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Cap Rate as the Interpretative Variable of the Urban Real Estate Capital Asset: A Comparison of Different Sub-Market Definitions in Palermo, Italy

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Abstract: Real estate capital is in constant competition with other capital assets due to its different and complementary economic functions such as direct use, productive investment, and speculative investment. These features and the resulting opportunities cannot be easily deduced from direct observation of the real estate markets, so some further insights need to be carried out in order to highlight the relationship between prices, rents and performances. This study aims at providing a multifaceted perspective of a specific urban real estate market to overcome the difficulties arising from opacities and informative asymmetries that hinder the decision of investors, by facilitating the comparison of different options such as capital value, income and performance. Within the mass appraisal approach, the study proposes a methodology for the analysis of the cap rate, intended as the expression of profitability and liquidity of the urban real estate capital asset. The methodology is based on a detailed survey of a sample of the housing market data, collected within a structured database, supported by statistical and territorial analyses of the sample, in order to display the range of cap rates featuring each sub-market, and the related distributions. The methodology is applied to a case study of nearly 1000 properties distributed in a vast urban area of the municipality of Palermo, Italy. The consistency of the relationships between the three variables has been tested with reference to two hypotheses about the sub-market definition, which has been carried out by cluster and by neighbourhood.

Keywords: cap rate; mass appraisal; cluster analysis; real estate market; theory of the capital

1. Introduction

1.1. General Issues and Aims

Cities. Big cities, as great palimpsests and due to the complexity of the functions and attributes that characterise them, are the most suitable places for the development of forms of economic communication (wealth exchange) capable of generating surpluses. On the other hand, this *autopoietic* ability—the ability to produce surplus by themselves—has as its flip side the possibility of overwhelming inequalities between central and marginal areas. The real estate industry is the cause and effect of such processes, and the representation of property market features reflects the way the economic surplus is distributed in the various forms of social capital or accumulated in the urban rent.

Rent. Urban housing is one of the shapes of the urban rent stock value, including single properties as well as urban aggregates, which the public investments in the infrastructural, architectural, social

and cultural capital is part of. In complex cities more than elsewhere, the main imperfections that result in overcrowding in urban housing are: the opacity of the real estate market (mostly due to the complexity of the characterisation of the properties); the difficulty of defining its submarkets [1]; the asymmetries of information privileges; the strong differentiation of agents by financial situation and income level; the different ability to meet risk and expectation; specific expertise in the exploitation of real estate cycles; and the stronger attractiveness of external investors. In a more general sense, a city is a place that maximises the creativity of more experienced real estate agents, who take advantage of their dominant position in both bullish and bearish trends. Consequently, the real estate market is the context of asymmetrical and in many ways overwhelming economic communication, to the detriment of agents with limited economic potential and driven by mainly utilitarian reasons.

Appraisal. The primary concern of the science of valuation is to recognise how much of urban rent could return to the city, e.g., by adapting real estate taxes to equalisation purposes. Valuation science assumes the real estate market as a privileged form of urban economic communication, and its representation as a premise of a possible contribution in terms of economic justice [2,3]. In this sense, it is committed in multiple market *representations* in the broad spectrum ranging between *observation* and *interpretation*: the former investigates unit prices, usually clustered according to conventional classifications, useful for a general awareness of macroeconomic and structural situation, rather than for the estimates; the second, as proposed in this study, finalises the direct investigation of prices and rents to calculate the cap rates of the properties whose price is known. The cap rate, in fact, includes the economic, monetary and financial characteristics of the real estate assets and, consequently, their ability to take part in the formation and distribution of the urban wealth stock.

Approaches. In terms of the general approach to the appraisal [4], some researchers have indicated important differences between Appraisal-Based and Transaction-Based Capitalisation Rates: appraisers base the price mostly on property-specific factors (e.g., location, state of maintenance, etc.) and micro-level risks (e.g., refurbishment risks); investors assume a wider perspective and are mostly concerned with the portfolio composition and the economic risk at the macro level (e.g., potential variation in expected Net Operating Income NOI) [5]. The data availability is the main support for real estate market analyses and property estimates, because they obviously affect the significance of the result; as a consequence, certain models have defined procedures operating in different cases by using a large sample of market prices, a few real estate data, a single datum (price), or data from other similar market areas [6,7]. Real estate market segmentation may be performed by applying various methodologies, for example multilevel hierarchical analysis permits to identify submarkets and calculate those index numbers representing the real estate dynamics [8]. The index numbers for the revaluation of the property market prices may also be appraised by using a methodology based on Simple Price Index Methods [9]. The effect of location on cap rate has been studied using different models of Multiple Regression Analysis MRA. With regard to the criticisms made of this approach for appraising real estate, a computerised probabilistic model is developed introducing the uncertainty in the appraisal and using Monte Carlo simulations expressing the appraiser's perception of the market [10]. The linear regression can also include as a variable an Iterative Location Adjustment Factor to introduce the influence of location on the mass appraisal [11].

Ethics. An informed and "critical representation" of the housing market should assert a sort of "real estate ethics" by stating some orientations that extend over space and time the constructive functions of the urban rent. A "real estate ethics" should drive urban renovation programmes according to environmental policy and reduction of energy use [12] to boost the social welfare of marginal neighbourhoods and the mobility between social strata, as well as reduce the typical filtering effects of regeneration and gentrification processes, of which the real estate market imperfection is a cause as well as an effect. Such a "real estate ethics" reduces the risks coming from real estate finance that turns properties into securities: property has the natural and original function of keeping homeowners' wealth safe; securities have an unnatural ability to drain this wealth for the benefit of the real estate companies.

Cap rate. From the point of view of evaluation science, the income method is far more significant than the direct comparison method; in fact, although the latter is generally considered much more accurate, it performs an *uncritical* analysis of the evidence, whereas the former, although more challenging due to the number of variables involved, sometimes imponderable, performs a *critical* interpretation of the possibilities that a capital asset has to produce income and increase in value as itself according to the determinants of the agents who get into a complementary relationship with it. As a result, the cap rate reveals the agents' motivations and as such is a psychological and prospective variable as well as the marginal efficiency of capital; since cap rate takes into account both the explicit liquidity (Net Operating Income) and the implicit one, i.e., psychological income coming from the propensity to hoard even in unfavourable circumstances, it should not be interpreted as a rate of profitability; this explains why some properties shows prices that are over- (or under-) estimated with respect to income.

Interpretations. Cap rate is a complex economic variable in which the interaction of numerous factors that act at micro- and macroeconomic as well as territorial levels is condensed [13]. Many studies concern the influence of these factors on cap rate, and the relationship between cap rate and property location has been developed specifically to get the cap rate differentiation for geographical or metropolitan statistical areas [14,15] using regression models (cross sectional/time series regression), or to estimate the range of cap rate as a function of the distance from the city centre using hedonic prices [16]. Other studies have instead applied regression models to examine the cap rate range by different property types (apartments, commercial retail, office building, commercial services, industrial and hotels) [14].

Many macroeconomic factors and fiscal policies also influence cap rate; consequently, the impact of the expected economic growth, of the liquidity [15], and of new tax laws on cap rate [17] has been analysed. In other studies cap rate is assumed to be the overall return on an investment and is expressed as WACC (Weighted Average Cost of Capital). In the case of real estate investments, the cap rate is defined as the WACC return on mortgage (loan to value) and return on equity (ROE) (return on alternative investments) [18].

In addition, the results of the capital asset pricing model (CAPM) indicate significant adjustment lags of cap rate to change in the capital market spreads [19]. According to a different approach, and using the traditional method of cash flow analysis, the cap rate may be appraised as Internal Rate of Return IRR and its range results correlated to the level of risk [20]. A stochastic approach to estimate the return rate sensitivity of real estate has also been based on the discounted cash flow (DCF) and Monte Carlo simulations. From this approach it results that, putting the analysis of the cap rate in a dynamic macroeconomic environment (defined by 15 determinants), the return rate is influenced by many factors such as risk premium, state of the macroeconomic system, property's remaining lifetime, etc., and that the uncertainty of the major parameters and the appraisal model must be included in the method to face unpredictable events such as severe recessions or periods of oversupply [21].

The city is a complex system of interrelations generating numerous opportunities, so that the Marginal Efficiency of Capital (*MEC*) approach can be considered preferable in comparison to the IRR approach, because the former takes into account the expectations of the investors in reference to the ask prices, rather than the pure observations of the market prices on which IRR is based.

1.2. Capital Theory and Cap Rate (Best) Practice

Cap rate is a variable involving real estate in the ground of the typical capital asset characteristics. As such, the role it plays in the description and interpretation of a real estate market needs to be specified referring to some issues of capital theory, as it was set forth by Francesco Rizzo in 1977 [22–25]. This dynamic approach focuses on the typical *autopoietic* capability of capital asset both in increasing and in decreasing value trends. As a consequence, capital value *V* is considered to differ from the current, ordinary or commonly assumed market price *k*; this plus/minus valuation is $\pm|a|k$; *a* measures

the expected (future discounted) increase in value, which can also be negative, so that the value of a capital asset is $V = k \pm |a|k$.

As mentioned in Section 1.1, such an approach is consistent with a particular vision of the current capital asset economy, mostly based on a speculative–financial paradigm, in which *causes* (unintentional current market prices k) are overcome by *motivations* (intentional perspective expected values V): we recognise that none acts without projecting to present its own future outlook. Thus, the capital value V of an enduring asset takes into account both the probability (k) and the possibility of overcoming or contradicting ($|a|k$), for better ($+|a|$) or for worse ($-|a|$). More precisely, we can state that the "possibility gives sense to probability".

This interpretation influences the implementation of the income approach and the capitalisation formula $V = Rn/r$, where Rn is the Net Operating Income and r is the cap rate. If $R_n/r = k \pm |a|k$, then $r = R_n/k(1 \pm |a|)$, and $r = f(\pm|a|)$, where, $(\pm|a|)$ is the expected increase/decrease in value. Therefore we distinguish $r = R_n/k(1 \pm |a|)$—as the *true cap rate* including expectation—from $r' = R_n/k$, which may be considered the average (or commonly esteemed) cap rate. Moreover, Rizzo re-interprets the Hicks concept of "crescendo" C and "diminuendo" D [24] as the positive or negative difference between current and expected cap rate, respectively: $r' > r \to C$; $r' < r \to D$. An income stream is "in crescendo" if its discounted expected value overcomes the current price, i.e., if it capitalises a greater income and vice versa. This capability does not belong to the asset itself, but mostly depends on the expectations expressed by the agents by means of their (dis-)investment decisions.

Although Rizzo also expresses the crescendo/diminuendo in terms of liquidity (expected capital gain/loss) and income (expected gain/loss of rent) [23], we recognise that the crescendo/diminuendo in terms of cap rate comprises and overcomes the former two, and that more explicitly includes the economic, financial, and monetary relationships between the microeconomic characteristics of the asset and the current macroeconomic climate and perspective.

The *crescendo in terms of income* is attributable to the Keynesian concept of marginal efficiency of capital *MEC* of an investment, equating the discounted *perspective yields* to the current supply price of a capital asset [26], so that, for our purposes: $MEC = r'(1 \pm |a|)$ in the case of a bullish/bearish trend; otherwise $MEC = r'$.

The *crescendo in terms of liquidity* refers to the three values—the *ask price*, *bid price* and *market price*—that are relevant for a capital asset transaction, so that the *true value* [27], the one that is relevant in an estimate, is in the range between k and V, and can be calculated as follows: $V = k(1 + r_v)^n/(1 + r_d)^n$, while r_v is the annual average increasing in value rate *expected*, r_d is the discount rate *observed*, and n is the time in which both agents forecast property can be sold; so, if $r_v > r_d$ then $V > k$, $V - k = +|a|k$ and general bullish behaviour prevails; if $r_v < r_d$ then $V < k$, $V - k = -|a|k$, and bearish behaviour prevails.

Summarising, Rizzo's relationship between the current cap rate r' and expected one r [25] can be interpreted as the relationship between the increase/decrease in value rate and the discount rate, respectively, indicating the degree of optimism and pessimism of each of the two agents involved.

Other similar approaches in the field of firm valuation come from Guatri, who proposes: 1. taking into account the uncertainty in the case of substantial immaterial investments [28]; 2. assuming the Wacc as the cap rate and reducing it according to the extra-income expectation; 3. integrating the valuation by taking into account the difference between economic capital W and potential capital W' [29], which may be considered a sort of capitalised goodwill.

A significant convergence between the theoretical approach expounded by Rizzo and the income method proposed by Forte [30] can better explain such a motivational and intentional approach to cap rate (Figure 1).

According to Forte [30], the average cap rate needs to be adjusted by taking into account some perspective circumstances concerning the variation of both value and income; similarly, according to Rizzo [22], the current value k needs to be adjusted by the expected value increase/decrease $\pm|a|k$, so that the cap rate is influenced by the plus/minus-valuation coefficient $\mp|a|$. Finally, according to

the Keynesian liquidity preference theory, the cap rate measures the different capability of a real asset to hoard liquidity, thus the prevailing of the implicit liquidity over the explicit one. We precise that the explicit liquidity is the ordinary NOI; the implicit one is the psychological income coming from owning the asset even in the extreme case of the absence of any real income.

Figure 1. Convergence between the capital theory of Rizzo and the income method of Forte.

1.3. Proposals

The proposed survey of the real estate market, involving a wide area of Palermo, covers a sample including nearly 1000 observations concerning properties for sale and for rent; with regard to the sub-sample of properties for sale, the subsequent observations and deductions converge in defining the degree of liquidity and the prevailing of the explicit or the implicit one, both over different neighbourhoods and different sub-markets, which are delimited within the sample of properties for sale by means of a cluster analysis performed by characteristics.

The definition of the sub-markets, regardless of the neighbourhood borders, has been carried out by *k-means* clustering. Then, according to the abovementioned approach combining the Rizzo and Forte perspectives, a procedure for associating a Net Operating Income with each property for sale in the various clusters has been implemented; later, the cap rates of all the properties have been calculated and some functions describing the relationship between cap rate and quality have been figured out. The proposed methodology can be applied when a significant amount of data is available to adequately represent a wide area and different neighbourhoods.

The results of the two analyses show the different distributions of the cap rate and the different inverse relationships between cap rate and the overall quality within each sub-market (by cluster and by neighbourhood).

2. Methodology for Appraising Cap Rate

The analysis of the urban real estate market (in urban areas or neighbourhoods) allows us to get structured information about the different characteristics of the properties and awareness of the relationships with market prices and rents that the cap rate depends on, so that the financial, economic, and monetary profile of each property—as well as of each consistent group of properties—can be outlined. This information can support the decisions investors make about their portfolios, and offers investors the opportunity to insert some urban real estate properties in their portfolios after having compared them with alternative investments. Furthermore, an overview of such profiles can help in urban equalisation policies concerning the renovation processes and cadastral taxation.

This study aims at analysing the complexity of the real estate market at the urban level by clusters and by neighbourhoods. In the first case, cluster analysis is applied to the real estate data to define the market segments (clusters of comparable properties), and to express their liquidity by estimating

the typical sizes and ranges of cap rate featuring each cluster. The results are compared with the same sizes and ranges featuring the different neighbourhoods (second case) according to the current territorial segmentation.

In both cases, however, the ranges of cap rate can support investors' decisions about the retention of their real estate ownership (hoarding), or about carrying out potential speculative–financial transactions (immediate or postponed purchase or sale).

Similarly, in the field of the urban and fiscal equalisation policies, cap rates characterise the properties of the submarkets more or less capable of capitalising liquidity, so more or less worthy of any tax allowance or increase.

This study performs a methodology of real estate market analysis, previously experienced [31–33], consisting of the following steps:

1. Real estate market survey and database;
2. Definition of the submarkets:

    2.1. *by cluster* (application of cluster analysis);
    2.2. *by neighbourhood* (selection on the basis of location);

3. Appraisal and comparison of the cap rates of the submarket (*by cluster* and *by neighbourhood*).

2.1. Real Estate Market Survey and Database

The first step of the housing market analysis concerns the market survey of prices and rents. According to the general purposes of this study—specifically interested in 1. the role that cap rate plays in the behaviour of users, investors and public administrations, and 2. the interpretation of a wide urban context performing the cap rate logic—ask prices and rents have been used. The survey provides a dataset with a high standard of quality in accordance with the International Valuation Standard [34], further expanded as much as detail and type of information collected allowed.

The database converts data into information so that each record is scored 1 to 5 according to 23 *attributes*, then aggregated in h ($h = e, v, t, a$) *main characteristics* (Table 1): k_e *location features* related to the neighbourhood in which the building is located, k_v *intrinsic features* associated to the position of the property in the building, k_t *technological features*, and k_a *architectural features* related both to building and property.

Table 1. Main characteristics and attributes of the properties surveyed.

Features (Main Characteristics)			
Location k_e	**Intrinsic** k_v	**Technological** k_t	**Architectural** k_a
k_{e1} Functional complexity k_{e2} Urban shape k_{e3} Facilities k_{e4} Centrality k_{e5} Functional mix k_{e6} Symbolic quality k_{e7} Settlement quality k_{e8} Societal mix	k_{v1} View k_{v2} Exposure k_{v3} Overlooking k_{v4} Brightness k_{v5} Security	k_{t1} Building maintenance status k_{t2} Property maintenance status k_{t3} Building structure k_{t4} Technologic equipment k_{t5} Building finishes	k_{a1} Architectural type k_{a2} Floor k_{a3} Size and functional adequacy k_{a4} Quality of finishing k_{a5} Accessories

The main characteristics are aggregated in a single score k^* expressing the overall quality associated to each jth property of the sample X. The overall score k^* is calculated by the following formula:

$$k_j^* = \sum_h k_{j_h} \lambda_h,\tag{1}$$

where λ_h is the weight of the hth characteristic, so that $\sum_h \lambda_h = 1$; the scores of all features related to each k_h are calculated similarly. The weights λ_h are empirically defined to maximise the R^2 of the

exponential simple regression function expressing the relationship between unit price and k^* displayed within the paragraph devoted to the application. This relationship only shows the qualitative trend of the economic variables (unit price and unit monthly rent) versus the overall quality k^*.

The whole sample X is divided into two sets, the sub-sample S of properties for sale, and the sub-sample F of the properties for rent.

2.2. Cluster Analysis and Housing Submarkets

Due to its the complexity and heterogeneity, the urban real estate market needs to be segmented into submarkets showing consistent relationships between performances (characteristics) and corresponding economic attributes (prices, rents or cap rates) according to the degree of similarity of the items [35–37].

In this case study two approaches have been performed in order to compare the above mentioned relationships that have been described both from a statistical and a territorial point of view. The first is carried out by applying cluster analysis; the second takes into account the conventional subdivision of the urban area in neighbourhoods that are assumed as the minimum territorial units.

2.2.1. Submarkets by Cluster

In reference to the real estate market, many studies have applied cluster analysis for various purposes, e.g., to represent the relations between real estate price and urban transformations [38–40], to introduce fiscal equalisation in mass appraisal [41], or to select projects for fair planning.

Cluster analysis is a particular data-mining technique used in many areas of knowledge and can be useful in the real estate field in order to articulate a large and heterogeneous market in submarkets sharing similar characteristics. Indeed, one of the most relevant difficulties of the empirical clustering is to distinguish and classify the different items if their qualitative profile depends on several characteristics, in some cases conflicting or compensating: in such cases, similar properties can report different prices, or different properties can have the same price. As a consequence, the attribute of similarity needs to be associated with a number of items according to the features that can be separately investigated.

The clustering based approaches are usually grouped as follows: hierarchical, non-hierarchical (partitions), grid-based and model-based [42].

In this study the non-hierarchical *k-means* method has been applied. This method aims at partitioning the items of the whole set into a consistent number k of clusters; the algorithm selects the elements minimising the distance from the pre-selected centroids for each characteristic [43–45]. The algorithm identifies the initial centroids, based on which it then selects the well outdistanced elements among them. At each iteration the algorithm provides a new partition, generating the new centroids of the groups, until convergence is reached and the partition becomes stable (or when the last partition is the same of the previous step) [46,47].

The procedure is articulated into the following steps:

Initialisation—Selection of temporary centroids;

Step 1—Partitioning (assigning each element to a centre minimising the Euclidean distance from it) and computing new group centroids.

Step 2—Formation of a new partition.

Step 3—Partition verification. If the last partition is different from the first, the procedure (step 1) is repeated; otherwise you jump to Step 4.

Step 4—The procedure is completed and the best possible partition is obtained.

The number of possible partitions p is calculated according to the following formula:

$$p = 2^{(n-1)} - 1, \tag{2}$$

where p is the number of possible partitions and n is the number of observations.

Since the number of possible partitions is high—e.g., for $n = 20$, p is 524.287—it is recommended to reduce the number by selecting a number of groups as appropriate. The Calinski–Harabasz index, *CH* [48,49], identifies the best subdivision, i.e., the one that maximises the external heterogeneity between the data groups and minimises the internal one.

$$CH(g) = \frac{B(g)/g - 1}{W(g)/n - g} \tag{3}$$

$$B(g) = \sum_{i=1}^{g} \overline{d(x_i, \bar{x})} \tag{4}$$

$$W(g) = \sum_{i=1}^{g} \sum_{j:x_j \in C_i} d(x_j, \overline{x_i}), \tag{5}$$

where CH_g is the Calinski–Harabasz index, B_g is the external deviance (between the groups), W_g is the internal deviance (within the group), g is the number of groups, $\overline{x_i}$ is the mean value of the observations belonging to the *i*th cluster C_i, $\bar{x}$ is the mean value of the entire sample, x_j is the *j*th observation, d is the Euclidean metric, and n is the number of the observations.

The more this index increases, the more the validity of the partition improves, since it represents the ratio between the external and the internal variance of the partition. The best partition provides, therefore, the number of submarkets S_c of the sample X.

2.2.2. Submarkets by Neighbourhood

The neighbourhoods may be considered the minimum urban units, whose quality influences in a remarkable way the real estate market prices, because of the presence of facilities, land use mix, etc. according to the accessibility and interaction principles and to the differential rent theory [16,50]. The city is a complex system arising from an *autopoietic* process [51–53], which causes continuous economic, cultural, and symbolic stratifications and modifications—over and above the physical and material ones. As a result, the neighbourhoods that have been forming over time (centuries or decades) lose their original homogeneity by interacting with each other and are influenced by the overall transformation process of the whole urban entity. In particular, the sudden development following the deep post-war economic crisis gave rise to a break in continuity in the urban shape, affected by the destruction of the identity of many neighbourhoods due to the unnatural increase of size and a poor architectural quality. This internal heterogeneity is reflected in range of rents, prices and, as a consequence, cap rates.

Nonetheless, according to such approach, the sets S (properties for sale) and F (properties for rent) are segmented by using the conventional urban subdivision in neighbourhoods.

2.3. Cap Rate Appraisal

The appraisal of the cap rate is performed in the two cases by using the well-known general capitalisation formula:

$$r = \frac{Rn}{P}, \tag{6}$$

where r is the cap rate, Rn is the Net Operating Income (NOI) of each property for sale, and P is the total price of the same property.

The *NOI* of each property for sale is: $NOI_{i_S} = l_{i_S} \cdot 12 - MOE$, where l_{i_S} is the gross monthly rent that has to be multiplied by 12 in order to calculate the gross annual operating income. The gross monthly rent l_{i_S} is the average of the gross monthly rents calculated both by surface and by rooms: the former is calculated by multiplying the unit gross monthly rent per sqm by the surface; the latter is calculated by multiplying the unit gross monthly rent per room by the number of rooms. *MOE* are the annual Management Owner's Expenditures calculated basing on the local customs, by reducing the

gross operating income by a percentage variable over the ranges indicated in Table 2, according to the specific situation of each property.

Table 2. Scheme for the Management Owner's Expenditures calculation.

	Maintenance	Insurance	Management	Services	Vacancy and Written-Off Amounts	Taxes	Total
Min	2.0%	0.5%	0.5%	0.5%	1.5%	30.0%	35.0%
Max	6.0%	1.0%	1.5%	1.5%	9.0%	30.0%	49.0%

Finally, the monthly rent l_{i_S} of each property for sale needs to be calculated in the two partitions, by cluster and by neighbourhood.

2.3.1. Appraisal of the Rent in the Partition by Cluster

In the partition by cluster l_{i_S} can be calculated with reference to cluster F_{i_S}, selected as a partition of F, which must be significant and well correlated to the corresponding ith property for sale.

The abovementioned correlation is defined on the basis of the distances of the vectors of the characteristics and the "neighbourhood relations", by a density-based clustering approach.

The computing process of a density-based clustering rests on six rules or definitions, creating two lemmas.

Definition 1: (*The Eps-neighbourhood of a point*)

$$N_{eps}(p) = \{q \epsilon D | dist(p,q) < Eps\} \tag{7}$$

For a point to belong to a cluster it needs to have at least one other point that lies closer to it than the distance *Eps*.

Definition 2: (*Directly density-reachable*)

There are two kinds of points belonging to a cluster: border points and core points:

The *Eps*-neighbourhood of a border point tends to have significantly fewer points than the *Eps*-neighbourhood of a core point.

The border points will still be a part of the cluster and in order to include these points, they must belong to the *Eps*-neighbourhood of a core point q.

1. $p \epsilon N_{Esp}(q)$

   In order for point q to be a core point it needs to have a minimum number of points within its *Eps*-neighbourhood.

2. $|N_{Eps}(q)| \geq MinPts |$ (*core point condition*).

Definition 3: (*Density-reachable*)

A point p is density-reachable from a point q with respect to *Eps* and *MinPts* if there is a chain of points $p_1, \ldots p_n$, $p_1 = q$, $p_n = p$ such that p_{i+1} is directly density-reachable from p_i.

Definition 4: (*Density-connected*)

If two border points belong to the same cluster but do not share a specific core point, they will not be density-reachable from each other. There must, however, be a core point q from which they are both density-reachable.

A point *p* is density-connected to a point *q* with respect to *Eps* and *MinPts* if there is a point *o* such that both *p* and *q* are density-reachable from *o* with respect to *Eps* and *MinPts*.

Definition 5: (*Cluster*)

If point *p* is a part of a cluster *C* and point *q* is density-reachable from point *p* with respect to a given distance and a minimum number of points within that distance, then *q* is also a part of cluster *C*.

1. $\forall\, p, q : if\ p \epsilon C$ and *q* is density-reachable from *p* with respect to *Eps* and *MinPts*, then $q \epsilon C$.

    Two points belongs to the same cluster *C*, i.e., *p* is density-connected to *q* with respect to the given distance and the number of points within that given distance.

2. $\forall\, p, q \epsilon C : p$ is density-connected to *q* with respect to *Eps* and *MinPts*.

Definition 6: (*Noise*)

Noise is the set of points in the database that do not belong to any of the clusters.

Lemma 1: *A cluster can be formed from any of its core points and will always have the same shape.*

Lemma 2: *Let p be a core point in cluster C with a given minimum distance Eps and a minimum number of points within that distance* (*MinPts*). *If the set O is density-reachable from p with respect to the same Eps and MinPts, then C is equal to the set O.*

To find a cluster, DBSCAN (Density-Based Spatial Clustering of Applications with Noise) starts with an arbitrary point *p* and retrieves all points density-reachable from *p* with respect to *Eps* and *MinPts*. If *p* is a core point, this procedure yields a cluster with respect to *Eps* and *MinPts*. If *p* is a border point then no points are density-reachable from *p* and DBSCAN visits the next point of the database.

According to this approach, for each point representative of a property of sample *S*, within a given "neighbourhood" there must exist a minimum number of points of the set *F* that form a subset F_{i_S}, which can be selected once a threshold density and the "neighbourhood" shape, such as the Euclidean distance, are fixed.

In the present case, a point belonging to F_{i_S} having coordinates $(k_e, k_v, k_t, k_a)_{F_{i_S}}$ is significantly reachable from a point of *S* having coordinates $(k_e, k_v, k_t, k_a)_{i_S}$, if their distance is less than an assigned ε (i.e., it is part of its ε-neighbourhood) and if the above point belonging to *S* is surrounded by a sufficient number of points belonging to F_{i_S}.

A subset F_{i_S} has to match the following properties:

1. all points inside are mutually density-connected;
2. if a point is density-connected to another point, it is also part of the cluster.

The procedure starts from any arbitrary point that has not yet been visited, by computing its ε-neighbourhood: if it contains a sufficient number of points, a cluster is created; if this does not occur, the point is labelled as noise.

If a point is associated with a cluster, the points of its ε-neighbourhood are also part of the cluster. Consequently, all the points found within its ε-neighbourhood are added to the cluster, as well as their ε-neighbourhoods. This process continues until the cluster is completed and all points in set *F* have been visited.

The procedure can be implemented several times in order to define for each of the properties of *S* the distance ε and $N_{F_{i_S}}$, that is, the number of *F* elements that can be considered appropriate to calculate the rent.

Obviously, the more the *i*th property of *S* is typical, the greater is the related $N_{F_{iS}}$ and it is possible to reduce the ε in view of the greater significance of the subset F_{i_S}; otherwise, it is necessary to increase ε in order to increase $N_{F_{iS}}$.

The procedure requires the calculation of the distance matrix between all elements of *S* and *F*.

Then, it is possible to calculate the monthly rent l_{i_S} of the *i*th property of *S* as the weighted average of $l_{F_{iS}}$—the monthly rents of the properties included in F_{i_S}—in relation to the distances determined for implementing the DBSCAN procedure:

$$l_{i_S} = \frac{\sum_{N_{F_{iS}}} (l \cdot d)_{F_{iS}}}{\sum_{N_{F_{iS}}} d_{F_{iS}}}. \tag{8}$$

Some final considerations support the suitability of ask prices and monthly fees for cap rates calculation, since some doubts might arise in this regard.

The first one concerns the internal consistency of the proposed survey aiming at comparing profitability and liquidity of real estate capital asset over the city according to two different segmentations of this market, by cluster and by neighbourhood.

The further considerations support the general consistency as well, since both prices and rents are affected by a similar degree of uncertainty affecting both prices and rents: the former due to the crisis of the real estate industry; the latter due to the more general climate of economic and employment crisis still existing in Italy in particular.

Due to the structural crisis in the real estate industry that started in 2007, real prices may be 25% (or more) less than the asking prices, as an effect of the "pathological money liquidity trap" and the consequent "credit crunch" due to weak economic growth perspectives and low inflation; as a consequence, real prices do not fit the moods of sellers as well as of buyers, who perceive the same transaction as a loss and as a risk, respectively. In such a climate, according to the speculative financial approach, professional investors take advantage of their own privileged financial situation and informative asymmetries, so increasing and performing their propensity to differ.

As for rent, although the difference between asked and real rent seems smaller, the effects of the economic crisis should be considered the main reason for the decrease in job opportunities discouraging workers' mobility and causing the demand for rental properties to fall, which extends the vacancy periods of the properties; on the other hand, a general reduction in household income leads to a decrease in tenants' solvency, thereby increasing the amounts written off.

2.3.2. Appraisal of the Rent in the Partition by Neighbourhood

In this case the monthly rents of the properties for sale belonging to the generic neighbourhood are calculated by means of a simple linear regression function connecting the unit monthly rent with the overall quality index k^*, defined by using the properties for rent belonging to the same neighbourhood.

3. The Case Study: Analysis of Cap Rate in the City of Palermo

The real estate market of 10 central and semi-central neighbourhoods of the city of Palermo was chosen as our case study because this area is the most densely populated and its real estate market is highly complex and heterogeneous. The neighbourhoods almost entirely correspond to the area delimited by the coast, Monte Pellegrino Mountain, and the beltway bordering the densest part of the city, Regione Siciliana Avenue, which introduces a deep break of continuity in the whole urban settlement (Figure 2).

Neighbourhoods 3 and 4 form the old town and coincide with the city that was stratified up to the eighteenth century, when it began to expand over the sixteenth-century city walls. In reference to the urban expansion along the north axis, Neighbourhoods 8 and 9 were developed between the end of the nineteenth and the beginning of the twentieth century, while Neighbourhoods 10 and 11 were founded in the second half of the twentieth century. In these neighbourhoods there are sporting

equipment, parks, schools, etc., as well as commercial activities; furthermore, it is important to note that high-income classes live mainly in these areas. On the other hand, the western (5, 6, and 7) and especially the southern (1 and 2) neighbourhoods have a lower quantity and quality of facilities, and are primarily inhabited by low-income and working classes.

1. Settecannoli – Brancaccio
2. Oreto – Stazione
3. Tribunali – Castellamare
5. Montegrappa – S. Rosalia
6. Cuba -Calatafimi
7. Zisa – Noce
8. Politeama
9. Libertà
10. Malaspina – Palagonia
11. Resuttana – S. Lorenzo
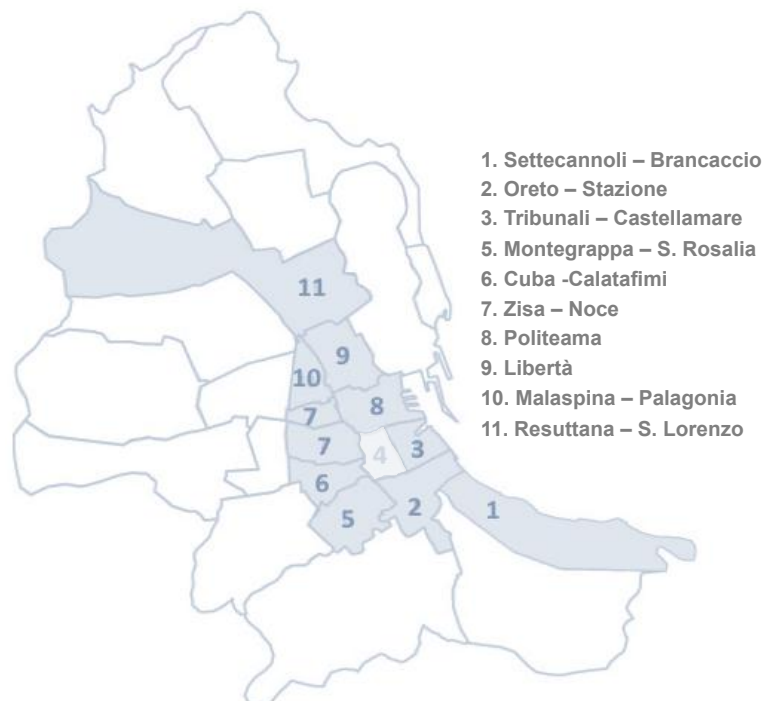
Figure 2. The 10 neighbourhoods in the case study.

3.1. The Real Estate Market and the Database of the Case Study

In reference to Phase 1 of the methodology (Section 2.1), altogether 948 properties, of which of 500 were for sale (*subset S*) and 448 properties were for rent (*subset F*), distributed in the 10 neighbourhoods, have been collected within a database (Table 3, Figure 3) (Neighbourhood 4 is excluded because no data were collected).

Table 3. Distribution of the samples of properties for sale and for rent by neighbourhood.

Neighbourhoods		1	2	3	4	5	6	7	8	9	10	11	*Tot.*
Sample *S*	n.	51	40	27	-	82	40	30	37	78	35	80	*500*
Sample *F*	n.	50	32	23	-	72	40	51	33	67	36	44	*448*

For each record, the data related to the characteristics (fields) of the real estate unit, of the building, of the micro-area, and of the neighbourhood have been arranged (see Table 1) and elaborated in order to calculate the fundamental characteristics scores k_h and the overall quality score k^* according to Equation (1). Some additional database fields report the total and unit market prices, €/sqm and €/room.

The analysis of the unit prices per room has been carried out in addition, in order to take into account the heterogeneity of the properties belonging to different typologies and ages; the reporting of the number of the main usable rooms (bedroom, living room, office) and accessories (to an extent of 25%) allows us to test the reliability and consistency of the reporting and to amend or exclude some unlikely data.

id	address (street, square)	n.	floor	main rooms	large kitchen	facilities	total rooms	inner surface area	outer surface	total surface	garage	renovated in	ke	kv	kt	ka	rooms	area (sqm)	ask price	overall quality	price per room	price per sqm
51	Zucchetto	6	3	4.0	1.0	2	5.0	131	15	134	2		1.07	4.40	2.70	2.28	5.0	134	€ 159,000	3.21	€ 31,800	€ 1,185
52	Oreto	306	1	4.0	1.0	1	4.8	100	33	108			2.85	2.20	2.70	2.30	4.8	108	€ 140,000	2.44	€ 29,474	€ 1,293
53	P.E.Giudici	4	3	5.0	1.0	2	6.0	149	24	155			2.73	2.20	3.50	2.23	6.0	155	€ 180,000	2.79	€ 30,000	€ 1,162
54	R. Jemma	88	1	3.0	1.0	1	3.8	95	10	98			2.73	2.20	3.50	2.20	3.8	98	€ 125,000	2.79	€ 33,333	€ 1,281
55	R. Jemma	56	6	3.0	1.0	1	3.8	114	12	116			2.73	3.60	3.30	2.20	3.8	116	€ 145,000	3.19	€ 38,667	€ 1,245
56	S. La Franca	98	4	4.0	1.0	1	4.8	170	20	175			2.73	3.60	3.30	2.17	4.8	175	€ 160,000	3.18	€ 33,684	€ 915
57	F. Corazza	77	1	3.0			1	3.3	70	4	71		2.73	1.60	1.10	1.62	3.3	71	€ 90,000	1.38	€ 27,692	€ 1,269
58	Campisi	0	4	5.0			2	5.5	110	15	114		2.09	3.00	4.10	2.65	5.5	114	€ 210,000	3.43	€ 38,182	€ 1,847
59	Arcoleo	15	4	6.0			1	6.3	164	35	173		2.73	3.80	1.90	2.71	6.3	173	€ 150,000	2.73	€ 24,000	€ 868
60	Magnanimo	22	3	3.0			1	3.3	103	14	106	2013	2.73	1.60	3.27	1.62	3.3	106	€ 103,000	2.35	€ 31,692	€ 968
61	Corso Tukory	240	0.5	4.0			1	4.3	85	14	89	2009	2.85	1.60	3.33	1.97	4.3	89	€ 110,000	2.45	€ 25,882	€ 1,241
62	Stazzone	7	3	4.0	1.0	1	4.8	153	12	156	2006	2.73	3.00	3.23	1.97	4.8	156	€ 155,000	2.90	€ 32,632	€ 995
63	P. E. Giudici	4	3	5.0	1.0	2	6.0	126	21	131		2.73	3.00	3.30	2.43	6.0	131	€ 180,000	3.02	€ 30,000	€ 1,375
64	Muratori	6	3	3.0			1	3.3	90	6	91		2.73	1.60	1.70	2.22	3.3	91	€ 70,000	1.77	€ 21,538	€ 768
65	Mortillaro	43	3	4.0			1	4.3	103	21	108		2.73	2.20	2.50	2.21	4.3	108	€ 80,000	2.34	€ 18,824	€ 739
66	F. P. Perez	133	0.5	4.0			1	4.3	91	12	94		2.73	1.60	2.50	1.81	4.3	94	€ 69,000	2.05	€ 16,235	€ 736
67	R. Pirri	11	3	6.0			2	6.5	200	8	202	1970	2.73	4.40	1.83	2.64	6.5	202	€ 200,000	2.89	€ 30,769	€ 991
68	G. Campisi	5	4	5.0			2	5.5	130	6	131	2006	2.09	1.60	4.63	2.65	5.5	131	€ 210,000	3.18	€ 38,182	€ 1,598
69	V. Reggio	20	3	4.0			1	4.3	120	2	120	1990	2.73	3.00	2.70	2.21	4.3	120	€ 125,000	2.71	€ 29,412	€ 1,038
70	A.Marinuzzi	143	2	3.0			1	3.3	80	4	81		2.73	1.60	2.70	2.04	3.3	81	€ 88,000	2.18	€ 27,077	€ 1,086
71	L.Manfredi	18	6	5.0	1.0	1	5.8	113	9	115	1994	2.73	3.00	2.83	2.15	5.8	115	€ 170,000	2.75	€ 29,565	€ 1,476
72	. F. Corazza	49	4	2.0			1	2.3	70	2	70		2.73	1.60	2.70	1.86	2.3	70	€ 79,000	2.15	€ 35,111	€ 1,123
73	F.Fortunato	15	5	5.0	1.0	2	6.0	205	22	211	1993	2.73	2.40	2.80	1.61	6.0	211	€ 270,000	2.42	€ 45,000	€ 1,283
74	F.sc P. Perez	84	3	2.0			1	2.3	65	2	65	1950	2.73	3.00	0.57	1.60	2.3	65	€ 65,000	1.62	€ 28,889	€ 994
75	V. Mortillaro	39	2	4.0			1	4.3	90	16	94		2.73	2.20	1.90	2.41	4.3	94	€ 80,000	2.11	€ 18,824	€ 851
76	Crs Dei Mille	178	1	5.0	1.0	2	6.0	137	8	139	2000	2.85	1.60	3.43	2.69	6.0	139	€ 130,000	2.64	€ 21,667	€ 935
77	Crs. Tukory	240	0.5	2.0	1.0	1	2.8	90	15	94	2000	2.85	2.40	3.03	2.08	2.8	94	€ 110,000	2.62	€ 40,000	€ 1,173
78	A. Elia	14	1	3.0	1.0	1	3.8	83	1	83	2008	2.73	1.00	2.50	1.96	3.8	83	€ 90,000	1.87	€ 24,000	€ 1,081
79	M. Cipolla	42	2	3.0	0.5	1	3.5	70	14	73	2004	2.73	2.20	3.17	2.62	3.5	73	€ 200,000	2.72	€ 57,143	€ 2,724
80	A.Marinuzzi	199	2	5.0			2	5.5	138	14	142		2.73	2.20	2.70	1.81	5.5	142	€ 150,000	2.35	€ 27,273	€ 1,060
81	M.S. Salamone	9	2	3.0	1.0	1	3.8	143	14	146		2.73	2.20	2.50	1.72	3.8	146	€ 150,000	2.24	€ 40,000	€ 1,024
82	R. Jemma	88	1	4.0	1.0	1	4.8	113	8	115	1990	2.73	3.00	2.70	1.55	4.8	115	€ 125,000	2.57	€ 26,316	€ 1,086
83	M. Corbino	1	5	3.0	1.0	2	4.0	105	20	110		2.73	2.20	2.70	2.35	4.0	110	€ 145,000	2.46	€ 36,250	€ 1,320
84	S. La Franca	98	4	4.0	1.0	1	4.8	122	10	125		2.73	2.40	2.70	1.97	4.8	125	€ 160,000	2.45	€ 33,684	€ 1,284
85	Oreto	252	5	4.0			1	4.3	95	12	98		2.85	3.00	2.50	1.59	4.3	98	€ 115,000	2.49	€ 27,059	€ 1,174
86	G. Patricolo	17	2	4.0			2	4.5	123	9	125	1990	2.73	3.60	3.10	1.70	4.5	125	€ 119,000	2.99	€ 26,444	€ 951
87	V. Errante	12	3	3.0	1.0	2	4.0	97	4	98	2	2012	2.73	3.60	4.73	3.13	4.0	98	€ 270,000	4.02	€ 67,500	€ 2,742
88	Oreto	456	1	3.0	1.0	3	4.3	91	11	94		1980	2.41	2.20	2.37	2.74	4.3	94	€ 120,000	2.38	€ 28,235	€ 1,278
89	S. Maria di Gesù 33/b		1	4.0	1.0	1	4.8	115	20	120	2		2.41	3.60	2.10	2.35	4.8	120	€ 180,000	2.67	€ 37,895	€ 1,500
90	Belm.Chiavelli	60	p.t.	3.0	1.0	1	3.8	73	42	84		1990	2.41	1.60	3.40	1.91	3.8	84	€ 60,000	2.47	€ 16,000	€ 716
91	S. Maria di Gesù 33/b		3	4.0	1.0	1	4.8	112	16	116		1998	2.41	3.60	3.17	2.97	4.8	116	€ 230,000	3.28	€ 48,421	€ 1,978
92	Aragona	4	2	5.0			1	5.3	147	12	150	2	2.71	2.40	2.50	1.99	5.3	150	€ 180,000	2.36	€ 34,286	€ 1,200
93	Chiesa Cocchieri	4	1	3.0	1.0	1	3.8	70	2	70		1952	2.71	3.00	-0.17	1.72	3.8	70	€ 89,000	1.32	€ 23,733	€ 1,271
94	Parlamento	31	3 e 4	4.0	1.0	1	4.8	109	2	110		2007	2.59	3.00	2.47	1.81	4.8	110	€ 120,000	2.52	€ 25,263	€ 1,093
95	Disc. Giudici	17	3 e 4	6.0	1.0	3	7.3	149	31	157		1975	2.71	2.40	0.60	2.40	7.3	157	€ 225,000	1.59	€ 31,034	€ 1,436
96	Giardinaccio	14	2	2.0	1.0	1	2.8	52	8	54		2010	2.39	1.60	4.77	2.64	2.8	54	€ 140,000	3.23	€ 50,909	€ 2,593
97	dei Cassari	61	2	4.0	1.0	1	4.8	108	5	110		2004	2.59	2.80	2.97	2.19	4.8	110	€ 160,000	2.75	€ 33,684	€ 1,461
98	Immacolatella	11	1	4.0	1.0	1	4.8	87	9	89			2.39	1.60	1.10	1.57	4.8	89	€ 78,000	1.37	€ 16,421	€ 874
99	N- Cervello	27	2	3.0	1.0	1	3.8	85	4	86		2007	2.39	1.00	2.47	2.22	3.8	86	€ 89,000	1.90	€ 23,733	€ 1,035
100	Vetriera	55	2	3.0	1.0	1	3.8	76	9	78		2006	2.39	3.00	4.43	2.22	3.8	78	€ 180,000	3.49	€ 48,000	€ 2,300

Figure 3. Portion of the database (from 51 to 100 items; the others items are omitted).

Figure 4 shows that the probability density functions of the unit prices, within the variation range, differ quite a bit from the Gaussian type, with the results being clearly asymmetrical. The "positive skewed distribution" indicates modal values in accordance with the low–medium unit price, while a long "tail" on the right of the curves indicates the presence of a few very high prices. This distribution reflects other asymmetries that interact with each other: the *urban asymmetry* reflects the complexity, discontinuity and heterogeneity of the quality distribution in the urban areas, as well as in the building patrimony; the *market asymmetry*, depending on the fact that prices are causal and not casual, and are influenced by macroeconomic (e.g., economic crisis, rise/fall of the real estate investments; difficult access to loans) and microeconomic factors (e.g., oversupply in the local real estate market, low solvency of local investors, low-income population), which can lower the real estate market prices.

The relationship between the unit price and the overall quality k^* is displayed in Figure 5, showing a weak positive trend given the scattered cloud of points and the consequent low value of R^2 (0.25 and 0.27); this representation is affected by the wide price range at the same score k^*. Such dispersion can depend on the low transparency of the market, on information asymmetry, and—especially at the current economic juncture characterised by uncertainty and a low level of demand—on a higher propensity to hoard; typically, the latter is displayed by the higher prices asked

by those owners particularly willing to wait, so displaying *bullish* behaviour; on the contrary, the lower prices of this range correspond to general scepticism about any possible market recovery, leading to *bearish* behaviour.
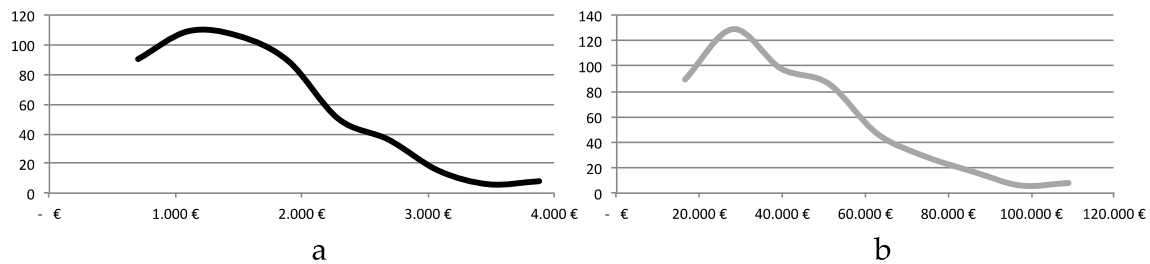

Figure 4. Frequency analysis of the prices of the sample euros/sqm (**a**) and euros/room (**b**).
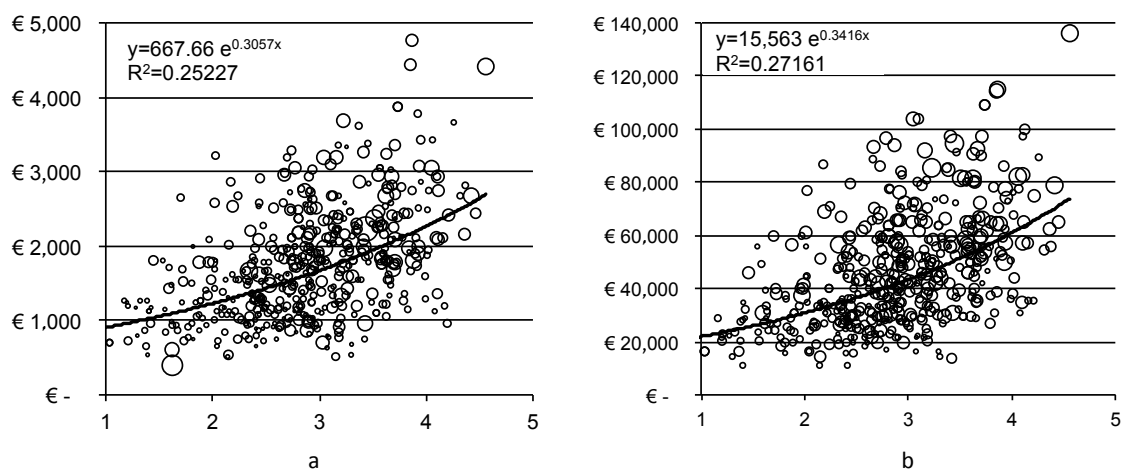

Figure 5. Relationship between (**a**) price euros/sqm (y-axis), or (**b**) price euros/room (y-axis) and the overall quality k^* of properties (x-axis).

The comparison between the distributions of the normalised k^* and p of S (Figure 6) highlights a significant gap between monetary and real variables, as the latter show a higher modal value.
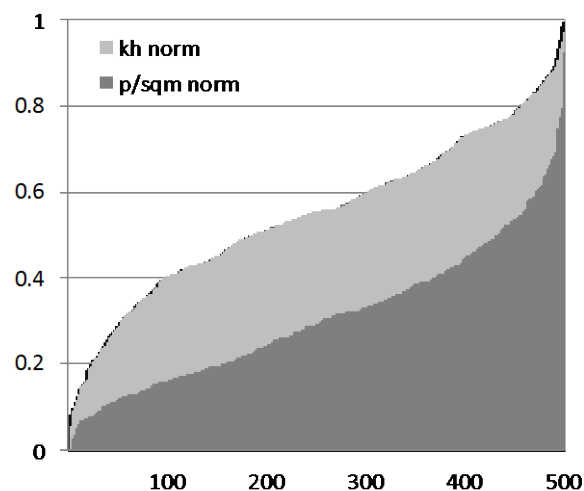

Figure 6. Normalised overall quality k* and normalised unit price of the sample (500 data points).

Similarly, it can be observed that the distribution of the normalised value of the main characteristics (Figure 7) differs: k_e distribution is more gradual than k_v, and both follow an almost linear trend, contrary to the non-linear trends of k_t and k_a, the former of which has the higher modal value.
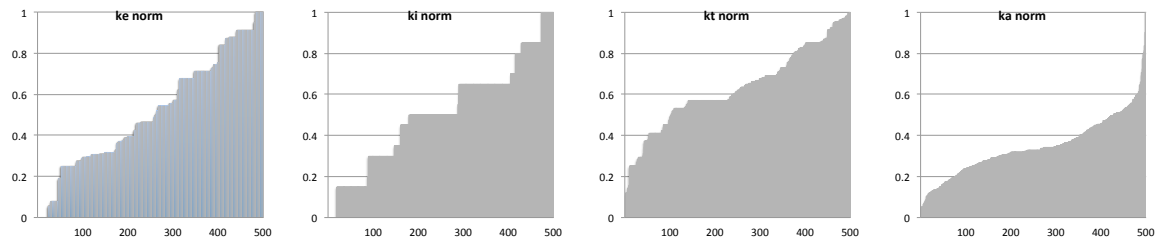

Figure 7. Distribution of the normalised scores of the fundamental characteristics of *S*.

4. Results and Discussion

The cap rates of the sub-sample *S* are appraised by applying the methodology outlined above (Section 2.3).

4.1. Cap Rates in the Submarkets by Cluster

4.1.1. Delimitation and Characteristics of the Clusters

The cluster analysis, carried out by characteristics, has been applied to the properties for sale in order to define submarkets featuring a high degree of similarity. The *k-mean* algorithm provides the best number of clusters in correspondence of the highest value of the Calinski–Harabasz index, which has been calculated for each of the eight hypotheses (from three to 10 clusters) and recorded the highest value for the nine-cluster hypothesis (Table 4).

Table 4. *CH* index (Calinski–Harabasz) for each of the eight hypotheses of clustering.

Number of Clusters	3	4	5	6	7	8	9	10
CH index	32.32	38.56	40.51	46.31	50.14	55.64	**63.60**	58.20

Table 5 shows the distribution of the sample *S* between the nine different clusters.

Table 5. Distribution of the sample of properties for sale in nine clusters.

Segments of Market (by Cluster)		1	2	3	4	5	6	7	8	9	*Tot.*
m_{S_c}—number of properties for sale	n.	45	40	69	29	80	151	47	19	20	*500*

In order to better understand the cluster analysis results, the location of the properties belonging to each cluster is shown in Figure 8. The properties of Cluster 1 are mostly located in the semi-central and peripheral neighbourhoods (except for few properties in Neighbourhoods 3 and 8). In Cluster 2 the properties are gathered in the central and semi-central neighbourhoods, but it never includes properties located in the southern neighbourhoods. The location of the properties belonging to Clusters 3, 4, 5 and 7 is quite scattered; instead, Cluster 6 includes the properties that are featured by the qualified location along the northern axis of expansion toward as well as in the central neighbourhoods (8, 9, 10, and 11). The properties of Clusters 8 and 9 share the quite low location quality that typically characterises the southern neighbourhoods.
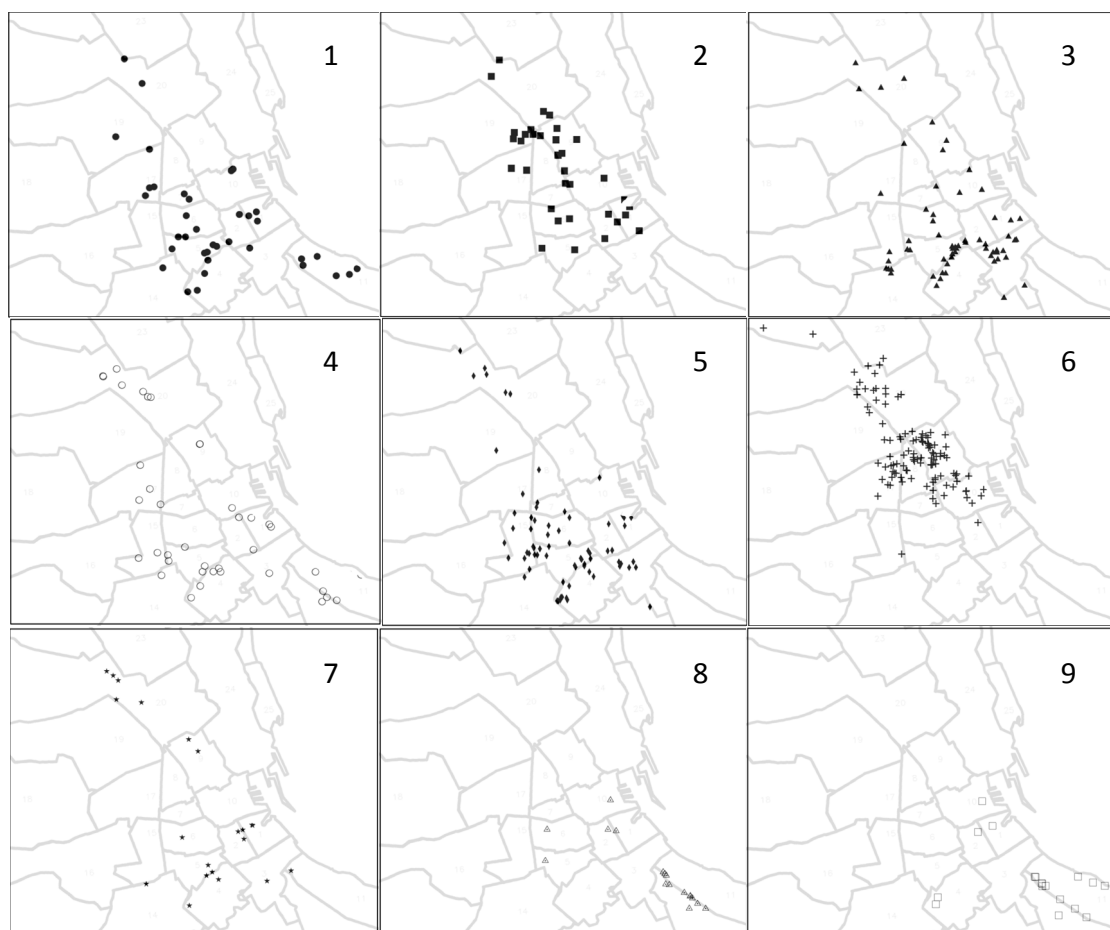
Figure 8. Location of the properties of the nine clusters.

For the purpose of a quantitative comparison of the clusters, the minimum, median, and maximum values, and the two intermediate quartiles of the principal characteristics of the properties have been calculated and represented with a colour shading from blue (lower score) to red (higher score), whereas the white background corresponds to the average score; Figure 9 shows that the quality of the characteristics in Cluster 1 is opposite to those in Cluster 6: the first cluster is formed of properties in which the values of all the characteristics are clearly below the median, and above all of intrinsic and technological characteristics (just a few properties have a location score higher than the median); Cluster 6 includes quite valuable properties that have high scores for all the characteristics.

The scores of the properties belonging to the remaining clusters range significantly over all the characteristics. Cluster 7, for example, is similar to Cluster 6 except for the average–low location score that characterises its properties. The properties in Cluster 2 have good location and intrinsic characteristics (higher than the median), but an average–low range of technological and architectural quality. Cluster 3 comprises properties with very poor intrinsic and architectural characteristics, while the quality of the other ones is acceptable. In Cluster 4 all the properties have good technological characteristics and bad intrinsic characteristics. Cluster 5 gathers properties of roughly average quality. Scores that diverge widely, from poor location quality (minimum scores) to good intrinsic characteristics (maximum scores), characterise Cluster 8. The properties of Cluster 9 also have a very low quality of location, but the other characteristics are acceptable.

	Clu1				Clu2				Clu3				Clu4				Clu5			
	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka
min	1.1	1.0	1.1	1.2	2.2	2.2	1.1	1.5	2.0	1.0	1.9	1.4	1.4	1.0	3.5	2.1	2.0	2.8	1.7	1.5
min-qle1	1.5	1.3	1.1	1.4	2.5	2.6	1.2	1.6	2.1	1.3	2.2	1.7	1.7	1.3	3.7	2.2	2.0	2.9	2.2	1.8
qle1	2.0	1.6	1.1	1.6	2.7	3.0	1.3	1.8	2.2	1.6	2.5	2.0	2.1	1.6	3.9	2.2	2.1	3.0	2.7	2.0
qle1-ql2	2.1	1.6	1.2	1.7	3.1	3.0	1.6	2.0	2.5	1.6	2.6	2.0	2.3	1.6	4.0	2.4	2.1	3.0	2.8	2.1
ql2	2.2	1.6	1.3	1.7	3.5	3.0	1.9	2.1	2.7	1.6	2.7	2.1	2.5	1.6	4.1	2.5	2.2	3.0	2.9	2.2
ql2-qle3	2.5	1.7	1.6	1.9	3.9	3.3	1.9	2.2	2.8	1.9	2.9	2.2	2.7	1.9	4.4	2.6	2.5	3.3	3.0	2.2
qle3	2.7	1.8	1.9	2.0	4.2	3.6	1.9	2.4	2.9	2.2	3.1	2.3	3.0	2.2	4.6	2.7	2.7	3.6	3.2	2.3
qle3-max	3.4	2.3	2.0	2.2	4.3	4.3	2.3	2.8	3.6	2.3	3.3	2.5	3.7	2.5	4.7	3.0	3.1	4.0	3.4	2.7
max	4.1	2.8	2.1	2.4	4.3	5.0	2.7	3.2	4.3	2.4	3.5	2.7	4.3	2.8	4.8	3.3	3.5	4.4	3.5	3.1
med	2.3	1.7	1.5	1.8	3.5	3.3	1.7	2.1	2.7	1.9	2.8	2.1	2.5	1.9	4.2	2.5	2.4	3.3	2.8	2.1
	1.8				2.7				2.3				2.8				2.7			

	Clu6				Clu7				Clu8				Clu9			
	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka
min	2.8	2.2	2.2	1.6	1.1	2.8	3.5	1.8	1.1	3.6	2.1	1.8	1.1	1.6	2.0	1.7
min-qle1	3.2	2.6	2.5	1.9	1.6	3.1	3.8	2.0	1.1	3.7	2.4	2.0	1.1	1.9	2.4	1.9
qle1	3.6	3.0	2.8	2.2	2.1	3.3	4.1	2.3	1.1	3.8	2.7	2.2	1.1	2.2	2.7	2.1
qle1-ql2	3.7	3.3	3.1	2.3	2.2	3.5	4.1	2.5	1.1	4.1	2.7	2.2	1.2	2.3	2.7	2.2
ql2	3.8	3.6	3.3	2.4	2.4	3.6	4.1	2.6	1.1	4.4	2.7	2.2	1.4	2.3	2.7	2.3
ql2-qle3	4.1	4.0	3.7	2.6	2.6	4.0	4.4	2.7	1.2	4.4	2.8	2.3	1.4	2.6	3.0	2.3
qle3	4.3	4.4	4.1	2.8	2.8	4.4	4.7	2.8	1.4	4.4	2.9	2.5	1.5	2.8	3.2	2.3
qle3-max	4.5	4.7	4.5	3.4	3.2	4.7	4.8	3.6	2.0	4.7	3.2	2.7	1.7	2.9	3.6	2.7
max	4.7	5.0	4.8	4.1	3.5	5.0	4.9	4.4	2.7	5.0	3.5	3.0	2.0	3.0	3.9	3.1
med	3.9	3.7	3.5	2.5	2.3	3.7	4.3	2.7	1.4	4.3	2.8	2.3	1.4	2.4	3.0	2.3
	3.4				3.2				2.7				2.3			

Figure 9. Comparison of the different clusters by minimum, average, maximum, and quartiles (normalised) scores of the main characteristics.

4.1.2. Appraisal of Cap Rates by Clusters

Replacing in Equation (6) the asking price and rents related to the 500 items of *S*, the corresponding cap rates by cluster have been appraised. The nine clusters differ significantly as far as distribution, frequency and range are concerned, and the results can be examined by merging information about the properties' location (Figure 8), the range and quartiles of the cap rates (Figure 10a), and the frequency and probability density functions of the cap rate (Figures 10b and 11).
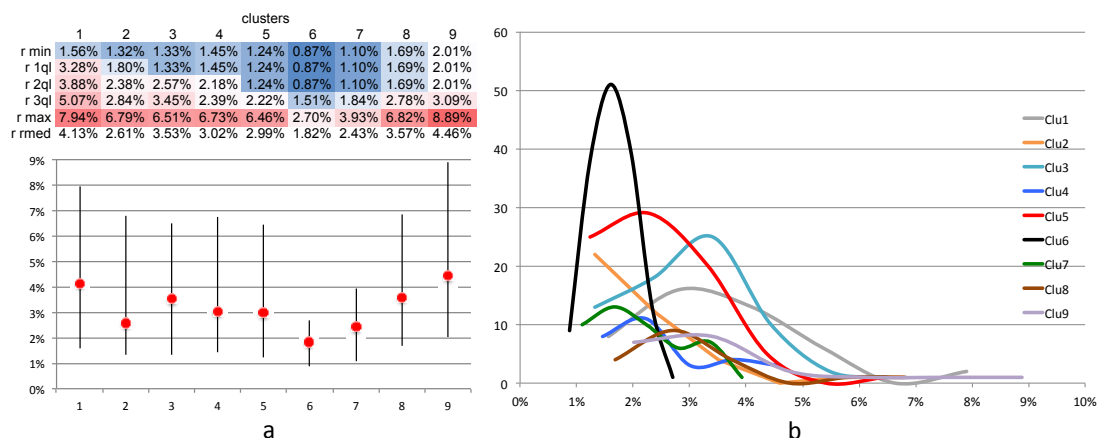
clusters	1	2	3	4	5	6	7	8	9
r min	1.56%	1.32%	1.33%	1.45%	1.24%	0.87%	1.10%	1.69%	2.01%
r 1ql	3.28%	1.80%	1.33%	1.45%	1.24%	0.87%	1.10%	1.69%	2.01%
r 2ql	3.88%	2.38%	2.57%	2.18%	1.24%	0.87%	1.10%	1.69%	2.01%
r 3ql	5.07%	2.84%	3.45%	2.39%	2.22%	1.51%	1.84%	2.78%	3.09%
r max	7.94%	6.79%	6.51%	6.73%	6.46%	2.70%	3.93%	6.82%	8.89%
r rmed	4.13%	2.61%	3.53%	3.02%	2.99%	1.82%	2.43%	3.57%	4.46%

Figure 10. Cap rate by cluster: (**a**) range and quartiles; (**b**) probability density functions.
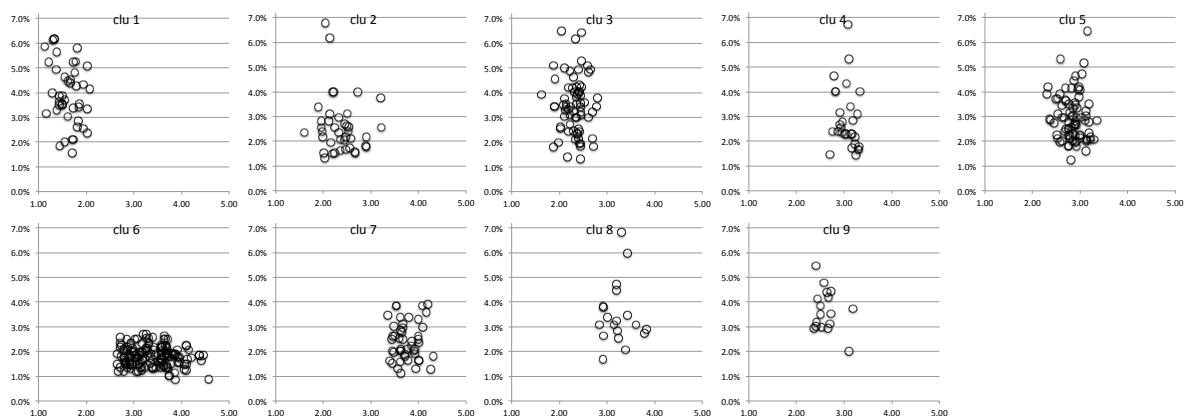
Figure 11. Cap rate by cluster: the relationship between cap rate and overall quality.

The cap rates of Cluster 6 are significant since it collects the highest number of properties (151 over 500); the latter are located in the central neighbourhoods and are worthy from the point of view of all the characteristics. In this cluster the range of cap rate is the narrowest (0.87–2.70%) and the modal value of cap rate is very low; such low cap rates usually express a high liquidity that is attributable to those properties being more suitable to meet long-term speculative expectations and promising to preserve and increase the capital asset value overtime.

The probability density functions of Clusters 5 and 3 have a similar shape; the former is a bit shifted to the right and has a more positive skewed distribution because the k_v score of Cluster 5 is medium–high; the higher cap rates depend on the diffused location of the data in various neighbourhoods and on the architectural characteristic ranging around the average score, whereas the good technological and intrinsic characteristics hold back the cap rate of most properties (up to the third quartile) within a narrow interval 1.24–2.22% in correspondence with Cluster 5 and a wider one 1.33–3.54% in correspondence with Cluster 3. The distributions of the cap rates of Clusters 8 and 9 show analogous shapes due to equivalent characteristics, e.g., the location in semi-central or peripheral neighbourhoods (the k_e score is lower than average), although the frequency of low cap rates could mostly depend on the relatively high rent in comparison to the technological and intrinsic characteristics. The cap rate function of Cluster 1 (in which all the characteristics have low scores and the location is diffused) is lengthened due to a great variability in both rents and prices; it could also be ascribable to different potential increases in value due to future renovation prospects. In many functions there are long "tails" that are partly caused by outliers (their frequencies are often very low) and partly by difficult access to market information, which hinders investment decisions.

4.2. Cap Rates in the Neighbourhoods

4.2.1. Characteristics of the Neighbourhoods

Assuming as submarkets the neighbourhoods (Figure 2), some considerations can be proposed similarly to the ones in Section 4.1.1, as seen in Figure 12.

	Neigh 1				Neigh 2				Neigh 3				Neigh 5				Neigh 6			
	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	ki	kt	ka
min	1.1	1.0	1.1	1.4	2.1	1.0	1.1	1.5	2.4	1.0	1.1	1.5	2.0	1.0	1.1	1.4	2.1	1.6	1.1	1.4
min-qle1	1.1	1.6	1.9	1.7	2.4	1.3	1.8	1.7	2.4	1.7	1.5	1.7	2.0	1.3	1.8	1.7	2.1	1.9	1.8	1.7
qle1	1.1	2.2	2.7	2.0	2.7	1.6	2.5	1.8	2.4	2.3	1.9	2.0	2.0	1.6	2.5	2.0	2.2	2.2	2.5	2.0
qle1-ql2	1.2	2.6	2.7	2.1	2.7	1.9	2.6	2.0	2.5	2.7	2.3	2.1	2.0	2.0	2.8	2.1	2.2	2.6	2.6	2.1
ql2	1.3	3.0	2.7	2.2	2.7	2.2	2.7	2.2	2.6	3.0	2.6	2.2	2.1	2.4	3.0	2.2	2.2	3.0	2.7	2.2
ql2-qle3	1.3	3.6	3.1	2.3	2.7	2.6	3.0	2.3	2.7	3.3	3.3	2.4	2.2	2.7	3.2	2.2	2.2	3.3	3.0	2.3
qle3	1.4	4.1	3.5	2.5	2.7	3.0	3.3	2.4	2.7	3.6	4.0	2.6	2.2	3.0	3.5	2.3	2.2	3.6	3.3	2.5
qle3-max	1.6	4.6	4.2	2.9	2.8	3.7	4.0	2.8	2.9	4.3	4.4	2.8	2.3	3.7	4.1	3.3	2.6	4.3	4.1	2.8
max	1.8	5.0	4.8	3.3	2.9	4.4	4.7	3.1	3.1	5.0	4.8	2.9	2.5	4.4	4.7	4.4	2.9	5.0	4.9	3.1
med	1.3	3.0	3.0	2.3	2.7	2.5	2.9	2.2	2.6	3.0	2.9	2.2	2.1	2.5	3.0	2.2	2.3	2.9	2.9	2.2
	2.4				2.6				2.7				2.5				2.6			

	Neigh 7				Neigh 8				Neigh 9				Neigh 10				Neigh 11			
	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka	ke	kv	kt	ka
min	2.1	1.0	1.1	1.6	3.1	1.0	1.1	1.6	3.3	1.6	1.1	1.6	1.0	1.6	1.3	1.2	1.0	1.6	1.3	1.2
min-qle1	2.1	1.6	1.7	1.7	3.4	1.6	1.7	1.8	3.8	2.3	1.9	1.9	1.8	2.6	2.0	1.7	1.8	2.6	2.0	1.7
qle1	2.1	2.2	2.2	1.9	3.7	2.2	2.3	2.0	4.2	3.0	2.7	2.2	2.6	3.6	2.7	2.2	2.6	3.6	2.7	2.2
qle1-ql2	2.1	2.6	2.5	2.0	3.9	2.9	2.6	2.2	4.3	3.3	2.7	2.3	2.9	3.6	3.0	2.3	2.9	3.6	3.0	2.3
ql2	2.2	3.0	2.7	2.2	4.1	3.6	2.9	2.3	4.3	3.6	2.7	2.4	3.2	3.6	3.3	2.4	3.2	3.6	3.3	2.4
ql2-qle3	2.2	3.3	3.0	2.3	4.1	4.0	3.3	2.4	4.3	3.9	3.3	2.6	3.4	3.6	3.7	2.6	3.4	3.6	3.7	2.6
qle3	2.2	3.6	3.3	2.3	4.2	4.4	3.7	2.4	4.3	4.2	3.9	2.9	3.7	3.6	4.1	2.8	3.7	3.6	4.1	2.8
qle3-max	2.3	4.3	3.7	2.9	4.3	4.7	4.3	3.0	4.5	4.6	4.3	3.5	4.3	4.0	4.5	3.3	4.3	4.0	4.5	3.3
max	2.5	5.0	4.1	3.5	4.5	5.0	4.8	3.5	4.7	5.0	4.8	4.1	5.0	4.4	4.8	3.8	5.0	4.4	4.8	3.8
med	2.2	2.9	2.7	2.2	3.9	3.3	2.9	2.3	4.3	3.5	3.1	2.5	3.2	3.6	3.3	2.4	3.2	3.6	3.3	2.4
	2.5				3.1				3.4				3.1				3.1			

Figure 12. Comparison of the 10 neighbourhoods by minimum, average, maximum, and quartiles (normalised) scores of the main characteristics.

4.2.2. Appraisal of Cap Rate by Neighbourhood

As in the Section 4.1.2, the cap rates by neighbourhood can be examined merging the information coming from the properties' location (Figure 2), the overall characteristics' scores (Figure 13a), and the probability density functions of the cap rate (Figure 13b), and to the relationship between cap rate and overall quality (Figure 14).
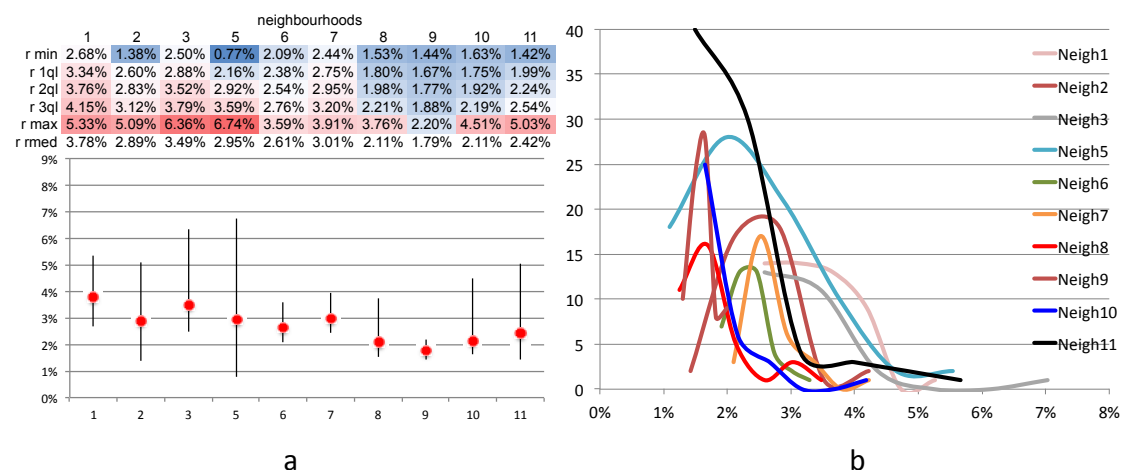
neighbourhoods	1	2	3	5	6	7	8	9	10	11
r min	2.68%	1.38%	2.50%	0.77%	2.09%	2.44%	1.53%	1.44%	1.63%	1.42%
r 1ql	3.34%	2.60%	2.88%	2.16%	2.38%	2.75%	1.80%	1.67%	1.75%	1.99%
r 2ql	3.76%	2.83%	3.52%	2.92%	2.54%	2.95%	1.98%	1.77%	1.92%	2.24%
r 3ql	4.15%	3.12%	3.79%	3.59%	2.76%	3.20%	2.21%	1.88%	2.19%	2.54%
r max	5.33%	5.09%	6.36%	6.74%	3.59%	3.91%	3.76%	2.20%	4.51%	5.03%
r rmed	3.78%	2.89%	3.49%	2.95%	2.61%	3.01%	2.11%	1.79%	2.11%	2.42%

a　　　　　　　　　　　　　　　　　　　　　　　　　b

Figure 13. Cap rates in the neighbourhoods: (**a**) ranges and quartiles; (**b**) probability density functions.
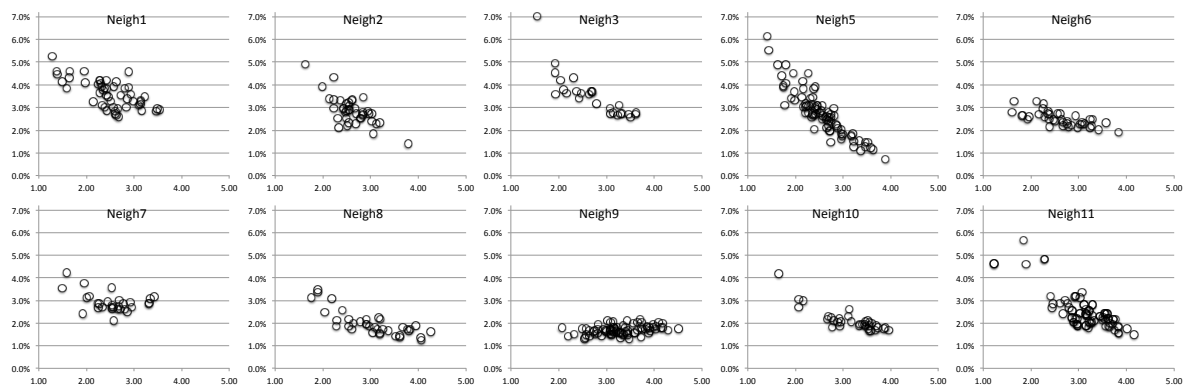
Figure 14. Cap rate and overall quality by neighbourhood.

The ranges of the cap rate are very narrow in some neighbourhoods, particularly 6, 7, and 9. In 8 (a central area), the cap rates have a similar form, except for a tail that corresponds to properties that are of a low quality, despite a good location.

In 1 and 3 the greater frequency of high cap rate is observed; it corresponds both to low market values and to relatively high potential rents in comparison to the overall quality of the properties.

In some cases, the wide variability of the cap rate depends on the "tails" undoubtedly due to the presence of outliers (e.g., in neighbourhoods 10 and 11); in other cases a wide range corresponds to the Gaussian distribution and is related to the neighbourhood peculiarities (e.g., 2 and 5); in such cases the overall quality of the properties, as well as of the real and potential rent resulting from the concentration of specific social groups (e.g., students, workers and migrants), is heterogeneous.

Figure 14 shows the different elasticity of cap rate in each neighbourhood, although $r(k^*)$ always has a negative correlation, as expected. In such a respect, Neighbourhood 9 can represent both an exception and an interesting achievement:

- as an *exception*. r is stable compared to the k^* range and this might contradict the observations and deductions performed in the other neighbourhoods;
- as an *achievement*. Two considerations can be made:

  ○ the first concerns the prevailing of the identity and symbolic value of the location over the other characteristics whose value does not significantly influence the liquidity and profitability profile of the investment in such areas; something similar also happens in Neighbourhood 7, except for some outliers and the higher cap rate featuring as riskier the investments in that area;

  ○ the second concerns a deeper and mostly theoretical issue; as previously mentioned, the characteristics generally influence the size of NOI and, as a consequence, the profitability of a real estate investment; cap rate, instead, is influenced by expectations; the latter are weakly represented by characteristics, as Rizzo addresses by distinguishing explicit (NOI) and implicit (expected capital gain/loss) liquidity [22], which is related to the concept of *crescendo* or *diminuendo*; furthermore, Forte [30] also distinguished the abovementioned 23 characteristics from the 36 influences (18 ascending and 18 descending), the latter affecting cap rate because of some prospective features of a property; as a consequence we conclude that in Neighbourhoods 7 and 9 the influences prevail over the characteristics, whereas in the other areas the characteristics interact with each other to overcome their additive value, embodying the contents of the influences and in some way replacing them.

A last comparison between the two analyses—the cluster and the territorial ones—provides a further perspective about the r/k^* function, involving the above concepts of crescendo and diminuendo, according to the interpretation by Rizzo [23].

Figure 15 displays for each cluster (15a) and for each neighbourhood (15b) the following five combinations of k^* (abscissa) and r (ordinate): $k^*_{min}, r_{max}; k^*_{qle1}, r_{qle3}; k^*_{qle2}, r_{qle2}; k^*_{qle3}, r_{qle1}; k^*_{max}, r_{min}$.
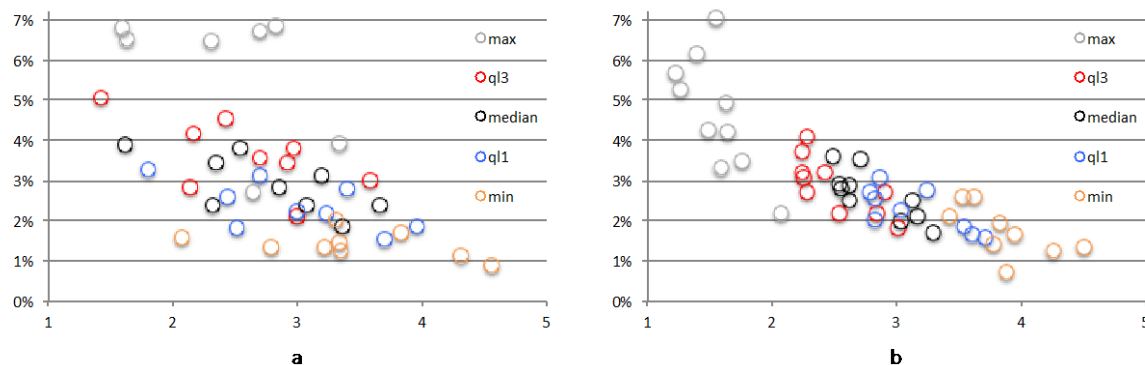

Figure 15. Cap rates and overall quality by cluster (**a**) and by neighbourhood (**b**).

Although in different ways, the two representations confirm the general inverse relation between cap rate and overall quality.

Assuming a functional point of view, most likely to be adopted by householders, the more scattered distribution of the values displayed in Figure 15a (partition by cluster) can likely be interpreted as the effect of clustering only by characteristics and not also by prices; this confirms that the cap rate needs to be represented by associating scores related to urban architectural performance as well as to economic performance, especially when such a wide statistical population is represented by means of a standard scoring pattern.

On the contrary, assuming an economic perspective more relevant to investors' behaviour or administration, a weak r/k^* relationship can suggest a more complex and uncertain market where the *crescendo* and *diminuendo* of the different properties need to be redefined by replacing the characteristics with the influences.

The Figure 15b, on the contrary, displays a stronger r/k^* relationship and a hard characterisation of the liquidity, confirming that the location in a specific neighbourhood is a symbolic feature likely embodying or somehow replacing the influences.

5. Conclusions

This study of the real estate market of Palermo, aimed at interpreting its main functional, architectural and economic-monetary performances, has been developed within the operational area of the mass appraisal procedures, so providing an overview useful to support private investors and public managers' choices and programmes [54–56].

The extension and heterogeneity of the studied context requested a detailed survey based on a wide sample, highlighting significant relationships between characteristics and prices first, and cap rates second.

With reference to the case study, concerning 948 properties located in the main neighbourhoods of Palermo, the cap rates of the 500 properties for sale have been calculated following two approaches: the statistical one, via cluster analysis; and the territorial one by neighbourhood.

The comparison of the relationships between cap rates and characteristics obtained in the two cases showed that, in general, the ranges of the cap rates calculated by clusters are wider than those calculated by neighbourhoods. As mentioned in the previous paragraphs, in both cases cap rates are inversely related to quality; this circumstance confirms the financial implications of the urban architectural performances for users as well as for investors and government.

Indeed, the original interest in cap rate from scientists and economic agents comes from the progressive relevance of the financial features of real estate, even when the functional motivations

seem to prevail in the trading decisions. As a consequence, the two representations of the cap rate ranges and distributions, by clusters and by neighbourhoods, support different points of view: narrower ranges indicate the prevailing of functional, architectural and urban performances, while a wider range suggests the prevailing of financial reasons pushing owners to perform a different degree of real estate hoarding.

In particular, the cap rate expresses both the profitability and the liquidity of the real estate asset, the former regarding its physical and location characteristics, the latter concerning the expectations and the investment programmes of private and public agents, both unavoidably performing a speculative–financial approach.

In reality, the perception of the capabilities that real estate industry has achieved in the current economic climate dominated by capitalism encourages us to take into account, even in the real estate market surveys, everything "beyond" simple functional and architectural performance that can be recognised in both agents' motivations and the public's determinations. This "beyondness" encourages the interest in cap rate, whose range defines the gap between the profitability and liquidity of real estate assets.

The two partitions of the sample surveyed—by cluster and by neighbourhood—highlight the relationship between this gap and the characteristics, and show that in the territorial approach the characteristics are able to embody the circumstances driving the agents to practice real estate hoarding, whereas in the cluster segmentation this relationship is weak, suggesting the inclusion within the set of characteristics ones explicitly indicating the liquidity of real estate assets, as they are relevant to the urban fate.

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