Supplementary Materials

Energy Harvester Based on a Rotational Pendulum Supported with FEM

Grzegorz Litak [1,*], Mirosław Kondratiuk [2], Piotr Wolszczak [1] , Bartłomiej Ambrożkiewicz [1] and Abhijeet Giri[1]

[1] Faculty of Mechanical Engineering, Lublin University of Technology, Nadbystrzycka 36, 20-618 Lublin, Poland; p.wolszczak@pollub.pl (P.W.); b.ambrozkiewicz@pollub.pl (B.A.); a.giri@pollub.pl (A.M.G.)

[2] Faculty of Mechanical Engineering, Białystok University of Technology, Wiejska 45C, 15-351 Białystok, Poland; m.kondratiuk@pb.edu.pl

* Correspondence: g.litak@pollub.pl

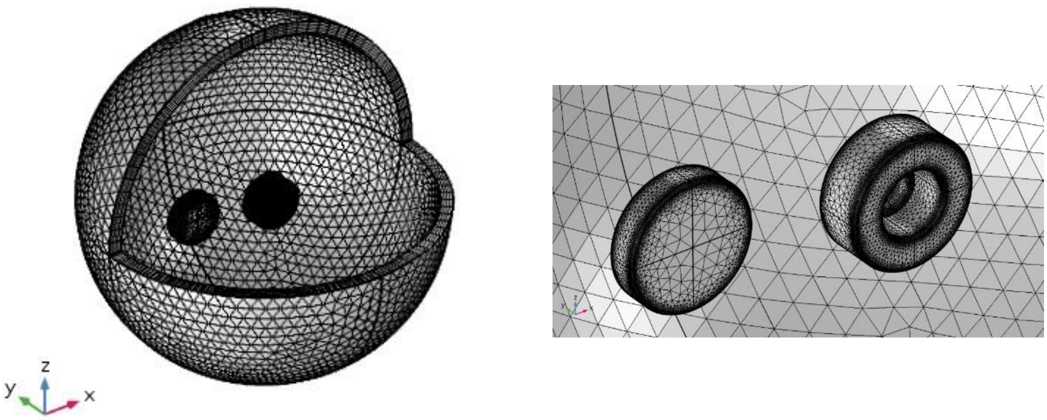
Details of Finite Element Analysis used for estimating magnetic interaction force and torques

A. Calculation of the components of magnetic forces for the pendulum magnet system and the magnet in the coil

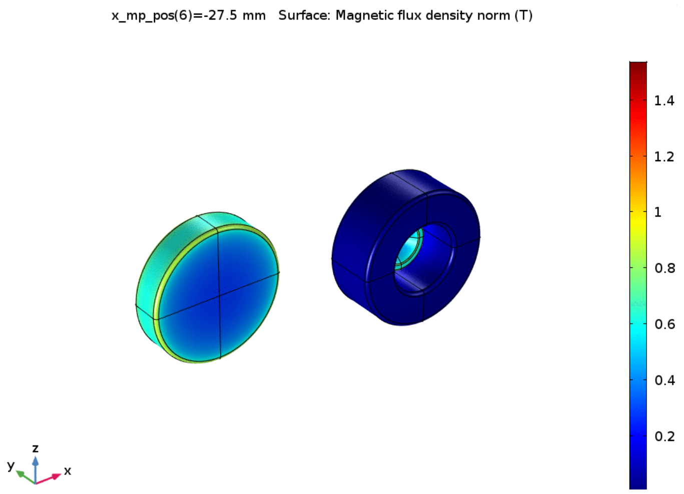
These calculations are performed for a stationary model in which the linear position of the pendulum-magnet relative to the magnet-coil pair was changed. Geometric parameters and other variables related to this analysis can be found in Table 1. In first case, there was no current flowing through the coil. The remnant induction for the pendulum magnet was considered as 1.23 Tesla and for the magnet it was considered as 1.19 Tesla. These FE calculations are performed for an automatically generated mesh with mesh refinements in the curvature areas.

As a result of the calculations, the components of the forces acting on the pendulum-magnet were determined.

It is worth noting that even though the position of the pendulum-magnet changes linearly relative to the coordinate system, the results can be translated into the rotational motion by projecting the components. Moreover, the results are symmetrical about the origin point for the Fm_x component and about the vertical axis for the Fm_y component. Having obtained the components of the magnetic forces for a single pendulum-magnet system, we can build a model consisting of one pendulum and many magnets around the circumference.

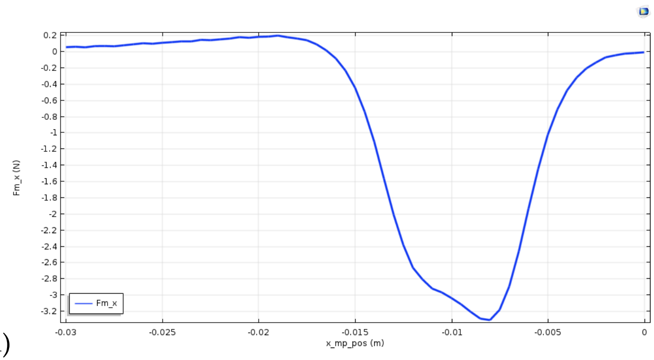


(a) (b)

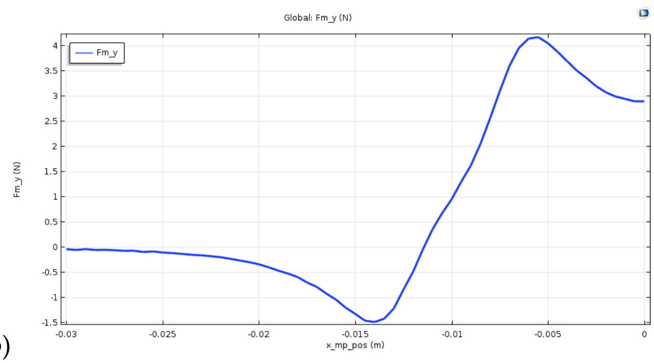


(c)

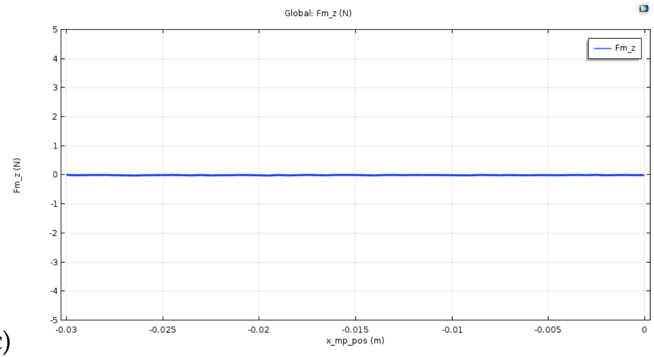
Figure S1. (a) Global mesh model of a stationary pendulum and magnet-coil pair. (b) Local enlarged mesh view of a stationary pendulum and a magnet-coil pair. (c) Contours of magnetic flux density [T] at a distance of -27.5 mm along x-axis.



a)



b)

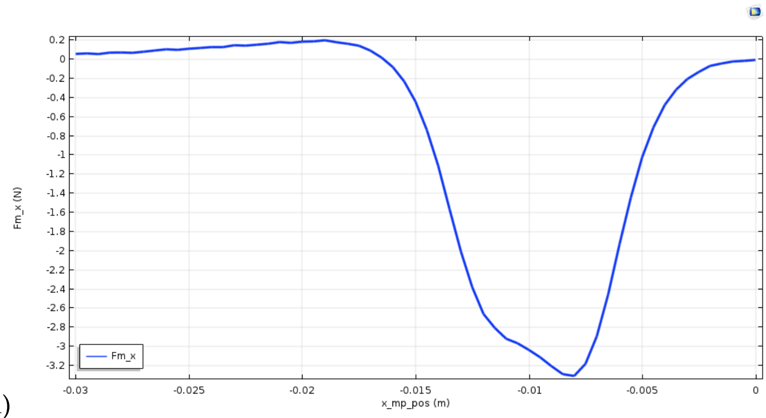


c)

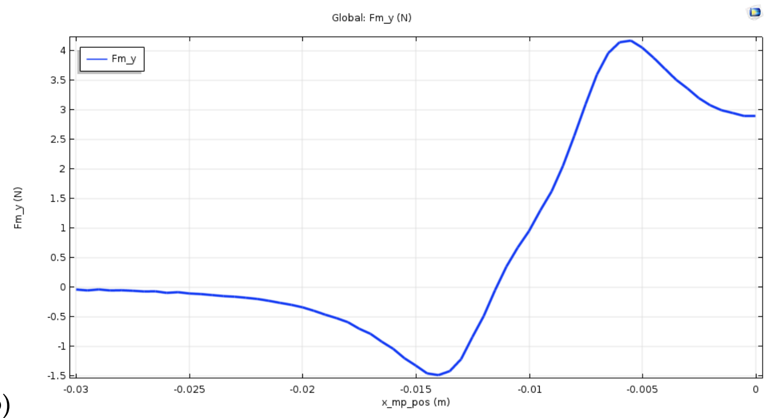
Figure S2. Variation of components of magnet force in x, y, and z axes with the distance between the magnets along x-axis are shown in (**a**)–(**c**), respectively.

B. Calculation of the components of magnetic forces for the pendulum-magnet and the magnet-coil with applied current

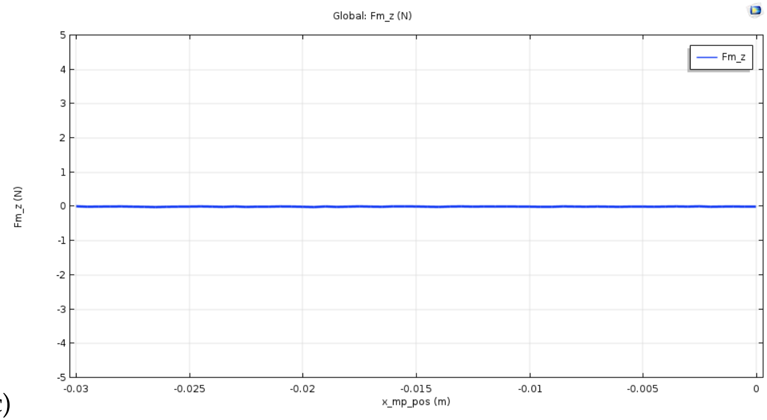
Analysis conditions are kept similar to previous case. However, current of 1 Ampere magnitude is applied to the coil as a boundary condition. When current flows through the coil, a magnetic field is generated, which acts on the pendulum-magnet. The strength of the magnetic interaction is proportional to the current. Therefore, assuming linearity, as the current magnitude is known using which the standard force components are computed, it is possible to determine the magnetic force between the coil and the pendulum at various positions. Again, it is worth noting that even though the position of the pendulum magnet changes linearly relative to the coordinate system, the reference characteristic, when multiplied by the coil current, can be translated into rotational path by projecting the components. Using these assumptions, the components of the forces acting on the pendulum-magnet were determined.



a)



b)



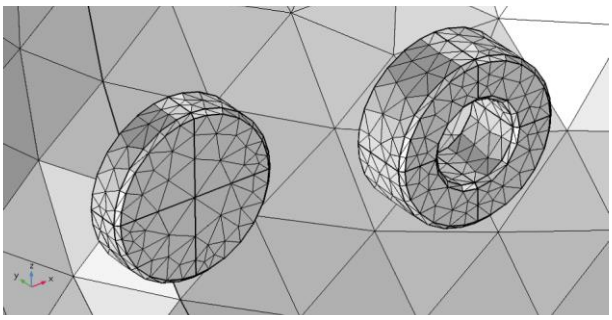
c)

Figure S3. Under the application of 1A current through coil, variation of components of magnet force in x, y, and z axes with the distance between the magnets along x-axis are shown in figures (a) to (c), respectively.

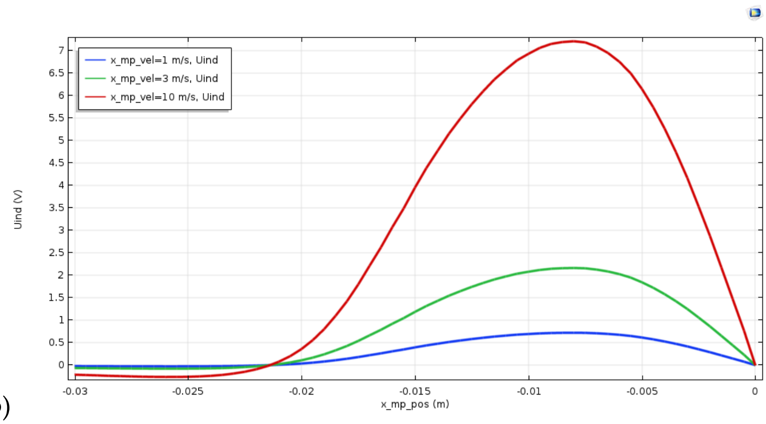
C. Calculation of the voltage induced in the coil

These calculations are performed for a quasi-dynamic model. The speed was taken into account in Maxwell's equations. In this model, linear position of the pendulum-magnet relative to the coil was changed at different speeds. No additional current flowing through the coil was assumed, and the remnant induction for the pendulum-magnet was assumed as 1.23 Tesla. The calculations are performed for an automatically generated mesh, but due to the nature of the calculations, a dense mesh was not necessary (see Figure S4(a)).

At different speeds, EMF and induced voltage are different as can be seen through Figure S4(b). After normalizing the result by the speed, a standard voltage characteristic was obtained as shown in Figure S4(c), which can be implemented into all the coils of the model. Once again, even though the position of the pendulum magnet changes linearly relative to the coordinate system (at different speeds), the reference characteristic, when multiplied by the linear speed of the pendulum, can be translated directly into the rotational motion.



a)



b)

c)

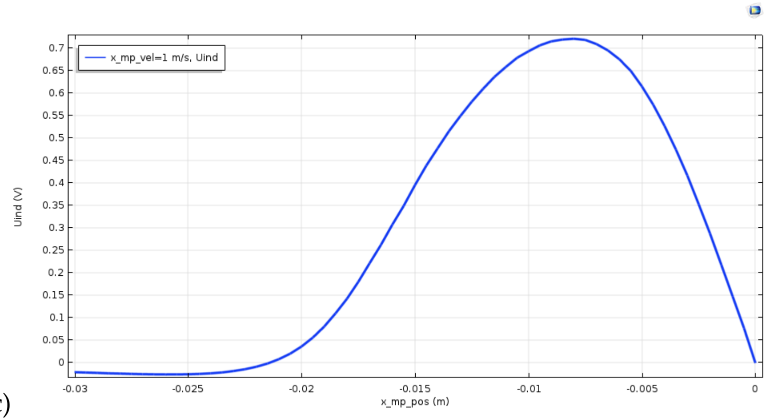


Figure S4. (a) Local mesh of a quasi-dynamic model for a pendulum-magnet and a coil with induced voltage. (b) Induced voltage in coil for different speeds of the pendulum. (c) Reference characteristics of induced voltage obtained after normalizing with speed.

The above-described stages in A, B, and C were used to build a pendulum model equipped with a system of 4 coils with magnets, and 8 coils without magnets. The geometric parameters of the entire system are presented in Table 1 below. The equations of the complete pendulum system include the following quantities:

- Projected characteristics of the magnetic force between the pendulum and the magnets of 4 coils.
- Projected standard characteristics of the mechanical force between the pendulum and 12 coils with current (depending on the induced voltage and load).
- Standard characteristics of the voltage induced by the moving magnet in relation to the 12 coils.

It is worth adding that the model ignores the mutual inductances of coils with currents due to their small values. Based on the characteristics obtained for magnetic force, magnetic torques for different magnet couplings as shown in Figure 6 of the manuscript can be estimated.

Table S1: List of variables used for the FEA simulation of magnetic force interactions

Variable used in COMSOL	Value[unit]	Variable Description
d_wi	0.32[mm]	Diameter of isolated wire
d_wn	0.31[mm]	Diameter of non-isolated wire
r_co	10[mm]	Coil outside radius
r_ci	5[mm]	Coil inside radius
y_c	8[mm]	Coil length
r_cfil	0.5[mm]	Coil fillet radius
N_c	253	Number of coil turns
I_c	1[A]	Coil current
r_mco	4[mm]	Coil magnet outside radius
r_mci_1	1.75[mm]	Coil magnet inside radius 1
r_mci_2	3[mm]	Coil magnet inside radius 2
y_mc	3[mm]	Coil magnet length
r_mc_fil	0.25[mm]	Coil magnet fillet radius
Br_mc	1.19[T]	Coil magnet remnant flux density
r_mp	10[mm]	Pendulum magnet radius
y_mp	5[mm]	Pendulum magnet length
r_mp_fil	0.5[mm]	Pendulum magnet fillet radius
Br_mp	1.23[T]	Pendulum magnet remnant flux density

x_mp_pos	-30[mm]	Pendulum magnet x-axis position
y_mp_pos	1[mm]	Pendulum magnet y-axis position
z_mp_pos	0[mm]	Pendulum magnet z-axis position
r_pp	5[mm]	Pendulum pivot radius
y_pp	5[mm]	Pendulum pivot length
a_p	4[mm]	Pendulum arm side length
h_p	33.25[mm]	Pendulum arm length
r_pph	2.5[mm]	Pendulum pivot hole radius
tr	2*pi/omega_p0	Pendulum reverse time
R_ct	21.5[ohm]	Total coils resistance
R_a	0[ohm]	Additional resistance
phi_p0	30[deg]	Pendulum initial deflection angle
omega_p0	10[deg/s]	Pendulum initial rotational speed