



Article Continuous-Time Perfect Control Algorithm—A State **Feedback Approach**

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Abstract: In this paper, a new approach to the continuous-time perfect control algorithm is given. Focusing on the output derivative, it is shown that the discussed control law can effectively be implemented in terms of state-feedback scenarios. Moreover, the application of nonunique matrix inverses is also taken into consideration during the perfect control design process. Simulation examples given within this work allow us to showcase the main properties obtained for continuoustime perfect control closed-loop plants.

Keywords: perfect control; continuous-time systems; state-space description; state feedback; nonunique matrix inverses; stability

1. Introduction

The design of control systems for multivariable plants is a wide research topic that is still subject to considerable worldwide interest. Systems with multiple input and output variables often deliver challenges and opportunities that are not available in the Single-Input Single-Output (SISO) plants. For example, in the Multi-Input Multi-Output (MIMO) approach, the Inverse Model Control (IMC) extends the potential of nonunique solutions for a given problem [1–7].

One of the most popular inverse model controls is the Minimum Variance Control (MVC) used for both discrete and continuous plants [8–11]. On the other hand, the perfect control, being a deterministic equivalent of MVC, has been developed mainly for the discrete-time framework. Therefore, some efforts to define the Continuous-Time Perfect Control (CTPC) have recently been undertaken [12]. The preliminary studies have shown that this topic is yet an unexplored area of control theory. Thus, a new approach to CTPC design is confirmed within this paper.

The key part of perfect control synthesis for multivariable systems is the proper solution of inverse problem derived from the control law. For decades, the unique Moore-Penrose inverse has been used due to its minimum-norm property [13–18]. On the other hand, admission of the nonunique inverses has resulted in measurable benefits in terms of control speed, energy minimization, or control stability [19]. Thus, in this paper, a short comparison of two selected inverses, namely, unique (Moore–Penrose) right T-inverse and nonunique right σ -inverse, is shown. Of course, this comparison will be conducted in the context of CTPC employment.

Stability of such a control scenario will be analyzed with the application of well-known state-feedback or pole-placement methods [20,21]. Additionally, in contrast to the preliminary studies [12], here, the state-feedback form is given in a straightforward manner, allowing us to obtain the closed-loop poles immediately for all classes of LTI MIMO state-space systems [22]. Of course, as in the discrete-time equivalent, the stability should be understood in terms of the Bounded-Input Bounded-State (BIBS) or input-to-state approach [23].

However, both control law analysis and synthesis will be made using continuoustime solvers provided by some incremental time-originated step Δt , which can be found,



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e.g., in the Matlab environment [24,25]. The influence of the time interval will be taken into account during the entirety of the process of perfect control realization and validation.

This paper is organized as follows. After a short introduction, the system description is given. In Section 3, a brief reminder considering unique and nonunique inverses of nonsquare matrices is presented. The crucial CTPC law is given in Section 4, together with the full calculation procedure covering its closed-loop behavior. Discussion over stability of such a perfect control plant is made with respect to the solver-related peculiarities, leading to interesting observations shown in Section 5. Simulation study given in the penultimate section shows that the proposed control law can successfully be applied in the Matlab/Simulink environment. Finally, the conclusions and open problems are manifested.

2. System Representation

As the perfect control algorithm reveals its characteristics for multivariable plants, the system is considered here as an LTI state-space plant described in the continuous-time domain as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), & \mathbf{x}(0) = \mathbf{x_0} \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) & , \end{cases}$$
(1)

where $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times n_u}$ and $\mathbf{C} \in \mathbb{R}^{n_y \times n}$ are system input and output matrices, respectively. Moreover $\mathbf{u}(t)$, $\mathbf{y}(t)$, and $\mathbf{x}(t)$ are n_u -input, n_y -output, and n-state vectors, respectively. As in all state-space-based scenarios, we additionally define the state initial condition \mathbf{x}_0 vector equivalent to the $\mathbf{x}(t)$ for t = 0.

Remark 1. Due to the nature of the perfect control law, we rather chose to omit the systems with $n_u < n_y$. For such plants, it is shown—e.g., in Ref. [26]—that the perfect control cannot be established.

Before we continue of the perfect control algorithm, let us recall the concept concerning nonunique matrix inverses. These inverses will later be used in order to establish the perfect control law; thus, inverse-oriented preliminaries are given in the next section.

3. (Non)unique Right Inverses

The idea of nonsquare matrix right inverses is built upon the statement of finding such a matrix \mathbf{M}^{R} that for a given matrix \mathbf{M} , the following relation

$$\mathbf{M}\mathbf{M}^{\mathrm{R}} = \mathbf{I},\tag{2}$$

holds. For nonsquare matrices, this can be accomplished using several frameworks, resulting in both unique and nonunique solutions [27,28]. The most popular is the unique minimum-norm Moore–Penrose *T*-inverse as follows:

$$\mathbf{M}_0^{\mathrm{R}} = \mathbf{M}^{\mathrm{T}} (\mathbf{M} \mathbf{M}^{\mathrm{T}})^{-1}, \tag{3}$$

where the full-rank matrix **M** needs to have more columns than rows to attain the property called right-invertibility. On the other hand, there is also a wide range of nonunique inverses available during the perfect control design. For instance, the right σ -inverse can be given in the following form [29]

$$\mathbf{M}_{\sigma}^{\mathrm{R}} = \beta^{\mathrm{T}} (\mathbf{M} \beta^{\mathrm{T}})^{-1}, \tag{4}$$

where β —being of the same sizes as **M**—provides a set of the so-called degrees of freedom. Of course, the additional limitation is only that the matrix (**M** β ^T) is expected to be a full rank. The application of right σ -inverse has already proven its usefulness during the perfect control design. For instance, in Refs. [19,28,30], it was shown that the proper degree-of-freedom selection enables to obtain, e.g., pole-free, robust, or minimum-energy perfect control behavior. However, the said advantages were obtained for discrete-time plants only;

thus, let us now continue with the perfect control algorithm for continuous-time systems. Therefore, this control strategy, defined in terms of the state-feedback system, is proposed in the next section. Observe that the discussed new approach is strictly dedicated to any MIMO plant with $n_u \ge n_y$. The predecessor provided by Ref. [12] has only employed the systems with right-invertible state-space-related matrices **B**. The additional improvement, in comparison to the algorithm given in Ref. [12], is the fact that there is no longer a need to switch between two operating-time-originated ranges.

4. New Perfect Control Law

The perfect control algorithm has already been developed for discrete-time systems. The main property obtained for such procedure is minimum possible control error that vanquishes immediately after a single simulation step (for plants with unit state delay). In this particular work, we are targeting similar plant behavior, but for systems described in the continuous-time state-space framework. In both cases, the main concern is to minimize a performance index, which, for the continuous-time scenario, can be described as follows:

$$J = ||\mathbf{y}_{ref}(t) - \mathbf{y}(t)||_2, \tag{5}$$

where symbols $\mathbf{y}_{ref}(t)$ and $\mathbf{y}(t)$ denote the reference value/setpoint and output of a system, respectively. Having such an index, we can attempt to obtain a similar minimum-error property as resulted in the discrete-time approach [31]. The study presented here is based on the important derivative of vector

$$\dot{\mathbf{y}}(t) = \mathbf{C}\dot{\mathbf{x}}(t),\tag{6}$$

allowing us to determine the operation of output variable. Now, aiming for the perfect control strategy with maximum accuracy (in terms of assumed norm (5)), we can introduce an equation minimizing the control error in the following manner

$$\dot{\mathbf{y}}(t) = \mathbf{y}_{ref}(t + \Delta t) - \mathbf{y}(t).$$
(7)

Of course, in such a scenario, the current control error is driven to zero immediately. However, at this moment, we need to acknowledge the fact, that solvers implemented in the Matlab environment are in fact based on nonzero fixed or adaptive step time Δt ; thus, we can unarguably write the subsequent formula

$$\dot{\mathbf{y}}(t) = \frac{d\mathbf{y}(t)}{dt} \approx \frac{\mathbf{y}(t + \Delta t) - \mathbf{y}(t)}{\Delta t}.$$
(8)

Naturally, this assumption is valid under $\Delta t \rightarrow 0$. It is clear now that the main aim here is to obtain system output behavior described by the following relation

$$\dot{\mathbf{y}}(t) = \frac{\mathbf{y}_{\mathsf{ref}}(t + \Delta t) - \mathbf{y}(t)}{\Delta t},\tag{9}$$

so the output error shall disappear right after the step time that is inherent to solver parameters. Once more, it is needed to emphasize that guaranteeing the output derivative equal to the current control error will minimize the performance index given in Equation (5). Now, using the output description from Equation (1), one can write the following outcome

$$\mathbf{C}\dot{\mathbf{x}}(t) = \frac{\mathbf{y}_{ref}(t + \Delta t) - \mathbf{C}\mathbf{x}(t)}{\Delta t},$$
(10)

where all parameters are known for systems with given initial conditions. Based on the above, one can obtain the perfect control formula by the proper mathematical manipulations

$$\mathbf{C}(\mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t))\Delta t = \mathbf{y}_{ref}(t + \Delta t) - \mathbf{C}\mathbf{x}(t).$$
(11)

Now, the issue is to extract the control signal $\mathbf{u}(t)$ from the above presented expression. This involves the crucial matrix inverse providing an infinite number of possible solutions for the discussed complex problem. Finally, let us introduce the perfect control law for continuous-time LTI MIMO state-space systems in the form of

$$\mathbf{u}(t) = -(\mathbf{C}\mathbf{B})^{\mathrm{R}}\mathbf{C}(\mathbf{A} + \mathbf{I}\frac{1}{\Delta t})\mathbf{x}(t) + (\mathbf{C}\mathbf{B})^{\mathrm{R}}\frac{1}{\Delta t}\mathbf{y}_{\mathrm{ref}}(t + \Delta t),$$
(12)

where superscript 'R' denotes any right inverse, particularly the right σ - and *T*-inverse covered in Section 3. Of course, for a zero reference value, the perfect control formula reduces to the perfect regulation as follows:

$$\mathbf{u}(t) = -(\mathbf{C}\mathbf{B})^{\mathrm{R}}\mathbf{C}(\mathbf{A} + \mathbf{I}\frac{1}{\Delta t})\mathbf{x}(t).$$
(13)

Note that the dynamics of a plant can similarly be examined for both zero- and nonzero-referenced setpoints.

Remark 2. Interestingly, the discrete-time perfect control scenario implies the input formula defined by

$$\mathbf{u}(k) = -(\mathbf{CB})^{\mathbf{R}} \mathbf{CAx}(k). \tag{14}$$

The just obtained similarity is remarkable; however, the continuous-time perfect control law converges only for $\Delta t \rightarrow \infty$ *.*

Referring to the conducted investigations, the formal conclusion is given below.

Theorem 1. Let us consider the LTI MIMO systems (1) with $n_u \ge n_y$ described in the continuoustime state-space framework. The continuous-time perfect control law for such a class of plants is defined as follows:

$$\mathbf{u}(t) = -(\mathbf{C}\mathbf{B})^{\mathrm{R}}\mathbf{C}(\mathbf{A} + \mathbf{I}\frac{1}{\Delta t})\mathbf{x}(t) + (\mathbf{C}\mathbf{B})^{\mathrm{R}}\frac{1}{\Delta t}\mathbf{y}_{\mathrm{ref}}(t + \Delta t),$$
(15)

under an arbitrary initial condition x_0 .

Proof. Immediately after combining the system's description (1) with expression (15), we receive

$$\dot{\mathbf{y}}(t) = -\mathbf{C}\frac{1}{\Delta t}\mathbf{x}(t) + \frac{1}{\Delta t}\mathbf{y_{ref}}(t+\Delta t),$$
(16)

which holds the examined perfect-control-oriented target in the form of

$$\dot{\mathbf{y}}(t) = \frac{\mathbf{y}_{ref}(t + \Delta t) - \mathbf{y}(t)}{\Delta t},\tag{17}$$

provided by $\Delta t \rightarrow 0$. \Box

Remark 3. Naturally, the condition (17) guarantees a possible minimum perfect control-related error derived from the step time Δt tending to zero. Thus, there is no dependence between the plant inertia and the time-step-size needed to reach the new reference value. Through the certain small Δt , the corresponding negligible steady-state error is obtained, which ultimately fades in the case of discrete-time perfect control instances.

Remark 4. It seems that operation $\Delta t \rightarrow 0$ generates the excessive energy of the perfect control input runs. This intriguing issue should be treated as an open problem.

Observe that the controlled output will stay at the reference value in any deterministic case, independently from the dynamics concealed within closed-loop equations. Thus, the stability of CTPC will be discussed in the next section.

5. Stability Properties of the New Perfect Control Law

The stability of perfect control algorithm has already been discussed widely for a class of discrete-time systems [26]. In the literature, it can be found that the stability property should be understood in terms of the BIBS approach. It is caused mainly by the fact that the perfect control output remains stable even for plants with closed-loop poles located outside the stable region. Thus, the input-to-state characteristics are the main concern here. Of course, according to Equation (13), the CTPC algorithm can be treated as the inverse model control-oriented state-feedback system occupied by

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t),\tag{18}$$

where $\mathbf{K} = (\mathbf{CB})^{R} \mathbf{C} (\mathbf{A} + \mathbf{I} \frac{1}{\Delta t})$, whose closed-loop control poles can be calculated in accordance with the following formula

$$det(\mathbf{A} - \mathbf{B}\mathbf{K}) = 0. \tag{19}$$

It is clear that the discussed stability depends on the part connecting vectors $\mathbf{x}(t)$ and $\dot{\mathbf{x}}(t)$; thus, the closed-loop poles can be obtained. The static non-autoregressive part disappears as soon as the output reaches its reference value, which essentially occurs after a single calculation step Δt . In conclusion, the stability of the closed-loop CTPC law is determined by the roots of characteristic equation as follows:

$$det(s\mathbf{I} - \mathbf{A} + \mathbf{B}(\mathbf{CB})^{\mathbf{R}}\mathbf{C}(\mathbf{A} + \mathbf{I}\frac{1}{\Delta t})) = 0.$$
 (20)

Here again, we need to emphasize that the numerical solution of stability problem can be considered in the context of the inverse matrix calculation, with an infinite number of possible outcomes—even with the ability to almost arbitrary allocation of the number of zero and nonzero closed-loop poles.

Remark 5. The pole-placement feature is connected with the rank of the closed-loop perfect control system matrix. The mentioned rank seems to be dependent on the size differences derived from **A**,**B**,**C** matrices. This intriguing phenomenon is still under investigation.

Interestingly, in comparison with the discrete-time instances, the zero closed-loop poles are not obtained here. This is caused by the fact that the additional part of state feedback $(I\frac{1}{\Delta t})$ entails a drift of the CTPC algorithm poles that were originally equal to zero. In this case, the zero eigenvalues of the closed-loop system matrix are transformed to those connected with the solver-derived Δt parameter. The influence of solver time seems to be an obvious consideration when aiming for proper output behavior. If the goal is to obtain the zero control error in a single solver step, it is clear that if the solver step takes a shorter time Δt , then the dynamics applied to the output and state variables should be faster to overcome the shorter calculation interval.

In order to explain the crucial role of the step time Δt , the closing statements are formulated below.

Theorem 2. The continuous-time perfect control law as in Equation (12) should be understood in terms of the single step time Δt tending to zero.

Proof. Immediately after substitution of formula (12) to right-invertible nondelayed plants (1), we obtain the expression

$$\dot{\mathbf{y}}(t) = \frac{1}{\Delta t} (\mathbf{y}_{ref}(t + \Delta t) - \mathbf{y}(t)),$$
(21)

which can be transformed, according to the relation (7), to

$$\left(\frac{\Delta t - 1}{\Delta t}\right) [\mathbf{y}_{ref}(t + \Delta t) - \mathbf{y}(t)] = \mathbf{0}.$$
(22)

For $\Delta t \to 0$, we receive $\mathbf{y}(t) \to \mathbf{y_{ref}}(t + \Delta t)$, which ends the proof. \Box

Corollary 1. The perfect regulation (13) having $\mathbf{y}_{ref}(t) = \mathbf{0}$ occurs iff the relation

$$\Delta t \mathbf{y}(t) = \mathbf{y}(t), \tag{23}$$

determined by $\Delta t \rightarrow 0$, $\mathbf{y}(t) \rightarrow 0$, holds.

Remark 6. The stability of the CTPC should be considered in the same manner.

Remark 7. It would be interesting to extend the new theory to the case of plants involving the time-delay-originated $\Gamma(s) = e^{-sT_0}$ term defined in the Laplace domain.

Having defined all crucial properties of CTPC systems, let us now continue with a simulation study. The proposed control schemes will be tested together with predicted closed-loop poles behavior. Instances made using the Matlab/Simulink environment are presented in the next section.

6. Simulation Examples

It has been shown in previous studies concerning the perfect control simulation for discrete systems that the application of nonunique right inverses is useful in the stabilization of nonminimum-phase plants [32]. In this section, we expect to observe similar properties for continuous-time objects. Moreover, there is a need to assess the system stability, especially in the context of the closed-loop poles presence. Thus, three different instances will be considered in this section in order to examine all analyzed behaviors. At this point, it is crucial to emphasize that the Matlab/Simulink environment was used in this study. Moreover, the sampling time of the *ode*1 solver was set to Δt for a reasonable data collection.

6.1. Two-By-One System

In the first simulation scenario, we consider the continuous-time LTI system with two inputs, one output, and two state variables described in the state-space framework that is under the following matrices $\mathbf{A} = \begin{bmatrix} -0.21 & -0.40 \\ 0.20 & -0.37 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1.10 & -0.70 \\ 0.50 & -1.00 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} -0.90 & -0.40 \end{bmatrix}$ and initial condition $\mathbf{x}_0^{\mathrm{T}} = \begin{bmatrix} 3 & -5 \end{bmatrix}$. Additionally, we declare that the simulation step time is equal to $\Delta t = 0.1$ s. As we can see, this plant is stable, since the open-loop eigenvalues are equal to $s_{1,2} = -0.2900 \pm 0.2713i$. For such a system, let us design the perfect control scheme guaranteeing the law from (12). In this scenario, we use the minimum-norm right *T*-inverse of (3).

As we can observe, a steady-state zero-error output is obtained here just after a single calculation step (see Figure 1). Once more, the calculation step Δt shall be recalled again,

as it affects the possible solution of stability criteria mentioned in the previous section. Thus, we obtain the closed-loop perfect control system matrix

$$\mathbf{A} - \mathbf{B}\mathbf{K} = \begin{bmatrix} -7.4965 & -3.2618\\ -5.6328 & -2.6609 \end{bmatrix},$$
(24)

and, according to Equation (20), our poles are equal to $s_1 = -10.0000$ and $s_2 = -0.1574$. Of course, with such a solution, this system is stable.



Figure 1. Perfect control plots: *T*-inverse, case $\Delta t = 0.1$ s.

The resulted stable state and control signals can be observed in Figure 1.

Remark 8. Alternatively, with the use of the classical closed-loop poles definition from (19), the eigenvalues $s_1 = 0$ and $s_2 = -0.1574$ are obtained. Moreover, for the simulation example repeated with calculation step time $\Delta t = 0.01s$, we receive $s_1 = -100.0000$ and $s_2 = -0.1574$. The connection between $\frac{1}{\Delta t}$ parameter and closed-loop poles is obvious here. It is clear that more aggressive calculations entail faster dynamic if the goal is to obtain the possibly smallest control time. This faster dynamics is mapped by the faster closed-loop perfect control poles.

In this scenario, a relatively simple second-order plant was considered in order to clearly introduce and show all peculiarities associated with the innovative continuous-time perfect control systems. Let us now continue this study with some more complex instances involving the nonunique generalized right matrix inverse.

6.2. Three-By-One System

Consider again the state-space continuous-time system, but now with three inputs, two state variables, and single output variable. Such a plant is described with the following matrices: $\mathbf{A} = \begin{bmatrix} 0.4000 & 0.1200 \\ 0.7000 & 1.1000 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & -1.1000 & 0.5000 \\ 0.4000 & 1.0000 & 1.8000 \end{bmatrix}$, and $\mathbf{C}^{\mathrm{T}} = \begin{bmatrix} 0.2000 \\ 0.6000 \end{bmatrix}$. Moreover, the initial condition is $\mathbf{x}_0 = \begin{bmatrix} 4 & -2 \end{bmatrix}^{\mathrm{T}}$ and $\Delta t = 0.2s$. With the application of the minimum-norm right *T*-inverse, we obtain the unstable perfect control system with the corresponding signals presented in Figure 2. In this scenario, the closed-loop poles obtained by the expression (20) are equal to $s_1 = 0.3307$ and $s_2 = -5.0000$, which also confirm the unstable behavior. Interestingly, the output variable remains at the reference value despite the obvious state instability. However, this feature was expected since, in the discrete-time manner, the perfect control algorithm has already revealed similar properties. It is also worth mentioning that the energy of control signal is equal $E = ||\mathbf{u}(t)||_2 = 2.2876 \cdot 10^6$.



Figure 2. Perfect control plots: *T*-inverse, case $\Delta t = 0.2$ s.

On the other hand, with the application of right σ -inverse with degrees of freedom in the form of

$$\beta = | -9.1000 - 0.3000 2.0000 |, \qquad (25)$$

the following stable state and control signals, presented in Figure 3, are obtained.



Figure 3. Perfect control plots: σ -inverse, case $\Delta t = 0.2$ s.

In this examination, the closed-loop perfect control poles are equal to $s_1 = -5.0000$ and $s_2 = -5.4748$, whilst the control energy equals $E = 6.0802 \cdot 10^6$. Here, the stability of closed-loop system is earned with higher energy expenditure. Of course, with a wider time horizon, the energy outcome will eventually favor the stable solution, as the steady state is obtained here after five calculation steps. Therefor, a stable continuous-time perfect control can be established using the generalized nonunique matrix inverse. This feature is coherent with results received for discrete-time systems.

7. Conclusions and Open Problems

In this paper, the issue concerning the continuous-time perfect control algorithm is discussed. It is shown that by using the proper estimation of output derivative, there is a possibility to derive the output variables right to the reference value in almost no time. Interestingly, the obtained dynamics are convergent with the time interval used by solvers implemented in the Matlab/Simulink environment. The stability-oriented simulation examples show that the closed-loop perfect control poles can be assigned in an arbitrary manner with respect to the crucial solver-originated step time and type of applied generalized right inverse. Finally, it would be interesting to extend the presented theory to a class of real-life systems having a different time delay blurred by the white noise disturbance. The energy-oriented studies also seem to be welcomed.

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Abbreviations

The following abbreviations are used in this manuscript:

- BIBS Bounded-Input Bounded-State
- CTPC Continuous-Time Perfect Control
- IMC Inverse Model Control
- MIMO Multi-Input Multi-Output
- MVC Minimum Variance Control
- SISO Single-Input Single-Output

References

- Molloy, T.L.; Inga, J.; Flad, M.; Ford, J.J.; Perez, T.; Hohmann, S. Inverse Open-Loop Noncooperative Differential Games and Inverse Optimal Control. *IEEE Trans. Autom. Control.* 2020, 65, 897–904. [CrossRef]
- Cao, F.; Yang, T.; Li, Y.; Tong, S. Adaptive Neural Inverse Optimal Control for a Class of Strict Feedback Stochastic Nonlinear Systems. In Proceedings of the 2019 IEEE 8th Data Driven Control and Learning Systems Conference (DDCLS), Dali, China, 24–27 May 2019; pp. 432–436.
- Ma, H.; Chen, M.; Wu, Q. Disturbance Observer-Based Inverse Optimal Tracking Control of the Unmanned Aerial Helicopter. In Proceedings of the 2019 IEEE 8th Data Driven Control and Learning Systems Conference (DDCLS), Dali, China, 24–27 May 2019; pp. 448–452.
- 4. Asadzadeh, M.Z.; Raninger, P.; Prevedel, P.; Ecker, W.; Mücke, M. Inverse model for the control of induction heat treatments. *Materials* **2019**, *12*, 2826. [CrossRef] [PubMed]
- 5. Li, Z.; Chen, H.; Wang, C.; Xue, K. Inverse model control for a quad-rotor aircraft using TS-fuzzy support vector regression. *J. Harbin Inst. Technol.* **2017**, *24*, 73–79.
- Zhiteckii, L.S.; Solovchuk, K.Y. Analysis of multivariable regulation systems using pseudo-inverse model-based controllers. In Proceedings of the 2017 IEEE First Ukraine Conference on Electrical and Computer Engineering (UKRCON), Kyiv, Ukraine, 29 May–2 June 2017; IEEE: Piscataway, NJ, USA 2017; pp. 894–899.
- 7. Li, Y.; Yao, Y.; Hu, X. Continuous-time inverse quadratic optimal control problem. Automatica 2020, 117, 108977. [CrossRef]
- 8. Filip, I.; Dragan, F.; Szeidert, I.; Albu, A. Minimum-Variance Control System with Variable Control Penalty Factor. *Appl. Sci.* 2020, 10, 2274. [CrossRef]
- 9. Lima, M.A.; Trierweiler, J.O.; Farenzena, M. A new approach to estimate the Minimum Variance Control law for Nonminimum phase Multivariable Systems. *IFAC-PapersOnLine* **2019**, *52*, 886–891. [CrossRef]
- Dube, D.Y.; Patel, H.G. Discrete time minimum variance control of satellite system. In International Conference on Mathematical Modelling and Scientific Computation; Springer: Singapore, 2018; pp. 337–346.
- 11. Silveira, A.; Silva, A.; Coelho, A.; Real, J.; Silva, O. Design and real-time implementation of a wireless autopilot using multivariable predictive generalized minimum variance control in the state-space. *Aerosp. Sci. Technol.* **2020**, *105*, 106053. [CrossRef]
- 12. Majewski, P.; Hunek, W.; Krok, M. Perfect Control for Continuous-Time LTI State-Space Systems: The Nonzero Reference Case Study. *IEEE Access* 2021, *9*, 82848–82856. [CrossRef]
- 13. Liang, M.; Zheng, B. Further results on Moore–Penrose inverses of tensors with application to tensor nearness problems. *Comput. Math. Appl.* **2019**, *77*, 1282–1293. [CrossRef]
- 14. Shen, S.; Liu, W.; Feng, L. Inverse and Moore-Penrose inverse of Toeplitz matrices with classical Horadam numbers. *Oper. Matrices* **2017**, *11*, 929–939. [CrossRef]
- 15. Zhang, W.; Wu, Q.J.; Yang, Y.; Akilan, T. Multimodel Feature Reinforcement Framework Using Moore-Penrose Inverse for Big Data Analysis. *IEEE Trans. Neural Netw. Learn. Syst.* 2020. [CrossRef]
- 16. Zhang, B.; Uhlmann, J. A Generalized Matrix Inverse with Applications to Robotic Systems. arXiv 2018, arXiv:1806.01776.
- 17. Kafetzis, I.S.; Karampetakis, N.P. On the algebraic structure of the Moore–Penrose inverse of a polynomial matrix. *IMA J. Math. Control. Inf.* **2021** doi:10.1109/CoDIT.2019.8820625. [CrossRef]
- 18. Nayan Bhat, K.; Karantha, M.P.; Eagambaram, N. Inverse complements of a matrix and applications. *J. Algebra Appl.* **2020**, 2150144. [CrossRef]

- Krok, M.; Hunek, W.P. Pole-Free vs. Minimum-Norm Right Inverse in Design of Minimum-Energy Perfect Control for Nonsquare State-Space Systems. In *Biomedical Engineering and Neuroscience, Proceedings of the 3rd International Scientific Conference on Brain-Computer Interfaces, BCI 2018, Opole, Poland, 13–14 March 2017*; Advances in Intelligent Systems and Computing; Springer: Cham, Switzerland, 2017; pp. 184–194. [CrossRef]
- De la Sen, M. On pole-placement controllers for linear time-delay systems with commensurate point delays. *Math. Probl. Eng.* 2005, 2005, 123–140. [CrossRef]
- 21. Cacace, F.; Conte, F.; Germani, A. State feedback stabilization of linear systems with unknown input time delay. *IFAC-PapersOnLine* **2017**, *50*, 1245–1250. [CrossRef]
- 22. Uthman, A.; Sudin, S. Antenna Azimuth Position Control System using PID Controller & State-Feedback Controller Approach. *Int. J. Electr. Comput. Eng. (IJECE)* 2018, *8*, 1539–1550.
- Cai, X.; Bekiaris-Liberis, N.; Krstic, M. Input-to-state stability and inverse optimality of predictor feedback for multi-input linear systems. *Automatica* 2019, 103, 549–557. [CrossRef]
- Postawa, K.; Szczygieł, J.; Kułażyński, M. A comprehensive comparison of ODE solvers for biochemical problems. *Renew. Energy* 2020, 156, 624–633. [CrossRef]
- 25. Torres-Del Carmen, F.d.J.; Jaramillo-Gernández, R.; Díaz-Sánchez, A.; Núñez-Altamirano, D.A. Comparison of numerical methods in code as solvers for simulation of robotic systems. *J. Appl. Comput.* **2020**, 4–15. [CrossRef]
- Hunek, W.P.; Krok, M. Towards a new minimum-energy criterion for nonsquare LTI state-space perfect control systems. In Proceedings of the 5th International Conference on Control, Decision and Information Technologies, CoDIT 2018, Thessaloniki, Greece, 10–13 April 2018; pp. 122–127. [CrossRef]
- 27. Ben-Israel, A.; Greville, T.N.E. Generalized Inverses, Theory and Applications, 2nd ed.; Springer: New York, NY, USA, 2003.
- Hunek, W.P.; Krok, M. A study on a new criterion for minimum-energy perfect control in the state-space framework. *Proc. Inst. Mech. Eng. Part J. Syst. Control. Eng.* 2019, 233, 779–787. [CrossRef]
- Hunek, W.P. An Application of New Polynomial Matrix *σ*-Inverse in Minimum-Energy Design of Robust Minimum Variance Control for Nonsquare LTI MIMO Systems. *IFAC-PapersOnLine* 2015, *48*, 150–154. [CrossRef]
- Hunek, W.P.; Krok, M. Pole-free perfect control for nonsquare LTI discrete-time systems with two state variables. In Proceedings of the 2017 13th IEEE International Conference on Control Automation (ICCA), Ohrid, Macedonia, 3–6 July 2017; pp. 329–334. [CrossRef]
- 31. Krok, M.; Hunek, W.P. Pole-Free Perfect Control: Theory vs. Simulation Examples. In Proceedings of the 23rd International Conference on Methods & Models in Automation & Robotics (MMAR'18), Międzyzdroje, Poland, 27–30 August 2018. [CrossRef]
- 32. Hunek, W.P. Towards a General Theory of Control Zeros for LTI MIMO Systems; Opole University of Technology Press: Opole, Poland, 2011.