

Supplementary Materials: Models for Facilitated Transport Membranes: A Review

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Goddard, Shultz and Bassett, 1969, On membrane diffusion with near equilibrium reaction.

$$G(Z) = \sqrt{\frac{Z^5}{1+Z^2}} \quad (S1)$$

$$\bar{Z} = \frac{\sigma}{1 + K_{eq} C_A^0} \quad (S2)$$

$$\underline{Z} = \frac{\sigma}{1 + K_{eq} C_A^L} \quad (S3)$$

$$\sigma = K_{eq} C_T \frac{D_{AC}}{D_A} \quad (S4)$$

Kreuzer and Hoofd, 1970, Facilitated diffusion of oxygen in the presence of hemoglobin.

$$E_1 = \frac{1}{2\alpha^2 D_{AC} D_A} \left(\frac{k_f C_C^0 J_A}{D_A \alpha} + k_r C_{AC}^0 - k_f C_A^0 C_C^0 \right) \quad (S5)$$

$$H_1 = \frac{1}{2\beta^2 D_{AC} D_A} \left(\frac{k_f C_C^L J_A}{D_A \beta} + k_r C_{AC}^L - k_f C_A^L C_C^L \right) \quad (S6)$$

Yung and Probstein, 1973, Similarity considerations in facilitated transport.

$$\alpha = \frac{C_T}{C_C^0} \quad (S7)$$

$$\eta = \frac{\left(\frac{C_C}{C_C^0} - 1 \right)}{\kappa} \quad (S8)$$

$$\eta_L = \frac{\left(\frac{C_C^L}{C_C^0} - 1 \right)}{\kappa} \quad (S9)$$

$$\Delta = \frac{D_{AC}}{D_A} K_{eq} C_T \quad (S10)$$

$$\eta_1 = B \left[1 + \sum_{n=1}^{\infty} \frac{\varepsilon^{3n}}{(3n)(3n-1)(3n-3)(3n-4) \times \dots 3 \times 2} \right] + C \left[\varepsilon + \sum_{n=1}^{\infty} \frac{\varepsilon^{3n+1}}{(3n)(3n+1)(3n-2)(3n-3) \times \dots 3 \times 4} \right] \quad (S11)$$

B, C are two expansion coefficients to be determined by BC's.

$$\beta = \kappa^{-1} \varepsilon - \alpha \varepsilon^{-1/3} \kappa^{-2/5} \quad (S12)$$

$$\varepsilon = [1 + \kappa(1 - x')](\varepsilon \kappa^2)^{-1/3} \quad (S13)$$

Smith, Meldon, Colton, 1973, An analysis of carrier facilitated transport.

$$J_{A,linearized}^* = \frac{1}{1 + (F_{eq} - 1) \left[Y \left(\frac{\tilde{\lambda}_0}{L} \right) + \frac{1}{Y} \left(\frac{\tilde{\lambda}_L}{L} \right) \right]} \quad (S14)$$

$$Y = \frac{k_f C_A^L + k_r}{k_f C_A^0 + k_r} \quad (S15)$$

$$G_0(0) - 2N_1 \left(\frac{\lambda_0}{\lambda_0} \right)^8 (J_A^*)^2 + \frac{1}{3} N_1 N_2 \left(\frac{\lambda_0}{\lambda_0} \right)^2 G_0^2(0) + \frac{1}{4} N_1 \left\{ 1 - \frac{N_2}{[1 - N_3 G_0(0)]^2} \right\} [1 - N_3 G_0(0)] \left(\frac{\lambda_0}{\lambda_0} \right)^5 J_A^* G_0(0) \left\{ = \left(\frac{\lambda_0}{\lambda_0} \right)^3 J_A^* \frac{1}{[1 - N_3 G_0(0)]} \right. \quad (S16)$$

$$G_L(0) + 2M_1 \left(\frac{\lambda_L}{\lambda_L} \right)^8 (J_A^*)^2 - \frac{1}{3} M_1 M_2 \left(\frac{\lambda_L}{\lambda_L} \right)^2 G_L^2(0) - \frac{1}{4} M_1 \left\{ 1 - \frac{M_2}{[1 + M_3 G_L(0)]^2} \right\} [1 + M_3 G_L(0)] \left(\frac{\lambda_L}{\lambda_L} \right)^5 J_A^* G_L(0) \left\{ = \left(\frac{\lambda_L}{\lambda_L} \right)^3 J_A^* \frac{1}{[1 + M_3 G_L(0)]} \right. \quad (S17)$$

$$N_1 = k_f \tilde{\lambda}_0^{-5} [C_A^0 - C_A^L] [1 + F_{eq}] [k_f C_A^0 + k_r] / (D_{AC}^2 L) \quad (S18)$$

$$N_2 = \frac{D_A k_f k_r C_T}{D_A (k_f C_A^0 + k_r)^2} \quad (S19)$$

$$N_3 = \frac{k_f^2 k_r C_T \tilde{\lambda}_0^{-3} [C_A^0 - C_A^L] [1 + F_{eq}]}{D_A (k_f C_A^0 + k_r)^2 L} \quad (S20)$$

$$N_4 = \frac{k_f^2 k_r C_T \tilde{\lambda}_0^{-3} [1 + F_{eq}]}{D_A (k_f C_A^0 + k_r) L} \quad (S21)$$

$$M_1 = k_f \tilde{\lambda}_L^{-5} [C_A^0 - C_A^L] [1 + F_{eq}] [k_f C_A^L + k_r] / (D_{AC}^2 L) \quad (S22)$$

$$M_2 = \frac{D_{AC} k_f k_r C_T}{D_A (k_f C_A^L + k_r)^2} \quad (S23)$$

$$M_3 = \frac{k_f^2 k_r C_T \tilde{\lambda}_L^{-3} [C_A^0 - C_A^L] [1 + F_{eq}]}{D_A (k_f C_A^L + k_r)^2 L} \quad (S24)$$

$$M_4 = \frac{k_f^2 k_r C_T \tilde{\lambda}_L^{-3} [1 + F_{eq}]}{D_A (k_f C_A^L + k_r) L} \quad (S25)$$

Basaran et al., 1989, Facilitated Transport with Unequal Carrier and Complex Diffusivities.

$$\delta = \frac{D_{AC}}{D_A} \quad (S26)$$

$$\sigma = \frac{1}{KT} \quad (S27)$$

$$\gamma_0 = \frac{1}{T} \quad (S28)$$

$$\gamma_1 = \frac{\overline{C_A^L}}{T} \quad (S29)$$

$$\eta_0 = \sigma + d\gamma_0 \quad (S30)$$

$$\eta_1 = \sigma + d\gamma_1 \quad (S31)$$

$$\omega = (\ln(\eta_0) - \ln(\eta_1))(\eta_0 - \eta_1)^{-1} \quad (S32)$$

$$p_0 = -\sigma\delta \frac{\sigma^2 + \gamma_0\gamma_1 + (\gamma_0 + \gamma_1)\sigma}{\eta_0\eta_1} \quad (S33)$$

$$p_1 = \sigma\delta - \eta_0\eta_1 \quad (S34)$$

$$p_2 = \eta_0\eta_1 \quad (S35)$$

$$\rho = \frac{(\eta_0^{-1} + \eta_1^{-1})}{2} \quad (S36)$$

Morales-Cabrera et al., 2002, Approximate Method for The Solution of Facilitated Transport Problems in Liquid Membranes.

$$A_L = \left[\frac{\varphi_R}{\varphi_L \tanh(\varphi_R) + \varphi_R \tanh(\varphi_L)} \right] \left[\left(\bar{C}_A^{-1} + \frac{\alpha_R}{\varphi_R^2} + \frac{\beta_R}{\varphi_R^2} \right) \frac{1}{\cosh(\varphi_R)} - \left(1 - \frac{\alpha_L}{\varphi_L^2} - \frac{\alpha_R}{\varphi_R^2} \right) \frac{\tanh(\varphi_R)}{\varphi_R} + \frac{\beta_L}{\varphi_L^2} - \frac{\beta_R}{\varphi_R^2} \right] \quad (S37)$$

$$A_R = \frac{\varphi_L}{\varphi_R} A_L - \frac{1}{\varphi_R} \frac{\alpha_L}{\varphi_L^2} + \frac{\alpha_R}{\varphi_R^2} \quad (S38)$$

$$B_L = \frac{1}{\cosh(\varphi_L)} \left[A_L \sinh(\varphi_L) + \left(1 - \frac{\alpha_L}{\varphi_L^2} + \frac{\beta_L}{\varphi_L^2} \right) \right] \quad (S39)$$

$$B_R = \frac{1}{\cosh(\varphi_R)} \left[-A_R \sinh(\varphi_R) + \left(\bar{C}_A^{-1} + \frac{\alpha_R}{\varphi_R^2} + \frac{\beta_R}{\varphi_R^2} \right) \right] \quad (S40)$$

$$\alpha_L = \phi^2 \left[\left(\frac{1}{r_C} + \frac{1}{Kr_{AC}} \right) \bar{J}_A \right] \quad (S41)$$

$$\alpha_R = \phi^2 \left[\left(\frac{\bar{C}_A^{-1}}{r_C} + \frac{1}{Kr_{AC}} \right) \bar{J}_A \right] \quad (S42)$$

$$\beta_L = -\phi^2 \left[\bar{C}_C^{-1} + \frac{T_C}{r_C} + \frac{T_{AC}}{Kr_{AC}} \right] \quad (S43)$$

$$\beta_R = -\phi^2 \left[\bar{C}_A^{-1} \bar{C}_C^{-1} + \frac{\bar{C}_A^{-1} T_C}{r_C} + \frac{T_{AC}}{Kr_{AC}} \right] \quad (S44)$$

$$r_{AC} = \frac{D_{AC}}{D_A} \quad (S45)$$

$$r_C = \frac{D_C}{D_A} \quad (S46)$$