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Delayed Fuzzy Output Feedback H_∞ Control for Offshore Structures

Hua-Nv Feng ¹, Bao-Lin Zhang ^{1,*}, Qing Li ² and Gong-You Tang ³¹ College of Science, China Jiliang University, Hangzhou 310018, China; s1808070102@cjl.u.edu.cn² College of Mechanical and Electrical Engineering, China Jiliang University, Hangzhou 310018, China; lqing@cjl.u.edu.cn³ College of Information Science and Engineering, Ocean University of China, Qingdao 266100, China; gtang@ouc.edu.cn

* Correspondence: zhangbl@cjl.u.edu.cn; Tel.: +86-571-8691-4469

Received: 15 May 2020; Accepted: 9 June 2020; Published: 12 June 2020



Abstract: Vibration damping of jacket platforms is among the significant issues in marine science and engineering, and the design of active vibration control schemes is very important to ensure the stability and safety of the jacket platforms against external loadings. This paper provides three fuzzy output feedback H_∞ controllers of the jacket platforms for irregular wave forces. By considering time-varying masses of jacket platforms, a Takagi-Sugeno (T-S) fuzzy dynamic model of the structure is established. Then fuzzy output feedback H_∞ control schemes are developed via using output signals of the platform with current and/or are delayed. Several existence conditions of fuzzy output feedback H_∞ controllers are derived. Simulation results demonstrate that the fuzzy output feedback H_∞ control strategies are remarkable to suppress the vibration of structure. Moreover, by choosing proper delayed output information of the system, the presented delayed fuzzy output feedback H_∞ control schemes outperform the conventional fuzzy output feedback H_∞ control approach.

Keywords: offshore structure; output feedback control; fuzzy control; time-delay; vibration control

1. Introduction

As basic infrastructures for the development of ocean resources, offshore structures are unavoidably affected by several external loadings besides waves [1–4]. In general, the unwanted loadings cause excessive vibration of the platform [5–7]. Notice that reducing oscillation amplitude of jacket platforms to 15% can prolong the service life significantly, thereby saving the cost of inspection and maintenance of the platform [8]. Therefore, it is important to find effective control schemes besides passive, active and semi-active control to attenuate the oscillation level of structures; one can see [9–11], and the references therein.

Active control has attracted increasing attention of researchers in recent decades, and several active control approaches have been extensively utilized in the vibration control of a structure [12]. For example, in [13], feedforward and feedback control approaches have been presented for offshore platforms affected by the wave force. In [14], a robust H_2 optimal control approach has been proposed for an offshore structure to mitigate the wave induced vibration. In [15], a neuro-based active controller has been utilized to control a fixed jacket platform under an earthquake. As an efficient control strategy for overcoming system uncertainty and the external disturbance, sliding mode control methods have been utilized in the jacket platform [16–18]. In [6,19,20], the sampled-data H_∞ control, robust stochastic sampled-data control and fault-tolerant H_∞ control of offshore structures have been discussed, respectively. In [21] and [22], event-triggered controllers have been designed to enhance

performance of the platform. Most recently, inspired by [23], active control schemes with delayed states or outputs have been investigated for the structure, see for example, [18,24,25] and references therein.

In the presented active control schemes mentioned above, most of results are based on the state signals of the jacket platform system, which requires all state signals of the system to be measurable. Specifically, to measure all state signals such as displacement, velocity and even acceleration of the system generally results in high control cost. In this case, output feedback control is one of feasible and economic options. In fact, in [17,24], output feedback controllers have been developed for jacket platforms with an AMD to resist self-excited hydrodynamic force. However, there are few results available concerning output feedback control for mitigating vibrations of the platform against irregular wave force. To design delayed output feedback controllers to depress the vibration of the jacket platform under the wave loading is our first motivation in this paper. Moreover, the presented controller designs for jacket structures are based on the basic assumption that the masse of the platform is time-invariant. However, due to the routine operation besides extraction, storage, loading or unloading of ocean oil resources, the masses of jacket platforms are time-varying. In this case, it is significant to establish a model of the jacket platform via taking time-varying masses of the system into account. Notice that Takagi-Sugeno (T-S) fuzzy models can effectively approximate nonlinear systems, and the issues regarding T-S fuzzy systems have been aroused much attention [26–28]. Inspired by [26], to establish a fuzzy offshore structure model and develop fuzzy control schemes to improve performance of the structure is our second motivation.

By considering the time-varying masses of the offshore structure, in this paper, a T-S fuzzy dynamic model of the offshore structure subject to the external wave loading is presented, and then fuzzy output feedback H_∞ controllers with delayed and/or current output information are proposed. Some novel sufficient conditions of the system of the platform are obtained via the Lyapunov-Krasovskii stability theory, and the design approaches of fuzzy output feedback H_∞ controllers are presented. Simulation results indicate that under the designed fuzzy output feedback H_∞ controllers, the performance of the jacket platform can be improved remarkably. Moreover, the delayed fuzzy output feedback H_∞ controllers are better than traditional one.

Notations: The superscript -1 and $+$ represent the inverse and Moore-Penrose inverse of the matrix, respectively. The symmetric term is defined by $*$, e.g., $\begin{bmatrix} U & V \\ * & W \end{bmatrix} = \begin{bmatrix} U & V \\ V^T & W \end{bmatrix}$.

2. Problem Formulation

Consider a damping control problem of offshore structures equipped with active mass damper (AMD) mechanisms shown in Figure 1 [14]. By taking the first vibration mode of the platform and the external wave loading acting on the structure into account, the dynamic motion equation can be described as:

$$\begin{cases} k_2(z_2(t) - z_1(t)) + c_2(\dot{z}_2(t) - \dot{z}_1(t)) - c_1\dot{z}_1(t) - k_1z_1(t) + f(t) - u(t) = m_1\ddot{z}_1(t) \\ -k_2(z_2(t) - z_1(t)) - c_2(\dot{z}_2(t) - \dot{z}_1(t)) + u(t) = m_2\ddot{z}_2(t) \end{cases} \quad (1)$$

where z_1 , m_1 , c_1 , and k_1 are the displacement, mass, damping coefficient, and stiffness coefficient of the offshore structure, respectively. z_2 , m_2 , c_2 , and k_2 represent the displacement, mass, damping coefficient, and stiffness coefficient of the AMD, respectively. f is the irregular wave loading, and u denotes control force.

Notice that on one hand, the extraction, storage, loading and/or unloading of ocean resources are implemented on the platform. On the other hand, the crew generally work and live on the platform. Consequently, the mass of the jacket platform may change to some degree over time rather than maintaining a constant. In this situation, it is necessary to model the mass m_1 of the dominant vibration mode of the jacket platform as a bounded time-varying one, i.e., $m_1(t)$. In terms of the AMD, which is

a main auxiliary equipment of passive control, its dynamic parameters can be determined in advance. Therefore, it is reasonable to take the mass m_2 of AMD as a constant.

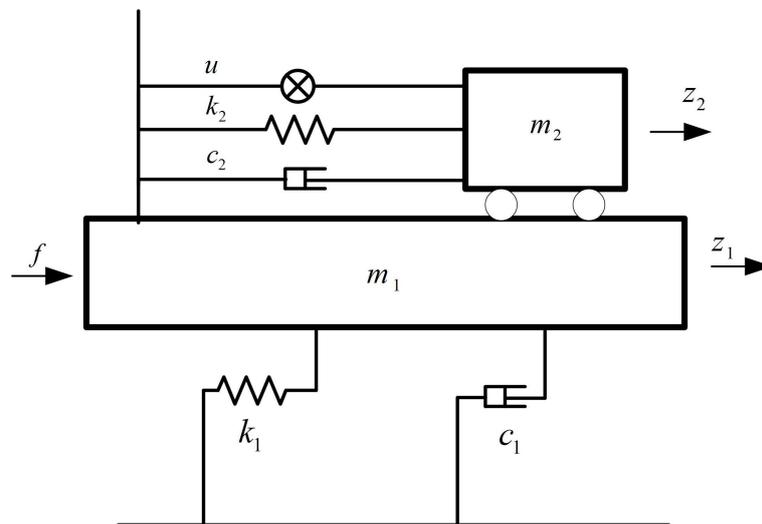


Figure 1. The sketch of an offshore structure with AMD mechanisms [14], with permission from Elsevier, 2003.

Let

$$x(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T \tag{2}$$

where

$$x_1(t) = z_1(t), \ x_2(t) = z_2(t), \ x_3(t) = \dot{z}_1(t), \ x_4(t) = \dot{z}_2(t) \tag{3}$$

Then, in (1), replacing the mass m_1 with the time-varying term $m_1(t)$ yields a state space model as:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + D(t)f(t) \tag{4}$$

where

$$A(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1(t)} & \frac{k_2}{m_1(t)} & -\frac{c_1+c_2}{m_1(t)} & \frac{c_2}{m_1(t)} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix}, \ B(t) = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_1(t)} \\ \frac{1}{m_2} \end{bmatrix}, \ D(t) = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1(t)} \\ 0 \end{bmatrix} \tag{5}$$

To simplify the controller design and the time-varying model (4) of the platform, the T-S fuzzy model is utilized to approximate the offshore structure. For this, define a fuzzy set $\{m_1^i | i = 1, 2, \dots, r\}$ subject to the mass $m_1(t)$ of the dominant vibration mode of the platform. Then, the fuzzy variable $m_1(t)$ is expressed as:

$$m_1(t) = \sum_{i=1}^r \psi_i(m_1(t))m_1^i \tag{6}$$

where $\psi_i(m_1(t)) \geq 0$ and satisfies

$$\sum_{i=1}^r \psi_i(m_1(t)) = 1$$

Based on (6) and (4), fuzzy model rules of the system are given as follows:

Fuzzy model rule for the plant i :

$$\text{IF } m_i(t) \text{ is } m_1^i \text{ THEN} \tag{7}$$

$$\dot{x}(t) = A_i x(t) + B_i u(t) + D_i f(t), \ i = 1, 2, \dots, r$$

where

$$\left\{ \begin{array}{l} A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1^i} & \frac{k_2}{m_1^i} & -\frac{c_1+c_2}{m_1^i} & \frac{c_2}{m_1^i} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix} \\ B_i = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_1^i} \\ \frac{1}{m_2} \end{bmatrix}, D_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1^i} \\ 0 \end{bmatrix}, i = 1, 2, \dots, r \end{array} \right. \quad (8)$$

Remark 1. The Equations (7) and (8) denote a new T-S fuzzy model of the offshore platform. In the presented model, the time-varying mass of the offshore structure is considered. If the mass of the offshore structure is taken as a constant, the dynamic model described by (7) and (8) reduces into the one in [5,7,14,21].

Using the center average defuzzifier method yields a whole dynamic fuzzy model as:

$$\dot{x}(t) = \sum_{i=1}^r \psi_i(m_1(t))(A_i x(t) + B_i u(t) + D_i f(t)) \quad (9)$$

The controlled output is given as:

$$z(t) = C_1 x(t) + H_1 f(t) \quad (10)$$

The measurable output is expressed as:

$$y(t) = Cx(t) \quad (11)$$

where C, C_1 and H_1 are given constant matrices.

In this paper, we tend to design delayed fuzzy output feedback H_∞ controllers and a delay-free fuzzy output feedback H_∞ controller such that under the designed fuzzy controllers, the system (9) with $f(t) = 0$ is asymptotically stable; and for wave loading $f(t) \in L_2[0, \infty]$, the following H_∞ performance index holds:

$$\|z(t)\| \leq \gamma \|f(t)\| \quad (12)$$

where $\gamma > 0$.

3. Fuzzy Output Feedback H_∞ Control Design

In this section, a delayed fuzzy output feedback H_∞ control approach, a pure delayed fuzzy output feedback H_∞ control approach, and a traditional fuzzy output feedback H_∞ control approach are developed respectively. Under the control laws, several sufficient conditions of asymptotic stability of the closed-loop offshore structure are investigated.

3.1. Mixed Delayed Fuzzy Output Feedback H_∞ Control Design

The fuzzy rules of a mixed delayed fuzzy output feedback H_∞ control law are designed as:
Control rule j :

$$\begin{array}{l} \text{IF } m_j(t) \text{ is } m_1^j \text{ THEN} \\ u_j(t) = K_j y(t) + S_j y(t-d), j = 1, 2, \dots, r \end{array} \quad (13)$$

where K_j and $S_j(j = 1, 2, \dots, r)$ are gain matrices, and $d > 0$ is a time-delay to be determined.

Further, the overall fuzzy control law is given as

$$u(t) = \sum_{j=1}^r \psi_j(m_1(t))(K_j y(t) + S_j y(t-d)) \tag{14}$$

Then, from (9), (11) and (14), we obtain the closed-loop system as:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t))((A_i + B_i K_j C)x(t) + B_i S_j Cx(t-d) + D_i f(t)) \tag{15}$$

where $v_{ij}(m_1(t)) = \psi_i(m_1(t))\psi_j(m_1(t))$, $i, j = 1, 2, \dots, r$.

The follow proposition provides an existence condition of the matrices K_j and $S_j(j = 1, \dots, r)$.

Proposition 1. For given scalars $\gamma > 0$ and $d > 0$, the closed-loop offshore structure system (15) with $f(t) = 0$ is asymptotically stable, and the H_∞ performance index (12) holds, if there exist matrices of appropriate dimensions K_j and $S_j(j = 1, 2, \dots, r)$ and 4×4 matrices $P_2 > 0, P_3 > 0, P > 0, R > 0, Q > 0$ such that

$$\begin{bmatrix} \Delta_{ij} & R + P_2 B_i S_j C & \tilde{\Delta}_{ij} & P_2 D_i & C_1^T \\ * & -R - Q & C^T S_j^T B_i^T P_3 & 0 & 0 \\ * & * & -2P_3 + d^2 R & P_3 D_i & 0 \\ * & * & * & -\gamma^2 I & H_1^T \\ * & * & * & * & -I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, r \tag{16}$$

where

$$\begin{cases} \Delta_{ij} = P_2(A_i + B_i K_j C) + (A_i + B_i K_j C)^T P_2 + Q - R \\ \tilde{\Delta}_{ij} = P - P_2 + (A_i + B_i K_j C)^T P_3, \quad i, j = 1, 2, \dots, r \end{cases} \tag{17}$$

Proof. A Lyapunov–Krasovskii functional candidate is selected as

$$V(t, x_t) = x^T(t)Px(t) + \int_{t-d}^t x^T(h)Qx(h)dh + d \int_{-d}^0 \int_{t+h}^t \dot{x}^T(\theta)R\dot{x}(\theta)d\theta dh \tag{18}$$

where $Q > 0, P > 0$, and $R > 0$.

Calculating the derivative of $V(t, x_t)$ and combing with (15) gets

$$\begin{aligned} \dot{V}(t, x_t) = & 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-d)Qx(t-d) \\ & + d^2 \dot{x}^T(t)R\dot{x}(t) - d \int_{t-d}^t \dot{x}^T(h)R\dot{x}(h)dh \end{aligned} \tag{19}$$

By Jensen’s inequality, one yields

$$-d \int_{t-d}^t \dot{x}^T(h)R\dot{x}(h)dh \leq -x^T(t)Rx(t) + 2x^T(t)Rx(t-d) - x^T(t-d)Rx(t-d) \tag{20}$$

Notice from (15) that

$$\begin{aligned} & 2[x^T(t)P_2 + \dot{x}^T(t)P_3][-\dot{x}(t) \\ & + \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t))((A_i + B_i K_j C)x(t) + B_i S_j Cx(t-d) + D_i f(t))] = 0 \end{aligned} \tag{21}$$

where $P_2 > 0$ and $P_3 > 0$.

Set

$$\eta^T(t) = \begin{bmatrix} x^T(t) & x^T(t-d) & \dot{x}^T(t) & f^T(t) \end{bmatrix} \tag{22}$$

Then by (19)–(21), one has

$$\dot{V}(t, x_t) \leq \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t)) \eta^T(t) \Omega \eta(t) \tag{23}$$

where

$$\Omega = \begin{bmatrix} \Delta_{ij} & R + P_2 B_i S_j C & \tilde{\Delta}_{ij} & P_2 D_i \\ * & -R - Q & C^T S_j^T B_i^T P_3 & 0 \\ * & * & -2P_3 + d^2 R & P_3 D_i \\ * & * & * & 0 \end{bmatrix} < 0 \tag{24}$$

To prove the asymptotic stability of offshore structure system (15), set $f(t) \equiv 0$ in (15), and denote $\theta^T(t) = [x^T(t) \ x^T(t-d) \ \dot{x}^T(t)]$. Then from (23), we have

$$\dot{V}(t, x_t) \leq \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t)) \theta^T(t) \tilde{\Omega} \theta(t) \tag{25}$$

where

$$\tilde{\Omega} = \begin{bmatrix} \Delta_{ij} & R + P_2 B_i S_j C & \tilde{\Delta}_{ij} \\ * & -R - Q & C^T S_j^T B_i^T P_3 \\ * & * & -2P_3 + d^2 R \end{bmatrix} \tag{26}$$

Note that if inequalities (16) hold, then we have $\tilde{\Omega} < 0$, which indicates that the closed-loop offshore structure system (15) with $f(t) = 0$ is asymptotically stable.

Now, we prove that the H_∞ index (12) can be guaranteed by the inequalities (16). In this case, notice from (23), (12), and (10) that

$$\dot{V}(t, x_t) + z^T(t)z(t) - \gamma^2 f^T(t)f(t) \leq \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t)) \eta^T(t) (\tilde{\Omega} + \Theta^T \Theta) \eta(t) \tag{27}$$

where

$$\left\{ \begin{array}{l} \tilde{\Omega} = \begin{bmatrix} \Delta_{ij} & R + P_2 B_i S_j C & \tilde{\Delta}_{ij} & P_2 D_i \\ * & -R - Q & C^T S_j^T B_i^T P_3 & 0 \\ * & * & -2P_3 + d^2 R & P_3 D_i \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ \Theta = \begin{bmatrix} C_1 & 0 & 0 & D_1 \end{bmatrix} \end{array} \right. \tag{28}$$

By the Schur Complement, inequalities (16) hold if and only if

$$\tilde{\Omega} + \Theta^T \Theta < 0 \tag{29}$$

Then, from (27), one has:

$$\dot{V}(t, x_t) + z^T(t)z(t) - \gamma^2 f^T(t)f(t) < 0 \tag{30}$$

Integrating both sides of (30) from 0 to ∞ , one gets

$$\int_0^\infty [z^T(t)z(t) - \gamma^2 f^T(t)f(t)]dt < 0$$

which shows that $\|z(t)\| \leq \gamma \|f(t)\|$ for nonzero $f(t) \in L_2[0, \infty]$. \square

Remark 2. Proposition 1 presents a bounded real lemma for the closed-loop offshore structure system (15) by employing the Jensen integral inequality and the simple Lyapunov–Krasovskii functional (18). If we apply recent developments on stability of time-delay systems, see, e.g., [29–32], some less conservative bounded real lemmas can be obtained. However, the following analysis shows that Proposition 1 is convenient and effective in designing suitable controller gains.

Note that in Proposition 1, the inequalities (16) are nonlinear. To obtain the gain matrices K_j and S_j in (14), set $P_3 = \varepsilon P_2$ in (16), and then pre- and post-multiply the right-hand side the (16) by $\text{diag}\{P_2^{-1}, P_2^{-1}, P_2^{-1}, I, I\}$ and its transpose, respectively, and denote $\bar{K}_j = K_j C P_2^{-1}$, $\bar{S}_j = S_j C P_2^{-1}$, $\bar{P}_2 = P_2^{-1}$, $\bar{R} = P_2^{-1} R P_2^{-1}$, $\bar{Q} = P_2^{-1} Q P_2^{-1}$, and $\bar{P} = P_2^{-1} P P_2^{-1}$, we gain the following result.

Proposition 2. For given scalars $\varepsilon > 0, d > 0$ and $\gamma > 0$, the closed-loop offshore structure system (15) with $f(t) = 0$ is asymptotically stable, and the H_∞ performance (12) is guaranteed, if there exist matrices of appropriate dimensions $\bar{K}_j, \bar{S}_j (j = 1, 2, \dots, r)$ and 4×4 matrices $\bar{P}_2 > 0, \bar{Q} > 0, \bar{P} > 0, \bar{R} > 0$ such that

$$\begin{bmatrix} \Xi_{ij} & B_i \bar{S}_j + \bar{R} & \tilde{\Xi}_{ij} & D_i & \bar{P}_2 C_1^T \\ * & -\bar{R} - \bar{Q} & \varepsilon \bar{S}_j^T B_i^T & 0 & 0 \\ * & * & -2\varepsilon \bar{P}_2 + d^2 \bar{R} & \varepsilon D_i & 0 \\ * & * & * & -\gamma^2 I & H_1^T \\ * & * & * & * & -I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, r \quad (31)$$

where

$$\begin{cases} \Xi_{ij} = A_i \bar{P}_2 + \bar{P}_2 A_i^T + B_i \bar{K}_j + \bar{K}_j^T B_i^T - \bar{R} + \bar{Q} \\ \tilde{\Xi}_{ij} = \bar{P} - \bar{P}_2 + \varepsilon \bar{P}_2 A_i^T + \varepsilon \bar{K}_j^T B_i^T, \quad i, j = 1, 2, \dots, r \end{cases} \quad (32)$$

Moreover, the controller gain matrices K_j and S_j are given by $K_j = \bar{K}_j \bar{P}_2^{-1} C^+$ and $S_j = -\bar{S}_j \bar{P}_2^{-1} C^+, j = 1, 2, \dots, r$.

3.2. Pure Delayed Fuzzy Output Feedback H_∞ Control Design

In (13) and (14), setting $K_j = 0$ leads to the following control rule as:

Control rule j :

$$\begin{aligned} \text{IF } m_j(t) \text{ is } m_1^j \text{ THEN} \\ u_j(t) = S_j y(t-d), \quad j = 1, 2, \dots, r \end{aligned} \quad (33)$$

and the overall pure delayed fuzzy output feedback H_∞ controller is in the form as

$$u(t) = \sum_{j=1}^r \psi_j(m_1(t)) S_j y(t-d) \quad (34)$$

In this case, the corresponding closed-loop offshore structure system is given as:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t)) (A_i x(t) + B_i S_j C x(t-d) + D_i f(t)) \quad (35)$$

To obtain the gain matrices $S_j, j = 1, 2, \dots, r$, by using Proposition 2, a Corollary is stated as follows.

Corollary 1. For given scalars $\varepsilon > 0, d > 0$ and $\gamma > 0$, if there exist matrices of appropriate dimensions \bar{S}_j ($j = 1, 2, \dots, r$) and 4×4 matrices $\bar{Q} > 0, \bar{P} > 0, \bar{P}_2 > 0, \bar{R} > 0$ such that

$$\begin{bmatrix} Y_i & B_i \bar{S}_j + \bar{R} & \bar{P} - \bar{P}_2 + \varepsilon \bar{P}_2 A_i^T & D_i & \bar{P}_2 C_1^T \\ * & -\bar{R} - \bar{Q} & \varepsilon \bar{S}_j^T B_i^T & 0 & 0 \\ * & * & -2\varepsilon \bar{P}_2 + d^2 \bar{R} & \varepsilon D_i & 0 \\ * & * & * & -\gamma^2 I & H_1^T \\ * & * & * & * & -I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, r \quad (36)$$

where

$$Y_i = A_i \bar{P}_2 + \bar{P}_2 A_i^T - \bar{R} + \bar{Q}, \quad i = 1, 2, \dots, r \quad (37)$$

Then the closed-loop system (35) with $f(t) = 0$ is asymptotically stable, and the prescribed H_∞ performance index (12) can be ensured, and the controller gain matrices S_j ($j = 1, 2, \dots, r$) are determined by $S_j = \bar{S}_j \bar{P}_2^{-1} C^+$.

3.3. Fuzzy Output Feedback H_∞ Control Design

In (13) and (14), setting $S_j = 0$ yields following control rules of a traditional fuzzy output feedback H_∞ control law as:

Control rule j :

$$\begin{aligned} \text{IF } m_j(t) \text{ is } m_1^j \text{ THEN} \\ u_j(t) = K_j y(t), \quad j = 1, 2, \dots, r \end{aligned} \quad (38)$$

The overall fuzzy output feedback H_∞ control law is given as

$$u(t) = \sum_{j=1}^r \psi_j(m_1(t)) K_j y(t) \quad (39)$$

Then, the closed-loop system is given as

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r v_{ij}(m_1(t)) ((A_i + B_i K_j C)x(t) + D_i f(t)) \quad (40)$$

From Proposition 2, we get following result:

Corollary 2. For given scalars $\gamma > 0$ and $\varepsilon > 0$, if there exist 1×4 matrices \bar{K}_j ($j = 1, 2, \dots, r$) and 4×4 matrices $\bar{P} > 0, \bar{P}_2 > 0$ such that

$$\begin{bmatrix} \Gamma_{ij} & \tilde{\Gamma}_{ij} & D_i & \bar{P}_2 C_1^T \\ * & -2\varepsilon \bar{P}_2 & \varepsilon D_i & 0 \\ * & * & -\gamma^2 I & H_1^T \\ * & * & * & -I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, r \quad (41)$$

where

$$\begin{cases} \Gamma_{ij} = A_i \bar{P}_2 + \bar{P}_2 A_i^T + B_i \bar{K}_j + \bar{K}_j^T B_i^T \\ \tilde{\Gamma}_{ij} = \bar{P} - \bar{P}_2 + \varepsilon \bar{P}_2 A_i^T + \varepsilon \bar{K}_j^T B_i^T, \quad i, j = 1, 2, \dots, r \end{cases} \quad (42)$$

Then the closed-loop offshore structure system (40) with $f(t) = 0$ is asymptotically stable, the H_∞ performance index (12) is ensured, and the controller gain matrices $K_j = \bar{K}_j \bar{P}_2^{-1} C^+$, $j = 1, 2, \dots, r$.

Remark 3. Corollaries 1 and 2 provide a simple and practical method to design fuzzy output feedback H_∞ controllers in case that several linear matrix inequalities are feasible. If we employ some recent developments on time-delay systems as stated in Remark 2, the controller design will be very complicated, see, for example [33]. As our future research, we will focus on seeking some novel criteria to design fuzzy output feedback controllers for the offshore platform using the developed skills on time-delay systems. Another future task is to extend the proposed method to repetitive control systems [34–36].

Remark 4. For the proposed T-S fuzzy model (7) of the offshore platform, three output feedback fuzzy control laws (14), (34) and (39) are developed, respectively. It should be pointed out that in (14), both present and delayed output of the structure are utilized to design fuzzy controller, while in (34) and (39), only the delayed output signals and the present output signals are used, respectively. In fact, the control laws (34) and (39) are special cases of the one (14). If the time-delay d is chosen properly, the delayed fuzzy output feedback H_∞ control schemes may have advantages of reducing vibration levels of the structure and the control cost, which will be demonstrated in the next Section.

4. Simulation Results

The designed fuzzy output feedback control schemes are used to the jacket platform for mitigating the wave excited vibration. Some comparison results of several active control schemes will be provided.

4.1. Parameters of an Offshore Structure

As [13], the parameters of an offshore structure subject to irregular wave force are used. The damping ratio of the structure is 0.02, the frequency of the structure is 2.0466 rads. The damping ratio of AMD is 2.00074, and the frequency of AMD is 0.2 rads. The mass m_2 of AMD is 78,253 kg, and the fuzzy set of mass of the offshore platform is given as {7,825,307, 8,126,682, 8,428,057, 8,729,432, 9,030,807} kg. Based on the above settings, the parameter matrices of the platform system (8) are obtained as:

$$\left\{ \begin{array}{l} A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.2289 & 0.0403 & -0.0899 & 0.0080 \\ 4.0297 & -4.0297 & 0.8030 & -0.8030 \end{bmatrix} \\ B_1 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ -1.2779 \\ -127.79 \end{bmatrix}, \quad D_1 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ 1.2779 \\ 0 \end{bmatrix} \end{array} \right. \quad (43)$$

$$\left\{ \begin{array}{l} A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.2274 & 0.0388 & -0.0896 & 0.0077 \\ 4.0297 & -4.0297 & 0.8030 & -0.8030 \end{bmatrix} \\ B_2 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ -1.2305 \\ -127.79 \end{bmatrix}, \quad D_2 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ 1.2305 \\ 0 \end{bmatrix} \end{array} \right. \quad (44)$$

$$\left\{ \begin{array}{l} A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.2260 & 0.0374 & -0.0893 & 0.0075 \\ 4.0297 & -4.0297 & 0.8030 & -0.8030 \end{bmatrix} \\ B_3 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ -1.1865 \\ -127.79 \end{bmatrix}, \quad D_3 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ 1.1865 \\ 0 \end{bmatrix} \end{array} \right. \quad (45)$$

$$\left\{ \begin{array}{l} A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.2247 & 0.0361 & -0.0891 & 0.0072 \\ 4.0297 & -4.0297 & 0.8030 & -0.8030 \end{bmatrix} \\ B_4 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ -1.1455 \\ -127.79 \end{bmatrix}, \quad D_4 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ 1.1455 \\ 0 \end{bmatrix} \end{array} \right. \quad (46)$$

$$\left\{ \begin{array}{l} A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4.2235 & 0.0349 & -0.0888 & 0.0070 \\ 4.0297 & -4.0297 & 0.8030 & -0.8030 \end{bmatrix} \\ B_5 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ -1.1073 \\ -127.79 \end{bmatrix}, \quad D_5 = 10^{-7} \times \begin{bmatrix} 0 \\ 0 \\ 1.1073 \\ 0 \end{bmatrix} \end{array} \right. \quad (47)$$

Set the displacement and velocity of the dominant vibration mode as the controlled and measurable output variables. In this case, the matrices C_1 , H_1 and C in (10) and (11) can be chosen as:

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad C = C_1 \quad (48)$$

To calculate the irregular wave force $f(t)$, the parameters concerning the wave are taken from [14], where the water depth is 218 m, the peak frequency of wave is 0.77 rad/s, the significant wave height is 7 m, the drag coefficient is 1.0, inertia coefficient is 1.5, and the peakedness coefficient is 3.3. Then the wave force can be simulated and the wave curve is shown in Figure 2.

For the offshore structure without control, the displacement and velocity response curves of the structure are shown in Figures 3 and 4, respectively. In this case, the jacket platform is in the dangerous state. In what follows, three fuzzy output feedback H_∞ controllers are designed to improve the performance of the system.

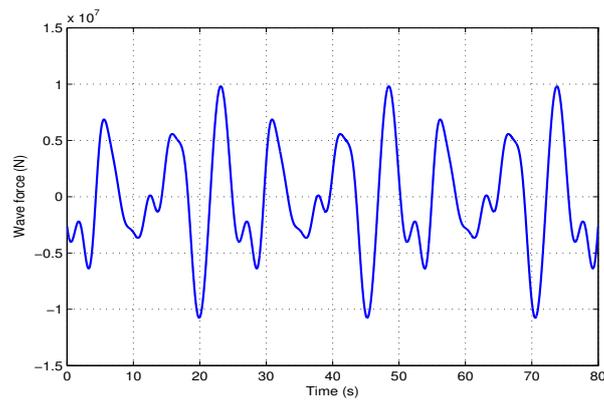


Figure 2. The irregular wave force.

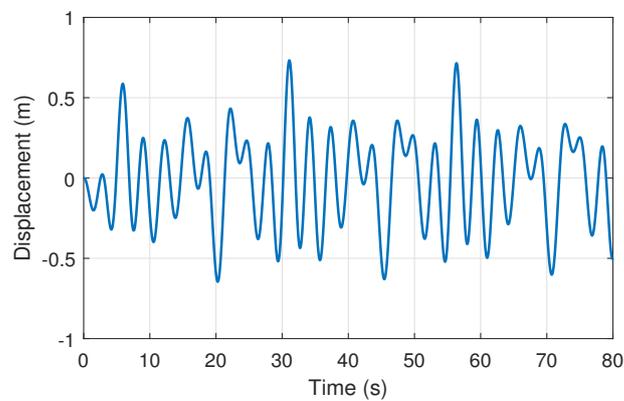


Figure 3. Displacement of the structure without control.

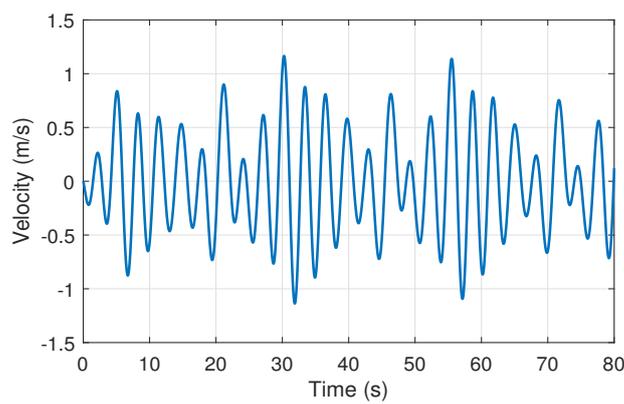


Figure 4. Velocity of the structure without control.

4.2. Responses of the Offshore Structure with Different Fuzzy Output Feedback H_∞ Control Schemes

We first design a fuzzy output feedback H_∞ controller (FOFC) for the platform. For this, set $\gamma = 9.8$ and $\varepsilon = 0.01$. Then by Corollary 2, solving the LMIs (41) using linear matrix inequality (LMI) toolbox of Matlab yields the gain matrices of an FOFC as:

$$K_j = 10^7 \times [-0.1468 \quad 2.0354], \quad j = 1, 2, 3, 4, 5 \tag{49}$$

As the designed FOFC is used to control the offshore structure, the response peak and root mean square (RMS) values of displacement and velocity of the structure and the control force are listed in Table 1. Clearly, the FOFC can weaken vibration level of the structure effectively.

Table 1. Response peak and RMS values of the structure and control force under FOFC and no control.

Controllers	Peak Values			RMS Values		
	Dis. (m)	Vel. (m/s)	u (10 ⁶ N)	Dis. (m)	Vel. (m/s)	u (10 ⁶ N)
No Control	0.7320	1.1648	–	0.2831	0.4785	–
FOFC	0.2120	0.1940	3.9925	0.1012	0.0841	1.7169

Then, by introducing proper time-delays, we design pure delayed fuzzy output feedback H_∞ controllers (PD-FOFCs). In fact, based on Corollary 1, for different given values of time-delay d , one can design different PD-FOFCs. For example, if $d = 0.03$, the gain matrices of the PD-FOFC can be obtained as

$$S_j = 10^7 \times [1.5238 \ 3.6675], \ j = 1, 2, 3, 4, 5 \tag{50}$$

For $d = 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.10, 0.15$ and 0.60 s, the corresponding PD-FOFCs can be designed respectively. Under these fuzzy controllers, the response peak and RMS values of the displacement and velocity of the offshore structure and the control force are provided by Table 2. From this Table, one can find that if $d \leq 0.15$ s, the designed PD-FOFCs can attenuate the vibration amplitudes of the system to different degrees. Specifically, if $d \leq 0.03$ s, the PD-FOFCs can attenuate the vibration of the structure significantly, while if the time-delay d is close to 0.60 s, the vibration amplitudes of the structure become large and tend to the ones without control.

Now, we design mixed delayed fuzzy output feedback H_∞ controllers (MD-FOFCs), where both delayed and current output information of the structure are used. Setting $d = 0.03$ and using Proposition 2 yields the matrices K_i and S_j as:

$$\left\{ \begin{array}{l} K_j = 10^7 \times [3.5235 \ 4.5598], \ j = 1, 2, 3 \\ K_4 = 10^7 \times [3.4803 \ 4.5242] \\ K_5 = 10^7 \times [0.7613 \ 1.0029] \\ S_j = 10^6 \times [-4.4597 \ -3.1073], \ j = 1, 2, 3 \\ S_4 = 10^6 \times [-4.4830 \ -3.163] \\ S_5 = 10^6 \times [3.7173 \ 5.5574] \end{array} \right. \tag{51}$$

Under the above MD-FOFC, the peak and RMS values of the structure response and the control force are provided by Table 3, where the response values of the system and control force with different MD-FOFCs are also listed. Table 3 shows that in the cases of $d \leq 5.50$ s, the MD-FOFCs can suppress the vibration of the structure. However, if d tends to 5.50 s, the peak value of the control force become very large.

Remark 5. Note that for PD-FOFCs, the maximum time-delay $d \leq 0.60$ s, and for MD-FOFCs, the maximum time-delay $d = 5.50$ s. It indicates that from the point of view of controller application, MD-FOFCs can provide more options to choose time-delays than PD-FOFCs.

Table 2. Response peak and RMS values of the structure and control force with PD-FOFCs.

<i>d</i> (s)	Peak Values			RMS Values		
	Dis. (m)	Vel. (m/s)	<i>u</i> (10 ⁶ N)	Dis. (m)	Vel. (m/s)	<i>u</i> (10 ⁶ N)
0.01	0.0393	0.0376	4.0354	0.0187	0.0162	1.9460
0.02	0.0518	0.0490	3.9232	0.0246	0.0211	1.8962
0.03	0.1064	0.0981	3.7403	0.0507	0.0425	1.7432
0.04	0.1927	0.1739	4.0412	0.0920	0.0760	1.7574
0.05	0.2679	0.2375	4.7104	0.1282	0.1050	2.0294
0.06	0.2956	0.2633	4.8960	0.1416	0.1159	2.1104
0.07	0.3597	0.3277	5.2523	0.1728	0.1421	2.2810
0.10	0.5282	0.5238	4.0183	0.2379	0.2171	1.8024
0.15	0.5321	0.5597	2.0533	0.2241	0.2422	0.8497
0.60	0.6887	1.0773	0.0847	0.2644	0.4325	0.3192

Table 3. Response peak and RMS values of structure and control force with MD-FOFCs.

<i>d</i> (s)	Peak Values			RMS Values		
	Dis. (m)	Vel. (m/s)	<i>u</i> (10 ⁶ N)	Dis. (m)	Vel. (m/s)	<i>u</i> (10 ⁶ N)
0.01	0.1506	0.1393	3.6803	0.0717	0.0602	1.6498
0.02	0.1806	0.1668	3.7523	0.0861	0.0720	1.6416
0.03	0.0787	0.0743	3.7035	0.0374	0.0319	1.7856
0.04	0.1462	0.1346	3.7199	0.0697	0.0583	1.6720
0.05	0.2081	0.1923	3.8471	0.0992	0.0830	1.6552
0.06	0.1690	0.1552	3.7840	0.0806	0.0672	1.6688
0.07	0.1118	0.1029	3.7337	0.0532	0.0446	1.7317
0.10	0.2640	0.2391	4.4164	0.1262	0.1042	1.8866
0.15	0.2547	0.2282	4.4678	0.1218	0.1002	1.9172
4.00	0.3074	0.6380	9.9099	0.0351	0.0627	3.2455
5.50	0.4896	0.8379	52.554	0.0696	0.1078	2.5903

4.3. Comparisons of FOFC, PD-FOFC, and MD-FOFC

It is found from Tables 2 and 3 that when different PD-FOFCs and MD-FOFCs are applied to the offshore structure, the damping effects of the system and the control cost may differ. Especially, if the time-delay *d* is chosen properly, the delayed fuzzy output feedback controllers may be better than the delay-free FOFC.

When the designed controllers FOFC, PD-FOFC (*d* = 0.03), and MD-FOFC (*d* = 0.03) are utilized to the offshore structure, the curves of displacement and velocity response of the structure are depicted in Figures 5 and 6, respectively. The curves of control force required by FOFC, PD-FOFC, and MD-FOFC are presented by Figure 7. The peak values and RMS values of the structure response, and the control forces by FOFC, PD-FOFC, and MD-FOFC are listed in Table 4.

Table 4. Response peak and RMS values of the structure and control force with different controllers.

Controllers	Peak Values			RMS Values		
	Dis. (m)	Vel. (m/s)	<i>u</i> (10 ⁶ N)	Dis. (m)	Vel. (m/s)	<i>u</i> (10 ⁶ N)
FOFC	0.2120	0.1940	3.9925	0.1012	0.0841	1.7169
PD-FOFC (<i>d</i> = 0.03)	0.1064	0.0981	3.7403	0.0507	0.0425	1.7432
MD-FOFC (<i>d</i> = 0.03)	0.0787	0.0743	3.7035	0.0374	0.0319	1.7856

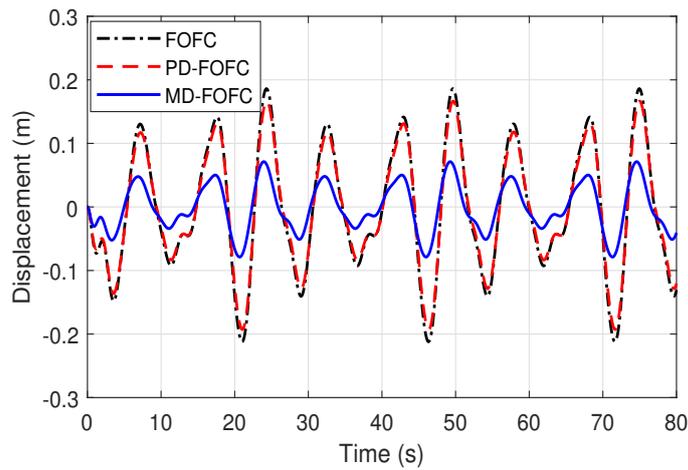


Figure 5. Displacement of the structure with FOFC, PD-FOFC, and MD-FOFC.

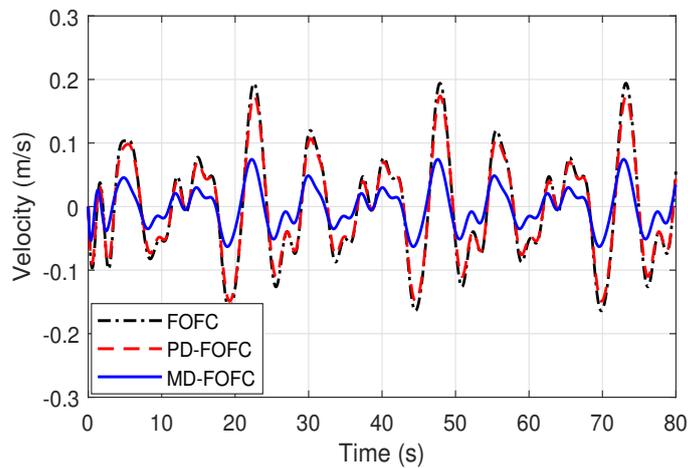


Figure 6. Velocity of the structure with FOFC, PD-FOFC, and MD-FOFC.

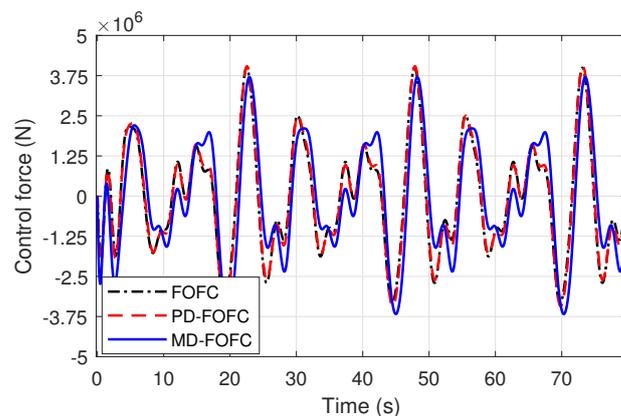


Figure 7. Control force by FOFC, PD-FOFC, and MD-FOFC.

Figures 5 and 6, and Table 4 show that the delay-free and delayed fuzzy output feedback controllers can mitigate vibration levels of the offshore structure significantly. Moreover, by choosing proper time-delay, the designed delayed fuzzy output feedback controllers, i.e., PD-FOFCs and MD-FOFCs, can further reduce the vibrations of the structure.

5. Conclusions

A T–S fuzzy dynamic model of an offshore structure with AMD mechanisms has been established, and then delay-free and delayed fuzzy output feedback H_∞ control schemes have been developed to weaken the vibration of the structure. Several conditions of existence of the delay-free and delayed fuzzy output feedback controllers have been obtained. The designed fuzzy output feedback H_∞ controllers are capable of improving performance and safety of the structure. Specifically, the delayed fuzzy output feedback H_∞ controllers have remarkable advantages of depressing the vibration levels of the platform, which has been demonstrated by simulation results.

Author Contributions: Conceptualization, G.-Y.T. and Q.L.; methodology, B.-L.Z. and H.-N.F.; software, H.-N.F.; validation, B.-L.Z. and H.-N.F.; formal analysis, H.-N.F.; investigation, H.-N.F. and Q.L.; resources, B.-L.Z. and H.-N.F.; writing—Original draft preparation, H.-N.F.; writing—Review and editing, B.-L.Z.; supervision, G.-Y.T.; project administration, B.-L.Z.; funding acquisition, B.-L.Z., Q.L. and G.-Y.T. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Key Project of Natural Science Foundation of Zhejiang Province of China grant number LZ19F030001, the Natural Science Foundation of China grant numbers 61773356 and 61673357, and the National Key Research and Development Program of China Grant number 2017YFC0804604.

Conflicts of Interest: The authors declare no conflict of interest.

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