

Article

An Accelerating Universe without Lambda: Delta Gravity Using Monte Carlo[†]

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Abstract: A gravitational field model based on two symmetric tensors, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, is studied, using a Markov Chain Monte Carlo (MCMC) analysis with the most updated catalog of SN-Ia. In this model, new matter fields are added to the original matter fields, motivated by an additional symmetry ($\tilde{\delta}$ symmetry). We call them $\tilde{\delta}$ matter fields. This theory predicts an accelerating Universe without the need to introduce a cosmological constant Λ by hand in the equations. We obtained a very good fit to the SN-Ia Data, and with this, we found the two free parameters of the theory called C and L_2 . With these values, we have fixed all the degrees of freedom in the model. The last H_0 local value measurement is in tension with the CMB Data from Planck. Based on an absolute magnitude $M_V = -19.23$ for the SN, Delta Gravity finds H_0 to be 74.47 ± 1.63 km/(s Mpc). This value is in concordance with the last measurement of the H_0 local value, 73.83 ± 1.48 km/(s Mpc).

Keywords: accelerated expansion; cosmological constant; cosmology; dark energy; Delta Gravity; modified gravity; phantom energy; supernovae

1. Introduction

General relativity (GR) is valid on scales larger than a millimeter to the solar-system scale [1,2]. Nevertheless, the theory is non-renormalizable, which prevents its unification with the other forces of nature. Trying to quantize GR is the main physical motivation of string theories [3,4]. Moreover, recent discoveries in cosmology [5–8] have revealed that most part of matter is in the form of unknown matter, dark matter (DM), and that the dynamics of the expansion of the Universe is governed by a mysterious component that accelerates the expansion, Dark Energy (DE). Although GR can accommodate both DM and DE, the interpretation of the dark sector in terms of fundamental theories of elementary particles is problematic.

Although some candidates exist that could play the role of DM, none have been detected yet. Also, an alternative explanation based on the modification of the dynamics for small accelerations cannot be ruled out [9,10]. On the other side, DE can be explained if a small cosmological constant (Λ) is present. In early times after the Big Bang, this constant is irrelevant, but at the later stages of the evolution of the Universe Λ will dominate the expansion, explaining the acceleration. Such small Λ is very difficult to generate in Quantum Field Theory (QFT) models, because Λ is the vacuum energy, which is usually very large [11].

One of the most important mysteries in cosmology and cosmic structure formation is to understand the nature of Dark Energy in the context of a fundamental physical theory [12,13].

In recent years there has been various proposals to explain the observed acceleration of the Universe. They involve the inclusion of some additional fields in approaches such as quintessence, chameleon, vector DE, or massive gravity; The addition of higher order terms in the Einstein-Hilbert action, such as $f(R)$ theories and Gauss-Bonnet terms and finally the introduction of extra dimensions for a modification of gravity on large scales (See [14]).

Other interesting possibilities are the search for non-trivial ultraviolet fixed points in gravity (asymptotic safety [15]) and the notion of induced gravity [16–19]. The first possibility uses exact renormalization-group techniques [20,21] together with lattice and numerical techniques such as Lorentzian triangulation analysis [22]. Induced gravity proposes that gravitation is a residual force produced by other interactions.

Recently, in [23,24] a field theory model explores the emergence of geometry by the spontaneous symmetry breaking of a larger symmetry where the metric is absent. Previous work in this direction can be found in [25,26].

In a previous work [27], we studied a model of gravitation that is very similar to classical GR, but could make sense at the quantum level. In the construction, we consider two different points. The first is that GR is finite on shell at one loop [28], so renormalization is not necessary at this level. The second is a type of gauge theories, $\tilde{\delta}$ Gauge Theories (Delta Gauge Theories), presented in [29,30], which main properties are: (a) New kind of fields are created, $\tilde{\phi}_I$, from the originals ϕ_I . (b) The classical equations of motion of ϕ_I are satisfied in the full quantum theory. (c) The model lives at one loop. (d) The action is obtained through the extension of the original gauge symmetry of the model, introducing an extra symmetry that we call $\tilde{\delta}$ symmetry, since it is formally obtained as the variation of the original symmetry. When we apply this prescription to GR we obtain Delta Gravity. Quantization of Delta Gravity is discussed in [31].

Here, we study the classical effects of Delta Gravity at the cosmological level. For this, we assume that the Universe is composed by non-relativistic matter (DM, baryonic matter) and radiation (photons, massless particles), which satisfy a fluid-like equation $p = \omega\rho$. Matter dynamics is not considered, except by demanding that the energy-momentum tensor of the matter fluid is covariantly conserved. This is required to respect the symmetries of the model. In contrast to [32], where an approximation is discussed, in this work we find the exact solution of the equations corresponding to the above suppositions. This solution is used to fit the SN-Ia Data and we obtain an accelerated expansion of the Universe in the model without DE.

It was noticed in [30] that the Hamiltonian of delta models is not bounded from below. Phantoms cosmological models [33,34] also have this property. Although it is not clear whether this problem will subsist or not in a diffeomorphism-invariant model as Delta Gravity. Phantom fields are used to explain the expansion of the Universe. Then, even if it could be said that our model works on similar grounds, the accelerated expansion of the Universe is really produced by a constant $L_2 \neq 0$ (it is a integration constant that comes from the Delta Field Equations), not by a phantom field.

It should be remarked that Delta Gravity is not a metric model of gravity because massive particles do not move on geodesics. Only massless particles move on null geodesics of a linear combination of both tensor fields.

2. Definition of Delta Gravity

In this section, we define the action as well as the symmetries of the model and derive the equations of motion.

These modified theories consist in the application of a variation represented by $\tilde{\delta}$. As a variation, it will have all the properties of a usual variation such as:

$$\begin{aligned}\tilde{\delta}(AB) &= \tilde{\delta}(A)B + A\tilde{\delta}(B) \\ \tilde{\delta}\delta A &= \delta\tilde{\delta}A \\ \tilde{\delta}(\Phi_{,\mu}) &= (\tilde{\delta}\Phi)_{,\mu}\end{aligned}\tag{1}$$

where δ is another variation. The particular point with this variation is that, when we apply it on a field (function, tensor, etc.), it will give new elements that we define as $\tilde{\delta}$ fields, which is an entirely new independent object from the original, $\tilde{\Phi} = \tilde{\delta}(\Phi)$. We use the convention that a tilde tensor is equal to the $\tilde{\delta}$ transformation of the original tensor when all its indexes are covariant.

First, we need to apply the $\tilde{\delta}$ prescription to a general action. The extension of the new symmetry is given by:

$$S_0 = \int d^n x \mathcal{L}_0(\phi, \partial_i \phi) \rightarrow S = \int d^n x (\mathcal{L}_0(\phi, \partial_i \phi) + \tilde{\delta} \mathcal{L}_0(\phi, \partial_i \phi)) \quad (2)$$

where S_0 is the original action, and S is the extended action in Delta Gauge Theories.

GR is based on Einstein-Hilbert action, then,

$$S_0 = \int d^4 x \mathcal{L}_0(\phi) = \int d^4 x \sqrt{-g} \left(\frac{R}{2\kappa} + L_M \right) \quad (3)$$

where $L_M = L_M(\phi_I, \partial_\mu \phi_I)$ is the Lagrangian of the matter fields ϕ_I , $\kappa = \frac{8\pi G}{c^2}$. Then, the Delta Gravity action is given by,

$$S = S_0 + \tilde{\delta} S_0 = \int d^4 x \sqrt{-g} \left(\frac{R}{2\kappa} + L_M - \frac{1}{2\kappa} (G^{\alpha\beta} - \kappa T^{\alpha\beta}) \tilde{g}_{\alpha\beta} + \tilde{L}_M \right) \quad (4)$$

where we have used the definition of the new symmetry: $\tilde{\phi} = \tilde{\delta} \phi$ and the metric convention of [5]¹.

Here:

$$\tilde{g}_{\mu\nu} = \tilde{\delta} g_{\mu\nu}, T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_M)}{\delta g_{\mu\nu}} \quad (5)$$

$$\tilde{L}_M = \tilde{\phi}_I \left(\frac{\delta L_M}{\delta \phi_I} \right) + (\partial_\mu \tilde{\phi}_I) \left(\frac{\delta L_M}{\delta(\partial_\mu \phi_I)} \right) \quad (6)$$

and $\tilde{\phi}_I = \tilde{\delta} \phi_I$ are the $\tilde{\delta}$ matter fields. Then, the equations of motion are:

$$G^{\mu\nu} = \kappa T^{\mu\nu} \quad (7)$$

$$F^{(\mu\nu)(\alpha\beta)\rho\lambda} D_\rho D_\lambda \tilde{g}_{\alpha\beta} + \frac{1}{2} g^{\mu\nu} R^{\alpha\beta} \tilde{g}_{\alpha\beta} - \frac{1}{2} \tilde{g}^{\mu\nu} R = \kappa \tilde{T}^{\mu\nu} \quad (8)$$

with:

$$\begin{aligned} F^{(\mu\nu)(\alpha\beta)\rho\lambda} &= P^{((\rho\mu)(\alpha\beta))} g^{\nu\lambda} + P^{((\rho\nu)(\alpha\beta))} g^{\mu\lambda} - P^{((\mu\nu)(\alpha\beta))} g^{\rho\lambda} - P^{((\rho\lambda)(\alpha\beta))} g^{\mu\nu} \\ P^{((\alpha\beta)(\mu\nu))} &= \frac{1}{4} (g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu} - g^{\alpha\beta} g^{\mu\nu}) \\ \tilde{T}^{\mu\nu} &= \tilde{\delta} T^{\mu\nu} \end{aligned}$$

where $(\mu\nu)$ denotes that μ and ν are in a totally symmetric combination. An important fact to notice is that our equations are of second order in derivatives which is needed to preserve causality. We can show that $(8)_{\mu\nu} = \tilde{\delta} [(7)_{\mu\nu}]$. The action (4) is invariant under (9) and (10) (extended general coordinate transformations), given by:

$$\tilde{\delta} g_{\mu\nu} = \tilde{\zeta}_{0\mu;\nu} + \tilde{\zeta}_{0\nu;\mu} \quad (9)$$

$$\tilde{\delta} \tilde{g}_{\mu\nu}(x) = \tilde{\zeta}_{1\mu;\nu} + \tilde{\zeta}_{1\nu;\mu} + \tilde{g}_{\mu\rho} \tilde{\zeta}_{0,\nu}^\rho + \tilde{g}_{\nu\rho} \tilde{\zeta}_{0,\mu}^\rho + \tilde{g}_{\mu\nu,\rho} \tilde{\zeta}_0^\rho \quad (10)$$

¹ In [35] can be found more about the formalism of the Delta Gravity action and the new symmetry $\tilde{\delta}$.

This means that two conservation rules are satisfied. They are:

$$D_\nu T^{\mu\nu} = 0 \quad (11)$$

$$D_\nu \tilde{T}^{\mu\nu} = \frac{1}{2} T^{\alpha\beta} D^\mu \tilde{g}_{\alpha\beta} - \frac{1}{2} T^{\mu\beta} D_\beta \tilde{g}_\alpha^\alpha + D_\beta (\tilde{g}_\alpha^\beta T^{\alpha\mu}) \quad (12)$$

It is easy to see that (12) is $\tilde{\delta}(D_\nu T^{\mu\nu}) = 0$.

3. Particle Motion in the Gravitational Field

We are aware of the presence of the gravitational field through its effects on test particles. For this reason, here we discuss the coupling of a test particle to a background gravitational field, such that the action of the particle is invariant under (9) and (10).

In Delta Gravity we postulate the following action for a test particle:

$$S_p = m \int \frac{dt}{\sqrt{-g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta}} \left(g_{\mu\nu} + \frac{1}{2} \tilde{g}_{\mu\nu} \right) \dot{x}^\mu \dot{x}^\nu \quad (13)$$

Notice that S_p is invariant under (9) and t -parametrizations.

Since far from the sources, we must have free particles in Minkowski space, i.e., $g_{\mu\nu} \sim \eta_{\mu\nu}$, $\tilde{g}_{\mu\nu} \sim 0$, it follows that we are describing the motion of a particle of mass m . Moreover, all massive particles fall with the same acceleration.

To include massless particles, we prefer to use the action [36]:

$$L = \frac{1}{2} \int dt \left(vm^2 - v^{-1} (g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu + \frac{m^2 + v^{-2} (g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu}{2v^{-3} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta} (m^2 + v^{-2} g_{\lambda\rho} \dot{x}^\lambda \dot{x}^\rho) \right) \quad (14)$$

This action is invariant under reparametrizations:

$$x'(t') = x(t); \quad dt'v'(t') = dtv(t); \quad t' = t - \varepsilon(t) \quad (15)$$

The equation of motion for v is:

$$v = - \frac{\sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}}{m} \quad (16)$$

Replacing (16) into (14), we get back (13).

Let us consider first the massive case. Using (15) we can fix the gauge $v = 1$. Introducing $mdt = d\tau$, we get the action:

$$\mathcal{L}_1 = \frac{1}{2} m \int d\tau \left(1 - (g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu + \frac{1 + (g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu}{2g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta} (1 + g_{\lambda\rho} \dot{x}^\lambda \dot{x}^\rho) \right) \quad (17)$$

plus the constraint obtained from the equation of motion for v :

$$g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -1 \quad (18)$$

From \mathcal{L}_1 the equation of motion for massive particles is derived. We define: $\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{2} \tilde{g}_{\mu\nu}$.

$$\frac{d(\dot{x}^\mu \dot{x}^\nu \bar{g}_{\mu\nu} \dot{x}^\beta g_{\alpha\beta} + 2\dot{x}^\beta \bar{g}_{\alpha\beta})}{d\tau} - \frac{1}{2} \dot{x}^\mu \dot{x}^\nu \bar{g}_{\mu\nu} \dot{x}^\beta \dot{x}^\gamma g_{\beta\gamma,\alpha} - \dot{x}^\mu \dot{x}^\nu \bar{g}_{\mu\nu,\alpha} = 0 \quad (19)$$

The motion of massive particles is discussed in [37].

The action for massless particles is:

$$\mathcal{L}_0 = \frac{1}{4} \int dt \left(-v^{-1} (g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu \right) \quad (20)$$

In the gauge $v = 1$, we get:

$$\mathcal{L}_0 = -\frac{1}{4} \int dt (g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu \quad (21)$$

plus the equation of motion for v evaluated at $v = 1$: $(g_{\mu\nu} + \tilde{g}_{\mu\nu}) \dot{x}^\mu \dot{x}^\nu = 0$. Therefore, the massless particle moves in a null geodesic of $\mathfrak{g}_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$.

4. Distances and Time Intervals

In this section, we define the measurement of time and distances in the model.

In GR the geodesic equation preserves the proper time of the particle along the trajectory. Equation (19) satisfies the same property: Along the trajectory $\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}$ is constant. Therefore we define proper time using the original metric $g_{\mu\nu}$,

$$d\tau = \sqrt{-g_{\mu\nu} dx^\mu dx^\nu} = \sqrt{-g_{00}} dx^0, \quad (dx^i = 0) \quad (22)$$

Following [38], we consider the motion of light rays along infinitesimally near trajectories and (22) to get the three dimensional metric:

$$dl^2 = \gamma_{ij} dx^i dx^j$$

$$\gamma_{ij} = \frac{g_{00}}{\mathfrak{g}_{00}} \left(\mathfrak{g}_{ij} - \frac{\mathfrak{g}_{0i} \mathfrak{g}_{0j}}{\mathfrak{g}_{00}} \right) \quad (23)$$

That is, we measure proper time using the metric $g_{\mu\nu}$ but the space geometry is determined by both metrics. In this model massive particles do not move on geodesics of a four-dimensional metric. Only massless particles move on a null geodesic of $\mathfrak{g}_{\mu\nu}$. Therefore, Delta Gravity is not a metric theory.

5. $T^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ for a Perfect Fluid

The Energy-Stress Tensors for a Perfect Fluid in Delta Gravity are [27] (assuming c is the speed of light equal to 1):

$$T_{\mu\nu} = p(\rho) g_{\mu\nu} + (\rho + p(\rho)) U_\mu U_\nu \quad (24)$$

$$\begin{aligned} \tilde{T}_{\mu\nu} = & p(\rho) \tilde{g}_{\mu\nu} + \frac{\partial p}{\partial \rho}(\rho) \tilde{\rho} g_{\mu\nu} + \left(\tilde{\rho} + \frac{\partial p}{\partial \rho}(\rho) \tilde{\rho} \right) U_\mu U_\nu + \\ & (\rho + p(\rho)) \left(\frac{1}{2} (U_\nu U^\alpha \tilde{g}_{\mu\alpha} + U_\mu U^\alpha \tilde{g}_{\nu\alpha}) + U_\mu^T U_\nu + U_\mu U_\nu^T \right) \end{aligned} \quad (25)$$

where $U^\alpha U_\alpha^T = 0$. p is the pressure, ρ is the density and U^μ is the four-velocity. For more details you can see [27].

6. Friedman-Lemaître-Robertson-Walker (FLRW) Metric

In this section, we discuss the equations of motion for the Universe described by the FLRW metric. We use spatial curvature equal to zero to agree with cosmological observations.

In the harmonic coordinate system, it is [27]:

$$g_{\mu\nu}dx^\mu dx^\nu = -c^2 dt^2 + R^2(t) (dx^2 + dy^2 + dz^2) \quad (26)$$

$$\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -3F_a(t)c^2 dt^2 + F_a(t)R^2(t) (dx^2 + dy^2 + dz^2) \quad (27)$$

$$\mathbf{g}_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu} = -c^2(1 + 3F_a(t))dt^2 + (1 + F_a(t))R^2(t) (dx^2 + dy^2 + dz^2) \quad (28)$$

Please note that $F_a(t)$ is an arbitrary function that remains after imposing homogeneity and isotropy of the space as well as the extended harmonic gauge $g^{\alpha\beta}\frac{1}{2}g^{\mu\lambda}(D_\beta\tilde{g}_{\lambda\alpha} + D_\alpha\tilde{g}_{\beta\lambda} - D_\lambda\tilde{g}_{\alpha\beta}) - \tilde{g}^{\alpha\beta}\Gamma_{\alpha\beta}^\mu = 0$. It is determined by solving the differential equations in (8).

These expressions represent an isotropic and homogeneous Universe. From (23) we already know that the proper time is measured only using the metric $g_{\mu\nu}$, but the space geometry in FLRW coordinates is determined by the modified null geodesic, given by (28), where both tensor fields, $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, are needed.

7. Delta Gravity Friedmann Equations

The equations of state for matter and radiation are:

$$p_m(R) = 0$$

$$p_r(R) = \frac{1}{3}\rho_r(R)$$

Then, from Equation (7) we obtain:

$$\rho(R) = \rho_m(R) + \rho_r(R) \quad (29)$$

$$p_r(R) = \frac{1}{3}\rho_r(R) \quad (30)$$

$$t(Y) = \frac{2\sqrt{C}}{3H_0\sqrt{\Omega_{r0}}} \left(\sqrt{Y+C}(Y-2C) + 2C^{3/2} \right) \quad (31)$$

$$Y(t) = \frac{R(t)}{R_0} \quad (32)$$

$$R_0 \equiv R(t = t_0) \equiv 1 \quad (33)$$

$$\Omega_r \equiv \frac{\rho_r}{\rho_c} \quad (34)$$

$$\Omega_m \equiv \frac{\rho_m}{\rho_c} \quad (35)$$

$$\rho_c \equiv \frac{3H^2}{8\pi G} \quad (36)$$

$$\Omega_{r0} + \Omega_{m0} \equiv 1 \quad (37)$$

$$\Omega_{r0} = \frac{1}{1 + \frac{1}{C}} \quad (38)$$

where t_0 is the age of the Universe (at the current time). It is important to highlight that t is the cosmic time, R_0 is the standard scale factor at the current time, $C \equiv \frac{\Omega_{r0}}{\Omega_{m0}}$, where Ω_{r0} and Ω_{m0} are the density energies normalized by the critical density at the current time, defined as the same as the Standard Cosmology. Furthermore, we have imposed that Universe must be flat ($k = 0$), so we require that $\Omega_r + \Omega_m \equiv 1$.

Using the Second Continuity Equation (12), where $\tilde{T}_{\mu\nu}$ is a new Energy-Momentum Tensor, two new densities called $\tilde{\rho}_M$ (Delta Matter Density) and $\tilde{\rho}_R$ (Delta Radiation Density) associated with this new tensor are defined. When we solve this equation, we find

$$\tilde{\rho}_M(Y) = -\frac{3\rho_{m0}}{2} \frac{F_a(Y)}{Y^3} \quad (39)$$

$$\tilde{\rho}_R(Y) = -2\rho_{r0} \frac{F_a(Y)}{Y^4} \quad (40)$$

Using the Second Field Equation (8) with the solutions (39) and (40) we found (and redefining with respect to Y):

$$F_a(Y) = -\frac{L_2}{3} Y \sqrt{Y+C} \quad (41)$$

Then, writing Equations (39) and (40) in terms of L_2 we have

$$\tilde{\rho}_m(Y) = \left(\frac{L_2}{2}\right) \rho_{m0} \frac{\sqrt{Y+C}}{Y^2} \quad (42)$$

$$\tilde{\rho}_r(Y) = \left(\frac{2L_2}{3}\right) \rho_{r0} \frac{\sqrt{Y+C}}{Y^3} \quad (43)$$

Thus, if we know the C and L_2 values, it is possible to know the Delta Densities $\tilde{\rho}_m$ and $\tilde{\rho}_r$.

Relation between the Effective Scale Factor Y_{DG} and the Scale Factor Y

The Effective Metric for the Universe is given by (28). From this expression, it is possible to define the Effective Scale Factor as follows:

$$R_{DG}(t) = R(t) \sqrt{\frac{1+F_a(t)}{1+3F_a(t)}} \quad (44)$$

Defining that $R(t_0) \equiv 1$, we have that $R(t) = Y(t)$. Furthermore, we define the Effective Scale Factor (normalized):

$$Y_{DG} \equiv \frac{R_{DG}(t)}{R_{DG}(t_0)} = \frac{Y}{Y(t_0)} \sqrt{\frac{1-L_2 \frac{Y}{3} \sqrt{Y+C}}{1-L_2 Y \sqrt{Y+C}}} \quad (45)$$

Please note that the denominator in Equation (45) is equal to zero when $1 = L_2 Y \sqrt{Y+C}$. Also remember that $C = \Omega_{r0}/\Omega_{m0} < 1$. Furthermore, we have imposed that $\tilde{\rho}_m > 0$ and $\tilde{\rho}_r > 0$, then L_2 must be greater than 0 [27]. Then the valid range for L_2 is approximately $0 \leq L_2 \leq 1$.

C must be positive, and (hopefully) is a very small value because the radiation is clearly not dominant in comparison with matter. Then, we can analyze cases close to the standard accepted value: $\Omega_{r0}/\Omega_{m0} \sim 10^{-4}$.

8. Useful Equations for Cosmology

Here we present the equations that are useful to fit the SN Data and obtain cosmological parameters that are presented in the Results Section.

8.1. Redshift Dependence

The relation between the cosmological redshift and the scale factor is preserved in Delta Gravity:

$$Y_{DG} = \frac{1}{1+z} \quad (46)$$

It is important to take into account that the current time is given by $t_0 \rightarrow Y(t_0) \rightarrow Y_{DG}(Y = 1) = 1$, where Y_{DG} is normalized.

8.2. Luminosity Distance

The proof is the same as GR, because the main idea is based on the light traveling through a null geodesic described by the Effective Metric given by (28) in Delta Gravity [32]. Taking into account that idea, we can obtain the following expression:

$$d_L(z, L_2, C) = c \frac{(1+z)\sqrt{C}}{100\sqrt{h^2\Omega_{r0}}} \int_{Y(t_1)}^1 \frac{Y}{\sqrt{Y+C}} \frac{dY}{Y_{DG}(t)} \quad (47)$$

Notice that $Y = 1$ today. To solve $Y(t_1)$ at a given redshift z , we need to solve (45) and (46) numerically. Furthermore, the integrand contains $Y_{DG}(t)$ that can be expressed in function of Y in (45). Do not confuse c (speed of light) with C , a free parameter to be fitted by SN Data.

The parameter $h^2\Omega_{r0}$ can be obtained from the CMB. The CMB Spectrum can be described by a Black Body Spectrum, where the energy density of photons is given by

$$\rho_{\gamma 0} = aT^4$$

From statistical mechanics, we know the neutrinos are related by [39]:

$$\rho_{\nu 0} = 3 \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \rho_{\gamma 0}$$

Then,

$$\Omega_{r0}h^2 = \Omega_{\gamma}h^2 + \Omega_{\nu}h^2 \quad (48)$$

Equation (48) is a value that only depends on the temperature of the Black Body Spectrum of the CMB. So we can add this value as a known Cosmological Parameter.

Thus, we only need to know the values C and L_2 . Take into account that it is impossible to know the value of Ω_{r0} without any other information.

8.3. Distance Modulus

The distance modulus is the difference between the apparent magnitude m and the absolute magnitude M of an astronomical object. Knowing this we can estimate the distance d to the object, provided that we know the value of the absolute magnitude M .

$$\mu = m - M = 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right) \quad (49)$$

8.4. Effective Scale Factor

The “size” of the Universe in Delta Gravity is given by $Y_{DG}(t)$, while in GR this is given by $a(t)$. Every cosmological parameter that in the GR theory was built up from the standard scale factor $a(t)$, in Delta Gravity will be built from $Y_{DG}(t)$. This value is equal to 1 at the current time, because it is the R_{DG} normalized by $R_{DG}(Y = 1)$.

8.5. Hubble Parameter

In Delta Gravity we will define the Hubble Parameter as follows:

$$H^{DG}(t) \equiv \frac{\dot{R}_{DG}(t)}{R_{DG}(t)} \quad (50)$$

Therefore, the Hubble Parameter is given by:

$$H^{DG}(t) = \frac{\frac{dR_{DG}}{dY} \left(\frac{dt}{dY} \right)^{-1}}{R_{DG}} \quad (51)$$

Notice that all the Delta Gravity parameters are written as function of Y .

8.6. Deceleration Parameter

In Delta Gravity we will define the deceleration parameter as follows:

$$q^{DG}(t) = -\frac{\ddot{R}_{DG} R_{DG}}{\dot{R}_{DG}^2} \quad (52)$$

Then,

$$q^{DG}(t) = -\frac{\frac{d}{dY} \left(\frac{dR_{DG}}{dY} \left(\frac{dt}{dY} \right)^{-1} \right) \left(\frac{dt}{dY} \right)^{-1} R_{DG}}{\left(\frac{dR_{DG}}{dY} \left(\frac{dt}{dY} \right)^{-1} \right)^2} \quad (53)$$

9. Fitting the SN Data

We are interested in the viability of Delta Gravity as a real Alternative Cosmology Theory that could explain the accelerating Universe without Λ , then it is natural to check if this model fits the SN Data.

9.1. SN Data

To analyze this, we used the most updated Type Ia Supernovae Catalog. We obtained the Data from Scolnic [40]. We only needed the distance modulus μ and the redshift z to the SN-Ia to fit the model using the Luminosity Distance d_L predicted from the theory.

The SN-Ia are very useful in cosmology [6] because they can be used as standard candles and allow to fit the Λ CDM model finding out free parameters such as Ω_Λ . We are interested in doing this in Delta Gravity. The main characteristic of the SN-Ia that makes them so useful is that they have a very standardized absolute magnitude close to -19 [41–45].

From the observations we only know the apparent magnitude and the redshift for each SN-Ia. Thus, we have the option to use a standardized absolute magnitude obtained by an independent method that does not involve Λ CDM model, or any other assumptions.

To fit the SN-Ia Data, we will use $M_V = -19.23 \pm 0.05$ [45]. The value was calculated using 210 SN-Ia Data from [45]. This value is independent from the model since it was calculated by building the distance ladder starting from local Cepheids measured by parallax and using them to calibrate the distance to Cepheids hosted in near galaxies (by Period-Luminosity relations) that are also SN-Ia host. Riess et al. calculated the M_V and the H_0 local value, and they did not use any particular cosmological model. Keep in mind that the value of M_V found by Riess et al. is an intrinsic property of SN-Ia and that is the reason they are used as standard candles.

We used 1048 SN-Ia Data in [40]². All the SN-Ia are spectroscopically confirmed. In this paper, we have used the full set of SN-Ia presented in [40]. They present a set of spectroscopically confirmed PS1 SN-Ia and combine this sample with spectroscopically confirmed SN-Ia from CfA1-4, CSP, PS1, SDSS, SNLS and HST SN surveys.

² Scolnic's Data are available at <https://archive.stsci.edu/hlsps/ps1cosmo/scolnic/>.

At [40] they used the SN Data to try to obtain a better estimation of the DE state equation. They define the distance modulus as follows:

$$\mu \equiv m_B - M + \alpha x_1 - \beta c + \Delta_M + \Delta_B \quad (54)$$

where μ is the distance modulus, Δ_M is a distance correction based on the host-galaxy mass of the SN and Δ_B is a distance correction based on predicted biases from simulations. Furthermore, α is the coefficient of the relation between luminosity and stretch, β is the coefficient of the relation between luminosity and color and M_V is the absolute B-band magnitude of a fiducial SN-Ia with $x_1 = 0$ and $c = 0$ [40].

In this work we are not interested in the specific corrections to observational magnitudes of SN-Ia. We only take the values extracted from [40] to analyze the Delta Gravity model. The SN Data are the redshift z_i and $(\mu + M)_i$ with the respective errors.

9.2. Delta Gravity Equations

We need to establish a relation between redshift and the apparent magnitude for the SN-Ia:

$$[\mu + M] - M = 5 \log_{10} \left(\frac{d_L(z, C, L_2)}{10 \text{ pc}} \right) \quad (55)$$

where $d_L(z, L_2, C)$ is given by (47) and $[\mu + M]$ are the SN-Ia Data given at [40].

In this expression we have as free parameters: C and L_2 to be found by fitting the model to the points $(z_i, [\mu + M]_i)$.

9.3. GR Equations

For GR we use the following expression

$$[\mu + M] - M = 5 \log_{10} \left(\frac{d_L(z, H_0, \Omega_{m0})}{10 \text{ pc}} \right) \quad (56)$$

where $d_L(z, H_0, \Omega_{m0})$ is given by:

$$d_L(z, H_0, \Omega_{m0}) = \frac{c(1+z)}{H_0} \int_{\frac{1}{1+z}}^1 \frac{du}{\sqrt{(1-\Omega_{m0})u^4 + \Omega_{m0}u}} \quad (57)$$

and $[\mu + M]$ are the SN Data given at [40]. Remember that we are always working on a flat Universe, and in GR standard model the Ω_{r0} is negligible. We have the same degrees of freedom as Delta Gravity.

Please note that we are including DE as $\Omega_{\Lambda 0} \equiv \Omega_{\Lambda} \equiv 1 - \Omega_{m0}$ in GR.

9.4. MCMC Method

To fit the SN-Ia Data to GR and Delta Gravity models, we used Markov Chain Monte Carlo (MCMC). This routine was implemented in Python 3.6 using PyMC2.³

Basically, MCMC consists on fitting a model, characterizing its posterior distribution. It is based on Bayesian Statistics. We used the Metropolis-Hastings algorithm.

We used a Bayesian approach because it allows us to know the posterior probability distribution for every parameter of the model [46,47]. Furthermore, it is possible to identify dependencies between the fitted parameters using MCMC, which it is not possible using another method such as the least-square used in [27].

Initially we propose initial distributions for the parameters that we want to fix, and then PyMC2 will give us the posterior probability distribution for these parameters.

³ <https://pymc-devs.github.io/pymc/>.

We want to find the best fitted parameters for Delta Gravity and GR models. These parameters will be C, L_2 for Delta Gravity and H_0, Ω_M for GR.

10. Results and Analysis

We present the results for Delta Gravity and GR fitted Data, and with these values we obtain different cosmological parameters. We divide the results into two fits: Delta Gravity Fit and GR Fit.

10.1. Fitted Curves

As we see in Figures 1 and 2, both models describe very well the m_B vs. z SN-Ia Data. It is important to note that, while in GR frame $\Lambda \neq 0$ is needed to find this well-behaved curve, in Delta Gravity Λ is not needed to fit the SN-Ia Data. Essentially, Delta Gravity predicts the same behavior, but the accelerating Universe appears explained without the need to include Λ , or anything like “Dark Energy”.

In Table 1, we present the coefficients of determination (r^2) and residual sum of squares (RSS) for both fitted models:

Table 1. Statistical parameters.

Model	r^2	RSS
Delta Gravity	0.99709	21.39
GR	0.99708	21.44

Both coefficients of determination are very good, and the RSS are similar for both cases.

The fitted parameters for GR and Delta Gravity models are shown in Tables 2 and 3 respectively.

Table 2. Fitted parameters using MCMC for Delta Gravity.

Delta Gravity	Value	Error
L_2	0.455	0.008
C	0.000169	0.000003

Table 3. Fitted parameters using MCMC for GR.

M_V Fixed GR Model	Value	Error
Ω_{m0}	0.28	0.01
h^2	0.549	0.004

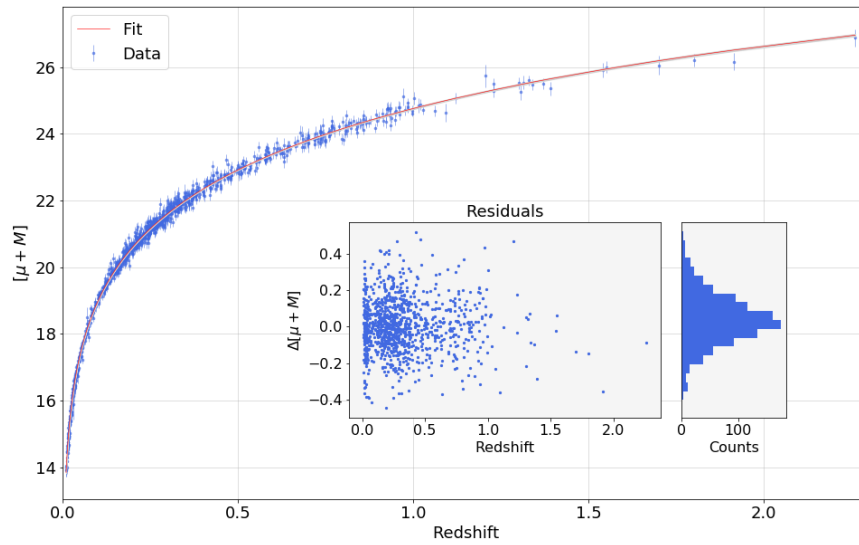


Figure 1. Fitted curve for Delta Gravity model assuming $M_V = -19.23$. On the right corner, the residual plot for the fitted Data.

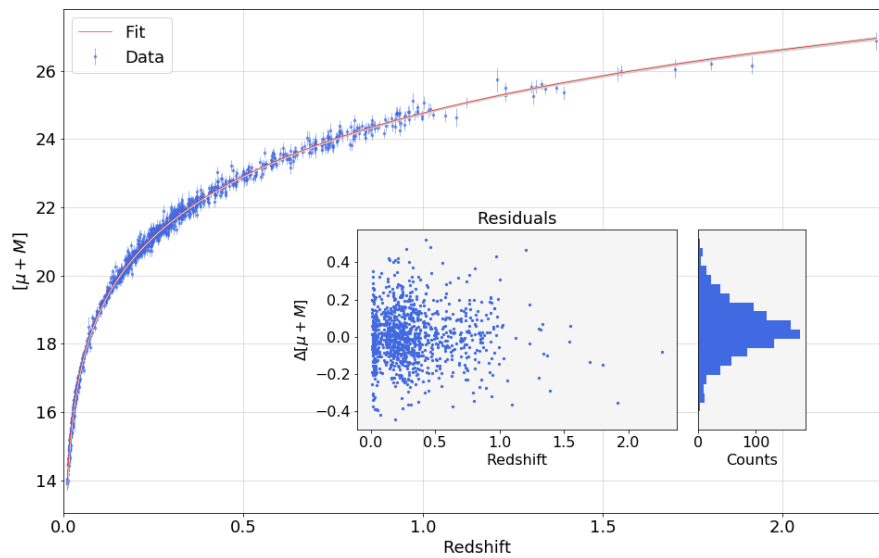


Figure 2. Fitted curve for GR standard model assuming $M_V = -19.23$. On the right corner, the residual plot for the fitted Data.

Furthermore, we present the posterior probability density maps for GR and Delta Gravity in Figure 3.

Please note that for both plots in Figure 3 the distributions are well defined, and for each parameter we obtain a Gaussian-like distribution. For both models, the combination of parameters constrained a region in the 2D-density plot. The fitted values for both models converged very well.

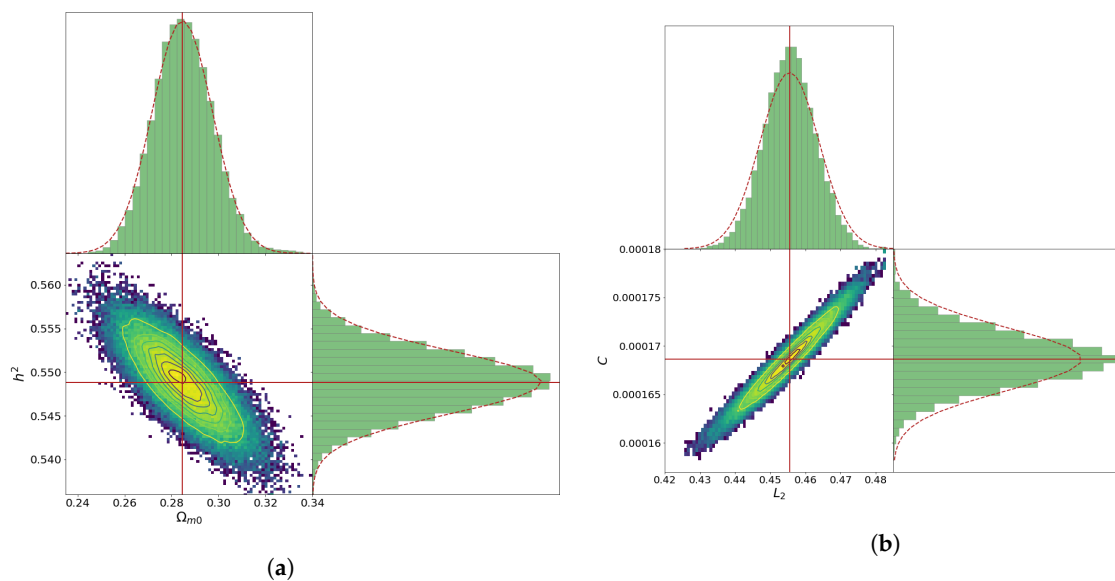


Figure 3. Posterior probability density plots obtained from MCMC for GR and Delta Gravity models. (a) Posterior probability density maps with for GR. Combination for Ω_{m0} and h^2 . (b) Posterior probability density maps for Delta Gravity. Combination for L_2 and C .

10.2. Convergence Tests

We applied two convergence tests for MCMC analysis. The first is known as Geweke [48]. This is a time-series approach that compares the mean and variance of segments from the beginning and end of a single chain. This method calculates values named z-scores (theoretically distributed as standard normal variates). If the chain has converged, the majority of points should fall within 2 standard deviations of zero⁴. The plots are shown in Figure 4.

In both plots it is possible to observe that the most part of the z-scores fall within 2σ , so the method is convergent for both models based on the Geweke criterion.

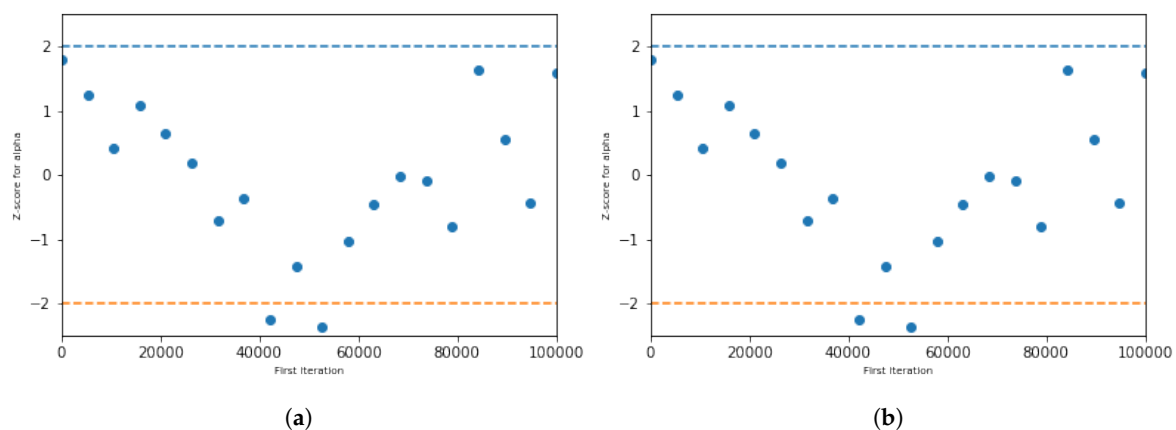


Figure 4. Convergence of values for GR and Delta Gravity. (a) Evolution of z-scores with steps in GR. (b) Evolution of z-scores with steps in Delta Gravity.

Another convergence test is the Gelman-Rubin statistic [49].

The Gelman-Rubin diagnostic uses an analysis of variance approach to assess convergence. This diagnostic uses multiple chains to check for lack of convergence, and is based on the notion that if

⁴ <https://media.readthedocs.org/pdf/pymcmc/latest/pymcmc.pdf>.

multiple chains have converged, by definition they should appear very similar to one another; if not, one or more of the chains has failed to converge (see PyMC 2 documentation).

In practice, we look for values of \hat{R} close to one because this is the indicator that shows convergence.

We ran 16 chains for Delta Gravity model. Figure 5 shows the L_2 and C predicted values for every chain of the Monte Carlo simulation. Figure 6a,b shows the convergence of L_2 and C . All the chains converge to a similar value assuming different priors. These final values predicted for every chain can be visualized in Figure 5. From all these chains, it is clear that the Delta Gravity MCMC analysis is convergent for the two free parameters.

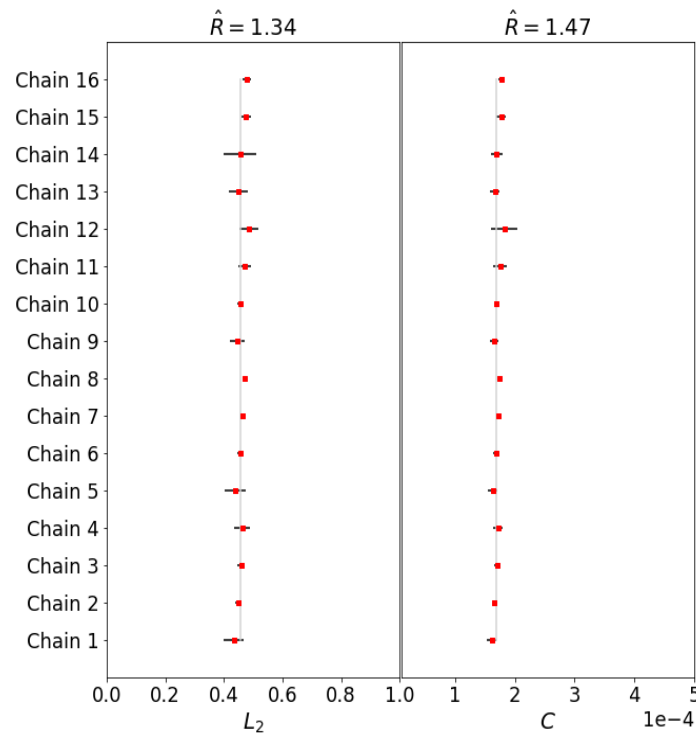


Figure 5. Gelman-Rubin test for Delta Gravity model assuming $M_V = -19.23$. The Gelman-Rubin test was run with 16 different chains, all with different L_2 and C priors. The \hat{R} coefficient (Gelman-Rubin coefficient) was calculated for each parameter.

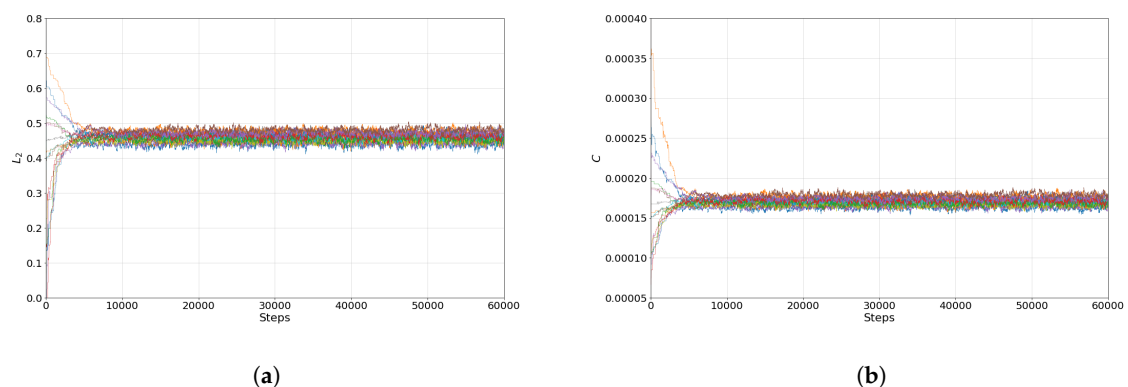


Figure 6. Gelman-Rubin test for Delta Gravity model. There are 16 chains with different priors. (a) All the chains converge to a $L_2 \approx 0.455$. (b) All the chains converge to a $C \approx 0.000169$.

10.3. Cosmic Time and Redshift

By using Equation (31) we obtain the Cosmic Time in Delta Gravity, where the redshift is obtained by numerical solution from Equation (46).

Meanwhile for GR model, we obtained the cosmic time from the integration of the first Friedmann equation and solving $t(\Omega_{m0}, H_0)$. Here we have included $\Omega_\Lambda = 1 - \Omega_{m0}$ and we did Ω_k ($k = 0$) and $\Omega_{r0} = 0$. The integral for the first Friedmann equation can be analytically solved:

$$t = \int_0^a \frac{1}{\sqrt{\frac{\Omega_{m0}}{x} + (1 - \Omega_{m0})x^2}} dx = \frac{2}{3\sqrt{1 - \Omega_{m0}}} \ln \left(\frac{\sqrt{-\Omega_{m0}a^3 + \Omega_{m0} + a^3} + \sqrt{1 - \Omega_{m0}a^3/2}}{\sqrt{\Omega_{m0}}} \right) \quad (58)$$

where t in (58) is the cosmic time for GR.

We plot the results in Figure 7:

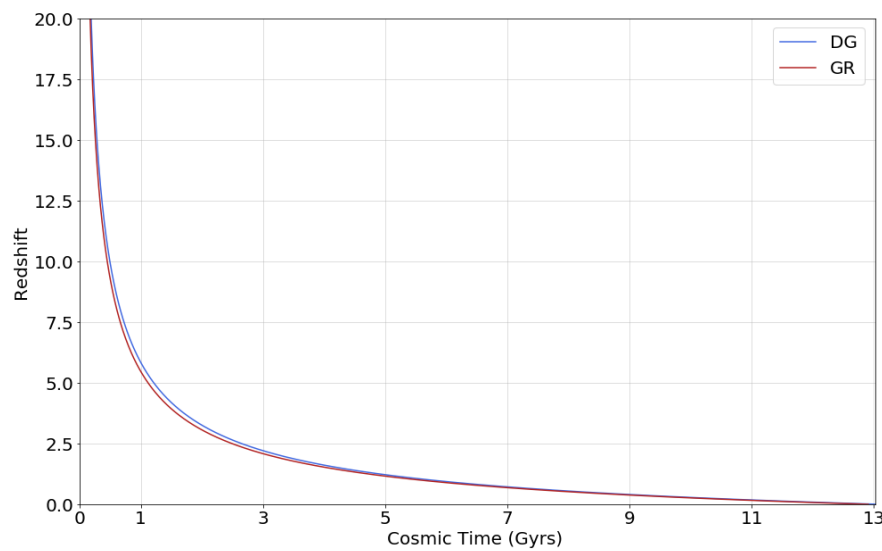


Figure 7. cosmic time for GR and Delta Gravity.

The behavior of cosmic time dependence with redshift for both models is very similar.

10.4. Hubble Parameter and H_0

With the fitted parameters found by MCMC for GR and Delta Gravity, we can find $H(t)$ and H_0 . Note the superscript for GR as GR and Delta Gravity as DG . For GR H_0 is easily obtained from the h^2 fitted ($H_0 = 100h$). $H^{GR}(t)$ can be obtained using the first Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left(\frac{\rho_{m0}}{a^3} + \frac{\rho_{r0}}{a^4} + \rho_{\Lambda 0} \right) \quad (59)$$

Taking into account that $\Omega_{m0} + \Omega_{r0} + \Omega_{\Lambda 0} = 1$, $\Omega_{r0} \approx 0$, and $\rho_{c0} = \frac{3H_0^2}{8\pi G}$, where $\Omega_{X_i 0} = \frac{\rho_{X_i 0}}{\rho_{c0}}$ for every X_i component in the Universe, we obtain

$$H^2 = H_0^2 \left(\frac{\Omega_{m0}}{a^3} + (1 - \Omega_{m0}) \right) \quad (60)$$

With (60), we obtain $H^{GR}(t)$ and using (51) we obtain $H^{DG}(t)$, Figure 8. For the actual time we evaluate H^{GR} at $a = 1$ and for Delta Gravity we evaluate H^{DG} at $Y_{DG} = 1$ obtaining the Hubble constant H_0^{GR} and H_0^{DG} .

We present the results for both models and we compare these values with previous measurements in Table 4.

Table 4. H_0 values found by MCMC with SN-Ia Data, assuming $M_V = -19.23$. Furthermore, we tabulate Planck [50] and Riess [51] H_0 values.

Model	H_0 (km/(s Mpc))	Error
Planck 2018 [50]	67.36	0.54
Riess 2018 ^a [51]	73.52	1.62
Riess 2018 ^b [51]	73.83	1.48
GR	74.08	0.24
Delta Gravity	74.47	1.63

^a The calibration was made including the new MW parallaxes from *HST* and *Gaia*; ^b The calibration was made considering the external constraints on the parallax offset based on Red Giants.

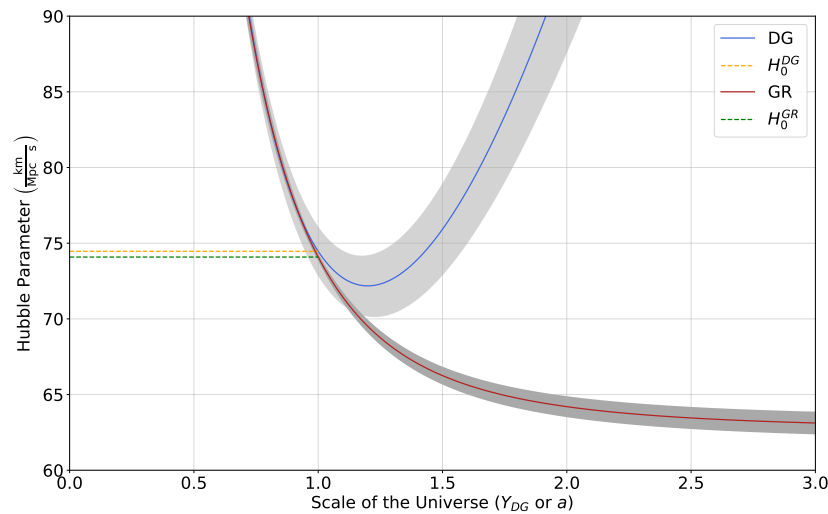


Figure 8. Hubble Parameter for Delta Gravity and GR fitted models assuming $M_V = -19.23$.

10.5. Age of the Universe

The age of the Universe in Delta Gravity is calculated using (31). $t(Y)$ only depends on C and not on L_2 . In GR we calculate the age of the Universe using (58).

With these expressions, we can compare the behavior between cosmic time and the scale factor in GR (or the effective scale factor in Delta Gravity).

In Figure 9, it is possible to see the evolution for $Y_{DG}(t)$ in time. At $t = 28.75$ Gyr, Y_{DG} goes to infinity, and the Universe ends with a Big Rip, then, in this model the Universe has an end (in time). Also, we see the dependence between the scale factor a and cosmic time t . The Universe has no end (in time) in GR.

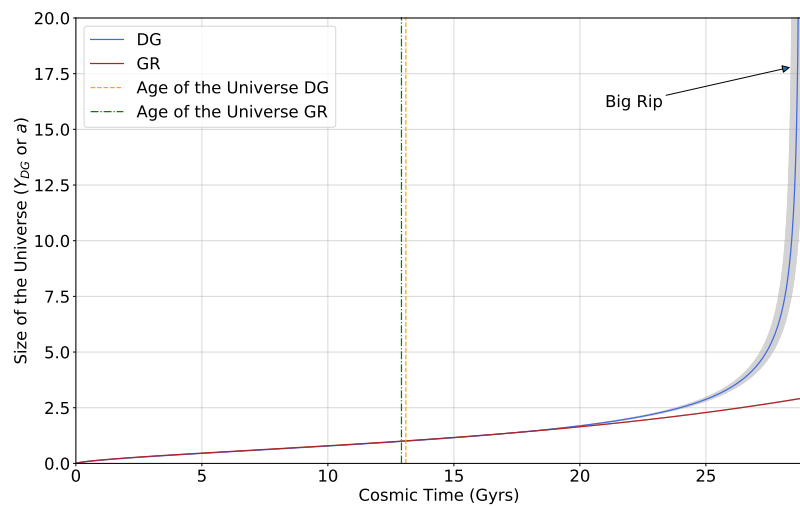


Figure 9. The size of the Universe vs. age of the Universe. In the Delta Gravity model, the size of the Universe Y_{DG} depends on cosmic time t and on C . The blue line indicates the effective scale factor in Delta Gravity. The gray zone shows the error associated with Y_{DG} . For GR, the scale factor a depends on cosmic time t and on Ω_{m0} . The red line indicates the scale factor evolution in GR. The gray zone shows the error associated with a (these are tiny).

10.6. Deceleration Parameter q_0

For Delta Gravity, we used Equation (53). For today, we evaluate $a = 1$ for GR, and $Y_{DG} = 1$ for Delta Gravity.

In Figure 10, we can see the evolution in time for both GR and Delta Gravity models.

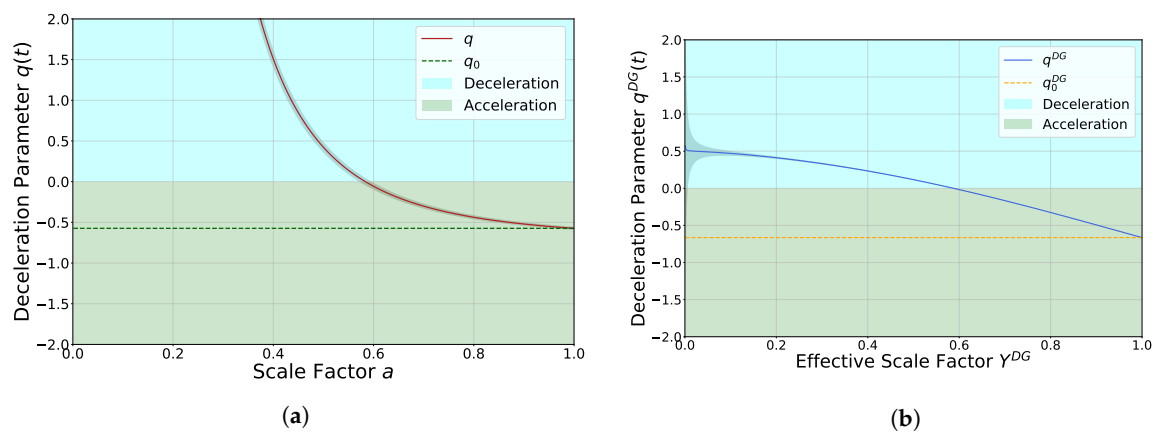


Figure 10. Deceleration parameter for both models. (a) Evolution of deceleration parameter in GR. (b) Evolution of deceleration parameter in Delta Gravity.

We tabulate the deceleration parameter for both models in Table 5.

Table 5. q_0 values found by MCMC with SN-Ia Data, assuming $M_V = -19.23$.

Model	q_0	Error
DG	−0.664	0.002
GR	−0.57	0.02

In both models $q_0 < 0$, then the Universe is accelerating.

10.7. Relation with Delta Components

In Delta Gravity we are interested in determining the Delta composition of the Universe. Using Equations (42) and (43), we can obtain the densities for Delta Matter and Delta Radiation with the C and L_2 fitted values.

$$\tilde{\rho}_{m0} = 0.22777\rho_{m0} = 0.22773\rho_{c0} \quad (61)$$

$$\tilde{\rho}_{r0} = 0.68330\rho_{r0} = 0.000115\rho_{c0} \quad (62)$$

In the expressions (62) and (61), we have obtained the current values for Delta Densities.

The Common Components are dominant compared with Delta components. Matter is always dominant compared with radiation (in both cases). See Figure 11.

Please note that the four components diverge (in density) at the beginning of the Universe, and the Delta Components show a “constant-like” behavior for $Y_{DG} > 0.4$. (Specially Delta Matter that is clearly dominant compared to the Delta Radiation).

In both the Common Components and Delta Components, there is a transition between matter and radiation that is indicated in the zoom in included in Figure 11. These transitions occur at very early stage of the Universe. Both transitions are indicated in Figure 11.

It is important to remember that in Delta Gravity we do not know the ρ_{c0} , but we know the densities of each component in units of ρ_{c0} , because they are given by C and L_2 fitted values from SN Data.

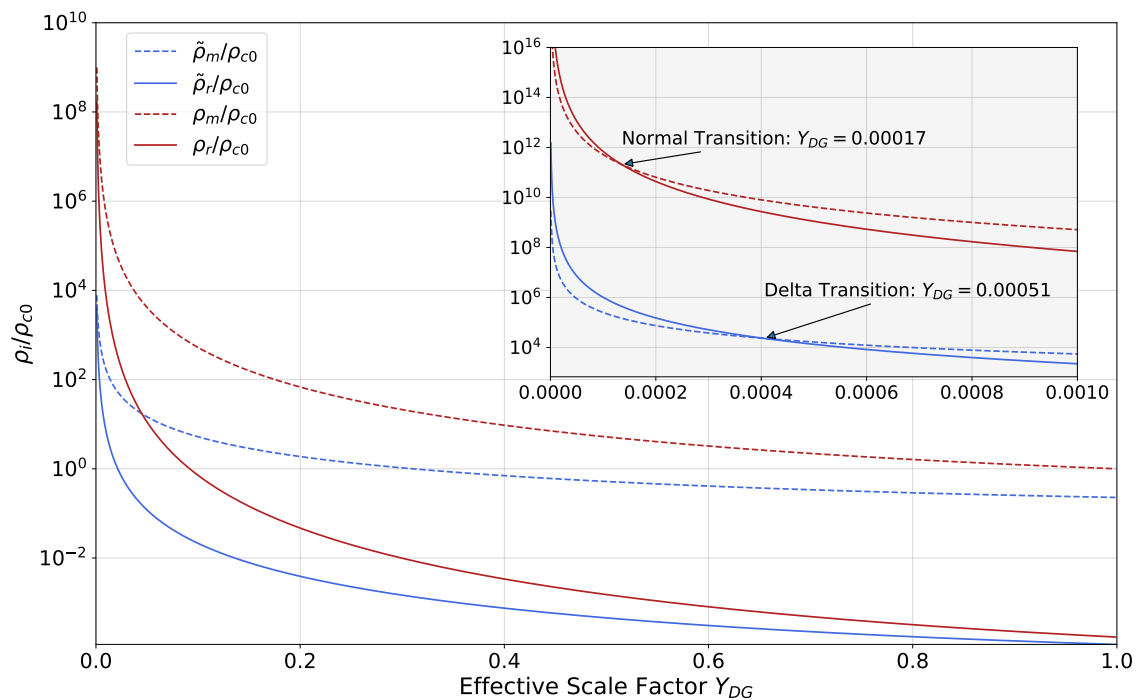


Figure 11. Temporal evolution of density components for Delta Gravity. The vertical axis is normalized by critical density at current time ρ_{c0} . On the top right corner, there is a zoom in very close to $Y_{DG} = 0$ showing the transition between Delta Matter and Delta Radiation (Delta components), and the transition between matter and radiation (common components). In general, the Common Density is higher than the Delta Density.

10.8. Importance of L_2 and C

To understand the role that L_2 and C are playing in the Delta Gravity model, we need to plot some cosmological parameters in function of both coefficients. We are interested in analyzing the

accelerating expansion of the Universe in function of these two parameters, so we plotted H_0^{DG} in Figure 12 and q_0^{DG} in Figure 13.

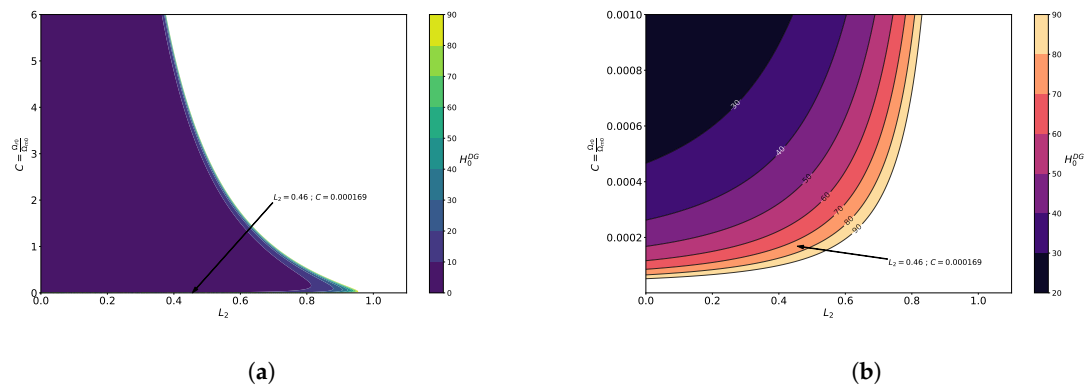


Figure 12. H_0^{DG} for a different combination of L_2 and C values. The fitted values found by MCMC analysis is indicated in the Figure. (a) C values go from 0 to 6 to explore various Universes, even a Universe wholly dominated by radiation. (b) The C values are bounded to very little values, nearly close to the C fitted value obtained by MCMC.

In Figure 12, we can see there is a big zone prohibited, because the results become complex values at certain level of the equations. The only allowed values are colored. Note that in Figure 12a almost all the allowed H_0^{DG} values are close to 0. Only the contour of the colored area shows $H_0^{DG} \neq 0$. The Figure 12b is the same as the left one, but with a big zoom in close the fitted values obtained from MCMC analysis. These range of C and L_2 are reasonable to make an analysis. Note that H_0^{DG} has a strong dependence of C and L_2 values.

Remember that L_2 has only sense between values 0 and 1, because we only want to allow positive Delta Densities and, from Equation (45), the denominator could be equal to 0.

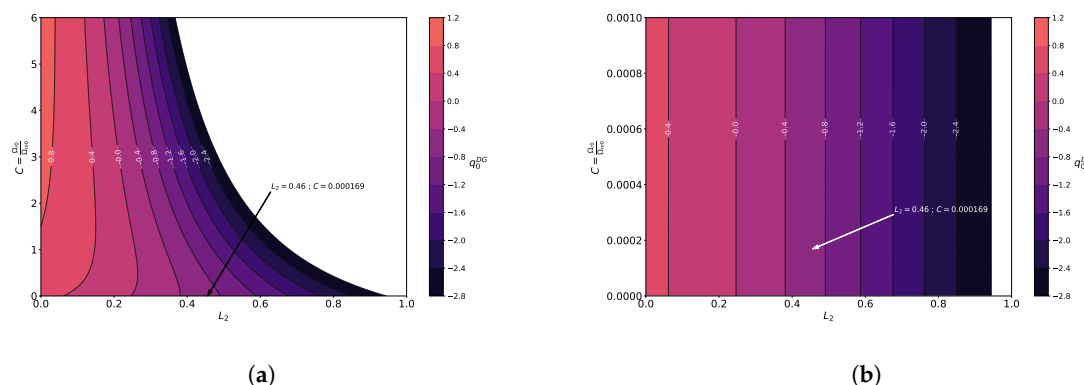


Figure 13. q_0^{DG} for different combination of L_2 and C values. The fitted values found by MCMC analysis is indicated in the Figure. (a) C values go from 0 to 6 to explore various Universes, even a Universe wholly dominated by radiation. (b) The C values are bounded to very little values, nearly close to the C fitted value obtained by MCMC.

The Figure 13 is very interesting because it shows the dependence of the current value of acceleration of the Universe expressed by the deceleration parameter q_0^{DG} . If we examine the parameters zone close to the fitted values in the Figure 13b, we can note that the acceleration of the Universe only depends on the value of L_2 . This is a very important result from the Delta Gravity model. The accelerating Universe is given by the L_2 parameter. This parameter appears naturally like an integration constant from the differential equations when we solved the field equations for Delta Gravity model. Then, in this model, and exploring the closest area to the Universe with a little

amount of radiation compared to matter, we found that a higher L_2 value, higher the acceleration of the Universe (current age): q_0^{DG} becomes more negative when $L_2 \rightarrow 1$ independently of C .

11. M Free

For completeness, we want to mention that we also did the MCMC analysis for M free in both GR and Delta Gravity.

From the MCMC analysis, we obtain a non-convergent result. In Delta Gravity model, the C and M parameters are dependent, but L_2 is independent. This can be visualized in Figure 14.

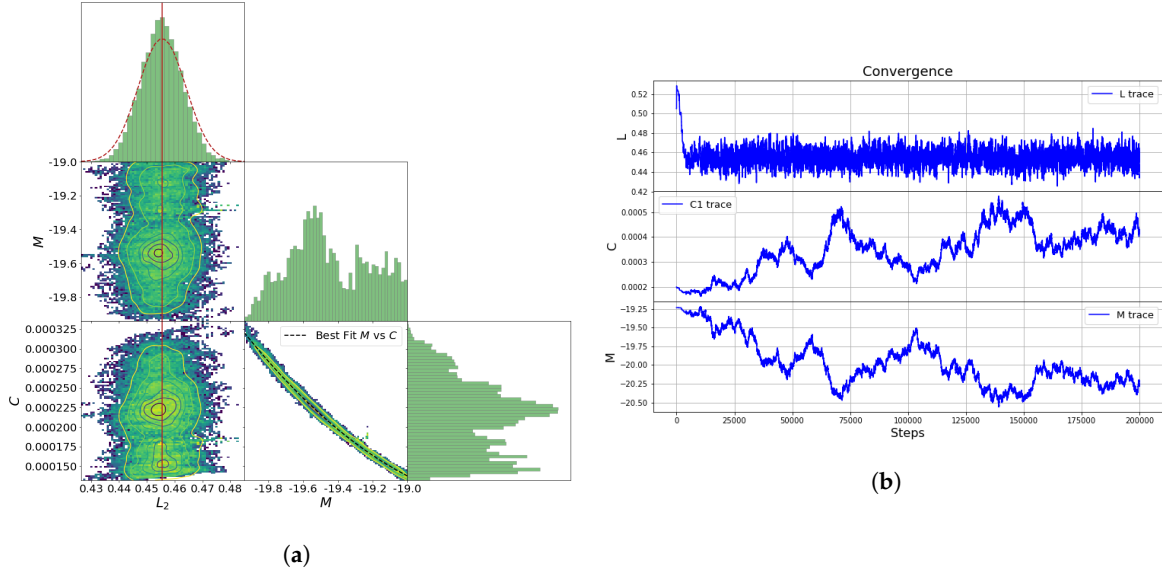


Figure 14. MCMC analysis assuming M as a free parameter in Delta Gravity. (a) Posterior probabilities densities. (b) Evolution of values with steps.

The dependence for Delta Gravity parameters can be fitted by a second order polynomial, as shown in Figure 14:

$$C = 8.59 \times 10^{-5} M^2 + 3.15 \times 10^{-3} M + 2.9 \times 10^{-2} \quad (63)$$

If we use $M = -19.23 \pm 0.05$ [45], we fix C which agrees with the results of the previous sections.

For GR, we did the same procedure, but in this model the dependence appears between h^2 and M . The polynomial is showed in Figure 15 and is given by:

$$h^2 = 0.177 M^2 + 7.335 M + 75.896 \quad (64)$$

Again, if we evaluate Equation (64) at $M = -19.23 \pm 0.05$, we obtain the h^2 value of previous sections.

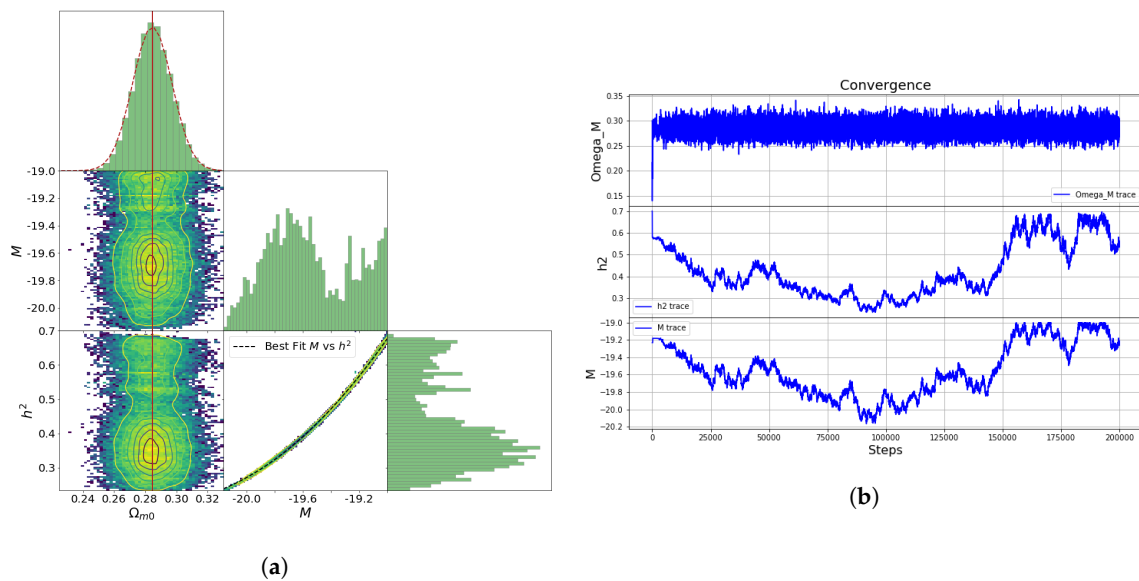


Figure 15. MCMC analysis assuming M as a free parameter in GR. (a) Posterior probabilities densities. (b) Evolution of values with steps.

12. Conclusions

Here we have studied the cosmological implications for a modified gravity theory, named Delta Gravity. The results from SN-Ia analysis indicate that Delta Gravity explains the accelerating expansion of the Universe without Λ or anything like “Dark Energy”. The acceleration is naturally produced by the Delta Gravity equations.

We assumed that $M_V = -19.23$ is a suitable value calculated from [45]. We want to emphasize the very important fact that this value was obtained by local measurements and calibrations of SN-Ia, and then, it is independent from any cosmological model. Assuming this, the procedure presented does not use Λ CDM assumptions. We only assume that the calibrations from Cepheids and SN-Ia are correct; therefore, the absolute magnitude $M_V = -19.23$ for SN-Ia is reasonably correct. In this case, the Universe is accelerating, and this result is stable under any change of the priors for the MCMC analysis. Note that the acceleration is highly determined by the L_2 value.

The acceleration in Delta Gravity is given by $L_2 \neq 0$. L_2 also determines that the Universe is made of Delta Matter and Delta Radiation. This can be associated with the new field: $\tilde{\phi}$. It is not clear if this Delta composition are real particles, or not.

Also, Delta Gravity can predict a high value for H_0 (assuming $M_V = -19.23$). This aspect is very important because the current H_0 value is in tension [45,51] between SN-Ia analysis and CMB Data. GR also predicts a high H_0 value with the same assumptions, but it needs to include Λ to fit the SN-Ia Data. The most important point about this, is that the local measurement of H_0 is independent of the model.⁵ Furthermore, the discrepancy about H_0 value could be indicating new physics beyond the Standard Cosmology Model Assumptions, and maybe, one possibility could be the modification of GR.

Another difference between Delta Gravity and GR models, is that Delta Gravity model predicts a Big Rip (as in phantom models [33,34]) that is dominated by the L_2 value. This is shown in Figure 9.

The most important difference between Delta Gravity and the Standard cosmological model is the explanation about “Dark Energy” (the relation of L_2 with the accelerated expansion of the Universe).

⁵ “The direct measurement is very model independent, but prone to systematics related to local flows and the standard candle assumption. On the other hand, the indirect method is very robust and precise, but relies completely on the underlying model to be correct. Any disagreement between the two types of measurements could in principle point to a problem with the underlying Λ CDM model.” [52].

Author Contributions: The theory has been developed by J.A. The methodology to obtain the cosmological parameters and fitting the SN-Ia data was made by M.S.M. The Python code to fit the SN-Ia data and obtain all the plots was made by M.S.M. and J.S. The analysis and writing was done by all the authors.

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Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

GR	General Relativity
DG	Delta Gravity
DM	Dark Matter
DE	Dark Energy
SN-Ia	Type Ia Supernova
CMB	Cosmic Microwave Background Radiation
Λ CDM	Λ Cold Dark Matter
MCMC	Markov Chain Monte Carlo

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