



Article Analysis of Dynamic Characteristics of Small-Scale and Low-Stiffness Ring Squeeze Film Damper-Rotor System

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Abstract: Rotor machinery supports are susceptible to fracture arising from multi-period or irregular transient vibration and even the inconspicuous fatigue phenomenon. The novel dynamic model of a rotor mounted on the floating ring squeeze film damper (SFD) was developed. The proposed SFD implements low stiffness and small scale to overcome the deficiency. Based on the theory of hydrodynamic lubrication, the Reynolds equations on the working principle of the floating ring are established. Then, the dynamic characteristics of the rotor system during maneuvers, with the floating ring SFD supports, are subsequently examined by adopting the finite difference method. In addition, the oil film whirl mechanism of the floating ring SFD is demonstrated according to the transient analysis of the fluid–structure interaction model. The results of the SFD simulation reveal that, with increasing eccentricity ratio, both the inner and outer oil film pressure tend to be larger due to the shrinkage of the effective coverage of oil film. The maximum oil film pressure and bearing capacity increase nonlinearly within a certain eccentricity ratio range. Through the comparisons of the results, the vibration suppression effects of the proposed SFD are analyzed. This work will provide the practical reference for the dynamic design of the rotor support system.

Keywords: squeeze film damper; rotor; fluid-structure interaction; small scale; low stiffness

1. Introduction

The working speed of modern aeroengines is generally above 10,000 rpm, while that of some small engines is as high as 40,000–50,000 rpm. Operation at high speeds induces severe dynamic loading with large amplitude journal motions at bearing supports [1]. Simultaneously, there are some large-amplitude response issues, which may cause serious accidents such as friction between the rotors and rotor fatigue cracks [2]. In order to meet the utmost desire of adequate critical-speed margin and high stable rotor speed in modern small aeroengines, squeeze film damper (SFD) bearings are an effective solution to add viscous damping that aids to attenuate vibration and also serve to isolate the rotors from the aeroengine frame [3]. Their simplicity of construction and consequent robustness means that they are commonplace in modern aircraft gas turbine engines [4,5]. Although SFDs have been frequently used in the turbo-machinery industry, the non-synchronous vibrations can result in some problems of fatigue [6,7]. Thus, it is a tremendous requirement to investigate and improve the ring SFD-rotor system dynamics for vibration attenuation of high-speed small rotating machinery.

The typical and more practical approach in SFD modeling is based on the incompressible Reynolds lubrication equation to describe the relationship between the pressure in a fluid film and the journal motion. Chen et al. [8] carried out the bifurcation behaviors of a rigid rotor-squeeze film damper considering the effect of fluid inertia. By changing



Citation: Shi, M.; Yang, Y.; Deng, W.; Wang, J.; Fu, C. Analysis of Dynamic Characteristics of Small-Scale and Low-Stiffness Ring Squeeze Film Damper-Rotor System. *Appl. Sci.* 2022, *12*, 7167. https://doi.org/ 10.3390/app12147167

Received: 16 May 2022 Accepted: 13 July 2022 Published: 16 July 2022

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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). the value of the Reynolds number that reflects the fluid inertia of SFD, the hysteresis set is moved obviously. Hamzehlouia [9] proposed the pressure distribution model of SFD with finite length, and the generalized Reynolds equation of hydrodynamic pressure distribution considering that the inertial effect is obtained. According to the simulation results, the pressure distribution and fluid film reaction forces are significantly influenced by fluid inertia effects even at small and moderate Reynolds numbers. As in a much earlier work [10], the pressure distribution in the full oil film was then described by a modified Reynolds equation. In Ref. [11], modified Reynold equation with short damper approximation was used to derive the SFD forces for 2π -film. Groves [12] proposed the use of Chebyshev polynomial fits to identify a finite difference (FD) solution of the incompressible Reynolds equation. Based on the complex fluid–structure interaction phenomenon inside the ring SFD, the pressure distribution of the damper oil film and the phenomenon of oil film whirl are worthy of further study for ring SFDs.

To improve the performance of SFDs, various new design concepts had been proposed. In Ref. [13], Han proposed a calculation procedure to investigate the dynamic characteristics and response of an ERSFD-supported rotor system. The study reveals that three effects of elastic cause the better dynamical performance of ERSFD than SFD and rigid ring squeeze film damper. Chen et al. [14–18] analyzed the influence of SFD oil film damping, oil film stiffness, and elastic ring stiffness on the rotor system through theoretical modeling, numerical calculation, finite element analysis, and so on. Andrés [19] studied the response of a rigid rotor system supported by SFD under the short-bearing approximation theory. The study showed that the oil film of SFD can withstand a certain negative pressure. Ding [20] studied the damping mechanism of SFD and showed that through the elastic ring deformation, the nonlinearity of SFD can be effectively lightened. Cao [21] established a rotor system model considering the nonlinear characteristics of SFD and analyzed the influence of SFD on the bistable response and non-coordinated precession response of the rotor system. Liu [22,23] used a rotor tester with an SFD to study the influence of the static eccentricity of SFD on the rotor vibration response and combined the design of SFD with the rotor dynamics design. Then, the design method of the aeroengine SFD was proposed. Zhou [24–26] used a two-way excitation tester to test the influence of the oil film width and oil film gap of SFD on its damping coefficient. In addition, experiments and numerical simulations on the damping mechanism of the floating ring SFD and the dynamic response of the rotor system were also carried out. As mentioned above, most works on ring deformation of SFD are based on vibration response analysis and identification. It is hard to observe the oil film distribution, bearing capacity, and deflection angle through experiments. Thus, this paper develops the dynamic analysis of the small-scale and low-stiffness floating ring SFD.

The SFD has highly nonlinear characteristics, which creates high-amplitude bi-stable and nonsynchronous responses for the rotors. The large-amplitude response can result in rotor rubs and fatigue [27]. The rotating machinery support of the aeroengine will endure a severe bad working condition when the aircraft operates at high speeds. Correspondingly, the rotor machinery support is wrecked by fatigue fracture under the effect of multi-period or irregular transient vibration and no apparent phenomenon before fracture. It majorly depends on support damper stiffness and damper mass. Therefore, the objective of the present work is to examine the stability of the small-scale and low-stiffness floating ring SFD mounted on the rotor system.

The research of this paper addresses the dynamic analysis of the small-scale and low-stiffness floating ring SFD. It combines the relative strengths of the small scale and low stiffness and uses two techniques to overcome their deficiency. Based on the perspective of theory and simulation, the vibration attenuation mechanism and the oil film whirl mechanism of the model are revealed according to the analysis of the transient characteristics. The numerical results provide useful insights into the design and development of a floating ring SFD for rotating machinery that operates at high speeds.

2. Theoretical Method and Physical Model

- 2.1. Solution of Oil Film Reynold Equation
- 2.1.1. Reynolds Equation

The Reynolds equation of incompressible fluid in a cylindrical coordinate system is defined by the classical lubrication theory [20].

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}(h^3\frac{\partial p}{\partial\theta}) + \frac{\partial}{\partial z}(h^3\frac{\partial p}{\partial z}) = 6\mu\dot{\psi}\frac{\partial h}{\partial\theta} + \frac{\partial h}{\partial t}$$
(1)

where *R* is radius, *P* the oil film pressure, *h* the oil film clearance, μ the fluid viscosity, *z* and θ are the axial and circumferential coordinates. *e* and ψ are the journal eccentricity and deflection angle.

Figure 1 illustrates the positional relationship of the various parts of the floating ring bearing when the journal whirl is eccentric. The solid line is the initial position, the dashed line is the changed position, C_1 and C_2 are the initial clearances of the inner and outer oil films, $r(\theta)$ is the radial deformation of the floating ring, and h_1 and h_2 are the oil film clearances of the inner and outer layers, respectively. *e* is the eccentricity of the journal with respect to the bearing.



Figure 1. The swirling form of the floating ring squeeze film damper.

The clearance of the inner oil film can be estimated as

$$\begin{cases} h_1(\theta) = C_1 + e\cos\theta + r(\theta) \\ \frac{\partial h_1}{\partial \theta} = \frac{\partial r}{\partial \theta} - e\sin\theta \\ \frac{\partial h_1}{\partial t} = \frac{\partial r}{\partial t} + \dot{e}\cos\theta \end{cases}$$
(2)

The clearance of the outer oil film can be expressed as

$$\begin{cases} h_2(\theta) = C_2 - r(\theta) \\ \frac{\partial h_2}{\partial \theta} = -\frac{\partial r}{\partial \theta} \\ \frac{\partial h_2}{\partial t} = 0 \end{cases}$$
(3)

For the convenience of the finite difference method, introducing non-dimensional parameters $\lambda = 2z/l$, $H_i = h_i/C_1$, $P = pC_1^2/2\Omega\mu r^2$, l is the axial width of the floating ring. The influence of the deformation of the floating ring is ignored here. Equation (1) can be rewritten as follows:

$$\frac{\partial}{\partial\theta}(H^3\frac{\partial P}{\partial\theta}) + \left(\frac{2R}{l}\right)^2\frac{\partial}{\partial\lambda}(H^3\frac{\partial P}{\partial\lambda}) = 6\frac{\partial H}{\partial\lambda} + \frac{6}{Rc\dot{\psi}}\frac{\partial H}{\partial t}$$
(4)

Equations (2) and (3) are estimated as

$$\begin{cases} H_1(\theta) = 1 + w(\theta) + \varepsilon \cos \theta \\ H_2(\theta) = c_3 - w(\theta) \end{cases}$$
(5)

where $\varepsilon = e/C_1$, $c_3 = C_2/C_1$ is the eccentricity ratio and $w(\theta)$ is the deformation function of the floating ring.

2.1.2. Equation of Motion

The inner and outer oil film pressures in the oil cavity can be equivalent to the resultant force in two vertical directions, radial force F_e , and circumferential F_{ϕ} , which can be converted into horizontal force F_x and vertical force F_y through coordinate transformation. Assuming that a single-disk symmetrical rigid rotor is supported by floating ring SFDs at both ends, the equation for the system dynamics becomes [20]

$$\begin{cases} m\ddot{x} + k_1 x + c_1 \dot{x} = em\Omega^2 \cos\Omega t + F_x \\ m\ddot{y} + k_2 y + c_2 \dot{x} = em\Omega^2 \sin\Omega t + F_y + G \end{cases}$$
(6)

where G = mg is half of the gravity of the turntable, k_1 and k_2 are the equivalent stiffness of the elastic support, c_1 and c_2 are equivalent damping which are under different pedestal contact statues of SFD, e is the eccentricity of the disc, and ω is the rotation without measuring the excitation frequency. F_x and F_y are oil film forces along x and y directions. If the displacement and time quantities $\tau = \omega t$ and X = 2x/l are introduced, and the simplified Equation (6) is substituted into the dimensionless system dynamics equation, it may be given that

$$\begin{cases} \ddot{X} + \omega_1^2 X + n_1 \dot{X} = \frac{2f}{ml} \cos \tau + \frac{2}{ml\Omega^2} F_x \\ \ddot{Y} + \omega_2^2 Y + n_2 \dot{Y} = \frac{2f}{ml} \sin \tau + \frac{2}{ml\Omega^2} F_y + \frac{2g}{l\Omega^2} \end{cases}$$
(7)
where $\omega_i = \sqrt{k_i/(m\Omega^2)}, n_i = c_i/(m\Omega^2), i = 1, 2.$

2.1.3. Solving Method

The internal surface of a cylindrical bearing is flattened into a two-dimensional rectangle and the rectangle plane is partitioned. Then, several meshes are divided along with the axial and circumferential directions. The pressure value on each grid node is constituted by the difference quotient of each order, which approximately replaces the partial derivative of each order in the Reynolds equation. The continuous partial differential equation is discretized into n algebraic equations. Then, the pressure value at each node is solved iteratively. The pressure value obtained by the solution can approximately represent the pressure distribution of the oil film in the oil cavity. Figure 2 depicts the obtained discrete grids of the oil film.



Figure 2. Discrete grid of the oil film.

The grid nodes are numbered sequentially in rows and columns. The number of rows along the axis is coded as *j*, and the number of columns along the circumferential direction is coded as *i*. It is uniformly divided into n grids along the axial direction, j = 1, 2, ..., n + 1, where j = 1 and j = n + 1 are the same points. It is evenly divided into *m* grids along the circumferential direction, i = 1, 2, ..., m + 1, where i = 1 and i = m + 1 are the same nodes. Then, the length of each grid in the axial direction is $\Delta \lambda = \lambda_j - \lambda_{j-1} = 2/n$ and the length of each grid in the circumferential direction is $\Delta \theta = \theta_i - \theta_{i-1} = 2\pi R/m$. The coordinate number of each node is (i, j), and the pressure value on each node is represented by $P_{i,j}$.

The partial derivative of the pressure on each node can be approximately equal to the pressure value of its adjacent nodes through the central difference, which can be deduced as

$$\left(\frac{\partial P}{\partial \theta}\right)_{i,j} = \frac{P_{i+1,j} - P_{i-1,j}}{2\Delta\theta}, \left(\frac{\partial P}{\partial\lambda}\right)_{i,j} = \frac{P_{i,j} - P_{i,j+1}}{2\Delta\lambda}$$
(8)

Equation (8) is the classical five-point difference format. In order to improve the calculation accuracy, it is required to use the half-step difference method, as shown in Figure 3.



Figure 3. Half-step difference method.

Equation (4) can be solved under the rectangular coordinate [28]

$$\frac{\partial}{\partial \theta} \left(H^3 \frac{\partial P}{\partial \theta} \right) + \left(\frac{2R}{l} \right)^2 \frac{\partial}{\partial \lambda} \left(H^3 \frac{\partial P}{\partial \lambda} \right) = 6 \frac{\partial H}{\partial \lambda} \tag{9}$$

The first derivative is obtained as

$$\begin{cases} \left(\frac{\partial P}{\partial \theta}\right)_{i,j} = \frac{P_{i+\frac{1}{2},j} - P_{i-\frac{1}{2},j}}{\Delta \theta}, \left(\frac{\partial P}{\partial \lambda}\right)_{i,j} = \frac{P_{i,j+\frac{1}{2}} - P_{i,j-\frac{1}{2}}}{\Delta \lambda} \\ \left(H^{3}\frac{\partial P}{\partial \theta}\right)_{i+\frac{1}{2},j} = H^{3}_{i+\frac{1}{2},j}\frac{P_{i+1,j} - P_{i,j}}{\Delta \theta}, \left(H^{3}\frac{\partial P}{\partial \theta}\right)_{i-\frac{1}{2},j} = H^{3}_{i+\frac{1}{2},j}\frac{P_{i,j} - P_{i-1,j}}{\Delta \theta} \\ \left(H^{3}\frac{\partial P}{\partial \theta}\right)_{i,j+\frac{1}{2}} = H^{3}_{i,j+\frac{1}{2}}\frac{P_{i,j+1} - P_{i,j}}{\Delta \lambda}, \left(H^{3}\frac{\partial P}{\partial \theta}\right)_{i,j-\frac{1}{2}} = H^{3}_{i,j-\frac{1}{2}}\frac{P_{i,j} - P_{i,j-1}}{\Delta \theta} \end{cases}$$
(10)

The second derivative gives the following expression as

$$\begin{cases} \left[\frac{\partial}{\partial\theta}(H^{3}\frac{\partial P}{\partial\theta})\right]_{i,j} = \frac{H^{3}_{i+\frac{1}{2},j}P_{i+1,j}+H^{3}_{i-\frac{1}{2},j}P_{i-1,j}-(H^{3}_{i+\frac{1}{2},j}+H^{3}_{i-\frac{1}{2},j})P_{i,j}}{(\Delta\theta)^{2}} \\ \left[\frac{\partial}{\partial\lambda}(H^{3}\frac{\partial P}{\partial\lambda})\right]_{i,j} = \frac{H^{3}_{i,j+\frac{1}{2}}P_{i,j+1}+H^{3}_{i,j-\frac{1}{2}}P_{i,j-1}-(H^{3}_{i,j+\frac{1}{2}}+H^{3}_{i,j-\frac{1}{2}})P_{i,j}}{(\Delta\lambda)^{2}} \\ \left(\frac{\partial H}{\partial\theta}\right) = \frac{H_{i+\frac{1}{2},j}-H_{i-\frac{1}{2},j}}{\Delta\theta} \tag{12}$$

 $(\frac{\partial}{\partial \theta}) = \Delta \theta$ (12) Substituting Equations (10)–(12) into (9), the Reynolds equation can be obtained as [20]

$$A_{i,j}(P_{i+1,j})_1 + B_{i,j}(P_{i-1,j})_1 + C_{i,j}(P_{i,j+1})_1 + D_{i,j}(P_{i,j-1})_1 + E_{i,j}(P_{i,j})_1 = (F_{i,j})_1$$
(13)

Among them, the following items are included

$$\begin{cases} A_{i,j} = \frac{H_{i+\frac{1}{2},j}^{3}}{\Delta\theta^{2}}, B_{i,j} = \frac{H_{i-\frac{1}{2},j}^{3}}{\Delta\theta^{2}}, C_{i,j} = \left(\frac{2R}{\Delta}\right)^{2} \frac{H_{i,j+\frac{1}{2}}^{3}}{\Delta\lambda^{2}}, D_{i,j} = \left(\frac{2R}{l}\right)^{2} \frac{H_{i,j-\frac{1}{2}}^{3}}{\Delta\lambda^{2}} \\ (F_{i,j}) = 6 \frac{H_{i+\frac{1}{2},j} - H_{i-\frac{1}{2},j}}{\Delta\theta} + \frac{6\dot{e}\cos\theta}{Rl\dot{\psi}} \\ E_{i,j} = A_{i,j} + B_{i,j} + C_{i,j} + D_{i,j} \end{cases}$$
(14)

where $(P_{i,j})_1$ and $(P_{i,j})_2$ are the oil film pressures of the inner and outer layers, respectively, and $A_{i,j}$ to $F_{i,j}$ are the equation coefficient matrices. The final dimensionless expression of the oil film pressures of the inner and outer layers is derived as

$$(P_{i,j}) = \frac{A_{i,j}(P_{i+1,j}) + B_{i,j}(P_{i-1,j}) + C_{i,j}(P_{i,j+1}) + D_{i,j}(P_{i,j-1}) - (F_{i,j})}{E_{i,j}}$$
(15)

 $P_{i,j}$ of all nodes is expressed by the above formula, and a set of (m - 1)(n - 1) inhomogeneous linear equations can be formed. According to the Reynolds boundary condition: the lubricating oil film is continuous between the start point and the end point of the oil film pressure in the circumferential direction of the journal. Then, some boundary conditions are defined, in which $\lambda = 0$, and at $\theta = 0$, P = 0. The equations are measured to get the pressure distribution $P_{i,j}^1$; then, it is regarded as the initial value of the next iteration. The iteration termination condition can be obtained as

$$\frac{\sum_{j=2}^{n} \sum_{i=2}^{m} \left| P_{i,j}^{(k)} - P_{i,j}^{(k-1)} \right|}{\sum_{j=2}^{n} \sum_{i=2}^{m} \left| P_{i,j}^{(k)} \right|} \le \delta, \ (\delta \text{ generally go to around } 10^{-3})$$
(16)

2.2. Numerical Results

2.2.1. Oil Film Pressure Distributions

The dimensionless oil film pressure *P* is obtained using the two parameters of widthto-diameter ratio and eccentricity. According to the foregoing, the oil film pressure inside and outside the floating ring bearing is derived.

Figures 4 and 5 present the selected pressure distribution of the inner and outer oil films during one period of the journal's whirl. After the cylindrical oil cavity is expanded into a plane, the three-dimensional distribution of the non-dimensional oil film pressure calculated is approximately a continuous parabola. In the wedge-shaped convergence zone, the dimensionless oil film pressure gradually increases from zero to the maximum pressure value, and then the oil film pressure begins to drop to zero. Still under the Reynolds boundary condition, the oil film pressure remains at zero in the divergence zone. The oil film pressure value is varied under different eccentricity conditions. Note that the oil film pressure is larger when the eccentricity is greater. The corresponding oil film damping will also increase, which leads to a subsequent reduction in journal vibration.



Figure 4. Oil film pressure distributions of the inner oil film. (a) e = 0.1, *ratio* = 0.5; (b) e = 0.3, *ratio* = 0.5; (c) e = 0.1, *ratio* = 0.3; (d) e = 0.3, *ratio* = 0.3.

2.2.2. Oil Film Bearing Capacity and Deflection Angle Calculation

The bearing capacity of the lubricating oil film versus for the journal and the floating ring provides lubrication and damping effects, with less friction and vibration. According to the oil film pressure distribution, note that the bearing capacity of the oil film inside and outside of the floating ring bearing is derived from the vector sum of the oil film force of each micro-element area of the bearing working surface. Based on the analysis of the working principle of the floating ring, the line of action of the bearing capacity of the inner oil film passes through the center of the journal and the center of the floating ring.



Figure 5. Oil film pressure distributions of the outer oil film. (a) e = 0.1, *ratio* = 0.5; (b) e = 0.3, *ratio* = 0.5; (c) e = 0.1, *ratio* = 0.3; (d) e = 0.3, *ratio* = 0.3.

After the dimensionless oil film pressure is measured in the oil cavity, the dimensionless bearing capacity *W* of the oil film can be obtained by integrating the oil film force on each micro-unit area of the bearing working surface. The calculation formula is written as

$$W_{r,\theta} = \int_{-1}^{1} \int_{0}^{2\pi} P d\varphi d\lambda \tag{17}$$

$$\begin{cases} W_x = \int_{-1}^1 \int_0^{2\pi} P \cos \varphi d\varphi d\lambda = \sum_{j=1}^n \sum_{i=1}^m P_{i,j} \cos \varphi_i \Delta \varphi \Delta \lambda \\ W_y = \int_{-1}^1 \int_0^{2\pi} P \sin \varphi d\varphi d\lambda = \sum_{j=1}^n \sum_{i=1}^m P_{i,j} \sin \varphi_i \Delta \varphi \Delta \lambda \end{cases}$$
(18)

where $W_{r,\theta}$ is the bearing capacity under the cartesian coordinates and $W_{x,y}$ is the bearing capacity under the polar coordinate.

The dimensionless oil film bearing capacity is defined as

$$W = \sqrt{W_x^2 + W_y^2} \tag{19}$$

The deflection angle of the sliding bearing can be obtained as

$$\theta = \operatorname{arctg} \frac{W_x}{W_y} \tag{20}$$

Figure 6 depicts the results of the maximum oil film pressure, bearing capacity, and deflection angle with eccentricity. The evolution of the maximum oil film pressure is close to linear growth in the case of a small eccentricity, as shown in Figure 6a. When the eccentricity exceeds 0.5, the maximum oil film pressure increases sharply with the

increase in eccentricity, showing a non-linear increase. Also shown in Figure 6b is that the dimensionless bearing capacity of the oil film increases linearly in the case of small eccentricity. In comparison with Figure 6a, the bearing capacity increases sharply and grows nonlinearly in this instance of the eccentricity exceeding 0.5. The oil film squeezing effect is significantly enhanced, and it will show strong nonlinear characteristics. In the case of large eccentricity, the oil film may rupture, and the journal and the floating ring may be in direct contact at some locations. Figure 6c depicts that the deflection angle decreases linearly with the increase in eccentricity.



Figure 6. The maximum oil film pressure, bearing capacity, and deflection angle. (**a**) The maximum oil film pressure. (**b**) The oil film bearing capacity. (**c**) The deflection angle.

The dimensionless maximum oil film pressure, bearing capacity, and deflection angle curves during one period are presented in Figure 7. Note that both the maximum oil film pressure and bearing capacity are closed due to the actions of the width-to-diameter. The wider the bearing, the greater the oil film pressure at the center of the wedge-shaped convergence zone. On the contrary, the deflection angle changes slightly with the increase in the width-to-diameter ratio.



Figure 7. The dimensionless maximum oil film pressure, bearing capacity, and deflection angle. (a) The maximum oil film pressure. (b) The oil film bearing capacity. (c) The deflection angle.

2.3. Process of Modeling

The structure of the simplified rotor-bearing system is shown in Figure 8. This model basically consists of two bearings including the ring SFD and a shaft, a disk 1, and a disk 2. Figure 9 shows that the ring SFD is composed of the floating ring, and the inner and outer oil films. Six holes are evenly spaced on the whole circumference of the floating ring. Based on the model proposed above, the lubricating oil flow channel between the inner and outer oil chambers is built. Then, the flow rate of lubricating oil between the inner and outer membranes through the oil holes and the pressure transmission characteristics of the oil film can be highlighted.



Figure 8. The floating ring bearing-rotor system FEM model.



Figure 9. Schematic diagram of the inner and outer oil film channels.

The connection between the rotor and the bearing is based on the "multi-physics coupling characteristics". The force transmission between the bearing and the rotor can be used to describe the behavior of the hydrodynamic pressure effect. In this simulation calculation, the stress and deformation of the floating ring are neglected. The centroids of the two disks are set to offset 6×10^{-7} m from the center to the *z*-axis direction. Due to factors such as gravity and errors of assembly, the position is set to be approximately equal to the mass unbalance and eccentricity of the rotor system. From the final calculation results, note that the system has reached a steady state before 0.05 s. Therefore, it is reasonable to set the analysis time as 0.2 s in the numerical simulation. Known from the result of the simulation, the rotor speed is 8000 rad/min while the time step is 5×10^{-4} s. Thereby, the backward difference formula solution method is selected. The floating ring bearing adopts a full-floating model. With the Sommerfeld hypothesis taken into account, the solution result can be compared with the Reynolds boundary conditions.

Table 1 shows the geometric parameters, material parameters, and settings of the floating ring bearing-rotor system, and the attribute parameters of the floating ring are shown in Table 2.

Parameter Name	Numerical Value	Parameter Name	Numerical Value
Young's modulus	205 GPa	Poisson's ratio	0.3
Rotor material density	7800 kg/m^3	Length of the rotor, L	0.15 m
Position of disk 1	0.1 L	Position of bearing 1	0.3 L
Position of disk 2	0.9 L	Position of bearing 2	0.7 L
Quality of disk 1	1.4 kg	Quality of disk 2	1.0 kg
Lateral moment of inertia of disk 1	$6.3 imes 10^{-4} \text{ kg} \cdot \text{m}^2$	Lateral moment of inertia of disk 2	$4.5 imes 10^{-4} \text{ kg} \cdot \text{m}^2$
Polar moment of inertia of disk 1	$1.26 \times 10^{-5} \text{ kg} \cdot \text{m}^2$	Polar moment of inertia of disk 2	$9 \times 10^{-4} \text{ kg} \cdot \text{m}^2$

Table 1. Model parameters of the rotor system.

Tal	ole	2.	FI	loating	ring	bearing	parameters.
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Parameter Name	Numerical Value	Parameter Name	Numerical Value	
Quality of the floating ring	0.02 kg	Outer gap	0.08 mm	
Outer radius of the floating ring	9 mm	Inner gap	0.02 mm	
Inner radius of the floating ring	6 mm	Lubricating oil viscosity	0.06 Pa·s	
Length of the floating ring	0.01 m	-	-	

The rotor model in this section is meshed with Timoshenko beam elements. The beam element is a one-dimensional line element in space, which can resist certain linear deformation and has high calculation efficiency. At the cantilever end of the beam rotor and the outer end of the disc, the rotor only bears its gravity, and the force is small. Thus, it can be divided into fewer beam elements. Finally, the entire rotor is divided into 100 beam elements.

The meshing of the oil film in the inner and outer layers of the floating ring: considering the existence of oil holes, the oil film in the inner and outer layers of the floating ring can be divided into two parts: the edge of the oil film and the middle flow channel. The edge of the inner and outer oil film uses a quadrilateral mesh, which is evenly divided into 15 nodes in the circumferential direction and four nodes in the axial direction. The free triangular mesh is used for the intermediate flow channel part, and the final meshing result of the floating ring-rotor system is shown in Figure 10. The partial diagram of the inner and outer oil film grid is shown in Figure 11. The largest element is 8.25×10^{-3} m, the smallest element is 6×10^{-4} m, the largest element growth rate is 1.4, and the curvature factor is 0.4. There are 2816 triangle elements, 960 quadrilateral elements, 1480 edge elements, and 167 vertex elements. The minimum element quality is 0.4615, the average element quality is 0.8627.



Figure 10. Overall meshing result.



Figure 11. Partial diagram of inner and outer oil film grid.

3. The Numerical Simulation of the Floating Ring Rotor System

3.1. Analysis of Oil Film Pressure Distribution and Fluid Load

The oil film pressure distribution of the inner and outer layers of the floating ring at the steady state of 8000 rad/min is shown in Figure 12. There are two obvious pressure concentration zones in each oil film. Blue represents the pressure divergence zone, namely negative pressure. Red represents the pressure convergence zone, namely positive pressure.



(a)

Figure 12. Oil film pressure distribution of the inner and outer layers of the floating ring. (a) Oil film pressure distribution of floating ring 1; (b) oil film pressure distribution of floating ring 2.

It can be observed from the pressure contour that the maximum oil film pressure of the inner and outer oil films is located in the middle area of the floating ring. From the inside to the outside, the pressure decreases gradually, and the pressure at the outer end of the floating ring is the smallest. The simulation results of oil film pressure are consistent with the pressure distribution obtained using the numerical method in Section 2. Additionally, in the area of the minimum oil film clearance, the positive pressure of the oil film transits to the negative pressure. The wedge-shaped convergence zone forms positive pressure, while the divergence zone forms negative pressure. In a clockwise direction, the pressure in the wedge-shaped zone gradually increases from 0 to the peak of positive pressure and then begins to decrease. It enters the divergence zone through the region of minimum oil film clearance, and the pressure gradually reaches the peak of negative pressure.

After calculating the oil film pressure distribution, the oil film pressure along the inner and outer surfaces of the floating ring is integrated. The fluid load components on the inner and outer surfaces of the floating ring in all directions are obtained, as shown in Figure 13.



Figure 13. Cont.





Figure 13. The fluid load of the inner and outer layers of the oil film on the floating ring. (**a**) The fluid load of the inner of the floating ring 1; (**b**) the fluid load of the outer of the floating ring 1; (**c**) the fluid load of the inner of the floating ring 2; (**d**) the fluid load of the outer of the floating ring 2.

The following conclusions can be drawn from the figure: (1) The *y*-direction and *z*-direction components of the oil film fluid load inside and outside the floating ring are distributed in periodic sine waves. There is only a small fluctuation when the initial instantaneous load increases. (2) There is a phase difference between the inner oil film load and the outer oil film load. When the inner oil film load reaches the maximum value, the outer oil film load reaches the minimum value. (3) The fluid load on the floating ring 1 with larger bearing capacity and obvious oil film squeezing effect is greater than that on the lightly loaded floating ring 2. After reaching the steady state, the peak values of the fluid load components in the y and z directions received by the inner and outer membranes of the floating ring 1 are both 17 N. The peak values of the load components received by the floating ring 2 in different directions are both 9 N.

3.2. Analysis of Floating Ring Fluid Torque and Rotation Speed

When the system is started, the journal starts to accelerate and rotate. The rotation will drive the oil film in contact with the journal surface to start the shearing flow and finally drives the inner oil film to flow as a whole. At this time, there is a relative slippage between the fluid–solid contact wall of the inner oil film and the floating ring. There is no relative sliding motion at the fluid–structure interface between the outer membrane and the floating ring. Therefore, the relative sliding speed difference between the inner membrane and the outer membrane produces a viscous moment acting on the floating ring to make the floating ring start to rotate.

The fluid torque change on the floating ring is shown in Figure 14. The maximum fluid torque produced by the inner oil film is $0.035 \text{ N} \cdot \text{m}$, which continuously decreases to $0.016 \text{ N} \cdot \text{m}$ with time. The fluid torque of the outer oil film is initially 0 and continues to increase to $0.016 \text{ N} \cdot \text{m}$ with time, which is a negative value. At this time, the inner and outer oil film fluid moments are in opposite directions. The total moment received decreases from 0.035 to $0 \text{ N} \cdot \text{m}$ with time, and the force of the floating ring reaches a balanced state.





The speed change of the floating ring is shown in Figure 15. Due to the net torque on the ring, it starts to accelerate axially, and the speed continues to increase. The subsequent rotation of the ring reduces the relative slip speed of the ring and the oil film. This result will lead to a decrease in the viscous force from the inner membrane and an increase in the viscous force from the outer membrane. The direction of the outer membrane torque and the inner membrane torque are opposite and cancel each other out. It can also be observed from the slope change of the ring velocity curve in Figure 15 that the net angular acceleration of the ring continuously decreases with the acceleration of the ring. Eventually, the floating ring will reach a stable rotation. When the torque of the outer membrane is equal and opposite, the net angular acceleration of the ring becomes zero. The force of the floating ring reaches a balanced state, and the fluid torque drops to 0. Then, the floating ring will keep rotating at this speed.



Figure 15. The speed of the floating ring.

Under the sub-working condition of 8000 rad/min, the floating ring 1 bears a greater load and the oil film squeezing effect is greater. As shown in Figure 15, the minimum oil film clearance is smaller and the viscous moment generated on the floating ring is greater, so the floating ring speed of ring 2 is higher than that of floating ring 1. The maximum angular velocity of floating ring 2 is 450 rad/min, the maximum rotating speed of floating ring 1 is 430 rad/min, and the rotating speed of floating ring 2 is slightly larger. The maximum angular acceleration of floating ring 2 is 2.5×10^4 (r/min²). Based on the working condition, the rotating speed ratio of the floating ring to the rotor is 5.6%. Compared with the rotor speed, the floating ring only rotates in the oil chamber at a very small speed.

3.3. Analysis of Rotor and Floating Ring Trajectory

The trajectory of the shaft journal center follows from Figure 16. In the initial transient stage, the rotor is subjected to the oil film force and oil film damping provided by the floating ring damper, and the vibration is continuously reduced. After a process of rapid amplitude convergence, the rotor finally reaches the equilibrium state, and a small amplitude convergence is performed around the equilibrium point of the ring. The trajectory of floating ring 2 converges faster than that of floating ring 1, but the amplitude of the floating ring 2 in the final equilibrium state is significantly greater than that of floating ring 1.



Figure 16. The trajectory of the shaft journal center.

The trajectory of the floating ring is drawn in Figure 17. In the initial transient phase, the decrease in the oil film force is evident when the rotation speed is small, and the floating ring sinks due to the gravity of the rotor and the turbine disk. It can be learnt from the plot of the trajectory of the floating ring that the relative maximum sinking amount of floating ring 1 is 0.5 mm. Simultaneously, the maximum sinking amount of floating ring 2 is 0.35 mm. With the increase in the rotor speed, the oil film pressure and the oil film bearing capacity tend to increase, and the floating ring starts to float. This process is basically synchronized with the convergence process of the rotor shaft trajectory and finally reaches a balanced state.



Figure 17. The trajectory of the floating ring center.

3.4. Analysis of Oil Film Clearance and Oil Film Flow

The flow rate of the oil hole channel is illustrated in Figure 18 along the squeeze zone of floating rings 1 and 2. The flow rate is defined as "the mass of fluid passing through the oil hole per unit time". During the very short transient start-up process of the system, the quasi-periodical behavior of the flow is obvious. One finds that the change in the flow rate is periodic when the system reaches a steady state. The flow rate changes from positive to negative during one period. The time point where the flow rate is positive indicates that the channel is located in the positive pressure zone, and the oil film is squeezed out. The lubricating oil film in the inner cavity is mainly affected by the squeezing effect from the channel to the outer cavity. On the contrary, the channel is in the negative pressure zone when the flow rate is negative. The lubricating oil flows from the outer oil cavity to the inner oil cavity, so as to keep the balance of the lubricating oil flow in the oil cavity. Simultaneously, one finds that within one period, the positive flow rate has a peak with a larger value while the negative flow rate has two valleys with a smaller value. Figure 12 illustrates that in the whole oil film pressure distribution, the area of the positive pressure convergence zone is small and the pressure is large, which includes only one channel, and the negative pressure zone contains two channels. The maximum flow rate of floating ring 2 can reach 2 kg/s. In addition, the maximum flow rate of floating ring 1 is 5 kg/s, which changed significantly and greatly impacts the squeezing performance.

The relationship between the minimum oil film clearance thickness and time is shown in Figure 19. In the model set, the bearing inner clearance is 0.02 mm, and the bearing outer clearance is 0.08 mm, which is the initial oil film clearance. From the analysis results of the above figure, note that the minimum oil film clearance of the inner film of the floating ring bearing 1 fluctuates very little due to the limitation of the initial gap; the minimum oil film clearance of the outer film undergoes a significant reduction within 0.02 s of the initial transient stage. The clearance thickness drops from 0.08 to 0.03 mm and reaches 0.045 mm within 0.02–0.05 s, and then reaches a steady state. The change rule of the minimum oil film clearance of bearing 2 is similar to that of bearing 1, but the fluctuation range of the minimum oil film clearance of the outer film is relatively small. The initial transient state drops from 0.08 to 0.041 mm and then returns to 0.058 mm, reaching a steady state. The minimum oil film clearance of bearing 2 is greater than that of bearing 1. Within 0.02 s of the initial transient phase, the minimum oil film clearance of the inner oil film decreases significantly, almost approaching zero. Due to the existence of oil holes on the floating ring, after squeezing out too much oil, the clearance of the oil film will continue to decrease and the pressure of the oil film will increase. This result may cause the oil film to rupture somewhere in the squeeze zone, and the journal and the floating ring are in direct contact and friction. At the same time, a strong nonlinear disturbance to the rotor system is generated, resulting in increased vibration; and the wear of the floating ring will shorten its longevity.



Figure 18. The flow rate of the oil hole channel.



Figure 19. The minimum oil film clearance. (**a**) Minimum oil film clearance of floating ring 1; (**b**) minimum oil film clearance of floating ring 2.

4. Conclusions and Future Work

This paper considered the modeling of the small-scale and low-stiffness floating ring SFD bearings commonly used in the aeroengine rotor support system. Based on the perspective of theory and simulation, the oil film pressure distribution, fluid–structure coupling dynamic characteristics, and elastic ring support stiffness of the ring SFD-rotor system were identified. The main research contents and conclusions are as follows:

 The inner and outer oil film pressure distribution of the floating ring was investigated: the maximum oil film pressure keeps increasing with the increase in eccentricity, and it increases linearly with small eccentricity and increases non-linearly with large eccentricity exceeding 0.5. The bearing capacity of the oil film increases with the increase in eccentricity. The squeeze effect of the oil film is significantly enhanced, showing a strong nonlinear effect. The deflection angle decreases linearly with the increase in eccentricity. With the increase in the aspect ratio, the maximum oil film pressure increases linearly. The bearing capacity of the oil film also increases linearly with the increase in the aspect ratio.

- (2) In the balanced state, compared with the rotor speed, the floating ring rotates in the oil cavity at a very small speed. The results of the shaft trajectory and floating ring trajectory changes during the rotor start-up speed-up process show that the ring SFD has a significant effect on regulating rotor vibration, and the rotor shaft trajectory quickly converges to a balanced state and then whirls slightly in the ring. These results may encourage further work based on enhancements of the theoretical model in test.
- (3) The conclusions will be used to verify the theoretical results, and the influence of other parameters of SFD will be evaluated in the future. To comprehensively and deeply study the nonlinear dynamic behavior of the small-scale and low-stiffness ring SFD, the structural parameters of the ring SFD should be carefully selected to acquire a good vibration reduction performance.

Author Contributions: M.S.: Conceptualization, methodology, validation, data analysis, investigation, writing—original draft. Y.Y.: Writing—review and editing, supervision, funding acquisition. W.D.: Validation, formal analysis, project administration. J.W.: Data curation, validation, formal analysis. C.F.: Data curation, project administration. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by National Natural Science Foundation of China, grant number 11972295, and the Key Laboratory of Aero-engine Vibration Technology, Aero Engine Corporation of China, grant number KY-1003-2020-0037.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- 1. Chen, X.; Gan, X.; Ren, G. Dynamic Modeling and Nonlinear Analysis of a Rotor System Supported by Squeeze Film Damper with Variable Static Eccentricity under Aircraft Turning Maneuver. J. Sound Vib. 2020, 485, 115551. [CrossRef]
- Yang, Y.; Wu, Q.; Wang, Y.; Qin, W.; Lu, K. Dynamic Characteristics of Cracked Uncertain Hollow-Shaft. *Mech. Syst. Signal Process.* 2019, 124, 36–48.
- Jäger, S.; Vogel, S. Validation of a Squeeze-Film-Damper Test Rig by Using Multibody Cosimulation. *Multibody Syst. Dyn.* 2015, 34, 243–257. [CrossRef]
- 4. Zapoměl, J.; Ferfecki, P.; Kozánek, J. The Mathematical Model for Analysis of Attenuation of Nonlinear Vibration of Rigid Rotors Influenced by Electromagnetic Effects. J. Sound Vib. 2018, 443, 167–177. [CrossRef]
- 5. Delgado, A.; Ertas, B. Dynamic Characterization of a Novel Externally Pressurized Compliantly Damped Gas-Lubricated Bearing with Hermetically Sealed Squeeze Film Damper Modules. *J. Eng. Gas Turbines Power* **2019**, *141*, 021028. [CrossRef]
- 6. Takeuchi, R.; Ishimaru, H.; Yamashita, H.; Yabui, S.; Inoue, T. Experimental Evaluation of Dynamic Characteristics of Circular Arc Spring Dampers for Rotating Machinery. *J. Eng. Gas Turbines Power* **2021**, *143*, 061008. [CrossRef]
- Fu, C.; Zhu, W.; Zheng, Z.; Sun, C.; Yang, Y.; Lu, K. Nonlinear Responses of a Dual-Rotor System with Rub-Impact Fault Subject to Interval Uncertain Parameters. *Mech. Syst. Signal Process.* 2022, 170, 108827. [CrossRef]
- Chen, H.; Chen, Y.; Hou, L.; Li, Z. Bifurcation Analysis of Rotor-Squeeze Film Damper System with Fluid Inertia. *Mech. Mach. Theory* 2014, *81*, 129–139. [CrossRef]
- 9. Hamzehlouia, S.; Behdinan, K. Squeeze Film Dampers Executing Small Amplitude Circular-Centered Orbits in High-Speed Turbomachinery. *Int. J. Aerosp. Eng.* 2016, 2016, 5127096. [CrossRef]
- Zapoměl, J.; Ferfecki, P.; Kozánek, J. Modelling of Magnetorheological Squeeze Film Dampers for Vibration Suppression of Rigid Rotors. Int. J. Mech. Sci. 2017, 127, 191–197. [CrossRef]
- 11. Shaik, K.; Dutta, B.K. Tuning Criteria of Nonlinear Flexible Rotor Mounted on Squeeze Film Damper Using Analytical Approach. *J. Vib. Eng. Technol.* **2020**, *9*, 325–339. [CrossRef]
- 12. Groves, K.H.; Bonello, P. Improved Identification of Squeeze-Film Damper Models for Aeroengine Vibration Analysis. *Tribol. Int.* **2010**, *43*, 1639–1649. [CrossRef]

- Han, Z.; Ding, Q.; Zhang, W. Dynamical Analysis of an Elastic Ring Squeeze Film Damper-Rotor System. *Mech. Mach. Theory* 2018, 131, 406–419. [CrossRef]
- 14. Chen, X.; Gan, X.; Ren, G. Effects of Journal Static Eccentricity on Dynamic Responses of a Rotor System under Base Motions Using FDM Inertia Model. *J. Sound Vib.* **2021**, *519*, 116591. [CrossRef]
- 15. Andres, L.S.; Koo, B.; Jeung, S.H. Experimental Force Coefficients for Two Sealed Ends Squeeze Film Dampers (Piston Rings and O-Rings): An Assessment of Their Similarities and Differences. *J. Eng. Gas Turbines Power* **2019**, *141*, 021024.
- Chen, H.; Hou, L.; Chen, Y.; Yang, R. Dynamic Characteristics of Flexible Rotor with Squeeze Film Damper Excited by Two Frequencies. *Nonlinear Dyn.* 2016, 87, 2463–2481. [CrossRef]
- 17. Cook, J.; Basinger, J.; Hazelwood, T.; Luttrell, C.; Van Hoy, B.; Delgado, A. Squeeze Film Damper Experimental and Numerical Correlation: Test Setup Description and Parameter Identification of Dry System. In *Rotating Machinery, Optical Methods & Scanning LDV Methods*; Conference Proceedings of the Society for Experimental Mechanics Series; Springer: Cham, Switzerland, 2022.
- Gary, K.; Ertas, B. Experimental Rotordynamic Force Coefficients for a Diffusion Bonded Compliant Hybrid Journal Gas Bearing Utilizing Fluid-Filled Hermetic Dampers. J. Eng. Gas Turbines Power 2020, 142, 041008. [CrossRef]
- 19. Andrés, L.S. Liquid Cavitation in Fluid Film Bearings. Mod. Hydrodyn. Lubr. Theory (Lecure Notes) 2009, 13–15.
- Zhang, W.; Ding, Q. Elastic Ring Deformation and Pedestal Contact Status Analysis of Elastic Ring Squeeze Film Damper. J. Sound Vib. 2015, 346, 314–327. [CrossRef]
- Cao, J.; Dimond, T.; Allaire, P. Numerical Analysis of Flexible Rotor with Nonlinear Bearings and Squeeze Film Dampers. In Proceedings of the ASME International Mechanical Engineering Congress and Exposition (IMECE), Montreal, QC, Canada, 14–20 November 2014; Volume 4B.
- 22. Liu, Z.C.; Liao, M.F.; Cong, P.H.; Li, Y.; Wang, J.; Wang, S.J. Experimental Investigation for Effects of Static Eccentricity on Vibration Attenuating Characteristics of Squeeze Film Damper. *Tuijin Jishu/J. Propuls. Technol.* **2016**, *37*. [CrossRef]
- 23. Liu, Z.C.; Liao, M.F.; Cong, P.H.; Wang, J.; Wang, S.J.; Shi, B.; Li, Y. Design Method of Squeeze Film Damper for Aero-Engine Rotors. *Hangkong Dongli Xuebao/J. Aerosp. Power.* **2015**, *30*. [CrossRef]
- Zhou, H.L.; Luo, G.H.; Chen, G.; Tian, H.T. Two Dynamic Models of Dual Clearance Squeeze Film Damper and Their Verification. *Tribol. Int.* 2013, 66, 187–193. [CrossRef]
- 25. Zhou, H.L.; Feng, G.Q.; Luo, G.H.; Ai, Y.T.; Sun, D. The Dynamic Characteristics of a Rotor Supported on Ball Bearings with Different Floating Ring Squeeze Film Dampers. *Mech. Mach. Theory* **2014**, *80*, 200–213. [CrossRef]
- Zhou, H.; Zhang, M.; Cheng, X.; Tang, Z.; Cao, P. Effects of Oil Supply Conditions on Equivalent Damping and Circumferential Position Damping of Squeeze Film Damper. *Jixie Gongcheng Xuebao/J. Mech. Eng.* 2018, 54, 215–223. [CrossRef]
- Singh, S.; Tiwari, R. Model-Based Fatigue Crack Identification in Rotors Integrated with Active Magnetic Bearings. J. Vib. Control 2016, 23, 980–1000. [CrossRef]
- Zhu, H.; Ding, Q. Numerical Analysis of Static Characteristics of Herringbone Grooved Hydrodynamic Journal Bearing. In *Applied Mechanics and Materials*; Trans Tech Publications Ltd.: Bäch, Switzerland, 2012.