

Article

Laboratory Characterization of a Compacted–Unsaturated Silty Sand with Special Attention to Dynamic Behavior

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Featured Application: The test results reveal the response of compacted sandy silt soil to dynamic loading. The derived parameters based on the test results enrich the database of equation constants and allow the designers to use them to model the engineered earth structure's behavior.

Abstract: The dynamic properties of compacted non-cohesive soils are desired not only because of the risk of natural sources of dynamic excitations such as earthquakes, but mostly because of the anthropogenic impact of machines that are working on such soils. These soils are often unsaturated, which positively affects the soil's mechanical properties. The information about the values of these parameters is highly desirable for engineers. In this article, we performed a series of tests, including oedometric tests, resonant column tests, bender element tests, and unsaturated triaxial tests, to evaluate those characteristic parameters. The results showed that sandy silt soil has a typical reaction to dynamic loading in terms of shear modulus degradation and the damping ratio curves' characteristics, which can be modeled by using empirical equations. We found that the compaction procedure caused an over-consolidation state dependent on the moisture content during compaction effort. The article analyzed the soil properties that impact the maximum shear modulus G_0 value. Those properties were suction s , confining pressure σ_3 , and compaction degree represented by the void ratio function $f(e)$.

Keywords: dynamic properties; silty sand; resonant column; unsaturated soils; compacted soils

1. Introduction

The development of geotechnical constructions such as roads, railways, levees, and foundations around the globe forces engineers to build road constructions on so far unsuitable subgrades for such purposes [1–5]. One of the easiest ways to improve soil properties is compaction. The traffic or railway loadings and industrial activities are connected to dynamic and cyclic excitations, which result in other soil responses in comparison to monotonic ones. The engineering constructions such as shallow foundations usually settle at an average strain equal to 1% [6].

Recently, the scientific focus of dynamic soil properties has turned from earthquake events to anthropogenic ones, which are characterized by low amplitude and low strain ranges. Soils, which are the subgrade of the foundation or road construction base, usually are compacted intentionally or

unintentionally by machines such as heavy trucks. Therefore, the change in the original structure of the soil due to static and repeated loads impacts the subgrade soil parameters, which forces engineers to have proper information about the differences between natural and compacted soil. The repeating loading and excitations due to cyclic and dynamic loads can be characterized by a small strain characteristic to solve the boundary value problem [7].

The research conducted by Papakyriakopoulos et al. [8] was focused on silty soils' laboratory investigations into the main factors affecting the dynamic soil properties with the application of the resonant column technique. The materials in this study were the undisturbed specimens (from three boreholes located in southern Greece), as well as two additional remolded ones for the resonant column and cyclic simple shear tests. The soil was composed of sand and silt fractions. After a series of tests, the authors concluded that the low amplitude shear modulus (G_0) increased with confining pressure (σ_0), when material damping (D_0) was slightly decreasing. The authors also observed that the high amplitude shear modulus (G) and material damping (D) could be normalized in terms of the G/G_0 and D/D_0 versus the γ/γ_0 relationship. The empirical relationship was proposed for estimation of the G_0 parameter, based on the proposition of Seed and Idriss [9] concerning the influence of the relative density described by coefficient $K_{2,max}$, which is a function of the void ratio ($K_{2,max} = 0.345(1000-430e)$) at very low strains ($\gamma \leq 10^{-3}\%$) (1):

$$G_0 = 345(1 - 0.43e) \cdot (\sigma_0)^{0.5}. \quad (1)$$

where G_0 and σ_0 are in MPa and e is the void ratio.

Studies of the dynamic properties of Piedmont residual soils [10] were done by performing resonant column and torsional shear tests. The residual soils tested in this research project were saprolites, which are micaceous sandy silts and silty sands obtained from depths from 0.9 m to 5.3 m. The results of the resonant column tests showed that the low amplitude shear modulus increased with confining pressure, which was also observed by Papakyriakopoulos et al. [8]. The maximum shear modulus varied by a power of less than 0.5 to confining pressure. The authors also observed that the shear modulus decreased with an increase in shear strain in the range of 0.001% to 0.002%. Factors such as the frequency or number of cycles have no significant influence on the shear modulus and damping ratio value.

Further studies on Piedmont residual silty soils extended the laboratory tests with an in situ test [11] with the conclusion that G_0 increased with increasing confining pressure. The article reported a general tendency of the normalized shear modulus G/G_0 characteristics to increase with the increase in confining stress. The research also showed that the overconsolidation ratio did not affect the G/G_0 relationship.

Another experimental study on sandy and silty soils showed that G_0 was related to the mean effective principal stress σ'_m and void ratio [12–15]. This relationship is presented by Equation (2):

$$G_0 = A \cdot F(e) \cdot (\sigma'_m)^n \quad (2)$$

where A is an empirical constant reflecting the soil fabric formed through various stress and strain histories, n is an empirically determined exponent, equal to 0.5, and $F(e)$ is the void ratio function defined as (3):

$$F(e) = \frac{(B - e)^2}{1 + e}, \quad (3)$$

where the constant B is usually taken as 2.17 for round-grained sands [16–18].

The tests performed by Salgado et al. [19] on silty sands revealed the impact of fines content on the small strain stiffness and shear strength of silty sand. They performed triaxial and bender element tests on the mix of Ottawa sand and silt with a known gradation component. The results showed that the small strain shear modulus G_0 was a function of the stress state and degree of compactness of the silty sand. The authors also noted that, although small strain stiffness dropped with fines

content, the critical-state strengths rose. The reason for that might be the freshly deposited silty sand fine particles that were not positioned in a way to provide optimum interlocking, so the small strain responses of soil resulted in low stiffness characteristics. The greater stresses would result in fine particles' rearrangement and therefore increase interlocking, dilatancy, and shear strength.

The properties of soil differ even in the same layer, which is locally subjected to other stress conditions or the groundwater flow regime. Therefore, the response of soil is strongly influenced by what are generally termed local site conditions [20].

The topic of non-cohesive soil physical characteristics' impact on the dynamic properties is still under development. Recently, an effort to understand the effect of mean grain size was conducted by Upereti and Leong [21] and by Wichtmann and Triantafyllidis [22]. The results of the tests showed that the increase of the coefficient of uniformity caused an increase in the non-linearity of the normalized shear modulus and damping ratio with shear strain. The normalized shear modulus and damping ratio also increased with the decrease in mean grain size. The authors employed the results of the bender element, as well as cyclic triaxial tests to derive an empirical equation for the shear modulus degradation curve based on the modified hyperbolic model [21].

The work of Lu and Kaya [23] aimed to study the elastic moduli change in unsaturated soils. They tested a wide range of soil types in a uniaxial compression apparatus. The results of the tests showed that the volumetric water content impacted the elastic moduli values. The results also indicated that the initial state of volumetric water content, as well as the drying and wetting path, would affect the values of elastic moduli.

A significant amount of knowledge concerning unsaturated non-cohesive soil response to cyclic loading was gathered recently [24–27]. The reason for that was the high demand for resilient modulus characteristics. Resilient modulus M_r is one of the well-known parameters used in pavement engineering, which specifies the resilient strain during one cycle of loading. During cyclic loading of the soil, hysteretic behavior is observed in the shearing strain range equal to $10^{-3}\%$. The response to such excitation in this strain range differs from this in the range $10^{-4}\%$ and $10^{-5}\%$, where we can observe that the elastic response and damping ratio are assumed to be equal to zero [28,29]. The resilient strain is defined as the soil elastic response to cyclic loading during the unloading phase [30]. Hence, we define the resilient modulus as the ratio of the repeated deviatoric stress to recoverable strain [31–33] (4):

$$M_r = \frac{\sigma_d}{\varepsilon_r}, \quad (4)$$

where σ_d is the applied repeated deviatoric stress and ε_r is the axial recoverable strain. In a small strain range, the resilient modulus is equivalent to Young's modulus. Therefore, data obtained from a torsional shear test can be easily transformed from the shear modulus to Young's modulus by applying the following relationship (5):

$$M_r = E = 2G(1 + \nu) \quad (5)$$

where ν is Poisson's ratio; hence, the knowledge about the shear modulus value can be easily translated to the resilient modulus value, and no additional cyclic loading tests are required.

To complement the existing literature, we performed tests on unsaturated silty soil in a small strain range to find the relationship between the unsaturated soil state parameters and the shear modulus characteristics. The tests were conducted to extend the knowledge about the soil behavior during dynamic loading. Moreover, the tested soil was compacted to simulate the subgrade conditions. To present the silty soil behavior fully, we performed laboratory tests to identify the following soil properties:

- Determination of the impact of compaction on the consolidation properties of the tested soil to verify if compaction creates a soil preconsolidation state and to ensure the same consolidation conditions during the tests.

- Characterization of the small strain behavior of the soil with the use of the resonant column test to obtain the initial shear modulus and shear modulus characteristics, as well as the damping ratio characteristics.
- Normalization of small strain data with respect to existing G_0 and D_0 empirical models.
- Undrained triaxial tests to measure the soil suction of silty soil specimens and the initial shear modulus value with the use of the bender element test.

Finally, we evaluated gathered data with existing empirical models, and we proposed the empirical model for unsaturated silty soil for initial shear modulus calculation.

2. Materials and Methods

2.1. Physical Properties of the Soil

The non-cohesive soil, used for these tests, was the silty sand from a road construction site in Pisa, Italy. The specific gravity (G_s) of the soil was 2.73. Based on the sieve analysis, we did the soil classification according to Eurocode 7, and we recognized the tested soil as sandy silt/silty sand (saSi/siSa). The results of the sieve analysis are shown in Figure 1. According to the value of C_c and C_u , the shape of the grading curve could be classified as medium-graded. The compaction test was performed with the application of Proctor's method with modified effort ($2700 \text{ kN}\cdot\text{mm}^{-3}$). The applied method consisted of 25 blows of a hammer on each of the five soil layers in a 1 dm^3 mold according to ASTM D1557-12e1. We performed the tests for seven different moisture contents, which were equal to 4.6%, 6.1%, 8.1%, 9.4%, 10.8%, 12.9%, and 14.8%.

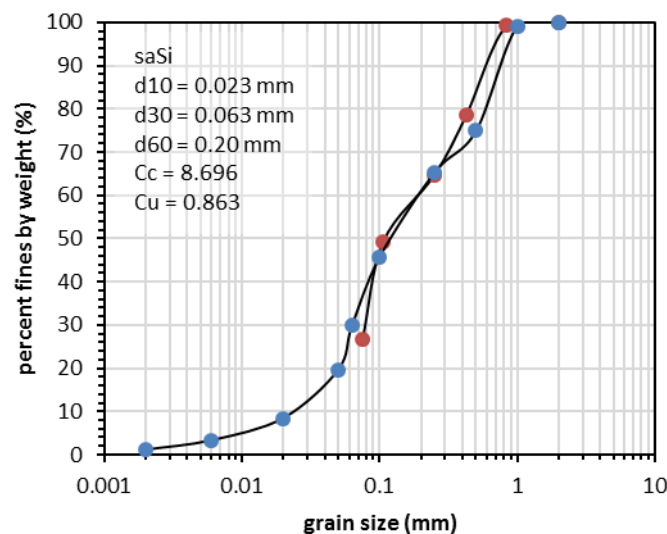
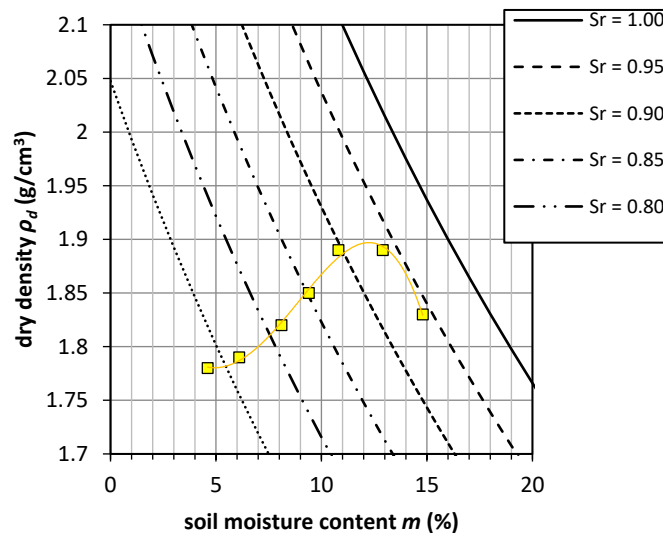


Figure 1. Grain-size distribution curves of tested soil (blue dots, dry sieve analysis and sedimentation analysis; orange dots, wet sieve analysis).

The results of the test showed maximal dry unit density ρ_{ds} equal to $1.89 \text{ g}\cdot\text{cm}^{-3}$, in optimum water content equal to 12.2%. Table 1 presents the results of the Proctor test with a corresponding degree of saturation and void ratio data. Figure 2 presents the optimal moisture content curve. We evaluated the bulk density with the use of the tapping fork test. The minimal bulk density $\rho_{d \min}$ was equal to $1.19 \text{ g}\cdot\text{cm}^{-3}$, and the maximum bulk density $\rho_{d \max}$ was equal to $1.57 \text{ g}\cdot\text{cm}^{-3}$. The Proctor test showed that the maximal bulk density was equal to $1.89 \text{ g}\cdot\text{cm}^{-3}$, and this value was taken for further calculations. $e_{\max}$ and $e_{\min}$ were equal to 1.288 and 0.444, respectively.

Table 1. Proctor's compaction test data and physical properties of the compacted samples.

m (%)	ρ_d (g·cm ⁻³)	e	S_r	I_D
4.6	1.78	0.53	0.73	0.89
6.1	1.79	0.53	0.76	0.90
8.1	1.82	0.50	0.81	0.93
9.4	1.85	0.48	0.85	0.96
10.8	1.89	0.44	0.9	1.00
12.9	1.89	0.44	0.94	1.00
14.8	1.83	0.49	0.95	0.94

**Figure 2.** Soil compaction curve of the tested soil.

2.2. Test Sample Preparation and Testing Methods

We performed the oedometer tests on compacted soil specimens prepared with the same method. The oven-dry soil with a known mass was sprayed with water to the appropriate moisture content. The compaction process was conducted with respect to the ASTM D1557-12e1 method. Molded soil was subsequently confined to a cylindrical steel confining ring with a diameter of 75 mm and 20 mm high and placed in a water bath. We subjected the soil samples to compressive stress by applying a vertical load that uniformly acted over the area of the sample. Compressive stress was equal to 12.5 kPa, 25.0 kPa, 50.0 kPa, 100.0 kPa, 200.0 kPa, 400.0 kPa, 800.0 kPa, 1600.0 kPa, and 3200.0 kPa.

We performed the resonant column tests on compacted soil samples (with respect to ASTM 4015-15). The resonant column used for the test was Stokoe's fixed-free-type device. In this type of resonant column, the torsional type of vibration is created. The samples were compacted in the Proctor mold, and from the compacted sample, the resonant column sample was timed with the height h equal to 10 cm and diameter d equal to 5 cm. During the resonant column test, soil samples remained unsaturated. The confining pressure σ_3 , applied in the resonant column cell on the sample was equal to 30, 45, and 60 and on one sample to 150 kPa. After consolidation in each step, the torsional shear and damping tests were performed.

The last test performed in the undrained triaxial apparatus aimed to establish the stress-dependent soil-water characteristic curve. We prepared the sample for this test by compaction in optimum moisture content. After the preparation of the sample, the soil was fully saturated and consolidated to σ'_3 equal to 100 kPa. After this phase, the unsaturated tests were conducted by using the axis translation technique. We also measured the G_0 moduli with the use of the bender elements peak-to-peak technique.

3. Results

3.1. Oedometer Test Results

We conducted the oedometer tests on compacted soil specimens in order to find the possible impact of the compaction process on the compressibility properties of the tested soil. The compaction might be responsible for the built-in vertical stress. The compressibility curves of the performed oedometer tests are shown in Figure 3. The test results showed a similar run of the stress–strain curve except for Test No. 3, where the possible inconsistency of side contact between the soil sample and steel ring occurred. The compressibility curves were recalculated to find the soil compressibility modulus M from the oedometer tests. The modulus M changed when applied stress increased as well (Figure 4).

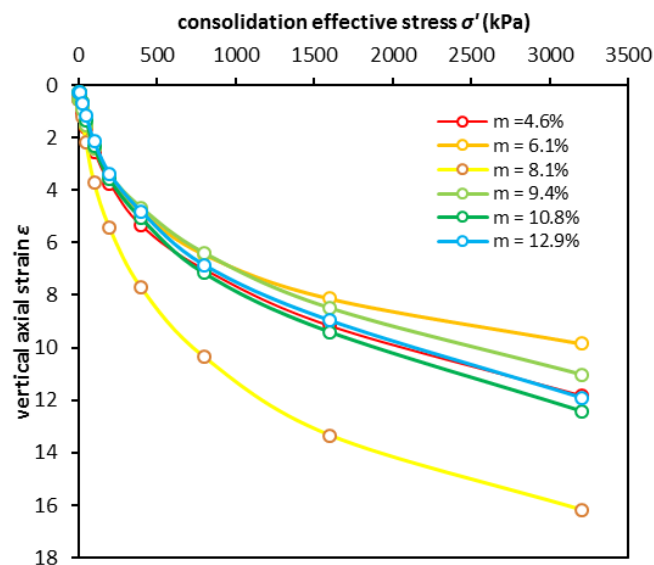


Figure 3. Laboratory oedometer test results on compacted soil samples with different moisture contents and with constant compaction effort.

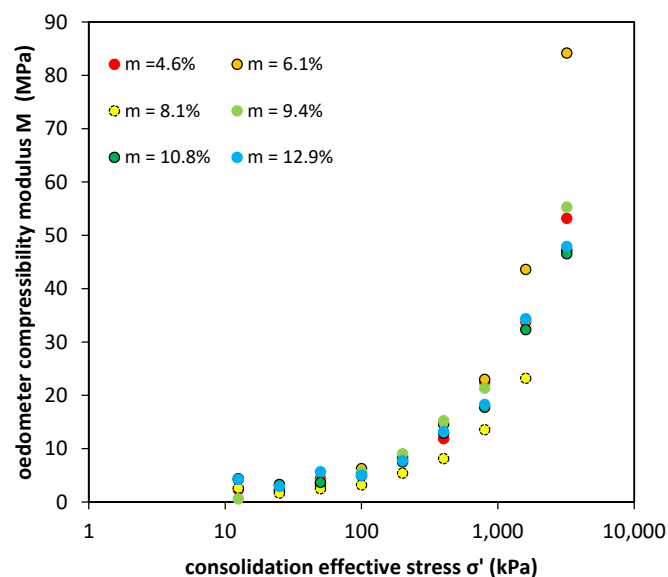


Figure 4. Compressibility modulus from the laboratory oedometer test results for compacted soil samples with different moisture contents and with constant compactive effort.

Soil samples in optimum moisture content were less stiff to the applied load than the soil specimens compacted in the dry site. This phenomenon became important when higher applied vertical stress

was imposed on the compacted sandy subgrade (for example, shallow foundation, or embankment, or levee construction).

The possible reason for such a response of soil specimens was the overconsolidation phenomena due to compaction. For the investigation of the overconsolidation impact on soil response to vertical applied stress, the determination of preconsolidation pressure was performed. In Figure 5, the compressibility characteristics are presented. The method of work was employed to calculate the preconsolidation pressure. The reason for that was the gentle curved $e-\sigma'$ characteristics where the graphic methods like the Casagrande method could not be applied with much confidence. The method of work proposed by Becker et al. [34] was based on the work input in the test. The sum of work ($\sum W$) was the sum of finite work ΔW_i in the i^{th} load step during the oedometric test. ΔW_i was the product of average stress σ'_{avg} on the sample ($\sigma'_{avg} = (\sigma'_i + \sigma'_{i+1})/2$) and the strain difference $\Delta \varepsilon$ ($\Delta \varepsilon = (\varepsilon_{i+1} - \varepsilon_i)$).

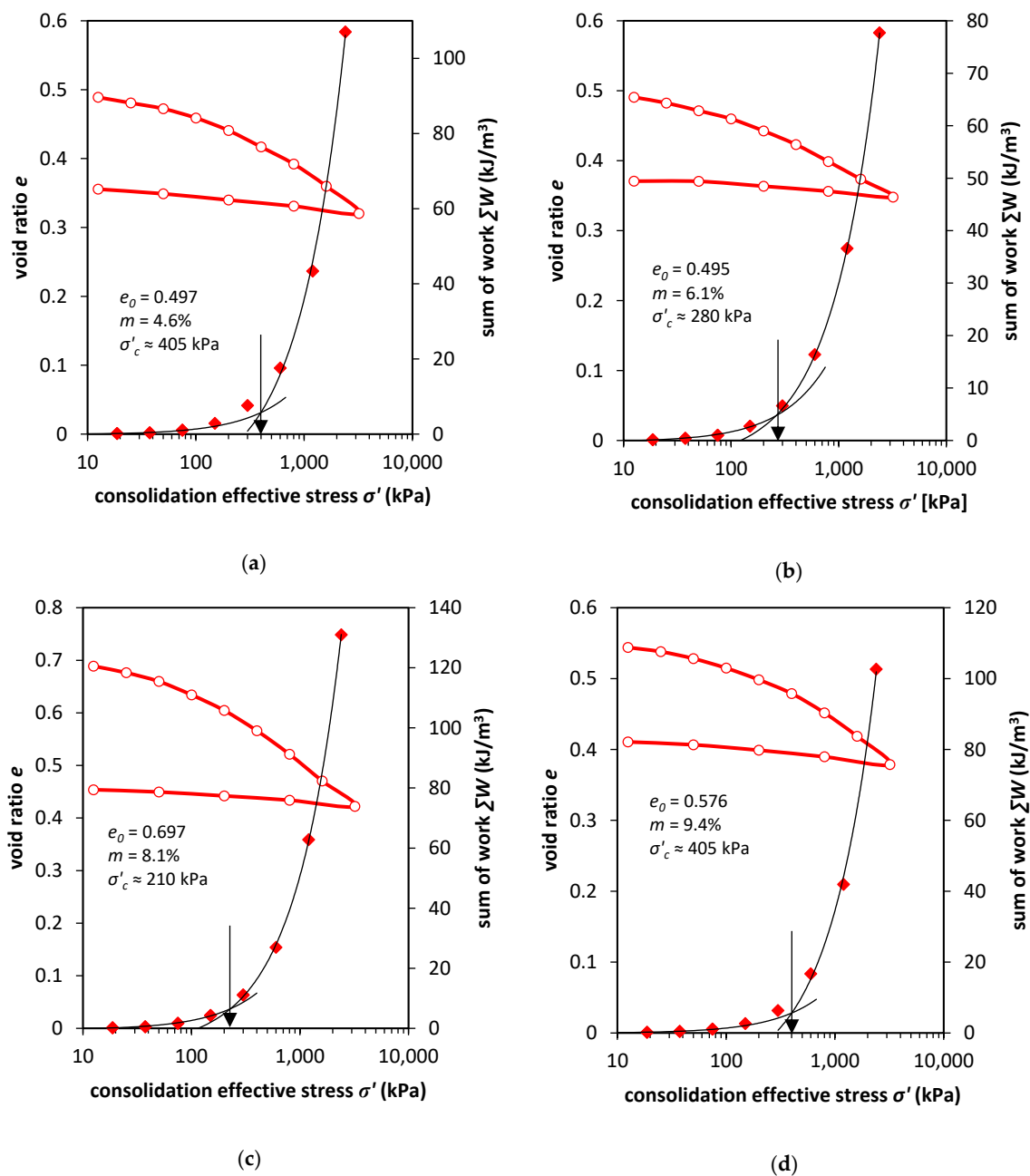


Figure 5. Cont.

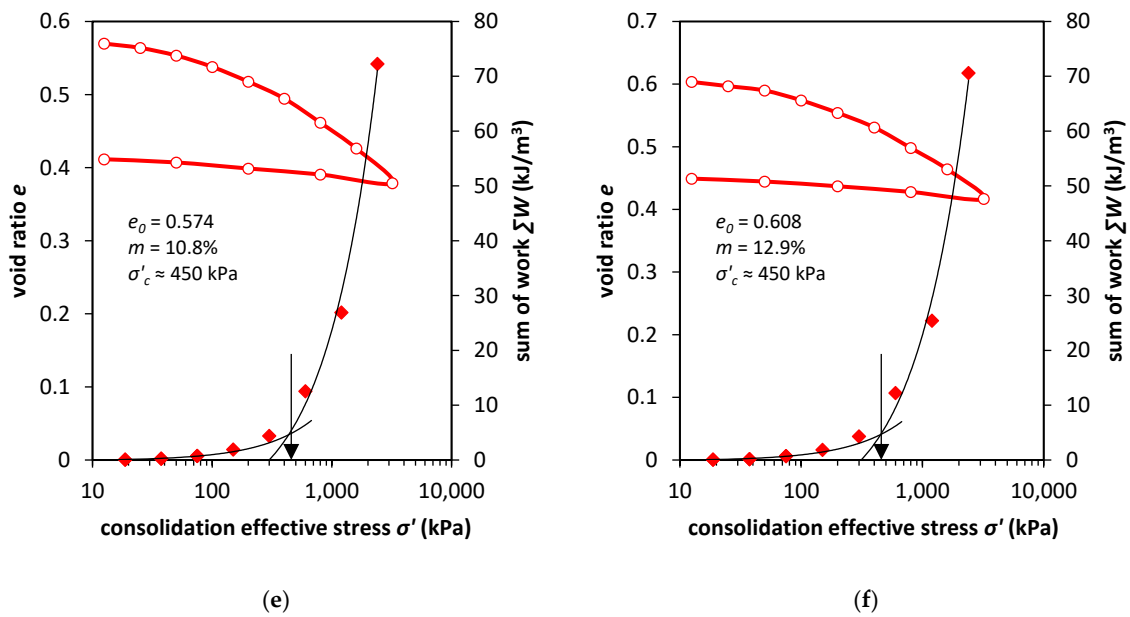


Figure 5. Soil compressibility characteristics (red line, empty points) with the method of work for preconsolidation pressure determination (red diamonds) in moisture content m equal to: (a) $m = 4.6\%$, (b) $m = 6.1\%$, (c) $m = 8.1\%$, (d) $m = 9.4\%$, (e) $m = 10.8\%$, and (f) $m = 12.9\%$.

The preconsolidation pressure σ'_c differed when different moisture content test results were compared. We estimated the highest preconsolidation pressure for peak moisture content ($\sigma'_c \sim 460$ kPa). The lowest one was observed for samples where the moisture content was equal to 8.1%, which was equal to 210 kPa. Other investigators also have concluded that compacted non-cohesive soils behave as if they were overconsolidated [35–37]. Some of them stated that the observed overconsolidation must be a function of remolding water content [38,39].

Figure 6 presents a three-dimensional view of consolidation from the oedometer test results under constant moisture conditions during the sample compaction process equal to 4.6%, 6.1%, 9.4%, 10.8%, and 12.9%. The influence of moisture content on soil consolidation properties could be seen by analyzing the fit to the mathematical data surface. The graph surface led to the derivation of the mathematical model, which allowed calculating the void ratio in specified loading conditions and moisture content. Equation (1) presents the formula as mentioned above for calculating the void ratio e , as a function of effective consolidation stress σ'_3 (kPa) and moisture content during the compaction process m (%) (6):

$$e = \frac{1}{2 - \frac{0.3 \cdot m^2}{100\%} + \frac{2 \cdot \sigma'^{0.5}_3}{100 \text{ kPa}}} \quad (6)$$

where R^2 is equal to 0.956. Based on the oedometer tests, the conclusion was that the sandy silt had preconsolidation pressure, which rose with moisture content, and this relation was somewhat logarithmic rather than linear, which meant in optimum moisture content, the preconsolidation pressure might be several dozen greater.

3.2. Resonant Column Test

The resonant column tests can measure rotational displacement and acceleration by the use of proximitors, an accelerometer that registers coil current. The resonant column measures resonant frequency and acceleration. Based on these measurements, we could calculate the modulus and strain levels during the tests. This type of test was performed in a fixed-free device, and excitations were imposed, as well as measurements were taken from a free end of the sample. The current that flowed through the coil provided the torque. The function generator and power amplifier devices were able to

impose harmonic signals even at high torque levels. The measurements were conducted in order to find the resonant frequency ω_n data, which later led to calculating the shear modulus G based on the shear wave velocity as in the following Equation (7):

$$G = \rho \cdot v_s = \rho \cdot \frac{\omega_n \cdot l}{\beta}, \quad (7)$$

where l is the length of the sample and β is a device constant calculated based on the specimen mass and geometries.

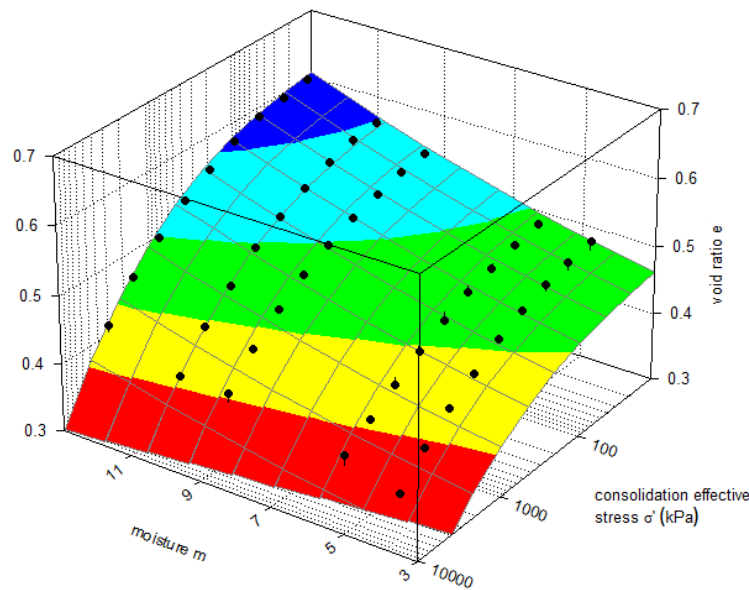


Figure 6. Void ratio change in different moisture and consolidation effective stress conditions for compacted sandy silt samples from odometer tests.

We calculated the damping from the resonant column test with the use of the logarithmic decrement method in which two acceleration maxims were used to calculate the damping based on Equation (8):

$$D = \frac{\delta_{ec}}{\sqrt{4\pi^2 + \delta_{ec}^2}} \quad (8)$$

where δ_{ec} is the logarithmic decrement calculated as (9):

$$\delta_{ec} = \ln\left(\frac{u_n}{u_{n+1}}\right) \quad (9)$$

where u is the amplitude of acceleration of the n and $n + 1$ maxims. The damping ratio can be presented as a dimensionless parameter or as a percentage [40].

Figure 7 presents the normalized shear modulus characteristics versus shear strain γ , and in Figure 8, the damping D characteristic is presented. The shear modulus change had close characteristics. These characteristics could be easily modeled if the G_0 modulus value were calculated. We could observe that the initial shear modulus also responded to the compaction curve. For samples prepared in moisture content equal to 10.8% (Figure 7c), the initial shear modulus G_0 had the highest values. The impact of consolidation pressure on the G_0 value was clear. With the rise of σ_3 , the G_0 also increased. The top value of shear modulus was equal to 164.08 MPa for σ_3 equal to 150 kPa. The lowest value of G_0 was equal to 82.02 MPa for the sample confined with a pressure equal to 30 kPa in moisture content $m = 12.9\%$.

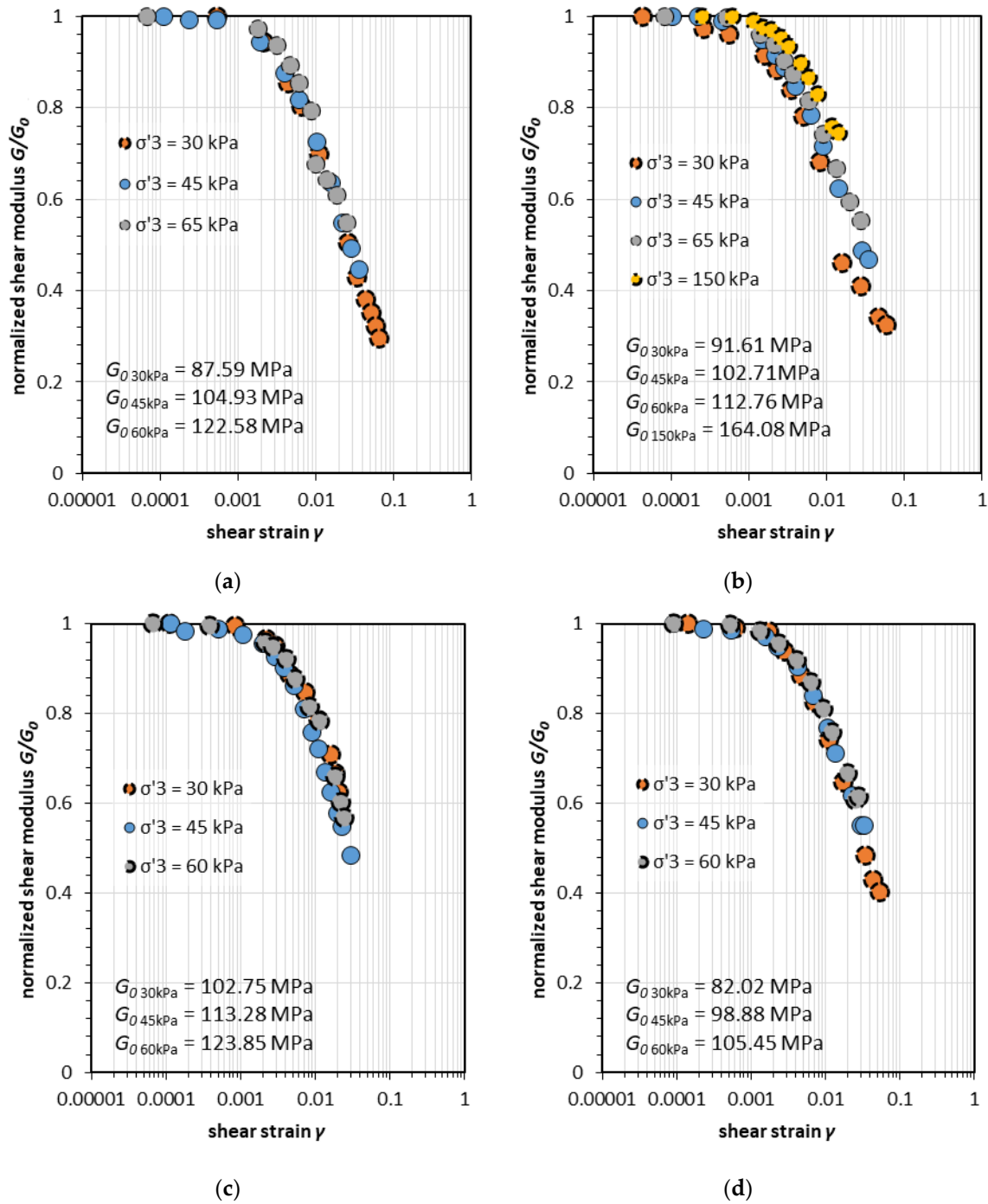
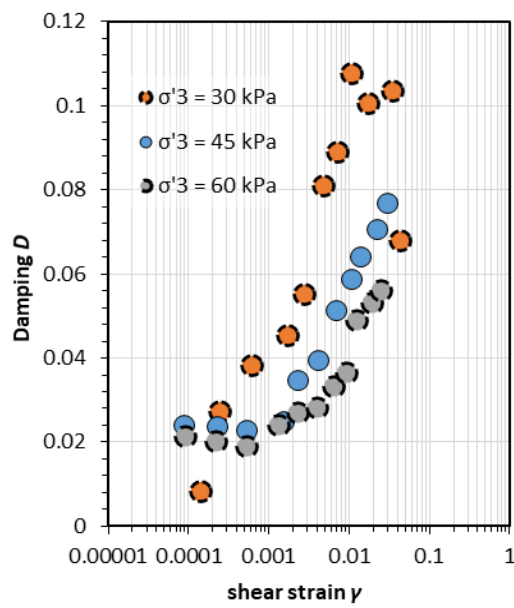


Figure 7. Modulus reduction curves with the normalized shear modulus G/G_0 for different moisture contents: (a) $m = 8.1\%$, (b) $m = 9.4\%$, (c) $m = 10.8\%$, and (d) $m = 12.9\%$.

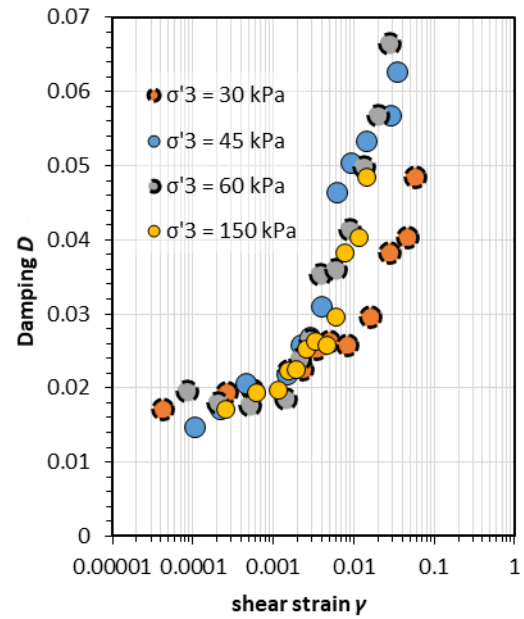
The soil samples were prepared in moisture contents equal to 8.1%, 9.4%, 10.8%, and 12.9%, which corresponded to the Proctor test results. Table 2 presents the soil parameters' values during the resonant column test. The compaction caused the preconsolidation state, and therefore, the overconsolidation ratio (OCR) could be calculated for the resonant column samples. The modeling of shear modulus degradation could be conducted with the application of the Stokoe et al. [41] modulus reduction curve (10):

$$\frac{G}{G_0} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_t}\right)^\alpha} \quad (10)$$

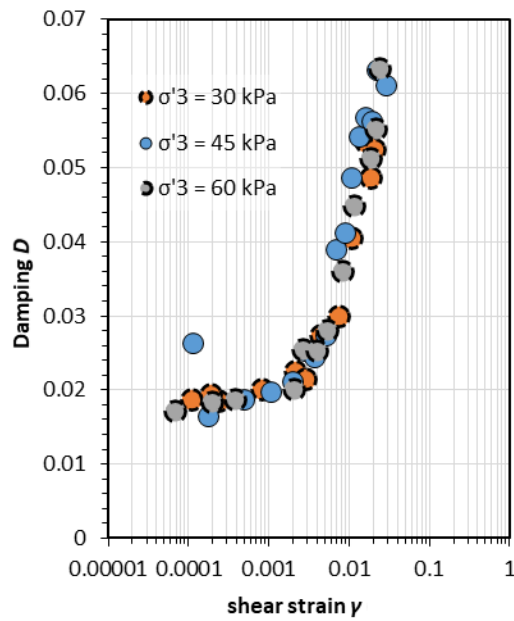
where α is a curvature parameter (usually $\alpha \approx 0.92$) and γ_t is called the pseudo-reference strain corresponding to $G/G_0 = 0.5$.



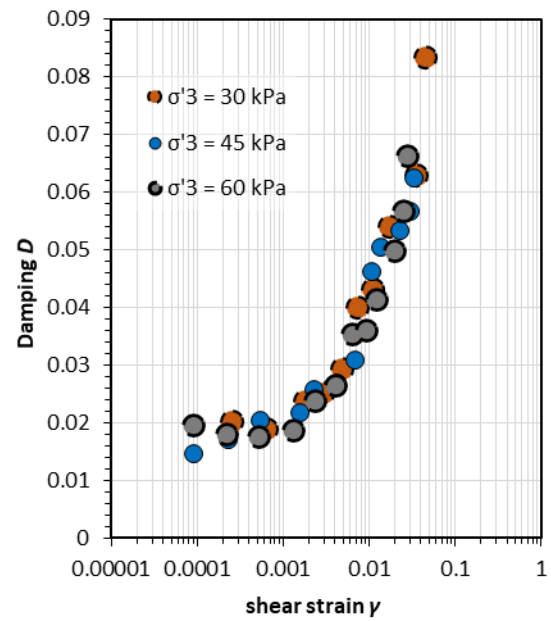
(a)



(b)



(c)



(d)

Figure 8. Material damping curve for different moisture m contents: (a) $m = 8.1\%$, (b) $m = 9.4\%$, (c) $m = 10.8\%$, and (d) $m = 12.9\%$.

Table 2. Physical properties, pressure in chamber, OCR and maximum shear modulus for tested samples.

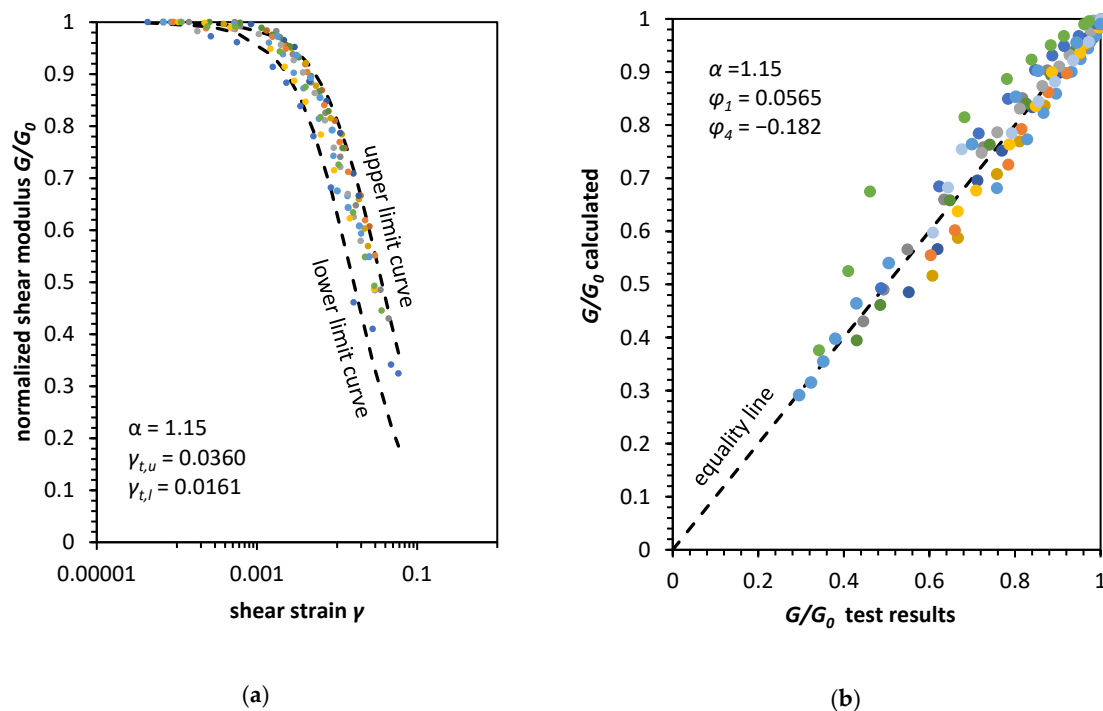
Sample No.	e_0	S_r	σ_3 (kPa)	OCR	G_0 (MPa)
1.1	0.53	0.76	30	16.7	87.6
1.2	0.53	0.76	45	11.1	104.9
1.3	0.53	0.76	60	8.3	122.6
2.1	0.5	0.81	30	19.0	91.6
2.2	0.5	0.81	45	9.5	102.7
2.3	0.5	0.81	60	9.5	112.8
2.4	0.5	0.81	150	3.8	163.1
3.1	0.46	0.87	30	20.0	102.0
3.2	0.46	0.87	45	13.3	113.3
3.3	0.46	0.87	60	10.0	123.9
4.1	0.44	0.93	30	20.3	82.0
4.2	0.44	0.93	45	13.6	98.9
4.3	0.44	0.93	60	10.2	105.5

For the test results, the modulus reduction curve was fit. For the purpose of accurately modeling the soil stiffness degradation, the upper and lower limit curves were derived. The soil pseudoreference stress was estimated from the test results, and based on this, and the curvature parameter was fit to the data. The value of α was equal to 1.15 for all test results. The results of the calculations are shown in Figure 9a. The pseudoreference stress could be estimated based on the empirical equation proposed by Darendeli [42] (11):

$$\gamma_t = (\varphi_1 + \varphi_2 \cdot I_P \cdot OCR^{\varphi_3}) \cdot \sigma'_3{}^{\varphi_4}, \quad (11)$$

where $\varphi_1 = 0.0352$, $\varphi_2 = 0.0010$, $\varphi_3 = 0.3246$, and $\varphi_4 = 0.3483$. The pseudoreference stress for non-cohesive soils reduced to the simpler Equation (12):

$$\gamma_t = \varphi_1 \cdot \sigma'_3{}^{\varphi_4}, \quad (12)$$

**Figure 9.** Modeling of the modulus reduction curve: the limit curves for mathematical models (a); pseudoreference stress parameters for the tested sandy silt (b).

For the presented power function, we found the value of the constants by regression analysis, which were equal to $\varphi_1 = 0.0565$ and $\varphi_4 = -0.182$ with the coefficient of determination R^2 equal to 0.238, which corresponded to low correlation, although the calculations with the constant curvature parameter and calculated γ_t based on Equation (12) showed good correlation with test data (Figure 9b).

Modeling of the damping gave us information about energy dissipation, which could occur even when the soil strain was at a low level. The soil damping could be related to the G/G_0 relationship. We modified the proposition of Hardin and Drnevich [43] and we were able to approximate the shape of the damping characteristic (13):

$$\frac{D}{D_{norm}} = 1 - \frac{G}{G_0}, \quad (13)$$

This simple relationship between the soil stiffness and damping characteristics was able to calculate the damping without special effort to perform the damping tests. D_{norm} is the normalization damping parameter. We analyzed the damping characteristics, and based on the test data, we found that for the compacted sandy silt soil tested in this article, D_{norm} did not have a constant value, but changed with the normalized shear modulus progress. This characteristic can be observed in Figure 10. Based on these results, the relationship between D_{norm} and G/G_0 was established as follows (14):

$$D_{norm} = \delta_1 - \frac{\delta_2}{\ln\left(\frac{G}{G_0}\right)}, \quad (14)$$

where δ_1 and δ_2 are constants equal to 0.094 and 0.0196, respectively, for this function's coefficient of determination, R^2 was equal to 0.988.

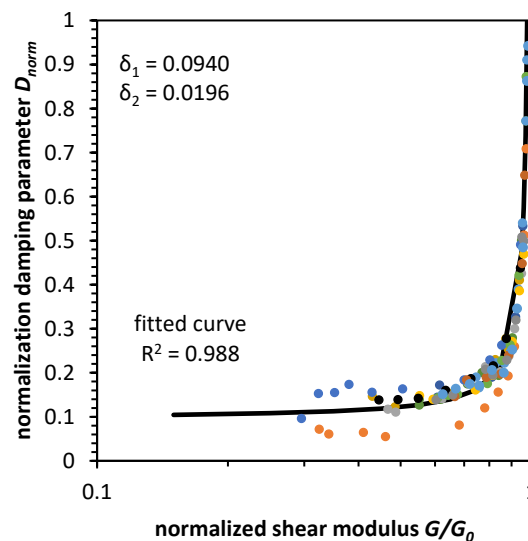


Figure 10. Maximum damping ratio characteristics for the sandy silt soil in this study.

3.3. Testing in a Triaxial Apparatus Equipped with Bender Elements for Unsaturated Soils

The unsaturated state of soil usually does not impact the resonant column test since the tests are shearing ones, so the pore water pressure is rarely measured during the resonant column test [44].

The tests performed in the triaxial apparatus for unsaturated soil led to determining a soil water characteristic experimental curve (Figure 11a). This characteristic experimental curve is presented in Figure 10. With the use of the Van Genuchten model, it was fit to the test results Equation (1):

$$S_r(s) = \left(\frac{1}{1 + (\beta_{Sr}s)^n} \right)^m, \quad (15)$$

where s is the suction and β_{Sr} , n , m ($m = 1 - 1/n$) are constants. For silty sand, the value of β_{Sr} was between 5.61 and 0.09 kPa⁻¹, and the value of m was between 0.73 and 0.11 [45,46]. In this case, the model parameters were equal to $\beta_{Sr} = 0.11$, $n = 1.29$, and $m = 0.225$. Following the unsaturated soil mechanistic approach, the soil sample was consolidated to 100 kPa net stress ($\sigma - u_w$), and then, to change the saturation degree, the axis-translation technique was used to control the soil suction. During the bender element tests to obtain reliable testing results, a considerably long period of time (a few days to weeks) was applied to achieve the equilibrium condition with respect to applied soil suction for the tested soil prior to determining the shear modulus G_0 values. We performed bender element tests, which indicated the impact of soil suction on initial shear modulus G_0 , and therefore, it indicated that we should pay more attention to the degree of saturation when we perform the dynamic soil response analysis. The results of the test are presented Figure 11b. The shear modulus G_0 from the bender element tests for S_r equal to 1.0 was 125.3 MPa, while when the suction rose to 200 kPa ($S_r = 0.4$), G_0 was equal to 222.7 MPa.

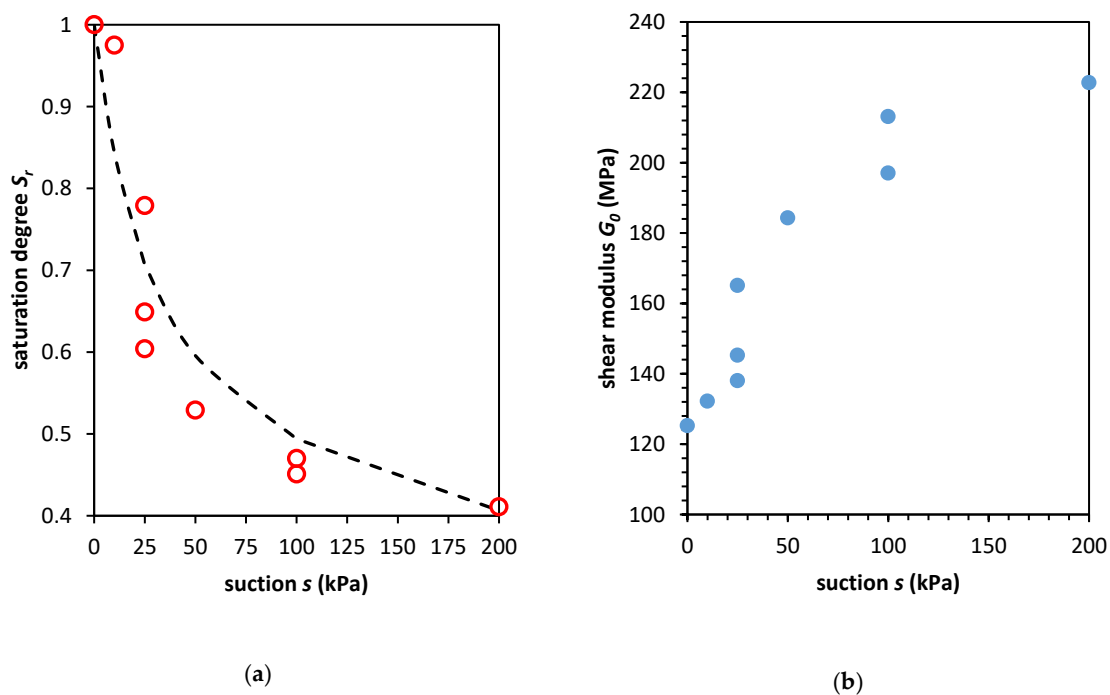


Figure 11. Results of the bender element test in the unsaturated triaxial apparatus: post-compaction suction in terms of the saturation ratio (a); maximum shear modulus in different suction conditions (b).

Since the G_0 value would impact the soil stiffness degradation and damping characteristics, the degree of saturation played an essential role in compacted silty soils. The optimum moisture content, which corresponded to S_r equal to 0.93–0.94, would cause suction equal to 4 kPa. Higher soil suction resulted in lower soil density. Soil compacted in optimum moisture content had very low soil suction in comparison to soil suction at $S_r = 0.41$, where s was equal to 200 kPa. This suction effect impacted the soil grain connection and therefore was the reason why soil was stiffer in small strain regimes.

3.4. G_0 Mathematical Modeling

The presented test results showed that the sandy silt in this study would respond to dynamic loadings in a manner that is well established in the literature. The mathematical modeling of dynamic soil response relies on the estimation of G_0 value. In this study, we found that the soil's initial shear modulus was dependent on soil suction. Another factor that impacted the soil's initial shear modulus was the pressure in the triaxial cell σ_3 . In the literature, we can find numerous equations for the

calculation of the G_0 modulus [12,13]. The most popular ones for the saturated soil in tests with uniform stress ($\sigma'_m = \sigma'_3$) are Equations (16) and (17):

$$G_0 = 625 \cdot (p_a \sigma'_3)^{0.5} / (0.3 + 0.7e^2), \quad (16)$$

$$G_0 = 22 \cdot K_{2, \max} \cdot (p_a \sigma'_3)^{0.5}, \quad (17)$$

where p_a is atmospheric pressure ($p_a = 100$ kPa), e is the void ratio, and $K_{2, \max}$ is the constant determined by the void ratio ($K_{2, \max} = -44.66 \cdot \ln(e) + 28.79$). The maximum shear modulus is calculated in both cases in kPa.

The equations that account for the unsaturated state are more complex than the equations for the saturated state. The equations have to consider the saturation ratio, as well as the soil suction; therefore, the equations are often limited to one or two types of soil. Examples of such equations are as follows [46,47], Equations (18) and (19):

$$\frac{G_0}{G_{r, \text{sat}}} = A \cdot f(e) \left[\frac{(p + S_r^\kappa s)}{p_a} \right]^n, \quad (18)$$

$$G_{0(\text{unsat})} = G_{0(\text{sat})} \left[1 + \zeta \left(\frac{s}{p_a / 101.3} \right) (S_r^\xi) \right], \quad (19)$$

where p is net pressure, ζ and ξ are constants of Equation (19), which relies on the coefficient of uniformity and soil classification, and κ and n (stiffness coefficient $n = 0.2$ for silty sand, $n = 0.6$ for sandy silt in this study) are constants of Equation (18) where κ is a parameter related to the plasticity index (for silty sand soil $\kappa = 1.75$, $\kappa = 5.0$ for sandy silt in this study). The $f(e)$ parameter is a void ratio function that can be an exponential function ($f(e) = e^{-\chi}$ where χ is the exponent parameter, equal to 1.7 for silty sand and sandy silt [48]). The $G_{r, \text{sat}}$ parameter is the reference shear modulus, which for silty sand was equal to 7 MPa and 13 MPa for sandy silt in this study. The A parameter is associated with the soil structure, which was further analyzed, and the correlation with the degree of saturation was found as follows (20):

$$A(S_r) = a \left(\frac{S_{\max}}{S_r} - 1 \right)^b, \quad (20)$$

where $S_{\max}$ is the maximum saturation ratio that can be obtained from compaction tests and a, b are constants ($a = 4.55$ and $b = 0.25$ for silty sand and $a = 4.0$ and $b = 0.15$ for sandy silt in this study). Additionally, we analyzed the resonant column and bender element tests, and we found that the relationship between the maximum shear modulus and stress state and the void ratio function can be described as (21):

$$G_0 = G_{0, \mu 0} + 150 \cdot f(e) \cdot (\sigma_3 + s), \quad (21)$$

where $G_{0, \mu 0}$ is the shear modulus value when $S_r = 0$ and $\sigma_3 = 0$ kPa.

For Equations (16)–(19) and (21), we evaluated the quality of G_0 prognosis with the use of ex-post methods. In order to do so, we used the test results as a reference. In Table 3, the data concerning the value of G_0 are presented. We also conducted the ex-post analysis to answer the question of how well the models performed if they were used before the tests that evaluated the models' reliability.

Based on the calculations performed in Table 3, the presented G_0 models' mean absolute percentage error (MAPE) was between 7.43% and 46.73%, and the mean error (ME), mean percentage error (MPE), mean absolute error (MAE), mean absolute percentage error (MAPE), and Theil index values are presented in Table 4.

Table 3. Results of the maximal shear modulus value calculations for the mathematical models.

Sample No.	Test Results	Model (16)	Model (17)	Model (18)	Model (19)	Model (21)
Bender Element test	125.3	149.6	150.9	0.0	0.0	137.0
	132.2	149.6	150.9	0.0	0.0	143.1
	138.0	151.7	153.4	203.2	103.5	152.2
	145.3	151.7	153.4	222.7	97.9	152.2
	165.1	151.7	153.4	228.8	95.8	152.2
	184.3	153.8	155.9	251.2	171.1	167.3
	197.1	153.8	155.9	261.3	316.3	197.6
	213.1	153.8	155.9	263.8	312.0	197.6
	222.7	153.8	155.9	271.4	591.8	258.1
1.1	87.6	68.9	68.9	67.2	102.7	100.7
1.2	104.9	84.4	84.3	82.6	102.7	107.4
1.3	122.6	97.5	97.4	96.4	102.7	114.0
2.1	91.6	72.1	72.0	68.7	68.0	98.4
2.2	102.7	88.3	88.2	84.9	68.0	105.7
2.3	112.8	101.9	101.8	99.4	68.0	113.0
2.4	163.1	161.2	161.0	167.2	68.0	156.9
3.1	102.0	76.4	76.5	71.0	50.6	98.9
3.2	113.3	93.6	93.7	88.0	50.6	107.3
3.3	123.9	108.0	108.2	103.0	50.6	115.8
4.1	82.0	78.6	78.9	55.2	32.3	97.7
4.2	98.9	96.3	96.6	68.9	32.3	106.7
4.3	105.5	111.2	111.5	81.0	32.3	115.8

Table 4. Ex-post analysis result for maximum shear modulus models.

Index	Model (16)	Model (17)	Model (18)	Model (19)	Model (21)
ME	14.83 MPa	14.07 MPa	−7.97 MPa	7.95 MPa	−2.80 MPa
MPE	10.37%	9.92%	−0.16%	14.27%	2.44%
MAE	20.97 MPa	20.79 MPa	36.11 MPa	68.18 MPa	9.84 MPa
MAPE	14.89%	14.85%	26.60%	46.73%	7.43%
I^2	0.0371	0.0356	0.0883	0.5243	0.0078
I_1^2	30.51%	28.72%	3.66%	0.61%	5.18%
I_2^2	6.77%	5.69%	77.67%	79.24%	0.01%
I_3^2	62.72%	65.59%	18.67%	20.15%	94.81%

The ME and MPE showed how precise the model was in the modeling of the G_0 value. For Model (21), the lowest error value was obtained. The G_0 models for saturated soils also had a low percentage of error. The MAE gave information about the absolute error for the calculated value of the maximum shear modulus. Finally, we calculated the Theil parameter I^2 to estimate the relative error of the models' results. The Theil parameter consists of three categories I_1^2 , I_2^2 , and I_3^2 . The first parameter describes how close the model is to the empirical data. The second parameter shows information about how precise the prognosis is, and the last parameter shows if the direction of the change was correctly calculated. Models (16), (17), and (21) had low Theil parameters. and in all three cases, the most significant amount of error came from the direction of change. For Models (18) and (19), the issue, in that case, was that the models were less precise and especially in the case of the Model (18), in which the Theil parameter had the same level as for the previous models. Model (19) performed poorly mostly because it did not consider the void ratio impact and relied too much on the empirical constants.

4. Conclusions

In this paper, the results of a wide range of tests were presented to evaluate the behavior of compacted sandy silt soil. Knowledge about soil response to dynamic loads is essential for road and earth construction designers. Based on these results, the following conclusions may be presented:

1. The soil compacted with the Proctor method in this study was tested to find preconsolidation stress. The oedometric test results clearly showed that the preconsolidation stress increased with the increase of moisture content during the compaction, and higher soil dry density resulted in higher preconsolidation pressure, which was equal to 450 kPa in optimal moisture content. The soil suction in optimum moisture content was equal to 4–5 kPa, which was also favorable during the compaction process.
2. The resonant column test revealed a high correlation of shear modulus degradation and damping characteristics with the maximum shear modulus value G_0 . As presented in this article, modeling of the soil's reaction to dynamic loading showed that if the information about G_0 were provided, the stiffness and damping characteristics could be modeled with the use of commonly known models.
3. The G_0 values also rose with the increase of soil compaction. The lowest values of G_0 were between 82 and 87 MPa (for samples compacted with moisture content other than optimum and in σ'_3 equal to 30 kPa). The highest G_0 was registered for the sample with optimum moisture content (equal to 123.9 MPa in σ'_3 equal to 60 kPa), but we also noted the impact of σ'_3 on the maximum shear modulus value where for σ'_3 equal to 150 kPa, G_0 was equal to 163.1 MPa.
4. The bender element suction-controlled unsaturated triaxial test showed the impact of the suction on G_0 where for $S_r = 0.41$ (suction s equal to 200 kPa), G_0 was equal to 222.7 MPa (the soil sample was compacted in optimum moisture content, $\sigma'_3 = 100$ kPa).
5. Finally, we made the effort in the mathematical modeling of G_0 using known equations for saturated and unsaturated soils. The results of the calculation were then evaluated with the use of ex post methods. We extended the analysis by the equation that was derived from the observations of soil behavior in this article. We recognized such factors impacting the soil stiffness as suction s , consolidation pressure σ'_3 , and compaction, which was represented by the void ratio function $f(e)$. The results of the test showed that the commonly known equation quite closely modelled the G_0 value (14.85% to 26.60% mean absolute percentage error) with one exception. The equation presented by us showed the best performance, but at this stage of development, it was limited only to sandy silt-type soils.

On the subject of compacted soil response to dynamic load modeling, much scientific effort has been conducted. The presented relationships proved that they could model the sandy silt soil's reaction to dynamic excitations accurately (modulus reduction curve, material damping curve) or close enough (G_0 equations). Nevertheless, those who use these relationships need the exact value of the parameters, which can only be obtained by the analysis of the test results. Our test results showed that the equations in the literature had to be treated with respect to the input parameters.

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