for enhancing the supplier development model. Gupta and Barua [24] worked on micro-small and medium-sized enterprises (MSMEs) in India, where they found the most significant enablers of technological innovation. Recently [25], the flexibility of information granularity is integrated with the best–worst method along with interval and type-2 fuzzy sets in linguistic terms. A novel model by Tavana et al. [26] has proposed a model which combines the compromise solution with the best–worst method. Malakoutikhah et al. [27] have incorporated fuzzy uncertainty in best–worst method and cognitive map and further applied it in the modeling of criteria and subcriteria affecting unsafe behaviors in organizational, individual, and socio-economic domains. Kharola et al. [28] utilized the best–worst method in prioritizing factors associated with green waste management in the food supply chain. They prioritized a total of 5 criteria and 25 sub-criteria. Bilbao et al. [29] incorporated multiple reference point concepts in the best–worst method and showed application in the assessment of non-life insurance companies. Sadaghiani et al. [30] studied the importance of external forces on the supply chain sustainability in the oil and gas industry; Groenendijk et al. [31] applied it to improve the quality of public transportation, water scarcity management [32], failure mode and effects analysis [33], the judgment of investment projects [34] etc. A good review of BWM articles is presented in [35]. Combination of BWM with other MCDM methods, such as TOPSIS [36], MULTIMOORA [37], VIKOR [38], FDA [39], and ELECTRE [40].

Researchers incorporated uncertainties, such as fuzzy, intuitionistic fuzzy, neutrosophic fuzzy [41], hesitant fuzzy [42], spherical fuzzy [43], probabilistic [44], interval type-2 fuzzy [45], and Bayesian [46] in the best–worst method. A Fuzzy hybrid BWM along with the geographic information system has been applied for the power station selection problem [47]. Fuzzy BWM has been applied to assess the potential environmental impacts of the process of ship recycling [48]. A rough BWM has been applied to the problem of prioritizing recovery solutions to the tourism sector after COVID-19 [49]. BWM has been applied for the land valuation model in three different scenarios [50]. An integrated model of BWM with superiority and inferiority ranking applied in an environment of probabilistic dual hesitant fuzzy sets to a Green supplier selection problem [51]. In [52], a probabilistic-based hybrid model has been proposed for solving group decision-making problems by combining BWM and Bayesian approaches to assess the quality index of medical devices. In [53], blockchain technology has been assessed using Bayesian BWM. A cost–benefit analysis of shale development in India has been carried out using the best–worst method approach [54]. In [55], identification and prioritization of criteria to tackle the COVID-19 outbreak has been carried out. Ref. [56] applied fuzzy BWM in prioritizing factors affecting ad hoc wireless networks. There are many applications of BWM, along with uncertainty, that exists in the literature.

The BWM model is a kind of linear mathematical programming model [15]. Linear mathematical programming refers to mathematical models having mathematical equations for an objective function that is needed to be achieved—with an optimal value under some set of constraints. The condition is that all equations should be of linear nature [57]. Multi-choice mathematical programming is a kind of mathematical programming having multi-choice parameters [18]. In the work of Hasan et al. [58], they have assumed a situation where the pairwise comparison parameter is considered multi-choice in nature. Hasan et al. [58] incorporated a type of uncertainty in the multi-choice form in the best–worst method. They have shown that having multiple choices for pairwise comparison of two criteria can be chosen by the experts instead of using any other kind of uncertainty, such as probabilistic, fuzzy, neutrosophic, etc. They have handled it using Lagrange interpolation and chosen those choices for which inconsistency has been minimized. Their approach has shown a significant decrease in inconsistency. This approach has not yet been explored in the case of group decision-making problems. So, on the basis of the above discussion, it can be observed that the researchers have not considered the most critical point, which is employing a multi-choice mathematical programming model to the MCDM problems in a group decision-making scenario. This has motivated me to work on such models, where multi-choice uncertainty can be incorporated into group decision-making problems. The issue, as mentioned above, is vital in the decision-making of real-world problems. Therefore, the present work focuses on a multi-choice mathematical programming model for group decision-making problems. This work presents two mathematical models. Both models are different with respect to the constraints and objective function. The authors have shown the approaches to solving group decision-making problems.

3. Preliminaries

As preliminary, this section provides an overview of Multi-choice mathematical programming models and the Lagrange interpolation method as follows:

Multi-choice mathematical programming:

It is a mathematical programming model to determine the solution set X, where X is $(x_1, x_2, x_3, \ldots, x_n)$. The objective is the maximization of the function denoted by Z. The model is as follows:

$$Max\ Z = \sum_{j=1}^{n} C_j x_j \tag{2}$$

$$subject\ to\ \sum a_{ij} x_j \leq \{b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, \ldots, b_i^{(k_i)}\},\ i = 1, 2, \ldots, m, \tag{3}$$

$$x_j \geq 0,\ j = 1, 2, \ldots, n. \tag{4}$$

The choice set of parameter b is $\{b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, \ldots, b_i^{(k_i)}\}$ with k_i number of choices. The solution set will consist of only one choice from the choice set, which is to be selected for the optimized model.

To determine the solution of the above model, it is necessary to convert it into the standard form of mathematical programming. There are many interpolating polynomial methods [17] for handling the multi-choice parameter b, such as using Lagrange, Newton's divided differences, Newton's forward difference, and Newton's backward difference interpolating polynomial. For the proposed extended methods, the Lagrange interpolation method is utilized.

Lagrange interpolation:

Let $0,\ 1,\ 2, \ldots,\ (k_i - 1)$ be k_i number of node points, where $b_i^{(1)},\ b_i^{(2)},\ \ldots,\ b_i^{(k_i)}$ are the associated functional values of the interpolating polynomial at k_i different node points. A polynomial $P_{k_{i-1}}(z^{(i)})$ of degree $(k_i - 1)$ which interpolates the given data is:

$$\begin{aligned} P_{k_{i-1}}(z^{(i)}) = & \frac{(z^{(i)}-1)(z^{(i)}-2)\ldots(z^{(i)}-k_i+1)}{(-1)^{k_i-1}(k_i-1)!} b_i^{(1)} + \frac{z^{(i)}(z^{(i)}-2)\ldots(z^{(i)}-k_i+1)}{(-1)^{k_i-2}(k_i-2)!} b_i^{(2)} \\ & + \frac{z^{(i)}(z^{(i)}-1)(z^{(i)}-3)\ldots(z^{(i)}-k_i+1)}{(-1)^{k_i-3}(k_i-3)!2!} b_i^{(3)} \\ & + \ldots + \frac{z^{(i)}(z^{(i)}-1)(z^{(i)}-2)\ldots(z^{(i)}-k_i+2)}{(k_i-1)!} b_i^{(k_i)}\ \forall\ i = 1,\ 2,\ 3, \ldots m. \end{aligned} \tag{5}$$

4. The Proposed Multi-Criteria Group Decision-Making Models in Multi-Choice Environment

Suppose k numbers of decision-makers are represented by the set $D = \{D_1, D_2, \ldots, D_k\}$, where D_k $(k \in 1, 2, \ldots, m)$ denotes the k-th expert. There are n numbers of decision criteria $C = \{c_1, c_2, \ldots, c_n\}$ with $n \geq 2$, where c_j $(j \in \{1, 2, \ldots, n\})$ indicates the j-th criteria. The expertise of the decision-makers in expressing their importance in comparing criteria may be identical or different. The criterion's importance is described using finite, pre-specified, and ordered linguistic terms sets, as shown in Table 1. The k-th decision-maker describes the multi-choice preference details $S_{ij}^k = \{1, 2, \ldots, s_{ij}\}^k$; $i, j \in 1, 2, \ldots, n$, where s_{ij} represents the linguistic term of the i-th criterion, over j-th criterion with T_k cardinalities, i.e., $S^k = \{s_1^k, s_2^k, s_3^k, \ldots, s_{T_k}^k\}$.

The notations used in this paper are presented in Table 2. The steps involved in the proposed group multi-choice best–worst method (GMCBWM) to obtain the criteria weights are described as follows.

Table 1. Linguistic expression for reference comparison.

Value	Linguistic Expression	Value	Linguistic Expression
1	EI: Equal important	2	IEM: Intermediate b/w EI and MI
3	MI: Moderate important	4	IMI: Intermediate b/w MI and I
5	I: Important	6	IVI: Intermediate b/w I and VI
7	VI: Very important	8	IEI: Intermediate b/w VI and EXI
9	EXI: Extreme important		

Table 2. Symbols/Notations used in this study.

Notations	Descriptions
n	Number of Criteria in group decision-making system
m	number of decision-makers in group decision-making system
$c_1, c_2, \ldots, c_n$	Criteria of group decision-making system with indices of $i, j = 1, 2, \ldots, n$
$D_1, D_2, \ldots, D_k$	Decision-makers of group decision-making system with index of $k = 1, 2, \ldots, m$
S^k	Number of multi-choices in the reference comparisons of k-th decision-maker
$\{x_{ij}^{s_{ij}}\}^k$	Multi-choice preference comparison of criterion i with j having $s_{ij} \geq 1$ number of possibilities for decision maker k
c_B	Best criterion with index B
c_W	Worst criterion with index W
X_B^k	Multi-choice preference comparisons best-to-others vector for k-th decision-maker
X_W^k	Multi-choice preference comparisons others-to-worst vector for k-th decision-maker
P^k	Interpolation polynomial of k-th decision-maker
λ_k	Weight of k-th decision-maker ($\lambda_k \in [0,1]$ and $\sum_k \lambda_k = 1$)
ξ_k	The consistency ratio for the k-th decision-maker
ω_i	The weight of criterion i
ξ	The consistency ratio group decision-making
CI	Consistency index
CR_k	Consistency ratio of k-th DM in the group decision-making problem
CR^G	Overall consistency ratio of the group decision-making problem

(I). Defining the system of decision-making mechanism.

The decision criteria system comprises a collection of decision criteria essential in making a judgment on diverse alternatives to consider—assuming n number of decision criteria, $\{c_1, c_2, \ldots, c_n\}$. Then, the decision matrix can be presented as follows.

$$X^k = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} \{x_{11}^{(1)}, x_{11}^{(2)}, \ldots, x_{11}^{(s_{11})}\}^k & \{x_{12}^{(1)}, x_{12}^{(2)}, \ldots, x_{12}^{(s_{12})}\}^k & \cdots & \{x_{1n}^{(1)}, x_{1n}^{(2)}, \ldots, x_{1n}^{(s_{1n})}\}^k \\ \{x_{21}^{(1)}, x_{21}^{(2)}, \ldots, x_{21}^{(s_{21})}\}^k & \{x_{22}^{(1)}, x_{22}^{(2)}, \ldots, x_{22}^{(s_{22})}\}^k & \cdots & \{x_{2n}^{(1)}, x_{2n}^{(2)}, \ldots, x_{2n}^{(s_{2n})}\}^k \\ \vdots & \vdots & \ddots & \vdots \\ \{x_{n1}^{(1)}, x_{n1}^{(2)}, \ldots, x_{n1}^{(s_{n1})}\}^k & \{x_{n2}^{(1)}, x_{n2}^{(2)}, \ldots, x_{n2}^{(s_{n2})}\}^k & \cdots & \{x_{nn}^{(1)}, x_{nn}^{(2)}, \ldots, x_{nn}^{(s_{nn})}\}^k \end{bmatrix} \end{array} \quad (6)$$

where, X^k represents the pairwise multi-choice comparison matrix of the k-th decision-maker and $\{x_{ij}^{(s_{ij})}\}^k$ denotes the relative importance of criteria i to criterion j with (s_{ij}) multiple reference comparisons. Additionally, $X_i^k = \left(\{x_{11}^{(1)}, \ldots, x_{11}^{(s_{11})}\}^k, \{x_{12}^{(1)}, \ldots, x_{12}^{(s_{12})}\}^k, \cdots, \{x_{1n}^{(1)}, \ldots, x_{1n}^{(s_{1n})}\}^k\right)$ is the vector of the i-th index's multi-choice preference relation to other indexes. Furthermore, the multi-choice pairwise comparison matrix X^k is deemed entirely consistent if and only if the following conditions are met:

$$\{x_{ip}^{s_{ip}}\}^k \times \{x_{pj}^{s_{pj}}\}^k = \{x_{ij}^{s_{ij}}\}^k \quad \forall i, j \tag{7}$$

(II). Finding the best and the worst criteria to use.

Decision-makers must recognize the best and worst, typically focused on the built-in decision criteria mechanism of the previous step. The best and worst selection by assigning c_B^k to the best and c_W^k to the worst criterion for the k-th decision-maker. Afterward, it can determine whether two criteria experts often used in the great majority, c_B^k and c_W^k, are the best and worst criteria. When several experts make recommendations, the best and worst criteria are often subjective, resulting in various c_B^k and c_W^k. To eventually incorporate the c_B^k and c_W^k of various experts, it is required to choose a set of widely accepted best and worst criteria in front.

(III). Conducting multi-choice preference comparisons to determine best-to-others and others-to-worst criteria.

This step defines pairwise multi-choice preference comparisons of best criteria B over other criteria $j(j = 1, 2, \ldots, n)$, designated by X_B^k, of the k-th decision-maker. The decision-makers pairwise multi-choice comparisons of all criteria j over the worst criterion W, represented by X_W^k, are termed others-to-worst. The decision-makers determine pairwise perceptions on the order of 1 to 9, as said in Table 1. Although there are several options possible when it comes to such judgments. Figure 1 depicts the best-to-others X_B^k and others-to-worst X_W^k vector multi-choice preferences with many alternatives. The resulting multi-choice best-to-others vector is denoted by X_B^k in Equation (8) as follows:

$$X_B^k = \left(x_{B1}^{S_{B1}}, x_{B2}^{S_{B2}}, \ldots, x_{Bn}^{S_{Bn}}\right), \text{ where } x_{Bj}^{S_{Bj}} = \{x_{Bj}^{(1)}, x_{Bj}^{(2)}, \ldots, x_{Bj}^{(s_{Bj})}\}^k \ \forall j \tag{8}$$

where $x_{Bj}^{S_{Bj}}$ is the set of multi-choice reference comparisons for the k-th expert evaluating the best criteria B over other criteria j, such that $x_{Bj}^{s_{Bj}} \geq 1 \forall j = 1, 2, \ldots, n$ and $s_{Bj} \geq 1$.

Similarly, the multi-choice preference comparisons of all criteria j over the worst criterion W are obtained. The multi-choice others-to-worst vector (X_W^k) for k-th decision-maker is expressed by Equation (9) as follows:

$$X_W^k = \left(x_{1W}^{S_{1W}}, x_{2W}^{S_{2W}}, \ldots, x_{nW}^{S_{nW}}\right), \text{ where } x_{jW}^{S_{jW}} = \{x_{jW}^{(1)}, x_{jW}^{(2)}, \ldots, x_{jW}^{(s_{jW})}\}^k \ \forall j \tag{9}$$

where $x_{jW}^{S_{jW}}$ is the multi-choice preference of the criterion $j(j = 1, 2, \ldots, n)$ over the worst criterion W, such that $x_{jW}^{s_{jW}} \geq 1 \forall j = 1, 2, \ldots, n$ and $s_{jW} \geq 1$.

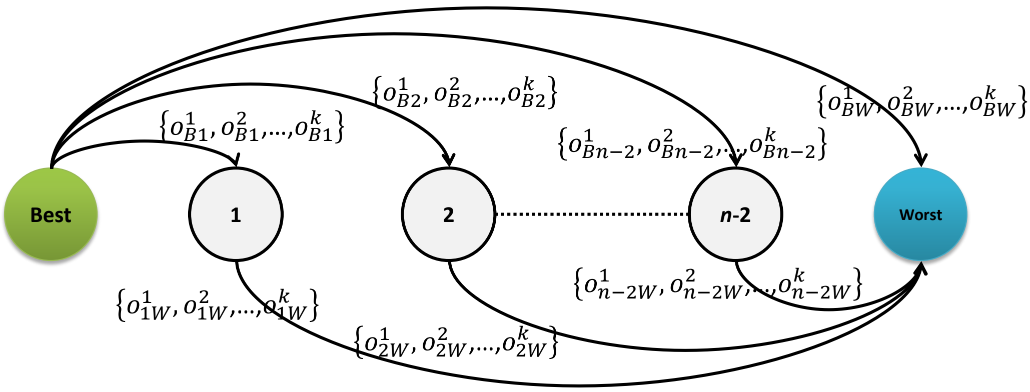

Figure 1. Multi-choice pairwise reference comparisons.

(IV). Incorporate multi-choice preference comparisons of decision-makers' evaluations.

Due to the fact that this work makes use of the input-based consistency ratio, first examine the consistency of every decision-maker's multiple-choice preference comparison vectors. After obtaining the assessment of each decision-maker, now create the decision-maker's multi-choice pairwise comparison matrix. The multi-choice comparison matrix for the k-th decision-maker is presented below:

$$X^k = \begin{bmatrix} - & - & \cdots & - & \cdots & \{x_{1W}^{(1)}, x_{1W}^{(2)}, \ldots, x_{1W}^{(s_{1W})}\}^k & \cdots & - \\ - & - & \cdots & - & \cdots & \{x_{2W}^{(1)}, x_{2W}^{(2)}, \ldots, x_{2W}^{(s_{2W})}\}^k & \cdots & \\ \vdots & \vdots & \vdots & - & \vdots & \vdots & \vdots & \vdots \\ \{x_{B1}^{(1)}, x_{B1}^{(2)}, \ldots, x_{B1}^{(s_{B1})}\}^k & \{x_{B2}^{(1)}, x_{B2}^{(2)}, \ldots, x_{B2}^{(s_{B2})}\}^k & \cdots & 1 & \cdots & \{x_{BW}^{(1)}, x_{BW}^{(2)}, \ldots, x_{BW}^{(s_{BW})}\}^k & \vdots & \{x_{Bn}^{(1)}, x_{Bn}^{(2)}, \ldots, x_{Bn}^{(s_{Bn})}\}^k \\ \vdots & \vdots & \vdots & - & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & - & \vdots & 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots & - & \vdots & \vdots & \vdots & \vdots \\ - & - & \cdots & - & \cdots & \{x_{nW}^{(1)}, x_{nW}^{(2)}, \ldots, x_{nW}^{(s_{nW})}\}^k & \cdots & - \end{bmatrix} \tag{10}$$

There are several techniques for combining the multi-choices of all the decision-makers. The associated method covered is the Lagrange interpolation method.

The main goal of such methods is the aggregation to obtain accurate results from either of the suitable choices from the multi-choice pairwise comparison matrix. There exist various acceptable approaches for handling multi-choice parameters. For multi-choice parameters, interpolating polynomials (IP) are defined by obtaining integer quantities referred to as nodal points or nodes. Each node represents a single functional significance of a multi-choice attribute. If a component includes s_{ij} possibilities, an exactly s_{ij} amount of nodes are required. The proposed interpolating polynomials aggregate the multi-choice comparison vector and obtain the non-linear function precisely at all nodes for multiple-choice. Substitute a multi-choice component with an appropriate polynomial. Lagrange method, Newton's divided difference method, Newton's forward difference method, and Newton's backward difference method are the four significant forms of interpolating polynomials-based methods.

Lagrangian method of polynomial interpolation

Lagrange's interpolating polynomial (LIP) is used to tackle the multi-choice preference comparisons vectors X_B^k and X_W^k from Equations (8) and (9), respectively. From Equation (8), for k-th decision-maker, the $x_{Bj}^{S_{Bj}} = \{x_{Bj}^{(1)}, x_{Bj}^{(2)}, \ldots, x_{Bj}^{(s_{Bj})}\}$ $\forall j$ where s_{Bj} $(s_{Bj} \geq 1)$ denotes the number of multiple choices in the comparison of criterion B to criterion $j (j = 1, 2, \ldots, n)$, assume the z^{Bj} variable representing the number of node points whose values are $(0, 1, 2, \ldots, s_{Bj} - 1)$. Derive a LIP $P_{LIP}^k(z^{Bj})$ of degree $(s_{Bj} - 1)$ as follows:

$$\begin{aligned} P_{LIP}^k(z^{Bj}) = & \frac{(z^{Bj}-1)(z^{Bj}-2)\ldots(z^{Bj}-s_{Bj}+1)}{(-1)^{s_{Bj}-1}(s_{Bj}-1)!} x_{Bj}^{(1)} + \frac{z^{Bj}(z^{Bj}-2)\ldots(z^{Bj}-s_{Bj}+1)}{(-1)^{s_{Bj}-2}(s_{Bj}-2)!} x_{Bj}^{(2)} \\ & + \frac{z^{Bj}(z^{Bj}-1)(z^{Bj}-3)\ldots(z^{Bj}-k+1)}{(-1)^{s_{Bj}-3}(s_{Bj}-3)!2!} x_{Bj}^{(3)} \\ & + \ldots + \frac{z^{Bj}(z^{Bj}-1)(z^{Bj}-2)\ldots(z^{Bj}-s_{Bj}+2)}{(s_{Bj}-1)!} x_{Bj}^{(s_{Bj})} \forall\ j \end{aligned} \tag{11}$$

In the same way, suppose z^{jW} represents node points with values $(0, 1, 2, \ldots, (s_{jW} - 1))$ with respect to X_W^k. From Equation (9), for the k-th decision-maker the $x_{jW}^{S_{jW}} = \left\{ x_{jW}^{(1)}, x_{jW}^{(2)}, \ldots, x_{jW}^{(s_{jW})} \right\}$ $\forall j$ where s_{jW} $(s_{jW} \geq 1)$ denotes the number of multiple choices in the other-to-worst, assume z^{jW} representing node points with values $(0, 1, 2, \ldots, (s_{jW} - 1))$. Derive a LIP $P^k_{LIP}(z^{jW})$ of degree $(s_{jW} - 1)$ as follows:

$$\begin{aligned} P^k_{LIP}(z^{jW}) = & \frac{(z^{jW}-1)(z^{jW}-2)\ldots(z^{jW}-s_{jW}+1)}{(-1)^{s_{jW}-1}(s_{jW}-1)!} x^{(1)}_{jW} + \frac{z^{jW}(z^{jW}-2)\ldots(z^{jW}-s_{jW}+1)}{(-1)^{s_{jW}-2}(s_{jW}-2)!} x^{(2)}_{jW} \\ & + \frac{z^{jW}(z^{jW}-1)(z^{jW}-3)\ldots(z^{jW}-k+1)}{(-1)^{s_{jW}-3}(s_{jW}-3)!2!} x^{(3)}_{jW} \\ & + \ldots + \frac{z^{jW}(z^{jW}-1)(z^{jW}-2)\ldots(z^{jW}-s_{jW}+2)}{(s_{jW}-1)!} x^{(s_{jW})}_{jW} \forall\ j \end{aligned} \tag{12}$$

Using the interpolating polynomials, model the multi-choice mathematical programming model as described in step V.

(V). Determine the optimal weights $(\omega_1^*, \omega_2^*, \ldots, \omega_n^*)$ by utilizing the optimization model.

In this step, determine the optimal weights $(\omega_1^*, \omega_2^*, \ldots, \omega_n^*)$ corresponding to each criteria $(c_1, c_2, \ldots, c_n)$. Then the non-linear mathematical optimization models that minimize the sum of the inconsistency variance for all decision-makers in accordance with the original group BWM are proposed. Following group BWM, we present two optimization models, MD-1 and MD-2. Both of them are discussed in detail.

MD-1: Mathematical model-1

The optimal weight for each criterion is where, for each multi-choice comparison, ω_B/ω_j and ω_j/ω_W should have $\omega_B/\omega_j = \{x^{(1)}_{Bj}, x^{(2)}_{Bj}, \ldots, x^{(s_{Bj})}_{Bj}\}^k$ and $\omega_j/\omega_W = \left\{x^{(1)}_{jW}, x^{(2)}_{jW}, \ldots, x^{(s_{jW})}_{jW}\right\}^k$ of k-th decision-maker. To identify the best criteria weights with the group decision-making, the maximal differences between the computed weights and the presentation of every decision-maker must be minimal. Every criterion j of the k-th decision-maker fulfills such requirements, the actual discrepancy $|\omega_B/\omega_j - X^k_B|$ and $|\omega_j/\omega_W - X^k_W|$ for all j could be established as follows.

$$\min \sum_{k \in D} \lambda_k \max_j \left\{ \left| \frac{\omega_B}{\omega_j} - \{x^{(1)}_{Bj}, x^{(2)}_{Bj}, \ldots, x^{(s_{Bj})}_{Bj}\}^k \right|, \left| \frac{\omega_B}{\omega_j} - \{x^{(1)}_{jW}, x^{(2)}_{jW}, \ldots, x^{(s_{jW})}_{jW}\}^k \right| \right\} \tag{13}$$

$$s.t. \begin{cases} \sum\limits_{j=1}^{n} \omega_j = 1 \\ \omega_j \geq 0 \\ j = 1, 2, \ldots, n \end{cases}$$

In the MD-1 model, presented in Equation (13), the objective function includes a parameter λ_k for $k - th$ expert having a value in the range $[0, 100]$ presenting the individual weight (importance) of the decision-makers. Further, we define $\xi_k = \max_j \left\{ \left| \frac{\omega_B}{\omega_j} - \{x^{(1)}_{Bj}, x^{(2)}_{Bj}, \ldots, x^{(s_{Bj})}_{Bj}\}^k \right|, \left| \frac{\omega_B}{\omega_j} - \{x^{(1)}_{jW}, x^{(2)}_{jW}, \ldots, x^{(s_{jW})}_{jW}\}^k \right| \right\}$ to simplify the proposed MD-1. Therefore, the proposed MD-1 model is transformed as:

$$\min \sum_{k \in D} \lambda_k \xi_k \tag{14}$$

$$s.t. \begin{cases} \left| \frac{\omega_B}{\omega_j} - \{x_{Bj}^{(1)}, x_{Bj}^{(2)}, \ldots, x_{Bj}^{(s_{Bj})}\}^k \right| \leq \xi_k \quad \forall j,\ k \\ \left| \frac{\omega_B}{\omega_j} - \{x_{jW}^{(1)}, x_{jW}^{(2)}, \ldots, x_{jW}^{(s_{jW})}\}^k \right| \leq \xi_k \quad \forall j,\ k \\ \sum_{j=1} \omega_j = 1 \quad \forall j \\ \omega_j \geq 0 \quad \forall j \\ \xi_k \geq 0 \quad \forall k \\ j = 1, 2, \ldots, n \\ k = 1, 2, \ldots, m \end{cases}$$

The above mathematical model represents a non-linear multi-choice optimization problem with the multi-choice comparison vectors as multi-choice parameters in the constraints. To tackle the multi-choice pairwise comparison parameters, we apply the polynomial interpolation methods with the aim of integer nodal points by using the previous Step (IV). Thus, the model presented in Equation (14) is transformed as follows:

$$\min \sum_{k \in D} \lambda_k \xi_k \tag{15}$$

$$s.t. \begin{cases} \left| \frac{\omega_B}{\omega_j} - P^k(z^{Bj}) \right| \leq \xi_k \quad \forall j,\ k \\ \left| \frac{\omega_B}{\omega_j} - P^k(z^{jW}) \right| \leq \xi_k \quad \forall j,\ k \\ \sum_{j=1} \omega_j = 1 \quad \forall j \\ \omega_j \geq 0 \quad \forall j \\ \xi_k \geq 0 \quad \forall k \\ z^{Bj} = 0, 1, 2, \ldots, (s_{Bj} - 1) \quad \forall j \\ z^{jW} = 0, 1, 2, \ldots, (s_{jW} - 1) \quad \forall j \\ j = 1, 2, \ldots, n \\ k = 1, 2, \ldots, m \end{cases}$$

The optimal criteria weights $(\omega_1^*, \omega_2^*, \ldots, \omega_n^*)$ are evaluated by solving the above model that is calculated as the actual values. Note that the proposed MD-1 model (as in Equation (15)) not only obtains the optimal criteria weights but also finds the position vectors, that is, integer nodal points z^{Bj}, and z^{jW} for all $j = 1, 2, \ldots, n$ of the polynomial P^k for all $k = 1, 2, \ldots, m$. Thus, the final solution includes the best pairwise comparison among the multiple choices that are provided by the decision-makers.

MD-2: Mathematical model-2

As described in MD-1, each decision-maker has an individual weight component, which is used to calculate their relative relevance in the proposed model. Consequently, the second mathematical model, MD-2, with a min–max objective, is presented as follows:

$$\min \max_{k} \quad \lambda_k \xi_k \tag{16}$$
$$s.t.\begin{cases} \left|\frac{\omega_B}{\omega_j} - \{x_{Bj}^{(1)}, x_{Bj}^{(2)}, \ldots, x_{Bj}^{(s_{Bj})}\}^k\right| \le \xi_k \quad \forall j,\ k \\ \left|\frac{\omega_B}{\omega_j} - \{x_{jW}^{(1)}, x_{jW}^{(2)}, \ldots, x_{jW}^{(s_{jW})}\}^k\right| \le \xi_k \quad \forall j,\ k \\ \sum_{j=1} \omega_j = 1 \quad \forall j \\ \omega_j \ge 0 \quad \forall j \\ \xi_k \ge 0 \quad \forall k \\ j = 1, 2, \ldots, n \\ k = 1, 2, \ldots, m \end{cases}$$

Here, similar to the mathematical model presented in MD-1 (as in Equation (14)), we simplify the MD-2 model in Equation (16) as $\xi = \max_k \lambda_k \xi_k$. Additionally, the multi-choice comparisons in the constraints of the model having the multi-choice parameter are interpolated using the previous Step (IV). We transform the model MD-2 as follows:

$$\min \xi \tag{17}$$
$$s.t.\begin{cases} \xi \ge \lambda_k \xi_k \quad \forall k \\ \left|\frac{\omega_B}{\omega_j} - P^k(z^{Bj})\right| \le \xi_k \quad \forall j,\ k \\ \left|\frac{\omega_B}{\omega_j} - P^k(z^{jW})\right| \le \xi_k \quad \forall j,\ k \\ \sum_{j=1} \omega_j = 1 \quad \forall j \\ \omega_j \ge 0 \quad \forall j \\ \xi_k \ge 0 \quad \forall k \\ z^{Bj} = 0, 1, 2, \ldots, (s_{Bj} - 1) \quad \forall j \\ z^{jW} = 0, 1, 2, \ldots, (s_{jW} - 1) \quad \forall j \\ j = 1, 2, \ldots, n \\ k = 1, 2, \ldots, m \end{cases}$$

The optimal criteria weights $(\omega_1^*, \omega_2^*, \ldots, \omega_n^*)$, integer nodal points z^{Bj}, and z^{jW} for all $j = 1, 2, \ldots, n$ are evaluated by solving the above proposed MD-2 model in Equation (17).

Following the solution of the mathematical models, MD-1 and MD-2, the optimum values of ξ_k are used to compute the consistency ratio (CR_k) for every k-th decision-maker that will apply to evaluate the group consistency ratio (CR^G) in a group decision-making system. If $x_{Bj} \times x_{jW} = x_{BW}\ \forall j$, where x_{Bj} is the preference of best criteria B over the all other criteria j and x_{jW} is the preference of criterion j over the worst criterion B, then comparisons are said to be entirely consistent. Thus, the consistency ratio of the proposed GMCBWM is calculated in the following step.

(VI). Consistency ratio for GMCBWM

The consistency ratio (CR) was used to demonstrate the validity of pairwise comparisons. The CR of the proposed GMCBWM of the k-th decision-maker CR_k and further the group decision-making CR^G are calculated using Equations (18) and (19) given as follows.

$$CR_k = \lambda_k \frac{\xi_k^*}{CI} \quad \forall k = 1, 2, \ldots, m \tag{18}$$
$$CR^G = \max_k \{CR_k\} \tag{19}$$

In Equation (18), ξ_k^* is the optimum value of inconsistency achieved by resolving mathematical models (MD-1 or MD-2) for the k-th decision-makers, and λ_k is the individual weight vector assigned to the k-th decision-maker depending on their degree of knowledge. The consistency index (CI) is a constant value presented in Table 3 for each decision-maker's value x_{BW}^{max}. The conclusive CR^G of the GMCBWM is the maximum CR_k among all decision-makers as in Equation (19). If CR^G is zero, the result in the form of optimal weights is totally consistent; nevertheless, as CR^G grows, the consistency diminishes.

Table 3. CI [14].

x_{BW}^{max}	1	2	3	4	5	6	7	8	9
CI (max ξ)	0.00	0.44	1	1.63	2.30	3	3.73	4.47	5.23

5. Numerical Studies

This section presents two numerical illustrations of group decision-making problems to demonstrate the applicability of the proposed GMCBWM in different circumstances and analyze the outcomes. The proposed models are formulated under each case and evaluated using the AMPL [59] and NEOS [60] to achieve the optimal weights criteria.

5.1. Numerical Example 1

In this example, the responses for four criteria (c_1, c_2, c_3, c_4) with $n = 4$ are taken from two decision-makers (DMs), i.e., $k = 2$ as DM1 and DM2 [61]. The criteria c_1 and c_3 are considered best (B) and worst (W) criteria by both experts having λ_1 and λ_2 individual weights, respectively. The multi-choice best-to-others X_B^k (Equation (8)) and others-to-worst X_W^k (Equation (9)) vector preferences on a scale of 1 to 9 are provided by each DM using Table 1. As $n = 4$, according to the basic BWM, we require at least $2n - 3 = 2 * 4 - 3 = 5$ preference responses from each DM [14]. The multi-choice pairwise preference comparing the best-to-others and others-to-worst criteria are presented in Table 4 for both DMs. In Table 4, the response x_{13} comparing c_1 to c_3 from DM1 is a multi-choice comparison with two choices 8 and 9, that is, $x_{13} = \{8,\ 9\}$ and for x_{14} comparing c_1 to c_4 from DM2 is 2 and 3, that is, $x_{14} = \{2,\ 3\}$ with $s_{13} = 2$ and $s_{14} = 2$, respectively.

Table 4. Multi-choice preference comparisons of four criteria with two decision-makers in Example 1.

DM	x_{12}	x_{13}	x_{14}	x_{23}	x_{43}
DM_1	2	{8, 9}	3	4	2
DM_2	2	8	{2, 3}	4	2

Thus, the multi-choice best-to-others vector is $X_B^1 = \{x_{12}, \{x_{13}^{(1)}, x_{13}^{(2)}\}, x_{14}\}\} = \{2, \{8,9\}, 3\}$ and $X_B^2 = \{x_{12}, x_{13}, \{x_{14}^{(1)}, x_{14}^{(2)}\}\} = \{2, 8, \{2, 3\}\}$ and others-to-worst vector is $X_W^1 = \{\{x_{13}^{(1)}, x_{13}^{(2)}\}, x_{23}, x_{43}\} = \{\{8,9\}, 4, 2\}$ and $X_W^2 = \{x_{13}, x_{23}, x_{43}\} = \{8, 4, 2\}$ using Table 4 from Step (III) of the proposed GMCBWM. The objective of the proposed models is to determine the best choice or the best pairwise comparison out of all the choices of pairwise comparisons assigned by DM1 and DM2. However, before formulating the mathematical model of the proposed GMCBWM, we first have to derive the polynomial functions that interpolate the multi-choice reference parameters associated with each comparison vector.

Thus, interpolation polynomials (IPs) are derived for the $x_{13} = \{x_{13}^{(1)}, x_{13}^{(2)}\} = \{8,\ 9\}$ and $x_{14} = \{x_{14}^{(1)}, x_{14}^{(2)}\} = \{3,\ 4\}$ multi-choice reference comparison taking integral values for the nodal points z^{13} and z^{14}. Each node will have $s_{13} = s_{14} = 2$ number of integer values. The Lagrange interpolating polynomial (LIP) functions are derived using Step 4 of Section 4. We formulate LIPs P_{LIP} for (z^{13}) of degree $(s_{13}-1)$ and for (z^{14}) of degree $(s_{14}-1)$ using Equations (11) and (12) as follows:

$$\begin{aligned}
P_{LIP}(z^{13}) =& \frac{(z^{13}-1)}{(-1)^{s_{13}-1}(s_{13}-1)!}x_{13}^{(1)} + \frac{z^{13}}{(-1)^{s_{13}-2}(s_{13}-2)!}x_{13}^{(2)} \\
=& \frac{(z^{13}-1)}{(-1)^{2-1}(2-1)!}8 + \frac{z^{13}}{(-1)^{2-2}(2-2)!}9 \\
=& z^{13}+8 \\
P_{LIP}(z^{14}) =& \frac{(z^{14}-1)}{(-1)^{s_{14}-1}(s_{14}-1)!}x_{14}^{(1)} + \frac{z^{14}}{(-1)^{s_{14}-2}(s_{14}-2)!}x_{14}^{(2)} \\
=& \frac{(z^{14}-1)}{(-1)^{2-1}(2-1)!}2 + \frac{z^{14}}{(-1)^{2-2}(2-2)!}3 \\
=& z^{14}+2
\end{aligned}$$

Now, to obtain the optimal weights $(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$ of the criteria, we present the two proposed mathematical programming model MD-1 and MD-2 from Step (V) of Section 4 using Equations (15) and (17) as follows:

$$\begin{aligned}
& MD-1 \\
& \min \ (\lambda_1\xi_1 + \lambda_2\xi_2) \\
& s.t. \begin{cases}
\left|\frac{\omega_1}{\omega_2} - 2\right| \le \xi_1 \\
\left|\frac{\omega_1}{\omega_2} - 2\right| \le \xi_2 \\
\left|\frac{\omega_1}{\omega_3} - (z^{13}+8)\right| \le \xi_1 \\
\left|\frac{\omega_1}{\omega_3} - 8\right| \le \xi_2 \\
\left|\frac{\omega_1}{\omega_4} - 3\right| \le \xi_1 \\
\left|\frac{\omega_1}{\omega_4} - (z^{14}+2)\right| \le \xi_2 \\
\left|\frac{\omega_2}{\omega_3} - 4\right| \le \xi_1 \\
\left|\frac{\omega_2}{\omega_3} - 4\right| \le \xi_2 \\
\left|\frac{\omega_4}{\omega_3} - 2\right| \le \xi_1 \\
\left|\frac{\omega_4}{\omega_3} - 2\right| \le \xi_2 \\
\omega_1+\omega_2+\omega_3+\omega_4 = 1 \\
\omega_1, \omega_2, \omega_3, \omega_4 \ge 0 \\
\xi_1, \xi_2 \ge 0 \\
z^{13}, z^{14} = 0, 1.
\end{cases}
\end{aligned} \tag{20}$$

$$\begin{aligned}
& MD-2 \\
& \min \ \xi \\
& s.t. \begin{cases}
\xi \ge \lambda_1\xi_1 \\
\xi \ge \lambda_2\xi_2 \\
\text{Rest of constraints are same as in Equation (20)}
\end{cases}
\end{aligned} \tag{21}$$

The non-linear mixed-integer mathematical programming models MD-1 and MD-2, as shown in Equations (20) and (21), are solved to obtain the optimal weights $(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$ and the nodal points (z^{13}, z^{14}) corresponding to the multi-choice reference comparison. The optimal ξ^* of the MDs are applied to evaluate the inconsistency of the group decision-making of Example 1. Further, the individual weights (λ_1, λ_2) of DMs have a substantial effect on the resulting consistency ratio (CR). Therefore, we investigate five different pairs (0.1,0.9), (0.3,0.7), (0.5,0.5), (0.7,0.3), and (0.9,0.1) of group decision instances to obtain optimal weights of the criteria. The sensitivity of weight values has been evaluated based on these pairs. Using the data provided in Table 4 of Example 1, we have solved the two models with different combinations of individual weight vectors of DMs.

The optimal results of Example 1 after solving MD-1 and MD-2 are provided in Tables 5 and 6, respectively. The outcomes of five pairs of (λ_1, λ_2) with optimal weights of critical, nodal points, inconsistency value (ξ_1, ξ_2), and consistency ratio (CR_1, CR_2) are presented for DM1 and DM2 with $k = 2$ in tables. Importantly, in the last column of Tables 5 and 6, the group consistency ratio (CR^G) is presented. It has been found that the optimal weight of criteria is the same for all pairs of λ_1 and λ_2 for both MD-1 and MD-2 for this numerical illustration. However, the CR_k of each DMs changes depending upon the values of λ_1 and λ_2. In other words, the ranking of the criteria is the same for each pair in both models and is $c_1 > c_2 > c_4 > c_3$. Moreover, the solutions of both models MD-1 and MD-2 for different combinations of λ_1 and λ_2 shows that the best nodal values are $z^{13} = 0$, i.e., $x_{13} = 8$ for DM1 comparing c_1 to c_3 and $z^{14} = 0$, i.e., $x_{14} = 2$ for DM2 comparing c_1 to c_4. Thus, the best choice of x_{BW}, i.e., x_{13} equals eight and remains unchanged for all models. Further, the group consistency ratio of Example 1 is equal to the minimum among all CR^G obtained for different values of λ_1 and λ_2. Therefore, the minimum consistency value of Example 1 using MD-1 is $CR^G = 0.002982886$ (see Table 5) and for MD-2 is $CR^G = 0.002982886$ (see Table 6) for $\lambda_1 = 0.5$ and $\lambda_2 = 0.5$. It is worth mentioning that the CR^G of the Example 1 with Group-BWM [61] with both model-1 and model-2 is equal to 0.096 with ranking $c_1 > c_2 > c_4 > c_3$. Hence, we can say that the proposed models of GMCBWM achieve the best consistency value for Example 1 and also validate the ranking of criteria.

Table 5. Result of Example 1 solving with MD-1.

No.	(λ_1, λ_2)	$(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$	z^{13}, z^{14}	ξ_k^*	CR_k	CR^G
1	(0.1, 0.9)	(0.506667, 0.266667	0	0.0266667	0.000596570	0.005369134
		0.0666667, 0.160000)	0	0.0266667	0.005369134	
2	(0.3, 0.7)	(0.506667, 0.266667	0	0.0266667	0.001789732	0.004176000
		0.0666667, 0.160000)	0	0.0266667	0.004176040	
3	(0.5, 0.5)	(0.506667, 0.266667	0	0.0266667	0.002982886	0.002982886
		0.0666667, 0.160000)	0	0.0266667	0.002982886	
4	(0.7, 0.3)	(0.506667, 0.266667	0	0.0266667	0.004176040	0.004176040
		0.0666667, 0.160000)	0	0.0266667	0.001789732	
5	(0.9, 0.1)	(0.506667, 0.266667	0	0.0266667	0.005369195	0.005369195
		0.0666667, 0.160000)	0	0.0266667	0.000596577	

Table 6. Result of Example 1 solving with MD-2.

No.	(λ_1, λ_2)	$(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$	z^{13}, z^{14}	ξ_k^*	CR_k	CR^G
1	(0.1, 0.9)	(0.506667, 0.266667	0	0.0266667	0.000596577	0.005368993
		0.0666667, 0.160000)	0	0.0266656	0.005368993	
2	(0.3, 0.7)	(0.506667, 0.266667	0	0.0266667	0.001789732	0.004175727
		0.0666667, 0.160000)	0	0.0266652	0.004175727	
3	(0.5, 0.5)	(0.506667, 0.266667	0	0.0266667	0.002982886	0.002982886
		0.0666667, 0.160000)	0	0.0266667	0.002982886	
4	(0.7, 0.3)	(0.506667, 0.266667	0	0.0266652	0.004175727	0.004175772
		0.0666667, 0.160000)	0	0.0622189	0.004175772	
5	(0.9, 0.1)	(0.506667, 0.266667	0	0.0266656	0.005368993	0.005368993
		0.0666667, 0.160000)	0	0.2399900	0.005368904	

5.2. Numerical Example 2

We have assumed three decision-makers (DM1, DM2, DM3), $k = 3$ with four criteria (c_1, c_2, c_3, c_4), $n = 4$. The c_1 and c_3 criteria are assumed as best (B) and worst (W) criterion by the all three experts having λ_1, λ_2, and λ_3 individual weights, respectively. The pairwise comparisons with the multi-choice response sets are presented in Table 7.

Table 7. Pairwise comparison of three decision-makers in Example 2.

DM	x_{12}	x_{13}	x_{14}	x_{23}	x_{43}
DM1	2	9	(3, 4)	4	2
DM2	2	(8, 9)	4	4	2
DM3	2	8	4	(3, 4)	2

The multi-choice best-to-others X_B^k (Equation (8)) and others-to-worst X_W^k (Equation (9)) vector preferences on a scale 1 to 9 are provided by each DM using Table 1. Table 4 shows the multi-choice pairwise comparisons of the worst and the best criteria than the other criteria for the two DMs.

The response from DM1 for x_{13} is a multi-choice parameter with two choices 8 and 9, that is, $x_{13} = \{8,\ 9\}$ and for DM2, the response is 2 and 3, that is, $x_{14} = \{2,\ 3\}$ with $s_{13} = 2$ and $s_{14} = 2$, respectively. The objective is to determine the best choice or the best pairwise comparison out of all the choices of pairwise comparisons assigned by DM1 and DM2.

Interpolating polynomials (IPs) are formulated for the $\{x_{13}^{(1)}, x_{13}^{(2)}\} = \{8,\ 9\}$ and $\{x_{14}^{(1)}, x_{14}^{(2)}\} = \{3,\ 4\}$ multi-choice reference comparison taking integral values for the nodal points z^{13} and z^{14}. Each node will have $s_{13} = s_{14} = 2$ number of integer values. The Lagrange IP (LIP) functions are formulated using step 4 for each multi-choice reference comparison. We derive LIPs P_{LIP} for (z^{13}) of degree $(s_{13} - 1)$ and for (z^{14}) of degree $(s_{14} - 1)$ using Equations (11) and (12) as follows:

$$\begin{aligned}
P_{LIP}(z^{13}) =& \frac{(z^{13} - 1)}{(-1)^{s_{13}-1}(s_{13} - 1)!} x_{13}^{(1)} + \frac{z^{13}}{(-1)^{s_{13}-2}(s_{13} - 2)!} x_{13}^{(2)} \\
=& \frac{(z^{13} - 1)}{(-1)^{2-1}(2 - 1)!} 8 + \frac{z^{13}}{(-1)^{2-2}(2 - 2)!} 9 \\
=& z^{13} + 8 \\
P_{LIP}(z^{14}) =& \frac{(z^{14} - 1)}{(-1)^{s_{14}-1}(s_{14} - 1)!} x_{14}^{(1)} + \frac{z^{14}}{(-1)^{s_{14}-2}(s_{14} - 2)!} x_{14}^{(2)} \\
=& \frac{(z^{14} - 1)}{(-1)^{2-1}(2 - 1)!} 2 + \frac{z^{14}}{(-1)^{2-2}(2 - 2)!} 3 \\
=& z^{14} + 2
\end{aligned}$$

Now, to obtain the optimal weights $(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$ of the criteria, we present the two proposed mathematical programming models MD-1 and MD-2 as follows:

$$
\begin{aligned}
&MD-1\\
&\min\ (\lambda_1\xi_1+\lambda_2\xi_2+\lambda_3\xi_3)\\
&s.t.\begin{cases}
\left|\frac{\omega_1}{\omega_2}-2\right|\le\xi_1,\left|\frac{\omega_1}{\omega_2}-2\right|\le\xi_2,\left|\frac{\omega_1}{\omega_2}-2\right|\le\xi_3\\
\left|\frac{\omega_1}{\omega_3}-9\right|\le\xi_1,\left|\frac{\omega_1}{\omega_3}-(z^{13}+8)\right|\le\xi_2,\left|\frac{\omega_1}{\omega_3}-8\right|\le\xi_3\\
\left|\frac{\omega_1}{\omega_4}-(z^{14}+3)\right|\le\xi_1,\left|\frac{\omega_1}{\omega_4}-4\right|\le\xi_2,\left|\frac{\omega_1}{\omega_4}-4\right|\le\xi_3\\
\left|\frac{\omega_2}{\omega_3}-4\right|\le\xi_1,\left|\frac{\omega_2}{\omega_3}-4\right|\le\xi_2,\left|\frac{\omega_2}{\omega_3}-(z^{23}+3)\right|\le\xi_3\\
\left|\frac{\omega_4}{\omega_3}-2\right|\le\xi_1,\left|\frac{\omega_4}{\omega_3}-2\right|\le\xi_2,\left|\frac{\omega_4}{\omega_3}-2\right|\le\xi_3\\
\omega_1+\omega_2+\omega_3+\omega_4=1\\
\omega_1,\omega_2,\omega_3,\omega_4\ge 0\\
\xi_1,\xi_2,\xi_3\ge 0\\
z^{13},z^{14},z^{23}=0,1.
\end{cases}
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&MD-2\\
&\min\ \xi\\
&s.t.\begin{cases}
\xi\ge\lambda_1\xi_1\\
\xi\ge\lambda_2\xi_2\\
\text{Rest of constraints are same as MD-1}
\end{cases}
\end{aligned}
\tag{23}
$$

The results of Models 1 and 2 are provided in Tables 8 and 9. In Table 8, the obtained weights are presented for six different sets of (λ_1, λ_2, λ_3). The change in weight values of the criterion has been evaluated based on these sets. It has been found that for model 2, no change is visible for five sets of (λ_1, λ_2, λ_3). Whereas, for (0.6, 0.2, 0.2), the weight values of the criterion are different from the other five sets. In Table 9, the inconsistency value (ξ_1, ξ_2, ξ_3) and consistency ratio (CR1, CR2, CR3) for decision maker 1, 2, and 3 were presented, respectively. Finally, in the last column, the Group CR is tabulated. All these are evaluated for six sets of λ_1, λ_2, and λ_3. For model 1, the minimum CRG is 0.002065 for set 3, whereas the maximum is for set 6 of λ_k. For model 2, the minimum CRG is for sets 2 and 3, i.e., 0.002065. The CRG is maximum for set 4, i.e., 0.002386 of $\lambda_1 = 0.3$, $\lambda_2 = 0.3$, and $\lambda_3 = 0.4$. The ranks of criteria for all models remain the same for all sets of λ_1, λ_2, and λ_3.

Table 8. Result of Example 2 for MD-1.

No.	λ_k	$(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$	z^{13}, z^{14}, z^{23}	ξ_k^*	CR_k	CR^G
1	0.2	(0.533333, 0.266667	1	0.066667	0.002549	0.002549
	0.2	0.066667, 0.133333)	0	0.000000	0.000000	
	0.6		1	0.000000	0.000000	
2	0.2	(0.533333, 0.266667	1	0.066667	0.002549	0.002549
	0.6	0.066667, 0.133333)	1	0.000000	0.000000	
	0.2		1	0.000000	0.000000	
3	0.6	(0.538462, 0.261538	1	0.015385	0.001765	0.002065
	0.2	0.061539, 0.138462)	1	0.015385	0.000688	
	0.2		0	0.046154	0.002065	
4	0.3	(0.533333, 0.266667	1	0.066667	0.003824	0.003824
	0.3	0.066667, 0.133333)	1	0.000000	0.000000	
	0.4		1	0.000000	0.000000	
5	0.3	(0.533333, 0.266667	1	0.066667	0.003824	0.003824
	0.4	0.066667, 0.133333)	1	0.000000	0.000000	
	0.3		1	0.000000	0.000000	
6	0.4	(0.533333, 0.266667	1	0.066667	0.005099	0.005099
	0.3	0.066667, 0.133333)	1	0.000000	0.000000	
	0.3		1	0.000000	0.000000	

Table 9. Result of Example 2 for MD-2.

No.	λ_k	$(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$	z^{13}, z^{14}, z^{23}	ξ_k^*	CR_k	CR^G
1	0.2	(0.528000, 0.272000	1	0.048000	0.001836	0.002148
	0.2	0.064000, 0.136000)	1	0.016000	0.000612	
	0.6		1	0.016000	0.002148	
2	0.2	(0.538462, 0.261538	0	0.046154	0.001765	0.002065
	0.6	0.061539, 0.138462)	1	0.015385	0.002065	
	0.2		1	0.046154	0.002065	
3	0.6	(0.538462, 0.261538	1	0.015385	0.001765	0.002065
	0.2	0.061539, 0.138462)	1	0.046154	0.001765	
	0.2		1	0.046154	0.002065	
4	0.3	(0.524444, 0.275556	1	0.035556	0.002040	0.002386
	0.3	0.062222, 0.137778)	1	0.026667	0.001530	
	0.4		1	0.026667	0.002386	
5	0.3	(0.527473, 0.272527	0	0.035165	0.002017	0.002360
	0.4	0.061539, 0.138462)	1	0.026374	0.002360	
	0.3		1	0.035165	0.002360	
6	0.4	(0.527473, 0.272527	1	0.026374	0.002017	0.002360
	0.3	0.061539, 0.138462)	1	0.035165	0.002017	
	0.3		1	0.035165	0.002360	

After solving all models, for example, 2, we have found that for all models, the ranks of criteria remain unchanged, but there is a little difference in values of weights for some sets of λ_1, λ_2, and λ_3. Additionally, the CR ratio for MD-1 is 0 whereas for MD-2, it is nearby to zero for all values of λ_1, λ_2, and λ_3. The CR values for models are acceptable.

6. Case Study

We have considered a real case study from [61] having 10 decision-makers about a piping selection method. The study consists of four criteria, i.e., total cost, security, social cost, and environmental cost. The total cost is the best criterion, and environmental cost is the worst criterion. The considered criteria, along with respective descriptions, are presented in Table 10. The pairwise comparison data for the case study is presented in Table 11. The multi-choice response sets were also presented in this table. The responses as multi-choice data are presented in the last row of Table 11. This multi-choice row consists of responses from all 10 decision-makers taken as choices. The objective is to determine the optimal weights of criteria by choosing that set of responses from the multi-choice sets, which will minimize the inconsistency. Two models, MD-1 and MD-2, are formulated and solved similarly to previous examples to determine the optimal weights of criteria.

Table 10. Criteria and their description for piping selection.

Title	Description
Total cost (C_1)	Direct costs employer pays to the contractor
Security (C_2)	Security of less damage to underground pipes
Social costs (C_3)	Problems due to noise and traffic limitations
Environmental costs (C_4)	Environmental pollution such as air and soil pollution.

Now modeling and solving, results are obtained. The criteria weights and their respective ranks obtained for MD-1 and MD-2 are presented in Table 12. In Table 12, we can see that in models 1 and 2, although the ranks of criteria are the same, there is a change in the weight values of criteria.

Table 11. Case study.

	x_{12}	x_{13}	x_{14}	x_{24}	x_{34}
$DM1$	{6, 7}	6	7	4	{2, 3}
$DM2$	7	7	{7,9}	{4, 6}	5
$DM3$	7	{6, 7}	9	{8, 9}	8
$DM4$	{2, 3, 5}	4	{6,7}	4	3
$DM5$	3	4	8	{3,4,5}	5
$DM6$	5	{6, 7}	7	5	{2, 3, 5}
$DM7$	9	{4, 6}	9	9	8
$DM8$	9	{7, 9}	9	{6, 8}	2
$DM9$	{8, 9}	9	9	{3, 4}	1
$DM10$	{7, 9}	9	9	{3, 5}	{1, 2, 3}

Table 12. Optimal weights obtained for case study using MD-1, and -2.

Criteria	MD-1	Rank	MD-2	Rank
1	0.754335	1	0.743833	1
2	0.0780347	3	0.0867027	3
3	0.0982659	2	0.103489	2
4	0.0693642	4	0.0659758	4

The individual consistency ratio for all 10 experts and respective consistency ratio along with the CR of the group is presented in Table 13. The CRG obtained for model 1 is 0.018015, and for model 2, it is 0.018901. There is a minor difference in obtained consistency ratios. Out of these two models, the consistency ratio is minimum for MD-1 and maximum for MD-2.

Table 13. Result of case study for MD-1, and -2.

	MD-1				MD-2			
Expert	**Weight**	x_{BW}	**CR**	**CRG**	**Weight**	x_{BW}	**CR**	**CRG**
1	0.268786	7	0.018015147		0.282003	7	0.018901005	
2	0.476879	7	0.003835488		0.441103	7	0.003547745	
3	0.476879	9	0.002735444		0.441103	9	0.002530228	
4	0.364162	6	0.004855493		0.570428	6	0.007605707	
5	0.520231	8	0.008146794	0.018015	0.483725	8	0.007575112	0.018901
6	0.364162	7	0.007810445		0.31032	7	0.006655657	
7	0.546243	9	0.010444417		0.507079	9	0.009695583	
8	0.754335	9	0.014423231		0.705007	9	0.013480057	
9	0.130058	9	0.003730153		0.187564	9	0.005379465	
10	0.33815	9	0.009698375		0.470004	9	0.013480038	

7. Limitations, Conclusions and Future Work

This study proposes two different mathematical programming models for solving group MCDM problems. Since there is a large number of variants of mcdm methods, where each method has its own suitability with respect to the assumptions and applications, the proposed methods also have some limitations. The proposed approaches can be applied when the pairwise comparisons collected are multi-choice in nature. The method has some difficulty in its application since no excel or easy-to-apply software is available. The decision-maker needs to code and solve it using optimization solvers, which is difficult for the management or non-technical persons. In the future, this can be rectified by developing easy to calculate excel file. The present work only proposes two models; in the future, these models can be extended to some more models by incorporating more information, such as the confidence level of experts, etc.

The two methods are proposed considering the assumption of multi-choice uncertainty in pairwise comparisons of criteria. The multi-choice uncertainty has been applied to determine the best choice out of multiple choices. It gives a real-life scenario to the decision-making problems. Although there are many other forms of uncertainty, such as rough set, fuzzy, intuitionistic fuzzy, neutrosophic, probabilistic, etc., it focuses on choices instead of the probabilistic or fuzzy nature of parameters. The parameters are considered multi-choice in the pairwise comparison. These parameters are handled by applying the Lagrange interpolating polynomial method. The proposed models are novel in terms of their mathematical structure and group decision-making approach. The models are formulated and further validated by solving numerical examples. It provides a framework for solving MCDM problems and provides weightage to the decision-makers as well. In the results of the numerical example 1 for MD-1 and MD-2, it can be seen that there is no effect on the weights obtained for the criteria due to the weights of the decision-makers, i.e., λ_k, also the ranking of criteria is similar to an earlier study conducted in [61]. For numerical example 2, It can be seen that there is no effect of λ_k on the weights of criteria using MD-1. For all variations of λ_k, weights are the same. Whereas in the case of MD-2, there is a change in weights due to variation in λ_k. It has been found that for all models, the ranks of criteria remain unchanged, but there is a little difference in values of weights for some sets of λ_1, λ_2, and λ_3. The CR values for all the models of example 1 and 2 has been found acceptable.

This work is an extension of multi-choice MCDM models to group decision-making models. In the future, the proposed models can be extended to models incorporating other kinds of uncertain parameters, such as rough set theory, fuzzy set theory, probabilistic, etc. The confidence level of decision-makers as probability or weights can also be incorporated in future studies. This can be applied to any case study following similar assumptions of proposed models. The study has shown the real-life application of proposed models in piping selection problem. Similar to it, the proposed models can be applied to other problems.

Author Contributions: All authors have equal contributions in writing this manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: The authors are thankful to Qassim University, represented by the "Deanship of Scientific Research", Saudi Arabia, for providing financial assistance to carry out this research work.

Data Availability Statement: The sources of secondary data are cited in the manuscript.

Acknowledgments: The author(s) gratefully acknowledge Qassim University, represented by the "Deanship of Scientific Research, on the financial support for this research under the number (10258-des-2020-1-3-I) during the academic year 1442 AH/2020 AD."

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Yu, X.; Zhang, S.; Liao, X.; Qi, X. Electre methods in prioritized mcdm environment. *Inf. Sci.* **2018**, *424*, 301–316. [CrossRef]
2. Roy, B. The outranking approach and the foundations of electre methods. In *Readings in Multiple Criteria Decision Aid*; Springer: Berlin/Heidelberg, Germany, 1990; pp. 155–183.
3. Ramón, N.; Ruiz, J.L.; Sirvent, I. Two-step benchmarking: Setting more realistically achievable targets in dea. *Expert Syst. Appl.* **2018**, *92*, 124–131. [CrossRef]
4. Charnes, A.; Cooper, W.W.; Rhodes, E. Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* **1978**, *2*, 429–444. [CrossRef]
5. Saaty, T.L. Analytic hierarchy process. *Encyclopedia of Biostatistics*, 1st ed.; John Wiley & Sons, Ltd.: Hoboken, NJ, USA, 2005.
6. Saaty, T.L. Group decision making and the ahp. In *The Analytic Hierarchy Process*; Springer: Berlin/Heidelberg, Germany, 1989; pp. 59–67.
7. Gao, R.; Nam, H.O.; Ko, W.I.; Jang, H. Integrated system evaluation of nuclear fuel cycle options in china combined with an analytical mcdm framework. *Energy Policy* **2018**, *114*, 221–233. [CrossRef]
8. Brans, J.-P.; Vincke, P. Note—A preference ranking organisation method: The promethee method for multiple criteria decision-making. *Manag. Sci.* **1985**, *31*, 647–656. [CrossRef]
9. Sennaroglu, B.; Celebi, G.V. A military airport location selection by ahp integrated promethee and vikor methods. *Transp. Res. Part D Transp. Environ.* **2018**, *59*, 160–173. [CrossRef]
10. Liu, Y.; Chen, Y.; Tzeng, G.-H. Identification of key factors in consumers' adoption behavior of intelligent medical terminals based on a hybrid modified madm model for product improvement. *Int. J. Med. Inform.* **2017**, *105*, 68–82. [CrossRef]

11. Tian, G.; Zhang, H.; Feng, Y.; Wang, D.; Peng, Y.; Jia, H. Green decoration materials selection under interior environment characteristics: A grey-correlation based hybrid mcdm method. *Renew. Sustain. Energy Rev.* **2018**, *81*, 682–692. [CrossRef]
12. Saaty, T.L. Decision making—the analytic hierarchy and network processes (ahp/anp). *J. Syst. Sci. Syst. Eng.* **2004**, *13*, 1–35. [CrossRef]
13. Wu, W.-Y.; Qomariyah, A.; Sa, N.T.T.; Liao, Y. The integration between service value and service recovery in the hospitality industry: An application of qfd and anp. *Int. J. Hosp. Manag.* **2018**, *75*, 48–57. [CrossRef]
14. Rezaei, J. Best-worst multi-criteria decision-making method. *Omega* **2015**, *53*, 49–57. [CrossRef]
15. Rezaei, J. Best-worst multi-criteria decision-making method: Some properties and a linear model. *Omega* **2016**, *64*, 126–130. [CrossRef]
16. Shameem, M.; Khan, A.A.; Hasan, M.G.; Akbar, M.A. Analytic hierarchy process based prioritisation and taxonomy of success factors for scaling agile methods in global software development. *IET Softw.* **2020**, *14*, 389–401. [CrossRef]
17. Biswal, M.; Acharya, S. Solving multi-choice linear programming problems by interpolating polynomials. *Math. Comput. Model.* **2011**, *54*, 1405–1412. [CrossRef]
18. Quddoos, A.; Hasan, M.G.u.; Khalid, M.M. Multi-choice stochastic transportation problem involving general form of distributions. *SpringerPlus* **2014**, *3*, 565. [CrossRef] [PubMed]
19. Singh, S.S. Multi-choice programming: An overview of theories and applications. *Optimization* **2017**, *66*, 1713–1738. [CrossRef]
20. Pamučar, D.; Stević, Ž.; Sremac, S. A new model for determining weight coefficients of criteria in mcdm models: Full consistency method (fucom). *Symmetry* **2018**, *10*, 393. [CrossRef]
21. Žižović, M.; Pamucar, D. New model for determining criteria weights: Level based weight assessment (lbwa) model, Decision Making. *Appl. Manag. Eng.* **2019**, *2*, 126–137. [CrossRef]
22. Pamucar, D.; Deveci, M.; Gokasar, I.; Işık, M.; Zizovic, M. Circular economy concepts in urban mobility alternatives using integrated dibr method and fuzzy dombi cocoso model. *J. Clean. Prod.* **2021**, *323*, 129096. [CrossRef]
23. Rezaei, J.; Wang, J.; Tavasszy, L. Linking supplier development to supplier segmentation using best worst method. *Expert Syst. Appl.* **2015**, *42*, 9152–9164. [CrossRef]
24. Gupta, H.; Barua, M.K. Identifying enablers of technological innovation for indian msmes using best–worst multi criteria decision making method. *Technol. Forecast. Soc. Chang.* **2016**, *107*, 69–79. [CrossRef]
25. Ma, X.; Qin, J.; Martínez, L.; Pedrycz, W. A linguistic information granulation model based on best-worst method in decision making problems. *Inf. Fusion* **2022**, *89*, 210–227. [CrossRef]
26. Tavana, M.; Shaabani, A.; Caprio, D.D.; Bonyani, A. A novel interval type-2 fuzzy best-worst method and combined compromise solution for evaluating eco-friendly packaging alternatives. *Expert Syst. Appl.* **2022**, *200*, 117188. [CrossRef]
27. Malakoutikhah, M.; Alimohammadlou, M.; Jahangiri, M.; Rabiei, H.; Faghihi, S.A.; Kamalinia, M. Modeling the factors affecting unsafe behaviors using the fuzzy best-worst method and fuzzy cognitive map. *Appl. Soft Comput.* **2022**, *114*, 108119. [CrossRef]
28. Kharola, S.; Ram, M.; Mangla, S.K.; Goyal, N.; Nautiyal, O.; Pant, D.; Kazancoglu, Y. Exploring the green waste management problem in food supply chains: A circular economy context. *J. Clean. Prod.* **2022**, *351*, 131355. [CrossRef]
29. Bilbao-Terol, A.; Arenas-Parra, M.; Quiroga-García, R.; Bilbao-Terol, C. An extended best–worst multiple reference point method: Application in the assessment of non-life insurance companies. *Oper. Res.* **2022**, *22*, 5323–5362. [CrossRef]
30. Sadaghiani, S.; Ahmad, K.W.; Rezaei, J.; Tavasszy, L. Evaluation of external forces affecting supply chain sustainability in oil and gas industry using best worst method. In Proceedings of the 2015 International Mediterranean Gas and Oil Conference (MedGO), Mechref, Lebanon, 16–18 April 2015; pp. 1–4.
31. Groenendijk, L.; Rezaei, J.; Correia, G. Incorporating the travellers' experience value in assessing the quality of transit nodes: A rotterdam case study. *Case Stud. Transp. Policy* **2018**, *6*, 564–576. [CrossRef]
32. Chitsaz, N.; Azarnivand, A. Water scarcity management in arid regions based on an extended multiple criteria technique. *Water Resour. Manag.* **2017**, *31*, 233–250. [CrossRef]
33. Ghoushchi, S.J.; Yousefi, S.; Khazaeili, M. An extended fmea approach based on the z-moora and fuzzy bwm for prioritization of failures. *Appl. Soft Comput.* **2019**, *81*, 105505. [CrossRef]
34. Hafezalkotob, A.; Hafezalkotob, A. A novel approach for combination of individual and group decisions based on fuzzy best-worst method. *Appl. Soft Comput.* **2017**, *59*, 316–325. [CrossRef]
35. Mi, X.; Tang, M.; Liao, H.; Shen, W.; Lev, B. The state-of-the-art survey on integrations and applications of the best worst method in decision making: Why, what, what for and what's next? *Omega* **2019**, *87*, 205–225. [CrossRef]
36. You, P.; Guo, S.; Zhao, H.; Zhao, H. Operation performance evaluation of power grid enterprise using a hybrid bwm-topsis method. *Sustainability* **2017**, *9*, 2329. [CrossRef]
37. Hafezalkotob, A.; Hafezalkotob, A.; Liao, H.; Herrera, F. Interval multimoora method integrating interval borda rule and interval best-worst-method-based weighting model: Case study on hybrid vehicle engine selection. *IEEE Trans. Cybern.* **2019**, *50*, 1157–1169. [CrossRef]
38. Gupta, H. Evaluating service quality of airline industry using hybrid best worst method and vikor.*J. Air Transp. Manag.* **2018**, *68*, 35–47. [CrossRef]
39. Maghsoodi, A.I.; Mosavat, M.; Hafezalkotob, A.; Hafezalkotob, A. Hybrid hierarchical fuzzy group decision-making based on information axioms and bwm: Prototype design selection. *Comput. Ind. Eng.* **2019**, *127*, 788–804. [CrossRef]

40. Yadav, G.; Mangla, S.K.; Luthra, S.; Jakhar, S. Hybrid bwm-electre-based decision framework for effective offshore outsourcing adoption: A case study. *Int. J. Prod. Res.* **2018**, *56*, 6259–6278. [CrossRef]
41. Vafadarnikjoo, A.; Tavana, M.; Botelho, T.; Chalvatzis, K. A neutrosophic enhanced best–worst method for considering decision-makers' confidence in the best and worst criteria. *Ann. Oper. Res.* **2020**, *289*, 391–418. [CrossRef]
42. Liao, H.; Mi, X.; Yu, Q.; Luo, L. Hospital performance evaluation by a hesitant fuzzy linguistic best worst method with inconsistency repairing. *J. Clean. Prod.* **2019**, *232*, 657–671. [CrossRef]
43. Liu, P.; Wang, D. An extended taxonomy method based on normal t-spherical fuzzy numbers for multiple-attribute decision-making. *Int. J. Fuzzy Syst.* **2022**, *24*, 73–90. [CrossRef]
44. Wu, Z.; Zhang, S.; Liu, X.; Wu, J. Best-worst multi-attribute decision making method based on new possibility degree with probabilistic linguistic information. *IEEE Access* **2019**, *7*, 133900–133913. [CrossRef]
45. Wu, Q.; Zhou, L.; Chen, Y.; Chen, H. An integrated approach to green supplier selection based on the interval type-2 fuzzy best-worst and extended vikor methods. *Inf. Sci.* **2019**, *502*, 394–417. [CrossRef]
46. Mohammadi, M.; Rezaei, J. Bayesian best-worst method: A probabilistic group decision making model. *Omega* **2019**, *96*, 102075. [CrossRef]
47. Fard, M.B.; Moradian, P.; Emarati, M.; Ebadi, M.; Chofreh, A.G.; Klemeŝ, J.J. Ground-mounted photovoltaic power station site selection and economic analysis based on a hybrid fuzzy best-worst method and geographic information system: A case study guilan province. Renew. *Sustain. Energy Rev.* **2022**, *169*, 112923. [CrossRef]
48. Soner, O.; Celik, E.; Akyuz, E. A fuzzy best–worst method (bwm) to assess the potential environmental impacts of the process of ship recycling. *Marit. Policy Manag.* **2022**, *49*, 396–409. [CrossRef]
49. Haqbin, A.; Shojaei, P.; Radmanesh, S. Prioritising covid-19 recovery solutions for tourism small and medium-sized enterprises: A rough best-worst method approach. *J. Decis. Syst.* **2022**, *31*, 102–115. [CrossRef]
50. Ertunç, E.; Uyan, M. Land valuation with best worst method in land consolidation projects. *Land Use Policy* **2022**, *122*, 106360. [CrossRef]
51. Wang, X.; Wang, H.; Xu, Z.; Ren, Z. Green supplier selection based on probabilistic dual hesitant fuzzy sets: A process integrating best worst method and superiority and inferiority ranking. *Appl. Intell.* **2022**, *52*, 8279–8301. [CrossRef]
52. Yazdi, M.; Adumene, S.; Zarei, E. Introducing a probabilistic-based hybrid model (fuzzy-bwm-bayesian network) to assess the quality index of a medical service, In *Linguistic Methods Under Fuzzy Information in System Safety and Reliability Analysis*; Springer: Berlin/Heidelberg, Germany, 2022; pp. 171–183.
53. Munim, Z.H.; Balasubramaniyan, S.; Kouhizadeh, M.; Hossain, N.U.I. Assessing blockchain technology adoption in the norwegian oil and gas industry using bayesian best worst method. *J. Ind. Inf. Integr.* **2022**, *28*, 100346. [CrossRef]
54. Ghaffar, A.R.A.; Nadeem, M.R.; Hasan, M.G. Cost-benefit analysis of shale development in india: A best-worst method based mcdm approach. *J. King Saud Univ. Sci.* **2022**, *33*, 101591. [CrossRef]
55. Ahmad, N.; Hasan, M.G.; Barbhuiya, R.K. Identification and prioritization of strategies to tackle covid-19 outbreak: A group-bwm based mcdm approach. *Appl. Soft Comput.* **2021**, *111*, 107642. [CrossRef]
56. Ahmad, N.; Hasan, M. Self-adaptive query-broadcast in wireless ad-hoc networks using fuzzy best worst method. *Wirel. Networks* **2021**, *27*, 765–780. [CrossRef]
57. Hillier, F.; Lieberman, G.J. Introduction to operations research. *J. Oper. Res. Soc.* **2006**, *57*, 330.
58. Hasan, M.G.; Ashraf, Z.; Khan, M.F. Multi-choice best-worst multi-criteria decision-making method and its applications. *Int. J. Intell. Syst.* **2022**, *37*, 1129–1156. [CrossRef]
59. Fourer, R.; Gay, D.M.; Kernighan, B.W. *AMPL: A Mathematical Programming Language*; AT & T Bell Laboratories: Murray Hill, NJ, USA, 1987.
60. Czyzyk, J.; Mesnier, M.P.; Moré, J.J. The neos server. *IEEE Comput. Sci. Eng.***1998**, *5*, 68–75. [CrossRef]
61. Safarzadeh, S.; Khansefid, S.; Rasti-Barzoki, M. A group multi-criteria decision-making based on best-worst method. *Comput. Ind. Eng.* **2018**, *126*, 111–121. [CrossRef]