Supplementary Materials

Analysis of WE Parameters of Life Using Adaptive-Progressively Type-II Hybrid Censored Mechanical Equipment Data

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Table S1: The MSs (1^{st} column) and MSEs (2^{nd} column) of γ.

T	(n,m)	Scheme	MLE		PSE		BE-ML		BE-PS	
2	(50,20)	1	0.0238	0.0075	0.0230	0.0071	0.0223	0.0050	0.0183	0.0045
		2	0.0245	0.0088	0.0224	0.0059	0.0216	0.0047	0.0179	0.0042
		3	0.0246	0.0072	0.0221	0.0061	0.0212	0.0045	0.0173	0.0041
	(50,40)	1	0.0223	0.0068	0.0221	0.0056	0.0202	0.0045	0.0170	0.0041
		2	0.0222	0.0067	0.0198	0.0048	0.0178	0.0042	0.0172	0.0037
		3	0.0219	0.0065	0.0194	0.0045	0.0171	0.0039	0.0157	0.0035
	(80,32)	1	0.0215	0.0055	0.0142	0.0042	0.0128	0.0025	0.0124	0.0022
		2	0.0212	0.0061	0.0145	0.0034	0.0131	0.0022	0.0122	0.0023
		3	0.0209	0.0051	0.0155	0.0039	0.0127	0.0023	0.0116	0.0021
	(80,64)	1	0.0202	0.0048	0.0129	0.0034	0.0119	0.0021	0.0112	0.0019
		2	0.0194	0.0048	0.0139	0.0035	0.0120	0.0019	0.0119	0.0017
		3	0.0192	0.0045	0.0141	0.0035	0.0115	0.0020	0.0111	0.0018
3	(50,20)	1	0.0235	0.0074	0.0198	0.0065	0.0134	0.0052	0.0092	0.0039
		2	0.0221	0.0086	0.0178	0.0059	0.0128	0.0041	0.0084	0.0032
		3	0.0228	0.0065	0.0165	0.0053	0.0121	0.0038	0.0090	0.0030
	(50,40)	1	0.0218	0.0068	0.0175	0.0052	0.0115	0.0037	0.0085	0.0028
		2	0.0201	0.0064	0.0164	0.0047	0.0108	0.0035	0.0081	0.0030
		3	0.0191	0.0061	0.0156	0.0041	0.0102	0.0034	0.0086	0.0028
	(80,32)	1	0.0205	0.0059	0.0168	0.0047	0.0102	0.0035	0.0078	0.0028
		2	0.0193	0.0056	0.0152	0.0041	0.0099	0.0032	0.0069	0.0026
		3	0.0183	0.0054	0.0145	0.0037	0.0089	0.0029	0.0068	0.0024
	(80,64)	1	0.0194	0.0048	0.0152	0.0038	0.0092	0.0024	0.0058	0.0023
		2	0.0182	0.0045	0.0143	0.0032	0.0084	0.0023	0.0050	0.0019
		3	0.0174	0.0047	0.0138	0.0033	0.0081	0.0019	0.0046	0.0017

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Table S2: The MSs (1^{st} column) and MSEs (2^{nd} column) of σ.

T	(n,m)	Scheme	MLE		PSE		BE-ML		BE-PS	
2	(50,20)	1	-0.1074	0.6506	-0.1391	1.0886	-0.0566	0.2324	-0.0678	0.2533
		2	-0.1237	0.7426	-0.3025	1.8802	-0.0637	0.2554	-0.0670	0.2482
		3	-0.1318	1.1300	-0.1561	1.0844	-0.0733	0.2624	-0.0772	0.2666
	(50,40)	1	-0.0945	0.6189	-0.1258	0.7698	-0.0507	0.2147	-0.0596	0.2493
		2	-0.1086	0.6837	-0.2770	0.9260	-0.0511	0.2358	-0.0538	0.2426
		3	-0.1274	0.6755	-0.1553	0.9864	-0.0549	0.2398	-0.0578	0.2524
	(80,32)	1	-0.0904	0.5294	-0.1019	0.4854	-0.0352	0.2268	-0.0371	0.2289
		2	-0.0812	0.6798	-0.2595	0.8386	-0.0422	0.2208	-0.0444	0.2387
		3	-0.0985	0.6446	-0.1349	0.9639	-0.0485	0.2140	-0.0534	0.2483
	(80,64)	1	-0.0805	0.4088	-0.0855	0.4085	-0.0304	0.1997	-0.0320	0.2161
		2	-0.0183	0.5205	-0.2533	0.7097	-0.0383	0.2072	-0.0403	0.2102
		3	-0.0494	0.5812	-0.1336	0.6038	-0.0422	0.2175	-0.0444	0.2181
3	(50,20)	1	-0.0801	0.6048	-0.1194	0.8635	-0.0159	0.0280	-0.0162	0.0326
		2	-0.0965	0.6472	-0.1884	1.0275	-0.0190	0.0297	-0.0195	0.0329
		3	-0.1360	1.1181	-0.2196	1.2190	-0.0222	0.0301	-0.0236	0.0334
	(50,40)	1	-0.0911	0.5344	-0.1139	0.5810	-0.0142	0.0276	-0.0144	0.0319
		2	-0.0720	0.6289	-0.1459	0.8267	-0.0160	0.0288	-0.0171	0.0323
		3	-0.0994	0.6329	-0.1789	0.9034	-0.0211	0.0292	-0.0225	0.0329
	(80,32)	1	-0.0702	0.3131	-0.1010	0.4942	-0.0130	0.0270	-0.0132	0.0310
		2	-0.0747	0.5316	-0.1376	0.7218	-0.0167	0.0282	-0.0163	0.0315
		3	-0.0790	0.5938	-0.1717	0.7731	-0.0178	0.0284	-0.0182	0.0320
	(80,64)	1	-0.0470	0.3071	-0.0866	0.3940	-0.0125	0.0261	-0.0126	0.0304
		2	-0.0489	0.4332	-0.1034	0.5727	-0.0128	0.0274	-0.0132	0.0308
		3	-0.0651	0.4589	-0.1429	0.6840	-0.0146	0.0286	-0.0151	0.0315

Table S3: The MSs (1^{st} column) and MSEs (2^{nd} column) of $R(t)$.

T	(n,m)	Scheme	MLE		PSE		BE-ML		BE-PS	
2	(50,20)	1	-0.0154	0.0456	-0.0136	0.0388	-0.0075	0.0337	-0.0071	0.0287
		2	-0.0137	0.0434	-0.0127	0.0372	-0.0064	0.0325	-0.0058	0.0278
		3	-0.0124	0.0418	-0.0115	0.0358	-0.0061	0.0309	-0.0054	0.0267
	(50,40)	1	-0.0131	0.0389	-0.0128	0.0331	-0.0062	0.0288	-0.0057	0.0249
		2	-0.0127	0.0376	-0.0121	0.0315	-0.0054	0.0276	-0.0045	0.0236
		3	-0.0111	0.0354	-0.0107	0.0301	-0.0049	0.0264	-0.0041	0.0228
	(80,32)	1	-0.0117	0.0341	-0.0111	0.0298	-0.0033	0.0258	-0.0031	0.0225
		2	-0.0102	0.0327	-0.0099	0.0279	-0.0031	0.0240	-0.0027	0.0216
		3	-0.0096	0.0299	-0.0087	0.0268	-0.0027	0.0226	-0.0023	0.0202
	(80,64)	1	-0.0105	0.0325	-0.0088	0.0286	-0.0023	0.0243	-0.0026	0.0219
		2	-0.0092	0.0316	-0.0084	0.0269	-0.0027	0.0234	-0.0023	0.0195
		3	-0.0087	0.0287	-0.0073	0.0254	-0.0025	0.0212	-0.0021	0.0188
3	(50,20)	1	-0.0258	0.0723	0.0234	0.0705	-0.0203	0.0556	0.0180	0.0523
		2	-0.0195	0.0708	0.0185	0.0684	-0.0149	0.0534	0.0141	0.0507
		3	-0.0176	0.0675	0.0168	0.0649	-0.0132	0.0512	0.0126	0.0487
	(50,40)	1	-0.0193	0.0694	0.0177	0.0671	-0.0142	0.0528	0.0134	0.0501
		2	-0.0174	0.0665	0.0169	0.0645	-0.0136	0.0494	0.0123	0.0482
		3	-0.0162	0.0637	0.0156	0.0612	-0.0125	0.0471	0.0107	0.0456
	(80,32)	1	-0.0176	0.0468	0.0154	0.0414	-0.0132	0.0361	0.0118	0.0315
		2	-0.0158	0.0375	0.0136	0.0336	-0.0123	0.0287	0.0105	0.0262
		3	-0.0149	0.0344	0.0121	0.0317	-0.01118	0.0265	0.0098	0.0240
	(80,64)	1	-0.0168	0.0369	0.0146	0.0348	-0.0126	0.0274	0.0110	0.0264
		2	-0.0151	0.0334	0.0125	0.0321	-0.0117	0.0245	0.0092	0.0245
		3	-0.0142	0.0315	0.0113	0.0295	-0.0103	0.0233	0.0089	0.0223

Table S4: The MSs (1^{st} column) and MSEs (2^{nd} column) of $h(t)$.

T	(n,m)	Scheme	MLE		PSE		BE-ML		BE-PS	
2	(50,20)	1	0.0158	0.0499	0.0164	0.0542	0.0119	0.0210	0.0128	0.0236
		2	0.0176	0.0531	0.0181	0.0575	0.0132	0.0235	0.0164	0.0259
		3	0.0191	0.0562	0.0198	0.0618	0.0143	0.0285	0.0187	0.0318
	(50,40)	1	0.0143	0.0416	0.0151	0.0459	0.0101	0.0185	0.0116	0.0215
		2	0.0155	0.0492	0.0173	0.0540	0.0113	0.0207	0.0137	0.0228
		3	0.0169	0.0543	0.0182	0.0593	0.0128	0.0225	0.0145	0.0254
	(80,32)	1	0.0126	0.0374	0.0148	0.0415	0.0094	0.0174	0.0108	0.0194
		2	0.0133	0.0426	0.0156	0.0461	0.0099	0.0198	0.0117	0.0215
		3	0.0146	0.0463	0.0163	0.0512	0.0108	0.0211	0.0123	0.0222
	(80,64)	1	0.0108	0.0323	0.0129	0.0359	0.0087	0.0165	0.0099	0.0182
		2	0.0121	0.0386	0.0131	0.0422	0.0089	0.0177	0.0108	0.0198
		3	0.0137	0.0417	0.0148	0.0445	0.0098	0.0185	0.0114	0.0209
3	(50,20)	1	0.0132	0.0449	0.0152	0.0491	0.0111	0.0179	0.0118	0.0205
		2	0.0141	0.0471	0.0176	0.0528	0.0121	0.0198	0.0136	0.0224
		3	0.0165	0.0526	0.0193	0.0567	0.0134	0.0245	0.0152	0.0271
	(50,40)	1	0.0127	0.0384	0.0145	0.0421	0.0098	0.0161	0.0111	0.0192
		2	0.0134	0.0433	0.0167	0.0486	0.0109	0.0173	0.0130	0.0205
		3	0.0148	0.0499	0.0174	0.0536	0.0116	0.0189	0.0137	0.0227
	(80,32)	1	0.0118	0.0337	0.0141	0.0388	0.0091	0.0152	0.0105	0.0175
		2	0.0128	0.0373	0.0147	0.0415	0.0095	0.0162	0.0113	0.0186
		3	0.0131	0.0407	0.0152	0.0456	0.0103	0.0175	0.0118	0.0203
	(80,64)	1	0.0102	0.0312	0.0117	0.0329	0.0082	0.0136	0.0093	0.0158
		2	0.0114	0.0357	0.0124	0.0394	0.0086	0.0145	0.0098	0.0171
		3	0.012	0.0365	0.0136	0.0409	0.0094	0.0158	0.0107	0.0184

Table S5: The ACLs of 95% ACI/HPD intervals of γ.

T	(n,m)	Scheme	ACI-ML	ACI-PS	HPD-ML	HPD-PS
2	(50,20)	1	0.3556	0.3325	0.2528	0.2275
		2	0.3273	0.3046	0.2483	0.2235
		3	0.3092	0.2985	0.2445	0.2201
	(50,40)	1	0.3154	0.2966	0.2419	0.2177
		2	0.3143	0.2956	0.2381	0.2143
		3	0.3033	0.2851	0.2345	0.2111
	(80,32)	1	0.2700	0.2664	0.1789	0.1610
		2	0.2687	0.2608	0.1785	0.1607
		3	0.2667	0.2561	0.1674	0.1507
	(80,64)	1	0.2607	0.2529	0.1594	0.1435
		2	0.2512	0.2509	0.1574	0.1417
		3	0.2460	0.2380	0.1531	0.1378
3	(50,20)	1	0.3544	0.3314	0.2372	0.2135
		2	0.3085	0.2863	0.2333	0.2100
		3	0.3032	0.2830	0.2257	0.2031
	(50,40)	1	0.3014	0.2974	0.2281	0.2053
		2	0.2952	0.2840	0.2224	0.2002
		3	0.2935	0.2819	0.2176	0.1958
	(80,32)	1	0.2659	0.2564	0.1588	0.1536
		2	0.2625	0.2539	0.1529	0.1476
		3	0.2527	0.2460	0.1707	0.1397
	(80,64)	1	0.2558	0.2508	0.1552	0.1429
		2	0.2493	0.2399	0.1473	0.1326
		3	0.2337	0.2355	0.1443	0.1299

Table S6: The ACLs of 95% ACI/HPD intervals of σ.

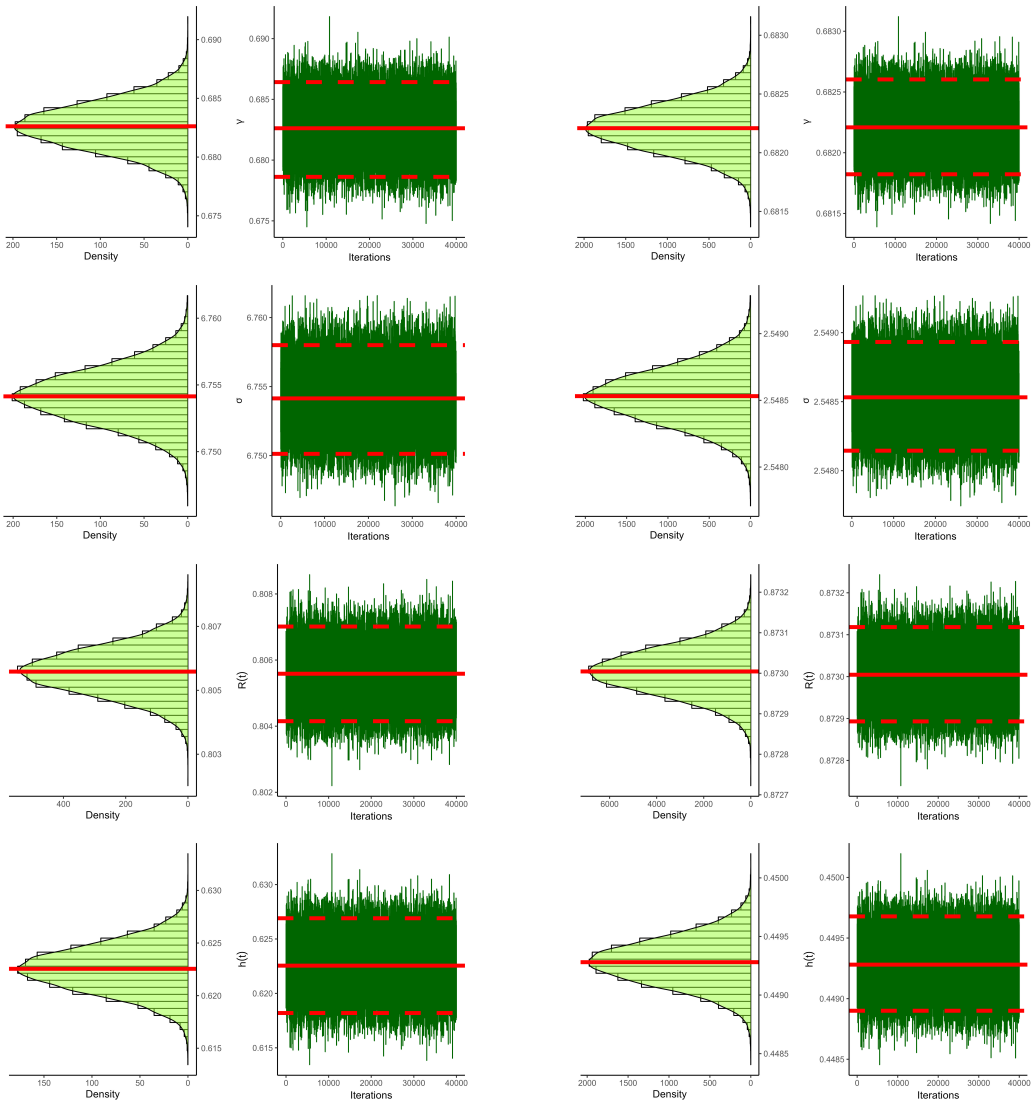
T	(n,m)	Scheme	ACI-ML	ACI-PS	HPD-ML	HPD-PS
2	(50,20)	1	3.1477	3.8090	1.8204	1.9162
		2	3.2346	4.1645	1.8572	1.9549
		3	3.8325	4.8884	1.9339	2.0357
	(50,40)	1	3.0621	3.2692	1.7810	1.8747
		2	3.1278	3.3790	1.8304	1.9267
		3	3.4818	3.7924	1.8599	1.9578
	(80,32)	1	2.4672	2.8295	1.7048	1.7945
		2	2.9648	3.2509	1.7625	1.8553
		3	3.1970	3.5482	1.7682	1.8613
	(80,64)	1	2.2823	2.4824	1.0886	1.0927
		2	2.7412	2.8166	1.6568	1.7440
		3	2.8115	3.1089	1.6603	1.7477
3	(50,20)	1	3.0208	3.0943	0.6750	0.6895
		2	3.1262	3.3984	0.6855	0.7009
		3	3.7354	3.8669	0.6984	0.7036
	(50,40)	1	2.9748	2.7082	0.6503	0.6746
		2	2.9767	2.9995	0.6514	0.6845
		3	3.0943	3.7882	0.6745	0.6995
	(80,32)	1	2.4405	2.1174	0.6375	0.6710
		2	2.8536	2.6026	0.6406	0.6743
		3	3.0680	3.2895	0.6644	0.6916
	(80,64)	1	2.1532	2.0408	0.6257	0.6586
		2	2.5517	2.2772	0.6330	0.6663
		3	2.6113	2.9889	0.6570	0.6894

Table S7: The ACLs of 95% ACI/HPD intervals of $R(t)$.

T	(n,m)	Scheme	ACI-ML	ACI-PS	HPD-ML	HPD-PS
2	(50,20)	1	0.2439	0.2273	0.2207	0.2154
		2	0.2389	0.2197	0.2179	0.2123
		3	0.2291	0.2161	0.2122	0.2097
	(50,40)	1	0.2313	0.2145	0.2079	0.2002
		2	0.2294	0.2094	0.2038	0.1995
		3	0.2254	0.2101	0.2015	0.1948
	(80,32)	1	0.2189	0.1714	0.1535	0.1507
		2	0.2154	0.1710	0.1502	0.1459
		3	0.2091	0.1606	0.1423	0.1409
	(80,64)	1	0.2052	0.1661	0.1413	0.1386
		2	0.2043	0.1614	0.1343	0.1319
		3	0.2032	0.1573	0.1358	0.1301
3	(50,20)	1	0.2147	0.2046	0.1984	0.1922
		2	0.2112	0.1977	0.1921	0.1893
		3	0.2038	0.1929	0.1861	0.1827
	(50,40)	1	0.2107	0.1931	0.1890	0.1879
		2	0.2075	0.1895	0.1848	0.1833
		3	0.1962	0.1854	0.1837	0.1745
	(80,32)	1	0.1887	0.1706	0.1437	0.1382
		2	0.1876	0.1681	0.1365	0.1330
		3	0.1840	0.1458	0.1327	0.1298
	(80,64)	1	0.1862	0.1565	0.1367	0.1349
		2	0.1841	0.1443	0.1279	0.1252
		3	0.1812	0.1383	0.1221	0.1194

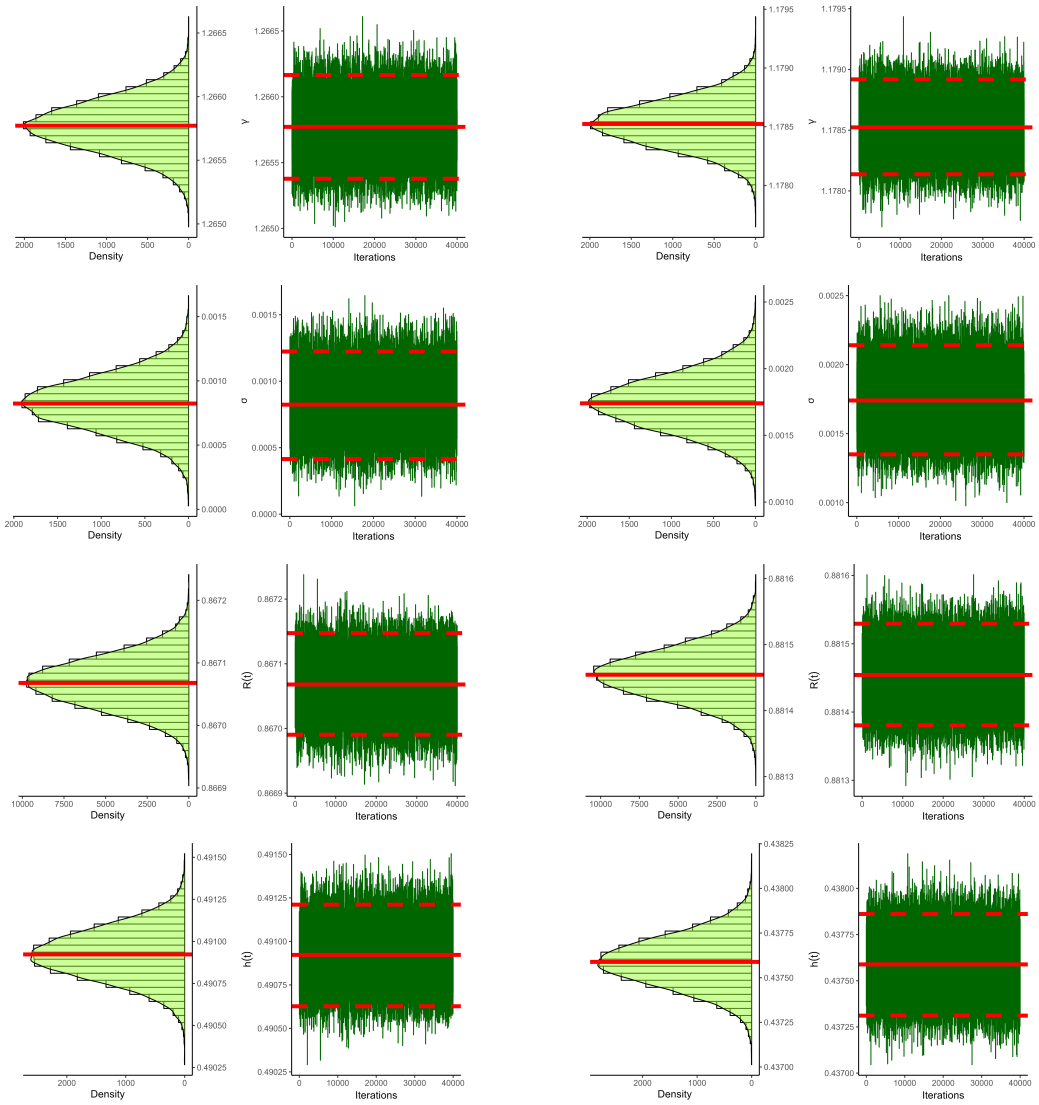
Table S8: The ACLs of 95% ACI/HPD intervals of $h(t)$.

T	(n,m)	Scheme	ACI-ML	ACI-PS	HPD-ML	HPD-PS
2	(50,20)	1	0.2727	0.2976	0.2397	0.2459
		2	0.2835	0.3110	0.2437	0.2546
		3	0.2927	0.3215	0.2579	0.2667
	(50,40)	1	0.2538	0.2784	0.2230	0.2174
		2	0.2625	0.2881	0.2315	0.2276
		3	0.2751	0.2978	0.2358	0.2393
	(80,32)	1	0.2173	0.2386	0.1606	0.1659
		2	0.2242	0.2426	0.1682	0.1754
		3	0.2476	0.2719	0.1702	0.1890
	(80,64)	1	0.1948	0.2108	0.1495	0.1536
		2	0.2034	0.2227	0.1507	0.1671
		3	0.2261	0.2467	0.1542	0.1698
3	(50,20)	1	0.2706	0.2857	0.2296	0.2346
		2	0.2874	0.2984	0.2363	0.2451
		3	0.2905	0.3173	0.2445	0.2501
	(50,40)	1	0.2508	0.2697	0.2119	0.2284
		2	0.2615	0.2794	0.2178	0.2315
		3	0.2713	0.2861	0.2288	0.2388
	(80,32)	1	0.1902	0.2276	0.1566	0.1579
		2	0.1985	0.2321	0.1595	0.1616
		3	0.2011	0.2616	0.1670	0.1679
	(80,64)	1	0.1870	0.2049	0.1444	0.1464
		2	0.1880	0.2148	0.1481	0.1496
		3	0.1933	0.2375	0.1527	0.1551



(b) Bayes LF-based (b) Bayes PS-based

Figure S1: Density (left) and Trace (right) plots of γ, σ, $R(t)$ and $h(t)$ based on S2 from RME data.



(b) Bayes LF-based (b) Bayes PS-based

Figure S2: Density (left) and Trace (right) plots of γ, σ, $R(t)$ and $h(t)$ based on S3 from RME data.