Definition of imaging features

First-order gray-level statistics

Let P define the first-order histogram and $P(i)$ the fraction of voxels with intensity level i. N_i is the number of discrete intensity levels.

Energy: the sum of all voxel values squared.

Entropy:

$$entropy = \sum_{i=1}^{N_i} P(i) log_2 P(i)$$

Kurtosis: the kurtosis of the first order histogram of voxel intensities.

Mean absolute deviation: the mean of the absolute deviations of all voxel intensities around the mean intensity value.

Median: the median intensity value.

Minimum: the minimum intensity value.

Range: the range of intensity values.

Root mean square: the quadratic mean, or the square root of the mean of squares of all voxel intensities.

Skewness: the skewness of the first order histogram of voxel intensity values.

Standard deviation: the standard deviation of all voxel intensity values.

Max: the maximum of all voxel intensity values.

Min: the maximum of all voxel intensity values.

Mean: the mean of all voxel intensity values.

Uniformity:

$$uniformity = \sum_{i=1}^{N_i} P(i)^2$$

Variance: the variance of all voxel intensity values.

Geometric features

Geometric features, describing the shape and

size of the volume of interest. Let V be the volume and A the surface area of the volume of interest.

Compactness 1:

$$compactness\ 1 = \frac{V}{\sqrt{\pi} A^{\frac{2}{3}}}$$

Compactness 2:

$$compactness\ 2 = 36\pi \frac{V^2}{A^3}$$

Spherical disproportion:

$$spherical\ disproportion = \frac{A}{4\pi R^2}$$

Sphericity:

$$sphericity = \frac{\pi^{\frac{1}{3}}(6V)3^{\frac{2}{}}}{A}$$

Surface-to-volume ratio: the surface area divided by the volume.

Gray level co-occurrence matrix-based features

$P(i, j)$ is the co-occurrence matrix

N_g is the number of discrete intensity levels in the image

μ is the mean of $P(i, j)$

$\mu_x(i)$ is the mean of row i

$\mu_y(j)$ is the mean of column j
$\sigma_x(i)$ is the standard deviation of row I
$\sigma_y(j)$ is the standard deviation of column j

$$p_x(i) = \sum_{j=1}^{N_g} P(i,j)$$
$$p_y(j) = \sum_{i=1}^{N_g} P(i,j)$$
$$HXY1 = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P(i,j)\log\left(p_x(i)p_y(j)\right)$$
$$HXY2 = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} p_x(i)p_y(j)\log\left(p_x(i)p_y(j)\right)$$

Autocorrelation:
$$autocorrelation = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g} ijP(i,j)$$

Cluster prominence:
$$cluster\ prominence = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\left[i + j - \mu_x(x) - \mu_y(j)\right]^4 P(i,j)$$

Cluster shade:
$$cluster\ shade = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\left[i + j - \mu_x(x) - \mu_y(j)\right]^3 P(i,j)$$

Cluster tendency:
$$cluster\ tendency = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\left[i + j - \mu_x(x) - \mu_y(j)\right]^2 P(i,j)$$

Contrast:
$$cluster\ contrast = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}|i - j|^2 P(i,j)$$

Correlation:
$$cluster\ tendency = \frac{\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} ijP(i,j) - \mu_x(i)\mu_y(j)}{\sigma_x(i)\sigma_y(j)}$$

Dissimilarity:
$$cluster\ contrast = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}|i - j| P(i,j)$$

Energy:
$$energy = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}[P(i,j)]^2$$

Entropy:
$$entropy = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g} P(i,j)\, log_2[P(i,j)]$$

Homogeneity1:
$$homogeneity1 = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\frac{P(i,j)}{1+|i-j|}$$

Homogeneity2:
$$homogeneity2 = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\frac{P(i,j)}{1+|i-j|^2}$$

IMC1:
$$IMC1 = \frac{H - HXY1}{max\{HX, HY\}}$$
where H is entropy

IMC2:
$$IMC2 = \sqrt{1 - e^{-2(HXY2 - H)}}$$
where H is entropy

Inverse Difference Moment Normalized:
$$IDMN = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\frac{P(i,j)}{1+\left(\frac{|i-j|^2}{N^2}\right)}$$

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Gray level co-occurrence cube-based features
$C(i, j, k)$ is the co-occurrence cube, which is an extension of the co-occurrence matrix to 3D
N_g is the number of discrete intensity levels in the image
μ is the mean of $C(i, j, k)$
$\mu_x(i)$ is the mean of row i
$\mu_y(j)$ is the mean of column j
$\mu_z(k)$ is the mean of slice k
$\sigma_x(i)$ is the standard deviation of row i
$\sigma_y(j)$ is the standard deviation of column j
$\sigma_z(k)$ is the standard deviation of slice k

$$c_x(i) = \sum_{j=1}^{N_g} c(i,j,k)$$
$$c_y(j) = \sum_{i=1}^{N_g} C(i,j,k)$$
$$c_z(k) = \sum_{k=1}^{N_g} C(i,j,k)$$
$$HXY1 = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g} C(i,j,k)\log\left(c_x(i)c_y(j)c_z(k)\right)$$
$$HXY2 = -\sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g} c_x(i)c_y(j)c_z\,(\mathrm{k})\log\left(c_x(i)c_y(j)c_z(k)\right)$$

Cube Autocorrelation:
$$cube\ autocorrelation = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g} ijkC(i,j,k)$$

Cube Cluster prominence:
$$cube\ cluster\ prominence = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}\left[i + j + k - \mu_x(x) - \mu_y(j) - \mu_z(k)\right]^4 C(i,j,k)$$

Cube Cluster shade:
$$cube\ cluster\ shade = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}\left[i + j + k - \mu_x(x) - \mu_y(j) - \mu_z(k)\right]^3 C(i,j,k)$$

Cube Cluster tendency:
$$cube\ cluster\ tendency = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}\left[i + j + k - \mu_x(x) - \mu_y(j) - \mu_z(k)\right]^2 C(i,j,k)$$

Cube Contrast:
$$cube\ luster\ contrast = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}|i - j - k|^2 C(i,j,k)$$

Cube Correlation:
$$cube\ cluster\ tendency = \frac{\sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g} ijkC(i,j,k) - \mu_x(i)\mu_y(j)\mu_z(k)}{\sigma_x(i)\sigma_y(j)\sigma_z(k)}$$

Cube Dissimilarity:
$$cube\ cluster\ contrast = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}|i - j - k| C(i,j,k)$$

Cube Energy:
$$cube\ energy = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}[C(i,j,k)]^2$$

Cube Entropy:
$$cube\ entropy = -\sum_{i=1}^{N_j}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g} C(i,j,k)\, log_2[c(i,j,k)]$$

Cube Homogeneity1:
$$cube\ homogeneity1 = \sum_{i=1}^{N_g}\sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\frac{C(i,j,k)}{1+|i-j-k|}$$

Cube Homogeneity2:
$$cube\ homogeneity2 = \sum_{i=1}^{N_g}\sum_{j=1}^{N_g}\sum_{k=1}^{N_g}\frac{C(i,j,k)}{1+|i-j-k|^2}$$

Cube IMC1:
$$IMC1 = \frac{H - HXY1}{max\{HX, HY\}}$$
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Cube Inverse Difference Moment Normalized:

$$IDMN = \sum_{i=1}^{N_g} \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} \frac{C(i,j)}{1+\left(\frac{|i-j-k|^2}{N^2}\right)}$$

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Gray level 3D features

The 3D features use the contoured tumor itself rather than creating a co-occurrence cube by counting neighboring gray levels. The gray levels from all tumor voxel are summed up. Subsequently, the gray levels in each voxel are derived by this sum, and thereby, a probability is calculated – *D(i,j,k)*. Similarly to the co-occurrence cube, different features are calculated based on the three-dimensional equations presented above.