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FE Model Updating of Continuous Beam Bridge Based on Response Surface Method

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Abstract: In this paper, A high-order response surface method is proposed for finite element model updating of continuous beam bridges. Firstly, based on visual inspection and environmental vibration testing, the peak picking (PP) method and random subspace identification (SSI) method are used to identify the dynamic characteristic parameters of the structure. Then, the finite element model of the continuous beam bridge is updated based on the third-order response surface method. It can be concluded that the results of the updated finite element model are in good agreement with the test results, and the maximum error between the calculated and measured frequency is less than 3%, with MAC values greater than 85%. Moreover, the updated finite element model can reflect the current situation of real bridges and serve as the basis for bridge health monitoring, damage detection, and safety assessment.

Keywords: continuous beam bridge; third-order response surface method; FE model updating; ambient vibration testing; damage

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1. Introduction

Since the 1990s, with the sustained and rapid development of China's economy, bridge construction has entered a 30 year period of rapid development. As of the end of 2021, China has built 961,100 highway bridges and 73.8021 million meters, of which over 80% are small and medium-sized bridges. With the rapid increase in the number of bridges and the emergence of innovative and breakthrough bridges, concerns about their safe operation are increasing. In the United States, approximately 40% of bridges require repair or reconstruction, and according to estimates from the Federal Highway Administration (FHWA), $90 billion is needed to address these issues. In China, with the increase in traffic flow, design and construction defects, overloading, insufficient operation and maintenance, and natural aging of bridges in recent years, the number of old and dangerous bridges remains high, but they operate with "diseases" all year round, posing huge safety hazards [1,2].

In response to the huge safety hazards existing in existing bridges, many scholars have attempted to detect and eliminate safety hazards based on structural health monitoring systems and have achieved certain results [3,4]. One of the main purposes of structural health monitoring is to obtain accurate finite element models, which can enable effective structural safety assessments [5].

Traditional finite element model updates have two shortcomings: (1) Sensitivity-analysis-based finite element model updates require iterative calculations, and each calculation requires calling a finite element model, which results in a large computational load and is not conducive to its application in engineering; (2) In the updating of finite element models of complex structures, there are many parameters that need to be modified, so a large number of finite element calculations are essential and not easy to implement on computers [6]. Finite element model updating based on the response surface method is a new method that differs from traditional models. Its basic concept [7,8] is that in the design space of variables, regression analysis methods will be used to fit the response values or test values of sample points to obtain surface responses that simulate real limit states. This will replace the finite element model or make the design or calculation of other complex models more effective. Ren et al. [9,10] introduced the updating of finite element models based on dynamic and static response surfaces. Fang et al. [11] used D-optimal design and first-order response surface models to predict the dynamic response and damage identification of intact and damaged systems. The effectiveness of this method was verified by the results of reinforced concrete frame model tests and I-40 bridge tests. Chen and Zhang et al. [12,13] introduced the uncertainties and correlation between reinforcement corrosion and concrete cracking, and a case study was employed to discuss the life-cycle modeling of concrete cracking and reinforcement corrosion. Zong et al. [14] used the central composite experimental design method (CCD) and second-order response surface model to update the finite element model based on the health monitoring of a large-span continuous rigid frame bridge, proving that finite element model updating based on a second-order response surface has a high accuracy [15–17]. In recent years, finite element modeling and correction technology based on artificial intelligence has attracted more and more attention; for instance, wavelet convolutional neural networks and deep-learning neural networks have been used for wind-induced vibration modeling and stress distribution prediction [18–20].

In this paper, a continuous beam bridge is employed as the engineering background, the FE model updating of the bridge was conducted based on the visual inspection and ambient vibration testing and third-order response surface model, after which the FE model can be further applied in bridge health monitoring and a safety evaluation.

2. Basic Methodology

The selection of the response surface function form, i.e., the response surface model, is an important part of the application of the response surface method. It should meet two requirements: (1) the expression of the response surface function should be as simple as possible while basically describing the relationship between the system input parameters and output response; (2) The number of undetermined coefficients in the response surface function expression should be minimized to reduce the number of system experiments or calculations [21].

Response surface models include complete and incomplete polynomial models, Kriging models, BP neural network models, radial basis functions (RBFs), and multivariate adaptive regression spline functions (MARSFs). The response surface model in this article adopts a polynomial response surface model [22].

Assuming that the system response for the dependent variable, $x_i (i = 1, 2, \cdots, k)$, is the design parameters, which is selected through the analysis of the variance method, the polynomial response surface model form is as follows:

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \beta_{iii} x_i^3 + \sum_i \sum_j \beta_{ij} x_i x_j + \sum_i \sum_j \beta_{iij} (x_i)^2 x_j + \sum_i \sum_j \sum_k \beta_{ijk} x_i x_j x_k \tag{1}$$

In Equation (1), $x_i \in [x_i^l, x_i^u]$, $(i \in (1, k))$, x_i^l, x_i^u are the upper boundary and lower boundary of the design parameter values, and $\beta_0, \beta_i, \beta_{ii}, \beta_{iii}, \beta_{ij}, \beta_{iij}, \beta_{ijk}$ are undetermined coefficients.

The fitting of the response surface function is the process of solving the undetermined coefficients in the response surface function. The least squares method is the basic method for solving undetermined coefficients, and its steps are as follows:

(1) Determine parameters and their range of values, and determine sample points (calculation points) through the experimental design;

(2) Calculate the response values of sample points $y_1, y_2, \cdots, y_N$ through finite element analysis to obtain sample data;
(3) Substitute the sample data into Equation (1), and then use regression analysis to calculate the undetermined coefficients $\beta_0, \beta_i, \beta_{ii}, \beta_{iii}, \beta_{ij}, \beta_{iij}, \beta_{ijk}$;
(4) Perform response surface model validation. If the accuracy of the response surface model meets the requirements, this response surface model can be used for model correction. If the accuracy of the response surface model does not meet the requirements, go back to step (1) and redo the experimental design until the accuracy meets the requirements.

After calculating the unknown parameters of each response surface function, it is necessary to verify the accuracy of the unknown parameters. The initial definition of model validation provided by the American Computer Simulation Association was that model validation is the process of validating conceptual models expressed by computational models within a given accuracy range. Among them, the conceptual model refers to the finite element analysis model, while the computational model is the response surface model of regression.

Based on the calculation of the finite element model and the response surface model, the standards for testing the accuracy of the response surface model include a normal distribution test of residuals, mean of residuals, relative root mean square error (RMSE) and R^2 test. For more complex models and response surface models with multiple responses, the latter two standards are usually used, and their expressions are shown in Equations (2) and (3), respectively [23–25].

$$R^2 = 1 - \frac{\sum_{j=1}^{N} [y_{RS}(j) - y(j)]^2}{\sum_{j=1}^{N} [y(j) - \overline{y}]^2} \tag{2}$$

$$RMSE = \frac{1}{N \cdot \overline{y}} \cdot \sqrt{\sum (y(j) - y_{RS}(j))^2} \tag{3}$$

where $y_{RS}(j)$ represents the calculated value of the response surface model, $y(j)$ represents the corresponding finite element analysis calculation results, $\overline{y}$ represents the average value of the finite element analysis calculation results, and N represents the number of inspection points in the design space.

The values of R^2 and $RMSE$ represent the difference between the response surface and the finite element analysis calculation, both taking values between 0 and 1. The closer the value of R^2 is to 1, the more accurately the response surface model of the regression describes the relationship between the system input and output in the experimental design space. On the other hand, the value of $RMSE$ is the opposite, and the closer the value of $RMSE$ is to 0, the more accurate the model is.

3. FE Model Updating Based on Response Surface Method

3.1. Ambient Vibration Testing

The continuous beam bridge is located in China. The upper structure is a 30 m partial prestressed concrete continuous combination box beam bridge, with a total of 12 sections and 72 spans and a length of 2168.20 m. The design load of the bridge is car-super level 20, trailer-120. It was built in 2001 and has run for 22 years (Figure 1).

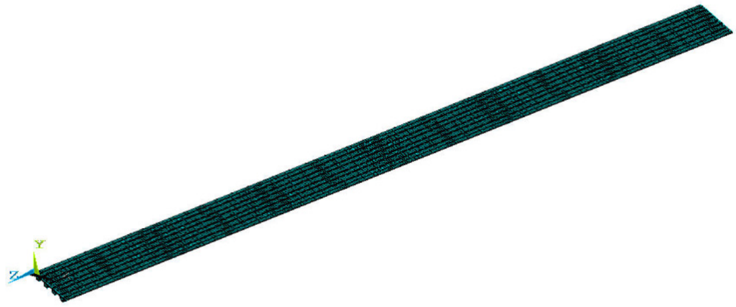
Figure 1. The continuous beam bridge.

Based on an appearance inspection of the bridge, it can be found that among these 12 bridges, the 9th bridge is significantly damaged, so the bridge was selected to conduct the ambient vibration testing. Each span arrangement was 9 point + 1 reference point (Figure 2), each point had a three-dimensional acceleration sensor, and each span had a station with a total of six stations. The sampling frequency was 200 Hz, filtering was 200 Hz, and the sampling time of each station was not less than 12 min. Based on the data of the test, the natural vibration frequency and vibration mode can be identified based on the peak picking (PP) method and stochastic subspace identification (SSI) method.

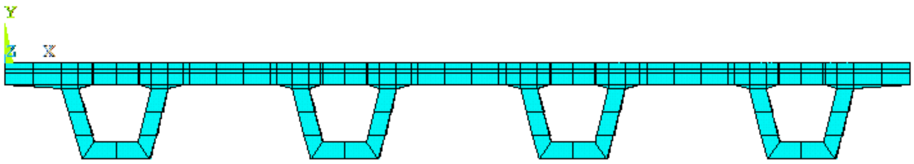
Figure 2. Sensor arrangement for environment vibration.

3.2. The Initial FE Model

The Initial FE model is based on the design and completion drawing, and the model used Cartesian coordinates: The Z-axis is along the longitudinal axis of the bridge, the Y-axis is vertically upwards, the X-axis is transverse to the bridge, and the longitudinal axis of the bridge is vertical. The entire bridge is simulated using solid45; combin14 is used for boundary constraints of the main bridge. For prestressed reinforcement, Jain et al. [17] proposed that the influence of prestressing on the vibration frequency of the structure is relatively small and can be ignored, so the finite element model in this article does not consider prestressed steel bars. According to relevant design material, the material parameter initial value of concrete is as follows: the elasticity modulus of box beam E = 3.45 × 104 MPa; the density is 2550 kg/m^3; the Poisson ratio is 0.167; and the whole bridge has 195,744 nodes and 49,992 units, as shown in Figure 3.



(**a**) Integral FE model

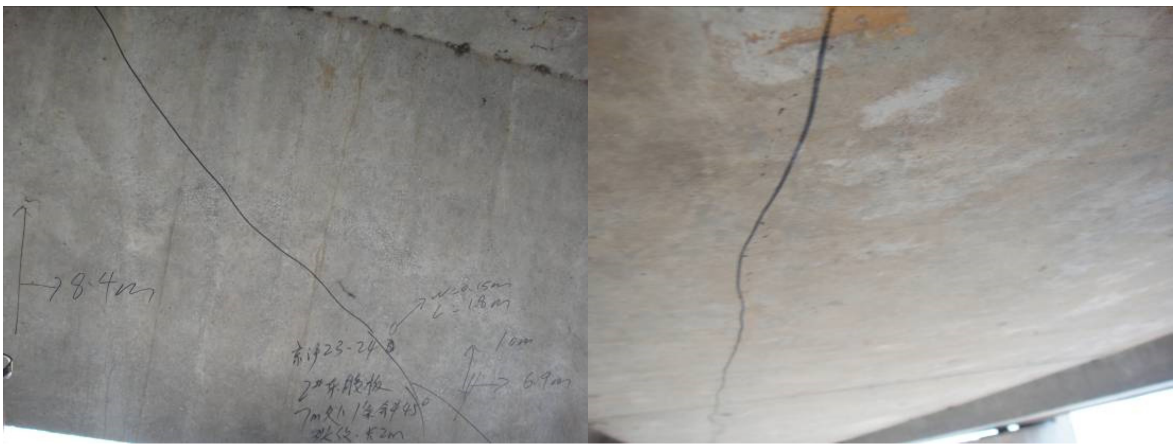


(**b**) side view

Figure 3. The initial FE model.

3.3. Parameters Selection

When establishing a finite element model, it is usually necessary to use a mathematical parameter to represent the actual structure, which includes geometric parameters, cross-sectional parameters, material parameters, load parameters, boundary conditions, etc. A reasonable selection of model parameters is another key point to obtain an accurate finite element model. The damage of this connecting bridge mainly manifests as longitudinal cracks in the concrete box beam web, numerous cracks in the bottom plate, and transverse cracks (Figure 4). Therefore, when updating the finite element model of a bridge with cracks or damage, it is necessary to consider the influence of bridge crack distribution and damage degree. Based on the characteristics of the parameters in the model and the update of the high-order response surface finite element model, this article takes modulus and support spring stiffness as the updated parameters, and selects the updated parameters based on the engineering experience, crack distribution and strength level distribution of the bridge, as shown in Table 1.



(**a**) cracks in the web of concrete box beam (**b**) longitudinal cracks

Figure 4. Typical cracks in the concrete box beams.

Table 1. Parameters selection.

Parameters	Modulus of Elasticity of Concrete				Spring Stiffness		
	E_1	E_2	E_3	E_4	K_1	K_2	K_3
Sections	No crack sections	2th, 3th, 5th span web sections (more fracture)	4th span web sections (less fracture)	2-3, 2-4, 5-1, 5-2, 5-3, 5-4 bottom slab sections (Fracture is homogeneous)	transverse spring stiffness at the support and expansion joints	longitudinal spring stiffness at the support	longitudinal spring stiffness at the expansion joints

Note: 2-3 refers to the third beam of the second span, and the rest are the same.

3.4. Experimental Design

The experimental design is closely related to the system response characteristics and the form of the response surface functions, and different experimental design methods are applicable to different systems and their response surface function forms. The experimental design methods of the response surface method include a full factor design, central composite experimental design, D-optimal, BBD design, orthogonal design, uniform design, etc. [14], among which the orthogonal design and uniform design are only applicable to low-order response surface models. Due to the existence of a large number of design points, the BBD design should not be applied to large models. The full factor design has a high accuracy, but the computational workload is too large. D-optimal is suitable for the high-order response surface modeling of large models, with the highest accuracy. This article introduces the updating process of finite element models based on the third-order response surface method: (1) it uses the D-optimal method to obtain sample points; (2) It substitutes experimental data into the initial finite element model to obtain sample values, as shown in Table 2.

Table 2. The sample values of the experimental design.

N	E_1	E_2	E_3	E_4	K_1	K_2	K_3	H_1	H_2	Z_1	S_1	S_2	S_3	S_4
1	3.60	2.50	3.60	2.50	0.80	2.00	3.67	0.91	1.88	1.18	2.87	3.06	3.60	4.27
2	2.50	2.50	3.60	2.87	0.80	7.00	5.33	0.91	1.87	2.21	2.86	3.06	3.58	4.24
3	2.50	3.23	3.60	3.60	0.67	7.00	2.00	0.86	1.77	2.21	2.90	3.10	3.62	4.30
4	3.60	3.60	2.50	2.50	0.60	2.00	2.00	0.83	1.71	1.18	2.86	3.06	3.58	4.25
5	3.33	2.78	2.78	3.33	0.70	5.75	4.50	0.87	1.81	2.00	2.90	3.11	3.62	4.31
6	2.50	3.23	3.23	3.60	0.80	7.00	7.00	0.91	1.88	2.21	2.89	3.10	3.61	4.29
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
125	2.87	3.60	2.87	3.60	0.40	7.00	2.00	0.73	1.47	2.21	2.91	3.11	3.62	4.23
126	3.05	3.60	2.50	2.50	0.80	7.00	7.00	0.91	1.88	2.21	2.85	3.05	3.57	4.23
127	3.23	2.50	3.60	2.50	0.80	7.00	7.00	0.91	1.88	3.21	2.87	3.07	3.59	4.26
128	3.23	2.50	2.50	3.60	0.40	3.67	2.00	0.73	1.47	1.60	2.90	3.11	3.61	4.24
129	3.60	2.87	3.60	3.60	0.40	7.00	2.00	0.74	1.48	2.21	2.94	3.14	3.66	4.29
130	3.05	3.33	3.33	2.78	0.70	5.75	3.25	0.87	1.80	2.00	2.87	3.07	3.60	4.26

3.5. Significance Test of Parameter

After the parameter selection, the response parameters of each significance are very important. Screening for parameters with a high response and ignoring parameters with a low response can effectively reduce the response surface function coefficients and computational complexity. Zong et al. [14] used the response surface method to update the finite element model of the Xiabaisi Bridge and applied the F-test method based on mathematical statistics to complete the significance test of the parameters. The basic principle is to decompose the square sum of total deviations of sample data into the regression sum of squares and square sum of errors, as shown in Equation (4).

$$F_m = \frac{SSE(x_1, x_2, \cdots x_{m-1}) - SSE(x_1, x_2, \cdots x_{m-1}, x_m)}{SSE(x_1, x_2, \cdots x_{m-1}, x_m)/(n-m-1)} \tag{4}$$

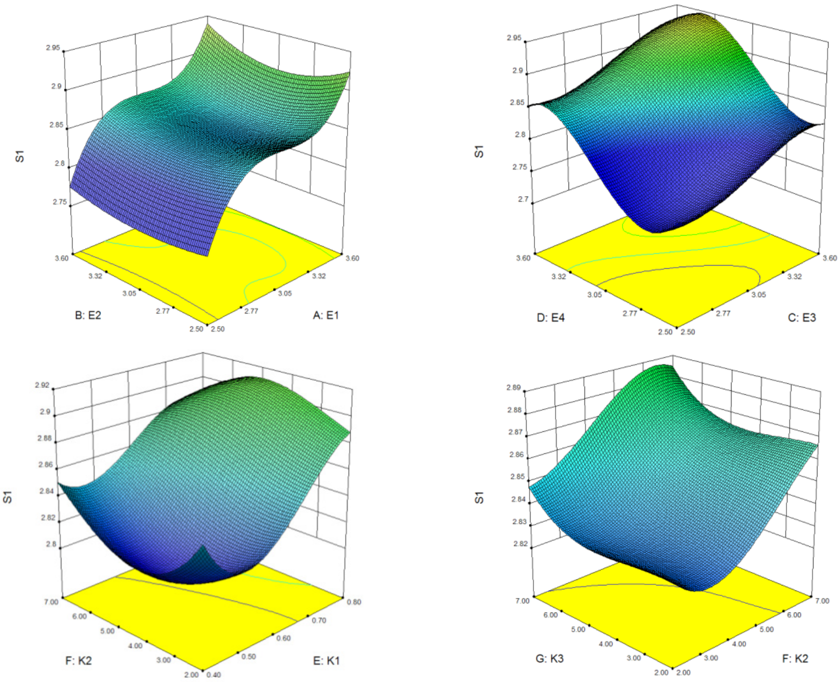
where $SSE(x_1, x_2, \cdots x_{m-1})$ is the sum of errors of the regression model including the independent variables $x_1, x_2, x_3 \cdots x_{m-1}$, $SSE(x_1, x_2, \cdots x_{m-1}, x_m)$ is the sum of errors of the regression model including the independent variables $x_1, x_2, x_3 \cdots x_{m-1}, x_m$, and n is the total number of all independent variables in the regression model. The stepwise regression method using F-test as the standard tests the significance of each variable and decides which independent variable to abandon. However, when considering all variables when calculating the response surface, the computational complexity added in this article is sustainable. Therefore, in the calculation, we will try to ensure accuracy as much as possible and consider the influence of all parameters [26].

3.6. Response Surface Fitting and Accuracy Inspection

In order to fully consider the influence of the randomness of the structural parameters on the structural dynamic properties, a third-order response surface function is selected for data fitting. The finite element model correction is based on the response surface model instead of the finite element model. Usually, the selected structural response feature is more than one; that is, there are multiple objective functions. The mathematical model for the multi-objective optimization problem is given as follows:

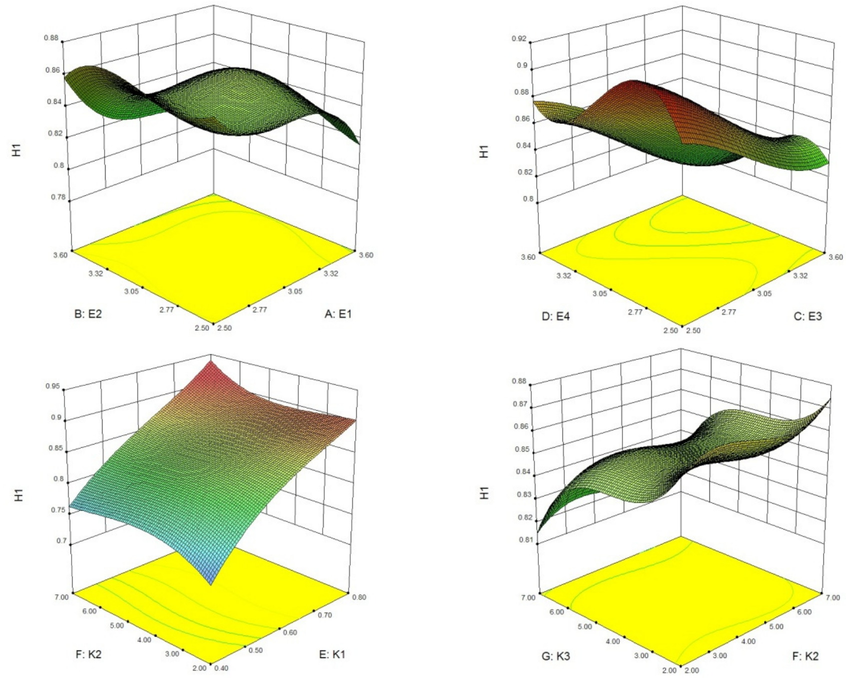
$$\min F(x) = \min\{f_1(x), f_2(x), \cdots f_p(x)\} \tag{5}$$

where $x = (x_1, x_2, \cdots, x_n)^T$ is the structural parameters of the bride, $f_i(x)$ $(i = 1, \cdots, p)$ is the error between the measured and calculated frequencies, and $F(x)$ is the objective function after transforming into a multi-objective problem. The response surface fitting functions for vertical, horizontal, and vertical fundamental frequencies are Equations (6)–(8). The relationship between the response surface model and parameters can be visually displayed using graphics, and the relationship between parameters and vertical, horizontal, and vertical first-order frequencies is shown in Figure 5.

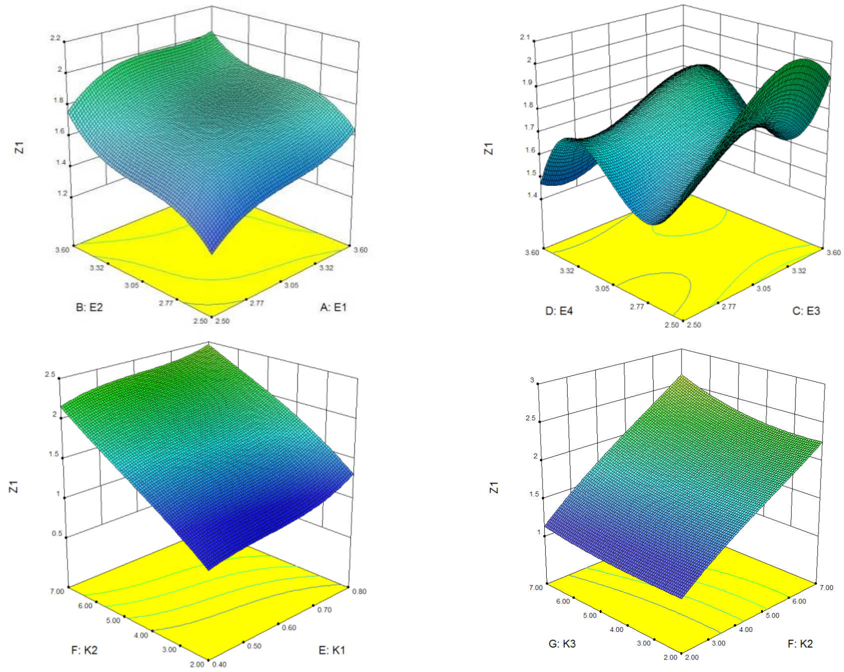


(**a**) Sketch map of vertical first-order response surface.

Figure 5. *Cont.*



(**b**) Sketch map of transversal first-order response surface.



(**c**) Sketch map of longitudinal first-order response surface.

Figure 5. Response surface of vertical, transversal, and longitudinal direction vibration.

Vertical first-order response surface function:

$$\begin{aligned} V_1 = {} & 25.9567 + 11.1342\mathrm{E}_1 - 0.38361\mathrm{E}_2 - 9.45490\mathrm{E}_3 - 21.0324\mathrm{E}_4 - 15.6256\mathrm{K}_1 - 0.18289\mathrm{K}_2 \\ & -0.36175\mathrm{K}_3 + 1.27610\mathrm{E}_1\mathrm{E}_4 + 2.62588\mathrm{E}_1\mathrm{K}_1 + 0.07253\mathrm{E}_1\mathrm{K}_3 + 0.66056\mathrm{E}_2\mathrm{E}_3 + 0.87598\mathrm{E}_3\mathrm{E}_4 \\ & +1.07189\mathrm{E}_3\mathrm{K}_1 + 1.26066\mathrm{E}_4\mathrm{K}_1 + \ldots - 4.23980\mathrm{E}_1^2 + 0.03922\mathrm{E}_2^2 + 2.23632\mathrm{E}_3^2 + 5.71977\mathrm{E}_4^2 \\ & +12.3050\mathrm{K}_1^2 + 0.04803\mathrm{K}_2^2 + 0.02439\mathrm{K}_3^2 + \ldots - 0.01957\mathrm{E}_1\mathrm{E}_2\mathrm{K}_1 - 0.09726\mathrm{E}_1\mathrm{E}_4\mathrm{K}_1 \\ & -0.02378\mathrm{E}_1\mathrm{K}_1\mathrm{K}_3 + 0.09432\mathrm{E}_2\mathrm{E}_4\mathrm{K}_1 + 0.13291\mathrm{E}_3\mathrm{E}_4\mathrm{K}_1 + 0.02108\mathrm{E}_3\mathrm{K}_1\mathrm{K}_2 - 0.01690\mathrm{E}_4\mathrm{K}_1\mathrm{K}_2 \\ & + \ldots - 0.35265\mathrm{E}_1^2\mathrm{K}_1 - 0.18027\mathrm{E}_3^2\mathrm{K}_1 - 0.35112\mathrm{E}_3\mathrm{K}_1^2 - 0.16553\mathrm{E}_4^2\mathrm{K}_1 - 0.43492\mathrm{E}_4\mathrm{K}_1^2 \\ & -0.18576\mathrm{K}_1^2\mathrm{K}_2 - 0.13063\mathrm{K}_1^2\mathrm{K}_3 + \ldots + 0.49168\mathrm{E}_1^3 - 0.00080\mathrm{E}_2^3 - 0.19202\mathrm{E}_3^3 - 0.51469\mathrm{E}_4^3 \\ & -4.43204\mathrm{K}_1^3 - 0.00077\mathrm{K}_2^3 + 0.00045\mathrm{K}_3^3 \end{aligned} \tag{6}$$

Transversal first-order response surface function:

$$
\begin{aligned}
H_1 = {} & -12.9501 - 3.17503\mathrm{E}_1 - 1.80070\mathrm{E}_2 + 4.41081\mathrm{E}_3 + 11.9143\mathrm{E}_4 + 8.31658\mathrm{K}_1 + 0.09156\mathrm{K}_2 \\
& + 0.36961\mathrm{K}_3 + 0.35748\mathrm{E}_1\mathrm{E}_2 - 0.46230\mathrm{E}_1\mathrm{E}_4 - 0.68350\mathrm{E}_2\mathrm{E}_3 - 0.74999\mathrm{E}_2\mathrm{K}_1 - 0.49871 \\
& \mathrm{E}_3\mathrm{E}_4 - 1.80669\mathrm{E}_3\mathrm{K}_1 - 0.27245\mathrm{K}_1\mathrm{K}_2 + \ldots + 1.15553\mathrm{E}_1^2 + 0.92512\mathrm{E}_2^2 - 0.73353\mathrm{E}_3^2 - 3.50574 \\
& \mathrm{E}_4^2 - 5.63844\mathrm{K}_1^2 - 0.04230\mathrm{K}_2^2 - 0.00136\mathrm{K}_3^2 - 0.02281\mathrm{E}_1\mathrm{E}_3\mathrm{E}_4 - 0.01767\mathrm{E}_1\mathrm{E}_3\mathrm{K}_1 \\
& + 0.02067\mathrm{E}_1\mathrm{E}_4\mathrm{K}_1 + 0.01882\mathrm{E}_1\mathrm{K}_1\mathrm{K}_3 + 0.04657\mathrm{E}_2\mathrm{E}_3\mathrm{K}_1 + 0.02315\mathrm{E}_2\mathrm{E}_4\mathrm{K}_1 - 0.03759\mathrm{E}_3\mathrm{E}_4\mathrm{K}_1 \\
& + \ldots - 0.08023\mathrm{E}_1\mathrm{E}_2^2 - 0.30160\mathrm{E}_1\mathrm{K}_1^2 + 0.09679\mathrm{E}_2^2\mathrm{E}_3 + 0.19840\mathrm{E}_3^2\mathrm{K}_1 + 0.08949\mathrm{E}_3\mathrm{E}_4^2 \\
& + 0.58062\mathrm{E}_3\mathrm{K}_1^2 + 0.10292\mathrm{E}_4\mathrm{K}_1^2 + \ldots - 0.16640\mathrm{E}_1^3 - 0.11656\mathrm{E}_2^3 + 0.06519\mathrm{E}_3^3 + 0.33159\mathrm{E}_4^3 \\
& + 1.77010\mathrm{K}_1^3 + 0.00084\mathrm{K}_2^3 - 0.00149\mathrm{K}_3^3
\end{aligned} \tag{7}
$$

Longitudinal first-order response surface function:

$$
\begin{aligned}
L_1 = {} & 100.278 + 10.7861\mathrm{E}_1 + 4.68355\mathrm{E}_2 - 15.9553\mathrm{E}_3 - 89.2199\mathrm{E}_4 - 34.6391\mathrm{K}_1 - 2.56437\mathrm{K}_2 \\
& - 0.25816\mathrm{K}_3 + 13.8429\mathrm{E}_1\mathrm{K}_1 + 4.42365\mathrm{E}_2\mathrm{E}_3 + 6.14491\mathrm{E}_2\mathrm{E}_4 - 3.91058\mathrm{E}_2\mathrm{K}_1 - 8.56477\mathrm{E}_3\mathrm{E}_4 \\
& + 6.13058\mathrm{E}_3\mathrm{K}_1 + 11.0166\mathrm{E}_4\mathrm{K}_1 + \ldots - 5.85373\mathrm{E}_1^2 - 7.76984\mathrm{E}_2^2 + 7.21838\mathrm{E}_3^2 + 28.7936\mathrm{E}_4^2 \\
& - 12.9758\mathrm{K}_1^2 + 0.09733\mathrm{K}_2^2 - 0.04939\mathrm{K}_3^2 + \ldots - 0.66806\mathrm{E}_1\mathrm{E}_3\mathrm{K}_1 + 0.33681\mathrm{E}_1\mathrm{E}_4\mathrm{K}_1 \\
& + 0.23196\mathrm{E}_1\mathrm{K}_1\mathrm{K}_2 + 0.07169\mathrm{E}_2\mathrm{E}_3\mathrm{E}_4 - 0.27307\mathrm{E}_2\mathrm{E}_3\mathrm{K}_1 - 0.11152\mathrm{E}_2\mathrm{K}_1\mathrm{K}_3 + 0.49799\mathrm{E}_3\mathrm{E}_4\mathrm{K}_1 \\
& + \ldots - 2.31562\mathrm{E}_1^2\mathrm{K}_1 - 0.77985\mathrm{E}_2\mathrm{E}_4^2 - 0.95213\mathrm{E}_2\mathrm{K}_1^2 - 1.05722\mathrm{E}_3^2\mathrm{K}_1 + 1.50974\mathrm{E}_3\mathrm{K}_1^2 \\
& - 2.09818\mathrm{E}_4^2\mathrm{K}_1 - 2.94197\mathrm{E}_4\mathrm{K}_1^2 + \ldots + 0.70305\mathrm{E}_1^3 + 1.09540\mathrm{E}_2^3 - 0.72481\mathrm{E}_3^3 - 2.74784\mathrm{E}_4^3 \\
& + 13.1539\mathrm{K}_1^3 + 0.00292\mathrm{K}_2^3 + 0.00082\mathrm{K}_3^3
\end{aligned} \tag{8}
$$

In order to obtain the correlation between the response surface model and the finite element model, the value of R^2 and the relative root mean square error (RMSE) can be used to test the accuracy of the response surface after fitting the response surface function. The calculation results are shown in Table 3, and it can be seen that the value of R^2 is close to 1, RMSE is close to 0, and the difference between the calculated value of the response surface function and the actual value is small. Therefore, in the parameter design space, the response surface function can effectively reflect the relationship between the structural response and parameters, and the response surface model can replace the finite element model for model updating.

Table 3. The accuracy inspection of each response surface.

Modal	1st-Order Transversal (H_1)	2nd-Order Transversal (H_2)	1st-Order Longitudinal (L_1)	1st-Order Vertical (V_1)	2nd-Order Vertical (V_2)	3rd-Order Vertical (V_3)	4th-Order Vertical (V_4)
R^2	1.0000	0.9989	0.9993	0.9998	0.9948	0.9998	0.9997
RMSE	0.0000	0.0007	0.0006	0.0001	0.0005	0.0001	0.0001

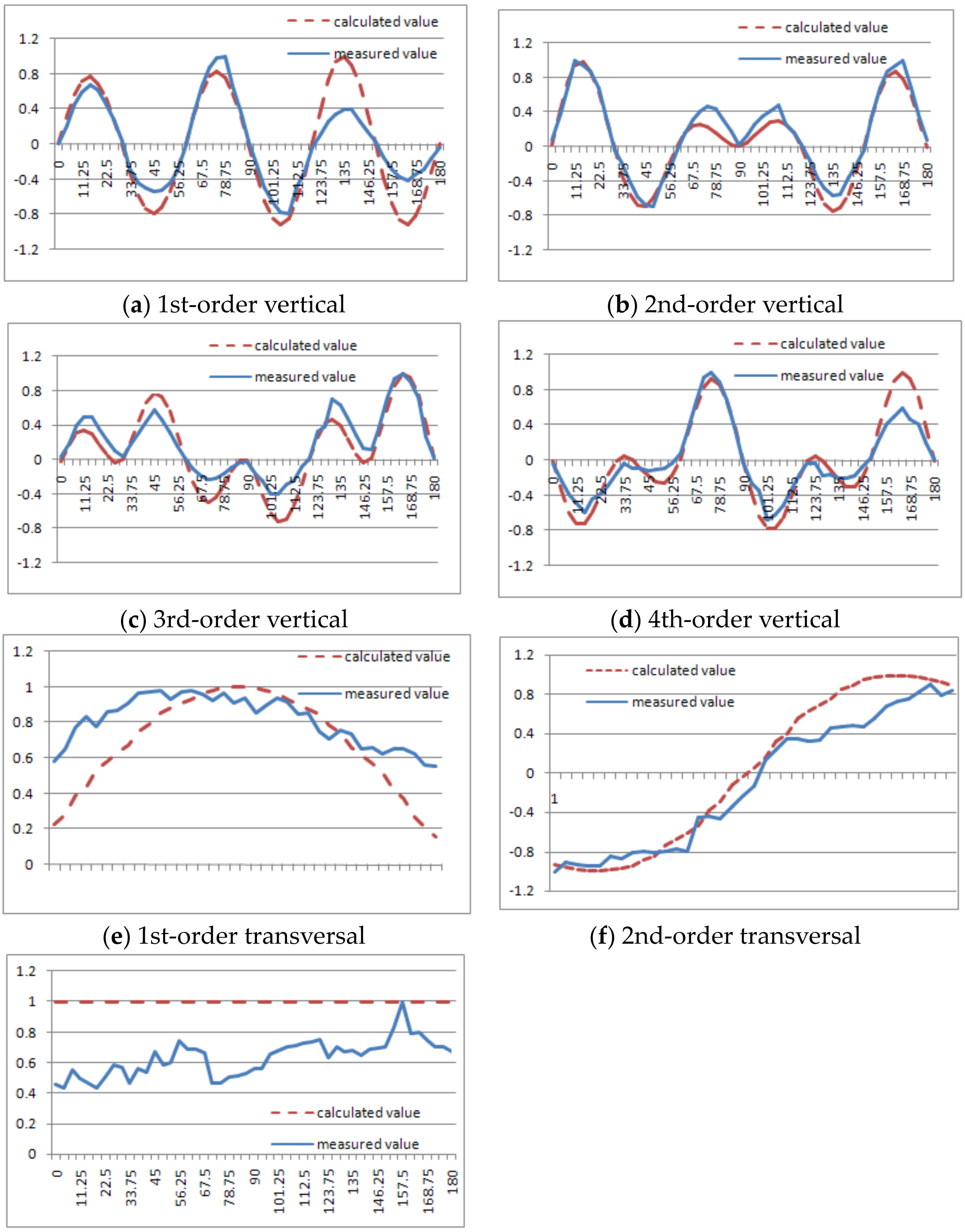
3.7. Model Updating

The frequency obtained through visual inspection and environmental vibration testing can be optimized using nonlinear programming methods to solve the nonlinear optimization of the response surface model. The updated parameters are shown in Table 4. The updated parameters still have the physical meaning shown in Table 4. Then, the updated parameters are substituted into the finite element model calculation, and the calculated results are compared with the measured results, as shown in Figure 6 and Table 5. It is known that the calculated frequency of the updated response surface model is close to the measured frequency, with a maximum error of less than 3%, and the modal assurance criterion (MAC) is greater than 85%, indicating a good correlation between the calculated mode and the measured mode. The finite element model based on the third-order response surface method can accurately and effectively simulate the actual bridge state.

Table 4. Parameter change before and after updating.

Parameter	E_1	E_2	E_3	E_4	K_1	K_2	K_3
Initial value	3.45	3.45	3.45	3.45	0.50	4.50	3.00
Updated value	3.62	3.49	3.21	2.51	0.60	3.08	3.07
Updated rates	4.92%	1.16%	−6.82%	−27.17%	20.01%	−31.51%	2.27%

Notice: E_1, E_2, E_3, E_4($\times 10^4$ MPa), K_1, K_2, K_3($\times 10^6$ N/m).



(**a**) 1st-order vertical (**b**) 2nd-order vertical (**c**) 3rd-order vertical (**d**) 4th-order vertical (**e**) 1st-order transversal (**f**) 2nd-order transversal (**g**) 1st-order longitudinal

Figure 6. Comparison of modal vibration shape between the testing and calculation.

Table 5. Comparison of updated frequencies and measured frequencies.

Modal	Updated Frequency (Hz)	Measurement Frequency (SSI) (Hz)	Relative Error (%)	MAC (%)
1st-order vertical (V_1)	2.879	2.891	−0.43%	86.3
2nd-order vertical (V_2)	3.081	3.025	1.84%	94.8
3rd-order vertical (V_3)	3.610	3.792	0.51%	91.4
4th-order vertical (V_4)	4.317	4.263	1.27%	85.2
1st-order transversal (H_1)	0.729	0.830	−0.16%	94.3
2nd-order transversal (H_2)	1.467	1.426	2.84%	94.2
1st-order longitudinal (L_1)	1.741	1.785	−2.45%	96.6

The updated finite element model can be used as a basic model for bridge structural damage identification, and combined with the finite element model and structural health monitoring data analysis, it can achieve a bridge structural safety assessment. At present, many scholars have begun to use finite element models to predict the future performance of bridges, in order to achieve a structural damage prognosis and help decision-makers develop appropriate maintenance and repair plans to ensure the safe use of bridges.

4. Conclusions

In this paper, a third-order response surface method is proposed to update the finite element model of bridge structures. Based on a continuous beam bridge experiment, the following conclusions can be drawn:

(1) The high-order response surface can better solve the problems of randomness and uncertainty in the model updating process. Through the continuous bridge experiments, this third-order response surface method can quickly and accurately achieve model correction of bridge structures. The calculated frequency of the updated response surface model is close to the measured frequency, with a maximum error of less than 3%. The modal assurance criterion (MAC) is greater than 85%, indicating a good correlation between the calculated mode and the measured mode.

(2) The updated model can better serve as the basis for bridge health monitoring, damage detection, and safety assessment. However, when using the polynomial response surface model, an increase in the polynomial order and an increase in the parameters to be corrected will cause an increase in the undetermined coefficients of the polynomial. Due to limitations in computation time and amount, the number of parameters to be corrected is limited to a certain extent, making it impossible to fully consider the impact of all parameters on the structural system. Therefore, faster high-order response surface calculation models, fast optimization iterative algorithms, and the development of corresponding software are all worth researching and developing.

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Nomenclature

CCD	central composite experimental design method
RBF	radial basis functions
MARSF	multivariate adaptive regression spline function
RMSE	relative root mean square error
PP	peak picking
SSI	stochastic subspace identification
MAC	Intrinsic mode function modal assurance criterion
RMS	The Response Surface Method

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