



Article

# **GRASP Manual for Users**

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**Abstract:** GRASP is a software package in Fortran 95, adapted to run in parallel under MPI, for research in atomic physics. The basic premise is that, given a wave function, any observed atomic property can be computed. Thus, the first step is always to determine a wave function. Different properties challenge the accuracy of the wave function in different ways. This software is distributed under the MIT Licence.

**Keywords:** GRASP; atomic properties; atomic wave function; multiconfigurational Dirac–Hartree–Fock; configuration interaction



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### 1. GRASP for Atomic Physics

#### 1.1. Relativistic vs. Non-Relativistic Calculations

The General Relativistic Atomic Structure Package (GRASP) is based on the fully relativistic (four-component) multiconfiguration Dirac–Hartree–Fock (MCDHF) method and is suitable for medium to heavy atomic systems. For light and near neutral systems, where relativistic effects often (though not always) are comparatively small, the ATSP2K Atomic Structure Package [1], based on the non-relativistic multiconfiguration Hartree–Fock (MCHF) method with Breit–Pauli (BP) relativistic corrections, may be a better choice. The MCHF-BP method allows *LS* symmetries to be used, which often makes it possible to include more electron correlation. In addition, semi-empirical fine-tuning of the energies can be done, that leads to more accurate results, especially in cases with closely degenerate states. ATSP2K and the corresponding manual can be downloaded from GitHub: <a href="https://github.com/compas">https://github.com/compas</a>, accessed on 5 November 2022.

## 1.2. Features of the Package

The first GRASP manual, distributed in 1980, described how a deck of cards needed to be assembled and submitted with the program deck that computed both the wave function and, say, a transition probability. Wave function expansions were just a few configuration state functions (CSFs). Its successor, GRASP92 was quite different. It divided the problem into stages so that all resources available could be used at every stage, and intermediate results were stored. The basic strategy was similar to that of ATSP2K, thereby it became a package rather than a single program. In time, the typical expansion size of a wave function has increased from 100–1000 to 5–50 millions today. What we are describing is the current version that still is evolving. This GRASP, like its predecessors, is based on the MCDHF method; see [2,3] for an account of the general theory. The package consists of a number of application programs and tools to compute approximate relativistic wave functions, from which atomic properties such as energy levels, hyperfine structures, Landé  $g_I$ -factors, isotope shifts, interactions with external fields, angular couplings for labeling purposes, radial electron density functions and transition energies and transition probabilities for many-electron atomic systems can be computed. There are also some graphical utilities. The application programs and tools, along with the underlying theory, are described in the original write-ups [4–16]. The present manual updates the previous version (GRASP2018[4]), to include also the most recent application programs. For convenience, the theory, as it applies to all the programs described in this manual, is presented in the accompanying paper [17] in the present Special Issue. The manual and the accompanying theory paper (or TP for short) go hand in hand, and we will refer to the latter in the coming sections. Using GRASP, research into highly accurate transition energies and transition rates as well as detailed electron nucleus interactions becomes feasible for a wide range of atomic systems.

The main features of the package are as follows:

- 1. There are efficient and easy to use programs to generate lists of CSFs that capture different electron correlation effects. The concepts of CSFs and electron correlation are discussed in TP Sections 2.4 and 4.
- 2. The interaction matrix, see TP Sections 2.2 and 2.8, is considered to be a series of sparse non-interacting blocks of given parity and *J* value, with selected eigenvalues and eigenvectors determined from each. For a description of the sparse Davidson eigenvalues library module, see [18].
- 3. Spin-angular integrations are based on second quantization in the coupled tensorial form, angular momentum theory in three spaces (orbital, spin and quasi-spin), and a generalized graphical technique. The theoretical background can be found in [19–21] as well as in TP Section 2. The spin-angular library is fully documented by Gaigalas [22] in the present Special Issue.
- 4. Wave functions in *jj*-coupling can be transformed to a basis of *LSJ*-coupled CSFs, see [23–26] and TP Section 2.9. Labels in *LSJ*-coupling are used by several programs in the package.

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5. Wave function in *jj*-coupling can be transformed to a basis of several other, e.g., *JK*, *LK*, coupling schemes CSFs, see [14]. Labels in different coupling schemes are used by many programs in the package.

- 6. Separately optimized initial and final state wave functions can be used to compute transition rates. The non-orthogonality between initial and final state radial orbitals is handled by an efficient biorthonormal transformation technique. The computation of transition rates and the use of transformation techniques are described in TP Section 3.5, see also [27].
- 7. The interaction between the electrons and extended and deformed nuclei can be described in a model independent way. The background assumptions are given in [12] as well as in TP Section 3.3.
- 8. MPI codes for parallel processing are available for the most time-consuming programs of the package.
- 9. Zero- and first-order perturbative methods can be used to handle large CSF expansions; see [2,28] and TP Section 2.8.

# 1.3. Downloading and Installing GRASP

GRASP is a series of libraries, application programs and tools written in Fortran 95 and adapted to run in parallel under MPI, a language-independent communication protocol. In addition, there are GNU Octave and Matlab M-files for graphical purposes. GRASP can be downloaded from GitHub: https://github.com/compas, accessed on 5 November 2022. The downloaded package contains the following directories:

```
directory where, after compilation, the executables reside
directory where, after compilation, the static library archives reside
src directory with the subdirectories appl, containing the source code
for the application programs, lib, containing the source code for the
libraries and tool, containing the source code for the tools.
grasptest directory containing scripts for all the test runs and examples in this manual
```

The package can be installed using CMake, that generates the necessary build files for either the GNU gfortran or Intel (ifort or ifx) compilers. For backward compatibility, the package can also be installed by running a pre-defined makefile. Detailed instructions can be found on GitHub: <a href="https://github.com/compas">https://github.com/compas</a>, accessed on 5 November 2022. Upon successful installation the following 6 static library files, where the suffix .a stands for archive, should appear in the lib directory

```
lib9290.a libmcp90.a libmpiu90.a libdvd90.a libmod.a librang90.a
```

The following 25 executable application programs should be found in the bin directory, where the extension \_mpi indicates that the executable can be run in parallel under MPI

```
jj2lsj
                                                    rangular
                                                                       rangular_mpi
                                   jjgen
                                                                       rcsfgenerate
rbiotransform rbiotransform_mpi
                                  rci
                                                    rci_mpi
               rcsfzerofirst
                                                                       hfszeeman95
rcsfinteract
                                  rdensity
                                                    rhfs
               rmcdhf
ris4
                                  rmcdhf_mem
                                                    rmcdhf_mpi
                                                                       rmcdhf_mem_mpi
rnucleus
               rtransition
                                  rtransition_mpi rtransition_phase
                                                                       rwfnestimate
```

The following 24 executable tools should also be found in the bin directory.

```
rasfsplit
            rcsfblock
                         rcsfmr
                                          rcsfsplit
                                                        rhfs_lsj
rlevelseV
            rlevels
                         rmixaccumulate
                                         rmixextract
                                                       rseqenergy
rseqhfs
            rseqtrans
                          rtabhfs
                                          rtablevels
                                                        rtabtrans1
rtabtrans2
            rtabtransE1
                         rwfnmchfmcdf
                                          rwfnplot
                                                        rwfnrelabel
rwfnrotate rwfntotxt
                          wfnplot
                                          fical
```

In addition, there are 4 script files

```
lscomp.pl mithit rsave rwfnpyplot
```

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The use of each of the application programs, tools, and script files will be discussed in the following sections.

The Coupling program, that is used to find the optimal coupling schemes, can be downloaded from <a href="https://github.com/compas/coupling">https://github.com/compas/coupling</a>, accessed on 5 November 2022. In this manual, we assume that the Coupling program has been installed and that the corresponding binary file is on the path.

# 1.4. Changing Parameters in the Package

The application programs are written in terms of some basic parameters. Most, but not all, are set in the directory GRASP2018/src/lib/libmod and can be changed by editing the file parameter\_def\_M.f90. These include parameters that define the grid, see TP Section 2.2. Often changes are with respect to the location of the first point away from the origin, defined in terms of a variable RNT that changes the number of points of the grid. The above installation sets the maximum number of grid points NNNP for representing the radial parts of the one-electron orbitals to the default value NNNP=590. This default value works fine in most cases. For heavy or super heavy elements, it is sometimes necessary to extend the number of grid points. Another parameter defining the grid is the step-size H. Reducing this parameter would improve the numerical accuracy of the calculations but, at the same time, might require an increase of the number of grid-points. To install the program with an extended grid, start by deleting the old executables and libraries in the GRASP2018/bin and GRASP2018/lib directories by issuing the make clean command in the GRASP2018/src directory and change the number of grid points from NNNP=590 to a larger value, say NNNP=1990. At the same time, set NNN1=2000 (NNN1 = NNNP + 10). Recompile all the package. After recompilation, all programs and tools in the GRASP2018/bin directory will be based on the extended grid. Unless explicitly stated, all examples in this guide are based on programs with the default grid NNNP=590. In Section 13.5, however, we have a specific example with an extended grid.

The rci programs (including the MPI version) have a parameter NINCOR that decides whether the eigenvalue problem stores the interaction matrix in memory or on disk, in terms of the memory requirement for all the non-zero matrix elements. This parameter has been increased to the number of double precision matrix elements that can be stored in 2 Gigabytes of memory. For the MPI version, this is a memory requirement per CPU. Another parameter is IOLPCK that determines whether matrices are stored in a sparse format and solved by the Davidson method or are small enough to be stored in the dense, symmetric matrix format and eigenvalues computed using a Lapack routine. This parameter is set to 2000. Both parameters can be modified by the user.

# 1.5. Citing the Package

Developing computational methods and programs is challenging, often requiring intensive effort. The work needs to be properly acknowledged and quoted in order to be continued.

1. Users of the current package should quote the most recent publication :

C. Froese Fischer, G. Gaigalas, P. Jönsson and J. Bieroń

GRASP 2018 - A Fortran 95 version of the General Relativistic Atomic Structure Package

Comput. Phys. Commun. 237, 184-187 (2019)

https://doi.org/10.1016/j.cpc.2018.10.032, accessed on 5 November 2022.

2. In addition, when applicable, users of ris4 should quote it as:

J. Ekman, P. Jönsson, M. Godefroid, C. Nazé, G. Gaigalas and J. Bieroń

RIS 4: A program for relativistic isotope shift calculations

Comput. Phys. Commun. 235, 433-446 (2019)

https://doi.org/10.1016/j.cpc.2018.08.017, accessed on 5 November 2022.

3. Users of COUPLING should quote it as:

G. Gaigalas

Coupling: The program for searching optimal coupling scheme in atomic theory

Comput. Phys. Commun. 247, 106960 (2020)

https://doi.org/10.1016/j.cpc.2019.106960, accessed on 5 November 2022.

4. Users of Hfszeeman95 should quote it as:

W. Li, J. Grumer, T. Brage and P. Jönsson

HFSZEEMAN95 — A program for computing weak and intermediate magnetic-field- and hyperfine-induced transition rates

Comput. Phys. Commun. 253, 107211 (2020)

https://doi.org/10.1016/j.cpc.2020.107211, accessed on 5 November 2022.

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5. Users of RDENSITY should quote it as:

S. Schiffmann, J.G. Li, J.Ekman, G. Gaigalas, M. Godefroid, P. Jönsson and J. Bieroń Relativistic radial electron density functions and natural orbitals from GRASP2018 Comput. Phys. Commun. 278, 108403 (2022) https://doi.org/10.1016/j.cpc.2022.108403, accessed on 5 November 2022.

#### 1.6. Reporting Errors

The programs and tools have been extensively tested and used, but as new calculations are tried, errors may be encountered. If you, the user of the program package, have reasons to believe that there is an error somewhere in the package, please send an email to one of the authors specifying the case that resulted in the error. Additionally, if there are sections in this manual that are unclear, please let us know. Better yet, if you find the needed correction, let us know so that others may benefit as well.

### 2. Package Structure and File Flow

#### 2.1. Program Naming Conventions

In multiconfiguration calculations, the wave function for an atomic state is approximated by an atomic state function (ASF). The ASF, in turn, is given as an expansion over CSFs

$$\Psi(\Gamma J M_J \pi) = \sum_{\alpha=1}^{N_{CSF}} c_{\alpha}^{\Gamma J} \Phi(\gamma_{\alpha} J M_J \pi). \tag{1}$$

Here  $\{\gamma_{\alpha}\}$  denote the configurations together with the angular coupling trees,  $\pi$  is parity, J is the total angular quantum number, and  $\{c_{\alpha}^{\Gamma}\}$  are the expansion (mixing) coefficients. The CSFs are given as coupled anti-symmetric products of one-electron orbitals

$$\psi_{n\kappa m}(r,\theta,\varphi) = \frac{1}{r} \begin{pmatrix} P_{n\kappa}(r) \ \Omega_{\kappa m}(\theta,\varphi) \\ i \ Q_{n\kappa}(r) \ \Omega_{-\kappa m}(\theta,\varphi) \end{pmatrix}, \tag{2}$$

where the radial parts of the orbitals (the radial wave functions)  $P(n\kappa;r)$ ,  $Q(n\kappa;r)$  are numerically represented on a grid, see TP Sections 2.1, 2.2 and 2.4 for a description of the CSFs and their construction. (In the guide the three terms radial orbital, radial part of the orbital, and radial wave function will be used intermixed meaning the same thing. Sometimes we will also loosely speak about the orbitals, meaning the radial parts of the orbitals.)

Given this description, we identify three main concepts:

- lists of CSFs defining the ASFs
- mixing coefficients
- radial parts of orbitals (radial wave functions)

These concepts provide the basis for the program naming conventions: programs generating or manipulating lists of CSFs have names starting with rcsf, programs generating or manipulating mixing coefficients have names starting with rmix, programs generating or manipulating the radial parts of the orbitals (radial wave functions) have names starting with rwfn. Other programs are named according to the atomic properties they compute. There are also a number of programs that produce output tables in LaTeX format. These programs all have names starting with rtab. Finally, there are programs that create GNU Octave and Matlab M-files for plotting properties along iso-electronic sequences. These programs have names starting with rseq.

#### 2.2. Application Programs and Tools

Below is a partial list of programs in the package. The extension \_mpi indicates that the program can be run in parallel under MPI:

- 1. rnucleus define nuclear data, including magnetic dipole and electric quadrupole moments, see TP Section 2.3
- 2. Routines that generate and manipulate lists of CSFs, see TP Section 4:
  - (a) rcsfgenerate generate a list of CSFs using rules for excitations.
  - (b) jjgen generate a list of CSFs. More general than rcsfgenerate, but more involved to run.
  - (c) resfinteract reduce a list of CSFs by retaining only CSFs that interact with CSFs of a reference list.
  - (d) rcsfsplit split a list of CSFs into a number of lists with CSFs that can be formed from different sets of active orbitals.

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(e) rcsfzerofirst — rearrange a list of CSFs in such a way that the most important CSFs are listed at the beginning, defining the zero-order space, and the less important are listed at the end, defining the first-order space, see TP Section 2.8.

- 3. rangular, rangular\_mpi perform angular integration and compute angular coefficients, see TP Section 2.6.
- 4. rwfnestimate estimate the radial parts of the orbitals (radial wave functions), see TP Section 2.7.
- 5. rmcdhf\_mem, rmcdhf, rmcdhf\_mem\_mpi, rmcdhf\_mpi determine radial parts of the orbitals and mixing coefficients of the CSFs in a relativistic self-consistent-field (SCF) procedure, see TP Section 2.7. The extension \_mem indicates that all angular data are kept in core and are not read from disk. rmcdhf\_mem and rmcdhf\_mem\_mpi are the preferred programs when enough RAM is available. Wave functions from these programs are referred to as rmcdhf wave functions.
- 6. rci, rci\_mpi perform relativistic configuration interaction (rci) calculation with transverse photon (Breit) interaction and vacuum polarization and self-energy (QED) corrections, see TP Sections 2.3 and 2.8. Wave functions from these programs are referred to as rci wave functions.
- 7. jj21sj a program for converting a portion of the wave function expansion in *jj*-coupled CSFs to a basis of *LSJ*-coupled CSFs for labeling purposes, see [23–26] and TP Section 2.9. Includes a feature to provide unique labels for all the considered states.
- 8. Coupling a program for searching the optimal coupling scheme, see [14].
- 9. Programs for computing transition probabilities,
  - (a) rbiotransform, rbiotransform\_mpi perform biorthonormal transformations of wave functions, see TP Section 3.5.
  - (b) rtransition, rtransition\_phase, rtransition\_mpi compute transition properties from transformed wave functions, see TP, Sections 3.5 and 3.6. The extension \_phase indicates that the program outputs additional phase information needed by the mithit program. If the jj2lsj program has been run, the labels of the states in the output files are in *LSJ*-coupling. If the Coupling program has been run, the levels of the states in the output files can be presented in other coupling schemes.
- 10. rhfs compute diagonal and off-diagonal hyperfine interaction constants and Landé  $g_J$ -factors, see [8] and TP Sections 3.1 and 3.2.
- 11. ris4 compute isotope shift and detailed electron and nucleus interactions, see [12] and TP Section 3.3.
- 12. hfszeeman95 compute reduced matrix elements for magnetic interactions as well as for hyperfine interactions, see [11] and TP Section 3.2.
- 13. mithit compute, given reduced matrix elements from hfszeeman95, and plot Zeeman splittings of fine- and hyperfine levels as functions of the magnetic field. Compute transition rates between magnetic fine- and hyperfine structure substates in the presence of an external magnetic field and the rates of hyperfine induced transitions in the field-free limit. Synthesizes spectral profiles, see [11] and TP Section 3.6.
- 14. rdensity compute the radial electron density function and transform to natural orbitals, see [16] and TP Section 3.4.

A number of generally short programs have been developed as tools to facilitate computational procedures.

- 1. rmixaccumulate accumulate CSFs corresponding to a specified fraction of the total wave function.
- rmixextract extract and print the numerical values of the expansion coefficients above a cut-off value along
  with the corresponding CSFs, in descending order of magnitude, if requested.
- 3. rcsfmr analyse the wave function expansion in *LSJ*-coupled CSFs and determine a multireference (MR).
- 4. hf perform a non-relativistic Hartree–Fock (HF) calculation to produce a radial wave function file wfn.out. The file wfn.out should be copied to wfn.inp for further processing by rwfnmchfmcdf.
- 5. rwfnmchfmcdf convert a non-relativistic Hartree—Fock radial wave function file, wfn.inp, to a GRASP radial wave function file, rwfn.out, that can be used with rwfnestimate.
- 6. rwfntotxt write radial wave functions in binary format to a text file.
- 7. rwfnpyplot Python script to plot radial wave functions from files produced by rwfntotxt.
- 8. rwfnplot extract radial wave functions from a radial wave function file and generate a GNU Octave/Matlab M-file that plots the radial wave functions as functions of  $\sqrt{r}$  or r.
- 9. wfnplot extract radial wave functions from the non-relativistic radial wave function file as produced by the hf program and generate a GNU Octave/Matlab M-file that plots the radial wave functions as functions of  $\sqrt{r}$  or r.
- 10. rwfnrotate a routine that rotates radial orbitals, useful for debugging purposes.

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11. rlevels — list the levels in a series of mixing files, in the order of increasing energy and report levels in cm<sup>-1</sup> relative to the lowest. If the jj2lsj program has been run, the levels are given in *LSJ*-coupling notation. If the Coupling program has been run, the levels are given in other coupling schemes, as determined by the user.

- 12. rlevelseV list the levels in a series of mixing files, in the order of increasing energy and report levels in eV relative to the lowest. If the program jj2lsj has been run, the levels are given in *LSJ*-coupling notation. If the program Coupling has been run, the levels are given in other coupling schemes.
- 13. rtablevels produce LaTeX and ASCII tables of energies from energy files produced by rlevels.
- 14. lscomp.pl perl script to produce LaTeX tables with LSJ composition and energies from energy files rlevels.
- 15. rtabtransE1 produce LaTeX and ASCII tables of transition parameters from files produced by rtransition (E1 transitions only).
- rtabtrans1 and rtabtrans2 produce LaTeX tables of transition parameters and lifetimes from files produced by rtransition.
- 17. rhfs\_lsj give the output from the rhfs program in *LSJ*-coupling notation.
- 18. rtabhfs produce LaTeX tables of hyperfine interaction constants.
- 19. rsequence produce GNU Octave/Matlab M-files that plot energies as functions of *Z* along an iso-electronic sequence.
- 20. rseqhfs produce GNU Octave/Matlab M-files that plot hyperfine interaction constants and Landé  $g_J$ -factors as functions of Z along an iso-electronic sequence.
- 21. rseqtrans produce GNU Octave/Matlab M-files that plot transition parameters as functions of *Z* along an iso-electronic sequence.
- 22. rsave a script file such that the command rsave name copies rwfn.out to name.w and rcsf.inp to name.c and moves rmix.out to name.m, rmcdhf.sum to name.sum, rangular.log to name.alog and rmcdhf.log to name.log.
- 23. rasfsplit splits the files defining a number of ASFs of different symmetry blocks (*J* and parity) into groups of files, one for each symmetry block.
- 24. rcsfblock splits the list produced by jjgen into block-form.
- 25. fical an auxiliary program that computes frequency isotope shifts given the output from ris4 and supplied nuclear data.

#### 2.3. File Naming Convention, Program, and Data Flow

The passing of information between different programs is done through files. This process is greatly simplified by a file naming convention. GRASP uses a convention similar to the one for ATSP2K [1]: a name is associated with the results from a calculation, and an extension defines the content and format of a file. Thus, the file name becomes name.extension. Common extensions are listed in Table 1. The tool rsave makes use of these default extensions to save the output files from an rmcdhf calculation. Most programs produce a file that keeps a record of the input data. This file is called a log-file.

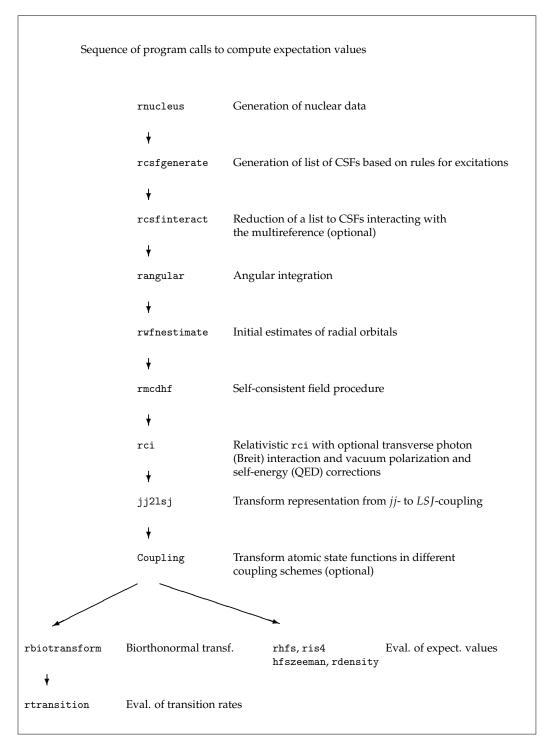
To perform a calculation, a number of programs need to be run in a predetermined sequence. Figure 1 shows a typical sequence of program calls to compute wave functions and different expectation values. The resulting flow of files is displayed in Figure 2.

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**Table 1.** Common file extensions.

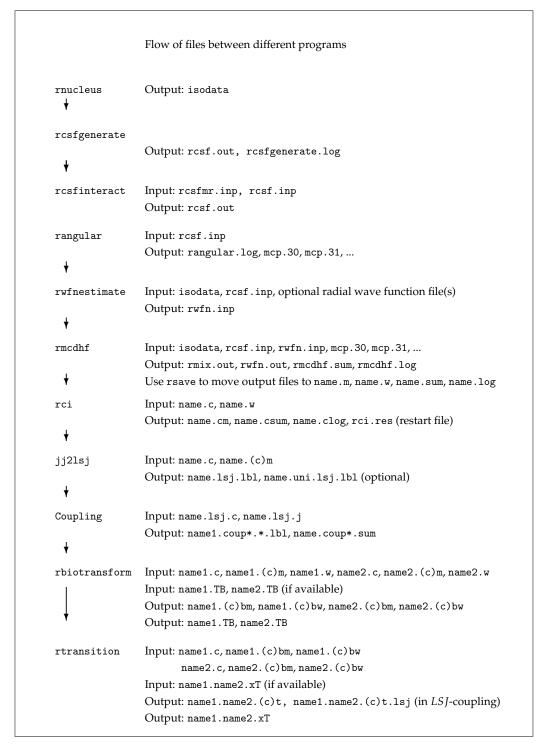
Extension	Type of file
С	List of CSFs.
W	Binary file of radial wave functions.
m	Binary file of expansion or mixing coefficients produced by rmcdhf.
sum	File containing information from an rmcdhf run.
cm	Binary file of mixing coefficients produced by rci.
csum	File containing information from an rci run.
bw	A .w file after biorthonormal transformation using rbiotransform.
bm	A .m file after biorthonormal transformation using rbiotransform.
cbm	A .cm file after biorthonormal transformation using rbiotransform.
lsj.lbl	File containing composition of wave functions in <i>LSJ</i> -coupling.
lsj.c	File containing the list of full CSFs in <i>LSJ</i> -coupling
3	(only after complete transformation from $jj$ to $LSJ$ -couplings).
lsj.j	File containing full set of composition of wave functions in <i>LSJ</i> -coupling
3 3	(only after complete transformation from <i>jj</i> to <i>LSJ</i> -couplings).
uni.lsj.lbl	File containing composition of wave functions in <i>LSJ</i> -coupling, but
J	arranged to give unique labels of all states.
uni.lsj.sum	File containing information from a jj21sj run in case of unique
J	labels identification.
coup*.AA.lbl	File containing composition of wave functions in AA-coupling.
55up	AA characters denote the corresponding coupling schemes:
	LS, jj, JJ, JK, LK, LScjj, LS3, LSJ3, JK3, LK3, cLSjj3.
coup*.sum	File containing information from a Coupling run.
t	Transition probability data from rmcdhf wave functions.
t.lsj	Transition probability data from rmcdhf wave functions. Labels in
0.15]	optional coupling scheme, e.g., LSJ-coupling.
ct	Transition probability data from rci wave functions.
ct.lsj	Transition probability data from rci wave functions. Labels in
cu.isj	optional coupling scheme, e.g., LSJ-coupling.
h	Hyperfine constants and Landé $g_I$ -factors from rmcdhf wave functions.
ch	Hyperfine constants and Landé $g_I$ -factors from rei wave functions.
hoffd	Diagonal- and off-diagonal hyperfine constants and matrix elements
norra	from rmcdhf wave functions.
choffd	Diagonal- and off-diagonal hyperfine constants and matrix elements
CHOITU	from rci wave functions.
i	Isotope shift data from rmcdhf wave functions.
	Isotope shift data from rci wave functions.
ci fi	Data from the program fical from rmcdhf wave functions.
cfi	
	Data from the program fical from rci wave functions.  Radial density distribution and function.
d	
nw mihfa	Binary file of radial parts of natural orbitals.
gjhfs	Reduced electronic hyperfine and magnetic matrix elements from
	rmcdhf wave functions.
cgjhfs	Reduced electronic hyperfine and magnetic matrix elements from
	rci wave functions.
zm	Energies and expansion coefficients of the magnetic sublevels from rmcdhf wave functions.
CEM	
czm	Energies and expansion coefficients of the magnetic sublevels from
<b></b>	rci wave functions.
trans	Transition probability data between the hyperfine structure states.
mtrans	Transition probability data between the magnetic substates.
S	Synthetic spectra data.
log	Log-file that keeps a record of program input.

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**Figure 1.** Typical sequence of program calls to compute expectation values and transition rates and to obtain labels in different coupling schemes.

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**Figure 2.** Flow of files for a normal sequence of program runs. Extensions (c) indicate data files based on rci mixing coefficients. For rtransition the extension x denotes the multipole.

#### 3. Important Concepts and Aspects of Processing

#### 3.1. Generating Lists of CSFs

Wave functions are expanded in CSFs, where effects beyond the single CSF approximation are referred to as correlation effects, see TP Section 4.1. Exploring different electron correlation models and generating lists of CSFs is a major task of the computation. To generate lists of CSFs based on the notion of excitations from orbitals in a MR to an active set of orbitals it is advantageous to use the rcsfgenerate program. (Please note that the word excitation might be a misuse of language in this context, since this term is in general used to indicate a physical process involving a change of state of the considered electron from a higher to a lower binding energy. When refering to constructions of configuration

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function states (CSFs) in the Dirac-Hartree-Fock theory, we should rather use the phrase substitution or replacement, for which the sign of the one-electron energy change is irrelevant. The latter terminology is preferably adopted in the accompanying theory paper [17]. However, the term excitation has been used in the GRASP community for many years, and it is still present in the fortran programs, as well on the outputs from these programs. Therefore, for the sake of consistency, as well as backward compatibility, in the present paper we continue to use excitation in the context of multiconfiguration expansions.) Different restrictions can be put on the excitations, and it is possible to generate CSFs that describe valence–valence, core–valence and core–core correlation in different combinations, see TP Sections 4.3 and 4.4. To make sure that the generated CSFs interact with the CSFs in the MR the program rcsfinteract should be used.

The reader is advised to work through the examples in Section 5 on how to use rcsfgenerate. The reader may also want to read the write-up of the jjgen program [10], the predecessor of rcsfgenerate. The write-up provides a number of examples on how to generate expansions capturing different correlation effects. The general theory, Z-dependent perturbation theory, for generating CSFs is described in [29], Sections 4 and 5. See also Section 4.3 of this manual and TP Section 4.2.

### 3.2. Lists of CSFs and Symmetry Blocks

A list of CSFs starts with a line that defines the core subshells (or orbitals). The core orbitals are fully occupied in all CSFs and need not be part of the specification of the CSFs. After the line with the core orbitals, there is a line of the remaining subshells (peel subshells). The specification of the orbitals is followed by the list of CSFs, where each CSF comprises three lines. The CSFs are arranged into symmetry blocks, where the different blocks are separated by an asterisk. We take a specific example.

```
Core subshells:
            2p-
       2s
  1s
                 2p
Peel subshells:
  3s
       3p-
           Зр
CSF(s):
  3s (1) 3p (2)
      1/2
                 0
                1/2+
  3s (1)
           3p-(1)
                    3p (1)
      1/2
               1/2
                         1/2+
  3s (1)
           3p-(2)
      1/2
                1/2+
  3s (1)
           3p (2)
      1/2
                 2
                3/2+
  3s (1)
           3p-(1)
                    3p (1)
      1/2
               1/2
                        3/2
                    0
                         3/2+
  3s (1)
           3p-(1)
                    3p (1)
      1/2
                        3/2
               1/2
                    1
                         3/2+
           3p (2)
  3s (1)
      1/2
                 2
                5/2+
  3s (1)
           3p-(1)
                    3p (1)
      1/2
               1/2
                        3/2
                    1
                         5/2+
```

There are four core subshells 1s, 2s, 2p-, 2p corresponding to a  $1s^22s^22p^6$  closed core (in non-relativistic notation) that is common to all CSFs. After the line with core subshells there is the line with the peel subshells, 3s, 3p-, 3p. The

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peel subshells (or orbitals) are the orbitals in the active set that are used in the construction of the CSFs in the list. The core subshells are not part of the active set. After the orbital specifications, the list of CSFs appear. Each CSF is written on three lines. The first line gives the configuration. The second line gives the *J* quantum number of each subshell. The third line shows how the *J* quantum numbers of each subshell are coupled together from left to right. Looking at the first CSF in the list

The 3s (1) subshell has J = 1/2 and the 3p (2) subshell is coupled to J = 0. The third line defines how the J quantum numbers of the different shells are coupled from left to right to a final J quantum number J = 1/2+, where + denotes positive (even) parity. In some cases, if needed, the second line displays more information than the single J quantum number of the  $j^N$  open subshell. For example, for  $4f^4$ , J = 2, resfgenerate produces the following:

The numbers 2; and 4; preceding the J=2 string specify unambiguously the CSF through the seniority number  $\nu$ . For convenience, a list of seniority numbers and other needed quantum numbers is given in Table 2 in the accompanying theory paper (TP).

In the current version of the codes, the CSFs are automatically arranged into symmetry blocks, where the different blocks are separated by an asterisk. In the example above, there are three symmetry blocks J = 1/2+, 3/2+, 5/2+ separated by an asterisk \*.

#### 3.3. Spectroscopic Orbitals and Convergence

Major contributors to an ASF define a MR set of CSFs. The orbitals building the reference CSFs of the targeted states are referred to as spectroscopic orbitals. A variational method is used that determines optimized radial functions for which the total energy is stationary with respect to all perturbations satisfying boundary and orthonormality conditions and leads to a non-linear system of equations, see TP Section 2.7. This requires that the radial functions have the same number of nodes as the corresponding hydrogen-like orbitals [29]. The radial equations are solved iteratively by the SCF method, which requires initial estimates that are then improved successively. Orthonormality and the associated Lagrange multipliers may lead to convergence problems, especially for near neutral systems where initial estimates from, e.g., screened hydrogenic functions are not sufficiently accurate.

In general, the program DBSR\_HF [30] is the most reliable method for getting started. This is a B-spline solution of the Dirac-Hartree–Fock equation in which orbitals are obtained from eigenvectors of a Dirac-Fock operator and orthogonality is achieved through the use of projection operators. Thus, the node-counting used by differential equation methods is avoided. The command — dbsr\_hf Li\_092 atom=U ion=Li out\_w=1 — will determine orbitals for Li-like Uranium, with orbitals output in GRASP format. When many CSFS are in the expansion, DBSR\_HF will provide orbitals for an EAL approximation. Suppose the calculation is for atom=Cu and the file Cu.c contains the expansion of 3d(10)4s, 3d(10)5s, 3d(9)4s(1)5s(1), in standard clist format, then the command dbsr\_hf Cu term=jj out\_w=1 will produce a file Cu.w that contains the EAL orbitals in GRASP format. Please note that such a calculation need not require a high-level of accuracy, and it might be desirable to reduce the convergence requirement. These generated orbitals can be directly used as input if all orbitals have been estimated.

Instead of the relativistic DBSR\_HF program, the non-relativistic HF program can be used, and the radial functions converted to relativistic form. In fact, it is the experience of the authors that the use of converted HF or MCHF radial wave functions generally give very good starting values, and that this may cut down on the number of needed iterations

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in the SCF procedure. The conversion of HF or MCHF radial wave functions to relativistic radial wave functions is done by rwfnmchfmcdf. In the present implementation, prior to normalization,

$$P(n\kappa;r) = P^{HF}(nl;r)$$

$$Q(n\kappa;r) = \frac{\alpha}{2} \left(\frac{d}{dr} + \frac{\kappa}{r}\right) P(n\kappa;r),$$

which means that the relativistic orbital pair is strictly kinetically matched [3].

The program rwfnestimate has the capability of combining initial estimates from many sources:

- GRASP wave function file. Each such file has information about the grid and atomic number so that the radial function can be scaled to the current case.
- Thomas Fermi potential orbitals from this simple potential are used as estimates.
- Screened hydrogenic functions these functions can be computed from analytic expressions.
- Screened hydrogenic functions with custom *Z* these functions can be computed from analytic expressions.

See Section 6.2 for an example using converted HF wave functions as initial estimates for rmcdhf. The use of screened hydrogenic functions with custom Z is exemplified in Section 6.8 and further discussed in Section 13.6.

## 3.4. Dealing with Convergence Problems

Most problems are encountered with outer spectroscopic radial functions. However, these orbitals can only converge if they are in an appropriate potential. It is customary to list orbitals in order of decreasing orbital energy so 4f orbital appears towards the end of a list. However, 4f may be a core orbital defining the potential of an outer orbital. So the first thing to do is remove the valence electrons and make sure core orbitals are adequately defined, see [2]. Then consider the following steps:

- 1. Start from relativistic DBSR\_HF or converted HF or MCHF radial wave functions as estimates.
- 2. Increase the nuclear charge *Z*. If convergence is achieved, decrease the nuclear charge in small steps. Remember that *Z* needs to have an integer value in quantum theory, but may have fractional values in GRASP. Use the converged radial wave functions from the previous rmcdhf run as input for the new rmcdhf calculation.
- 3. Use the above strategies together with non-default options in rmcdhf allowing direct control of damping and orbital updates.
- 4. If nothing helps, see if it is possible to start with a different MR set.

Convergence will be further discussed in Section 13 in connection with some practical examples on how convergence of spectroscopic orbitals can be achieved in problematic cases.

### 3.5. Correlation Orbitals and Layer-by-Layer Calculations

Orbitals introduced to build CSFs that correct the reference CSFs are called correlation orbitals. These are corrections to the wave function due to electron-electron interactions and may no longer have spectroscopic nodal structure. Initial estimates are not as critical. In fact, the mean radius of a converged correlation orbital is similar to that of the occupied orbital in the MR set. Thus, the initial estimate of, say, a 10s correlation orbital may need to be a contracted orbital, something most readily achieved by increasing the nuclear charge of a screened hydrogenic orbital (the custom *Z*-option for the program rwfnestimate).

Although desirable, it is often not possible to optimize all radial orbitals, spectroscopic and correlation orbitals, simultaneously because of orthonormality constraints. Instead, the calculations can be done layer-by-layer in a procedure that is described as follows:

- 1. Perform calculation for the MR where the orbitals are required to be spectroscopic.
- 2. Use the active set approach to generate the list of CSFs. Increase the active set systematically by adding a layer of correlation orbitals (a layer is a set of correlation orbitals such that there are no two orbitals with the same symmetry). Optimize only the outermost layer and keep the remaining orbitals fixed from the previous calculation.
- 3. Monitor the convergence of the calculated properties such as energy differences, transition rates, hfs, isotope shift, as the active set is increased.
- 4. Stop the calculations when the properties are converged at some level and when it is not meaningful to extend the active set further.

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5. Relax the rules for generating CSF, perform calculations using rci and check if the calculated properties are converged also with respect to the type (valence–valence, core–valence and core–core, etc.) of included electron correlation, see TP Section 4.4 and [2] for a general discussion of systematic methodologies.

#### 3.6. Simultaneous Calculations for Many Levels

In GRASP, calculations can be done for many levels (states) simultaneously, sometimes referred to as 'all levels' calculations or, if both even and odd parity levels are targeted at the same time, spectrum calculations. Although the wave function for each individual level (state) may not be the most accurate, simultaneous calculations lead to a balanced description of the levels with accurate energy separations. Simultaneous calculations are often done by term, which determines all the levels of an *LS*-term, by configuration, which determines all the levels of a configuration, or by parity, which determines all the desired levels with the same parity. Simultaneous calculations can be done also in other ways and may include all desired levels of both parities. Studies have been performed where hundreds of levels in an atomic spectrum have been determined simultaneously [31,32].

In rmcdhf, simultaneous calculations of many levels are done in the so-called extended optimal level (EOL) mode. Here a weighted energy functional of a selected set of levels is constructed, and by applying the variational principle both the radial wave functions and the corresponding expansion coefficients are determined, see [2] and TP Section 2.7. As an example we consider  $1s^22s2p$ . We want to do the calculation by parity and determine the four levels  $1s^22s2p$   $^3P_{0,1,2}^o$  and  $1s^22s2p$   $^1P_1^o$  simultaneously. The J=0 and J=2 levels are the lowest of their symmetry. The two J=1 are the lowest and the second lowest of their symmetry. In the rmcdhf calculation, we would specify this by saying that we want the serial number 1 of symmetry J=0, the serial numbers 1 and 2 of symmetry J=1, and the serial number 1 of symmetry J=1. In previous studies, levels entering the construction of the energy functional have been equally weighted [33] and also weighted by the statistical weight 2J+1 [31,32]. Depending on the case, other weights may be useful.

# 3.7. Transverse Photon Interaction and Self-Energy Correction

Relativistic corrections beyond the Dirac–Coulomb approximation for a many-electron system are implemented using assumptions based on one-electron concepts. For example, in the transverse photon interaction

$$\mathcal{H}_{TP} = -\sum_{i < j}^{N} \left[ \boldsymbol{\alpha}_{i} \cdot \boldsymbol{\alpha}_{j} \frac{\cos(\omega_{ij} r_{ij} / c)}{r_{ij}} (\boldsymbol{\alpha}_{i} \cdot \boldsymbol{\nabla}_{i}) (\boldsymbol{\alpha}_{j} \cdot \boldsymbol{\nabla}_{j}) \frac{\cos(\omega_{ij} r_{ij} / c) - 1}{\omega_{ij}^{2} r_{ij} / c^{2}} \right], \tag{3}$$

which is the leading correction to the electron-electron Coulomb interaction, the frequency  $\omega_{ij}$  is assumed to be the difference between the diagonal orbital energy parameters. This may be an appropriate assumption for singly occupied orbitals, but is not correct for multiply occupied ones and certainly is not true for correlation orbitals. For these reasons, transverse photon interaction is often computed in the low-frequency limit by multiplying the frequency  $\omega_{ij}$  with a scale factor. The scale factor is often set to  $10^{-6}$ . The transverse photon interaction with scaled frequencies is sometimes referred to as the Breit interaction, see TP Section 2.3.

Similarly, the self-energy correction is computed from a screened-hydrogenic approximation, a model that does not apply well to correlation orbitals that are far from hydrogenic. The rci code allows the user to specify the largest principal quantum number for which CSFs are to be considered in the self-energy corrections. For small calculations with a few correlation orbitals, this cut-off is set to the largest principal quantum number of the included orbitals. In large calculations with many correlation orbitals, the cut-off is typically set to a number somewhat larger than the highest principal quantum number of the spectroscopic orbitals. In many research articles, the vacuum polarization and the self-energy correction are referred to as the leading quantum electrodynamic (QED) corrections.

# 3.8. Biorthonormal Transformations for Transition Calculations

Transition parameters, such as rate and weighted oscillator strength, for a multipole transition of rank L from  $\Gamma'J'$  to  $\Gamma J$ , are related to the reduced transition matrix element

$$\langle \Gamma J \| \mathbf{O}^{(L)} \| \Gamma' J' \rangle$$
 (4)

where  $\mathbf{O}^{(L)}$  is the transition operator, see TP Section 3.5. This matrix element is very time-consuming to evaluate between separately determined initial and final state wave functions, since the non-orthogonalities of the initial and final state orbital sets prevent Racah-algebra to be used. Provided the CSF expansions for the initial and final states are closed under de-excitation (cud), it is possible to change the wave function representation of the two states in such a way that

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Racah-algebra can be used for evaluating the matrix elements in the new representation [27]. This cud property is satisfied if for each CSF based on a configuration that is part of the list, all the CSFs based on the configurations where the orbitals are de-excited to orbitals with lower principal quantum numbers are also part of the list. Please note that (i)- an expansion based on the active set approach is closed under de-excitation if the MR is closed under de-excitation and (ii)- CSF lists based on the active set approach from a single core-excited configuration may not be closed under de-excitation although additional CSFs can be introduced to satisfy the cud condition. See also TP Section 3.5.

The procedure for calculating the oscillator strength can be summarized as follows:

- 1. Perform separate rmcdhf or rci calculations for the initial and the final states.
- 2. Change the initial and final state wave function representations by transforming the radial orbital sets to a biorthonormal orbital set. This is followed by a counter-transformation of the initial and final state expansion coefficients to leave the total wave functions invariant.
- 3. Calculate the transition matrix element with the transformed wave functions, for which now the Racah-algebra can be used

The biorthonormal transformation is very fast and is performed with the program rbiotransform. The evaluation of the transition parameters from the transformed initial and final wave functions is then performed with rtransition.

## 3.9. Angular Data from rbiotransform and rtransition

The rbiotransform and rtransition programs and their MPI variants save angular data on file to speed up calculations for an iso-electronic sequence. If angular files are available, the programs read these files and the execution time is reduced considerably. If, for some reason, there are incomplete files with angular coefficients, these programs will end with some error message when trying to process the angular data files. In these cases, the user should remove the angular files (they all have a capital T in the extension) and rerun the case again.

#### 3.10. Managing Large Expansions — Zero- and First-Order Calculations

Often the CSFs expansions grow so large that they can not be handled with the available computational resources. In these cases an approximate computational scheme can be employed in which the CSF list is rearranged into zero- and a first-order spaces:

$$\underbrace{\Phi(\gamma_1^0 J), \Phi(\gamma_2^0 J), \dots, \Phi(\gamma_M^0 J)}_{\text{zero-order space, } P}, \underbrace{\Phi(\gamma_1^1 J), \Phi(\gamma_2^1 J), \dots, \Phi(\gamma_N^1 J)}_{\text{first-order space, } Q}$$

where M+N is the total number of CSFs in the original list. The zero-order space, P, contains the most important CSFs, while the first-order space, Q, contain less important CSFs that can be regarded as minor corrections. Normally  $M \ll N$ . Associated with the rearrangement of the CSFs is a decomposition of the Hamiltonian interaction matrix in submatrices

$$\left(\begin{array}{cc} H^{(PP)} & H^{(PQ)} \\ H^{(QP)} & H^{(QQ)} \end{array}\right).$$

The energy expression, on which to optimize, is now obtained from the limited interaction matrix where the full  $H^{(PP)}$ ,  $H^{(QP)}$ ,  $H^{(QP)}$  submatrices are included (interactions within the zero-order space and between the zero- and first-order spaces) but only the diagonal part of  $H^{(QQ)}$ , see TP Section 2.8. The rearrangement of the list of CSFs in zero- and first-order spaces is done by the program rcsfzerofirst. In the programs rangular and rci, which set up expressions for the Hamiltonian, there is a question if full interaction should be considered or not. If not full interaction, the user can specify the size of the zero-order space for each symmetry block. See [28] for recent applications of this methodology. The handling of large expansions is discussed and exemplified in Section 14.

### 3.11. Running Parallel Programs Using MPI

Some of the more time-consuming programs in GRASP have been converted to run in parallel under MPI, a language-independent communication protocol used to program parallel computers. In order to compile the programs, MPI libraries need to be installed. For cases where the MPI codes can be used, the increase of speed is often substantial. In Section 6.4 we show in detail how to set up the computational environment and use the MPI codes.

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#### 3.12. Restarting rci

rci and rci\_mpi produce a file rci.res containing, in sparse representation, the matrix elements of the Hamiltonian. If, for some reason, an rci or rci\_mpi run stalls, then the programs can be restarted. During a restart, the rci.res file is read, and the computation continues at the place where the original computation stalled. The restart option is described in Section 6.7.

#### 4. Lists of CSFs

# 4.1. Configurations, Configuration State Functions

A configuration is a number of orbitals with occupation numbers, e.g.,

$$1s^22s^2(2p-)^2$$
,  $1s^22s^2(2p-)2p$ ,  $1s^22s^22p^2$ ,

where we use the notation 1s, 2s, 2p-, 2p for  $1s_{1/2}$ ,  $2s_{1/2}$ ,  $2p_{3/2}$ . Frequently, the non-relativistic notation is used, and the configuration is then

$$1s^22s^22p^2$$
.

CSFs are formed by angular couplings of the orbitals in a relativistic configuration. Depending on the structure of the configuration, i.e., number of open shells, there may be many angular couplings and thus CSFs for each configuration. An angular coupling is sometimes referred to as a coupling tree.

In GRASP the CSFs are given in rcsf.inp. The CSFs comprise three lines in the file. The first line gives the configuration, and lines two and three define the coupling tree, see TP Section 2.4. The CSFs are ordered in blocks specified by parity and J symmetry, the blocks being separated by an asterisk \*. Below are all the CSFs of even parity belonging to the configuration  $1s^22s^22p^2$ .

In the case above we have three symmetry blocks with even parity corresponding to J = 0, 1, 2.

GRASP handles expansions with hundreds of thousands of CSFs, even on a small scalar computer. On a cluster, expansions with millions of CSFs can be used. The success of a calculation depends on judiciously chosen CSFs.

# 4.2. Multireference

The starting point for a study is normally a calculation for a number of important CSFs that define the MR. The CSFs in the MR are those that can be formed from nearly degenerate configurations, see [29] chapter 4, ref. [2] and TP Sections 4.1 and 4.4. (When talking about the MR we will, somewhat loosely, refer to both the set of CSFs and the set of configurations from which the CSFs are formed) The wave function based on the CSFs in the MR is the first approximation, and it is the starting point for further refinements. The concept of an MR is best illustrated by some examples.

Suppose we want to compute the wave function for the ground state  $3s^2$   $^1S_0$  of Mg I. The  $3s^2$ ,  $3p^2$  and  $3d^2$  configurations are formed by orbitals with the same principal quantum numbers and the configurations are closely degenerate. An MR

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in this case could consist of the CSFs that can be formed from these configurations.

Suppose we want to compute the wave functions for the 3s3p  $^3P_{0,1,2}^o$ ,  $^1P_1^o$  excited states of Mg I. The 3s3p and 3p3d configurations are formed by orbitals with the same principal quantum numbers, and these configurations are closely degenerate. An MR in this case could consist of the CSFs that can be formed from these configurations. However, it turns out that 3s4p is important, and thus a more suitable MR should consist of CSFs also from the latter configuration.

We want to compute the wave functions for the states of  $2s^22p^2$  and  $2s^2p^3$ . The  $2s^22p^2$  and  $2p^4$  configurations are formed by orbitals with the same principal quantum numbers, and the configurations are closely degenerate. Thus, the MR for the even states would consist of the CSFs that can be formed from these two configurations. It turns out that the  $2s^2p^23d$  and  $2s^23d^2$  configurations are important, and a better MR includes CSFs also from these configurations. Looking at  $2s^2p^3$  there is no other configuration of the same parity that can be formed by orbitals with the same principal quantum numbers. In this case, the MR would consist of CSFs formed from this single configuration. However, also  $2p^33d$ ,  $2s^22p3d$  and  $2s^2p^3d^2$  are important and the MR should consist of CSFs also from these configurations.

We see that the selection of the MR in advance or *a priori* is far from trivial, and it often requires a number of exploratory calculations to find a good MR. In other words, the MR is best determined after some correlation studies have been performed. The program rcsfmr, described in Section 6.6, is designed to support the exploratory process.

For 'all levels' calculations or spectrum calculations, see Section 6.4, where wave functions are determined for a number of states belonging to several configurations the MR is often taken as the set of CSFs that can be formed from these configurations. Suppose that we want to determine the wave functions for states of the 3l3l', 3l4l' and 3l5s configurations in Mg-like ions. The MR in this case would be the CSFs that can be formed from these configurations. If we do the calculations by parity, the MR for the even parity states would be the CSFs formed from even parity configurations and the MR for the odd parity states would be the CSFs formed from odd parity configurations.

# 4.3. Active Set Approach

CSFs are often generated using the active set approach. In the active set approach, CSFs of a specified parity and *J* symmetry are obtained from angular couplings of configurations generated by excitations from orbitals of one or more configurations in the MR to orbitals in an active set (AS). Orbitals of a reference configuration are classified as closed (c), inactive (i), active (\*), or active having minimal occupation (m). The active set consists of the active orbitals in the reference configuration together with orbitals up to a given limit specified by the highest principal quantum number of each orbital symmetry. Closed orbitals are fully occupied and make up the core. No excitations are allowed from inactive orbitals of the reference configuration. Excitations are allowed from the active orbitals of the reference configuration to orbitals in the active set. Excitations from active orbitals having minimal occupations are such that the occupations after the excitations are always larger or equal to the specified minimal occupation.

Based on perturbation theory one can show that the major electron correlation effects are captured by including, in the ASF, the CSFs that can be formed from configurations obtained by allowing single (S) and double (D) excitations from the most important configurations, defining the MR, to an extended active set of orbitals [29].

For small systems, e.g., nominal three and four electron systems, it is sometimes advantageous to include CSFs that can be formed from all possible excitations: single (S), double (D), triple (T), (Q) quadruple, etc. This expansion is referred to as the complete active space (CAS).

#### 4.4. Different Types of Correlation Effects

For complex systems it may not be possible, or even desirable, to allow excitations from all orbitals of the MR. Often excitations are done only from outer orbitals, and the corresponding CSFs are said to describe valence-valence correlation. If one excitation is from a core orbital and one from an outer orbital, then the corresponding CSFs are said to describe core-valence correlation. If both excitations are from the core, the corresponding CSFs are said to describe core-core correlation, see [2] and TP Section 4.3. A discussion about different correlation effects and their relation to the orbital basis can be found in [34].

As an example of correlation effects, we look at the ground state of Mg  $1s^22s^22p^63s^2$   $^1S_0$ . To make things simple, we consider only a single reference.

# Valence-valence correlation

CSFs based on configurations of the type  $1s^22s^22p^6nln'l'$  represent valence–valence correlation. In the active set approach,

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these configurations can be formed by starting from  $1s^22s^22p^63s^2$  and classifying the 1s, 2s, 2p orbitals as inactive (i) and the 3s orbital as active (\*). In our notation, we have

$$1s(2,i)2s(2,i)2p(6,i)3s(2,*)$$

SD-excitations are then allowed to orbitals in the active set.

#### Core-valence correlation

CSFs based on configurations of the type  $1s^22s^22p^5nl3sn'l'$ ,  $1s^22snl2p^63sn'l'$  represent core–valence correlation involving the  $2s^22p^6$  core. In the active set approach, the configurations of the first type can be formed by starting from  $1s^22s^22p^63s^2$  and classifying the 1s, 2s orbitals as inactive (i), the 2p orbital as active having a minimal occupation 5 and the 3s orbital as active having a minimal occupation 1. In our notation, we have

SD-excitations are then allowed to orbitals in the active set. Configurations of the second type can be formed by starting from  $1s^22s^22p^63s^2$  and classifying the 1s, 2p orbitals as inactive (i), the 2s orbital as active having a minimal occupation 1 and the 3s orbital as active having a minimal occupation 1. In our notation, this is

SD-excitations are then allowed to orbitals in the active set. In practical applications one most often treats valence—valence and core—valence correlation together and this is achieved by classifying the 3s orbital as active instead of active with minimal occupation 1. This corresponds to

and

$$1s(2,i)2s(2,1)2p(6,i)3s(2,*)$$

#### Core-core correlation

CSFs based on configurations of the type  $1s^22s^22p^4nln'l'3s^2$ ,  $1s^22snl2p^5n'l'3s^2$ ,  $1s^2nln'l'2p^63s^2$  represent core–core correlation in the  $2s^22p^6$  core. In the active set approach, these configurations can be formed by starting from  $1s^22s^22p^63s^2$  and classifying the 1s orbital as inactive (i), the 2s, 2p orbitals as active (\*) and the 3s orbital as inactive (i). In our notation

$$1s(2,i)2s(2,*)2p(6,*)3s(2,i)$$

SD-excitations are then allowed to orbitals in the active set. In practical applications, we very seldom treat core–core correlation alone. Instead, we treat valence–valence, core–valence and core–core correlation together and this is achieved by classifying the 2s, 2p, 3s orbitals active (\*)

and allowing SD-excitations to orbitals in the active set.

Atomic properties depend in various ways on electron correlation effects. For transition rate calculations, it is important to include valence–valence and core–valence correlation [35]. For calculations of hyperfine structure and isotope shift, it is important to include also deep core–valence correlation effects [36].

#### 4.5. Doubly Occupied Correlation Orbitals

Accounting for electron correlation effects including core—core often leads to very large expansions. Imposing the restriction that correlation orbitals are doubly occupied reduces the expansion size. For example, if 3s, 4s, 5s, 3p-, 3p, 4p-, 4p, 5p-, 5p, 3d-, 3d, 4d-, 4d, 5d-, 5d are correlation orbitals in relativistic notation only excitation pairs  $3s^2$ ,  $4s^2$ ,  $5s^2$ , (3p-) $^2$ , (3p-) $^3p$ ,  $3p^2$ , (4p-) $^2$ , (4p-)4p,  $4p^2$  etc. are allowed. Such an expansion still describes a fair part of the correlation. A practical example of how to use the restriction that correlation orbitals are doubly occupied is given in Section 5.6.

# 4.6. CSFs Interacting with the MR

It is important to realize that the active set approach, as we have described it above, is based on generation of configurations that are then coupled to form CSFs. However, not all CSFs generated in this way have non-zero Hamiltonian matrix elements (interact) with CSFs in the MR. Generated CSFs not interacting with the CSFs of the MR can often, though not always, be removed from the list of CSFs without any major loss of accuracy [29,37]. This is done by the program rcsfinteract. The reduction of CSFs is important mainly for complex systems, where the list of CSFs grows very rapidly with the increasing active set of orbitals.

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# 5. Running the CSFs Generation Programs

5.1. First Example: Valence-Valence, Core-Valence and Core-Core for 1s<sup>2</sup>2s<sup>2</sup> <sup>1</sup>S<sub>0</sub>

We want to generate an expansion for the  $1s^22s^2$   $^1S_0$  state. In this example, the CSFs are generated by SD-excitations from the  $\{1s^22s^2, 1s^22p^2\}$  MR set to an active set characterized by a maximal principal quantum number n=4. The expansion accounts for valence–valence, core–valence and core–core correlation.

```
RUN RCSFGENERATE
          OUTPUT FILES: rcsf.out, rcsfgenerate.log
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
       0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                           = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                           = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                           = 36 electrons)
       5: Xe([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,*)2s(2,*)
 Give configuration 2
>>1s(2,*)2p(2,*)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
    . . . . . .
```

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#### 1 blocks were created

block	J/P	NCSF
1	0+	361

Please note that by answering 2 for the number of excitations, we will include both single (S) and double (D) excitations. By default, the orbitals will be in the order 1s, 2s, 2p, 3s, 3p, 3d etc. There is also the possibility to have a reverse orbital order  $\ldots 3d$ , 3p, 3s, 2p, 2s, 1s, a symmetry order 1s, 2s, 3s,  $\ldots$ , 2p, 3p,  $\ldots$ , 3d, 4d,  $\ldots$  or a user defined order. We will look at these options in Section 5.9. The generated file rcsf.out with the CSF list looks like

Core subshells:

```
Peel subshells:
                                                 4p- 4p
 1s
      2s
           2p- 2p
                    3s 3p- 3p 3d- 3d
                                            4s
                                                          4d- 4d
                                                                    4f- 4f
CSF(s):
 1s (2) 2s (2)
 1s (2) 2s (1) 3s (1)
              1/2
                         0+
 1s (2) 2s (1) 4s (1)
              1/2
                      1/2
 1s (2) 2p (2)
                0+
 1s (2) 2p-(2)
                 0+
   . . . . . . . . . . . . . .
```

In addition to the file rcsf.out with the list of CSFs, the generation program produces a log-file rcsfgenerate.log that mirrors the input. The latter looks like

In practical work, it is often convenient to edit the log-file and use this as input for additional runs of rcsfgenerate.

5.2. Second Example: Valence–Valence, Core–Valence for 
$$1s^22s^22p^63s3p$$
  $^3P_{0,1,2}^o$  ,  $^1P_1^o$ 

We want to generate expansions for  $1s^22s^22p^63s3p$   $^3P^o_{0,1,2}$ ,  $^1P^o_1$ . In this example, the CSFs are generated by SD-excitations from  $\{1s^22s^22p^63s3p, 1s^22s^22p^63p3d\}$  to an active set n=5 with the restrictions that 1s is closed and that there is at most one excitation from orbitals with n=2. The expansions account for valence–valence and core–valence correlation.

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```
************************************
          RUN RCSFGENERATE
          OUTPUT FILES: rcsf.out, rcsfgenerate.log
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                           = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                           = 18 electrons)
                                           = 36 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,1)2p(6,i)3s(1,*)3p(1,*)
 Give configuration 2
>>2s(2,i)2p(6,5)3s(1,*)3p(1,*)
 Give configuration 3
>>2s(2,1)2p(6,i)3p(1,*)3d(1,*)
 Give configuration 4
>>2s(2,i)2p(6,5)3p(1,*)3d(1,*)
 Give configuration 5
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>2
 Generate more lists ? (y/n)
>>n
    . . . . . .
  3 blocks were created
```

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block	J/P	NCSF
1	0-	1912
2	1-	5210
3	2-	7122

# 5.3. Third Example: Valence-Valence, Core-Valence and Intercore for $1s^22s^22p^63s3p$ $^3P_{0,1,2}^0$ , $^1P_0^0$

We want to generate expansions for  $1s^22s^22p^63s3p$   $^3P_{0,1,2}^o$ ,  $^1P_1^o$ . In this example, the CSFs are generated by SD-excitations from  $\{1s^22s^22p^63s3p, 1s^22s^22p^63p3d\}$  to an active set n=5 with the restrictions that 1s is closed (and hence inactive) and that there is at most one excitation from 2s and 2p, respectively. In this case, in addition to valence–valence and corevalence correlation, also intercore correlation are accounted for through configurations of the form  $1s^22snl2p^5n'l'3s3p$   $1s^22snl2p^5n'l'3p3d$ , where  $1s^2$  is inactive. Please note how much the number of CSFs has increased.

```
RUN RCSFGENERATE
          OUTPUT FILES: rcsf.out, rcsfgenerate.log
**********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
        0: No core
        1: He (
                     1s(2)
                                            = 2 electrons)
        2: Ne ([He] + 2s(2)2p(6)
                                            = 10 electrons)
        3: Ar ([Ne] + 3s(2)3p(6)
                                            = 18 electrons)
        4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                            = 36 electrons)
        5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                            = 54 electrons)
        6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,1)2p(6,5)3s(1,*)3p(1,*)
 Give configuration 2
>>2s(2,1)2p(6,5)3p(1,*)3d(1,*)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
 Number of excitations (if negative number e.g., -2, correlation
```

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```
orbitals will always be doubly occupied)
>>2
Generate more lists ? (y/n)
>>n
......

3 blocks were created

block J/P NCSF
1 0- 10743
2 1- 29589
3 2- 41500
```

Give configuration 4

# 5.4. Fourth Example: Valence-Valence and Core-Valence and Large Multireference

We want to generate CSF expansions that describe all 92 states with symmetries J = 0, 1, 2, 3, 4, 5 of the configurations  $\{2s^22p^2, 2p^4, 2s^22p3p, 2s2p^23s, 2s2p^23d\}$ . In this example, the CSFs are generated by SD-excitations from  $\{2s^22p^2, 2p^4, 2s^22p3p, 2s2p^23s, 2s2p^23d\}$  to an active set n = 5 with the restriction that there is at most one excitation from 1s. The expansions account for valence–valence and core–valence correlation.

```
***********************************
          RUN RCSFGENERATE
          OUTPUT FILES: rcsf.out, rcsfgenerate.log
**********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                   1s(2)
                                          = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                         = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                         = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                         = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                         = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,1)2s(2,*)2p(2,*)
 Give configuration 2
>>1s(2,1)2p(4,*)
 Give configuration 3
>>1s(2,1)2s(2,*)2p(1,*)3p(1,*)
```

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```
>>1s(2,1)2s(1,*)2p(2,*)3s(1,*)
 Give configuration 5
>>1s(2,1)2s(1,*)2p(2,*)3d(1,*)
 Give configuration 6
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,10
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
  6 blocks were created
       block J/P
                            NCSF
           1
              0+
                            14351
           2
               1+
                            38928
           3
              2+
                           53645
              3+
                           56147
           5
               4+
                            48973
                            36562
```

## 5.5. Fifth Example: CSFs Interacting with CSFs in the MR

In this example, we show how to reduce the number of CSFs in the previous list by retaining only the CSFs that interact with the CSFs of the MR through the Dirac–Coulomb or Dirac–Coulomb–Breit Hamiltonian. We start by copying rcsf.out from the previous run to rcsf.inp. After that, we generate the list of CSFs for the MR. For an additional example, see Section 6.3. Please note that the orbital order needs to be the same for the MR file and the file with CSFs that should be reduced, to ensure that this is the case it is sometimes necessary to invoke the user specified orbital ordering, see Section 6.6.

```
***********************************
       COPY FILE
>>cp rcsf.out rcsf.inp
**********************************
       RUN RCSFGENERATE
       OUTPUT FILES: rcsf.out, rcsfgenerate.log
**********************************
>>rcsfgenerate
RCSFGENERATE
This program creates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (*) or has a minimal
occupation e.g., 1s(2,1)2s(2,*)
Outputfiles: rcsf.out, rcsfgenerate.log
```

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```
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                    1s(2)
                                          = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                          = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                          = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,i)2s(2,i)2p(2,i)
 Give configuration 2
>>1s(2,i)2p(4,i)
 Give configuration 3
>>1s(2,i)2s(2,i)2p(1,i)3p(1,i)
 Give configuration 4
>>1s(2,i)2s(1,i)2p(2,i)3s(1,i)
Give configuration 5
>>1s(2,i)2s(1,i)2p(2,i)3d(1,i)
 Give configuration 6
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
    . . . . . .
  6 blocks were created
      block J/P
                          NCSF
              0+
                             14
          2
                             25
             1+
          3
              2+
                             28
          4
              3+
                             16
          5
                              7
              4+
************************************
          COPY RCSF.OUT TO RCSFMR.INP
**************************************
```

>>cp rcsf.out rcsfmr.inp

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#### >>rcsfinteract

RCSFinteract: Determines all the CSFs (rcsf.inp) that interact with the CSFs in the multireference (rcsfmr.inp)

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(Fortran 95 version) NIST (2017).

Input files: rcsfmr.inp, rcsf.inp

Output file: rcsf.out

Reduction based on Dirac-Coulomb (1) or Dirac-Coulomb-Breit (2) Hamiltonian?

. . . . .

There are 25 relativistic subshells;

Block	MR NCSF	Before NCSF	After NCSF
1	14	14351	7765
2	25	38928	24492
3	28	53645	33925
4	16	56147	29299
5	7	48973	17134
6	2	36562	7542

RCSFINTERACT: Execution complete

Comparing with what we had before, we see that there is a great reduction in the number of CSFs, where the removed CSFs are relatively unimportant. The reduction based on the Dirac–Coulomb–Breit Hamiltonian gives somewhat more CSFs compared to the reduction based on the Dirac–Coulomb Hamiltonian. There is, however, not a big difference.

# 5.6. Sixth Example: Core-Core and Doubly Occupied Orbitals

Allowing SD-excitations from all subshells of an MR without restrictions leads to large expansions. We may impose different restrictions allowing, for example, at most one excitation from the core. The resulting expansion accounts for valence–valence and core–valence electron correlation. Another restriction is to require that all correlation orbitals are doubly occupied in the generated CSFs. This cuts down the expansion size quite substantially, but still efficiently accounts for much of the correlation.

We generate a CSF expansion that describes the states with symmetries J=0,1,2 of the configuration  $2s^22p^63s3p$ . CSFs are generated by SD-excitations from  $\{2s^22p^63s3p,2s^22p^63p3d\}$  to an active set n=8 and symmetry l=h with the restriction that there is at most one excitation from  $2s^22p^6$ . The expansion accounts for valence–valence and core–valence correlation. In addition, there are SD-excitations from  $\{2s^22p^63s3p,2s^22p^63p3d\}$  to an active set n=8 and symmetry l=h with the restriction that the correlation orbitals are doubly occupied (see section 5.5). This part of the expansion accounts for part of the core–core correlation.

*****	<i>╅</i> ┍┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲┲	т
*	RUN RCSFGENERATE	*
*	OUTPUT FILES: rcsf.out, rcsfgenerate.log	*
******	**************************	*

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# >>rcsfgenerate RCSFGENERATE This program creates a list of CSFs Configurations should be entered in spectroscopic notation with occupation numbers and indications if orbitals are closed (c), inactive (i), active (\*) or has a minimal occupation e.g., 1s(2,1)2s(2,\*)Outputfiles: rcsf.out, rcsfgenerate.log Default, reverse, symmetry or user specified ordering? (\*/r/s/u) >>\* Select core 0: No core 1: He ( = 2 electrons) 1s(2)2: Ne ([He] + 2s(2)2p(6)= 10 electrons) 3: Ar ([Ne] + 3s(2)3p(6)= 18 electrons) 4: Kr ([Ar] + 3d(10)4s(2)4p(6)= 36 electrons) 5: Xe ([Kr] + 4d(10)5s(2)5p(6)= 54 electrons) 6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons) >>1 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (\*) Give configuration >>2s(2,i)2p(6,5)3s(1,\*)3p(1,\*)Give configuration >>2s(2,1)2p(6,i)3s(1,\*)3p(1,\*)Give configuration >>2s(2,i)2p(6,5)3p(1,\*)3d(1,\*)Give configuration >>2s(2,1)2p(6,i)3p(1,\*)3d(1,\*) Give configuration >> Give set of active orbitals, as defined by the highest principal quantum number per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d >>8s,8p,8d,8f,8g,8h Resulting 2\*J-number? lower, higher (J=1 -> 2\*J=2 etc.) Number of excitations (if negative number e.g., -2, correlation orbitals will always be doubly occupied) Generate more lists ? (y/n) Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (\*) Give configuration >>2s(2,\*)2p(6,\*)3s(1,\*)3p(1,\*) Give configuration >>2s(2,\*)2p(6,\*)3p(1,\*)3d(1,\*)Give configuration >> Give set of active orbitals, as defined by the highest principal quantum number

per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d

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```
>>8s,8p,8d,8f,8g,8h
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>-2
 Generate more lists ? (y/n)
>>n
    . . . . . .
 3 blocks were created
       block J/P
                             NCSF
           1 0-
                            21399
                1-
                            59512
           3
                2-
                            85284
```

# 5.7. Running rcsfgenerate More Than Once

We may merge CSF expansions by running rcsfgenerate more than once. In this example, we first generate a CAS expansion for  $1s^22p$  to the orbital set 5s, 5p, 5d, 5f, 5g. This is then merged by an SD expansion to a larger orbital set.

```
*******************************
         RUN RCSFGENERATE
         OUTPUT FILES: rcsf.out, rcsfgenerate.log
********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                   1s(2)
                                       = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                       = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                       = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                       = 36 electrons)
```

```
3: Ar ([Ne] + 3s(2)3p(6) = 18 electrons)
4: Kr ([Ar] + 3d(10)4s(2)4p(6) = 36 electrons)
5: Xe ([Kr] + 4d(10)5s(2)5p(6) = 54 electrons)
6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)

>>0
Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)

Give configuration 1

>>1s(2,*)2p(1,*)
Give configuration 2

>>
```

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```
Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,3
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>y
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>1s(2,*)2p(1,*)
 Give configuration
                               2
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>7s,7p,7d,7f,7g,7h,7i
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
. . . . . . . . .
 2 blocks were created
       block J/P
                             NCSF
           1 1/2-
                             2408
                             4174
           2 3/2-
```

As expected, we get the same number of CSFs in the two runs. Please note that the resulting *J* number needs to be the same when running rcsfgenerate several times for the same parity.

#### 5.8. Running rcsfgenerate for Even and Odd Parity

occupation e.g., 1s(2,1)2s(2,\*)

Outputfiles: rcsf.out, rcsfgenerate.log

We want to generate CSFs for odd states with J = 1/2, 3/2 by allowing all SDT-excitations from  $1s^22p$  and for even states with J = 1/2 by allowing all SDT-excitations from  $1s^22s$ . In both cases, the excitations are to an active set with n = 5.

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```
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
        0: No core
        1: He (
                      1s(2)
                                              = 2 electrons)
        2: Ne ([He] + 2s(2)2p(6)
                                              = 10 electrons)
        3: Ar ([Ne] + 3s(2)3p(6)
                                              = 18 electrons)
        4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                              = 36 electrons)
        5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                              = 54 electrons)
        6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
                              1
>>1s(2,*)2p(1,*)
 Give configuration
                               1
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,3
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>3
 Generate more lists ? (y/n)
>>y
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>1s(2,*)2s(1,*)
 Give configuration
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,1
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
. . . . . . . . .
  3 blocks were created
       block J/P
                             NCSF
           1 1/2+
                             1463
           1 1/2-
                             1454
           2 3/2-
                             2478
```

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## 5.9. User Defined Orbital Ordering

In Ce III the ground configuration is  $5s^25p^64f^2$ , where 4f is to the right of the 5s and 5p orbitals and a user defined orbital order is needed. To illustrate the user defined orbital ordering, we generate a list of CSFs by allowing SD-excitations from  $4s^24p^64d^{10}5s^25p^64f^2$  to an active orbital set  $\{1s, 2s, 3s, 4s, 5s, 6s, 2p, 3p, 4p, 5p, 6p, 3d, 4d, 5d, 4f, 5f\}$  (or 6s, 6p, 5d, 5f in the notation of the rcsfgenerate program).

To generate a list of CSFs where, in the configurations, 4f is to the right of the 5s and 5p orbitals, start by creating a file clist.ref with the desired orbital order; one orbital per line, left justified and with a non-relativistic notation.

```
1s
2s
2p
3s
Зр
3d
4s
4p
4d
5ธ
5p
4f
5d
5f
68
6p
Then run rcsfgenerate as usual, but select the user defined orbital order.
```

```
RUN RCSFGENERATE USING USER DEFINED ORBITAL ORDERING
      INPUT FILE: clist.ref
      OUTPUT FILES: rcsf.out, rcsfgenerate.log
```

# >>rcsfgenerate

#### RCSFGENERATE

```
This program creates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (*) or has a minimal
occupation e.g., 1s(2,1)2s(2,*)
Outputfiles: rcsf.out, rcsfgenerate.log
```

Default, reverse, symmetry or user specified ordering? (\*/r/s/u) >>11

### Select core

```
0: No core
1: He (
              1s(2)
                                     = 2 electrons)
2: Ne ([He] + 2s(2)2p(6)
                                     = 10 electrons)
3: Ar ([Ne] + 3s(2)3p(6)
                                     = 18 electrons)
4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                     = 36 electrons)
5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                     = 54 electrons)
6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
```

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```
Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>3d(10,c)4s(2,*)4p(6,*)4d(10,*)5s(2,*)5p(6,*)4f(2,*)
 Give configuration
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>6s,6p,5d,5f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>2
 Generate more lists ? (y/n)
>>n
. . . . . . . . .
 7 blocks were created
        block
              J/P
                            NCSF
           1
               0+
                           26477
           2
                           74434
               1+
           3
               2+
                          112054
           4
               3+
                          133012
           5
               4+
                          137871
                          127297
               5+
                          107194
                6+
The produced output file rcsf.out looks like this
Core subshells:
  1s
       2s
            2p-
                2p
                     3s
                          3p-
                               Зр
                                    3d-
                                         3d
Peel subshells:
  4s
       4p- 4p
                4d-
                     4d
                          5s
                               5p-
                                    5p
                                         4f-
                                              4f
                                                   5d-
                                                       5d
                                                             5f- 5f
                                                                       6s
                                                                           6p-
CSF(s):
  4s (2) 4p-(2) 4p (4) 4d-(4)
                                     4d (6)
                                              5s (2)
                                                       5p (4)
                                                               4f (4)
                                                                       0+
  4s (2) 4p-(2) 4p (4) 4d-(4)
                                     4d (6)
                                              5s (2)
                                                       5p-(1)
                                                                5p (3) 4f (4)
                                                                    3/2
                                                           1/2
                                                                          2;
                                                                               2
                                                                         2
  4s (2) 4p-(2) 4p (4) 4d-(4)
                                     4d (6)
                                              5s (2)
                                                       5p-(1)
                                                                5p (3)
                                                                         4f (4)
                                                                    3/2
                                                           1/2
                                                                          4;
                                                                         2
                                                                               0+
          4p-(2)
                   4p (4)
                            4d-(4)
                                     4d (6)
                                              5s (2)
                                                       5p-(2)
                                                                5p (2)
                                                                         4f (4)
                                                                      0
                                                                               0
                                                                               0+
  4s (2) 4p-(2) 4p (4) 4d-(4) 4d (6) 5s (2)
                                                       5p-(2)
                                                                5p (2)
                                                                         4f (4)
                                                                          2;
                                                                               0+
                                                       5p-(2) 5p (2)
                                                                         4f (4)
  4s (2) 4p-(2) 4p (4) 4d-(4) 4d (6) 5s (2)
                                                                          4;
                                                                               2
                                                                               0+
  4s (2) 4p-(2) 4p (4) 4d-(4) 4d (6) 5p-(2) 5p (4) 4f (4)
```

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Comment: when using rcsfinteract make sure that you have the same orbital order (and core) for both rcsf.inp and rcsfmr.inp. The additional quantum numbers 2; and 4; for the 4f (4) subshell are the seniority quantum numbers.

# 5.10. Running jjgen

The jjgen program is a more flexible generation program than rcsfgenerate. It has several useful properties, but the input is somewhat longer and more involved. The use of jjgen is described in detail in the original write-up [10]. Please note that after generating a CSF list with jjgen the list needs to be put in block form by rcsfblock.

### 6. Running the Application Programs

In this section we demonstrate the use of the application programs of GRASP in six cases. The use of the tools of GRASP is described in Section 7. All data written to the output files are shown, explained and discussed in detail in Section 8. Scripts for example 1 are found in grasptest/example1/script, scripts for example 2 in grasptest/example2/script, etc. Please note that the executables must be on the path! When running the application programs and the tools, the user is encouraged to look at all the output files and use the information in Section 8 to correctly interpret the output data.

# 6.1. First Example: $1s^22s$ <sup>2</sup>S and $1s^22p$ <sup>2</sup>P<sup>0</sup> in Li I

The first example is for  $1s^22s$   $^2S_{1/2}$  and  $1s^22p$   $^2P_{1/2,3/2}^o$  in Li. The example shows the computation of rmcdhf and rci wave functions, and the subsequent evaluation of hyperfine structure constants, Landé  $g_J$ -factors, and isotope shift parameters. In addition, the biorthogonal transformation is applied, and the transition rates computed from the transformed wave functions. The example also illustrates the use of jj21sj for labeling purposes.

#### Overview

- Define nuclear data.
- 2. Obtain common spectroscopic orbitals for the MR set.
  - (a) Generate configuration state list containing three CSFs:  $1s^22s^2S_{1/2}$ ,  $1s^22p^2P_{1/2,3/2}^0$ .
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of  $1s^22s$   $^2S_{1/2}$ ,  $1s^22p$   $^2P_{1/2,3/2}^0$ .
  - (e) Save output to 2s\_2p\_DF.
- 3. Improve even states
  - (a) Generate n = 3 complete active space (CAS) expansion for  $1s^2 2s \, ^2S_{1/2}$ .
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on  $1s^2 2s {}^2S_{1/2}$ .
  - (e) Save output to 2s\_3.
  - (f) Perform RCI calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
- 4. Transform from *jj* to *LSJ*-coupling
- 5. Improve odd states
  - (a) Generate n = 3 complete active space (CAS) expansion for  $1s^2 2p^2 P_{1/2,3/2}^0$ .
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of  $1s^22p$   $^2P_{1/2,3/2}^o$ .
  - (e) Save output to 2p\_3.

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(f) Perform RCI calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.

- 6. Transform from *jj* to *LSJ*-coupling
- 7. Run rlevels to view energy separations.
- 8. Calculate properties
  - (a) Calculate hyperfine structure using the rci wave functions.
  - (b) Calculate isotope shift using the rci wave functions.
  - (c) Compute the transition rates from the rci wave functions. Calculation in two steps: biorthonormal transformation and evaluation of transition matrix elements using standard Racah algebra methods.

### **Program Input**

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

```
********************************
        RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
        OUTPUT FILE: isodata
*************************************
>>rnucleus
 Enter the atomic number:
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>7
 The default root mean squared radius is
                                       2.4440000057220459
                                                                 (Angeli)
                                                            fm:
  the default nuclear skin thickness is
                                       2.29999999999998
                                                            fm:
 Revise these values?
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
>>6.941
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1.5
 Enter the nuclear dipole moment (in nuclear magnetons):
>>3.2564268
 Enter the nuclear quadrupole moment (in barns):
>>-0.040
***********************************
        RUN RCSFGENERATE TO GENERATE LIST OF CSFs FOR 2S
        AND 2P WITH THREE CSFs: 1s(2)2s J=1/2, 1s(2)2p- J=1/2,
                                               1s(2)2p J=3/2
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 OUTPUT FILES: rcsf.out, rcsfgenerate.log
```

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```
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
        0: No core
        1: He (
                      1s(2)
                                             = 2 electrons)
        2: Ne ([He] + 2s(2)2p(6)
                                             = 10 electrons)
        3: Ar ([Ne] + 3s(2)3p(6)
                                             = 18 electrons)
        4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                             = 36 electrons)
        5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                             = 54 electrons)
        6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,i)2s(1,i)
 Give configuration 2
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>2s
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,1
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
>>y
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*).
 Give configuration 1
>>1s(2,i)2p(1,i)
 Give configuration 2
 Give set of active orbitals in a comma delimited list ordered by 1-symmetry, e.g., 5s,4p,3d
>>1s,2p
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
        . . . . . . . . .
 3 blocks were created
       block J/P
                             NCSF
           1 1/2+
                                1
           2 1/2-
                                1
           3 3/2-
                                1
```

```
**********************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
************************************
>>cp rcsfgenerate.log 2s_2p_DF.exc
>>cp rcsf.out rcsf.inp
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
           rangular.log
Full interaction? (y/n)
>>y
 . . . . .
RANGULAR: Execution complete.
*************************************
        RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
        * MEANS ALL ORBITALS
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp, rwfnestimate.log
***********************************
>>rwfnestimate
RWFNESTIMATE
This program estimates radial wave functions
for orbitals
Input files: isodata, rcsf.inp, optional rwfn file
Output file: rwfn.inp
Default settings ?
>>y
Loading CSF file ... Header only
                    4 relativistic subshells;
The following subshell radial wavefunctions remain to be estimated:
1s 2s 2p- 2p
```

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```
Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
                      p0
Shell
           е
                                gamma
                                             <r>
                                                      MTP
                                                          SRC
      0.2476D+01 0.9246D+01 0.1000D+01 0.5691D+00
  1s
                                                      332
  2s
      0.2895D+00 0.2308D+01 0.1000D+01 0.3010D+01
                                                      355
                                                           T-F
  2p- 0.2173D+00 0.1444D-03 0.1000D+01 0.3019D+01
      0.2173D+00 0.1204D+01 0.2000D+01 0.3020D+01 358 T-F
  2p
 RWFNESTIMATE: Execution complete
Comment: \langle r \rangle is the mean orbital radius in a.u. (a_0). MTP is the extension of the orbitals on the grid, for which the upper
limit in the default installation is 590 points. SRC is the source of the estimate, in this case T-F (Thomas-Fermi).
***********************************
         RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p AND THEY ARE ALL
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         * MEANS ALL ORBITALS
*************************************
>>rmcdhf_mem
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
 Loading CSF file ... Header only
                        4 relativistic subshells;
 There are/is
 Loading CSF File for ALL blocks
 There are
                     3 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
```

3 blocks (block

1

2 1/2-

J/Parity

3 3/2-

NCF):

1

There are

1 1/2+

1

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Enter ASF serial numbers for each block

```
Block 1 ncf = 1 id = 1/2+
>>1
Block
          2 	 ncf = 1 	 id = 1/2-
>>1
               3 	 ncf = 1 	 id = 3/2-
Block
>>1
level weights (1 equal; 5 standard; 9 user)
Radial functions
1s 2s 2p- 2p
Enter orbitals to be varied (Updating order)
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
       . . . . . . . . . .
RMCDHF: Execution complete.
*******************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                   name.alog, name.log
*******************************
>>rsave 2s_2p_DF
Created 2s_2p_DF.w, 2s_2p_DF.c, 2s_2p_DF.m, 2s_2p_DF.sum, 2s_2p_DF.alog and 2s_2p_DF.log
*************************************
        RUN RCSFGENERATE TO GENERATE n = 3 CAS LIST
        OF CSFs FOR 2S
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
RCSFGENERATE
This program creates a list of {\tt CSFs}
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (*) or has a minimal
occupation e.g., 1s(2,1)2s(2,*)
Outputfile: rcsf.out, rcsfgenerate.log
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
Select core
      0: No core
      1: He (
                 1s(2)
                                    = 2 electrons)
                                   = 10 electrons)
      2: Ne ([He] + 2s(2)2p(6)
```

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```
3: Ar ([Ne] + 3s(2)3p(6)
                                        = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                       = 36 electrons)
                                   = 54 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,*)2s(1,*)
 Give configuration 2
 Give set of active orbitals, as defined by the highest principal quantum number
per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,1
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
  1 blocks were created
                        NCSF
      block J/P
         1 1/2+
**************************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
************************************
>>cp rcsfgenerate.log 2s_3.exc
>>cp rcsf.out rcsf.inp
********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
************************************
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
            rangular.log
 Full interaction? (y/n)
```

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```
>>y
  . . . . . . . . . . .
 RANGULAR: Execution complete.
**********************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                        9 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
 Enter the file name (Null then "rwfn.out")
 Enter the list of relativistic subshells:
>>*
 The following subshell radial wavefunctions remain to be estimated:
 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                                                      MTP SRC
                      p0
                                gamma
                                             <r>
  1s
      0.2518D+01 0.9280D+01 0.1000D+01 0.5732D+00 355
```

0.1963D+00 0.1452D+01 0.1000D+01 0.3873D+01

2s

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```
      2p-
      0.1287D+00
      0.5116D-04
      0.1000D+01
      0.4796D+01
      366
      rwf

      2p
      0.1287D+00
      0.4265D+00
      0.2000D+01
      0.4796D+01
      366
      rwf

      3s
      0.9128D-01
      0.9783D+00
      0.1000D+01
      0.8483D+01
      369
      T-F

      3p-
      0.7531D-01
      0.6591D-04
      0.1000D+01
      0.9267D+01
      371
      T-F

      3p-
      0.7531D-01
      0.5494D+00
      0.2000D+01
      0.9267D+01
      371
      T-F

      3d-
      0.6228D-01
      0.3234D-05
      0.2000D+01
      0.9127D+01
      373
      T-F

      3d-
      0.6228D-01
      0.3237D-01
      0.3000D+01
      0.9128D+01
      373
      T-F

      RWFNESTIMATE:
      Execution complete
```

Comment: please note how we used the wild card \* twice. We start by reading the orbitals from a GRASP file (previous run rwfn.out). Using the wild card \* the program reads as many orbitals as possible, i.e., 1s, 2s, 2p-, 2p. The orbitals 3s, 3p-, 3p, 3d-, 3d then remain to be estimated, and we use Thomas-Fermi estimates. By again using the wild card \* all the remaining orbitals will be Thomas-Fermi estimates. Instead of Thomas-Fermi estimates, we could have used option 4, screened hydrogenic with custom Z and adjusted the charge until the radii <r>
of the 1s and 2s spectroscopic orbitals, see Section 6.8 for an example of the use of option 4.

\*

```
RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
         NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
         VARY THE CORRELATION ORBITALS 3s,3p, 3d. NONE OF THESE ARE
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         3* MEANS 3s, 3p-, 3p, 3d-, 3d
>>rmcdhf_mem
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-consistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                       9 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                   79 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    1 blocks (block
                                       J/Parity
                                                  NCF):
  1 1/2+
            79
 Enter ASF serial numbers for each block
 Block
                 1 ncf =
                               79 \text{ id} = 1/2 +
>>1
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
>>3*
```

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```
Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
   . . . . . . . . . .
 RMCDHF: Execution complete.
*************************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                     name.alog, name.log
********************************
>>rsave 2s_3
 Created 2s_3.w, 2s_3.c, 2s_3.m, 2s_3.sum, 2s_3.alog and 2s_3.log
***********************************
        RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
        INPUT FILES : isodata, 2s_3.c, 2s_3.w
        OUTPUT FILES: 2s_3.cm, 2s_3.csum, 2s_3.clog, rci.res
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
**************************************
>>rci
R.C.T
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog, rci.res
 Default settings?
>>y
 Name of state:
>>2s_3
                1 , ncf =
Block
                                   79
Loading CSF file ... Header only
 There are/is
                     9 relativistic subshells;
 Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
>>y
Enter the scale factor:
>>1.d-6
Include H (Vacuum Polarisation)?
>>y
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
>>n
```

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```
Estimate self-energy?
 Largest n quantum number for including self-energy for orbital
 n should be less or equal 8
>>3
 Loading Radial WaveFunction File ...
 There are
                    1 blocks (block
                                       J/Parity
                                                NCF):
  1 1/2+
 Enter ASF serial numbers for each block
 Block
                1 	 ncf =
                              79 \text{ id} = 1/2 +
>>1
   . . . . . .
 RCI: Execution complete.
**********************************
         RUN JJ2LSJ TO TRANSFORM FROM JJ- TO LSJ-COUPLING
         INPUT FILES: 2s_3.c, 2s_3.cm
         OUTPUT FILE: 2s_3.lsj.lbl, 2s_3.uni.lsj.lbl
*******************************
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2021).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
              name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>2s_3
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 79 relativistic CSFs;
  ... load complete;
Mixing coefficients from a CI calc.?
>>7
 Do you need a unique labeling? (y/n)
>>y
   nelec =
                      3
   ncftot =
                     79
                      9
   nblock =
                      1
   block
            ncf
                   nev
                          2j+1 parity
            79
      1
                     1
                             2
                                    1
```

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```
Default settings? (y/n)
>>y
 jj2lsj: Execution Complete
************************************
         RUN RCSFGENERATE TO GENERATE n = 3 CAS LIST
         OF CSFs FOR 2P
                                                                          *
         OUTPUT FILES: rcsfgenerate.log, rcsf.out
*************************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                     1s(2)
                                          = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                          = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,*)2p(1,*)
 Give configuration 2
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
```

. . . .

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#### 2 blocks were created

block J/P NCSF 1 1/2-76 2 3/2-110

```
********************************
      COPY FILES
********************************
>>cp rcsfgenerate.log 2p_3.exc
>>cp rcsf.out rcsf.inp
********************************
      RUN RANGULAR TO GENERATE ENERGY EXPRESSION
      INPUT FILE : rcsf.inp
      OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
*********************************
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
```

Outputfiles: mcp.30, mcp.31, ....

rangular.log

Full interaction? (y/n)

>>y

RANGULAR: Execution complete.

\* RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS \* WE CAN USE WILD CARDS \* TO SPECIFY ORBITALS \* MEANS ALL ORBITALS WE TAKE THE SPECTROSCOPIC ORBITALS FROM OUR DF CALCULATION INPUT FILES: isodata, rcsf.inp, previous rwfn files OUTPUT FILE: rwfn.inp \*

## >>rwfnestimate

## RWFNESTIMATE

This program estimates radial wave functions

for orbitals

Input files: isodata, rcsf.inp, optional rwfn file

Output file: rwfn.inp

Default settings ?

>>y

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```
Loading CSF file ... Header only
 There are/is
                       9 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
>>2s_2p_DF.w
Enter the list of relativistic subshells:
>>*
 The following subshell radial wavefunctions remain to be estimated:
 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                                                   MTP SRC
          е
                     p0
                               gamma
                                           <r>
     0.2518D+01 0.9280D+01 0.1000D+01 0.5732D+00 355
  1s
                                                        2s_
     0.1963D+00 0.1452D+01 0.1000D+01 0.3873D+01 361
  2p- 0.1287D+00 0.5116D-04 0.1000D+01 0.4796D+01 366
                                                        2s_
      0.1287D+00 0.4265D+00 0.2000D+01 0.4796D+01
                                                   366
                                                        2s_
      0.9128D-01 0.9783D+00 0.1000D+01 0.8483D+01 369
                                                        T-F
  3p- 0.7531D-01 0.6591D-04 0.1000D+01 0.9267D+01 371 T-F
      0.7531D-01 0.5494D+00 0.2000D+01 0.9267D+01 371
  3p
                                                        T-F
  3d- 0.6228D-01 0.3234D-05 0.2000D+01 0.9127D+01
                                                   373
                                                        T-F
      0.6228D-01 0.3237D-01 0.3000D+01 0.9128D+01 373 T-F
 RWFNESTIMATE: Execution complete.
************************************
         RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
         NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
         VARY THE CORRELATION ORBITALS 3s,3p, 3d. NONE OF THESE ARE
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         3* MEANS 3s, 3p-, 3p, 3d-, 3d
**************************************
```

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```
RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
                      9 relativistic subshells;
 There are/is
 Loading CSF File for ALL blocks
 There are
                  186 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    2 blocks (block
                                      J/Parity NCF):
 1 1/2-
            76
                    2 3/2-
                             110
 Enter ASF serial numbers for each block
 Block
                1
                    ncf =
                                   76 \text{ id} = 1/2-
>>1
Block
                    ncf =
                                  110 id = 3/2-
>>1
 level weights (1 equal; 5 standard; 9 user)
>>5
Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
>>3*
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
  . . . . . .
 RMCDHF: Execution complete.
******************************
         RUN RSAVE TO SAVE OUTPUT FILES
******************************
>>rsave 2p_3
 Created 2p_3.w, 2p_3.c, 2p_3.m, 2p_3.sum, 2p_3.alog and 2p_3.log
********************************
         RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
         INPUT FILES : isodata, 2p_3.c, 2p_3.w
         OUTPUT FILES: 2p_3.cm, 2p_3.csum, 2p_3.clog, rci.res
         THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
         LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
```

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```
THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
>>rci
 RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog, rci.res
Default settings?
>>y
 Name of state:
>>2p_3
               1 , ncf =
Block
                                  76
               2 , ncf =
                                 110
 Block
Loading CSF file ... Header only
 There are/is 9 relativistic subshells;
 Include contribution of H (Transverse)?
>>y
Modify all transverse photon frequencies?
>>y
Enter the scale factor:
>>1.d-6
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
Include H (Specific Mass Shift)?
Estimate self-energy?
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
>>3
Loading Radial WaveFunction File ...
 There are
           2 blocks (block J/Parity NCF):
 1 1/2- 76
                  2 3/2- 110
 Enter ASF serial numbers for each block
          1 	 ncf =
Block
                                76 \text{ id} = 1/2
>>1
           2 	 ncf = 110 	 id = 3/2-
Block
>>1
 RCI: Execution complete.
```

\* RUN JJ2LSJ TO TRANSFORM FROM JJ- TO LSJ-COUPLING

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```
INPUT FILES: 2p_3.c, 2p_3.cm
        OUTPUT FILE: 2p_3.lsj.lbl, 2p_3.uni.lsj.lbl
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2021).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
             name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>2p_3
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 186 relativistic CSFs;
  ... load complete;
Mixing coefficients from a CI calc.?
>>y
 Do you need a unique labeling? (y/n)
>>y
   nelec =
                     3
   ncftot =
                   186
   nw
                     9
   nblock =
                     2
  block
           ncf
                         2j+1 parity
                  nev
            76
                            2
                                 -1
      1
                    1
      2
           110
                    1
                            4
 Default settings? (y/n)
>>y
jj2lsj: Execution Complete
************************************
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS.
        IF DESIRED WE CAN INSTEAD RUN RLEVELSEV TO GET THE SEPARATION IN EV *
>> rlevels 2s_3.cm 2p_3.cm
 nblock =
                     ncftot =
                                           nw =
                                                          9 nelec =
                   2
                     ncftot =
                                      186 nw =
                                                                               3
 nblock =
                                                              nelec =
 Energy levels for ...
 Rydberg constant is 109737.31569
 Splitting is the energy difference with the lower neighbour
```

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No Pos J Parity	Energy Total (a.u.)			Configuration	
1 1 1/2 +	-7.4719740	0.00	0.00		
2 1 1/2 -	-7.4719740 -7.4042610	14861.28	14861.28	1s(2).2p_2P	
3 1 3/2 -	-7.4042597	14861.57	0.29	1s(2).2p_2P	
	************** S FOR 2s_3	*******	*******	**************************************	
	_	9s 3.c. 2s	3.w. 2s 3.cm		
* INPUT FILES: isodata, 2s_3.c, 2s_3.w, 2s_3.cm					
	•			**********	•
>>rhfs					
RHFS					
This is the hype					
Input files: is			name.w		
Output files: na	me.(c)h, name.(	c)hoffd			
Default settings	?				
>>y					
Name of state					
>>2s_3					
Mixing coefficie	nts from a CI ca	alc.?			
>>y					
DUEC. Evenution	acmmlete				
RHFS: Execution	complete.				
				*********	:
	AGONAL HFS CONST			*	•
	SLIGHTLY EDITED			**************************************	
*****	******	*****	******	• * * * * * * * * * * * * * * * * * * *	•
>> more 2s_3.ch					
Nuclear spin		1 5000	000000000000000000000000000000000000000	0+00 au	
Nuclear magnetic	dipole moment		126800000000001		
Nuclear electric	-				
Interaction cons	tante				
interaction cons	tants:				
Level1 J Parity	A (MHz)	)	B (MHz)	total g_J	
1 1/2 +	3.8844184122	2D+02 -0.	.000000000D-	+00 2.0023047262D+00	
******	******	*******	******	*********	:
* RUN RHF	S FOR 2p_3			*	•

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```
INPUT FILES: isodata, 2p_3.c, 2p_3.w, 2p_3.cm
        OUTPUT FILE: 2p_3.ch, 2p_3.choffd
************************************
>>rhfs
 RHFS
 This is the hyperfine structure program
 Input files: isodata, name.c, name.(c)m, name.w
 Output files: name.(c)h, name.(c)hoffd
 Default settings?
>>y
 Name of state
>>2p_3
Mixing coefficients from a CI calc.?
   . . . . .
 RHFS: Execution complete.
***********************************
        VIEW DIAGONAL HFS CONSTANTS AND GJ FACTORS
        OUTPUT SLIGHTLY EDITED TO DISPLAY ONLY THE TOTAL GJ
************************************
>> more 2p_3.ch
                                 1.50000000000000D+00 au
Nuclear spin
                                 3.25642680000000D+00 n.m.
Nuclear magnetic dipole moment
Nuclear electric quadrupole moment -4.00000000000000D-02 barns
 Interaction constants:
 Level1 J Parity
                       A (MHz)
                                         B (MHz)
                                                         total g_J
         1/2 -
                  4.4821853986D+01
                                   -0.00000000D+00
                                                      6.6588395646D-01
   1
   1
         3/2 -
                  -3.5378452915D+00
                                   -1.7729096327D-01
                                                      1.3340987050D+00
```

Please note that rhfs computes both diagonal and off-diagonal hyperfine interaction constants. The latter are available in the name.choffd file. The off-diagonal parameters are sometimes available from experiment. For Li I, the  $A_{3/2,1/2}$  interaction constant is for example measured from level-crossing spectroscopy [38]. For systems with small fine-structure separations, the off-diagonal hyperfine parameters are of crucial importance in order to model the observed hyperfine line profiles [39]. For systems with large fine structure separations, the off-diagonal hyperfine constants may be neglected.

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```
RIS: Execution begins ...
 Default settings?
>>y
Name of state
>>2s_3
Mixing coefficients from a CI calc.?
>>y
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 79 relativistic CSFs;
  ... load complete;
 Loading Radial WaveFunction File ...
   nelec =
                    .3
   ncftot =
                   79
   nw =
                     9
   nblock =
                     1
  block
          ncf nev
                         2j+1 parity
           79
                  1
 RIS_CAL: Execution Begins ...
 Compute higher order field shift electronic factors?
>>y
 One-body angular file not available
 Two-body angular file not available
 Save ang. coefficients of one- and two-body op.?
>>y
   . . . . .
 RIS: Execution complete.
**********************************
        VIEW SPECIFIC MASS SHIFT AND FIELD SHIFT PARAMETERS
        OUTPUT EDITED TO FIT THE PAGE
*******************************
>> more 2s_3.ci
 Number of eigenvalues: 1
 Level J Parity Energy
  1
         1/2 +
                    -0.7471973983D+01 (a.u.)
 Level J Parity Normal mass shift parameter
```

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```
<K^1>
                                                <K^2+K^3>
                                                                   <K^1+K^2+K^3>
  1
         1/2 +
                        0.7475765524D+01
                                            -0.6760181109D-02
                                                                  0.7469005343D+01 (a.u.)
                        0.2698364414D+05
                                           -0.2440075478D+02
                                                                  0.2695924338D+05
                                                                                     (GHz u)
Level J Parity Specific mass shift parameter
                             <K^1>
                                                <K^2+K^3>
                                                                   <K^1+K^2+K^3>
         1/2 +
                        0.3072684862D+00
                                                                  0.3070570663D+00 (a.u.)
  1
                                           -0.2114198685D-03
                        0.1109080195D+04
                                           -0.7631162959D+00
                                                                  0.1108317079D+04
                                                                                    (GHz u)
Level J Parity Electron density in atomic units
                        Dens. (a.u.)
  1
         1/2 +
                        0.1388454525D+02
Level J Parity Field shift electronic factors and average point discrepancy in fit
                        FO (GHz/fm<sup>2</sup>)
                                             F2 (GHz/fm<sup>4</sup>)
                                                                  F4 (GHz/fm<sup>6</sup>)
                        0.2049813242D+00
         1/2 +
                                           -0.3342886617D-05
                                                                  0.5289532830D-07
  1
                                             Disc. (per mille)
                        F6 (GHz/fm^8)
         1/2 +
                        -0.7068539282D-09
                                              0.0000
Level J Parity Field shift electronic factors (corrected for varying density inside nucleus)
                                             FOVED1 (GHz/fm<sup>4</sup>)
                        FOVEDO (GHz/fm^2)
  1
         1/2 +
                        0.2049364945D+00
                                           -0.2839055991D-05
```

The normal and specific mass shift parameters are those of the three terms defined in TP Sections 3.3, Equations (73) and (74). The field shift electronic factors  $F_0, F_2, \ldots, F_6$  are the ones defined in TP Section 3.3, Equation (79).  $F_{\Gamma J,0}^{(0)\text{ved}}$  and  $F_{\Gamma J,0}^{(1)\text{ved}}$  are the parameters defined in TP Section 3.3, Equation (83).

>>2p\_3

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```
Mixing coefficients from a CI calc.?
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 186 relativistic CSFs;
  ... load complete;
 Loading Radial WaveFunction File ...
   nelec =
   ncftot =
                   186
                      9
   nw
   nblock =
                      2
  block
          ncf nev
                         2j+1 parity
            76
                   1
                          2
                                  -1
           110
                     1
 RIS_CAL: Execution Begins ...
 NRNUC:
                91
  Compute higher order field shift electronic factors?
>>y
 One-body angular file not available
 Two-body angular file not available
 Save ang. coefficients of one- and two-body op.?
 Column 100 complete;
 Column 100 complete;
 RIS: Execution complete.
*************************************
         VIEW SPECIFIC MASS SHIFT AND FIELD SHIFT PARAMETERS
         OUTPUT EDITED TO FIT THE PAGE
************************************
>> more 2p_3.ci
 Number of eigenvalues:
 Level J Parity Energy
         1/2 -
   1
                    -0.7404260995D+01 (a.u.)
   1
         3/2 -
                     -0.7404259683D+01 (a.u.)
 Level J Parity Normal mass shift parameter
                          <K^1>
                                           <K^2+K^3>
                                                           <K^1+K^2+K^3>
         1/2 -
                      0.7409611828D+01
                                      -0.6671237484D-02
                                                          0.7402940590D+01 (a.u.)
                                                          0.2672078382D+05 (GHz u)
                      0.2674486353D+05
                                      -0.2407971433D+02
```

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```
<K^1>
                                               <K^2+K^3>
                                                                 <K^1+K^2+K^3>
  1
         3/2 -
                       0.7409602908D+01
                                          -0.6657064450D-02
                                                                0.7402945843D+01 (a.u.)
                                         -0.2402855701D+02
                       0.2674483134D+05
                                                                0.2672080278D+05 (GHz u)
Level J Parity Specific mass shift parameter
                            <K^1>
                                               <K^2+K^3>
                                                                 <K^1+K^2+K^3>
         1/2 -
                       0.2425644688D+00
  1
                                         -0.1746264308D-03
                                                                0.2423898424D+00 (a.u.)
                       0.8755321826D+03 -0.6303110296D+00
                                                                0.8749018716D+03 (GHz u)
                                                                 <K^1+K^2+K^3>
                            <K^1>
                                               <K^2+K^3>
  1
         3/2 -
                       0.2425741100D+00
                                          -0.1915018511D-03
                                                                0.2423826081D+00 (a.u.)
                       0.8755669823D+03
                                          -0.6912225626D+00
                                                                0.8748757597D+03 (GHz u)
Level J Parity Electron density in atomic units
                       Dens. (a.u.)
         1/2 -
                       0.1372240739D+02
  1
  1
         3/2 -
                       0.1372240990D+02
Level J Parity Field shift electronic factors and average point discrepancy in fit
                       FO (GHz/fm<sup>2</sup>)
                                           F2 (GHz/fm<sup>4</sup>)
                                                                F4 (GHz/fm<sup>6</sup>)
         1/2 -
                       0.2025876387D+00 -0.3303847114D-05
                                                                0.5227748000D-07
  1
         3/2 -
  1
                       0.2025876757D+00 -0.3303847831D-05
                                                                0.5227749057D-07
                       F6 (GHz/fm^8)
                                           Disc. (per mille)
         1/2 -
                       -0.6985943239D-09
                                             0.0000
         3/2 -
                       -0.6985944586D-09
                                             0.0000
  1
Level J Parity Field shift electronic factors (corrected for varying density inside nucleus)
                       FOVEDO (GHz/fm^2)
                                          FOVED1 (GHz/fm<sup>4</sup>)
         1/2 -
  1
                       0.2025433326D+00
                                           -0.2805899138D-05
         3/2 -
                       0.2025433696D+00
                                          -0.2805899756D-05
```

Comment: Given the information in 2s\_3.ci and 2p\_3.ci together with isotopic data, the frequency isotope shift can be computed using the fical program, see Section 12.2.

```
*******

* RUN RBIOTRANSFORM FOR 2s_3 AND 2p_3 TO TRANSFORM WAVE FUNCTIONS

* INPUT FILES: isodata, 2s_3.c, 2s_3.w, 2s_3.cm, *

* 2p_3.c, 2p_3.w, 2p_3.cm *

* OUTPUT FILES: 2s_3.cbm, 2s_3.bw, 2p_3.cbm, 2p_3.bw *

* 2s_3.TB, 2p_3.TB (angular files) *

* NOTE THAT THE ORDER OF INITIAL AND FINAL STATE DOES NOT MATTER *
```

>>rbiotransform

RBIOTRANSFORM

```
This program transforms the initial and final wave
 functions so that standard tensor albegra can be
 used in evaluation of the transition parameters
 Input files: isodata, name1.c, name1.w, name1.(c)m
              name2.c, name2.w, name2.(c)m
              name1.TB, name2.TB (optional angular files)
 Output files: name1.bw, name1.(c)bm,
              name2.bw, name2.(c)bm
              name1.TB, name2.TB (angular files)
 Default settings?
>>y
 Input from a CI calculation?
>>y
  Name of the Initial state
>>2s_3
  Name of the Final state
>>2p_3
  Transformation of all J symmetries?
 BIOTRANSFORM: Execution complete.
******************************
         RUN RTRANSITION FOR 2s_3 and 2p_3 TO COMPUTE TRANSITION PARAMETERS
         INPUT FILES: isodata, 2s_3.c, 2s_3.bw, 2s_3.cbm
                      2p_3.c, 2p_3.bw, 2p_3.cbm
         OUTPUT FILES: 2s_3.2p_3.ct
                       2s_3.2p_3.-1T (angular file)
         NOTE THAT THE ORDER OF INITIAL AND FINAL STATE DOES NOT MATTER
>>rtransition
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
              name2.c, name2.bw, name2.(c)bm
              optional, name1.lsj.lbl, name2.lsj.lbl
              name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
              optional, name1.name2.(c)t.lsj
              name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
 Default settings?
>>y
 Input from a CI calculation?
>>y
 Name of the Initial state
>>2s_3
 Name of the Final state
```

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```
>>2p_3
MRGCSL: Execution begins ...
Loading Configuration Symmetry List File ...
There are 9 relativistic subshells;
There are 79 relativistic CSFs;
 ... load complete;
Loading Configuration Symmetry List File ...
There are 9 relativistic subshells;
There are 186 relativistic CSFs;
 ... load complete;
         1 s
         2 s
         2 p-
         2 p
         3 s
         3 p-
         3 p
         3 d-
         3 d
         1
        79
         2
        76
                 186
Loading Configuration Symmetry List File ...
 there are 9 relativistic subshells;
 there are 265 relativistic CSFs;
 ... load complete;
Enter the list of transition specifications
 e.g., E1,M2 or E1 M2 or E1;M2:
>>E1
RTRANSITION: Execution complete.
******************************
        VIEW COMPUTED TRANSITION PARAMETERS
>>more 2s_3.2p_3.ct
Transition between files:
f1 = 2s_3
f2 = 2p_3
Electric 2**( 1)-pole transitions
_____
Upper
          Lower
Lev J P Lev J P
                  E (Kays)
                              A (s-1)
                                                               S
                                                   gf
f2 1 1/2 - f1 1 1/2 + 14861.28 C 3.81311D+07 5.17671D-01 1.14676D+01
                                    B 3.74756D+07 5.08773D-01 1.12705D+01
```

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f2 1 3/2 - f1 1 1/2 + 14861.57 C 3.81334D+07 1.03537D+00 2.29353D+01 B 3.74782D+07 1.01758D+00 2.25413D+01 \* VIEW COMPUTED TRANSITION PARAMETERS IN LSJ COUPLING \* >>more 2s\_3.2p\_3.ct.1sj Transition between files:  $2s_3$ 2p\_3 -7.47197398 1s(2).2s\_2S -7.40426099 1s(2).2p\_2P 6728.20 ANGS(AIR) 14861.28 CM-1 6728.89 ANGS(VAC) E1 S = 1.12705D+01 GF = 5.08773D-01AKI = 3.74756D+07dT = 0.017191.14676D+01 5.17671D-01 3.81311D+07 -7.47197398 1s(2).2s\_2S 1 -7.40425968 1s(2).2p\_2P 14861.57 CM-1 6728.76 ANGS(VAC) 6728.06 ANGS(AIR) GF = 1.01758D+00AKI = 3.74782D+07dT = 0.01718E1 S = 2.25413D+01

Comment: the values in Babushkin gauge are now shown on the first line. In addition, the uncertainty parameter

$$dT = \frac{|A_C - A_B|}{\max(A_C, A_B)}$$

3.81334D+07

is given, see TP Section 3.5.

2.29353D+01

6.2. Second Example:  $1s^22s2p$   $^3P_{0,1,2}^o$ ,  $^1P_1^o$  for B II in Different Coupling Schemes – HF Initial Estimates

1.03537D+00

The second example is  $1s^22s2p$   $^3P_{0,1,2}^o$ ,  $^1P_1^o$  for B II in different coupling schemes and aims to illustrate the use of the Coupling program. In this example, we also illustrate how we can use converted HF wave function as starting estimates for the radial orbitals.

#### Overview

- 1. Define nuclear data
- 2. Obtain common spectroscopic orbitals for the MR set
  - (a) Generate configuration list containing 4 CSFs belonging to  $1s^22s2p^{-1.3}P^0$
  - (b) Perform angular integration
  - (c) Perform HF calculation
  - (d) Convert HF orbitals to relativistic orbitals. We do not need to run rwfnestimate since all orbitals have been estimated
  - (e) Perform SCF calculation on the weighted average on the state belonging to  $1s^22s2p^{-1.3}p^0$
  - (f) Save output to 2s2p\_DF
- 3. Transform from *jj* to *LSJ*-coupling
- 4. Run rlevels to view energy separations.
- 5. Run jj2lsj, Coupling, and rlevels to define energy spectra in different coupling scheme.

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# Program Input

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

```
*********************************
* RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
* OUTPUT FILE: isodata
***********************************
>>rnucleus
 RNUCLEUS
 This program defines nuclear data and the radial grid
 Outputfile: isodata
 Enter the atomic number:
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>11
 The default root mean squared radius is
                                       2.4059998989105225
                                                             fm;
                                                                  (Angeli)
  the default nuclear skin thickness is
                                       2.299999999999998
                                                             fm;
 Revise these values?
>>n
Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1.5
 Enter the nuclear dipole moment (in nuclear magnetons):
>>2.6886489
 Enter the nuclear quadrupole moment (in barns):
>>1
************************************
        RUN RCSFGENERATE TO GENERATE LIST FOR
        1P_1 AND 3P_0,1,2 WITH FOUR CSFs: 2s2p- J=0, 2s2p- J=1,
                                        2s2p J=1, 2s2p J = 2
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
**********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
```

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```
1s(2)
       1: He (
                                       = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                      = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                      = 18 electrons)
                                      = 36 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                   = 54 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,i)2s(1,i)2p(1,i)
 Give configuration 2
>>
 Give set of active orbitals, as defined by the highest principal quantum number
per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>2s,2p
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 3 blocks were created
       block J/P
                         NCSF
         1
             0-
                            1
          2
              1-
            2-
**********************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
***********************************
>>cp rcsfgenerate.log 2s2p_DF.exc
>>cp rcsf.out rcsf.inp
*******************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
**************************************
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
```

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```
rangular.log
Full interaction? (y/n)
>>y
RANGULAR: Execution complete.
******************************
        RUN HF PROGRAM TO GENERATE NON-RELATIVISTIC RADIAL ORBITALS
        THAT CAN BE CONVERTED TO RELATIVISTIC ORBITALS
        OUTPUT FILE: wfn.out
>>hf
                   H A R T R E E - F O C K . 96
                   _____
             THE DIMENSIONS FOR THE CURRENT VERSION ARE:
                     NWF= 20
                                 NO=220
 START OF CASE
 =========
 Enter ATOM, TERM, Z
 Examples: 0,3P,8. or Oxygen,AV,8.
>>B,AV,5.
 List the CLOSED shells in the fields indicated (blank line if none)
  ... ... ... ... ... ... ... etc.
     (Please note that the closed shells should be entered right-justified with
>> 1s
        respect to the dots on the line above!!!)
 Enter electrons outside CLOSED shells (blank line if none)
 Example: 2s(1)2p(3)
>>2s(1)2p(1)
 There are 3 orbitals as follows:
    1s 2s 2p
 Orbitals to be varied: ALL/NONE/=i (last i)/comma delimited list/H
>>all
 Default electron parameters ? (Y/N/H)
 Default values for remaining parameters? (Y/N/H)
>>y
```

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WEAK ORTHOGONALIZATION DURING THE SCF CYCLE T

SCF CONVERGENCE TOLERANCE (FUNCTIONS) = 1.00D-08

NUMBER OF POINTS IN THE MAXIMUM RANGE = 220

ITERATION NUMBER 1

-----

. . . . . . . . . . . . . . . .

ITERATION NUMBER 6

-----

SCF CONVERGENCE CRITERIA (SCFTOL\*SQRT(Z\*NWF)) = 1.2D-06

C(1s 2s) = 0.00000 V(1s 2s) = -7.06535 EPS = 0.000000

 $E(2s 1s) = 0.02654 \quad E(1s 2s) = 0.01327$ 

EL F.D A 7. NORM DPM 20.8332819 1.0000000 16.3418222 1s 1.93D-08 2s 1.8579695 4.7336947 1.0000000 1.38D-08 2p 1.4015370 4.0799511 1.0000000 1.74D-08

< 1s| 2s >= 8.0D-09

TOTAL ENERGY (a.u.)

-----

 Non-Relativistic
 -24.06678870
 Kinetic
 24.06678852

 Relativistic Shift
 -0.00587815
 Potential
 -48.13357722

 Relativistic
 -24.07266685
 Ratio
 -2.000000008

Additional parameters ? (Y/N/H)

>>n

Do you wish to continue along the sequence ? >>n

END OF CASE

=======

>>cp wfn.out wfn.inp

\*

\* RUN RWFNMCHFMCDF TO CONVERT NON-RELATIVISTIC RADIAL ORBITALS TO

\* RELATIVISTIC ONES \*

\* INPUT FILE: wfn.inp \*

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```
OUTPUT FILE: rwfn.out
***********************************
>>rwfnmchmcdf
RWFNMCHFMCDF
This program converts non-relativistic radial
orbitals to relativistic ones in GRASP format
Input file: wfn.inp
Output file: rwfn.out
********************************
        COPY FILES
        WE DONT NEED TO INVOKE RWFNESTIMATE SINCE ALL ORBITALS HAVE
        BEEN ESTIMATED THROUGH THE MCHF MCDF CONVERSION
************************************
>>cp rwfn.out rwfn.inp
***********************************
        RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS.
        INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
        OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
        NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
        THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
        ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p AND THEY ARE ALL
        SPECTROSCOPIC. WE CAN USE WILD CARDS FOR SPECIFYING ORBITALS
                ************************
>>rmcdhf_mem
RMCDHF
This program determines the radial orbitals
and the expansion coefficients of the CSFs
in a self-onsistent field proceedure
Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
>>y
Loading CSF file ... Header only
There are/is
                    4 relativistic subshells;
Loading CSF File for ALL blocks
There are
           4 relativistic CSFs... load complete;
Loading Radial WaveFunction File ...
There are
                  3 blocks (block
                                   J/Parity
                                             NCF):
   0-
                      1-
                                              1
Enter ASF serial numbers for each block
         1 	 ncf =
                           1 id =
Block
>>1
Block
          2 ncf = 2 id =
```

1-

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```
>>1,2
               3 ncf =
Block
                                1 id =
                                          2-
>>1
level weights (1 equal; 5 standard; 9 user)
>>5
Radial functions
1s 2s 2p- 2p
Enter orbitals to be varied (Updating order)
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
 . . . . .
RMCDHF: Execution complete.
***********************************
        RUN RSAVE TO SAVE OUTPUT FILES
*************************************
>>rsave 2s2p_DF
Created 2s2p_DF.w, 2s2p_DF.c, 2s2p_DF.m, 2s2p_DF.sum, 2s2p_DF.alog and 2s2p_DF.log
RUN JJ2LSJ TO GET THE LSJ-COMPOSITION
        INPUT FILE: 2s2p_DF.c, 2s2p_DF.m
        OUTPUT FILE: 2s2p_DF.lsj.lbl, 2s2p_DF.uni.lsj.lbl
>>jj21sj
jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
       into an LSJ-coupled CSF basis (Fortran 95 version)
       (C) Copyright by G. Gaigalas and Ch. F. Fischer,
       (2021).
Input files: name.c, name.(c)m
Output files: name.lsj.lbl
  (optional) name.lsj.c, name.lsj.j,
            name.uni.lsj.lbl, name.uni.lsj.sum
Name of state
>>2s2p_DF
Loading Configuration Symmetry List File ...
There are 4 relativistic subshells;
There are 4 relativistic CSFs;
 ... load complete;
Mixing coefficients from a CI calc.?
 Do you need a unique labeling? (y/n)
   nelec =
                    4
```

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```
4
  ncftot =
  nblock =
 block
       ncf nev 2j+1 parity
                    1 -1
    1
        1
              1
               2
                    3
    2
          2
                          -1
         1
    3
                     5
                          -1
               1
Default settings? (y/n)
```

. . . .

jj2lsj: Execution complete.

>> rlevels 2s2p\_DF.m

```
nblock = 3 ncftot = 4 nw = 4 nelec = 4
```

Energy levels for ...

Rydberg constant is 109737.31569

Splitting is the energy difference with the lower neighbor

No	F	os	J	Parity	Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	Configuration	
1		1	0	-	-24.1270878	0.00	0.00	1s(2).2s_2S.2p_3P	
2		1	1	-	-24.1270404	10.39	10.39	1s(2).2s_2S.2p_3P	
3		1	2	-	-24.1269457	31.17	20.79	1s(2).2s_2S.2p_3P	
4		2	1	-	-23.9154061	46458.75	46427.58	1s(2).2s_2S.2p_1P	

For interpretation of *LSJ*-coupling notation produced by jj21sj see Section 8.2, where we discuss in detail the transformation from jj- to *LSJ*-coupling for the  $1s^22s^22p^3$  and  $1s^22p^5$  configurations in Si VIII.

>>jj21sj

```
jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
  into an LS-coupled CSF basis (Fortran 95 version)
  (C) Copyright by G. Gaigalas and Ch. F. Fischer,
  (2021).
```

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```
Input files: name.c, name.(c)m
        Ouput files: name.lsj.lbl,
         (optional) name.lsj.c, name.lsj.j,
                    name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>2s2p_DF
  Loading Configuration Symmetry List File ...
 There are 4 relativistic subshells;
 There are 4 relativistic CSFs;
  ... load complete;
  Mixing coefficients from a CI calc.?
>>n
Do you need a unique labeling? (y/n)
>>n
   nelec =
   ncftot =
                      4
                      4
   nblock =
                     3
  block
           ncf nev 2j+1 parity
      1
            1
                   1
                           1 -1
      2
                     2
                            3
                                   -1
                            5
                                   -1
      3
             1
                    1
 Default settings? (y/n)
  All levels (Y/N)
>>y
 Maximum % of omitted composition
 What is the value below which an eigenvector composition
 is to be neglected for printing?
>>0.01
 jj2lsj: Execution complete.
***********************************
         RUN COUPLING TO GET THE IDENTIFICATION STATES IN DIFFERENT
         COUPLING SCHEMES
         INPUT FILES: 2s2p_DF.lsj.c, 2s2p_DF.lsj.j
         OUPUT FILES: 2s2p_DF.coup3.LK3.1b1, 2s2p_DF.coup3.JK3.1b1
                                             2s2p_DF.coup3.LS3.1b1
                     2s2p_DF.coup3.LS.1b1,
                     2s2p_DF.coup3.LSJ3.1bl, 2s2p_DF.coup3.jj.1bl
                     2s2p_DF.coup3.cLSJ3.1b1, 2s2p_DF.coup3.sum
>>Coupling
 Coupling: Transformation of ASFs from a LS-coupled CSF basis
          into differete coupled CSF bases (Fortran 95)
          (C) (2022)
                                   G. Gaigalas, A. Kramida.
 Input files: *.lsj.c, *.lsj.j (ATSP (CPC) or GRASP2K types)
```

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```
Output files: *.coup*.*.lbl, *.coup*.sum
 Name of state
>>2s2p_DF
 Default settings ? (Y/N)
 Specify the number of coupled shells for evaluation (1,2 or 3):
>>3
 3
 What is the value below which an eigenvector composition
 is to be neglected for printing?
>>0
                               0.000000000000000
 Specify shells for recoupling (no more than 12)
>>1s,2s,2p
  All transformations completed
 There is one-to-one classification for LS coupling
 There is one-to-one classification for LS3 coupling
 There is one-to-one classification for LSJ3 coupling
 There is one-to-one classification for LK3 coupling
 There is one-to-one classification for JK3 coupling
 There is one-to-one classification for cLSJ3 coupling
 There is one-to-one classification for jj3 coupling
    end subroutine generate_classification_data
 Coupling: Execution complete.
******************************
        COPY 2s2p_DF.coup3.LK3.1b1 TO 2s2p_DF.lsj.1b1.
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
        IN LK3-COUPLING. COMMENT: RLEVELS TAKES <name.lsj.lbl>
        FOR THIS REASON WE COPY <name.coup3.LK3.lbl> TO <name.lsj.lbl>
**********************************
>>cp 2s2p_DF.coup3.LK3.lbl 2s2p_DF.lsj.lbl
>>rlevels 2s2p_DF.m
 nblock =
                  3 ncftot =
                                     4 	 nw =
                                                       4 	ext{ nelec} =
 Energy levels for ...
 Rydberg constant is 109737.31569
 Splitting is the energy difference with the lower neighbor
 No Pos J Parity Energy Total Levels
                                      Splitting
                                                 Configuration
                  (a.u.)
                            (cm^-1)
                                       (cm^-1)
-----
  1 1 0 -
               -24.1270878
                              0.00
                                         0.00 1s2_ 2s_2p_(3P) P_3[1]<0>
  2 1 1 -
                              10.39
                                        10.39 1s2_ 2s_2p_(3P) P_3[1]<1>
               -24.1270404
  3 1 2 -
                            31.17
               -24.1269457
                                        20.79 1s2_ 2s_2p_(3P) P_3[1]<2>
  4 2 1 -
               -23.9154061 46458.75 46427.58 1s2_ 2s_2p_(1P) P_1[1]<1>
```

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```
**************************************
       COPY 2s2p_DF.coup3.JK3.1bl TO 2s2p_DF.lsj.lbl.
       RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
       IN JK3-COUPLING
*************************************
>>cp 2s2p_DF.coup3.JK3.lbl 2s2p_DF.lsj.lbl
>>rlevels 2s2p_DF.m
nblock = 3 ncftot = 4 nw =
                                             4 nelec =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
______
No Pos J Parity Energy Total Levels Splitting Configuration (a.u.) (cm^-1)
______
 4 2 1 -
            -23.9154061 46458.75 46427.58 1s2_<0>2s_2p_(1P) 1[1]<1>
   ************************************
       COPY 2s2p_DF.coup3.LS3.1bl TO 2s2p_DF.lsj.lbl.
       RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
       IN LS3-COUPLING
*********************************
>>cp 2s2p_DF.coup3.LS3.1bl 2s2p_DF.lsj.lbl
>>rlevels 2s2p_DF.m
                                4 	 nw =
nblock =
         3 ncftot =
                                              4 nelec =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
_____
No Pos J Parity Energy Total Levels Splitting Configuration
         (a.u.) (cm^-1) (cm^-1)
-----

    1
    1
    0
    -
    -24.1270877
    0.00
    0.00
    1s2_ 2s_2p_(3P)
    3P<0>

    2
    1
    1
    -
    -24.1270404
    10.39
    10.39
    1s2_ 2s_2p_(3P)
    3P<1>

    3
    1
    2
    -
    -24.1269457
    31.17
    20.79
    1s2_ 2s_2p_(3P)
    3P<2>

             -23.9154061 46458.75 46427.58 1s2_ 2s_2p_(1P) 1P<1>
 4 2 1 -
  *******************************
       COPY 2s2p_DF.coup3.LSJ3.1b1 TO 2s2p_DF.lsj.1b1.
       RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
       IN LSJ3-COUPLING
```

\*

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>>cp 2s2p\_DF.coup3.LSJ3.1bl: 2s2p\_DF.lsj.1bl >>rlevels 2s2p\_DF.m nblock = 3 ncftot = 4 nw = 4 nelec = Energy levels for ... Rydberg constant is 109737.31569 Splitting is the energy difference with the lower neighbor ------No Pos J Parity Energy Total Levels Splitting Configuration (a.u.)  $(cm^-1)$  $(cm^-1)$ \_\_\_\_\_\_ 1 1 0 - -24.1270877 0.00 0.00 1s2\_2s\_2p\_(3P) (0,0)<0> -24.1270404 10.39 10.39 1s2\_ 2s\_2p\_(3P) (0,1)<1> 2 1 1 -31.17 20.79 1s2\_ 2s\_2p\_(3P) (0,2)<2> 3 1 2 --24.1269457 -23.9154061 46458.75 46427.58 1s2\_ 2s\_2p\_(1P) (0,1)<1> 4 2 1 -\_\_\_\_\_\_ COPY 2s2p\_DF.coup3.jj.lbl TO 2s2p\_DF.lsj.lbl. RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS IN jj-COUPLING \* >>cp 2s2p\_DF.coup3.jj.lbl 2s2p\_DF.lsj.lbl >>rlevels 2s2p\_DF.m nblock = 3 ncftot = 4 nw = 4 nelec = Energy levels for ... Rydberg constant is 109737.31569 Splitting is the energy difference with the lower neighbor \_\_\_\_\_\_ No Pos J Parity Energy Total Levels Splitting Configuration (a.u.) (cm^-1) (cm^-1) 1 1 0 - -24.1270877 0.00 0.00 1s+2\_2s+\_<1/2>.2p-\_(1/2) <0>
2 1 1 - -24.1270404 10.39 10.39 1s+2\_2s+\_<1/2>.2p-\_(1/2) <1>
3 1 2 - -24.1269457 31.17 20.79 1s+2\_2s+\_<1/2>.2p+\_ <2> 4 2 1 --23.9154061 46458.75 46427.58 1s+2\_2s+\_<1/2>.2p+\_(3/2) <1> \* COPY 2s2p\_DF.coup3.cLSJ3.1bl TO 2s2p\_DF.lsj.1bl. RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS IN cLSJ3-COUPLING >>cp 2s2p\_DF.coup3.cLSJ3.1bl 2s2p\_DF.lsj.1bl >>rlevels 2s2p\_DF.m

 $4 \quad \text{nw} =$ 

nblock =

3 ncftot =

4 nelec =

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Energy levels for ...

Rydberg constant is 109737.31569

Splitting is the energy difference with the lower neighbor

No Pos J Parity	y Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	Configuration
1 1 0 - 2 1 1 - 3 1 2 - 4 2 1 -	-24.1270877 -24.1270404 -24.1269457 -23.9154061	0.00 10.39 31.17 46458.75	10.39 20.79	1s+2_ (0,0)<0> 2s_2p_(3P)<0> (0,0)<0> 1s+2_ (0,0)<0> 2s_2p_(3P)<1> (0,1)<1> 1s+2_ (0,0)<0> 2s_2p_(3P)<2> (0,2)<2> 1s+2_ (0,0)<0> 2s_2p_(1P)<1> (0,1)<1>

For definition of different coupling schemes see in [14] and for interpretation of different coupling schemes notation produced by Coupling see Section 8.2.

6.3. Third Example:  $2s^22p^3$  and  $2p^5$  for Si VIII in Different Coupling Schemes–Condensing the CSF List

The third example is  $2s^22p^3$  and  $2p^5$  in Si VIII, where we compute M1 transition rates and give the transition data in different coupling schemes. This example also illustrates the use of the rcsfinteract program to reduce the expansion sizes by retaining only the CSFs that interact with the CSFs in the MR.

### Overview

- Define nuclear data
- 2. Obtain common spectroscopic orbitals for the MR set
  - (a) Generate configuration list belonging to  $2s^22p^3$  and  $2p^5$
  - (b) Perform angular integration
  - (c) Generate initial estimates of radial orbitals
  - (d) Perform SCF calculation on the weighted average of all states belonging to  $2s^22p^3$  and  $2p^5$  (there are two states with J = 1/2, four states with J = 3/2 and one state with J = 5/2, see NIST Tables)
  - (e) Save output to 2s22p3\_2p5\_DF
- 3. Improve states
  - (a) Generate CSF list from SD-excitations from  $2s^22p^3$  and  $2p^5$  to n=3
  - (b) Run resfinteract to extract CSFs that interact with CSFs belonging to  $2s^22p^3$  or  $2p^5$
  - (c) Perform angular integration
  - (d) Generate initial estimates of radial orbitals
  - (e) Perform SCF calculation on the weighted average of all states belonging to  $2s^22p^3$  and  $2p^5$
  - (f) Save output to 2s22p3\_2p5\_3
  - (g) Perform rci calculation in which Breit and QED effects are added.
- 4. Transform from *jj* to *LSJ*-coupling
- 5. Run rlevels to view energy separations.
- 6. Run jj21sj, Coupling, and rlevels to define energy spectra in different coupling schemes for those levels which have 1s, 2s, 2p shells in identification.
- 7. Calculate properties
  - (a) Compute the M1 transition rates from the rci wave functions. Biorthonormal transformation not needed in this case since the states are described using the same orthonormal orbital set. Copy files and run the transition program.
  - (b) Compute the M1 transition rates in different coupling schemes for those levels which have 1s, 2s, 2p shells in identification. Display the transition file.

### **Program Input**

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```
***********************************
* RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
* OUTPUT FILE: isodata
*************************************
>>rnucleus
 RNUCLEUS
 This program defines nuclear data and the radial grid
 Outputfile: isodata
 Enter the atomic number:
>>14
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>28
 The default root mean squared radius is
                                       3.1224000453948975
                                                             fm;
                                                                  (Angeli)
  the default nuclear skin thickness is
                                       2.299999999999998
                                                             fm;
 Revise these values?
>>n
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
>>27.9769271
Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
 Enter the nuclear dipole moment (in nuclear magnetons):
 Enter the nuclear quadrupole moment (in barns):
>>1
Comment: if we are not interested in the hyperfine structure constants we may just set nuclear spin and electromagnetic
moments (magnetic dipole and electric quadrupole) to 1.
   RUN RCSFGENERATE TO GENERATE LIST FOR ALL
        STATES OF 2s(2)2p(3) + 2p(5)
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                   1s(2)
                                         = 2 electrons)
```

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```
2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                         = 36 electrons)
       5: \text{Xe } ([Kr] + 4d(10)5s(2)5p(6) = 54 \text{ electrons})
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,i)2s(2,i)2p(3,i)
 Give configuration 2
>>1s(2,i)2p(5,i)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>2s,2p
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
>>n
        . . . . . . . . .
  3 blocks were created
      block J/P
                          NCSF
          1 1/2-
                            2
          2 3/2-
          3 5/2-
                              1
************************************
         COPY FILES
         NOTE THAT WE COPY THE FILE TO RCSFMR.INP FOR FUTURE USE
         TOGETHER WITH RCSFINTERACT
*************************************
>>cp rcsfgenerate.log 2s22p3_2p5_DF.exc
>>cp rcsf.out rcsf.inp
>>cp rcsf.out rcsfmr.inp
         RUN RANGULAR TO GENERATE ENERGY EXPRESSION
         INPUT FILE : rcsf.inp
         OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
>>rangular
```

RANGULAR

This program performs angular integration

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```
Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
            rangular.log
 Full interaction? (y/n)
>>y
 RANGULAR: Execution complete.
***********************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp, rwfnestimate.log
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                      4 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>3
 Enter the list of relativistic subshells:
>>*
 Orbital Z_eff for hydrogenic orbitals
        14.00
 1s
        14.00
 2s
        14.00
 2p-
        14.00
 All required subshell radial wavefunctions have been estimated:
Shell
          е
                     р0
                              gamma
                                          <r>
                                                 MTP SRC
  1s
      0.9826D+02 0.1033D+03 0.1000D+01 0.1068D+00 328
                                                      Hyd
  2s
     0.2458D+02 0.3670D+02 0.1000D+01 0.4269D+00
                                                  344
                                                      Hyd
  2p- 0.2458D+02 0.8338D-01 0.1000D+01 0.3555D+00
                                                      Hyd
  2p
      0.2452D+02 0.1492D+03 0.2000D+01 0.3568D+00
                                                  343
                                                      Hyd
```

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RWFNESTIMATE: Execution complete.

```
RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p AND THEY ARE ALL
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         NOTE: INSTEAD OF SAYING THAT WE WILL OPTIMIZE ON, FOR EXAMPLE,
         STATES 1,2,3,4 WE CAN WRITE 1-4 MEANING THE SAME THING
********************************
>>rmcdhf_mem
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                       4 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                    7 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
                    3 blocks (block J/Parity NCF):
 There are
 1 1/2-
                    2 3/2-
                               4
                                      3 5/2-
                                                  1
 Enter ASF serial numbers for each block
 Block
                1 	 ncf =
                                    2 \text{ id} = 1/2
>>1-2
Block
                2 ncf =
                                   4 \text{ id} = 3/2
>>1-4
Block
                     ncf =
                                    1 id = 5/2-
 level weights (1 equal; 5 standard; 9 user)
Radial functions
 1s 2s 2p- 2p
Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
```

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. . . . . . RMCDHF: Execution complete. \* RUN RSAVE TO SAVE OUTPUT FILES \* >>rsave 2s22p3\_2p5\_DF Created 2s22p3\_2p5\_DF.w, 2s22p3\_2p5\_DF.c, 2s22p3\_2p5\_DF.m, 2s2p3\_2p5\_DF.sum, 2s2p3\_2p5\_DF.alog and 2s22p3\_2p5\_DF.log \* RUN RCSFGENERATE TO GENERATE LIST OBTAINED BY SD-EXCITATIONS FROM 1s(2)2s(2)2p(3) + 1s(2)2p(5) TO n = 3OUTPUT FILES: rcsfgenerate.log, rcsf.out \* >>rcsfgenerate RCSFGENERATE This program creates a list of CSFs Configurations should be entered in spectroscopic notation with occupation numbers and indications if orbitals are closed (c), inactive (i), active (\*) or has a minimal occupation e.g., 1s(2,1)2s(2,\*)Outputfiles: rcsf.out, rcsfgenerate.log Default, reverse, symmetry or user specified ordering? (\*/r/s/u) >>\* Select core 0: No core 1: He ( 1s(2)= 2 electrons) 2: Ne ([He] + 2s(2)2p(6)= 10 electrons) 3: Ar ([Ne] + 3s(2)3p(6)= 18 electrons) 4: Kr ([Ar] + 3d(10)4s(2)4p(6)= 36 electrons) 5: Xe ([Kr] + 4d(10)5s(2)5p(6)= 54 electrons) 6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons) >>0 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (\*) Give configuration 1 >>1s(2,\*)2s(2,\*)2p(3,\*)Give configuration 2 >>1s(2,\*)2p(5,\*)Give configuration 3 >> Give set of active orbitals, as defined by the highest principal quantum number per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d >>3s,3p,3dResulting 2\*J-number? lower, higher (J=1 -> 2\*J=2 etc.) >>1,5

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```
Number of excitations (if negative number e.g., -2, correlation
orbitals will always be doubly occupied)
>>2
Generate more lists ? (y/n)
>>n
      . . . . . . . . .
3 blocks were created
     block J/P
                      NCSF
        1 1/2-
                        595
         2 3/2-
                        914
         3 5/2-
                        847
********************************
        COPY FILES
>>cp rcsfgenerate.log 2s22p3_2p5_3.exc
>>cp rcsf.out rcsf.inp
RUN RCSFINTERACT PROGRAM TO DETERMINE WHICH OF THE CSFs IN THE
        rcsf.inp LIST INTERACTS WITH THE CSFs IN rcsfmr.inp
        THE INTERACTING CSFs ARE WRITTEN TO rcsf.out
        INPUT FILES: rcsfmr.inp, rcsf.inp
        OUTPUT FILE: rcsf.out
***********************************
>>rcsfinteract
RCSFinteract: Determines all the CSFs (rcsf.inp) that interact
            with the CSFs in the multireference (rcsfmr.inp)
            (C) Copyright by G. Gaigalas and Ch. F. Fischer
            (Fortran 95 version)
                                        NIST (2017).
            Input files: rcsfmr.inp, rcsf.inp
            Output file: rcsf.out
Reduction based on Dirac-Coulomb (1) or
Dirac-Coulomb-Breit (2) Hamiltonian?
>>1
There are 9 relativistic subshells;
 Block
        MR NCSF Before NCSF After NCSF
             2
                     595
                                 274
   1
   2
             4
                      914
                                 591
                                 300
   3
                      847
             1
```

RCSFINTERACT: Execution complete

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Please note that the orbital orders in rcsfmr.inp and rcsf.inp are required to be the same. In the case above, this requirement was fulfilled. In more complex cases, to meet the above requirement, one needs to prescribe the orbital order in the clist.ref file that is used when generating the rcsf.inp list, see Section 6.6.

```
***********************************
        COPY FILES
************************************
>>cp rcsf.out rcsf.inp
************************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
***********************************
>>rangular
RANGULAR.
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
           rangular.log
Full interaction? (y/n)
>>y
RANGULAR: Execution complete.
***********************************
        RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp, rwfnestimate.log
************************************
>>rwfnestimate
RWFNESTIMATE
This program estimates radial wave functions
for orbitals
Input files: isodata, rcsf.inp, optional rwfn file
Output file: rwfn.inp
Default settings ?
Loading CSF file ... Header only
There are/is
                    9 relativistic subshells;
The following subshell radial wavefunctions remain to be estimated:
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
Read subshell radial wavefunctions. Choose one below
```

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```
1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
>>
 Enter the list of relativistic subshells:
 The following subshell radial wavefunctions remain to be estimated:
 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
    3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>3
 Enter the list of relativistic subshells:
>>*
 Orbital Z_eff for hydrogenic orbitals
        14.00
 3s
 3р-
        14.00
       14.00
 Зр
        14.00
 3d-
 3d
        14.00
  All required subshell radial wavefunctions have been estimated:
Shell
                                           <r>
                                                   MTP SRC
          е
                     p0
                               gamma
     0.7698D+02 0.1056D+03 0.1000D+01 0.1109D+00 347 rwf
  1s
     0.1236D+02 0.3088D+02 0.1000D+01 0.5172D+00 351
  2p- 0.1089D+02 0.5761D-01 0.1000D+01 0.4660D+00 352 rwf
  2p
      0.1086D+02 0.1007D+03 0.2000D+01 0.4675D+00 352
                                                        rwf
      0.1092D+02 0.1998D+02 0.1000D+01 0.9615D+00 354
                                                        Hyd
  3p- 0.1092D+02 0.4942D-01 0.1000D+01 0.8901D+00
     0.1090D+02  0.8855D+02  0.2000D+01  0.8918D+00
  Зр
                                                    354
                                                        Hyd
  3d- 0.1090D+02 0.4311D-01 0.2000D+01 0.7490D+00
                                                    353
                                                        Hyd
      0.1089D+02 0.9250D+02 0.3000D+01 0.7496D+00
                                                   353
                                                        Hyd
 RWFNESTIMATE: Execution complete.
************************************
         RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
         NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
         VARY THE CORRELATION ORBITALS 3s,3p, 3d. NONE OF THESE ARE
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         NOTE: INSTEAD OF SAYING THAT WE WILL OPTIMIZE ON, FOR EXAMPLE,
         STATES 1,2,3,4 WE CAN WRITE 1-4 MEANING THE SAME THING
**********************************
```

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```
>>rmcdhf_mem
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file \dots Header only
 There are/is
                       9 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                 1164 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    3 blocks (block
                                       J/Parity
                                                 NCF):
 1 1/2-
           274
                    2 3/2-
                            590
                                       3 5/2-
                                                300
 Enter ASF serial numbers for each block
 Block
               1 	 ncf =
                                  274 \text{ id} = 1/2 -
>>1-2
                 2
                                  590 \text{ id} = 3/2
                   ncf =
 Block
>>1-4
                                   300 \text{ id} = 5/2-
                 3
                     ncf =
Block
 level weights (1 equal; 5 standard; 9 user)
Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
 . . . . .
 RMCDHF: Execution complete.
************************************
         RUN RSAVE TO SAVE OUTPUT FILES
************************************
>>rsave 2s22p3_2p5_3
 {\tt Created~2s22p3\_2p5\_3.w,~2s22p3\_2p5\_3.c,~2s22p3\_2p5\_3.m,~2s22p3\_2p5\_3.sum,}
        2s22p3_2p5_3.alog and 2s22p3_2p5_3.log
*************************************
         RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
         INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.w
```

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```
OUTPUT FILES: 2s22p3_2p5_3.cm, 2s22p3_2p5_3.csum, 2s22p3_2p5_3.clog *
                       rci.res
         THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
         LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
         THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
         HIGH N.
         NOTE: INSTEAD OF SAYING THAT WE WILL COMPUTE EIGENVALUES FOR
         STATES 1,2,3,4 WE CAN WRITE 1-4 MEANING THE SAME THING
>>rci
RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog. rci.res
 Default settings?
>>y
 Name of state:
>>2s22p3_2p5_3
                 1 , ncf =
                                    274
Block
                 2 , ncf =
                                     590
 Block
                 3 , ncf =
 Block
                                     300
Loading CSF file ... Header only
                        9 relativistic subshells;
 There are/is
 Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
>>y
 Enter the scale factor:
>>1.d-6
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
>>n
Estimate self-energy?
>>y
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
>>3
 Loading Radial WaveFunction File ...
                     3 blocks (block
                                         J/Parity
                                                   NCF):
 There are
                                         3 5/2-
           274
                    2 3/2- 590
 1 1/2-
                                                   300
 Enter ASF serial numbers for each block
 Block
                 1 	 ncf =
                                    274 \text{ id} = 1/2
>>1-2
                 2 ncf =
Block
                                  590 \text{ id} = 3/2
>>1-4
```

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```
Block
               3 	 ncf =
                                300 \text{ id} = 5/2
>>1
 . . . .
 RCI: Execution complete.
********************************
        RUN JJ2LSJ TO GET THE LSJ-COMPOSITION
        INPUT FILE: 2s22p3_2p5_3.c, 2s22p3_2p5_3.cm
        OUTPUT FILE: 2s22p3_2p5_3.1sj.1b1, 2s22p3_2p5_3.uni.1sj.1b1
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2017).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
             name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>2s22p3_2p5_3
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
 Mixing coefficients from a CI calc.?
>>y
Do you need a unique labeling? (y/n)
>>y
   nelec =
                    7
   ncftot =
                  1164
   nw
                     9
   nblock =
                   3
                        2j+1 parity
  block
        ncf
                  nev
           274
                   2
      1
                         2
                              -1
                                -1
      2
          591
                   4
                          4
      3
          300
                   1
                          6
                                 -1
 Default settings? (y/n)
>>y
     . . . . . . . . . . .
 jj2lsj: Execution complete.
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
```

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```
NOTE: SINCE LSJ-INFORMATION NOW IS AVAILABLE OUTPUT LABELS
       WILL BE IN LSJ-COUPLING
       IF DESIRED WE CAN INSTEAD RUN RLEVELSEV TO GET THE SEPARATION IN EV *
**********************************
>>rlevels 2s22p3_2p5_3.cm
                                                  9 nelec =
               3 ncftot =
                                1165 nw =
nblock =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
No Pos J Parity Energy Total Levels
                                   Splitting
                                              Configuration
        (a.u.) (cm^-1) (cm^-1)
 1 1 3/2 - -263.2797841
                           0.00
                                     0.00 1s(2).2s(2).2p(3)4S3_4S
 2 \quad 2 \quad 3/2 \quad - \quad -262.9550555 \qquad 71269.67 \qquad 71269.67 \qquad 1s(2).2s(2).2p(3)2D3\_2D
 3 1 5/2 - -262.9538206 71540.71 271.04 1s(2).2s(2).2p(3)2D3_2D
 7 2 1/2 - -259.4979399 830018.86 5745.41 1s(2).2p(5)_2P
To interpret the LSJ-coupling notation produced by jj21sj, see Section 8.2.
***********************************
       THE ABOVE JJ2LJS RUN TRANSFORMED ALL LEVELS TO LSJ COUPLING.
       BELOW WE WILL TRANSFORM A SUBSET OF THE LEVELS TO OTHER COUPLING
       SCHEMES. FOR TECHNICAL REASONS WE HAVE TO ADD INFORMATION ALSO
       FOR THE UNTRANSFORMED LEVELS IN ORDER FOR THE PROGRAMS TO WORK
       THE LABELS FOR THE UNTRANSFORMED LEVELS WILL BE THOSE FROM THE
       ABOVE RUN. FOR THIS REASON WE HAVE TO SAVE A COPY OF THE
       2s22p3_2p5_3.1sj.1bl LABEL FILE
************************************
>>cp 2s22p3_2p5_3.lsj.lbl 2s22p3_2p5_3.lsj.lbl_SAVE
************************************
       RUN JJ2LSJ TO GET THE INPUT FOR COUPLING PROGRAM FOR THOSE
       LEVELS WHICH HAVE 1s, 2s, AND 2p SHELLS IN IDENTIFICATION
       INPUT FILES: 2s22p3_2p5_3.c, 2s22p3_2p5_3.cm
       OUPUT FILES: 2s22p3_2p5_3.1sj.c, 2s22p3_2p5_3.1sj.j,
                  2s22p3_2p5_3.1sj.1bl
       THE LEVELS WE ARE INTERESTED IN ARE
       BLOCK 1, J = 1/2, LEVEL 1
       BLOCK 2, J = 3/2, LEVEL 1, 2, 3 (1-3)
       BLOCK 3, J = 5/2, LEVEL 1
***********************************
```

>>jj21sj

jj2lsj: Transformation of ASFs from a jj-coupled CSF basis

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```
into an LS-coupled CSF basis (Fortran 95 version)
         (C) Copyright by G. Gaigalas and Ch. F. Fischer,
         (2017).
         Input files: name.c, name.(c)m
         Ouput files: name.lsj.lbl,
          (optional) name.lsj.c, name.lsj.j,
                      name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>2s22p3_2p5_3
  Loading Configuration Symmetry List File ...
 There are 4 relativistic subshells;
 There are 4 relativistic CSFs;
  ... load complete;
   Mixing coefficients from a CI calc.?
>>y
 Do you need a unique labeling? (y/n)
    nelec =
                        4
    ncftot =
                        4
    nw
                        4
                        3
    nblock =
   block
                            2j+1 parity
             ncf
                     nev
               1
                       1
                               1
               2
                       2
       2
                               3
                                      -1
               1
                               5
                       1
                                      -1
 Default settings? (y/n)
>>n
  All levels (Y/N)
>>n
 Maximum number of ASFs is:
 Enter the level numbers of the ASF which are to be transformed,
 Enter the block number
  The block number is:
  e.g., 1 3 4 7--20 48 69--85:
>>1
 Do you need to include more levels? (y/n)
>>y
  Enter the block number
>>2
 The block number is:
  e.g., 1 3 4 7--20 48 69--85 :
>>1-3
 Do you need to include more levels? (y/n)
>>y
 Enter the block number
>>3
 The block number is:
  e.g., 1 3 4 7--20 48 69--85:
```

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```
>>1
 Do you need to include more levels? (y/n)
>>n
 Maximum % of omitted composition
>>0
 What is the value below which an eigenvector composition
 is to be neglected for printing?
>>0.01
 jj2lsj: Execution complete.
         RUN COUPLING TO GET THE IDENTIFICATION STATES IN DIFFERENT
         COUPLING SCHEMES FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
         SHELLS IN IDENTIFICATION
         INPUT FILES: 2s22p3_2p5_3.lsj.c, 2s22p3_2p5_3.lsj.j
         OUPUT FILES: 2s22p3_2p5_3.coup3.LK3.lbl, 2s22p3_2p5_3.coup3.JK3.lbl *
                      2s22p3_2p5_3.LS.1b1,
                                                  2s22p3_2p5_3.coup3.LS3.1b1 *
                      2s22p3_2p5_3.coup3.LSJ3.1b1, 2s22p3_2p5_3.coup3.jj.1b1 *
                      2s22p3_2p5_3.coup3.cLSJ3.1b1, 2s22p3_2p5_3.coup3.sum
************************************
>>Coupling
 Coupling: Transformation of ASFs from a LS-coupled CSF basis
           into differete coupled CSF bases
                                                (Fortran 95)
           (C) (2022)
                                    G. Gaigalas, A. Kramida.
 Input files: *.lsj.c, *.lsj.j (ATSP (CPC) or GRASP2K types)
 Output files: *.coup*.*.lbl, *.coup*.sum
 Name of state
>>2s22p3_2p5_3
 Default settings ? (Y/N)
 Specify the number of coupled shells for evaluation (1,2 or 3):
>>3
 3
 What is the value below which an eigenvector composition
 is to be neglected for printing?
>>0
                                   0.000000000000000
 Specify shells for recoupling (no more than 12)
>>1s,2s,2p,3s,3p,3d
   All transformations completed
 There is one-to-one classification for LS coupling
 There is one-to-one classification for LS3 coupling
 There is one-to-one classification for LSJ3 coupling
 There is one-to-one classification for LK3 coupling
 There is one-to-one classification for JK3 coupling
 There is one-to-one classification for cLSJ3 coupling
```

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There is one-to-one classification for jj3 coupling end subroutine generate\_classification\_data

Coupling: Execution complete.

```
**********************************
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
        IN LK3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
        SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
        OBSERVE!
        ABOVE WE TRANSFORMED ONLY A SUBSET OF THE LEVELS
        WE NEED, HOWEVER, TO HAVE LABELS FOR ALL LEVELS. WE WILL USE
        THE LSJ LABELS IN THE FILE 2s22p3_2p5_3.1sj.1b1_SAVE FOR THE
        UNTRANSFORMED LEVELS. WE CAN BY HAND EDIT THE
        2s22p3_2p5_3.coup3.LK3.1b1 FILE AND PASTE THE INFORMATION FOR
        THE UNTRANSFORMED LEVELS FROM THE 2s22p3_2p5_3.1sj.lbl_SAVE
        FILE AT THE APPROPRIATE PLACE IN 2s22p3_2p5_3.coup3.LK3.1b1
        ALTERNATIVELY, AND THIS IS WHAT WE WILL DO BELOW, WE CAN USE THE
        sed COMMAND TO COPY THE APPROPRIATE INFORMATION FOR THE TWO
        UNTRANSFORMED LEVELS FROM 2s22p3_2p5_3.1sj.1b1_SAVE AND PUT
        THE INFORMATION IN PATCH1 and PATCH2.
        WE NOW USE sed TO PASTE THE INFORMATION IN PATCH1 AND PATCH2
        AT THE APPROPRIATE PLACE IN 2s22p3_2p5_3.coup3.LK3.lbl
        THE USER IS ADVICED TO OPEN AND INSPECT BOTH THE
        2s22p3_2p5_3.lsj.lbl_SAVE FILE AND THE 2s22p3_2p5_3.coup3.LK3.lbl
        FILE TO UNDERSTAND WHAT IS GOING ON
**********************************
>>sed -n 25,28p 2s22p3_2p5_3.lsj.lbl_SAVE >patch1
>>sed -i 327rpatch1 2s22p3_2p5_3.coup3.LK3.1bl
>>sed -n 6,10p 2s22p3_2p5_3.lsj.lbl_SAVE >patch2
>>sed -i 57rpatch2 2s22p3_2p5_3.coup3.LK3.1bl
>>cp 2s22p3_2p5_3.coup3.LK3.1b1 2s22p3_2p5_3.1sj.1b1
>>rlevels 2s22p3_2p5_3.cm
nblock =
          3 \quad \text{ncftot} = 1165 \quad \text{nw} =
                                                      9 nelec =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
______
No Pos J Parity Energy Total Levels
                                     Splitting
                                                 Configuration
              (a.u.) (cm<sup>-1</sup>)
                                      (cm^-1)
______
 1 1 3/2 - -263.2797841
                            0.00
                                      0.00 1s2_ 2s2_.2p3(4S)(4S) S_4[0]<3/2>
 2 2 3/2 - -262.9550555 71269.67 71269.67 1s2_ 2s2_.2p3(2D)(2D) D_2[2]<3/2>
              -262.9538206 71540.71 271.04 1s2_ 2s2_.2p3(2D)(2D) D_2[2]<5/2>
 3 1 5/2 -
              -262.7906339 107356.06 35815.34 1s2_2s2_.2p3(2P)(2P) P_2[1]<1/2>
 4 1 1/2 -
 5 3 3/2 - -262.7882742 107873.94
                                      517.88 1s2_ 2s2_.2p3(2P)(2P) P_2[1]<3/2>
 6 4 3/2 - -259.5241179 824273.45 716399.51 1s(2).2p(5)_2P
 7 2 1/2 -
              -259.4979399 830018.86 5745.41 1s(2).2p(5)_2P
```

Please note that the labels for No 1–5 are in LK3 coupling and the rest (No 6 and 7) in LSJ coupling.

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```
*********************************
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
        IN JK3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
        SHELLS IN IDENTIFICATION AND FOR THE REST LSJ-COUPLING
        INFORMATION FROM 2s22p3_2p5_3.1sj.1b1_SAV ADDED
**********************************
>>sed -i 327rpatch1 2s22p3_2p5_3.coup3.JK3.1bl
>>sed -i 57rpatch2 2s22p3_2p5_3.coup3.JK3.lbl
>>cp 2s22p3_2p5_3.coup3.JK3.lbl 2s22p3_2p5_3.lsj.lbl
>>rlevels 2s22p3_2p5_3.cm
nblock = 3 ncftot = 1165 nw =
                                             9 nelec =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
------
No Pos J Parity Energy Total Levels Splitting Configuration
               (a.u.) (cm^-1) (cm^-1)
 1 1 3/2 - -263.2797841 0.00 0.00 1s2_<0>2s2_.2p3(4S)(4S) 4[0]<3/2>
 2 2 3/2 - -262.9550555 71269.67 71269.67 1s2_<0>2s2_.2p3(2D)(2D) 2[2]<3/2>
 3 1 5/2 - -262.9538206 71540.71 271.04 1s2_<0>2s2_.2p3(2D)(2D) 2[2]<5/2>
 4 1 1/2 - -262.7906339 107356.06 35815.34 1s2_<0>2s2_.2p3(2P)(2P) 2[1]<1/2> 5 3 3/2 - -262.7882742 107873.94 517.88 1s2_<0>2s2_.2p3(2P)(2P) 2[1]<3/2>
  6 \quad 4 \quad 3/2 \quad - \quad -259.5241179 \quad 824273.45 \quad 716399.51 \quad 1s(2).2p(5)\_2P 
 7 2 1/2 - -259.4979399 830018.86 5745.41 1s(2).2p(5)_2P
*********************************
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
       IN LS3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
       SHELLS IN IDENTIFICATION AND FOR THE REST LSJ-COUPLING
        INFORMATION FROM 2s22p3_2p5_3.1sj.1b1_SAV ADDED
**********************************
>>sed -i 320rpatch1 2s22p3_2p5_3.coup3.LS3.1bl
>>sed -i 56rpatch2 2s22p3_2p5_3.coup3.LS3.1b1
>>cp 2s22p3_2p5_3.coup3.LS3.1b1 2s22p3_2p5_3.1sj.1b1
>>rlevels 2s22p3_2p5_3.cm
           3 ncftot =
                                                   9 nelec =
                                 1165 nw =
nblock =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
_____
No Pos J Parity Energy Total
                           Levels Splitting Configuration
             (a.u.)
                           (cm^-1)
                                     (cm^-1)
 1 1 3/2 - -263.2797858
                            0.00
                                      0.00 1s2_ 2s2_.2p3(4S)(4S) 4S<3/2>
 2 2 3/2 - -262.9550573 71269.67 71269.67 1s2_ 2s2_.2p3(2D)(2D) 2D<3/2>
 3 1 5/2 - -262.9538223 71540.71 271.04 1s2_ 2s2_.2p3(2D)(2D) 2D<5/2>
 4 1 1/2 - -262.7906356 107356.06 35815.35 1s2_ 2s2_.2p3(2P)(2P) 2P<1/2>
```

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```
5 3 3/2 -
           -262.7882760 107873.94
                                    517.88 1s2_ 2s2_.2p3(2P)(2P) 2P<3/2>
 6 4 3/2 - -259.5241195 824273.48 716399.54 1s(2).2p(5)_2P
 7 2 1/2 - -259.4979415 830018.89 5745.41 1s(2).2p(5)_2P
 -----
*************************************
       RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
       IN LSJ3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
       SHELLS IN IDENTIFICATION AND FOR THE REST LSJ-COUPLING
       INFORMATION FROM 2s22p3_2p5_3.1sj.1b1_SAV ADDED
*******************************
>>sed -i 327rpatch1 2s22p3_2p5_3.coup3.LSJ3.lbl
>>sed -i 57rpatch2 2s22p3_2p5_3.coup3.LSJ3.1b1
>>cp 2s22p3_2p5_3.coup3.LSJ3.1b1 2s22p3_2p5_3.1sj.1b1
>>rlevels 2s22p3_2p5_3.cm
                                1165 nw =
                                                  9 nelec =
nblock =
               3 ncftot =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
-----
No Pos J Parity Energy Total Levels Splitting Configuration
              (a.u.) (cm^-1)
                                   (cm^-1)
                                         _____
 1 1 3/2 - -263.2797841
                           0.00
                                     0.00 1s2_ 2s2_.2p3(4S)(4S) (0,3/2)<3/2>
 2 2 3/2 - -262.9550555 71269.67 71269.67 1s2_2s2_.2p3(2D)(2D)(0,3/2)<3/2>
 3 1 5/2 - -262.9538206 71540.71 271.04 1s2_ 2s2_.2p3(2D)(2D) (0,5/2)<5/2>
 4 1 1/2 - -262.7906339 107356.06 35815.34 1s2_2s2_.2p3(2P)(2P) (0,1/2)<1/2>
 5 3 3/2 - -262.7882742 107873.94 517.88 1s2_ 2s2_.2p3(2P)(2P) (0,3/2)<3/2>
6 4 3/2 - -259.5241179 824273.45 716399.51 1s(2).2p(5)_2P
 7 2 1/2 -
             -259.4979399 830018.86 5745.41 1s(2).2p(5)_2P
 ***********************************
       RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
       IN jj-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
       SHELLS IN IDENTIFICATION AND FOR THE REST LSJ-COUPLING
       INFORMATION FROM 2s22p3_2p5_3.1sj.1b1_SAV ADDED
**********************************
>>sed -i 329rpatch1 2s22p3_2p5_3.coup3.jj.lbl
>>sed -i 59rpatch2 2s22p3_2p5_3.coup3.jj.lbl
>>cp 2s22p3_2p5_3.coup3.jj.lbl 2s22p3_2p5_3.lsj.lbl
>>rlevels 2s22p3_2p5_3.cm
                                1165 nw =
nblock = 3 ncftot =
                                                  9 nelec =
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
______
No Pos J Parity Energy Total Levels Splitting (a.u.) (cm^-1) (cm^-1)
                                  Splitting Configuration
```

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```
1 1 3/2 - -263.2797841 0.00 0.00 1s+2_2s+2_<0>.2p-_<1/2>.2p+2(2) <3/2>
  2 2 3/2 - -262.9550555 71269.67 71269.67 1s+2_2s+2_<0>.2p-_<1/2>.2p+2(2) <3/2>
  3 1 5/2 - -262.9538206 71540.71 271.04 1s+2_2s+2_<0>.2p-_<1/2>.2p+2(2) <5/2>
                -262.7906339 107356.06 35815.34 1s+2_2s+2_<0>.2p-_<1/2>.2p+2(0) <1/2>
  4 1 1/2 -
  5 3 3/2 - -262.7882742 107873.94 517.88 1s+2_2s+2_<0>.2p+3(3/2) <3/2>
  6 4 3/2 - -259.5241179 824273.45 716399.51 1s(2).2p(5)_2P
  7 2 1/2 -
                -259.4979399 830018.86 5745.41 1s(2).2p(5)_2P
RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
        IN cLSJ3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
        SHELLS IN IDENTIFICATION AND FOR THE REST LSJ-COUPLING
        INFORMATION FROM 2s22p3_2p5_3.lsj.lbl_SAV ADDED
*****************************
>>sed -i 327rpatch1 2s22p3_2p5_3.coup3.cLSJ3.lbl
>>sed -i 57rpatch2 2s22p3_2p5_3.coup3.cLSJ3.1b1
>>cp 2s22p3_2p5_3.coup3.cLSJ3.1b1 2s22p3_2p5_3.1sj.1b1
>>rlevels 2s22p3_2p5_3.cm
           3 ncftot = 1165 nw =
nblock =
                                                      9 nelec =
 Energy levels for ...
 Rydberg constant is 109737.31569
 Splitting is the energy difference with the lower neighbor
_____
 No Pos J Parity Energy Total Levels Splitting Configuration
                (a.u.) (cm<sup>-1</sup>) (cm<sup>-1</sup>)
              ______
 1 1 3/2 - -263.2797841 0.00 0.00 1s+2_(0,0)<0> 2s2_.2p3(4S)(4S)<3/2> (0,3/2)<3/2>
 2 2 3/2 - -262.9550555 71269.67 71269.67 1s+2_ (0,0)<0> 2s2_.2p3(2D)(2D)<3/2> (0,3/2)<3/2>
3 1 5/2 - -262.9538206 71540.71 271.04 1s+2_ (0,0)<0> 2s2_.2p3(2D)(2D)<5/2> (0,5/2)<5/2>
4 1 1/2 - -262.7906339 107356.06 35815.34 1s+2_ (0,0)<0> 2s2_.2p3(2P)(2P)<1/2> (0,1/2)<1/2>
5 3 3/2 - -262.7882742 107873.94 517.88 1s+2_ (0,0)<0> 2s2_.2p3(2P)(2P)<3/2> (0,3/2)<3/2>
 6 4 3/2 - -259.5241179 824273.45 716399.51 1s(2).2p(5)_2P
 7 2 1/2 - -259.4979399 830018.86 5745.41 1s(2).2p(5)_2P
```

For definition of different coupling schemes see in [14] and for interpretation of different coupling schemes notation produced by Coupling see Section 8.2, where we discuss in detail the transformation in different coupling schemes for  $1s^22s2p$   $^3P_{0.1.2}^0$ ,  $^1P_1^0$  in B II.

>>cp 2s22p3\_2p5\_3.lsj.lbl\_SAVE 2s22p3\_2p5\_3.lsj.lbl

```
************************************
         RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
         INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                               2s22p3_2p5_3.cbm,
         OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                      2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                      2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
         NOTE THAT THE LATTER OUTPUT FILE HAS ALL THE LABELS IN LSJ-
         COUPLING WHICH IS VERY CONVENIENT
         PLEASE OBSERVE!! IF WE ARE GOING TO RUN RTRANSITION FOR AN RCI WAVE *
         FUNCTIONS THEN THE LSJ-INFORMATION SHOULD BE AVAILABLE FOR THE SAME *
         WAVE FUNCTION. IF FOR EXAMPLE THE LSJ-INFORMATION FROM JJ2LSJ IS
         IS AVAILABLE FROM AN RMCDHF RUN AND WE RUN RTRANSITION ON THE RCI
         WAVE FUNCTION THEN RTRANSITION WILL STOP. IN THIS CASE JUST RERUN
         JJ2LSJ FOR THE RCI WAVE FUNCTION AND START RTRANSITION AGAIN FOR
         THE SAME WAVE FUNCTION. IN OUR EXAMPLE JJ2LJS AND RTRANSITION ARE
         RUN FOR RCI WAVE FUNCTIONS AND EVERYTHING IS OK.
************************************
>>rtransition
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
              name2.c, name2.bw, name2.(c)bm
              optional, name1.lsj.lbl, name2.lsj.lbl
              name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
              optional, name1.name2.(c)t.lsj
              name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
  Default settings?
>>7
  Input from a CI calculation?
>>y
  Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 {\tt MRGCSL} \colon \ {\tt Execution begins} \ \dots
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
```

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```
1 s
          2 s
          2 p-
          2 p
          3 s
          3 p-
          3 p
          3 d-
          3 d
          3
        274
                   864
                             1164
          3
        274
                   864
                             1164
 Loading Configuration Symmetry List File \dots
  there are 9 relativistic subshells;
          1
          2
                     2
 there are 2328 relativistic CSFs;
  ... load complete;
 Enter the list of transition specifications
 e.g., E1,M2 or E1 M2 or E1;M2:
>>M1
 M1 transitions only between levels with different J?
>>n
 . . . . .
 RTRANSITION: Execution complete.
************************************
         VIEW THE TRANSITION FILE WHERE THE LABELS ARE IN LSJ COUPLING
*************************************
>>more 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
 Transition between files:
 2s22p3_2p5_3
 2s22p3_2p5_3
   1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
   1 -259.49793990 1s(2).2p(5)_2P
                   138.38 ANGS(VAC)
                                        138.38 ANGS(AIR)
 722662.80 CM-1
 M1 S = 1.18001D-11 GF = 3.44839D-16 AKI = 6.00621D-05
   1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
  3 -262.78827423 1s(2).2s(2).2p(3)2P1_2P
    517.88 CM-1
                  193094.26 ANGS(VAC)
                                      193074.30 ANGS(AIR)
 M1 S = 1.31018D+00 GF = 2.74383D-08 AKI = 1.22716D-03
   1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
   3 -259.52411791 1s(2).2p(5)_2P
```

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```
716917.39 CM-1 139.49 ANGS(VAC)
                                     139.49 ANGS(AIR)
M1 S = 1.73052D-06 GF = 5.01695D-11 AKI = 4.29992D+00
  3 -263.27978407 1s(2).2s(2).2p(3)4S3_4S
  1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
 107356.06 CM-1
                  931.48 ANGS(VAC)
                                     931.48 ANGS(AIR)
M1 S = 1.86549D-03 GF = 8.09867D-09 AKI = 3.11300D+01
  3 -263.27978407 1s(2).2s(2).2p(3)4S3_4S
  1 -259.49793990 1s(2).2p(5)_2P
 830018.86 CM-1 120.48 ANGS(VAC) 120.48 ANGS(AIR)
M1 S = 5.92158D-07 GF = 1.98756D-11 AKI = 4.56677D+00
  5 -262.95382060 1s(2).2s(2).2p(3)2D3_2D
  3 -262.78827423 1s(2).2s(2).2p(3)2P1_2P
  36333.23 CM-1
                2752.30 ANGS(VAC)
                                      2752.01 ANGS(AIR)
M1 S = 3.63575D-02 GF = 5.34186D-08 AKI = 1.17593D+01
  5 -262.95382060 1s(2).2s(2).2p(3)2D3_2D
  3 -259.52411791 1s(2).2p(5)_2P
 752732.73 CM-1 132.85 ANGS(VAC) 132.85 ANGS(AIR)
M1 S = 1.88251D-06 GF = 5.73023D-11 AKI = 5.41422D+00
***********************************
        GIVE THE TRANSITION FILE A NEW APPROPRIATE NAME FOR LATER USE
************************************
>>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_SAVE
*********************************
        RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
        IN LK3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
        SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
        INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                          2s22p3_2p5_3.cbm,
        OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                   2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                   2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
        COPY THE 2s22p3_2p5_3.coup3.LK3.1b1 TO 2s22p3_2p5_3.lsj.1b1
        IT IS THE LATTER FILE THAT IS READ AND USED BY THE TRANSITION
        PROGRAM
>>cp 2s22p3_2p5_3.coup3.LK3.lbl 2s22p3_2p5_3.lsj.lbl
>>rtransition
```

RTRANSITION

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```
This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
               name2.c, name2.bw, name2.(c)bm
               optional, name1.lsj.lbl, name2.lsj.lbl
               name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
               optional, name1.name2.(c)t.lsj
               name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
  Default settings?
>>y
  Input from a CI calculation?
  Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 {\tt MRGCSL} \colon \ {\tt Execution begins} \ \dots
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
           1 s
           2 s
           2 p-
           2 p
           3 s
           3 p-
           3 p
           3 d-
           3 d
           3
         274
                      864
                                 1164
           3
         274
                      864
                                 1164
 Loading Configuration Symmetry List File ...
  there are 9 relativistic subshells;
           1
  there are 2328 relativistic CSFs;
  ... load complete;
 Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
>>M1
 M1 transitions only between levels with different J?
>>n
```

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. . . . .

RTRANSITION: Execution complete. \* VIEW THE TRANSITION FILE WHERE THE LABELS ARE IN LK3 COUPLING THE DATA IS THE SAME AS ABOVE, ONLY THE LABELS OF THE STATES DIFFER \* >>more 2s22p3\_2p5\_3.2s22p3\_2p5\_3.ct.lsj Transition between files: 2s22p3\_2p5\_3 2s22p3\_2p5\_3 1 -262.79063388 1s2\_ 2s2\_.2p3(2P)(2P) P\_2[1]<1/2> 1 -259.49793990 1s(2).2p(5)\_2P 138.38 ANGS(VAC) 138.38 ANGS(AIR) 722662.80 CM-1 M1 S = 1.18001D-11 GF = 3.44839D-16 AKI = 6.00621D-051 -262.79063388 1s2\_ 2s2\_.2p3(2P)(2P) P\_2[1]<1/2> 3 -262.78827423 1s2\_ 2s2\_.2p3(2P)(2P) P\_2[1]<3/2> 517.88 CM-1 193094.26 ANGS(VAC) 193074.30 ANGS(AIR) M1 S = 1.31018D+00 GF = 2.74383D-08 AKI = 1.22716D-031 -262.79063388 1s2\_ 2s2\_.2p3(2P)(2P) P\_2[1]<1/2> 3 -259.52411791 1s(2).2p(5)\_2P 716917.39 CM-1 139.49 ANGS(VAC) 139.49 ANGS(AIR) M1 S = 1.73052D-06 GF = 5.01695D-11 AKI = 4.29992D+003 - 263.27978407  $1s2_ 2s2_.2p3(4S)(4S) S_4[0]<3/2>$ 1 -262.79063388 1s2\_ 2s2\_.2p3(2P)(2P) P\_2[1]<1/2> 107356.06 CM-1 931.48 ANGS(VAC) 931.48 ANGS(AIR) M1 S = 1.86549D-03 GF = 8.09867D-09 AKI = 3.11300D+013 - 263.27978407  $1s2_ 2s2_.2p3(4S)(4S) S_4[0]<3/2>$ 1 -259.49793990 1s(2).2p(5)\_2P 830018.86 CM-1 120.48 ANGS(VAC) 120.48 ANGS(AIR) M1 S = 5.92158D-07 GF = 1.98756D-11 AKI = 4.56677D+00. . . . . . . . . . 5 -262.95382060 1s2\_ 2s2\_.2p3(2D)(2D) D\_2[2]<5/2>

3 -262.78827423 1s2\_ 2s2\_.2p3(2P)(2P) P\_2[1]<3/2>

36333.23 CM-1 2752.30 ANGS(VAC) 2752.01 ANGS(AIR) M1 S = 3.63575D-02 GF = 5.34186D-08 AKI = 1.17593D+01

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```
5 -262.95382060 1s2_ 2s2_.2p3(2D)(2D) D_2[2]<5/2>
  3 -259.52411791 1s(2).2p(5)_2P
 752732.73 CM-1
                    132.85 ANGS(VAC)
                                         132.85 ANGS(AIR)
 M1 S = 1.88251D-06 GF = 5.73023D-11
                                        AKI = 5.41422D+00
*******************************
        GIVE THE TRANSITION FILE A NEW APPROPRIATE NAME FOR LATER USE
************************************
 >>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_LK3
***********************************
        RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
        IN JK3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
        SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
         INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                             2s22p3_2p5_3.cbm,
        OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                    2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                    2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
        COPY THE 2s22p3_2p5_3.coup3.JK3.1b1 TO 2s22p3_2p5_3.1sj.1b1
        IT IS THE LATTER FILE THAT IS READ AND USED BY THE TRANSITION
        PROGRAM
***********************************
>>cp 2s22p3_2p5_3.coup3.JK3.lbl 2s22p3_2p5_3.lsj.lbl
>>rtransition
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
             name2.c, name2.bw, name2.(c)bm
             optional, name1.lsj.lbl, name2.lsj.lbl
             name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
             optional, name1.name2.(c)t.lsj
             name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
 Default settings?
  Input from a CI calculation?
>>y
 Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 MRGCSL: Execution begins ...
```

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```
Loading Configuration Symmetry List File \dots
There are 9 relativistic subshells;
There are 1164 relativistic CSFs;
  ... load complete;
Loading Configuration Symmetry List File ...
There are 9 relativistic subshells;
There are 1164 relativistic CSFs;
  ... load complete;
           1 s
          2 s
           2 p-
          2 p
          3 s
          3 p-
          3 p
          3 d-
          3 d
          3
         274
                    864
                               1164
           3
         274
                    864
                               1164
Loading Configuration Symmetry List File \dots
  there are 9 relativistic subshells;
 there are 2328 relativistic CSFs;
  ... load complete;
Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
M1 transitions only between levels with different J?
>>n
  . . . . .
RTRANSITION: Execution complete.
>>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_JK3
*********************************
         RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
         IN LS3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
         SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
         INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                               2s22p3_2p5_3.cbm,
         OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                      2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                      2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
         COPY THE 2s22p3_2p5_3.coup3.LS3.lb1 TO 2s22p3_2p5_3.lsj.lb1
         IT IS THE LATTER FILE THAT IS READ AND USED BY THE TRANSITION
         PROGRAM
```

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### >>rtransition

```
RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
               name2.c, name2.bw, name2.(c)bm
               optional, name1.lsj.lbl, name2.lsj.lbl
               name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
               optional, name1.name2.(c)t.lsj
               name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
  Default settings?
  Input from a CI calculation?
>>y
  Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 MRGCSL: Execution begins ...
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
           1 s
           2 s
           2 p-
           2 p
           3 s
           3 p-
           3 p
           3 d-
           3 d
                     864
         274
                                1164
           3
         274
                     864
                                1164
 Loading Configuration Symmetry List File ...
  there are 9 relativistic subshells;
  there are 2328 relativistic CSFs;
  ... load complete;
 Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
```

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```
>>M1
 M1 transitions only between levels with different J?
>>n
  . . . . .
 RTRANSITION: Execution complete.
********************************
        GIVE THE TRANSITION FILE A NEW APPROPRIATE NAME FOR LATER USE
***********************************
 >>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_LS3
**********************************
        RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
        IN LSJ3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
        SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
        INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                             2s22p3_2p5_3.cbm,
        OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                    2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                    2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
        COPY THE 2s22p3_2p5_3.coup3.LSJ3.1b1 TO 2s22p3_2p5_3.lsj.1b1
        IT IS THE LATTER FILE THAT IS READ AND USED BY THE TRANSITION
        PROGRAM
***********************************
>>cp 2s22p3_2p5_3.coup3.LSJ3.1b1 2s22p3_2p5_3.lsj.1b1
>>rtransition
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
             name2.c, name2.bw, name2.(c)bm
             optional, name1.lsj.lbl, name2.lsj.lbl
             name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
             optional, name1.name2.(c)t.lsj
             name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
 Default settings?
  Input from a CI calculation?
 Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 MRGCSL: Execution begins ...
```

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```
Loading Configuration Symmetry List File \dots
There are 9 relativistic subshells;
There are 1164 relativistic CSFs;
  ... load complete;
Loading Configuration Symmetry List File ...
There are 9 relativistic subshells;
There are 1164 relativistic CSFs;
  ... load complete;
          1 s
          2 s
          2 p-
          2 p
          3 s
          3 p-
          3 p
          3 d-
          3 d
          3
                    864
        274
                              1164
          3
        274
                    864
                              1164
Loading Configuration Symmetry List File \dots
  there are 9 relativistic subshells;
 there are 2328 relativistic CSFs;
  ... load complete;
Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
M1 transitions only between levels with different J?
>>n
  . . . . .
RTRANSITION: Execution complete.
>>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_LSJ3
*********************************
         RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
         IN jj-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
         SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
         INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                              2s22p3_2p5_3.cbm,
         OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                     2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                     2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
         COPY THE 2s22p3_2p5_3.coup3.jj.lbl TO 2s22p3_2p5_3.lsj.lbl
         IT IS THE LATTER FILE THAT IS READ AND USED BY THE TRANSITION
         PROGRAM
  ************************************
```

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### >>rtransition

```
RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
               name2.c, name2.bw, name2.(c)bm
               optional, name1.lsj.lbl, name2.lsj.lbl
               name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
               optional, name1.name2.(c)t.lsj
               name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
  Default settings?
  Input from a CI calculation?
>>y
  Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 MRGCSL: Execution begins ...
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
           1 s
           2 s
           2 p-
           2 p
           3 s
           3 p-
           3 p
           3 d-
           3 d
                     864
         274
                                1164
           3
         274
                     864
                                1164
 Loading Configuration Symmetry List File ...
  there are 9 relativistic subshells;
  there are 2328 relativistic CSFs;
  ... load complete;
 Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
```

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```
>>M1
 M1 transitions only between levels with different J?
>>n
  . . . . .
 RTRANSITION: Execution complete.
********************************
         GIVE THE TRANSITION FILE A NEW APPROPRIATE NAME FOR LATER USE
************************************
 >>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_jj
**********************************
         RUN RTRANSITION FOR 2s22p3_2p5_3 TO COMPUTE M1 TRANSITION PARAMETERS*
         IN cLSJ3-COUPLING FOR THOSE LEVELS WHICH HAVE 1s, 2s, AND 2p
         SHELLS IN IDENTIFICATION AND FOR THE REST LS-COUPLING
         INPUT FILES: isodata, 2s22p3_2p5_3.c, 2s22p3_2p5_3.bw,
                             2s22p3_2p5_3.cbm,
         OUTPUT FILE: 2s22p3_2p5_3.2s22p3_2p5_3.ct,
                     2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
                     2s22p3_2p5_3.2s22p3_2p5_3.+1T (angular file)
         COPY THE 2s22p3_2p5_3.coup3.cLSJ3.1b1 TO 2s22p3_2p5_3.lsj.1b1
         IT IS THE LATTER FILE THAT IS READ AND USED BY THE TRANSITION
         PROGRAM
***********************************
>>cp 2s22p3_2p5_3.coup3.cLSJ3.lbl 2s22p3_2p5_3.lsj.lbl
>>rtransition
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
             name2.c, name2.bw, name2.(c)bm
             optional, name1.lsj.lbl, name2.lsj.lbl
             name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
             optional, name1.name2.(c)t.lsj
             name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
 Default settings?
  Input from a CI calculation?
>>y
 Name of the Initial state
>>2s22p3_2p5_3
  Name of the Final state
>>2s22p3_2p5_3
 MRGCSL: Execution begins ...
```

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```
Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 1164 relativistic CSFs;
  ... load complete;
          1 s
          2 s
          2 p-
          2 p
          3 s
          3 p-
          3 p
          3 d-
          3 d
          3
                   864
        274
                             1164
          3
        274
                   864
                             1164
 Loading Configuration Symmetry List File \dots
  there are 9 relativistic subshells;
  there are 2328 relativistic CSFs;
  ... load complete;
 Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
 M1 transitions only between levels with different J?
>>n
  . . . . .
 RTRANSITION: Execution complete.
************************************
         GIVE THE TRANSITION FILE A NEW APPROPRIATE NAME FOR LATER USE
********************************
 >>mv 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj 2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj_cLSJ3
```

# 6.4. Fourth Example: 3l3l' States in Fe XV Using MPI

The fourth example is to determine the energies for the 10 states belonging to the three even configurations  $3s^2$ ,  $3p^2$ , 3s3d and the 16 states belonging to the two odd configurations 3s3p, 3p3d. In addition, the E1, M2 and M1 transition data should be computed. The NIST table for these states is shown below.

Configuration	Term	l	J	 	Level	1
		1				- 1
		1				
2p6.3s2	1S	1	0	1	0	1
	1	1		1		- 1
3s.3p	3P*	İ	0	Ì	233842	l

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		1	239660
		2	253820
			1
3s.3p	1P*	1	351911
		I	I
3p2	3P	0	554524
		I	I
3p2	1D	2	559600
3p2	3P	1	564602
		2	581803
	1 4 0		(50007
3p2	1S	0	659627
3s.3d	   3D	   1	678772
35.3u	ן טט	2	679785
		3	681416
			001410
3s.3d	1D	2	762093
		i	
3p.3d	3F*	2	928241
		3	938126
			1
3p.3d	1D*	2	948513
		1	1
3p.3d	3F*	4	949658
			I
3p.3d	3D*	1	982868
		I	I
3p.3d	3P*	2	983514
3p.3d	3D*	3	994852
0 01	0.0.		005000
3p.3d	3P*	0	995889
		1	996243
3p.3d	3D*	2	996623
5p.5u	ן טער ן	<u>2</u>   	990023
3p.3d	1F*	3	1062515
Jr. 04			1002010
3p.3d	1P*	1	1074887
· 			

The starting point is two separate rmcdhf calculations for the even and odd reference states, respectively. Then one layer of correlation orbitals is included describing valence–valence and core–valence correlation.

# Overview

- 1. Define nuclear data.
- 2. Obtain common spectroscopic orbitals for the even parity MR set
  - (a) Generate list of CSFs describing the even states belonging to  $3s^2$ ,  $3p^2$ , 3s3d
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of the even states.
  - (e) Save output to evenmr.

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- 3. Improve even states
  - (a) Generate n = 4 valence–valence and core–valence CSF expansions
  - (b) Perform angular integration using MPI.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF MPI calculation on the weighted average of the even states.
  - (e) Save output to even4.
  - (f) Perform rci MPI calculation in which Breit and QED effects are added.
- 4. Run jj21sj to transform to *LSJ*-coupling.
- 5. Obtain common spectroscopic orbitals for the odd parity MR set
  - (a) Generate list of CSFs describing the even states belonging to 3s3p,3p3d
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of the odd states.
  - (e) Save output to oddmr.
- 6. Improve odd states.
  - (a) Generate n = 4 valence-valence and core-valence CSF expansions.
  - (b) Perform angular integration using MPI.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF MPI calculation on the weighted average of the odd states.
  - (e) Save output to odd4.
  - (f) Perform rci MPI calculation in which Breit and QED effects are added.
- 7. Run jj21sj to transform to *LSJ*-coupling.
- 8. Run rlevels, rlevelseV to view energy separations in LSJ-coupling scheme.
- 9. Compute properties.
  - (a) Compute transition rates from the rci wave functions. Computation in two steps: biorthonormal transformation and then evaluation of transition matrix elements using standard Racah algebra methods. Both steps use MPI code.

### Preparation for the MPI Run

We intend to run rangular\_mpi, rmcdhf\_mpi, and rci\_mpi using four processors and a disks file defining the location of the directory (on the disk) in which temporary data should be stored. On our computer, we will run the MPI jobs on four processors from a directory called

/home/tspejo/GRASP2018/grasptest/example4/script

and store the temporary data in

/home/tspejo/GRASP2018/grasptest/example4/tmp\_mpi

The disks file corresponding to this case is shown below.

- '/home/tspejo/GRASP2018/grasptest/example4/script'
- '/home/tspejo/GRASP2018/grasptest/example4/tmp\_mpi'
- '/home/tspejo/GRASP2018/grasptest/example4/tmp\_mpi'
- '/home/tspejo/GRASP2018/grasptest/example4/tmp\_mpi'
- '/home/tspejo/GRASP2018/grasptest/example4/tmp\_mpi'

If we use four processors for the MPI run, the full path to the directory storing temporary data should be given four times in the disks file. If we use eight processors for the MPI run, the full path to the directory storing temporary data should be given eight times, etc. The directory storing temporary data can be anywhere in the file system, and need not be on the same level in the file system as the working directory.

Provided the disks file is set up correctly according to the file structure of the local computer, the cpath.f90 routine of the GRASP mpi90 library automatically creates the directory in which temporary data are stored along with subdirectories 000, 001, 002 etc. named after the processors, starting with 0. On our system cpath.f90 creates

/home/tspejo/GRASP2018/grasptest/example4/tmp\_mpi

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along with four subdirectories 000, 001, 002, 003 named after the four processors, starting with 0. If cpath.f90 fails to create the directories specified in the disks file then temporary data are stored in the directory specified by the MPI\_TMP environment variable.

On some computer systems, the MPI libraries need to be loaded before the calculation starts. The commands for this depend on the system, but could look like

```
module add openmpi
```

Make sure you load MPI libraries for gfortran. For additional runs using the MPI codes, see Section 9.7.

### Program Input

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

```
**********************************
* RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
* OUTPUT FILE: isodata
*************************
>>rnucleus
RNUCLEUS
This program defines nuclear data and the radial grid
Outputfile: isodata
Enter the atomic number:
>>26
Enter the mass number (0 if the nucleus is to be modelled as a point source:
The default root mean squared radius is 3.7376999855041504
                                                          fm;
                                                              (Angeli)
  the default nuclear skin thickness is 2.29999999999998
                                                          fm;
Revise these values?
Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
Enter the nuclear dipole moment (in nuclear magnetons):
Enter the nuclear quadrupole moment (in barns):
>>1
**********************************
        RUN RCSFGENERATE TO GENERATE LIST OF CSFs FOR
        CONFIGURATIONS 3s(2), 3p(2), 3s3p
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
RCSFGENERATE
This program creates a list of CSFs
```

Configurations should be entered in spectroscopic notation with occupation numbers and indications if orbitals are

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```
closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                           = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                           = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                           = 36 electrons)
       5: Xe([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,i)3s(2,i)
 Give configuration 2
>>2s(2,i)2p(6,i)3p(2,i)
 Give configuration 3
>>2s(2,i)2p(6,i)3s(1,i)3d(1,i)
 Give configuration 4
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,6
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
        . . . . . . . . .
  4 blocks were created
              J/P
                           NCSF
      block
               0+
                              3
                              2
          2
               1+
          3
               2+
********************************
         IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
         RECORD ON HOW THE LIST OF CSFs WAS CREATED
**************************************
```

>>cp rcsfgenerate.log evenmr.exc

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```
>>cp rcsf.out rcsf.inp
************************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
**********************************
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
            rangular.log
 Full interaction? (y/n)
>>y
  . . . . .
 RANGULAR: Execution complete.
*********************************
        RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp, rwfnestimate.log
***********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                     9 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                    p0
                                         <r>
                                                 MTP SRC
         е
                             gamma
```

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```
1s
     0.3098D+03 0.2951D+03 0.1000D+01 0.5759D-01 328 T-F
  2s 0.6428D+02 0.1015D+03 0.1000D+01 0.2385D+00 346
                                                       T-F
  2p- 0.6284D+02 0.7744D+00 0.1000D+01 0.2003D+00 346
                                                       T-F
  2p 0.6217D+02 0.6941D+03 0.2000D+01 0.2028D+00 346
                                                       T-F
  3s
     0.2358D+02 0.5152D+02 0.1000D+01 0.5708D+00 358 T-F
  3p- 0.2295D+02 0.4217D+00 0.1000D+01 0.5370D+00 358 T-F
     0.2278D+02 0.3794D+03 0.2000D+01 0.5409D+00
  Зр
                                                  358
                                                       T-F
  3d- 0.2170D+02 0.5565D+00 0.2000D+01 0.4629D+00
                                                  358
                                                       T-F
     0.2165D+02 0.6259D+03 0.3000D+01 0.4642D+00
                                                  358 T-F
 RWFNESTIMATE: Execution complete.
************************************
         RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s,2s,2p,3s,3p,3d AND THEY ARE ALL
         SPECTROSCOPIC. WE CAN USE WILD CARDS FOR SPECIFYING ORBITALS
         NOTE: INSTEAD OF SAYING THAT WE SHOULD OPTIMIZE ON, FOR EXAMPLE,
         THE STATES 1,2,3,4 WE CAN WRITE 1-4 MEANING THE SAME THING
*************************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
                      9 relativistic subshells;
 There are/is
 Loading CSF File for ALL blocks
 There are
                   10 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
                    4 blocks (block
                                                 NCF):
 There are
                                       J/Parity
 1
    0+
                         1+
 Enter ASF serial numbers for each block
                                    3 id =
 Block
               1 	 ncf =
                                               0+
>>1-3
                2 	 ncf =
                                   2 id =
 Block
>>1-2
```

4 id =

2+

Block

3 ncf =

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```
>>1-4
               4 ncf =
Block
                                 1 id =
                                            3+
>>1
 level weights (1 equal; 5 standard; 9 user)
>>5
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
. . . . . . . . . . . . . .
 RMCDHF: Execution complete.
*************************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                    name.alog name.log
>>rsave evenmr
 Created evenmr.w, evenmr.c, evenmr.m, evenmr.sum, evenmr.alog and evenmr.log
***********************************
        RUN RCSFGENERATE TO GENERATE n = 4 VALENCE-VALENCE AND
        CORE-VALENCE LIST
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                  1s(2)
                                       = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                      = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                      = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                      = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6) = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
```

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```
Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,5)3s(2,*)
 Give configuration 2
>>2s(2,1)2p(6,i)3s(2,*)
 Give configuration 3
>>2s(2,i)2p(6,5)3p(2,*)
 Give configuration 4
>>2s(2,1)2p(6,i)3p(2,*)
 Give configuration 5
>>2s(2,i)2p(6,5)3s(1,*)3d(1,*)
 Give configuration 6
>>2s(2,1)2p(6,i)3s(1,*)3d(1,*)
 Give configuration 7
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,6
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 4 blocks were found
      block J/P
                        NCSF
             0+
                          556
          2
              1+
                          1448
          3
              2+
                          1898
              3+
                          1810
*************************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
>>cp rcsfgenerate.log even4.exc
>>cp rcsf.out rcsf.inp
RUN RANGULAR_MPI USING 4 PROCESSES TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,..IN 000, 001, 002, 003 \ast
***********************************
>>mpirun -np 4 rangular_mpi
```

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```
RANGULAR_MPI: Execution Begins ...
 ______
 Participating nodes:
  Host: atom1 ID: 000
  Host: atom1
               ID: 001
  Host: atom1 ID: 002
  Host: atom1 ID: 003
 Date and Time:
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
   atom1: Date: 20140812 Time: 011040.566 Zone: +0200
 Start Dir:
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
   atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
 Serial I/O Dir (node-0 only):
   atom1: /home/tspejo/GRASP2018/grasptest/example4/script
 Work Dir (Parallel I/O):
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
Full interaction? (y/n)
>>y
                          O from RANGULAR_MPI: Execution complete.
mpi stopped by node-
mpi stopped by node-
                           2 from RANGULAR_MPI: Execution complete.
                           1 from RANGULAR_MPI: Execution complete.
 mpi stopped by node-
 mpi stopped by node-
                            3 from RANGULAR_MPI: Execution complete.
***********************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp
************************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
```

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```
>>y
 Loading CSF file ... Header only
 There are/is
                      16 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
 Enter the list of relativistic subshells:
 The following subshell radial wavefunctions remain to be estimated:
 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
    3--Screened Hydrogenic
     4---Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                                                    MTP SRC
          е
                      p0
                               gamma
                                            <r>
     0.2803D+03 0.2923D+03 0.1000D+01 0.5839D-01 354
  1s
                                                         rwf
     0.4802D+02 0.9081D+02 0.1000D+01 0.2623D+00 358
  2p- 0.4353D+02 0.6335D+00 0.1000D+01 0.2298D+00
                                                    358
                                                         rwf
      0.4305D+02 0.5671D+03 0.2000D+01 0.2326D+00
                                                    358
      0.1667D+02 0.4072D+02 0.1000D+01 0.6859D+00 362
  3p- 0.1543D+02 0.2995D+00 0.1000D+01 0.6765D+00
                                                   363
     0.1534D+02 0.2693D+03 0.2000D+01 0.6810D+00
  Зр
                                                    363 rwf
  3d- 0.1358D+02 0.2668D+00 0.2000D+01 0.6260D+00
                                                    364
  3d
     0.1356D+02 0.3005D+03 0.3000D+01 0.6270D+00
                                                    364
                                                         rwf
  4s
     0.1141D+02 0.3095D+02 0.1000D+01 0.1086D+01
                                                    366
                                                         T-F
  4p- 0.1110D+02 0.2579D+00 0.1000D+01 0.1061D+01
                                                    367
                                                         T-F
      0.1104D+02 0.2325D+03 0.2000D+01 0.1066D+01
                                                    367
                                                         T-F
  4d- 0.1053D+02 0.3857D+00 0.2000D+01 0.1002D+01
                                                    367
                                                         T-F
     0.1051D+02 0.4342D+03 0.3000D+01 0.1004D+01
                                                    367
                                                         T-F
  4f- 0.9897D+01 0.2129D+00 0.3000D+01 0.8833D+00
                                                    367
      0.9890D+01 0.2598D+03 0.4000D+01 0.8842D+00
                                                    367
                                                         T-F
 RWFNESTIMATE: Execution complete.
************************************
         RUN RMCDHF_MEM_MPI USING 4 PROCESSES TO OBTAIN SELF CONSISTENT
         SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
```

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```
NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
        NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
        VARY THE CORRELATION ORBITALS 4s,4p,4d,4f. NONE OF THESE ARE
        SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
        4* MEANS 4s, 4p-, 4p, 4d-, 4d, 4f-, 4f
***********************************
>>mpirun -np 4 rmcdhf_mem_mpi
______
      RMCDHF_MPI: Execution Begins ...
______
Participating nodes:
  Host: atom1 ID: 000
  Host: atom1 ID: 001
  Host: atom1 ID: 002
  Host: atom1 ID: 003
Date and Time:
  atom1: Date: 20140812 Time: 011653.965 Zone: +0200
  atom1: Date: 20140812 Time: 011653.965 Zone: +0200
  atom1: Date: 20140812 Time: 011653.965 Zone: +0200
  atom1: Date: 20140812 Time: 011653.965 Zone: +0200
Start Dir:
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
Serial I/O Dir (node-0 only):
  atom1: /home/tspejo/GRASP2018/grasptest/example4/script
Work Dir (Parallel I/O):
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
Default settings? (y/n)
>>y
_____
Spin-angular coefficient are putting into the memory:
_____
                             755.00 Gb
    Total memory on computer:
                             630.17 Gb
    Free memory on computer:
    Allocation for mcp.30:
      Free memory on computer
                             630.16 Gb
    Allocation for mcp.31:
      Free memory on computer
                             630.16 Gb
    Allocation for mcp.32:
```

630.16 Gb

Free memory on computer

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```
Allocation for mcp.33:
      Free memory on computer
                               630.15 Gb
    Allocation for mcp.34:
      Free memory on computer
                               630.14 Gb
    Allocation for mcp.35:
                               630.13 Gb
      Free memory on computer
    Allocation for mcp.36:
      Free memory on computer
                               630.13 Gb
    Allocation for mcp.37:
      Free memory on computer
                               630.13 Gb
    Allocation for mcp.38:
      Free memory on computer
                               630.13 Gb
    Allocation for mcp.39:
      Free memory on computer
                               630.13 Gb
 Loading CSF file ... Header only
 There are/is
                    16 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                5712 relativistic CSFs... load complete;
 There are
                   4 blocks (block J/Parity
                                                 NCF):
                    2
    0+
                        1+ 1448
                                           2+ 1898
           556
                                       3
                                                              3+ 1810
 Enter ASF serial numbers for each block
               1 ncf =
                             556 id =
>>1-3
 Block
                   ncf =
                                 1448 id =
                                               1+
>>1-2
                     ncf =
Block
                                  1898 id =
>>1-4
                     ncf =
                                  1810 id =
 Block
>>1
 level weights (1 equal; 5 standard; 9 user)
  Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Enter orbitals to be varied (Updating order)
>>4*
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
 . . . . .
 mpi stopped by node-
                            1 from RMCDHF_MPI: Execution complete.
 mpi stopped by node-
                             0 from RMCDHF_MPI: Execution complete.
                             3 from RMCDHF_MPI: Execution complete.
 mpi stopped by node-
 mpi stopped by node-
                            2 from RMCDHF_MPI: Execution complete.
*************************************
         RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                      name.alog, name.log
**********************************
```

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```
>>rsave even4
 Created even4.w, even4.c, even4.m, even4.sum, even4.alog and even4.log
************************************
        RUN RCI MPI USING 4 PROCESSES TO INCLUDE BREIT AND QED EFFECTS
        INPUT FILES : isodata, even4.c, even4.w
        OUTPUT FILES: even4.cm, even4.csum
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HTGH N.
***********************************
>>mpirun -np 4 rci_mpi
_____
       RCI_MPI: Execution Begins ...
 ______
 Participating nodes:
  Host: atom1 ID: 000
  Host: atom1
               ID: 001
  Host: atom1 ID: 002
  Host: atom1 ID: 003
 Date and Time:
  atom1: Date: 20140812 Time: 012250.312 Zone: +0200
  tart Dir:
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
 Serial I/O Dir (node-0 only):
  atom1: /home/tspejo/GRASP2018/grasptest/example4/script
 Work Dir (Parallel I/0):
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
Default settings?
>>y
Name of state:
>>even4
               1 , ncf =
 Block
                                 556
 Block
               2 , ncf =
                                1448
 Block
               3 , ncf =
                                1898
               4 , ncf =
 Block
                                1810
```

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```
Loading CSF file ... Header only
 There are/is
                      16 relativistic subshells;
 Include contribution of H (Transverse)?
>>y
 Modify all transverse photon frequencies?
 Enter the scale factor:
>>1.d-6
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
>>n
 Include H (Specific Mass Shift)?
>>n
Estimate self-energy?
>>y
 Largest n quantum number for including self-energy for orbital
 n should be less or equal 8
>>4
 There are
                    4 blocks (block
                                       J/Parity
                                                 NCF):
                    2
                                                              3+ 1810
 1
      0+
           556
                         1+ 1448
                                       3
                                           2+ 1898
 Enter ASF serial numbers for each block
                1 ncf =
                             556 id =
>>1-3
 Block
                   ncf =
                                 1448 id =
                                               1+
>>1-2
                3 	 ncf =
Block
                                 1898 id =
                                               2+
>>1-4
                     ncf =
                                  1810 id =
 Block
>>1
                            0 from RCI_MPI: Execution complete.
mpi stopped by node-
 mpi stopped by node-
                            3 from RCI_MPI: Execution complete.
                            1 from RCI_MPI: Execution complete.
 mpi stopped by node-
 mpi stopped by node-
                             2 from RCI_MPI: Execution complete.
***********************************
         RUN JJ2LSJ TO GET THE LSJ-COMPOSITION
         INPUT FILE: even4.c, even4.cm
         OUTPUT FILE: even4.lsj.lbl, even4.uni.lsj.lbl
**********************************
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2017).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
```

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```
name.uni.lsj.lbl, name.uni.lsj.sum
```

```
Name of state
>>even4
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
 There are 5712 relativistic CSFs;
  ... load complete;
 Mixing coefficients from a CI calc.?
>>7
 Do you need a unique labeling? (y/n)
>>y
   nelec =
                   12
   ncftot =
                  5712
                   16
   nblock =
                     4
          ncf nev 2j+1 parity
  block
                          1
      1
          556
                   3
                                  1
         1448
                           3
      2
                   2
                                   1
      3
         1898
                    4
                          5
                                   1
                          7
      4
          1810
                    1
                                   1
 Default settings? (y/n)
>>y
 jj2lsj: Execution complete.
*************************************
        RUN RCSFGENERATE TO GENERATE LIST OF CSFs FOR
        CONFIGURATIONS 3s3p, 3p3d
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
                                        = 2 electrons)
       1: He (
                  1s(2)
       2: Ne ([He] + 2s(2)2p(6)
                                        = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                        = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                       = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                        = 54 electrons)
```

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```
6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,i)3s(1,i)3p(1,i)
 Give configuration 2
>>2s(2,i)2p(6,i)3p(1,i)3d(1,i)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 5 blocks were created
      block J/P
                        NCSF
         1 0-
                           2
         2
            1-
         3
             2-
                           5
         4
             3-
                           3
         5
             4-
                            1
**********************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
***********************************
>>cp rcsfgenerate.log oddmr.exc
>>cp rcsf.out rcsf.inp
**********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
*********************************
```

## >>rangular

## RANGULAR

This program performs angular integration

Input file: rcsf.inp

Outputfiles: mcp.30, mcp.31, ....

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rangular.log Full interaction? (y/n)>>y RANGULAR: Execution complete. \* RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS \* INPUT FILES: isodata, rcsf.inp, previous rwfn files OUTPUT FILE: rwfn.inp, rwfnestimate.log \* >>rwfnestimate RWFNESTIMATE This program estimates radial wave functions for orbitals Input files: isodata, rcsf.inp, optional rwfn file Output file: rwfn.inp Default settings ? >>y Loading CSF file ... Header only There are/is 9 relativistic subshells; The following subshell radial wavefunctions remain to be estimated: 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d Read subshell radial wavefunctions. Choose one below 1--GRASP2K File 2--Thomas-Fermi 3--Screened Hydrogenic 4--Screened Hydrogenic [custom Z] >>2 Enter the list of relativistic subshells: All required subshell radial wavefunctions have been estimated: Shell p0 gamma <r> MTP SRC 0.3098D+03 0.2951D+03 0.1000D+01 0.5759D-01 328 T-F 1s 0.6428D+02 0.1015D+03 0.1000D+01 0.2385D+00 346 T-F 2p- 0.6284D+02 0.7744D+00 0.1000D+01 0.2003D+00 346 T-F 2p 0.6217D+02 0.6941D+03 0.2000D+01 0.2028D+00 346 T-F 0.2358D+02 0.5152D+02 0.1000D+01 0.5708D+00 358 T-F 3s 3p- 0.2295D+02 0.4217D+00 0.1000D+01 0.5370D+00 358 T-F Зр 0.2278D+02 0.3794D+03 0.2000D+01 0.5409D+00 358 T-F 3d- 0.2170D+02 0.5565D+00 0.2000D+01 0.4629D+00 358 T-F 0.2165D+02 0.6259D+03 0.3000D+01 0.4642D+00 358 T-F RWFNESTIMATE: Execution complete.

\* RUN RMCDHF\_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS

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```
INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s,2s,2p,3s,3p,3d AND THEY ARE ALL
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
************************************
>>rmcdhf_mem
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                       9 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                   16 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
                                                  NCF):
 There are
                     5 blocks (block
                                        J/Parity
      0-
                         1-
                                5
                                             2-
                                                   5
      4-
  5
             1
 Enter ASF serial numbers for each block
 Block
           1 	 ncf =
                               2 id =
>>1-2
 Block
                 2
                   ncf =
                                     5 id =
>>1-5
Block
                 3 	 ncf =
                                     5 id =
                                                 2-
>>1-5
 Block
                 4
                      ncf =
                                      3 id =
>>1-3
Block
                 5
                      ncf =
                                     1 id =
                                                4-
 level weights (1 equal; 5 standard; 9 user)
>>5
Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
>>*
 Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
```

. . . . . . . . . . . . . .

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RMCDHF: Execution complete. \* RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum name.alog, name.log \* >>rsave oddmr Created oddmr.w, oddmr.c, oddmr.m, oddmr.sum, oddmr.alog and oddmr.log \* RUN RCSFGENERATE TO GENERATE n = 4 VALENCE-VALENCE AND CORE-VALENCE LIST OUTPUT FILES: rcsfgenerate.log, rcsf.out \* >>rcsfgenerate RCSFGENERATE This program creates a list of CSFs Configurations should be entered in spectroscopic notation with occupation numbers and indications if orbitals are closed (c), inactive (i), active (\*) or has a minimal occupation e.g., 1s(2,1)2s(2,\*)Outputfiles: rcsf.out, rcsfgenerate.log Default, reverse, symmetry or user specified ordering? (\*/r/s/u) >>\* Select core 0: No core = 2 electrons) 1: He ( 1s(2) 2: Ne ([He] + 2s(2)2p(6)= 10 electrons) 3: Ar ([Ne] + 3s(2)3p(6)= 18 electrons) 4: Kr ([Ar] + 3d(10)4s(2)4p(6)= 36 electrons) 5: Xe ([Kr] + 4d(10)5s(2)5p(6)= 54 electrons) 6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons) >>1 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (\*) Give configuration 1 >>2s(2,i)2p(6,5)3s(1,\*)3p(1,\*)Give configuration 2 >>2s(2,1)2p(6,i)3s(1,\*)3p(1,\*) Give configuration 3 >>2s(2,i)2p(6,5)3p(1,\*)3d(1,\*)Give configuration 4 >>2s(2,1)2p(6,i)3p(1,\*)3d(1,\*)Give configuration 5 Give set of active orbitals, as defined by the highest principal quantum number per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d >>4s,4p,4d,4f

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```
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,8
Number of excitations (if negative number e.g., -2, correlation
orbitals will always be doubly occupied)
>>2
Generate more lists ? (y/n)
>>n
 5 blocks were created
     block J/P
                     NCSF
        1 0-
                      546
        2 1-
                      1456
        3 2-
                     1891
        4 3-
                     1814
        5
           4-
                      1393
COPY FILES
       IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
       RECORD ON HOW THE LIST OF CSFs WAS CREATED
************************************
>>cp rcsfgenerate.log odd4.exc
>>cp rcsf.out rcsf.inp
*********************************
       RUN RANGULAR_MPI USING 4 PROCESSES TO GENERATE ENERGY EXPRESSION
       INPUT FILE : rcsf.inp
       OUTPUT FILES: rangular.log, mcp.30, mcp.31,...IN 000, 001, 002, 003 *
***********************************
>>mpirun -np 4 rangular_mpi
_____
      RANGULAR_MPI: Execution Begins ...
 ______
Participating nodes:
  Host: atom1 ID: 000
            ID: 001
  Host: atom1
  Host: atom1 ID: 002
  Host: atom1 ID: 003
Date and Time:
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
  atom1: Date: 20140812 Time: 011040.566 Zone: +0200
Start Dir:
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
```

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```
atom1: /GRASP2018/grasptest/example4/script
   atom1: /GRASP2018/grasptest/example4/script
 Serial I/O Dir (node-0 only):
   atom1: /home/tspejo/GRASP2018/grasptest/example4/script
 Work Dir (Parallel I/0):
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
Full interaction? (y/n)
>>y
 mpi stopped by node-
                              O from RANGULAR_MPI: Execution complete.
 mpi stopped by node-
                             2 from RANGULAR_MPI: Execution complete.
 mpi stopped by node-
                             1 from RANGULAR_MPI: Execution complete.
 mpi stopped by node-
                              3 from RANGULAR_MPI: Execution complete.
************************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp
*************************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                       16 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
 Enter the list of relativistic subshells:
 The following subshell radial wavefunctions remain to be estimated:
```

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```
4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
                                                 MTP SRC
Shell
                                         <r>
         е
                    p0
                             gamma
     0.2803D+03 0.2923D+03 0.1000D+01 0.5838D-01 351
  1s
                                                     rwf
  2s
     0.4797D+02 0.9077D+02 0.1000D+01 0.2624D+00 357
  2p- 0.4349D+02 0.6336D+00 0.1000D+01 0.2298D+00 358 rwf
     0.4301D+02 0.5672D+03 0.2000D+01 0.2326D+00 358
  3s
      0.1680D+02  0.4086D+02  0.1000D+01  0.6840D+00  362
                                                     rwf
  3p- 0.1542D+02 0.2993D+00 0.1000D+01 0.6773D+00 363
  Зр
     0.1533D+02 0.2692D+03 0.2000D+01 0.6818D+00
                                                363 rwf
  3d- 0.1358D+02 0.2660D+00 0.2000D+01 0.6264D+00
                                                 364
                                                     rwf
  3d
      0.1357D+02 0.2997D+03 0.3000D+01 0.6274D+00
                                                364
                                                     rwf
  4s
     0.1141D+02 0.3095D+02 0.1000D+01 0.1086D+01 366
                                                     T-F
  4p- 0.1110D+02 0.2579D+00 0.1000D+01 0.1061D+01 367
                                                     T-F
     0.1104D+02 0.2325D+03 0.2000D+01 0.1066D+01 367
  4p
                                                     T-F
  4d- 0.1053D+02 0.3857D+00 0.2000D+01 0.1002D+01 367
                                                     T-F
     0.1051D+02 0.4342D+03 0.3000D+01 0.1004D+01
  4d
                                                367
                                                     T-F
  4f- 0.9897D+01 0.2129D+00 0.3000D+01 0.8833D+00
                                                 367
                                                     T-F
      0.9890D+01 0.2598D+03 0.4000D+01 0.8842D+00
                                                367
                                                     T-F
 RWFNESTIMATE: Execution complete.
**********************************
        RUN RMCDHF_MEM_MPI USING 4 PROCESSES TO OBTAIN SELF CONSISTENT
        SOLUTIONS
        INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
        OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
        NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
        NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
        VARY THE CORRELATION ORBITALS 4s,4p,4d,4f. NONE OF THESE ARE
        SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
        4* MEANS 4s, 4p-, 4p, 4d-, 4d, 4f-, 4f
>>mpirun -np 4 rmcdhf_mem_mpi
       RMCDHF_MPI: Execution Begins ...
 Participating nodes:
                ID: 000
  Host: atom1
  Host: atom1
               ID: 001
               ID: 002
  Host: atom1
  Host: atom1
               ID: 003
```

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```
Date and Time:
   atom1: Date: 20140812 Time: 020654.423 Zone: +0200
   atom1: Date: 20140812 Time: 020654.422 Zone: +0200
   atom1: Date: 20140812 Time: 020654.422 Zone: +0200
   atom1: Date: 20140812 Time: 020654.422 Zone: +0200
 Start Dir:
   atom1: /GRASP2018/grasptest/example4/script
   atom1: /GRASP2018/grasptest/example4/script
   atom1: /GRASP2018/grasptest/example4/script
   atom1: /GRASP2018/grasptest/example4/script
 Serial I/O Dir (node-0 only):
   atom1: /home/tspejo/GRASP2018/grasptest/example4/script
 Work Dir (Parallel I/O):
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
Default settings? (y/n)
>>y
Spin-angular coefficient are putting into the memory:
_____
     Total memory on computer:
                                 755.00 Gb
                                 630.12 Gb
     Free memory on computer:
     Allocation for mcp.30:
       Free memory on computer
                                 630.12 Gb
     Allocation for mcp.31:
                                 630.12 Gb
       Free memory on computer
     Allocation for mcp.32:
                                 630.12 Gb
       Free memory on computer
     Allocation for mcp.33:
                                 630.11 Gb
       Free memory on computer
     Allocation for mcp.34:
       Free memory on computer
                                 630.09 Gb
     Allocation for mcp.35:
       Free memory on computer
                                 630.08 Gb
     Allocation for mcp.36:
       Free memory on computer
                                 630.08 Gb
     Allocation for mcp.37:
                                 630.08 Gb
       Free memory on computer
     Allocation for mcp.38:
       Free memory on computer
                                 630.08 Gb
     Allocation for mcp.39:
       Free memory on computer
                                 630.08 Gb
```

Loading CSF file ... Header only

16 relativistic subshells;

There are/is

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```
Loading CSF File for ALL blocks
There are 7100 relativistic CSFs... load complete;
                 5 blocks (block J/Parity NCF):
There are
                 2 1- 1456 3 2- 1891 4 3- 1814
 1 0- 546
     4- 1393
Enter ASF serial numbers for each block
Block
              1 	 ncf =
                              546 id =
>>1-2
            2 	 ncf = 1456 	 id =
Block
>>1-5
          3 ncf = 1891 id =
Block
>>1-5
Block
            4 	 ncf =
                            1814 id =
>>1-3
             5 	 ncf =
                             1393 id =
Block
                                          4-
level weights (1 equal; 5 standard; 9 user)
>>5
 Radial functions
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
Enter orbitals to be varied (Updating order)
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
. . . . . . . . . . . .
mpi stopped by node-
mpi stopped by node-
2 from RMCDHF_MPI: Execution complete.
mpi stopped by node-
1 from RMCDHF_MPI: Execution complete.
                         3 from RMCDHF_MPI: Execution complete.
mpi stopped by node-
******************************
       RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                  name.alog, name.log
*********************************
>>rsave odd4
Created odd4.w, odd4.c, odd4.m, odd4.sum, odd4.alog and odd4.log
RUN RCI_MPI USING 4 PROCESSES TO INCLUDE BREIT AND QED EFFECTS
        INPUT FILES : isodata, odd4.c, odd4.w
       OUTPUT FILES: odd4.cm, odd4.csum
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
```

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```
>>mpirun -np 4 rci_mpi
 ______
       RCI_MPI: Execution Begins ...
 ______
 Participating nodes:
  Host: atom1 ID: 000
  Host: atom1 ID: 001
  Host: atom1 ID: 002
  Host: atom1 ID: 003
 Date and Time:
  atom1: Date: 20140812 Time: 021251.038 Zone: +0200
  atom1: Date: 20140812 Time: 021251.038 Zone: +0200
  atom1: Date: 20140812 Time: 021251.038 Zone: +0200
   atom1: Date: 20140812 Time: 021251.038 Zone: +0200
 Start Dir:
  atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
   atom1: /GRASP2018/grasptest/example4/script
  atom1: /GRASP2018/grasptest/example4/script
 Serial I/O Dir (node-0 only):
  atom1: /home/tspejo/GRASP2018/grasptest/example4/script
 Work Dir (Parallel I/0):
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
  atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
   atom1: /home/tspejo/GRASP2018/grasptest/example4/tmp_mpi
Default settings?
>>y
Name of state:
>>odd4
               1 , ncf =
 Block
                                 546
 Block
                2 , ncf =
                                 1456
 Block
               3 , ncf =
                                 1891
 Block
               4 , ncf =
                                 1814
               5 , ncf =
                                 1393
 Block
 Loading CSF file ... Header only
 There are/is
                     16 relativistic subshells;
 Include contribution of H (Transverse)?
>>y
 Modify all transverse photon frequencies?
>>y
 Enter the scale factor:
>>1.d-6
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
>>n
```

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```
Include H (Specific Mass Shift)?
 Estimate self-energy?
>>y
 Largest n quantum number for including self-energy for orbital
 n should be less or equal 8
>>4
 There are
                    5 blocks (block
                                      J/Parity
                                                NCF):
  1
      0-
           546
                        1- 1456
                                           2- 1891
                                                             3- 1814
      4- 1393
 Enter ASF serial numbers for each block
 Block
                1
                     ncf =
                                 546 id =
>>1-2
Block
                    ncf =
                                 1456 id =
>>1-5
 Block
                3
                     ncf =
                                  1891 id =
                                              2-
>>1-5
                4
Block
                     ncf =
                                  1814 id =
                                              3-
>>1-3
                5
 Block
                     ncf =
                                  1393 id =
>>1
 mpi stopped by node-
                             0 from RCI_MPI: Execution complete.
 mpi stopped by node-
                             2 from RCI_MPI: Execution complete.
 mpi stopped by node-
                             3 from RCI_MPI: Execution complete.
 mpi stopped by node-
                             1 from RCI_MPI: Execution complete.
***********************************
         RUN JJ2LSJ TO GET THE LSJ-COMPOSITION
         INPUT FILE: odd4.c, odd4.cm
         OUTPUT FILE: odd4.lsj.lbl, odd4.uni.lsj.lbl
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2017).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
             name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>odd4
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
 There are 7100 relativistic CSFs;
  ... load complete;
```

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```
Mixing coefficients from a CI calc.?
Do you need a unique labeling? (y/n)
>>y
   nelec =
                  12
   ncftot =
                 7100
                  16
   nw =
   nblock =
                   5
  \verb|block| & \verb|ncf| & \verb|nev| & 2j+1 & \verb|parity| \\
                        1 -1
3 -1
5 -1
7 -1
     1
          546 2
      2 1456 5
3 1891 5
      4 1814 3
5 1393 1
                         7 -1
9 -1
Default settings? (y/n)
>>y
jj2lsj: Execution complete.
*********************************
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS
        NOTE: SINCE LSJ-INFORMATION NOW IS AVAILABLE OUTPUT LABELS
        WILL BE IN LSJ-COUPLING
***********************************
>>rlevels even4.cm odd4.cm
nblock =
                                   5712 nw = 16 nelec = 7100 nw = 16 nelec =
                4 ncftot =
                                                                           12
                 5 ncftot =
nblock =
                                                                           12
Energy levels for ...
Rydberg constant is 109737.31569
Splitting is the energy difference with the lower neighbor
```

No	Pos	J	Par:	ity Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	Configuration
1	1	0	+	-1182.4117992	0.00	0.00	2s(2).2p(6).3s(2)_1S
2	1	0	_	-1181.3459632	233923.97	233923.97	2s(2).2p(6).3s_2S.3p_3P
3	1	1	-	-1181.3193175	239772.02	5848.05	2s(2).2p(6).3s_2S.3p_3P
4	1	2	-	-1181.2548318	253925.01	14152.99	2s(2).2p(6).3s_2S.3p_3P
5	2	1	-	-1180.8025239	353195.12	99270.11	2s(2).2p(6).3s_2S.3p_1P
6	2	0	+	-1179.8828042	555050.24	201855.12	2s(2).2p(6).3p(2)3P2_3P
7	1	2	+	-1179.8592139	560227.72	5177.47	2s(2).2p(6).3p(2)1D2_1D
8	1	1	+	-1179.8374539	565003.49	4775.77	2s(2).2p(6).3p(2)3P2_3P
9	2	2	+	-1179.7587696	582272.71	17269.22	2s(2).2p(6).3p(2)3P2_3P
10	3	0	+	-1179.3935747	662423.72	80151.02	2s(2).2p(6).3p(2)1S0_1S
11	2	1	+	-1179.3158027	679492.71	17068.98	2s(2).2p(6).3s_2S.3d_3D
12	3	2	+	-1179.3111395	680516.16	1023.45	2s(2).2p(6).3s_2S.3d_3D
13	1	3	+	-1179.3038352	682119.26	1603.10	2s(2).2p(6).3s_2S.3d_3D
14	4	2	+	-1178.9201602	766326.18	84206.92	2s(2).2p(6).3s_2S.3d_1D
15	2	2	-	-1178.1773931	929344.72	163018.54	2s(2).2p(6).3p_2P.3d_3F
16	1	3	-	-1178.1321370	939277.28	9932.56	2s(2).2p(6).3p_2P.3d_3F

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17	3	2	-	-1178.0860896	949383.53	10106.25	2s(2).2p(6).3p_2P.3d_1D
18	1	4	-	-1178.0797665	950771.28	1387.75	2s(2).2p(6).3p_2P.3d_3F
19	3	1	-	-1177.9273160	984230.30	33459.02	2s(2).2p(6).3p_2P.3d_3D
20	4	2	-	-1177.9244263	984864.51	634.21	2s(2).2p(6).3p_2P.3d_3P
21	2	3	-	-1177.8730161	996147.75	11283.24	2s(2).2p(6).3p_2P.3d_3D
22	2	0	-	-1177.8671996	997424.33	1276.58	2s(2).2p(6).3p_2P.3d_3P
23	4	1	-	-1177.8658739	997715.29	290.96	2s(2).2p(6).3p_2P.3d_3P
24	5	2	-	-1177.8645522	998005.36	290.07	2s(2).2p(6).3p_2P.3d_3D
25	3	3	-	-1177.5443213	1068287.93	70282.57	2s(2).2p(6).3p_2P.3d_1F
26	5	1	-	-1177.4837515	1081581.45	13293.52	2s(2).2p(6).3p_2P.3d_1P

------

We compare with the energy levels given in the NIST database.

Configuration	Term		Level
	   	<u> </u>	İ
2p6.3s2	1S 	0	0
3s.3p	3P*	0	233842
	1	1	239660
		2	253820   
3s.3p	1P*	1 1	351911
3p2	   3P		 554524
Op2			
3p2	l 1D	2	559600
3p2	   3P		564602
•	1	1 2 1	581803
3p2	   1S		659627
G <sub>F</sub> =			
3s.3d	3D	1	678772
	l	2     3	679785   681416
	İ		001410
3s.3d	1D	2	762093
3p.3d	   3F*	1 2 1	928241
1	İ	3	938126
3p.3d	   1D*		948513
op.ou	10*		940313
3p.3d	3F*	4	949658
3p.3d	   3D*		982868
	Ī		ĺ
3p.3d	3P* 	2	983514
3p.3d	3D*	3	994852
	1	1 1	1

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3p.3d	3P*	 	0   1	995889 996243	1
3p.3d	l I 3D∗	1	2	996623	1
op.3u	554	i		330023	i
3p.3d	1F* 	 	3	1062515	1
3p.3d	1P*	İ	1	1074887	Ì

Please note that this is just a very small example calculation. The agreement between theory and experiment is improved when the active set is extended. If desired, we may display the energy separations in eV by running rlevelseV.

\*

>>rlevelseV even4.cm odd4.cm

```
nblock = 4 ncftot = 5712 nw = 16 nelec = 12 nblock = 5 ncftot = 7100 nw = 16 nelec = 12
```

Energy levels for ...

Rydberg constant is 109737.31569

Splitting is the energy difference with the lower neighbor

-		_				•	
No	Pos	J	Par	ity Energy Total (a.u.)	Levels (eV)	Splitting (eV)	Configuration
1	 1	0	+	 -1182.4117992	0.00000	0.00000	2s(2).2p(6).3s(2)_1S
2	1	0	_	-1181.3459632	29.00288	29.00288	-
3	1	1	_	-1181.3193175	29.72794	0.72507	
4	1	2	_	-1181.2548318	31.48269	1.75475	
5	2	1	_	-1180.8025239	43.79061	12.30792	
6	2	0	+	-1179.8828042	68.81746	25.02685	
7	1	2	+	-1179.8592139	69.45938	0.64192	
8	1	1	+	-1179.8374539	70.05150	0.59212	
9	2	2	+	-1179.7587696	72.19261	2.14111	
10	3	0	+	-1179.3935747	82.13007	9.93746	2s(2).2p(6).3p(2)1S0_1S
11	2	1	+	-1179.3158027	84.24636	2.11628	2s(2).2p(6).3s_2S.3d_3D
12	3	2	+	-1179.3111395	84.37325	0.12689	2s(2).2p(6).3s_2S.3d_3D
13	1	3	+	-1179.3038352	84.57201	0.19876	2s(2).2p(6).3s_2S.3d_3D
14	4	2	+	-1178.9201602	95.01234	10.44033	2s(2).2p(6).3s_2S.3d_1D
15	2	2	-	-1178.1773931	115.22406	20.21172	2s(2).2p(6).3p_2P.3d_3F
16	1	3	-	-1178.1321370	116.45554	1.23148	2s(2).2p(6).3p_2P.3d_3F
17	3	2	-	-1178.0860896	117.70856	1.25302	2s(2).2p(6).3p_2P.3d_1D
18	1	4	-	-1178.0797665	117.88061	0.17206	2s(2).2p(6).3p_2P.3d_3F
19	3	1	-	-1177.9273160	122.02900	4.14839	2s(2).2p(6).3p_2P.3d_3D
20	4	2	-	-1177.9244263	122.10764	0.07863	2s(2).2p(6).3p_2P.3d_3P
21	2	3	-	-1177.8730161	123.50658	1.39894	2s(2).2p(6).3p_2P.3d_3D
22	2	0	-	-1177.8671996	123.66486	0.15828	2s(2).2p(6).3p_2P.3d_3P
23	4	1	-	-1177.8658739	123.70093	0.03607	2s(2).2p(6).3p_2P.3d_3P
24	5	2	-	-1177.8645522	123.73689	0.03596	2s(2).2p(6).3p_2P.3d_3D

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```
25 3 3 - -1177.5443213 132.45082
26 5 1 - -1177.4837515 134.09901
                                               8.71393 2s(2).2p(6).3p_2P.3d_1F
                                               1.64819 2s(2).2p(6).3p_2P.3d_1P
        RUN RBIOTRANSFORM_MPI USING 4 PROCESSES
        INPUT FILES: even4.c, even4.w, even4.cm
                    odd4.c, odd4.w, odd4.cm, isodata
        OUPUT FILES: even4.bw, even4.cbm, odd4.bw, odd4.cbm
***********************************
>>mpirun -np 4 rbiotransform_mpi
 ______
       RBIOTRANSFORM_MPI: Execution Begins ...
 ______
 Participating nodes:
  Host: per-vaio ID: 000
  Host: per-vaio ID: 001
  Host: per-vaio ID: 002
  Host: per-vaio ID: 003
 Date and Time:
  per-vaio: Date: 20141120 Time: 002539.729 Zone: +0100
  per-vaio: Date: 20141120 Time: 002539.729 Zone: +0100
  per-vaio: Date: 20141120 Time: 002539.729 Zone: +0100
  per-vaio: Date: 20141120 Time: 002539.729 Zone: +0100
 Start Dir:
  per-vaio: /home/per/graspruns/FeXIII
  per-vaio: /home/per/graspruns/FeXIII
  per-vaio: /home/per/graspruns/FeXIII
  per-vaio: /home/per/graspruns/FeXIII
 Serial I/O Dir (node-0 only):
  per-vaio: /home/per/graspruns/FeXIII
 Work Dir (Parallel I/0):
  per-vaio: /home/per/tmp_mpi
  per-vaio: /home/per/tmp_mpi
  per-vaio: /home/per/tmp_mpi
  per-vaio: /home/per/tmp_mpi
 Default settings?
>>y
 Input from a CI calculation?
>>y
 Name of the Initial state
>>even4
 Name of the Final state
>>odd4
 Transformation of all J symmetries?
>>y
```

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. . . . . O from RBIOTRANSFORM\_MPI: Execution complete. mpi stopped by nodempi stopped by node-1 from RBIOTRANSFORM\_MPI: Execution complete. mpi stopped by node-2 from RBIOTRANSFORM\_MPI: Execution complete. mpi stopped by node-3 from RBIOTRANSFORM\_MPI: Execution complete. \* RUN RTRANSITION\_MPI USING 4 PROCESSES INPUT FILES: even4.c, even4.bw, even4.cbm odd4.c, odd4.bw, odd4.cbm, isodata OUPUT FILES: even4.odd4.ct >>mpirun -np 4 rtransition\_mpi RTRANSITION\_MPI: Execution Begins ... \_\_\_\_\_ Participating nodes: Host: per-vaio ID: 000 Host: per-vaio ID: 001 Host: per-vaio ID: 002 Host: per-vaio ID: 003 Date and Time: per-vaio: Date: 20141120 Time: 003050.621 Zone: +0100 per-vaio: Date: 20141120 Time: 003050.621 Zone: +0100 per-vaio: Date: 20141120 Time: 003050.621 Zone: +0100 per-vaio: Date: 20141120 Time: 003050.621 Zone: +0100 Start Dir: per-vaio: /home/per/graspruns/FeXIII per-vaio: /home/per/graspruns/FeXIII per-vaio: /home/per/graspruns/FeXIII per-vaio: /home/per/graspruns/FeXIII Serial I/O Dir (node-0 only): per-vaio: /home/per/graspruns/FeXIII Default settings? >>y Input from a CI calculation? >>y Name of the Initial state >>even4 Name of the Final state >>odd4 Enter the list of transition specifications e.g., E1,M2 or E1 M2 or E1;M2:

>>E1,M2

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. . . . .

```
mpi stopped by node-
mpi stopped by node-
3 from RTRANSITION_MPI: Execution complete.
mpi stopped by node-
1 from RTRANSITION_MPI: Execution complete.
mpi stopped by node-
2 from RTRANSITION_MPI: Execution complete.
mpi stopped by node-
```

Comment: it does not matter in which order the files even4 and odd4 are specified.

# 6.5. Fifth Example: The Study of Energy Spectra for Ni XIV, Obtaining Unique Labels

A wave function or a corresponding energy level is often designated by the label of the CSF with the largest expansion coefficient. This example presents a study of energy spectra for Ni XIV in which a few levels have the same identification. To get the energy spectra with unique labels, we use the unique option in the jj2lsj program. The program uses the algorithm described in [26,40,41]: for a given set of wave functions for the same J and parity, the CSF with the largest expansion coefficient is used as the label for the function containing this largest component. Once a label is assigned, the corresponding CSF is removed from consideration in the determination of the next label. The last remaining label for a wave function may be based on a contribution that is tiny.

#### Overview

- 1. Define nuclear data.
- 2. Obtain common spectroscopic orbitals for the MR set.
  - (a) Generate list of CSFs describing the even states belonging to the  $3s3p^4$ ,  $3s^23p^23d$  configurations and the odd states belonging to the  $3s^23p^3$  configuration.
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of all states belonging to  $3s3p^4$ ,  $3s^23p^23d$ , and  $3s^23p^3$ .
  - (e) Save output to Ni\_mr.
- 3. Improve even states
  - (a) Generate CSF list from SD-excitations from  $3s3p^4$  and  $3s^23p^23d$  to n=4.
  - (b) Run resfinteract to extract CSFs that interact with CSFs belonging to  $3s3p^4$  and  $3s^23p^23d$ .
  - (c) Perform angular integration.
  - (d) Generate initial estimates of radial orbitals.
  - (e) Perform SCF calculation on the weighted average of all states belonging to  $3s3p^4$  and  $3s^23p^23d$ .
  - (f) Save output to Ni\_even\_n4.
  - (g) Perform rci calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
- 4. Transform from *jj* to *LSJ*-coupling
- 5. Improve odd states
  - (a) Generate CSF list from SD-excitations from  $3s^23p^3$  to n=4.
  - (b) Run resfinteract to extract CSFs that interact with CSFs belonging to  $3s^23p^3$ .
  - (c) Perform angular integration.
  - (d) Generate initial estimates of radial orbitals.
  - (e) Perform SCF calculation on the weighted average of all states belonging to  $3s^23p^3$ .
  - (f) Save output to Ni\_odd\_n4.
  - (g) Perform rci calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
- 6. Transform from *jj* to *LSJ*-coupling using the unique label option.
- 7. Run rlevels to view energy separations (several states have the same label).
- 8. Copy files so that rlevels will display unique labels.
- 9. Run rlevels to view energy separations for levels now with unique labels.

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10. Compute transition rates from the rci wave functions. Computation in two steps: biorthonormal transformation and then evaluation of transition matrix elements using standard Racah algebra methods.

## **Program Input**

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

```
*******************************
        RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
        OUTPUT FILE: isodata
***********************************
>>rnucleus
 RNUCLEUS
 This program defines nuclear data and the radial grid
 Outputfile: isodata
 Enter the atomic number:
>>28
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>61
 The default root mean squared radius is
                                       3.8224999904632568
                                                                 (Angeli)
                                                             fm:
  the default nuclear skin thickness is
                                       2.299999999999998
                                                             fm;
 Revise these values?
>>n
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
>>58.6934
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
 Enter the nuclear dipole moment (in nuclear magnetons):
 Enter the nuclear quadrupole moment (in barns):
>>1
*************************************
        RUN RCSFGENERATE TO GENERATE LIST FOR ALL
        STATES OF 3s3p(4), 3s(2)3p(2)3d and 3s(2)3p(3)
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
*******************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 OUTPUT FILES: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
```

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```
0: No core
        1: He (
                      1s(2)
                                             = 2 electrons)
        2: Ne ([He] + 2s(2)2p(6)
                                             = 10 electrons)
        3: Ar ([Ne] + 3s(2)3p(6)
                                             = 18 electrons)
        4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                             = 36 electrons)
                                             = 54 electrons)
        5: Xe([Kr] + 4d(10)5s(2)5p(6)
        6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,i)3s(1,i)3p(4,i)
 Give configuration 2
>>2s(2,i)2p(6,i)3s(2,i)3p(2,i)3d(1,i)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*).
 Give configuration 1
>>2s(2,i)2p(6,i)3s(2,i)3p(3,i)
 Give configuration 2
>>
 Give set of active orbitals in a comma delimited list ordered by 1-symmetry, e.g., 5s,4p,3d
>>3s,3p
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
        . . . . . . . . .
 8 blocks were created
       block J/P
                             NCSF
           1 1/2+
                                8
           2 1/2-
                                1
           3 3/2+
                               11
           4 3/2-
                                3
           5 5/2+
                               10
           6 5/2-
                                1
```

7 7/2+

5

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8 9/2+ COPY FILES IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A RECORD ON HOW THE LIST OF CSFs WAS CREATED >>cp rcsfgenerate.log Ni\_mr.exc >>cp rcsf.out rcsf.inp \* RUN RANGULAR TO GENERATE ENERGY EXPRESSION INPUT FILE : rcsf.inp OUTPUT FILES: rangular.log, mcp.30, mcp.31,.... \* >>rangular RANGULAR This program performs angular integration Input file: rcsf.inp Outputfiles: mcp.30, mcp.31, .... rangular.log Full interaction? (y/n)>>y . . . . . RANGULAR: Execution complete. \* RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS \* INPUT FILES: isodata, rcsf.inp, previous rwfn files OUTPUT FILE: rwfn.inp, rwfnestimate.log \* >>rwfnestimate RWFNESTIMATE This program estimates radial wave functions for orbitals Input files: isodata, rcsf.inp, optional rwfn file Output file: rwfn.inp Default settings ? >>y Loading CSF file ... Header only There are/is 9 relativistic subshells; The following subshell radial wavefunctions remain to be estimated: 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d

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```
Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
Shell
                     p0
                                                   MTP
                                                       SRC
                              gamma
                                          <r>
      0.3531D+03 0.3348D+03 0.1000D+01 0.5348D-01
  1s
                                                   329
                                                       T-F
     0.7017D+02 0.1144D+03 0.1000D+01 0.2231D+00
  2s
                                                   346
                                                       T-F
  2p- 0.6820D+02 0.1007D+01 0.1000D+01 0.1878D+00
                                                   346
 2p
     0.6732D+02 0.8291D+03 0.2000D+01 0.1905D+00
                                                   346
                                                       T-F
      0.2444D+02 0.5706D+02 0.1000D+01 0.5420D+00
                                                  358
                                                       T-F
  3p- 0.2358D+02 0.5370D+00 0.1000D+01 0.5120D+00
                                                   359
                                                       T-F
      0.2336D+02 0.4440D+03 0.2000D+01 0.5164D+00
                                                   359
                                                       T-F
  3d- 0.2191D+02 0.7313D+00 0.2000D+01 0.4446D+00
                                                   359
                                                       T-F
      0.2185D+02 0.7607D+03 0.3000D+01 0.4461D+00
                                                   359
                                                       T-F
RWFNESTIMATE: Execution complete.
RUN RMCDHF_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s,2s,2p,3s,3p,3d AND THEY ARE ALL
         SPECTROSCOPIC. WE CAN USE WILD CARDS FOR SPECIFYING ORBITALS
         NOTE: INSTEAD OF SAYING THAT WE SHOULD OPTIMIZE ON, FOR EXAMPLE,
         THE STATES 1,2,3,4 WE CAN WRITE 1-4 MEANING THE SAME THING
***********************************
>>rmcdhf_mem
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                       9 relativistic subshells;
 Loading CSF File for ALL blocks
                   41 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
                   8 blocks (block
                                       J/Parity
                                                 NCF):
 There are
  1 1/2+
                    2 1/2-
                               1
                                       3 3/2+
                                                 11
                                                         4 3/2-
                                                                    3
```

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```
6 5/2- 1 7 7/2+ 5
 5 5/2+
           10
                                                8 9/2+
                                                               2
Enter ASF serial numbers for each block
                                 8 \text{ id} = 1/2 +
Block
               1
                 ncf =
>>1-8
               2
                 ncf =
                                 1 \text{ id} = 1/2-
Block
>>1
               3 	 ncf =
                                11 \text{ id} = 3/2 +
Block
>>1-11
               4 	 ncf =
                                3 \text{ id} = 3/2
Block
>>1-3
               5 	 ncf =
                                 10 id = 5/2+
Block
>>1-10
Block
               6 	 ncf =
                                 1 \text{ id} = 5/2
>>1
               7
                                5 \text{ id} = 7/2 +
                 ncf =
Block
>>1-5
               8
                   ncf =
                                 2 id = 9/2+
Block
>>1-2
level weights (1 equal; 5 standard; 9 user)
Radial functions
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
Enter orbitals to be varied (Updating order)
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>999
. . . . . . . . . . . . . .
RMCDHF: Execution complete.
*************************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                   name.alog, name.log
**********************************
>>rsave Ni_mr
Created Ni_mr.w, Ni_mr.c, Ni_mr.m, Ni_mr.sum, Ni_mr.alog and Ni_mr.log
*******************************
        RUN RCSFGENERATE TO GENERATE LIST FOR ALL
        STATES OF 3s3p(4), 3s(2)3p(2)3d
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
>>rcsfgenerate
RCSFGENERATE
This program generates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
```

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```
closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 OUTPUT FILES: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
                    1s(2)
       1: He (
                                         = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                         = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                         = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                         = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                         = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,i)3s(1,i)3p(4,i)
 Give configuration 2
>>2s(2,i)2p(6,i)3s(2,i)3p(2,i)3d(1,i)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,9
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 5 blocks were created
      block J/P
                         NCSF
          1 1/2+
                             8
          2 3/2+
                            11
          3 5/2+
                            10
          4 7/2+
                             5
          5 9/2+
********************************
         COPY FILES
         NOTE THAT WE COPY THE FILE TO RCSFMR.INP FOR USE
         TOGETHER WITH RCSFINTERACT
```

>>cp rcsf.out rcsfmr.inp

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```
************************************
         RUN RCSFGENERATE TO GENERATE LIST OBTAINED BY
         SD-EXCITATIONS FROM 3s3p(4) and 3s(2)3p(2)3d TO n = 4
         OUTPUT FILES: rcsfgenerate.log, rcsf.out
************************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 OUTPUT FILES: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
                                           = 36 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,i)3s(1,*)3p(4,*)
 Give configuration 2
>>2s(2,i)2p(6,i)3s(2,*)3p(2,*)3d(1,*)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,9
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
        . . . . . . . . .
  5 blocks were created
      block J/P
                           NCSF
          1 1/2+
                            1664
          2 3/2+
                            2837
```

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	3	5/2+	3271			
	4	7/2+	2972			
	5	9/2+	2264			
******	****	******	*****	******	*********	**
*	COP	Y FILES				
******	****	*******	******	******	**********	**
>>cp rcsf	.out	rcsf.inp				
*****	****	******	******	******	*********	**
*					WHICH OF THE CSFs IN THE	
*		-	T INTERACTS WIT		-	
*			ING CSFs ARE WI		sf.out	
*			rcsfmr.inp, ro	csf.inp		
*			rcsf.out		*********	
*****	****	******	*****	******	*********	**
>>rcsfint	erac	t				
RCSFinte	ract	: Determi	nes all the CS	Fs (rcsf.inp)	) that interact	
				=	ce (rcsfmr.inp)	
		(C) Cc	pyright by G.	Gaigalas and	Ch. F. Fischer	
		(Fortra	n 95 version)		NIST (2017).	
		Input f	iles: rcsfmr.i	np, rcsf.inp		
		Output	file: rcsf.out			
			rac-Coulomb (1	•		
	ulom	b-Breit (	2) Hamiltonian	?		
>>2						
••••						
There ar	e 16	relativi	stic subshells	:		
Block				ter NCSF		
1		8	1664	1047		
2		11	2837	1862		
3		10	3271	2112		
4		5	2972	1537		
5		2	2264	801		
RCSFINTE	RACT	: Executi	on complete			
******	****	******	******	******	*********	**
*		Y FILES				
*****	****	******	******	*******	*********	**
>>cp rcsf	.out	rcsf.inp				
*****	****	******	******	******	*********	**
*			TO GENERATE EI	NERGY EXPRESS	SION	
*			: rcsf.inp			
*	UUT	PUT FILES	: rangular.log	, mcp.30, mcp	0.31,	

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```
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
             rangular.log
 Full interaction? (y/n)
>>y
  . . . . .
 RANGULAR: Execution complete.
*************************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp, rwfnestimate.log
*********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                      16 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
>>Ni_mr.w
 Enter the list of relativistic subshells:
>>*
 The following subshell radial wavefunctions remain to be estimated:
 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
```

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### 4--Screened Hydrogenic [custom Z]

>>2
 Enter the list of relativistic subshells:

>>*						
Shell	е	p0	gamma	<r></r>	MTP	SRC
1s	0.3241D+03	0.3333D+03	0.1000D+01	0.5375D-01	357	Ni_
2s	0.5342D+02	0.1015D+03	0.1000D+01	0.2420D+00	359	Ni_
2p-	0.4833D+02	0.8507D+00	0.1000D+01	0.2110D+00	360	Ni_
2p	0.4768D+02	0.6996D+03	0.2000D+01	0.2140D+00	360	Ni_
3s	0.1702D+02	0.4580D+02	0.1000D+01	0.6442D+00	364	Ni_
3р-	0.1557D+02	0.3893D+00	0.1000D+01	0.6379D+00	364	Ni_
3р	0.1545D+02	0.3217D+03	0.2000D+01	0.6430D+00	364	Ni_
3d-	0.1320D+02	0.3700D+00	0.2000D+01	0.5968D+00	366	Ni_
3d	0.1319D+02	0.3854D+03	0.3000D+01	0.5980D+00	366	Ni_
4s	0.1130D+02	0.3354D+02	0.1000D+01	0.1050D+01	368	T-F
4p-	0.1090D+02	0.3205D+00	0.1000D+01	0.1031D+01	368	T-F
4p	0.1083D+02	0.2655D+03	0.2000D+01	0.1038D+01	368	T-F
4d-	0.1015D+02	0.4907D+00	0.2000D+01	0.9852D+00	369	T-F
4d	0.1013D+02	0.5108D+03	0.3000D+01	0.9872D+00	369	T-F
4f-	0.9324D+01	0.2669D+00	0.3000D+01	0.8778D+00	369	T-F
4f	0.9316D+01	0.3018D+03	0.4000D+01	0.8787D+00	369	T-F

RWFNESTIMATE: Execution complete.

## >>rmcdhf\_mem

# RMCDHF

This program determines the radial orbitals and the expansion coefficients of the CSFs in a self-onsistent field proceedure

Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...

Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log

Default settings? (y/n)

>>y

Loading CSF file ... Header only

There are/is 16 relativistic subshells;

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```
Loading CSF File for ALL blocks
               7359 relativistic CSFs... load complete;
Loading Radial WaveFunction File ...
                  5 blocks (block J/Parity NCF):
There are
 1 1/2+ 1047
                  2 3/2+ 1862
                                   3 5/2+ 2112
                                                4 7/2+ 1537
 5 9/2+ 801
Enter ASF serial numbers for each block
Block
               1 	 ncf =
                              1047 \text{ id} = 1/2+
>>1-8
               2 	 ncf =
                             1862 \text{ id} = 3/2+
Block
>>1-11
               3 ncf =
                               2112 id = 5/2+
Block
>>1-10
               4 	 ncf =
                               1537 \text{ id} = 7/2+
Block
>>1-5
               5 	 ncf =
                               801 \text{ id} = 9/2+
Block
>>1-2
level weights (1 equal; 5 standard; 9 user)
>>5
Radial functions
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
Enter orbitals to be varied (Updating order)
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
. . . . . . . . . . . . . .
RMCDHF: Execution complete.
**********************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                   name.alog, name.log
>>rsave Ni_even_n4
Created Ni_even_n4.w, Ni_even_n4.c, Ni_even_n4.m, Ni_even_n4.sum, Ni_even_n4.alog
 and Ni_even_n4.log
RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
        INPUT FILES : isodata, Ni_even_n4.c, Ni_even_n4.w
        OUTPUT FILES: Ni_even_n4.cm, Ni_even_n4.csum, Ni_even_n4.clog,
        rci.res
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
***********************************
```

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```
>>rci
 RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog, rci.res
 Default settings?
>>y
 Name of state:
>>Ni_even_n4
 Block
                  1 , ncf =
                                     1047
 Block
                  2 , ncf =
                                     1862
 Block
                  3 , ncf =
                                     2112
 Block
                  4 , ncf =
                                     1537
                                      801
 Block
                  5 , ncf =
 Loading CSF file \dots Header only
                        16 relativistic subshells;
 There are/is
 Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
 Include H (Vacuum Polarisation)?
>>y
 Include H (Normal Mass Shift)?
>>n
 Include H (Specific Mass Shift)?
>>n
 Estimate self-energy?
>>y
 Largest n quantum number for including self-energy for orbital
 {\tt n} should be less or equal 8
>>4
 Loading Radial WaveFunction File \dots
 There are
                    5 blocks (block
                                          J/Parity NCF):
  1 1/2+ 1047
                                                              4 7/2+ 1537
                      2 3/2+ 1862
                                          3 5/2+ 2112
  5 9/2+
           801
 Enter ASF serial numbers for each block
 Block
                 1
                    ncf =
                                    1047 \text{ id} = 1/2+
>>1-8
                       ncf =
                                     1862 \text{ id} = 3/2+
 Block
>>1-11
                  3
                       ncf =
                                     2112 id = 5/2+
 Block
>>1-10
 Block
                  4
                       ncf =
                                     1537 id = 7/2+
>>1-5
 Block
                  5
                       ncf =
                                      801 id = 9/2+
>>1-2
```

. . . .

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RCI: Execution complete. \* RUN JJ2LSJ TO GET THE LSJ-COMPOSITION INPUT FILE: Ni\_even\_n4.c, Ni\_even\_n4.cm OUTPUT FILE: Ni\_even\_n4.lsj.lbl, Ni\_even\_n4.uni.lsj.lbl \* >>jj21sj jj2lsj: Transformation of ASFs from a jj-coupled CSF basis into an LSJ-coupled CSF basis (Fortran 95 version) (C) Copyright by G. Gaigalas and Ch. F. Fischer, (2017).Input files: name.c, name.(c)m Output files: name.lsj.lbl (optional) name.lsj.c, name.lsj.j, name.uni.lsj.lbl, name.uni.lsj.sum Name of state >>Ni\_even\_n4 Loading Configuration Symmetry List File ... There are 16 relativistic subshells; There are 7359 relativistic CSFs; ... load complete; Mixing coefficients from a CI calc.? >>y Do you need a unique labeling? (y/n)>>y nelec = 15 7359 ncftot = 16 5 nblock = block ncf nev 2j+1 parity 1 1047 8 2 1 2 1862 11 4 1 3 2112 10 6 1 4 1537 5 8 1 2 10 5 801 1 Default settings? (y/n). . . . . . . . . . . jj2lsj: Execution complete. \* RUN RCSFGENERATE TO GENERATE LIST FOR ALL STATES OF 3s(2)3p(3)OUTPUT FILES: rcsfgenerate.log, rcsf.out

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\* >>rcsfgenerate RCSFGENERATE This program generates a list of CSFs Configurations should be entered in spectroscopic notation with occupation numbers and indications if orbitals are closed (c), inactive (i), active (\*) or has a minimal occupation e.g., 1s(2,1)2s(2,\*)OUTPUT FILES: rcsf.out, rcsfgenerate.log Default, reverse, symmetry or user specified ordering? (\*/r/s/u) >>\* Select core 0: No core 1: He ( 1s(2)= 2 electrons) 2: Ne ([He] + 2s(2)2p(6)= 10 electrons) 3: Ar ([Ne] + 3s(2)3p(6)= 18 electrons) 4: Kr ([Ar] + 3d(10)4s(2)4p(6)= 36 electrons) 5: Xe ([Kr] + 4d(10)5s(2)5p(6)= 54 electrons) 6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons) >>1 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (\*) Give configuration 1 >>2s(2,i)2p(6,i)3s(2,i)3p(3,i)Give configuration 2 Give set of active orbitals, as defined by the highest principal quantum number per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d >>3s,3p Resulting 2\*J-number? lower, higher (J=1 -> 2\*J=2 etc.) Number of excitations (if negative number e.g., -2, correlation orbitals will always be doubly occupied) >>0 Generate more lists ? (y/n) >>n 3 blocks were created block J/P NCSF 1 1/2-1 2 3/2-3 3 5/2-\* COPY FILES NOTE THAT WE COPY THE FILE TO RCSFMR.INP FOR USE

TOGETHER WITH RCSFINTERACT

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```
*********************************
>>cp rcsf.out rcsfmr.inp
***********************************
        RUN RCSFGENERATE TO GENERATE LIST OBTAINED BY
        SD-EXCITATIONS FROM 3s(2)3p(3) TO n = 4
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 OUTPUT FILES: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                   1s(2)
                                        = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                        = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                        = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                        = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                        = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,i)2p(6,i)3s(2,*)3p(3,*)
 Give configuration 2
>>
 Give set of active orbitals, as defined by the highest principal quantum number
per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,5
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>2
 Generate more lists ? (y/n)
>>n
 3 blocks were created
```

block J/P

NCSF

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	1	1/2-	48	31				
		3/2-	80	2				
	3	5/2-	86	8				
******			********	*****	******	******	******	**********
*		FILES						
*****	****	*****	*****	*****	*****	******	******	**********
>>cp rcsf	.out	rcsf.i	np					
******	****	*****	******	*****	******	******	******	********
*	RUN	RCSFIN	TERACT PROGR	AM TO D	ETERMINE	WHICH O	F THE CSF	s IN THE
*	rcsf	.inp L	IST INTERACT	S WITH '	THE CSFs	IN rcsfr	mr.inp	
*			CTING CSFs A			csf.out		
*	INPUT FILES: rcsfmr.inp, rcsf.inp							
*			E: rcsf.out					
*****	****	*****	*****	*****	******	******	******	********
>>rcsfint	eract	;						
RCSFinte	ract:	Deter	mines all th	e CSFs	(rcsf.in	p) that	interact	
			the CSFs in			_		
		(C)	Copyright by	G. Gai	galas an	nd Ch. F.	Fischer	
		(Fort	ran 95 versi	on)		NIST	(2017).	
		_	files: rcsf	_	rcsf.in	ıp		
		Outpu	t file: rcsf	.out				
			Dirac-Coulom		r			
m)	4.0							
There ar Block			vistic subsh Befor NCSF		NCCE			
1 DIOCK	MU	1 1	481	After	237			
2		3	802		577			
3		1	868		480			
RCSFINTE	RACT:	Execu	tion complet	е				
******		***** FILES	*****	*****	*****	******	******	**********
******			******	*****	******	******	******	·***********
>>cp rcsf	.out	rcsf.i	np					
******	****	*****	******	*****	******	******	******	·**********
*	RUN RANGULAR TO GENERATE ENERGY EXPRESSION							
*	INPUT FILE : rcsf.inp							
*	OUTP	UT FIL	ES: rangular	.log, m	cp.30, m	cp.31,		
******	****	*****	******	*****	******	******	******	******

>>rangular

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```
RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
             rangular.log
 Full interaction? (y/n)
>>y
  . . . . .
 RANGULAR: Execution complete.
********************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp, rwfnestimate.log
********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
                       16 relativistic subshells;
 There are/is
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
     2--Thomas-Fermi
    3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
>>Ni_mr.w
Enter the list of relativistic subshells:
>>*
 The following subshell radial wavefunctions remain to be estimated:
 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
    3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
```

>>2

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Enter the list of relativistic subshells:

All required subshell radial wavefunctions have been estimated: Shell **0**q gamma

<r> MTP SRC

1s 0.3241D+03 0.3333D+03 0.1000D+01 0.5375D-01 357 Ni\_ 0.5342D+02 0.1015D+03 0.1000D+01 0.2420D+00 359 Ni 2p- 0.4833D+02 0.8507D+00 0.1000D+01 0.2110D+00 360 Ni\_ 0.4768D+02 0.6996D+03 0.2000D+01 0.2140D+00 360 Ni\_ 2σ 0.1702D+02 0.4580D+02 0.1000D+01 0.6442D+00 364 3s  $\mathtt{Ni}_{-}$ 3p- 0.1557D+02 0.3893D+00 0.1000D+01 0.6379D+00 364  $\mathtt{Ni}_{-}$ 

0.1545D+02 0.3217D+03 0.2000D+01 0.6430D+00 364 Зр Νi 3d- 0.1320D+02 0.3700D+00 0.2000D+01 0.5968D+00 366 Ni\_

3d 0.1319D+02 0.3854D+03 0.3000D+01 0.5980D+00 366  $\mathtt{Ni}_-$ 4s 0.1130D+02 0.3354D+02 0.1000D+01 0.1050D+01 368 T-F

4p- 0.1090D+02 0.3205D+00 0.1000D+01 0.1031D+01 368 T-F

0.1083D+02 0.2655D+03 0.2000D+01 0.1038D+01 368 T-F

4d- 0.1015D+02 0.4907D+00 0.2000D+01 0.9852D+00 369 T-F 4d 0.1013D+02 0.5108D+03 0.3000D+01 0.9872D+00 369 T-F

4f- 0.9324D+01 0.2669D+00 0.3000D+01 0.8778D+00 369 T-F 4f 0.9316D+01 0.3018D+03 0.4000D+01 0.8787D+00 369 T-F

RWFNESTIMATE: Execution complete.

RUN RMCDHF\_MEM TO OBTAIN SELF CONSISTENT SOLUTIONS

INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,... OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log

NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE VARY THE CORRELATION ORBITALS 4s, 4p, 4d, 4f. NONE OF THESE ARE SPECTROSCOPIC. WE CAN USE WILD CARDS \* FOR SPECIFYING ORBITALS

NOTE: INSTEAD OF SAYING THAT WE SHOULD OPTIMIZE ON, FOR EXAMPLE,

THE STATES 1,2,3,4 WE CAN WRITE 1-4 MEANING THE SAME THING

\*

>>rmcdhf\_mem

RMCDHF

This program determines the radial orbitals and the expansion coefficients of the CSFs in a self-onsistent field proceedure

Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ... Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log

Default settings? (y/n)

>>y

Loading CSF file ... Header only

16 relativistic subshells; There are/is

Loading CSF File for ALL blocks

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```
1294 relativistic CSFs... load complete;
 There are
 Loading Radial WaveFunction File ...
 There are
                    3 blocks (block J/Parity
                                                NCF):
 1 1/2-
           237
                    2 3/2- 577
                                       3 5/2-
                                                480
 Enter ASF serial numbers for each block
 Block
               1 	 ncf =
                                  237 \text{ id} = 1/2-
>>1
 Block
                2 ncf =
                                 577 \text{ id} = 3/2
>>1-3
                3 	 ncf =
                                480 \text{ id} = 5/2
Block
>>1
 level weights (1 equal; 5 standard; 9 user)
>>5
Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
>>
 Enter the maximum number of SCF cycles:
>>100
. . . . . . . . . . . . . .
 RMCDHF: Execution complete.
**********************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                      name.log
*************************************
>>rsave Ni_odd_n4
 Created Ni_odd_n4.w, Ni_odd_n4.c, Ni_odd_n4.m, Ni_odd_n4.sum and Ni_odd_n4.log
************************************
         RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
         INPUT FILES : isodata, Ni_odd_n4.c, Ni_odd_n4.w
         OUTPUT FILES: Ni_odd_n4.cm, Ni_odd_n4.csum, Ni_odd_n4.clog, rci.res *
         THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
         LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
         THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
         HIGH N.
>>rci
 RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog, rci.res
 Default settings?
```

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```
>>y
 Name of state:
>>Ni_odd_n
                1 , ncf =
Block
                                  237
                2 , ncf =
 Block
                                  577
 Block
                3 , ncf =
                                  480
 Loading CSF file ... Header only
 There are/is
                   16 relativistic subshells;
 Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
>>n
 Include H (Vacuum Polarisation)?
>>y
 Include H (Normal Mass Shift)?
>>n
 Include H (Specific Mass Shift)?
>>n
 Estimate self-energy?
>>y
 Largest n quantum number for including self-energy for orbital
n should be less or equal 8
Loading Radial WaveFunction File ...
                   3 blocks (block
                                      J/Parity
                                               NCF):
 There are
 1 1/2-
          237
                   2 3/2- 577
                                      3 5/2-
 Enter ASF serial numbers for each block
                                 237 \text{ id} = 1/2
 Block
                1 	 ncf =
>>1
               2 ncf =
                              577 \text{ id} = 3/2 -
Block
>>1-3
                3 ncf =
                                 480 \text{ id} = 5/2
Block
>>1
 RCI: Execution complete.
**********************************
        RUN JJ2LSJ TO GET THE LSJ-COMPOSITION
        INPUT FILE: Ni_odd_n4.c, Ni_odd_n4.cm
        OUTPUT FILE: Ni_odd_n4.lsj.lbl, Ni_odd_n4.uni.lsj.lbl
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2017).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
```

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```
(optional) name.lsj.c, name.lsj.j,
            name.uni.lsj.lbl, name.uni.lsj.sum
Name of state
>>Ni_odd_n4
Loading Configuration Symmetry List File ...
There are 16 relativistic subshells;
There are 1294 relativistic CSFs;
 ... load complete;
Mixing coefficients from a CI calc.?
>>y
Do you need a unique labeling? (y/n)
>>y
   nelec =
                  15
   ncftot =
                1294
                 16
   nblock =
  block
         ncf nev
                       2j+1 parity
         237
                 1
                            -1
     1
                       2
                        4
                               -1
     2
         577
                   3
                        6
                              -1
     3
          480
                 1
Default settings? (y/n)
>>y
     . . . . . . . . . . .
jj2lsj: Execution complete.
************************************
        RUN RLEVELS TO VIEW ENERGIES AND ENERGY SEPARATIONS.
        IF DESIRED WE CAN INSTEAD RUN RLEVELSEV TO GET THE SEPARATION IN EV *
************************************
>> rlevels Ni_even_n4.cm Ni_odd_n4.cm
             5 ncftot =
                            7359 nw =
                                              16 nelec =
                                                              15
nblock =
nblock =
            3 ncftot =
                             1294 nw =
                                               16 nelec =
Energy levels for ...
Rydberg constant is 109737.31534
Splitting is the energy difference with the lower neighbor
No Pos J Parity Energy Total Levels Splitting Configuration
                 (a.u.)
                           (cm^-1)
                                     (cm^-1)
______
 1 1 3/2 - -1443.2224318
                           0.00
                                    0.00 2s(2).2p(6).3s(2).3p(3)4S3_4S
 2 2 3/2 - -1443.0055953 47590.12 47590.12 2s(2).2p(6).3s(2).3p(3)2D3_2D
            -1442.9699841 55405.86
 3 1 5/2 -
                                    7815.74 2s(2).2p(6).3s(2).3p(3)2D3_2D
 4 1 1/2 - -1442.8231291 87636.81 32230.95 2s(2).2p(6).3s(2).3p(3)2P1_2P
 5 3 3/2 - -1442.7718374 98894.05 11257.25 2s(2).2p(6).3s(2).3p(3)2P1_2P
 6 1 5/2 + -1441.7819193 316155.95 217261.89 2s(2).2p(6).3s_2S.3p(4)3P2_4P
```

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```
1 \ 3/2 +
 7
               -1441.7163490
                               330546.97
                                            14391.02 2s(2).2p(6).3s_2S.3p(4)3P2_4P
 8
   1 1/2 +
               -1441.6893375
                               336475.31
                                             5928.35
                                                     2s(2).2p(6).3s_2S.3p(4)3P2_4P
 9
   2 3/2 +
               -1441.4303647
                               393313.26
                                            56837.95
                                                     2s(2).2p(6).3s_2S.3p(4)1D2_2D
   2 5/2 +
10
               -1441.4141226
                               396877.99
                                            3564.73
                                                     2s(2).2p(6).3s_2S.3p(4)1D2_2D
    3 \ 3/2 +
               -1441.1719219
                               450034.90
                                            53156.91 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
11
   2 1/2 +
                                                     2s(2).2p(6).3s_2S.3p(4)1S0_2S
12
               -1441.1457552
                               455777.83
                                            5742.94
13
    3 1/2 +
               -1441.0407477
                               478824.32
                                            23046.48
                                                     2s(2).2p(6).3s_2S.3p(4)1S0_2S
   4 \ 3/2 +
               -1441.0025311
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
14
                               487211.88
                                            8387.57
15
    3 5/2 +
               -1440.9762734
                               492974.79
                                            5762.91 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
16
   1 7/2 +
               -1440.9366782
                               501664.93
                                            8690.14 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
   4 5/2 +
17
               -1440.9108204
                               507340.07
                                             5675.14 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2F
   1 9/2 +
18
               -1440.8905994
                               511778.07
                                            4437.99
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
    2
      7/2 +
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
19
               -1440.8808794
                               513911.35
                                             2133.28
20
   4 1/2 +
               -1440.8807431
                               513941.26
                                              29.92
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
21
   5 \ 3/2 +
               -1440.8744144
                               515330.27
                                             1389.01
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
   5 5/2 +
22
                                             6632.21
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
               -1440.8441958
                              521962.48
23
   37/2 +
               -1440.7788694
                               536299.97
                                            14337.49
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
24
   4 7/2 +
               -1440.6229634
                               570517.38
                                            34217.41 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2G
25 2 9/2 +
               -1440.5997042
                               575622.19
                                            5104.81 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2G
26
   6 3/2 +
               -1440.5811088
                               579703.40
                                             4081.21 2s(2).2p(6).3s_2S.3p(4)3P2_2P
    6 5/2 +
27
               -1440.5348164
                               589863.39
                                            10159.99
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
28
   7 \ 3/2 +
               -1440.5083509
                               595671.90
                                             5808.50
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
29
   5 1/2 +
               -1440.5059857
                               596191.02
                                             519.12 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
30
   6 1/2 +
               -1440.4811030
                               601652.14
                                             5461.12 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
   8 3/2 +
                                                     2s(2).2p(6).3s(2).3p(2)1S0_1S.3d_2D
31
               -1440.4611905
                               606022.41
                                            4370.27
32
   7
      5/2 +
               -1440.3895597
                               621743.56
                                           15721.15 2s(2).2p(6).3s(2).3p(2)1S0_1S.3d_2D
33 9 3/2 +
                                            18756.70 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2D
               -1440.3040979
                               640500.26
   8 5/2 +
34
               -1440.2991882
                               641577.81
                                            1077.56 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2D
35
   7 1/2 +
               -1440.2296306
                                            15266.12 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
                               656843.93
   9 5/2 +
                                            7470.39 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2F
36
               -1440.1955930
                               664314.33
37 10 3/2 +
                                            4769.29 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
              -1440.1738626
                               669083.62
   8 1/2 +
                                             2969.61 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
              -1440.1603320
                               672053.23
  5 7/2 +
                                             274.77
                                                     2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2F
39
              -1440.1590801
                               672328.00
40 10 5/2 +
               -1440.0334583
                               699898.79
                                            27570.79 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2D
41 11 3/2 +
              -1440.0284354
                              701001.19
                                            1102.41 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2D
```

The rlevels program reads the name.cm files along with the name.lsj.lbl files, which display the *LSJ* composition as generated by jj2lsj. From the energy spectra we see that some even levels have the same identification. These pairs of levels are 12 and 13; 19 and 23; 29 and 30. If a unique label was required, jj2lsj also outputs files name.uni.lsj.lb in which unique labels are determined according to the prescription given in [26,40,41]. To display the energies with unique labels, we should copy name.cm to name.uni.cm and rerun rlevels with name.uni.cm as the input file.

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# >> rlevels Ni\_even\_n4.uni.cm Ni\_odd\_n4.cm

nblock = 5 ncftot = 7359 nw = 16 nelec = 15 nblock = 3 ncftot = 1294 nw = 16 nelec = 15

Energy levels for  $\dots$ 

Rydberg constant is 109737.31534

Splitting is the energy difference with the lower neighbor

No	Pos	J P	arity	Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	Configuration
 1	 1	3/2	 1	 1443.2223116	0.00	0.00	2s(2).2p(6).3s(2).3p(3)4S3_4S
2	2	3/2		1443.0054751	47590.12	47590.12	2s(2).2p(6).3s(2).3p(3)2D3_2D
3	1	5/2		1442.9698639	55405.86	7815.74	2s(2).2p(6).3s(2).3p(3)2D3_2D
4	1	1/2		1442.8230089	87636.81	32230.95	2s(2).2p(6).3s(2).3p(3)2P1_2P
5	3	3/2	1	1442.7717172	98894.05	11257.24	2s(2).2p(6).3s(2).3p(3)2P1_2P
6	1	5/2		1441.7818000	316155.75	217261.70	2s(2).2p(6).3s_2S.3p(4)3P2_4P
7	1	3/2	+ -1	1441.7162297	330546.77	14391.02	2s(2).2p(6).3s_2S.3p(4)3P2_4P
8	1	1/2	+ -1	1441.6892182	336475.11	5928.35	2s(2).2p(6).3s_2S.3p(4)3P2_4P
9	2	3/2	+ -1	1441.4302453	393313.09	56837.98	2s(2).2p(6).3s_2S.3p(4)1D2_2D
10	2	5/2	+ -1	1441.4140033	396877.81	3564.72	2s(2).2p(6).3s_2S.3p(4)1D2_2D
11	3	3/2	+ -1	1441.1718022	450034.79	53156.98	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
12	2	1/2	+ -1	1441.1456357	455777.69	5742.90	2s(2).2p(6).3s_2S.3p(4)3P2_2P
13	3	1/2	+ -1	1441.0406282	478824.17	23046.48	2s(2).2p(6).3s_2S.3p(4)1S0_2S
14	4	3/2	+ -1	1441.0024109	487211.89	8387.72	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
15	3	5/2	+ -1	1440.9761532	492974.79	5762.91	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
16	1	7/2	+ -1	1440.9365580	501664.94	8690.14	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
17	4	5/2	+ -1	1440.9107001	507340.08	5675.14	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2F
18	1	9/2	+ -1	1440.8904792	511778.07	4437.99	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
19	2	7/2	+ -1	1440.8807592	513911.35	2133.28	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2F
20	4	1/2	+ -1	1440.8806230	513941.26	29.91	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
21	5	3/2	+ -1	1440.8742942	515330.26	1389.01	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
22	5	5/2	+ -1	1440.8440756	521962.48	6632.21	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
23	3	7/2	+ -1	1440.7787492	536299.97	14337.49	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
24	4	7/2	+ -1	1440.6228432	570517.38	34217.42	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_20
25	2	9/2	+ -1	1440.5995839	575622.19	5104.81	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_20
26	6	3/2	+ -1	1440.5809891	579703.29	4081.10	2s(2).2p(6).3s_2S.3p(4)3P2_2P
27	6	5/2	+ -1	1440.5346963	589863.37	10160.08	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
28	7	3/2		1440.5082308	595671.88	5808.50	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
29	5	1/2	+ -1	1440.5058658	596190.95	519.07	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
30	6	1/2	+ -1	1440.4809830	601652.08	5461.13	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
31	8	3/2		1440.4610704	606022.40	4370.32	2s(2).2p(6).3s(2).3p(2)1S0_1S.3d_2D
32	7	5/2	+ -1	1440.3894396	621743.54	15721.14	2s(2).2p(6).3s(2).3p(2)1S0_1S.3d_2D
33	9	3/2		1440.3039779	640500.22	18756.68	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2D
34	8	5/2	+ -1	1440.2990681	641577.79	1077.57	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2D
35	7	1/2		1440.2295105	656843.92	15266.12	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
36	9	5/2		1440.1954728	664314.33	7470.41	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2F
37		3/2		1440.1737424	669083.60	4769.27	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
38	8	1/2		1440.1602120	672053.18	2969.58	2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
39	5	7/2		1440.1589598	672328.00	274.82	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2F
	10	5/2		1440.0333381	699898.78	27570.78	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2D
41	11	3/2	+ -1	1440.0283152	701001.19	1102.41	2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2D

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Comment: we now see that all states have different labels that allow for unambiguous identifications.

Now we can use these files with unique labels in further calculations, e.g., transition properties.

# >>rbiotransform

#### RBIOTRANSFORM

```
This program transforms the initial and final wave
 functions so that standard tensor albegra can be
 used in evaluation of the transition parameters
 Input files: isodata, name1.c, name1.w, name1.(c)m
               name2.c, name2.w, name2.(c)m
               name1.TB, name2.TB (optional angular files)
 Output files: name1.bw, name1.(c)bm,
               name2.bw, name2.(c)bm
               name1.TB, name2.TB (angular files)
 Default settings?
>>y
 Input from a CI calculation?
>>y
  Name of the Initial state
>>Ni_odd_n4
  Name of the Final state
>>Ni_even_n4.uni
  Transformation of all J symmetries?
>>y
```

BIOTRANSFORM: Execution complete.

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```
************************************
         RUN RTRANSITION FOR Ni_odd_n4 AND Ni_even_n4.uni
         TO COMPUTE TRANSITION PARAMETERS
         INPUT FILES: isodata, Ni_odd_n4.c, Ni_odd_n4.bw, Ni_odd_n4.cbm,
                      Ni_even_n4.uni.c, Ni_even_n4.uni.bw, Ni_even_n4.uni.cbm*
         OUTPUT FILES: Ni_odd_n4.Ni_even_n4.uni.ct
                      Ni_odd_n4.Ni_even_n4.uni.-1T (angular files)
*******************************
>>rtransition
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
              name2.c, name2.bw, name2.(c)bm
              optional, name1.lsj.lbl, name2.lsj.lbl
              name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
              optional, name1.name2.(c)t.lsj
              name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
 Default settings?
>>y
 Input from a CI calculation?
 Name of the Initial state
>>Ni_odd_n4
 Name of the Final state
>>Ni_even_n4.uni
 MRGCSL: Execution begins ...
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
 There are 1294 relativistic CSFs;
  ... load complete;
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
 There are 7359 relativistic CSFs;
  ... load complete;
          1 s
          2 s
          2 p-
          2 p
          3 s
          3 p-
          3 p
          3 d-
          3 d
          4 s
          4 p-
          4 p
```

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```
4 d-
           4 d
           4 f-
           4 f
           3
                                 1294
         237
                      814
           5
        1047
                     2909
                                                          7359
                                 5021
                                              6558
Loading Configuration Symmetry List File ...
  there are 16 relativistic subshells;
  there are 8653 relativistic CSFs;
  ... load complete;
Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
>>E1
   . . . . .
```

RTRANSITION: Execution complete.

Transition data are in Ni\_odd\_n4.Ni\_even\_n4.uni.ct.lsj file in which all levels have the unique labels.

An alternative way to get unique labels than the one described above is to denote the states by the *LS* composition. This can be done with the PERL script lscomp.pl and it is described in more detail in Section 7.10.

## 6.6. Sixth Example: The Study of Energy Spectra for Ni XIV, Extended MR Using rcsfmr

To obtain good transition energies, it is often necessary to extend the MR. This is facilitated by the program rcsfmr. The rcsfmr program reads the name.lsj.lbl file produced by jj2lsj and extracts the configurations that give rise to LSJ-coupled CSFs with weights exceeding a user defined cut-off. Below is part of the Ni\_even\_n4.lsj.lbl file from the fifth example.

```
J
Pos
          Parity
                      Energy Total
                                         Comp. of ASF
            +
                      -1441.689593921
                                            99.941%
 1 1/2
       -0.92754342
                      0.86033679
                                    2s(2).2p(6).3s_2S.3p(4)3P2_4P
                      0.10013822
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
       -0.31644623
       -0.13107223
                      0.01717993
                                    2s(2).2p(6).3s_2S.3p(4)1S0_2S
       -0.06808224
                      0.00463519
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)1S0_4P
       -0.06306024
                      0.00397659
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)1D2_4P
       -0.06139607
                      0.00376948
                                    2s(2).2p(6).3p(4)3P2_3P.3d_4P
       -0.04384478
                      0.00192236
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3P2_4P
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
        0.04315453
                      0.00186231
        0.04160917
                      0.00173132
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_4P
   1/2
                      -1441.146026942
                                            99.870%
        0.55236001
                      0.30510158
                                    2s(2).2p(6).3s_2S.3p(4)1S0_2S
        0.54901778
                      0.30142053
                                    2s(2).2p(6).3s_2S.3p(4)3P2_2P
       -0.51850029
                      0.26884256
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
       -0.25241177
                      0.06371170
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
        0.14974129
                      0.02242245
        0.08843416
                      0.00782060
                                    2s(2).2p(6).3p(4)1D2_1D.3d_2P
       -0.07913818
                      0.00626285
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
        0.06957348
                      0.00484047
                                    2s(2).2p(6).3s_2S.3p(2)1S0_2S.3d(2)1S0_2S
       -0.06792804
                      0.00461422
                                    2s(2).2p(6).3s_2S.3p(4)3P2_4P
       -0.04635416
                      0.00214871
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2P
       -0.04439733
                      0.00197112
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2P
        0.03795472
                      0.00144056
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3P2_2P
```

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```
2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2S
      -0.03450153
                     0.00119036
      -0.03371402
                     0.00113663
                                  2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
      -0.03274764
                     0.00107241
                                  2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2S
      0.03171981
                     0.00100615
                                  2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3F2_2P
3
  1/2
           +
                     -1441.041027919
                                          99.883%
      0.69856599
                                  2s(2).2p(6).3s_2S.3p(4)1S0_2S
                     0.48799445
      0.44943909
                     0.20199550
                                  2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
                                  2s(2).2p(6).3s_2S.3p(4)3P2_2P
      -0.37641525
                     0.14168844
      -0.31029154
                     0.09628084
                                  2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
      -0.14516094
                     0.02107170
                                  2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
      0.11017096
                     0.01213764
                                  2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
      -0.10592606
                     0.01122033
                                  2s(2).2p(6).3s_2S.3p(4)3P2_4P
                                  2s(2).2p(6).3s_2S.3p(2)1S0_2S.3d(2)1S0_2S
      0.08894930
                     0.00791198
      -0.06646514
                     0.00441762
                                  2s(2).2p(6).3p(4)1D2_1D.3d_2P
      -0.04537257
                     0.00205867
                                  2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
      -0.04336944
                     0.00188091
                                  2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2S
      -0.04274245
                     0.00182692
                                  2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2S
                                  2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2P
      0.04115897
                     0.00169406
      0.03553871
                     0.00126300
                                  2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2P
      1 9/2
          +
                     -1440.890865311
                                          99.130%
      0.96823013
                     0.93746959
                                  2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
                                  2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2G
                     0.03576556
      -0.18911784
      0.09063449
                     0.00821461
                                  2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3F2_4F
                    0.00196711
                                  2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)1G2_4F
      0.04435216
      -0.04085208
                     0.00166889
                                  2s(2).2p(6).3s_2S.3p(2)3P2_2P.3d(2)3F2_4F
      -0.03263660
                     0.00106515
                                  2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)1D2_4F
  9/2
                     -1440.599976411
                                          99.058%
                                  2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2G
      0.96757720
                     0.93620564
                                  2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
      0.18856813
                     0.03555794
      -0.08465547
                     0.00716655
                                  2s(2).2p(6).3s_2S.3p(2)3P2_2P.3d(2)1G2_2G
      -0.06715953
                     0.00451040
                                  2s(2).2p(6).3s_2S.3p(2)3P2_2P.3d(2)3F2_2G
      -0.04376060
                     0.00191499
                                  2s(2).2p(6).3s(2).3d(3)2G3_2G
      -0.04015861
                     0.00161271
                                  2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2G
```

We see that the states are strongly mixed and that it is desirable to extend the MR. The size of the extended MR is a compromise between available computational resources and the desired accuracy of computed properties. Often an exploratory approach is needed. In this example, we will somewhat ad hoc determine an MR from the *LSJ*-coupled CSFs with weights larger than 0.03.

# Overview

- 1. Run rcsfmr for Ni\_even\_n4.lsj.lbl with a cut-off 0.03.
- 2. Use the output from rcsfmr as an input to rcsfgenerate with no excitations. Copy to rcsfmr.inp
- 3. Use the output from rcsfmr as an input to rcsfgenerate and allow SD excitations from the extended MR. Copy to rcsf.inp
- 4. Run rcsfinteract

#### **Program Input**

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```
RCSFMR
This program reads the name.lsj.lbl file and extracts a
set of MR configurations that give rise to LSJ coupled
CSFs with absolute weights larger than a specified cut-off
 Input file: namel.lsj.lbl
Ouput is written to screen
Name of state
>>Ni_even_n4
Give cut-off for weight
>>0.03
Configurations in the MR
2s(2,*)2p(6,*)3s(1,*)3p(4,*)
2s(2,*)2p(6,*)3s(2,*)3p(2,*)3d(1,*)
2s(2,*)2p(6,*)3s(1,*)3p(2,*)3d(2,*)
2s(2,*)2p(6,*)3p(4,*)3d(1,*)
2s(2,*)2p(6,*)3s(2,*)3d(3,*)
2s(2,*)2p(6,*)3s(2,*)3p(1,*)3d(1,*)4f(1,*)
2s(2,*)2p(6,*)3s(1,*)3p(3,*)4f(1,*)
************************************
         RUN RCSFGENERATE USING THE OBTAINED CONFIGURATIONS FROM RCSFMR
         BY REQUESTING ZERO EXCITATIONS WE WILL GET THE CSFs OF THE MR
**********************************
>>rcsfgenerate
RCSFGENERATE
This program generates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (*) or has a minimal
occupation e.g., 1s(2,1)2s(2,*)
Outputfiles: rcsf.out, rcsfgenerate.log
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
Select core
       0: No core
                                           = 2 electrons)
       1: He (
                     1s(2)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                           = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                          = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
```

Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (\*)

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```
Give configuration
>>2s(2,i)2p(6,i)3s(1,*)3p(4,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,*)3p(2,*)3d(1,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(1,*)3p(2,*)3d(2,*)
 Give configuration
>>2s(2,i)2p(6,i)3p(4,*)3d(1,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,*)3d(3,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,*)3p(1,*)3d(1,*)4f(1,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(1,*)3p(3,*)4f(1,*)
 Give configuration
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
. . . . . . . . . . . . . . . . . .
 Group CSFs into symmetry blocks
 5 blocks were created
      block J/P
                          NCSF
         1 1/2+
                           61
         2 3/2+
                           104
          3 5/2+
                           116
          4 7/2+
                           96
          5 9/2+
                           67
**********************************
        COPY FILES
```

```
>>cp rcsf.out rcsfmr.inp
```

It is very important to realize that the orbital order in rcsfmr.inp needs to be the same as in the larger list rcsf.inp to be reduced. For this reason, we need a user defined orbital ordering that starts with the orbitals in the MR and then adds the correlation orbitals. The clist.ref file is, thus

1s 2s

2p

3s

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```
Зр
3d
4f
4s
4p
4d
We are now in the position to run rcsfgenerate.
*******************************
         RUN RCSFGENERATE USING THE OBTAINED CONFIGURATIONS FROM RCSFMR
         REQUEST TWO EXCITATIONS. USER DEFINED ORBITAL ORDERING
*************************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>u
 Select core
       0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                          = 36 electrons)
       5: Xe([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
                            1
>>2s(2,i)2p(6,i)3s(1,*)3p(4,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,*)3p(2,*)3d(1,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(1,*)3p(2,*)3d(2,*)
 Give configuration
>>2s(2,i)2p(6,i)3p(4,*)3d(1,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,*)3d(3,*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,*)3p(1,*)3d(1,*)4f(1,*)
 Give configuration
                            7
>>2s(2,i)2p(6,i)3s(1,*)3p(3,*)4f(1,*)
```

Give configuration

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```
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>1,9
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
. . . . . . . . . . . . . . . . . . .
 Group CSFs into symmetry blocks
 5 blocks were created
      block J/P
                         NCSF
         1 1/2+
                         5061
         2 3/2+
                         8907
          3 5/2+
                         10810
          4 7/2+
                        10604
                         8889
         5 9/2+
***********************************
        COPY FILES
********************************
>>cp rcsf.out rcsf.inp
**************************************
        RUN RCSFINTERACT
****************************
 RCSFinteract: Determines all the CSFs (rcsf.inp) that interact
             with the CSFs in the multireference (rcsfmr.inp)
             (C) Copyright by G. Gaigalas and Ch. F. Fischer
             (Fortran 95 version)
                                            NIST (2017).
             Input files: rcsfmr.inp, rcsf.inp
             Output file: rcsf.out
 Reduction based on Dirac-Coulomb (1) or
 Dirac-Coulomb-Breit (2) Hamiltonian?
>>2
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
         MR NCSF
  Block
                  Before NCSF
                              After NCSF
   1
             61
                       5061
                                   3551
   2
             104
                       8907
                                   6489
   3
             116
                       10810
                                   7824
   4
              96
                       10604
                                   7398
   5
              67
                       8889
                                   5936
```

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```
Wall time:
5 seconds

Finish Date and Time:
Date (Yr/Mon/Day): 2018/05/17
Time (Hr/Min/Sec): 00/54/36.490
Zone: +0200

RCSFinteract: Execution complete.
```

The same procedure can be applied to Ni\_even\_n4.lsj.lbl.

#### 6.7. Seventh Example: Restarting rci

Follow the fifth example up to the rci calculation for Ni\_even\_n4. During the rci calculation the Hamiltonian matrix elements, in sparse representation, are successively written to the file rci.res. If the calculation stalls at some point, the rci program can be restarted. During a restart, all radial integrals are recomputed, and then the computation starts with computing the matrix elements following the last matrix element that was saved to rci.res. In this example, we assume that the rci calculation for Ni\_even\_n4 stalled in the middle of block 3, and we show how to make a restart.

#### Overview

- 1. Run rci for Ni\_even\_n4 (run assumed to stall in the middle of block 3)
- 2. Use the restart file rci.res to restart the rci run.

#### **Program Input**

```
RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
         INPUT FILES : isodata, Ni_even_n4.c, Ni_even_n4.w, rci.res
         OUTPUT FILES: Ni_even_n4.cm, Ni_even_n4.csum, Ni_even_n4.clog,
         rci.res
         This is a restart that reads the rci.res file
***********************************
>>rci
 RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog
             rci.res (can be used for restart)
 Default settings?
>>n
 Name of state:
>>Ni_even_n4
 Block
                1 , ncf =
                                  1047
 Block
                2 , ncf =
                                  1862
 Block
                3 , ncf =
                                  2112
                4 , ncf =
                                   1537
 Block
 Block
                5 , ncf =
                                   801
 Loading CSF file ... Header only
 There are/is
                    16 relativistic subshells;
 Restarting RCI90 ?
>>y
 Calling lodres ...
 Estimate contributions from the self-energy?
>>y
```

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```
J/Parity NCF):
 There are
                      5 blocks (block
                        2 3/2+
  1 1/2+
             1047
                                    1862
                                               3 5/2+
                                                          2112
                                                                     4 7/2+
                                                                                 1537
  5 9/2+
              801
 Enter ASF serial numbers for each block
 Block
                  1
                       ncf =
                                      1047 \text{ id} = 1/2+
>>1-8
                  2
                       ncf =
                                      1862 \quad id = 3/2 +
 Block
>>1-11
                  3
                       ncf =
                                      2112 id = 5/2+
 Block
>>1-10
                  4
                       ncf =
                                      1537 \text{ id} = 7/2+
 Block
>>1-5
                  5
 Block
                       ncf =
                                       801 \text{ id} = 9/2+
>>1-2
 Calling STRSUM...
 Calling FACTT...
 Calling GENINTRK...
 Allocating space for
                              6071 Rk integrals
 Calling GENINTBREIT1...
 Computing
                 53106 Breit integrals of type 1
 Calling GENINTBREIT2...
 Computing
                 26494 Breit integrals of type 2
 Calling MATRIX...
 Loading CSF File for block
 There are
                   1047 relativistic CSFs... load complete;
 Entering QED ...
                             1047 ) rows read from .res
        1047 (total
 LAPACK routine DSPEVX selected for eigenvalue problem.
 RCI92 MIXing coefficients File generated.
 Loading CSF File for block
                   1862 relativistic CSFs... load complete;
 There are
 Entering QED ...
        1862 (total
                              1862 ) rows read from .res
 LAPACK routine DSPEVX selected for eigenvalue problem.
 RCI92 MIXing coefficients File generated.
 Loading CSF File for block
 There are
                   2112 relativistic CSFs... load complete;
 Entering QED ...
         739 (total
                              2112 ) rows read from .res
 Calling setham ...
                              283 nonzero elements; block =
 Row
              : 008
                                                                          3
 Row
              900:
                              258 nonzero elements; block =
                                                                          3
             1000 :
                             393 nonzero elements; block =
                                                                          3
 Row
                                                                          3
 Row
             1100 :
                             224 nonzero elements; block =
             1200 :
                             250 nonzero elements; block =
                                                                          3
 R<sub>ow</sub>
             1300 :
                                                                          3
 Row
                              104 nonzero elements; block =
                             297 nonzero elements; block =
                                                                          3
 Row
             1400 :
 Row
             1500 :
                             375 nonzero elements; block =
                                                                          3
                             224 nonzero elements; block =
                                                                          3
             1600 :
 Row
 Row
             1700 :
                             106 nonzero elements; block =
                                                                          3
             1800 :
                             141 nonzero elements; block =
                                                                          3
 R.ow
             1900 :
                             128 nonzero elements; block =
                                                                          3
 Row
                             208 nonzero elements; block =
 Row
             2000:
                                                                          3
```

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```
2100 :
                                                                       3
R.ow
                            111 nonzero elements; block =
Row
            2111 :
                             74 nonzero elements; block =
                                                                       3
Row
            2112 :
                            113 nonzero elements; block =
                                                                       3
                        436242
nelmnt =
 Sparse - Memory, iniest2
RCI92 MIXing coefficients File generated.
Loading CSF File for block
There are
                  1537 relativistic CSFs... load complete;
Entering QED ...
          0
            (total
                            1537 ) rows read from .res
Calling setham ...
               1:
                              1 nonzero elements; block =
R.ow
                                                                       4
             100:
                                                                       4
Row
                             67
                                 nonzero elements;
                                                    block =
             200:
Row
                             62 nonzero elements; block =
                                                                       4
Row
             300:
                            100 nonzero elements; block =
                                                                       4
             400 :
                            83 nonzero elements; block =
                                                                       4
R.ow
Row
             500:
                            139 nonzero elements; block =
                                                                       4
             600:
                            223 nonzero elements; block =
                                                                       4
ROW
                                                                       4
Row
            700 :
                            234 nonzero elements; block =
R.ow
            800:
                            321 nonzero elements; block =
                                                                       4
                                nonzero elements; block =
Row
            900 :
                            341
                                                                       4
            1000 :
                            289 nonzero elements; block =
                                                                       4
Row
                                                                       4
Row
            1100 :
                            337 nonzero elements; block =
Row
            1200 :
                           311 nonzero elements: block =
                                                                       4
                                                                       4
            1300 :
                            192 nonzero elements; block =
Row
Row
            1400 :
                            191 nonzero elements; block =
                                                                       4
            1500 :
                            282 nonzero elements; block =
                                                                       4
R.ow
R.ow
            1536 :
                            150 nonzero elements; block =
Row
            1537 :
                            176 nonzero elements; block =
                                                                       4
LAPACK routine DSPEVX selected for eigenvalue problem.
RCI92 MIXing coefficients File generated.
Loading CSF File for block
                   801 relativistic CSFs... load complete;
There are
Entering QED ...
          0 (total
                             801 ) rows read from .res
Calling setham ...
               1:
                             1 nonzero elements; block =
Row
                                                                       5
Row
             100:
                             13
                                 nonzero elements;
                                                    block =
                                                                       5
             200:
                             81 nonzero elements; block =
                                                                       5
R.ow
Row
             300:
                             48 nonzero elements; block =
                                                                       5
             400:
                            183 nonzero elements; block =
                                                                       5
Row
                                                                       5
Row
             500:
                            207 nonzero elements; block =
                                                                       5
Row
             600:
                            118 nonzero elements; block =
                                                                       5
             700:
                            162 nonzero elements; block =
Row
Row
             : 008
                            163 nonzero elements; block =
                                                                       5
             801:
                            184 nonzero elements;
                                                                       5
Row
                                                    block =
LAPACK routine DSPEVX selected for eigenvalue problem.
RCI92 MIXing coefficients File generated.
```

Finish time, Statistics

Wall time:

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#### 54 seconds

Finish Date and Time:

Date (Yr/Mon/Day): 2018/07/20 Time (Hr/Min/Sec): 23/40/40.589

Zone: +0200

RCI: Execution complete.

During the restart, all matrix elements for blocks 1 and 2 were read from rci.res. For block 3 matrix elements up to row 739 were read and the restarted computation carries on from this point. Transforming from jj to LSJ coupling and displaying energies with rlevels shows that all the energies from the restarted rci calculation are identical to the ones in the fifth example. The restart option works the same for rci\_mpi with the difference that the rci.res are read from the files defined in the disks file.

6.8. Eighth Example:  $2s^2$  <sup>1</sup>S in Be I, Transforming to Natural Orbitals, Using Option 4 in rwfnestimate

The eighth example is for  $1s^22s^2$   $^1S_0$  in Be I. The example shows the computation of rmcdhf and rci wave functions, and the subsequent transformation to natural orbitals TP Section 3.4. The rci calculation is redone in the natural orbitals basis, and we see how the expansion coefficients are concentrated to relatively fewer CSFs, potentially leading to smaller MR sets. In addition, we plot the radial density function D(r), see TP Section 3.4.

#### Overview

- 1. Define nuclear data.
- 2. Obtain spectroscopic orbitals for the MR set.
  - (a) Generate configuration state list containing three CSFs generated from the  $1s^22s^2$ ,  $1s^22p^2$  configurations.
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation
  - (e) Save output to DF.
- 3. Improve the wave function
  - (a) Generate n = 3 valence correlation expansion.
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation.
  - (e) Save output to n3.
  - (f) Generate n = 4 valence correlation expansion.
  - (g) Perform angular integration.
  - (h) Generate initial estimates of radial orbitals, use option 4.
  - (i) Perform SCF calculation.
  - (j) Save output to n4.
  - (k) Perform rci calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
  - (l) Display the expansion coefficients for the rci wave function
- 4. Transform to natural orbitals
- 5. Perform rci calculation in the natural orbital basis in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
- 6. Display the expansion coefficients for the rci wave functions in the natural orbital basis.
- 7. Plot the radial density distribution.

### Program Input

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

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```
*********************************
        RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
        OUTPUT FILE: isodata
***********************************
>>rnucleus
 Enter the atomic number:
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
 The default root mean squared radius is
                                      2.5190000534057617
                                                            fm; (Angeli)
  the default nuclear skin thickness is
                                      2.299999999999998
                                                            fm;
 Revise these values?
>>n
Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
Enter the nuclear dipole moment (in nuclear magnetons):
>>1
 Enter the nuclear quadrupole moment (in barns):
>>1
RUN RCSFGENERATE TO GENERATE LIST OF CSFs FOR 1s(2)2s(2) 1S J = 0. *
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 OUTPUT FILES: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
                                       = 2 electrons)
       1: He (
                  1s(2)
       2: Ne ([He] + 2s(2)2p(6)
                                       = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                       = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                      = 36 electrons)
                                       = 54 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
```

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```
Give configuration 1
>>2s(2,i)
 Give configuration 2
>>2p(2,i)
 Give configuration 3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>2s,2p
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 1 blocks were created
                       NCSF
      block J/P
         1 0+
***********************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
***********************************
>>cp rcsfgenerate.log DF.exc
>>cp rcsf.out rcsf.inp
************************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
************************************
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
            rangular.log
 Full interaction? (y/n)
>>y
```

RANGULAR: Execution complete.

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```
RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         * MEANS ALL ORBITALS
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp, rwfnestimate.log
************************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
 Loading CSF file \dots Header only
 There are/is
                       4 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
                     p0
                                           <r> MTP SRC
Shell
                               gamma
  1s 0.4307D+01 0.1434D+02 0.1000D+01 0.4246D+00 332 T-F
     0.4126D+00 0.3424D+01 0.1000D+01 0.2336D+01 357
  2s
                                                        T-F
  2p- 0.2827D+00 0.3699D-03 0.1000D+01 0.2430D+01 361 T-F
     0.2827D+00 0.2310D+01 0.2000D+01 0.2431D+01 361 T-F
 RWFNESTIMATE: Execution complete.
```

\*

Comment: <r> is the mean orbital radius in a.u. MTP is the extension of the orbitals on the grid, for which the upper limit in the default installation is 590 points. SRC is the source of the estimate, in this case T-F (Thomas-Fermi).

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```
* MEANS ALL ORBITALS
***********************************
>>rmcdhf_mem
RMCDHF
This program determines the radial orbitals
and the expansion coefficients of the CSFs
in a self-onsistent field proceedure
Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
>>y
Loading CSF file ... Header only
                     4 relativistic subshells;
There are/is
Loading CSF File for ALL blocks
                  3 relativistic CSFs... load complete;
There are
Loading Radial WaveFunction File ...
There are
                  1 blocks (block J/Parity
                                            NCF):
     0+
             3
Enter ASF serial numbers for each block
          1 ncf = 3 id =
>>1
Radial functions
1s 2s 2p- 2p
Enter orbitals to be varied (Updating order)
>>*
Which of these are spectroscopic orbitals?
>>*
Enter the maximum number of SCF cycles:
>>100
       . . . . . . . . . .
RMCDHF: Execution complete.
*********************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                   name.alog, name.log
********************************
>>rsave DF
Created DF.w, DF.c, DF.m, DF.sum DF.alog and DF.log
*********************************
        RUN RCSFGENERATE TO GENERATE n = 3 VV CORRELATION LIST
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
**************************************
```

>>rcsfgenerate

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```
RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfile: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
       0: No core
       1: He (
                    1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                          = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                          = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,*)
 Give configuration 2
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,0
Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
        . . . . . . . . .
  1 blocks were created
      block J/P
                           NCSF
          1 1/2+
                             11
COPY FILES
         IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
         RECORD ON HOW THE LIST OF CSFs WAS CREATED
>>cp rcsfgenerate.log n3.exc
>>cp rcsf.out rcsf.inp
```

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```
**********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
************************************
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
           rangular.log
Full interaction? (y/n)
>>y
  . . . . . . . . . . .
RANGULAR: Execution complete.
RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp
>>rwfnestimate
RWFNESTIMATE
This program estimates radial wave functions
for orbitals
Input files: isodata, rcsf.inp, optional rwfn file
Output file: rwfn.inp
Default settings ?
Loading CSF file ... Header only
There are/is
                    9 relativistic subshells;
The following subshell radial wavefunctions remain to be estimated:
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>1
Enter the file name (Null then "rwfn.out")
Enter the list of relativistic subshells:
The following subshell radial wavefunctions remain to be estimated:
```

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```
3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                                                     MTP SRC
                      p0
                                gamma
                                            <r>
      0.4712D+01 0.1475D+02 0.1000D+01 0.4128D+00 357
  1s
                                                          rwf
  2s
      0.3498D+00 0.2557D+01 0.1000D+01 0.2582D+01
                                                     360
  2p- 0.4927D+00 0.2396D-03 0.1000D+01 0.2498D+01 358
                                                         rwf
      0.4927D+00 0.1497D+01 0.2000D+01 0.2498D+01
                                                    358
  3s
      0.1104D+00 0.1312D+01 0.1000D+01 0.7134D+01 372
                                                         T-F
  3p- 0.8539D-01 0.1497D-03 0.1000D+01 0.8182D+01 375
      0.8538D-01 0.9348D+00 0.2000D+01 0.8183D+01
  Зр
                                                     375
                                                         T-F
  3d- 0.6409D-01 0.6710D-05 0.2000D+01 0.8778D+01
                                                     378
                                                          T-F
  3d
      0.6409D-01 0.5036D-01 0.3000D+01 0.8778D+01 378 T-F
 RWFNESTIMATE: Execution complete.
```

Comment: please note how we used the wild card \* twice. We start by reading the orbitals from a GRASP file (previous run rwfn.out). Using the wild card \* the program reads as many orbitals as possible, i.e., 1s, 2s, 2p-, 2p. The orbitals 3s, 3p-, 3p-, 3d-, 3d then remain to be estimated, and we use Thomas-Fermi estimates. By again using the wild card \* all the remaining orbitals will be Thomas-Fermi estimates.

```
>>rmcdhf_mem
```

```
RMCDHF
```

```
This program determines the radial orbitals and the expansion coefficients of the CSFs in a self-consistent field proceedure

Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...

Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log

Default settings? (y/n)

>>y

Loading CSF file ... Header only

There are/is 9 relativistic subshells;

Loading CSF File for ALL blocks
```

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```
There are
                  11 relativistic CSFs... load complete;
Loading Radial WaveFunction File ...
                  1 blocks (block J/Parity NCF):
There are
 1 1/2+
         11
Enter ASF serial numbers for each block
                         11 id = 1/2+
              1 	 ncf =
Radial functions
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
Enter orbitals to be varied (Updating order)
>>3*
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
   . . . . . . . . . .
RMCDHF: Execution complete.
********************************
       RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                   name.alog, name.log
**********************************
>>rsave n3
Created n3.w, n3.c, n3.m, n3.sum n3.alog and n3.log
RUN RCSFGENERATE TO GENERATE n = 4 VV CORRELATION LIST
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
**********************************
>>rcsfgenerate
RCSFGENERATE
This program creates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (*) or has a minimal
occupation e.g., 1s(2,1)2s(2,*)
Outputfile: rcsf.out, rcsfgenerate.log
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
Select core
       0: No core
       1: He (
                  1s(2)
                                      = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                     = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                     = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                      = 36 electrons)
```

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```
5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                     = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>2s(2,*)
 Give configuration 2
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,0
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
  1 blocks were created
      block J/P
                          NCSF
          1 1/2+
                            26
********************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
>>cp rcsfgenerate.log n4.exc
>>cp rcsf.out rcsf.inp
**********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
******************************
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
            rangular.log
 Full interaction? (y/n)
>>y
```

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. . . . . . . . . . . RANGULAR: Execution complete. \* RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS \* INPUT FILES: isodata, rcsf.inp, previous rwfn files OUTPUT FILE: rwfn.inp \* >>rwfnestimate RWFNESTIMATE This program estimates radial wave functions for orbitals Input files: isodata, rcsf.inp, optional rwfn file Output file: rwfn.inp Default settings ? >>y Loading CSF file ... Header only There are/is 9 relativistic subshells; The following subshell radial wavefunctions remain to be estimated: 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f Read subshell radial wavefunctions. Choose one below 1--GRASP2K File 2--Thomas-Fermi 3--Screened Hydrogenic 4--Screened Hydrogenic [custom Z] >>1 Enter the file name (Null then "rwfn.out") Enter the list of relativistic subshells: >>\* The following subshell radial wavefunctions remain to be estimated: 4s 4p- 4p 4d- 4d 4f- 4f Read subshell radial wavefunctions. Choose one below 1--GRASP92 File 2--Thomas-Fermi 3--Screened Hydrogenic 4--Screened Hydrogenic [custom Z] >>4 Enter the list of relativistic subshells: Enter increase in Z for correlation orbitals Orbital Z\_eff for hydrogenic orbitals 7.00 7.00 4p-7.00 4p

4d-

7.00

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```
4d 7.00
4f- 7.00
4f 7.00
```

```
All required subshell radial wavefunctions have been estimated:
                                                   MTP SRC
Shell
                     p0
                               gamma
                                           <r>
      0.4712D+01 0.1475D+02 0.1000D+01 0.4128D+00 357
 1s
                                                        n3.
      0.3498D+00 0.2557D+01 0.1000D+01 0.2582D+01 360
                                                        n3.
 2p- 0.4927D+00 0.2396D-03 0.1000D+01 0.2498D+01
                                                   358
                                                        n3.
 2p
     0.4927D+00 0.1497D+01 0.2000D+01 0.2498D+01 358
                                                        n3.
      0.8407D+00 0.4206D+01 0.1000D+01 0.3214D+01 361
                                                        n3.
 3s
 3p- 0.1383D+01 0.3979D-03 0.1000D+01 0.3091D+01
                                                   358
                                                        n3.
 Зр
      0.1382D+01 0.2507D+01 0.2000D+01 0.3093D+01 358
                                                        n3.
 3d- 0.1001D+01 0.1148D-04 0.2000D+01 0.2655D+01 357
                                                        n3.
     0.1001D+01 0.8634D-01 0.3000D+01 0.2655D+01 357
 3d
                                                        n3.
 4s
      0.1532D+01 0.4624D+01 0.1000D+01 0.3427D+01 350
                                                        Hyd
 4p- 0.1532D+01 0.2924D-02 0.1000D+01 0.3284D+01 350
                                                        Hyd
     0.1532D+01 0.1046D+02 0.2000D+01 0.3285D+01
                                                   350
                                                        Hyd
 4d- 0.1532D+01 0.1478D-02 0.2000D+01 0.2999D+01
                                                   350
                                                        Hyd
 4d
      0.1531D+01 0.6343D+01 0.3000D+01 0.3000D+01
                                                   350
                                                        Hyd
 4f- 0.1531D+01 0.3041D-03 0.3000D+01 0.2571D+01
                                                   349
                                                        Hyd
      0.1531D+01 0.1399D+01 0.4000D+01 0.2571D+01
                                                        Hyd
RWFNESTIMATE: Execution complete.
```

Please note how we use option 4. We have tested an increase in Z, in this case 5, so that the mean radii <r> of the new orbitals overlap approximately the region in space where we expect them. Since they describe valence correlation, they should have about the same radii as the <math>n = 2,3 orbitals.

```
>>rmcdhf_mem
```

```
RMCDHF
```

```
This program determines the radial orbitals and the expansion coefficients of the CSFs in a self-consistent field proceedure
Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log

Default settings? (y/n)

>>y
Loading CSF file ... Header only
There are/is 16 relativistic subshells;
```

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```
Loading CSF File for ALL blocks
                  26 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
                  1 blocks (block J/Parity NCF):
 There are
    0+
             26
 Enter ASF serial numbers for each block
                            26 id =
              1 	 ncf =
                                             0+
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Enter orbitals to be varied (Updating order)
>>4*
 Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
   . . . . . . . . . .
 RMCDHF: Execution complete.
********************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                    name.alog, name.log
**********************************
>>rsave n4
 Created n4.w, n4.c, n4.m, n4.sum n4.alog and n4.log
***********************************
        RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
        INPUT FILES: isodata, n4.c, n4.w
        OUTPUT FILES: n4.cm, n4.csum, n4.clog, rci.res
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
**********************************
>>rci
RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog, rci.res
Default settings?
>>y
Name of state:
>>n4
               1 , ncf =
                                  26
 Loading CSF file \dots Header only
```

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```
There are/is
                      16 relativistic subshells;
 Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
 Include H (Vacuum Polarisation)?
>>v
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
>>n
 Estimate self-energy?
>>y
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
 Loading Radial WaveFunction File ...
                                       J/Parity NCF):
 There are
                    1 blocks (block
      0+
              26
 Enter ASF serial numbers for each block
 Block
                1 	 ncf =
                              26 id =
                                               0+
>>1
 RCI: Execution complete.
************************************
         RUN RMIXEXTRACT TO DISPLAY THE MIXING COEFFICIENTS
*************************************
>>rmixextract
 RMIXEXTRACT
 Extract and prints mixing coefficient above a
 cut-off. Corresponding CSFs written to screen and
 to rcsf.out
 Input files: name.c, name.(c)m
 Output file: rcsf.out
 Name of state
>>n4
 Mixing coefficients from CI calc. ?
 Enter the cut-off value for the coefficients [0--1]
 Sort extracted CSFs according to mixingcoeffcients? (y/n)
>>n
  nblock =
                ncftot =
                                 26
                                              16
                                                   nelec =
  nb =
       1 \text{ ncfblk} =
                            26 \text{ nevblk} =
                                         1 	 2J+1 =
                                                       1 parity =
```

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```
nb = 1 \quad ncfblk = 26 \quad nevblk = 1 \quad 2J+1 = 1 \quad parity = 1
______
Average Energy = -12.602608908837407
                              ncf_reduced =
                                                 26
Energy = -14.620897436670143 Coefficients and CSF:
    1 0.953738
 2s (2)
    0+
     2 -0.001117
 2s ( 1) 3s ( 1)
   1/2 1/2
         0+
     3 -0.001846
 2s (1) 4s (1)
   1/2 1/2
     4 0.242750
 2p (2)
    0
    0+
     5 0.171674
 2p-(2)
    0+
     6 0.000254
 2p (1) 3p (1)
   3/2 3/2
          0+
    7 0.000302
 2p (1) 4p (1)
   3/2 3/2
          0+
     8 0.000178
 2p-(1) 3p-(1)
    1/2 1/2
         0+
     9 0.000214
 2p-(1) 4p-(1)
  1/2 1/2
    10 -0.039770
 3s (2)
     0+
     11 -0.001052
 3s (1) 4s (1)
   1/2 1/2
          0+
     12 0.004905
 3p (2)
    0
      0+
```

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```
13 0.003467
3p-(2)
   0+
  14 -0.000333
3p (1) 4p (1)
3/2 3/2
0+
 15 -0.000237
3p-(1) 4p-(1)
1/2 1/2 0+
 16 -0.013120
3d (2)
 0
  0+
17 -0.010712
3d-(2)
 0
   0+
  18 0.000530
3d (1) 4d (1)
 5/2 5/2
0+
  19 0.000432
3d-(1) 4d-(1)
3/2 3/2
0+
 20 -0.004103
4s (2)
  0+
21 0.001628
4p (2)
 0
  0+
22 0.001150
4p-(2)
   0+
   23 -0.002808
4d (2)
 0
 0+
24 -0.002291
4d-(2)
  0
  0+
25 0.004766
4f (2)
   0+
  26 0.004127
4f-(2)
```

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```
0
        0+
RMIXEXTRACT: Execution complete.
*************************************
        RUN RDENSITY TO COMPUTE THE RADIAL DENSITY FUNCTION AND TRANSFORM
        TO NATURAL ORBITALS.
        INPUT FILES: isodata, n4.c, n4.w, n4.cm
        OUTPUT FILE: n4.cd (density), n4.nw (natural orbitals)
********************************
>>redensity
RDENSITY: Execution begins ...
Default settings?
>>y
Name of state
>>n4
Mixing coefficients from a CI calc.?
>>y
Loading Configuration Symmetry List File ...
There are 16 relativistic subshells;
There are 26 relativistic CSFs;
 ... load complete;
Loading Radial WaveFunction File ...
   nelec =
                   4
   ncftot =
                   26
                   16
   nblock =
                    1
  block
         ncf
                 nev
                       2j+1 parity
          26
                  1
                          1
 How do you want to order your egvc ?
              1) By looking at the dominant component
              2) Following the decreasing order of the egvl
>>2
  . . . . . .
RDENSITY: Execution complete.
************************************
        COPY FILES TO PERFORM RCI CALCULATION IN THE NATURAL ORBITAL BASIS. *
>>cp n4.c n4NO.c
>>cp n4.nw n4N0.w
        RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
```

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```
INPUT FILES : isodata, n4NO.c, n4NO.w
         OUTPUT FILES: n4NO.cm, n4NO.csum, n4NO.clog, rci.res
         THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
         LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
         THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
         HIGH N.
************************************
>>rci
R.C.T
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog, rci.res
Default settings?
>>y
Name of state:
>>n4N0
                                    26
 Block
                 1 , ncf =
Loading CSF file \dots Header only
 There are/is
                      16 relativistic subshells;
Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
 Include H (Vacuum Polarisation)?
>>y
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
>>n
Estimate self-energy?
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
>>3
Loading Radial WaveFunction File ...
 There are
                    1 blocks (block
                                       J/Parity
                                                  NCF):
      0+
              26
 Enter ASF serial numbers for each block
             1 \quad \text{ncf} =
Block
                              26 id =
                                                0+
>>1
   . . . . . .
 RCI: Execution complete.
************************************
         RUN RMIXEXTRACT TO DISPLAY THE MIXING COEFFICIENTS
```

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# >>rmixextract

```
RMIXEXTRACT
Extract and prints mixing coefficient above a
cut-off. Corresponding CSFs written to screen and
to rcsf.out
Input files: name.c, name.(c)m
Output file: rcsf.out
Name of state
>>n4N0
Mixing coefficients from CI calc. ?
>>y
Enter the cut-off value for the coefficients [0--1]
Sort extracted CSFs according to mixingcoeffcients? (y/n)
 nblock = 1 ncftot = 26 nw = 16
                                        nelec = 4
______
 nb = 1 ncfblk = 26 nevblk = 1 2J+1 = 1 parity =
 nb = 1 ncfblk =
                     26 nevblk = 1 2J+1 = 1 parity = 1
______
Average Energy = -12.602609084238223
                                     ncf_reduced =
Energy = -14.620897382271725
                        Coefficients and CSF :
        1 0.953740
  2s (2)
       0+
       2 -0.000000
  2s (1) 3s (1)
     1/2 1/2
              0+
        3 -0.000000
  2s (1) 4s (1)
     1/2
          1/2
        4 0.242750
  2p (2)
      0
       0+
        5 0.171674
  2p-(2)
       0+
        6 0.000000
  2p (1) 3p (1)
     3/2 3/2
             0+
           0.000000
```

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```
2p (1) 4p (1)
 3/2 3/2
0+
 8 0.000000
2p-(1) 3p-(1)
 1/2 1/2 0+
    9 0.000000
2p-(1) 4p-(1)
 1/2 1/2
  10 -0.039787
3s (2)
  0+
11 -0.000000
3s (1) 4s (1)
1/2 1/2
0+
 12 0.004922
3p (2)
 0
0+
13 0.003479
3p-(2)
   0+
  14 -0.000000
3p ( 1) 4p ( 1)
  3/2 3/2 0+
  15 -0.000000
3p-(1) 4p-(1)
1/2 1/2 0+
 16 -0.013134
3d (2)
 0
  0+
17 -0.010723
3d-(2)
  0
   0+
18 -0.000000
3d (1) 4d (1)
  5/2 5/2 0+
  19 -0.000000
3d-(1) 4d-(1)
3/2 3/2 0+
 20 -0.004089
4s (2)
```

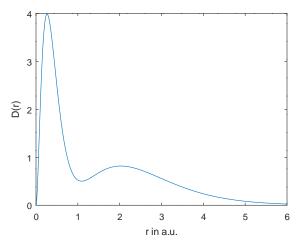
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```
0+
             0.001611
       21
4p (2)
      0
       0+
             0.001138
       22
4p-(2)
            -0.002794
       23
4d (2)
       0+
            -0.002280
       24
4d-(2)
      0
       0+
       25
             0.004766
4f (2)
      0
       0+
       26
             0.004127
4f-(2)
      0
       0+
```

RMIXEXTRACT: Execution complete.

We see that the energy is invariant, but the weights have been concentrated to relatively fewer CSFs. The weights for many CSFs are now zero.

The rdensity program also outputs the file n4.cd file that contains the radial density distribution D(r) for each grid point, see Section 8.4 for a discussion of the file structure. In Figure 3 we have plotted D(r) as a function of r in a.u.



**Figure 3.** Radial density function D(r) for  $1s^22s^2$   $^1S_0$  in Be I.

6.9. Ninth Example: Magnetic-Field- and Hyperfine-Induced  $2s2p\ ^3P_0^o-2s^2\ ^1S_0$  Transitions in Ni XXV

The ninth example is for the unexpected transition 2s2p  $^3P_0^o - 2s^2$   $^1S_0$  in Ni XXV, see [42]. The example shows the computation of Zeeman and hyperfine interaction matrix using the hfszeeman95 program, with given rci wave functions,

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and the use of Matlab program mithit to compute the transition rates between magnetic fine-structure substates in the presence of an external magnetic field and the rates of hyperfine induced transitions in the field-free limit.

# Overview

- 1. Define nuclear data.
- 2. Obtain common spectroscopic orbitals for the MR set.
  - (a) Generate configuration state list for MR set  $\{1s^22s^2, 1s^22p^2, \text{ and } 1s^22s2p\}$ .
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of all states belonging to  $1s^22s^2$  and  $1s^22s^2p$ .
  - (e) Save output to mr.
- 3. Improve even states
  - (a) Generate n = 3 valence-valence CSF expansions.
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of the even state.
  - (e) Save output to even\_n3.
  - (f) Perform rci calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
- 4. Transform from *jj* to *LSJ*-coupling
- 5. Improve odd states
  - (a) Generate n = 3 valence-valence CSF expansions.
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of the odd states.
  - (e) Save output to odd\_n3.
  - (f) Perform rci calculation in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.
- 6. Transform from *jj* to *LSJ*-coupling
- 7. Calculate properties
  - (a) Calculate Zeeman and hyperfine interaction matrix using the rci wave functions.
  - (b) Compute the transition rates from the rci wave functions. Calculation in two steps: biorthonormal transformation and evaluation of transition matrix elements using standard Racah algebra methods. The latter procedure is done using rtransition\_phase.
  - (c) Compute the magnetic-field-induced transition  $2s2p \,^{3}P_{0}^{o} 2s^{2} \,^{1}S_{0}$  rate at B = 3 tesla.
  - (d) Compute the hyperfine-induced transition  $2s2p^{3}P_{0}^{0} 2s^{2}{}^{1}S_{0}$  rate in the field-free limit.

# **Program Input**

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

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```
the default nuclear skin thickness is
                                          2.299999999999998
                                                                 fm:
 Revise these values?
>>n
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
>>58.6934
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
 Enter the nuclear dipole moment (in nuclear magnetons):
>>-0.75002
 Enter the nuclear quadrupole moment (in barns):
>>0.162
***********************************
         RUN RCSFGENERATE TO GENERATE LIST OF CSFs FOR
         CONFIGURATIONS 2s(2), 2p(2), 2s2p
         OUTPUT FILES: rcsfgenerate.log, rcsf.out
************************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                           = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                           = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>1s(2,i)2s(2,i)
 Give configuration
>>1s(2,i)2p(2,i)
 Give configuration
                             3
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
```

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```
>>0,0
Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
>>y
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>1s(2,i)2s(1,i)2p(1,i)
 Give configuration
>>
 Give set of active orbitals, as defined by the highest principal quantum number
per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>0
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 4 blocks were created
      block J/P
                          NCSF
          1
             0+
                             3
          2
              0-
                             1
              1-
                             2
          3
 **********************************
         COPY FILES
**********************************
>>cp rcsf.out rcsf.inp
**********************************
         RUN RANGULAR TO GENERATE ENERGY EXPRESSION
         INPUT FILE : rcsf.inp
         OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
>>rangular
 RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
            rangular.log
```

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```
Full interaction? (y/n)
>>y
 RANGULAR: Execution complete.
RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
        * MEANS ALL ORBITALS
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp, rwfnestimate.log
***********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                     4 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                                        <r> MTP SRC
                    p0
         е
                             gamma
     0.3902D+03 0.3381D+03 0.1000D+01 0.5289D-01 328 T-F
    0.9484D+02 0.1211D+03 0.1000D+01 0.2123D+00 344 T-F
  2s
  2p- 0.9460D+02 0.1104D+01 0.1000D+01 0.1766D+00 344 T-F
      0.9357D+02 0.9101D+03 0.2000D+01 0.1791D+00 344 T-F
 RWFNESTIMATE: Execution complete.
******************************
        RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
        INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
        OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
        NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
```

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```
THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p AND THEY ARE ALL
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         * MEANS ALL ORBITALS
*************************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                      4 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                    7 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    4 blocks (block
                                      J/Parity
                                                 NCF):
    0+
              3
                      2 0-
                                               1-
                                                       2
                                                                   2-
 Enter ASF serial numbers for each block
 Block
               1 	 ncf =
                              3 id =
                                               0+
>>1
             2 ncf =
 Block
                                   1 id =
>>1
                3 	 ncf =
                                    2 id =
Block
>>1,2
                4
Block
                     ncf =
                                    1 id =
                                               2-
level weights (1 equal; 5 standard; 9 user)
>>5
Radial functions
 1s 2s 2p- 2p
 Enter orbitals to be varied (Updating order)
>>*
 Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
       . . . . . . . . . .
 RMCDHF: Execution complete.
*************************************
```

\* RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum \* name.alog, name.log \*

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```
**********************************
>>rsave mr
Created mr.w, mr.c, mr.m, mr.sum mr.alog and mr.log
***********************************
        RUN RCSFGENERATE TO GENERATE n = 3 VALENCE-VALENCE
        CORRELATION LIST FOR EVEN STATE
                                                                       *
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
       0: No core
       1: He (
                   1s(2)
                                        = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                        = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                        = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                       = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                        = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>1s(2,i)2s(2,*)
 Give configuration
                           2
>>1s(2,i)2p(2,*)
                           3
 Give configuration
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
```

>>n

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```
. . . . . . . . .
1 blocks were created
                       NCSF
     block J/P
        1 0+
                         11
**********************************
        COPY FILES
********************************
>>cp rcsf.out rcsf.inp
***********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
************************************
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
           rangular.log
Full interaction? (y/n)
>>y
 . . . . . . . . . . .
RANGULAR: Execution complete.
***********************************
        RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp
*************************************
>>rwfnestimate
RWFNESTIMATE
This program estimates radial wave functions
for orbitals
Input files: isodata, rcsf.inp, optional rwfn file
Output file: rwfn.inp
Default settings ?
>>y
Loading CSF file ... Header only
There are/is
                   9 relativistic subshells;
```

The following subshell radial wavefunctions remain to be estimated:

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```
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
>>mr.w
 Enter the list of relativistic subshells:
>>*
 The following subshell radial wavefunctions remain to be estimated:
 3s 3p- 3p 3d- 3d
 Read subshell radial wavefunctions. Choose one below
     1--GRASP2K File
     2--Thomas-Fermi
    3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
                                           <r> MTP SRC
Shell
          е
                      p0
                               gamma
     0.3671D+03 0.3339D+03 0.1000D+01 0.5367D-01 341 mr.
  1s
  2s 0.8431D+02 0.1134D+03 0.1000D+01 0.2235D+00 347
  2p- 0.8236D+02 0.9734D+00 0.1000D+01 0.1894D+00 347
                                                         mr.
  2p
      0.8154D+02 0.8034D+03 0.2000D+01 0.1919D+00 347
                                                         mr.
     0.4070D+02 0.6530D+02 0.1000D+01 0.4835D+00 355
  3s
                                                         T-F
  3p- 0.4058D+02 0.6470D+00 0.1000D+01 0.4481D+00 354 T-F
      0.4028D+02 0.5359D+03 0.2000D+01 0.4517D+00 355 T-F
  Зр
  3d- 0.4007D+02 0.1028D+01 0.2000D+01 0.3798D+00
                                                    354
                                                         T-F
      0.3997D+02 0.1070D+04 0.3000D+01 0.3812D+00
                                                    354 T-F
 RWFNESTIMATE: Execution complete.
***********************************
         RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
         NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
         VARY THE CORRELATION ORBITALS 3s,3p, 3d. NONE OF THESE ARE
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         3* MEANS 3s, 3p-, 3p, 3d-, 3d
>>rmcdhf
 RMCDHF
```

This program determines the radial orbitals

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```
and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                      9 relativistic subshells;
 Loading CSF File for ALL blocks
                  11 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
                   1 blocks (block J/Parity NCF):
 There are
    0+
            11
 Enter ASF serial numbers for each block
                           11 id = 0+
           1 	 ncf =
>>1
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
>>3*
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
    . . . . . . . . . .
 RMCDHF: Execution complete.
************************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                      name.alog, name.log
***********************************
>>rsave even_n3
Created even_n3.w, even_n3.c, even_n3.m, even_n3.sum even_n3.alog and even_n3.log
********************************
         RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
         INPUT FILES : isodata, even_n3.c, even_n3.w
         OUTPUT FILES: even_n3.cm, even_n3.csum, even_n3.clog, rci.res
         THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
         LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
         THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
>>rci
 RCI
 This is the configuration interaction program
```

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```
Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog
             rci.res (can be used for restart)
Default settings?
>>y
Name of state:
>>even_n3
 Block
                1 , ncf =
Loading CSF file \dots Header only
                       9 relativistic subshells;
 There are/is
 Include contribution of H (Transverse)?
>>y
 Modify all transverse photon frequencies?
Enter the scale factor:
>>1.d-6
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
>>n
Estimate self-energy?
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
 Loading Radial WaveFunction File ...
                    1 blocks (block
                                       J/Parity
 There are
                                                 NCF):
  1
      0+
              11
 Enter ASF serial numbers for each block
 Block
                1 	 ncf =
                                 11 id =
>>1
   . . . . . .
 RCI: Execution complete.
**********************************
         RUN JJ2LSJ TO TRANSFORM FROM JJ- TO LSJ-COUPLING
         INPUT FILES: even_n3.c, even_n3.cm
         OUTPUT FILE: even_n3.lsj.lbl, even_n3.uni.lsj.lbl
************************************
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2021).
 Input files: name.c, name.(c)m
```

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```
Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
             name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
 >>even_n3
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 11 relativistic CSFs;
  ... load complete;
Mixing coefficients from a CI calc.?
>>y
Do you need a unique labeling? (y/n)
>>y
   nelec =
                     4
   ncftot =
                    11
                     9
   nblock =
                     1
  block
                         2j+1 parity
           ncf
                  nev
      1
           11
                   1
                            1
 Default settings? (y/n)
>>y
 jj2lsj: Execution Complete
RUN RCSFGENERATE TO GENERATE n = 3 VALENCE-VALENCE
        CORRELATION LIST FOR ODD STATE
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
***********************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
       0: No core
       1: He (
                   1s(2)
                                        = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                        = 10 electrons)
```

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```
3: Ar ([Ne] + 3s(2)3p(6)
                                      = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                      = 36 electrons)
                                 = 54 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
Give configuration
>>1s(2,i)2s(1,*)2p(1,*)
Give configuration
>>
Give set of active orbitals, as defined by the highest principal quantum number
per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
Number of excitations (if negative number e.g., -2, correlation
orbitals will always be doubly occupied)
>>2
Generate more lists ? (y/n)
>>n
       . . . . . . . . .
3 blocks were created
      block J/P
                       NCSF
         1
            0-
                           6
         2
             1-
                          14
             2-
                          12
************************************
        COPY FILES
*****************************
>>cp rcsf.out rcsf.inp
***********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.log, mcp.30, mcp.31,....
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
            rangular.log
Full interaction? (y/n)
>>y
```

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. . . . . . . . . . . RANGULAR: Execution complete. \* RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS \* INPUT FILES: isodata, rcsf.inp, previous rwfn files OUTPUT FILE: rwfn.inp \* >>rwfnestimate RWFNESTIMATE This program estimates radial wave functions for orbitals Input files: isodata, rcsf.inp, optional rwfn file Output file: rwfn.inp Default settings ? >>y Loading CSF file ... Header only There are/is 9 relativistic subshells; The following subshell radial wavefunctions remain to be estimated: 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d Read subshell radial wavefunctions. Choose one below 1--GRASP2K File 2--Thomas-Fermi 3--Screened Hydrogenic 4--Screened Hydrogenic [custom Z] >>1 Enter the file name (Null then "rwfn.out") >>mr.w Enter the list of relativistic subshells: >>\* The following subshell radial wavefunctions remain to be estimated: 3s 3p- 3p 3d- 3d Read subshell radial wavefunctions. Choose one below 1--GRASP2K File 2--Thomas-Fermi 3--Screened Hydrogenic 4--Screened Hydrogenic [custom Z] >>2 Enter the list of relativistic subshells: >>\* All required subshell radial wavefunctions have been estimated: Shell MTP SRC е p0 gamma <r> 0.3671D+03 0.3339D+03 0.1000D+01 0.5367D-01 341 1 s mr. 0.8431D+02 0.1134D+03 0.1000D+01 0.2235D+00

2p- 0.8236D+02 0.9734D+00 0.1000D+01 0.1894D+00

347

mr.

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```
0.8154D+02 0.8034D+03 0.2000D+01 0.1919D+00 347
  2p
      0.4070D+02 0.6530D+02 0.1000D+01 0.4835D+00 355
  3p- 0.4058D+02 0.6470D+00 0.1000D+01 0.4481D+00 354
     0.4028D+02 0.5359D+03 0.2000D+01 0.4517D+00
  3p
                                                   355
                                                       T-F
  3d- 0.4007D+02 0.1028D+01 0.2000D+01 0.3798D+00 354
                                                       T-F
      0.3997D+02 0.1070D+04 0.3000D+01 0.3812D+00 354 T-F
 RWFNESTIMATE: Execution complete.
***********************************
         RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: FOR CORRELATION ORBITALS THERE ARE NO RESTRICTIONS ON THE
         NUMBER OF NODES, I.E. THEY ARE NOT SPECTROSCOPIC. IN THIS RUN WE
         VARY THE CORRELATION ORBITALS 3s,3p, 3d. NONE OF THESE ARE
         SPECTROSCOPIC. WE CAN USE WILD CARDS * FOR SPECIFYING ORBITALS
         3* MEANS 3s, 3p-, 3p, 3d-, 3d
*************************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
 Loading CSF file \dots Header only
 There are/is
                       9 relativistic subshells;
 Loading CSF File for ALL blocks
                   32 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    3 blocks (block
                                       J/Parity NCF):
     0-
              6
                    2 1-
                                 14
                                          3
                                               2-
                                                      12
 Enter ASF serial numbers for each block
                                    6 id =
 Block
                1 	 ncf =
>>1
                2 ncf =
                                    14 id =
Block
                                               1_
>>1,2
                3
                     ncf =
 Block
                                    12 id =
>>1
 level weights (1 equal; 5 standard; 9 user)
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
 Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
```

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```
>>
Enter the maximum number of SCF cycles:
>>100
   . . . . . . . . . .
 RMCDHF: Execution complete.
********************************
        RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                    name.alog, name.log
*************************************
>>rsave odd_n3
Created odd_n3.w, odd_n3.c, odd_n3.m, odd_n3.sum odd_n3.alog and odd_n3.log
******************************
        RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
        INPUT FILES : isodata, odd_n3.c, odd_n3.w
        OUTPUT FILES: odd_n3.cm, odd_n3.csum, odd_n3.clog, rci.res
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
**********************************
>>rci
 RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog
            rci.res (can be used for restart)
Default settings?
>>7
Name of state:
>>odd n3
Block
               1 , ncf =
Block
               2 , ncf =
                                  14
               3 , ncf =
Block
                                  12
Loading CSF file ... Header only
                     9 relativistic subshells;
 There are/is
 Include contribution of H (Transverse)?
 Modify all transverse photon frequencies?
>>y
Enter the scale factor:
>>1.d-6
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
>>n
```

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```
Include H (Specific Mass Shift)?
 Estimate self-energy?
>>y
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
 Loading Radial WaveFunction File ...
 There are
                    3 blocks (block
                                      J/Parity NCF):
    0-
                         1- 14
                                          3
                                                      12
                                               2-
 Enter ASF serial numbers for each block
 Block
                1 ncf =
                                     6 id =
>>1
Block
                2 ncf =
                                    14 id =
>>1,2
                3 ncf =
                                   12 id =
Block
                                               2-
>>1
   . . . . . .
 RCI: Execution complete.
************************************
         RUN JJ2LSJ TO TRANSFORM FROM JJ- TO LSJ-COUPLING
         INPUT FILES: odd_n3.c, odd_n3.cm
         OUTPUT FILE: odd_n3.lsj.lbl, odd_n3.uni.lsj.lbl
*********************************
>>jj21sj
 jj2lsj: Transformation of ASFs from a jj-coupled CSF basis
        into an LSJ-coupled CSF basis (Fortran 95 version)
        (C) Copyright by G. Gaigalas and Ch. F. Fischer,
        (2021).
 Input files: name.c, name.(c)m
 Output files: name.lsj.lbl
   (optional) name.lsj.c, name.lsj.j,
              name.uni.lsj.lbl, name.uni.lsj.sum
 Name of state
>>odd_n3
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 11 relativistic CSFs;
  ... load complete;
Mixing coefficients from a CI calc.?
Do you need a unique labeling? (y/n)
>>y
   nelec =
                      4
   ncftot =
                     32
```

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```
9
   nblock =
                   3
         ncf nev
                     2j+1 parity
  block
     1
          6
                 1
                        1 -1
     2
          14
                  2
                       3
                              -1
     3
          12
                  1
                       5
                              -1
Default settings? (y/n)
>>y
jj2lsj: Execution Complete
********************************
       RUN HFSZEEMAN95 FOR even_n3
       INPUT FILES: isodata, even_n3 .c, even_n3 .w, even_n3 .cm
       OUTPUT FILE: even_n3 .ch, even_n3 .cgjhfs
>>hfszeeman95
HFSZEEMAN95
This is the magnetic interaction program
Input files: isodata, name.c, name.(c)m, name.w
Output files: name.(c)h, name.(c)gjhfs
HFSZEEMAN95: Execution begins ...
Default settings?
>>y
Name of state
>>even_n3
Mixing coefficients from a CI calc.?
Calculate off-diagonal matrix elements?
>>y
HFSZEEMAN95: Execution complete.
*************************
       RUN HFSZEEMAN95 FOR odd_n3
       INPUT FILES: isodata, odd_n3 .c, odd_n3 .w, odd_n3 .cm
       OUTPUT FILE: odd_n3 .ch, odd_n3 .cgjhfs
**************************************
>>hfszeeman95
HFSZEEMAN95
This is the magnetic interaction program
Input files: isodata, name.c, name.(c)m, name.w
Output files: name.(c)h, name.(c)gjhfs
```

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```
HFSZEEMAN95: Execution begins ...
Default settings?
>>y
Name of state
>>odd_n3
Mixing coefficients from a CI calc.?
Calculate off-diagonal matrix elements?
>>y
  . . . .
HFSZEEMAN95: Execution complete.
*************************************
        VIEW ZEEMAN AND HYPERFINE INTERACTION MATRX FOR ODD STATES
***********************************
>>more odd_n3.cgjhfs
 Number of relativistic eigenvalues
        J Parity
 Lev
                     Ε
                 -944.099340681
  1
       2.0
               -944.694737339
  1
       1.0 -
       1.0 -
                 -942.723168239
  1
       0.0
                 -944.876941705
 Zeeman interaction matrix
 0.18322E+01 -0.34691E+00 0.68227E-01 0.00000E+00
 0.44786E+00 0.10439E+01 -0.67174E-01 0.40125E+00
 -0.88081E-01 -0.67174E-01 0.71718E+00 -0.78350E-01
 0.00000E+00 -0.69499E+00 0.13571E+00 0.00000E+00
 HFI-matrix for the magnetic dipole operator
 0.36367E+02 -0.10508E+02 0.27000E+02 0.00000E+00
 0.13566E+02 0.36112E+02 0.22640E+02 0.15292E+02
 -0.34857E+02 0.22640E+02 -0.18141E+01 0.81431E+01
 0.00000E+00 -0.26486E+02 -0.14104E+02 0.00000E+00
 HFI-matrix for the electric quadrupole operator
 -0.41925E+03 -0.22145E+03 0.14396E+03 0.00000E+00
 0.76944E+02 0.14396E+03 0.46833E+03 0.00000E+00
 -0.49632E+03 -0.00000E+00 -0.00000E+00 0.00000E+00
***********************************
        RUN RBIOTRANSFORM FOR even_n3 AND odd_n3 TO TRANSFORM WAVE FUNCTIONS *
        INPUT FILES: isodata, even_n3.c, even_n3.w, even_n3.cm,
                             odd_n3.c, odd_n3.w, odd_n3.cm
        OUTPUT FILES: even_n3 .cbm, even_n3 .bw, odd_n3.cbm, odd_n3.bw
                    even_n3.TB, odd_n3.TB (angular files)
        NOTE THAT THE ORDER OF INITIAL AND FINAL STATE DOES NOT MATTER
   ************************************
```

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#### >>rbiotransform

```
RBIOTRANSFORM
 This program transforms the initial and final wave
 functions so that standard tensor albegra can be
 used in evaluation of the transition parameters
 Input files: isodata, name1.c, name1.w, name1.(c)m
             name2.c, name2.w, name2.(c)m
             name1.TB, name2.TB (optional angular files)
 Output files: name1.bw, name1.(c)bm,
             name2.bw, name2.(c)bm
             name1.TB, name2.TB (angular files)
 Default settings?
>>y
 Input from a CI calculation?
>>y
  Name of the Initial state
>>even_n3
 Name of the Final state
>>odd n3
  Transformation of all J symmetries?
>>y
 BIOTRANSFORM: Execution complete.
RUN RTRANSITION_PHASE FOR even_n3 and odd_n3 TO COMPUTE TRANSITION PARAMETERS *
        INPUT FILES: isodata, even_n3.c, even_n3.bw, even_n3.cbm
                    odd_n3.c, odd_n3.bw, odd_n3.cbm
        OUTPUT FILES: even_n3.odd_n3.ct
                     odd_n3.odd_n3.-1T (angular file)
        NOTE THAT THE ORDER OF INITIAL AND FINAL STATE DOES NOT MATTER
>>rtransition_phase
 RTRANSITION
 This program computes transition parameters from
 transformed wave functions
 Input files: isodata, name1.c, name1.bw, name1.(c)bm
             name2.c, name2.bw, name2.(c)bm
             optional, name1.lsj.lbl, name2.lsj.lbl
             name1.name2.KT (optional angular files)
 Output files: name1.name2.(c)t
             optional, name1.name2.(c)t.lsj
             name1.name2.KT (angular files)
 Here K is parity and rank of transition: -1,+1 etc
 Default settings?
>>y
  Input from a CI calculation?
>>y
```

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```
Name of the Initial state
>>even_n3
 Name of the Final state
>>odd_n3
 MRGCSL: Execution begins ...
 Loading Configuration Symmetry List File \dots
 There are 9 relativistic subshells;
 There are 11 relativistic CSFs;
 ... load complete;
 Loading Configuration Symmetry List File ...
 There are 9 relativistic subshells;
 There are 32 relativistic CSFs;
  ... load complete;
          1 s
          2 s
          2 p-
          2 p
          3 s
          3 p-
          3 p
          3 d-
          3 d
          1
         11
          3
                    20
                              32
 Loading Configuration Symmetry List File ...
 there are 9 relativistic subshells;
 there are 43 relativistic CSFs;
  ... load complete;
 Enter the list of transition specifications
  e.g., E1,M2 or E1 M2 or E1;M2:
>>E1,M2
   . . . . .
 RTRANSITION: Execution complete.
********************************
         VIEW COMPUTED TRANSITION PARAMETERS
*************************************
>>more even_n3.odd_n3.ct
 Transition between files:
 f1 = even_n3
 f2 = odd_n3
 Electric 2**( 1)-pole transitions
 _____
 Upper
          Lower
 Lev J P
         Lev J P
                        E (Kays)
                                      A (s-1)
                                                       gf
                                                                    S
                                                                                 М
```

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```
f2 1 1 - f1 1 0 +
                         419612.86 C 1.04269D+08 2.66340D-03 2.08960D-03 -4.57121D-02
                             B 8.48871D+07 2.16832D-03 1.70118D-03 -4.12454D-02
f2 2 1 - f1 1
                          852322.26 C 2.56686D+10 1.58918D-01 6.13827D-02 -2.47755D-01
                                 B 2.38815D+10 1.47854D-01 5.71090D-02 -2.38975D-01
Magnetic 2**( 2)-pole transitions
Upper
        Lower
Lev J P Lev J P
                                                gf
                   E (Kays)
                                 A (s-1)
       2 - f1 1
f2 1
                  0 + 550287.32 M 1.40086D+01 3.46773D-10 9.30999D-01 -9.64883D-01
```

Comment: The reduced transition matrix elements M given in the even\_n3.odd\_n3.ct are need by the Matlab program mithit for the computation of magnetic-field- and hyperfine-induced transitions.

```
RUN MATLAB PROGRAM MITHIT FOR MAGNETIC-FIELD-INDUCED TRANSITION AT *
        MAGNETIC FIELD STRENGTH B = 3 TESLA
        INPUT FILES: even_n3.cgjhfs, odd_n3.cgjhfs, even_n3.odd_n3.ct
        OUTPUT FILES: even_n3.czm, odd_n3.czm, even_n3.odd_n3.fs.mit.mtrans *
************************************
>>mithit
Name of the Initial state:
>>even_n3
Name of the Final state:
>>odd_n3
Are the calculations based on a relativistic CI calculation? (Y/N)
MIT-fs(0), HIT(1) or MIT-hfs(2):
B-field in Tesla (0) or Gauss (1):
>>0
Give the upper limit for the B-field:
>>3
Energies in a.u. (0), cm-1 (1) or MHz (2) ?
>>1
Start Computation of Energies and Mixing Coefficients
of the Magnetic Sublevels of Initial States
level
     E_fs (a.u.)
                     _____
     -946.606634183
                    0
Would you like a plot of Zeeman splitting with B field? (Y/N)
>>n
```

Finished even\_n3

Start Computation of Energies and Mixing Coefficients

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of the Magnetic Sublevels of Final States

```
level
     E_fs (a.u.)
                 J
_____
    -944.099340681 2
1
    -944.694737339
3
    -942.723168239
                   1
    -944.876941705
Would you like a plot of Zeeman splitting with B field? (Y/N)
>> n
Finished odd_n3
Would you like to compute the transition rates? (Y/N)
>>y
level E_BP (a.u.)
_____
Initial levels:
     -946.606634183
Final levels:
                     2
1 -944.099340681
2
    -944.694737339
                       1
    -942.723168239
3
                       1
     -944.876941705
Give an index vector of the initial levels(lower level):
>>1
Give an index vector of the final levels(upper level):
Would you like a plot of synthetic spectra? (Y/N)
>>n
MITHIT finished
**********************************
       VIEW COMPUTED MAGNETIC-FIELD-INDUCED TRANSITION RATE
************************************
>>more even_n3.odd_n3.fs.mit.mtrans
Magnetic field
 B = 3.0000000 Tesla
Fine structure energies in a.u.
even_n3
level J E_BP (a.u.)
  0.0 -946.606634
odd_n3
level J
           E_BP (a.u.)
    2.0 -944.099341
1
      1.0 -944.694737
      1.0 -942.723168
3
```

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#### 4 0.0 -944.876942

Transition rates and wavenumbers in Kays

Upper Lower

level J M\_J E\_hfs (a.u.) FS-LEV level J M\_J E\_hfs (a.u.) FS-LEV A (s-1) E (Kays)

4 0.0 0.0 -944.876941705 4 1 0.0 0.0 -946.606634183 1 4.0607E-02 379623.6178

\*

\* RUN MATLAB PROGRAM MITHIT FOR HYPERFINE-INDUCED TRANSITION

\* INPUT FILES: even\_n3.cgjhfs, odd\_n3.cgjhfs, even\_n3.odd\_n3.ct \*

\* OUTPUT FILES: even\_n3.czm, odd\_n3.czm, even\_n3.odd\_n3.hfs.hit.trans \*

\*

>>mithit

Name of the Initial state:

>>even\_n3

Name of the Final state:

>>odd\_n3

Are the calculations based on a relativistic CI calculation? (Y/N) >>y

MIT-fs(0), HIT(1) or MIT-hfs(2):

>>1

Nuclear spin I:

>>1.5

Nuclear magnetic dipole moment mu:

>> -0.75002

Nuclear electric quadrupole moment  $\mathbb{Q}$ :

>>0.162

Start Computation of Energies and Mixing Coefficients of the Magnetic Sublevels of Initial States  $\begin{tabular}{ll} \hline \end{tabular}$ 

level	E_hfs (a.u.)	FS-LEV	J	F	
1	-946.606634183	1	0	3/2	

Finished even\_n3

Start Computation of Energies and Mixing Coefficients of the Magnetic Sublevels of Final States

E_hfs (a.u.)	FS-LEV	J	F	
-944.099384680	1	2	7/2	
-944.694775514	2	1	5/2	
-944.099333945	1	2	5/2	
-942.723166078	3	1	5/2	
-944.876941710	4	0	3/2	
	-944.099384680 -944.694775514 -944.099333945 -942.723166078	-944.099384680 1 -944.694775514 2 -944.099333945 1 -942.723166078 3	-944.099384680 1 2 -944.694775514 2 1 -944.099333945 1 2 -942.723166078 3 1	-944.099384680 1 2 7/2 -944.694775514 2 1 5/2 -944.099333945 1 2 5/2 -942.723166078 3 1 5/2

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6	-944.694711495	2	1	3/2
7	-944.099296427	1	2	3/2
8	-942.723170502	3	1	3/2
9	-944.694674496	2	1	1/2
10	-944.099273405	1	2	1/2
11	-942.723170188	3	1	1/2

Finished odd\_n3

Would you like to compute the transition rates? (Y/N) >>y

level	E_hfs (a.u.)	FS-LEV	J	F		
Initi	Initial levels:					
1	-946.606634183	1	0	3/2		
Final	Final levels:					
1	-944.099384680	1	2	7/2		
2	-944.099333945	1	2	5/2		
3	-944.099296427	1	2	3/2		
4	-944.099273405	1	2	1/2		
5	-944.694775514	2	1	5/2		
6	-944.694711495	2	1	3/2		
7	-944.694674496	2	1	1/2		
8	-942.723166078	3	1	5/2		
9	-942.723170502	3	1	3/2		
10	-942.723170188	3	1	1/2		
11	-944.876941710	4	0	3/2		

Give an index vector of the initial levels(lower level):

>>1

Give an index vector of the final levels(upper level):

>>11

Would you like a plot of synthetic spectra? (Y/N)

>>n

MITHIT finished

>>more even\_n3.odd\_n3.hfs.hit.trans

Nuclear data

Nuclear spin 1.500000 au

Nuclear magnetic dipole moment -0.750020 n.m.

Nuclear electric quadrupole moment 0.162000 barns

Hyperfine structure energies in a.u.

even\_n3

level J F E\_hfs (a.u.) FS-LEV 1 0.0 1.5 -946.606634 1

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odd_n3					
level	J	F	E_hfs (a.u.)	FS-LEV	
1	2.0	3.5	-944.099385		1
2	2.0	2.5	-944.099334		1
3	2.0	1.5	-944.099296		1
4	2.0	0.5	-944.099273		1
5	1.0	2.5	-944.694776		2
6	1.0	1.5	-944.694711		2
7	1.0	0.5	-944.694674		2
8	1.0	2.5	-942.723166		3
9	1.0	1.5	-942.723171		3
10	1.0	0.5	-942.723170		3
11	0.0	1.5	-944.876942		4

```
Transition rates and wavenumbers in Kays
```

```
Upper Lower

level J F E_hfs (a.u.) FS-LEV level J F E_hfs (a.u.) FS-LEV A (s-1) E (Kays)

11 0.0 1.5 -944.876941710 4 1 0.0 1.5 -946.606634183 1 2.6057E+00 379623.6167
```

# 7. Running the Tools

# 7.1. Splitting a List of CSFs

When using scripts, see case studies in Sections 9–11, it is often convenient to generate a list of CSFs based on a large active set of orbitals and then split this list into a number of lists with CSFs that can be formed from different subsets of active orbitals.

# Overview

- 1. Generate a list of CSFs for  $1s^2 2p \,^2 P_{1/2,3/2}^o$  by allowing SDT excitations (CAS expansion) to the active set n=5.
- 2. Split into three lists with CSFs that can be formed by the active sets n = 3, n = 4, and n = 5.

# Program Input

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

#### >>rcsfgenerate

# RCSFGENERATE

This program creates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (\*) or has a minimal
occupation e.g., 1s(2,1)2s(2,\*)
Outputfiles: rcsf.out, rcsfgenerate.log

Default, reverse, symmetry or user specified ordering? (\*/r/s/u) >>\*

Select core

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```
0: No core
       1: He (
                   1s(2)
                                        = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                       = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                        = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                       = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                        = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>0
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration 1
>>1s(2,*)2p(1,*)
 Give configuration 2
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>3
 Generate more lists ? (y/n)
>>n
 2 blocks were created
      block J/P
                          NCSF
         1 1/2-
                          1454
          2 3/2-
                          2478
 **********************************
* COPY FILES
**********************************
>>cp rcsf.out odd.c
*********************************
* RUN RCSFSPLIT AND SPLIT INTO LISTS CORRESPONDING TO ACTIVE SETS
* n = 3, n = 4, n = 5
* INPUT FILE : odd.c
* OUTPUT FILE: odd3.c, odd4.c, odd5.c
***********************************
>>rcsfsplit
 RCSFSPLIT
 Splits a list name.c of CSFs into a number of lists with CSFs that
 can be formed from different sets of active orbitals.
 Orbital sets are specified by giving the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
 Input file: name.c
```

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```
Output files: namelabel1.c, namelabel2.c, ...
 Name of state
>>odd
 Number of orbital sets
>>3
 Orbital set
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
 Give file label
 >>3
 Orbital set
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Give file label
>>4
                       3
 Orbital set
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>5s,5p,5d,5f,5g
 Give file label
>>5
 List odd3.c based on orbital set
                  186 CSFs
 contains
 List odd4.c based on orbital set
                                             2
                 1048 CSFs
 contains
 List odd5.c based on orbital set
                                             3
                 3932 CSFs
 contains
```

#### 7.2. Extracting and Condensing

The program rmixextract extracts and displays, for each state in a calculation, the CSFs with absolute values of the mixing coefficients larger than a certain cut-off. The program rmixaccumulate works somewhat differently, it accumulates the CSFs that account for a given percentage of the wave functions in a calculation. The algorithm is described as follows:

Start from a calculation targeting one or more states, thus start from a number of ASFs

$$ASF_1: \Psi(\gamma_1 PJ) = \sum_{i=1}^{N} c_i^1 \Phi(\gamma_i PJ)$$

$$ASF_{M}: \Psi(\gamma_{M}PJ) = \sum_{i=1}^{N} c_{i}^{M} \Phi(\gamma_{i}PJ)$$

2. For *i* from 1 to *N* compute

$$s_i = (c_i^1)^2 + (c_i^2)^2 + \ldots + (c_i^M)^2.$$

3. Sort  $s_1, s_2, \ldots, s_N$  in descending order.

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Accumulate the sorted s values until a specified fraction of the total squared weight

$$M = s_1 + \ldots + s_N = \sum_{i,j} (c_i^j)^2$$

is attained. The corresponding CSFs gives a condensed list.

#### Overview

- 1. Extract and display CSFs from the rci wave functions 2p\_3 in the first example, see Section 6.1.
- 2. Accumulate CSFs from the rci wave functions 2p\_3 in the first example, accounting for 99.99 % of the total wave function content.

# Program Input

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

```
MAKE SURE YOU ARE IN THE DIRECTORY WHERE THE FILES FROM THE FIRST
       EXAMPLE ARE
*************************************
********************************
       RUN RMIXEXTRACT TO EXTRACT AND PRINT MIXING COEFFICIENTS FOR 2p_3
       CORRESPONDING CSFs WRITTEN to rcsf.out
       INPUT FILES: 2p_3.c, 2p_3.cm
       OUTPUT FILE: rcsf.out
***********************************
>>rmixextract
RMIXEXTRACT
Extract and prints mixing coefficient above a
cut-off. Corresponding CSFs written to screen and
to rcsf.out
Input files: name.c, name.(c)m
Output file: rcsf.out
Name of state
>>2p_3
Mixing coefficients from CI calc. ?
Enter the cut-off value for the coefficients [0--1]
>>0.01
Sort extracted CSFs according to mixing coeffcients? (y/n)
        2
                         186
 nblock =
            ncftot =
                                nw =
                                         nelec =
______
                     76 \text{ nevblk} = 1 2J+1 =
 nb = 1 ncfblk =
                                            2 parity = -1
                                            2 parity = -1
 nb = 1 ncfblk =
                   76 \text{ nevblk} = 1 2J+1 =
 ______
 Average Energy = 8.4326211886741014
                                     ncf_reduced =
```

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```
Energy = -7.4042609950173972 Coefficients and CSF:
     1 0.998449
 1s (2) 2p-(1)
         1/2
          1/2-
       2 -0.034328
 2p-(1) 3s (2)
    1/2
          1/2-
     3 0.032596
 2p-(1) 3p (2)
    1/2 0
          1/2-
     4 0.023063
 2p-(1) 3p-(2)
    1/2
          1/2-
      5 0.010751
 2s (1) 2p-(1) 3s (1)
    1/2
         1/2
               1/2
               1/2-
             1
_____
 -----
Average Energy = 9.6081889296591605 ncf_reduced =
Energy = -7.4042596826542209 Coefficients and CSF:
       1 0.998449
 1s (2) 2p (1)
         3/2
          3/2-
       2 -0.034328
 2p (1) 3s (2)
    3/2
          3/2-
      3 0.032590
 2p (1) 3p (2)
    3/2 0
          3/2-
     4 0.023072
 2p (1) 3p-(2)
    3/2
           3/2-
RMIXEXTRACT: Execution complete.
*******************************
      RUN RMIXACCUMULATE TO ACCUMULATE CSFs CONTRIBUTING TO 99.99\%
      OF THE TOTAL WAVE FUNCTION CONTENT. CORRESPONDING CSFs WRITTEN TO
      rcsf.out
      INPUT FILES: 2p_3.c, 2p_3.cm
      OUTPUT FILE: rcsf.out
```

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\*

>>rmixaccumulate

\*

Welcome to program rmixaccumulate

The program accumulates dominating CSFs by mixing coefficients up to a user defined fraction of the total wave function.

The CSFs in the output list can be sorted by mixing coefficients to provide better initial estimates for the subsequent diagonalisation. of CI matrices.

Input files: <state>.(c)m, <state>.c

Output file: rcsf.out

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Give name of the state:

>>2p\_3

Expansion coefficients resulting from CI calculation (y/n)?

>>y

Fraction of total wave function [0-1] to be included in reduced list:

>>0.9999

CSFs in output file sorted by mixing coefficients (y/n)?

>>y

Block data read from mixing file

block	ncf	nev	2j+1	parity
1	76	1	2	-1
2	110	1	4	-1

Number of CSF:s written to rcsf.out

block ncf

WARNING! Not all peel subshells are occupied in the output CSF list:

Remove the following peel subshells:

3d-

For each symmetry block, only 7 CSFs are needed to account for 99.99% of the total wave function content. The extracted list of CSFs is shown below.

Core subshells:

Peel subshells:

1/2

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Indeed, we see that the 3*d*- orbital is not present in any of the extracted CSFs. If we are going to use the list of extracted CSFs for some other purposes, we should remove 3*d*- from the list of peel subshells. In Section 14.2 we discuss the use of rmixaccumulate for handling large CSF expansions.

# 7.3. Extracting Radial Orbitals for Plotting

The program rwfnplot extracts specified orbitals from a binary radial orbital file name. w and generates a Matlab/GNU Octave M-file (and also an Xmgrace file for convenience) that plots the large components of the radial orbitals as functions of r or  $\sqrt{r}$ . The Matlab/GNU Octave M-file is easy to edit to modify the appearances of the plots. By editing the Matlab/GNU Octave M-file, also the small component of the radial orbitals can be plotted. In this example, we extract the 1s, 2s and 2p orbitals from  $2s_2p_DF$ . w generated in the example 1, see Section 6.1.

# Program Input

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

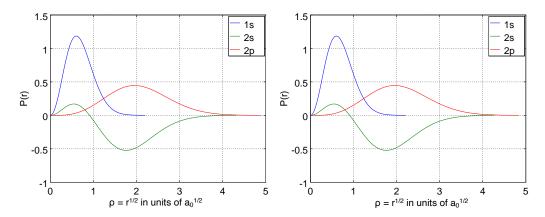
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******	***************************************	<b>k</b> *
*	MAKE SURE YOU ARE IN THE DIRECTORY WHERE THE FILES FROM THE FIRST EXAMPLE ARE	*
	**************************************	۳ *د
****	***************************************	. ~
	***************************************	*
*	RUN RWFNPLOT TO PLOT THE 1s, 2s, 2p ORBITALS INPUT FILE: 2s_2p_DF.w	*
*	OUTPUT FILE: octave_2s_2p_DF.m (Matlab/GNU Octave M-file)	*
******	***************************************	*
>>rwfnpl	ot	
RWFNPLO	T	
_	to generate Matlab/GNU Octave and	
•	e files that plot radial orbitals	
-	<pre>file: name.w files: octave_name.m, xmgrace_name.agr</pre>	
-	orbital: press enter ove orbital: type "d" or "D" and press enter	
	Jorgen Ekman Jun 2015	
Name of	state:	
>>2s_2p_	DF	
To have >>n	r on x-axis: type "y" otherwise "n" for sqrt(r)	
1s =		
>>		
2s =		
>>		
2p- =		
>>d 2p =		
>>		
FINISH	ED	
******	************************	*
*	START GNU OCTAVE BY TYPING octave	*
*****	***************************************	*
>>octave		
******	************************	*
*	AT THE GNU OCTAVE COMMAND LINE octave:1> INVOKE THE M-FILE	*
*	NOTE THAT ONLY THE FILE NAME AND NOT THE EXTENSION SHOULD BE GIVEN	*
******	***************************************	*

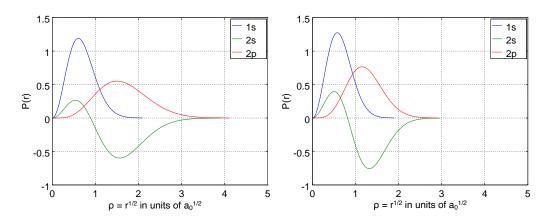
octave:1> octave\_2s\_2p\_DF

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Executed in Matlab or GNU Octave, the octave\_2s\_2p\_DF.m M-file gives the radial orbitals to the left in Figure 4. To the right, we have plotted converted MCHF orbitals. In Figure 5 we have, for comparison, plotted the Thomas-Fermi and the screened hydrogenic initial estimates of the radial orbitals. (To save an Octave figure to file click on File and Save As in the upper left corner of the graphical window. By saving the figures in pdf format, they can easily be imported in a LaTeX document.) By comparing Figures 4 and 5 one sees that Thomas-Fermi gives better initial estimates of the radial orbitals than do the screened hydrogenic estimates. Best initial estimates, however, are obtained from converted HF or MCHF orbitals.



**Figure 4.** Left: plot of the large components of the 1*s*, 2*s* and 2 $p_{3/2}$  orbitals in Li I as functions of  $\rho = \sqrt{r}$ , where *r* is in units of the Bohr radius  $a_0$ . Right: large components from converted MCHF orbitals.



**Figure 5.** Left: plot of the large components of the 1*s*, 2*s* and 2 $p_{3/2}$  orbitals in Li I as functions of  $\rho = \sqrt{r}$ , where *r* is in units of the Bohr radius  $a_0$ , from Thomas-Fermi estimates: **Right**: large components from screened hydrogenic estimates.

# 7.4. Output from rhfs with LSJ Labels

The program rhfs\_lsj reads data from name.(c)h. Using data available from name.lsj.lbl output is produced with LSJ-labels. We will use the program on the files from example 1, see Section 6.1.

#### **Program Input**

In the test-runs, prompt marked by >> or >>3, for example, indicates that the user should input 3 and then strike the return key. When >> is followed by blanks, just strike the return key.

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```
************************************
       RUN RHFS_LSJ TO OBTAIN OUTPUT WITH LSJ LABELS
       INPUT FILES: 2p_3.ch, 2p_3.lsj.lbl
       OUTPUT FILE: 2p_3.chlsj
**********************************
>>rhfs_lsj
RHFS_LSJ
This program prints output from the rhfs program
using LSJ lables. Output can be energy sorted
Input files: name.(c)h, name.lsj.lbl
Output file: name.(c)hlsj
Name of the state
>>2p_3
Hfs data from a CI calc?
>>y
Energy sorted output?
>>y
```

```
Energy State J P A(MHz) B(MHz) gJ -7.4042610 1s(2).2p_2P 1/2 - 4.482D+01 -0.000D+00 6.666573D-01 -7.4042597 1s(2).2p_2P 3/2 - -3.538D+00 -1.773D-01 1.333325D+00
```

# 7.5. Producing HFS Tables in LaTeX

The produced file 2p\_3.chlsj is shown below.

The program rtabhfs produces LaTeX files from output files from rhfs\_lsj. Before running rtabhfs, run rhfs\_lsj on 2s\_3 so that 2s\_3.chlsj is available. Now we will invoke rtabhfs to make a LaTeX file with hyperfine interaction constants and Landé  $g_I$ -factors for all states in the above files.

#### >>rtabhfs

#### RTABHFS

```
This program reads the output from rhfs_lsj and produces LaTeX tables of hfs data
Input files: name.(c)hlsj produced by rhfs_lsj
Output file: hfs.tex

How many HFS files ?
>>2
Full name of HFS file 1
```

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```
>>2s_3.chlsj
Full name of HFS file 2
>>2p_3.chlsj

Inspect the name.(c)hlsj files and determine how many positions should be skipped in the string that determines the label. For example if the string is 1s(2).2s_2S.2p(2)3P2_4P and 1s(2) is a core then you want to skip 1s(2). i.e., 6 positions

How many positions should be skipped?
>>0
```

The produced LaTeX file hfs.tex is shown below

```
\documentclass[10pt]{article}
\usepackage{longtable}
\begin{document}
\begin{longtable}{lrrrr} \midrule
State & $E$(a.u.) & $A$(MHz) & $B$(MHz) &
g_J \\ \midrule
$1s^2 \,2s^^2\!S_{ 1/2}$ &
                               -7.4719740 &
                                               3.885D+02 &
                                                              -0.000D+00 & 1.999985D+00 \\
1s^2 ,2p^2!P_{1/2}^o \
                                 -7.4042610 &
                                                4.482D+01 &
                                                                -0.000D+00 & 6.666573D-01 \\
1s^2 ,2p^2 !P_{ 3/2}^o &
                                 -7.4042597 &
                                                -3.538D+00 &
                                                                -1.773D--01 & 1.333325D+00 \\
\midrule\\
\caption{Hyperfine interaction constants}
\end{longtable}
\end{document}
```

The above LaTeX file generates Table 2.

**Table 2.** Hyperfine interaction constants.

State	E(a.u.)	A(MHz)	B(MHz)	g <sub>J</sub>
$1s^2 2s {}^2S_{1/2}$	-7.4719740	3.885D+02	-0.000D+00	1.999985D+00
$1s^2 2p {}^2P_{1/2}^o$	-7.4042610	4.482D+01	-0.000D+00	6.666573D-01
$1s^2 2p  ^2P_{3/2}^{o'}$	-7.4042597	-3.538D+00	-1.773D-01	1.333325D+00

# 7.6. Producing Energy Tables in LaTeX

The program rtablevels produces LaTeX and ASCII files from output files from rlevels. As a first simple example, we make a table of the energies from the calculations in example 1, see Section 6.1. It is possible to read many files from rlevels and make, for example, tables that show convergence of energy levels with respect to the increasing active set, see Section 9.4.

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```
RUN RTABLEVELS TO PRODUCE LATEX AND ASCII TABLES
         INPUT FILE: energy3
         {\tt OUTPUT\ FILES:\ energy table latex.tex,\ energy table ascii.txt}
*********************************
>>rtablevels
  RTABLEVELS
  Makes LaTeX and ASCII tables of energy files produced by
  rlevels (in ljs format)
  Multiple energy files can be used as input
  Energies from file 1 fills column 1, energies from file 2
  fills column 2 etc. Checks are done to see if the labels
  if the labels in the files are consistent
  Input file: name1, name2, ...
  Output files: energylabellatex.tex, energylabelascii.txt
  Inspect energy files and determine how many positions
  should be skipped in the string that determines the label
  e.g., if the string is 1s(2).2s_2S.2p(2)3P2_4P and 1s(2) is a core
  then you would like to skip 1s(2). i.e., 6 positions and determine
  the label from 2s_2S.2p(2)3P2_4P
  How many positions should be skipped?
>>0
 Give the number of energy files from rlevels
>>1
 Name of file 1
>>energy3
The produced file energytablelatex.tex is shown below
\documentclass[10pt]{article}
\usepackage{longtable}
\begin{document}
\begin{longtable}{lr} \midrule
$1s^2 \,2s^^2\!S_{ 1/2}$
                                          //
1/2 ^2 \p^2 !P_{ 1/2}^o  
                                          //
                                 14861
1s^2 ,2p^2 !P_{ 3/2}^o  
                                 14861
\midrule\\
\caption{Energies from the files energy3,}
\end{longtable}
\end{document}
```

\*

#### 7.7. Producing E1 Transition Tables in LaTeX

The program rtabtransE1 produces LaTeX and ASCII files from the nam1.name2.(c)t.lsj output file from rtransition (E1 transitions only). As an example, we make a table of the transition data from the 2s\_3.2p\_3.ct.lsj file in example 1, see Section 6.1. For additional use of rtabtrans1E1, see Section 9.5

```
********************************
      MAKE SURE YOU ARE IN THE DIRECTORY WHERE THE FILES FROM THE FIRST
      EXAMPLE ARE
*********************************
```

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```
**********************************
         RUN RTABTRANSE1 TO PRODUCE LATEX AND ASCII TABLES
         INPUT FILE: 2s_3.2p_3.ct.lsj
         OUTPUT FILES: transitiontable.tex, transitiontableascii.txt
**************************************
>>rtabtransE1
  RTABTRANSE1
  Makes LaTeX tables of transition data from transition files
  name1.name2.ct.lsj
  Input file: name1.name2.ct.lsj
  Output file: transitiontable.tex
  Specify table format
  (1). Lower & Upper & Energy diff. & wavelength & S & gf & A & dT
  (2). Lower & Upper & Energy diff. & wavelength & gf & A & dT
  (3). Lower & Upper & Energy diff. & wavelength & gf & A
  (4). Lower & Upper & Energy diff. & S & gf & A & dT
  (5). Lower & Upper & Energy diff. & gf & A & dT
  (6). Lower & Upper & Energy diff. & gf & A
>>5
  Inspect the name1.name2.ct.lsj file and determine how many positions
  should be skipped in the string that determines the label
  e.g., if the string is 1s(2).2s_2S.2p(2)3P2_4P and 1s(2) is a core
  then you would like to skip 1s(2). i.e., 6 positions and determine
  the label from 2s_2S.2p(2)3P2_4P
  How many positions should be skipped?
  Name of file
>>2s_3.2p_3.ct.lsj
The produced file transitiontable.tex is shown below
\documentclass[10pt]{article}
\usepackage{longtable}
\begin{document}
\begin{longtable}{llrrrr}
Lower state & Upper state & \C (cm^{-1}\) \ & $gf$ & $A$ (s$^{-1}\) & $dT$ \\ \midrule
$1s^2 \,2s^^2\!S_{1/2}$ & $1s^2 \,2p^^2\!P_{1/2}$ & 14861 & 5.087D-01 & 3.747D+07 &
                                                                                     0.017\\
1s^2 \,2s^2\!S_{1/2} & 1s^2 \,2p^2\!P_{3/2} &
                                                    14861 & 1.017D+00 & 3.747D+07 &
\midrule\\
\caption{Transition data from the file 2s_3.2p_3.ct.lsj}
\end{longtable}
\end{document}
```

**Table 3.** Transition data from the file 2s\_3.2p\_3.ct.1sj.

The above LaTeX file generates Table 3.

Lower State	Upper State	$\Delta E  (\mathrm{cm}^{-1})$	gf	$A (s^{-1})$	dT
$1s^2 2s {}^2S_{1/2}$	$1s^2 2p^2 P_{1/2}$	14861	5.087D-01	3.747D+07	0.017
$1s^2 2s  ^2S_{1/2}$	$1s^2 2p {}^2P_{3/2}$	14861	1.017D+00	3.747D+07	0.017

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#### 7.8. Handling Levels with the Same Quantum Labels

For more complex systems, it sometimes happens that two levels have the same dominating LSJ term. The two levels will then get the same quantum labels in the output from rlevels and rtransition. The user of the program thus needs to pay attention to this and make sure that the labels are unique. If two or more levels have the same quantum labels, open the name.lsj.lbl file and edit it so that all levels have unique quantum labels. After having done these changes in the label file, rerun rlevels and rtransition and the changes will be reflected in the output files. Subsequent runs of rtablevels and rtabtransE1 will use the new quantum labels. Alternatively, to avoid problems with quantum states having the same label, use the unique label option of jj2lsj as described in Section 6.5. If the user does not want to edit the name.lsj.lbl file manually to get unique quantum labels or use the unique label option of jj2lsj he or she can still run rtablevels and rtabtrans1 (to be discussed in the next section). These two programs identify all cases with labels that are not unique and resolve them by adding indices a, b, c etc. at the end of the quantum labels so that the labels are unique.

#### 7.9. Producing Transition and Lifetime Tables in LaTeX

The programs rtabtrans1 and rtabtrans2 produce a LaTeX transition table file and a lifetime file from the \*.(c)t output file from rtransition. In case of several runs for different multipoles with rtransition all the different \*.(c)t output files can be concatenated before processing. For example: if we have one file name1.name2.ct with transition data from a run for E1 transitions between states in the two files name1 and name2 and one file name1.ct from a run for M1 transitions between states in the file name1 the transition files can be concatenated (any name can be used for the concatenated file)

```
cat name1.name2.ct name1.name1.ct > E1M1.ct
```

By processing the concatenated file E1M1.ct a LaTeX transition table is produced for all E1 and M1 transitions.

As an example, we make a table of the transition data from the even4.odd4.ct file in the fourth example, see Section 6.4. In addition, we produce a lifetime table. More accurate values of the lifetime would require that we also include M1, E2 transitions between states of the same parity. We neglect this for simplicity. First, we create energy label data for all the levels involved in the transitions. These data are obtained by processing the even4.cm and odd4.ct files.

```
MAKE SURE YOU ARE IN THE DIRECTORY WHERE THE FILES FROM THE FOURTH *
        EXAMPLE ARE
***********************************
*************************************
        RUN RTABTRANS1 TO PRODUCE AN ENERGY LABEL FILE NEEDED FOR
        FURTHER PROCESSING WITH RTABTRANS2
        INPUT FILE: even4.cm, odd4.cm
        OUTPUT FILES: energylabel.latex
***********************************
>>rtabtrans1
 RTABTRANS1
 This program creates a file energylabel that is
 used by RTABTRANS2 to produce LaTeX tables of
 transition data
 Input files: mixing coefficient files
      name1.(c)m, name2.(c)m,.... for the wave-
      functions that are used to compute the
      transition data
 Output file: energylabel.latex(ascii)
```

Type the full input file name, one for each line (NULL to terminate)

\*

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```
File name ?
>>even4.cm
  File name ?
>>odd4.cm
  File name ?
>>
```

Inspect the labels of the states and determine how many positions should be skipped in the string that determines the label. For example if all the states have a common core 1s(2) in the label then 6 positions should be skipped

How many positions should be skipped?

>>12

Output labels in LaTeX or ASCII format (0/1)?

>>0

Energy label data written to file
energylabel.latex(ascii)

The produced file energylabel.latex is shown below

```
      nblock =
      4 ncftot =
      5712 nw =
      16 nelec =
      12

      nblock =
      5 ncftot =
      7100 nw =
      16 nelec =
      12
```

Energy levels for ...

Rydberg constant is 109737.31534

No - Serial number of the state; Pos - Position of the state within the

J/P block;

No	Pos	J Parity Energy Total (a.u.)		<i>y</i> 0 <i>y</i>	Levels (cm^-1)	File	Configuration
1	1	0	+	-1182.4117992	0.00	even4	\$3s^2 ~^1\!S_{ 0 }\$
2	1	0	-	-1181.3459632	233923.97	odd4	\$3s~^2\!S3p~^3\!P_{ 0 }^o\$
3	1	1	-	-1181.3193175	239772.02	odd4	\$3s~^2\!S3p~^3\!P_{ 1 }^o\$
4	1	2	-	-1181.2548318	253925.01	odd4	\$3s~^2\!S3p~^3\!P_{{ 2 }^o\$
5	2	1	-	-1180.8025239	353195.12	odd4	\$3s~^2\!S3p~^1\!P_{ 1 }^o\$
6	2	0	+	-1179.8828042	555050.24	even4	\$3p^2 (^3_2P)~^3\!P_{ 0 }\$
7	1	2	+	-1179.8592139	560227.72	even4	\$3p^2 (^1\!_2D)~^1\!D_{ 2 }\$
8	1	1	+	-1179.8374539	565003.49	even4	\$3p^2 (^3_2P)~^3\!P_{ 1 }\$
9	2	2	+	-1179.7587696	582272.71	even4	\$3p^2 (^3_2P)~^3\!P_{ 2 }\$
10	3	0	+	-1179.3935747	662423.72	even4	\$3p^2 (^1\!_0S)~^1\!S_{ 0 }\$
11	2	1	+	-1179.3158027	679492.71	even4	\$3s~^2\!S3d~^3\!D_{ 1 }\$
12	3	2	+	-1179.3111395	680516.16	even4	\$3s~^2\!S3d~^3\!D_{ 2 }\$
13	1	3	+	-1179.3038352	682119.26	even4	\$3s~^2\!S3d~^3\!D_{ 3 }\$
14	4	2	+	-1178.9201602	766326.18	even4	\$3s~^2\!S3d~^1\!D_{ 2 }\$
15	2	2	-	-1178.1773931	929344.72	odd4	\$3p~^2\!P3d~^3\!F_{ 2 }^o\$
16	1	3	-	-1178.1321370	939277.28	odd4	\$3p~^2\!P3d~^3\!F_{ 3 }^o\$
17	3	2	-	-1178.0860896	949383.53	odd4	\$3p~^2\!P3d~^1\!D_{ 2 }^o\$
18	1	4	-	-1178.0797665	950771.28	odd4	$3p^2!P,3d^{3}!F_{4} $
19	3	1	-	-1177.9273160	984230.30	odd4	\$3p~^2\!P3d~^3\!D_{ 1 }^o\$
20	4	2	-	-1177.9244263	984864.51	odd4	\$3p~^2\!P3d~^3\!P_{ 2 }^o\$
21	2	3	-	-1177.8730161	996147.75	odd4	\$3p~^2\!P3d~^3\!D_{ 3 }^o\$
22	2	0	-	-1177.8671996	997424.33	odd4	\$3p~^2\!P3d~^3\!P_{ 0 }^o\$

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```
-1177.8658739
23 4
      1 –
                         997715.29 odd4
                                                     3p^2 !P\,3d^3 !P_{ 1 }^o
24 5
      2 -
           -1177.8645522
                         998005.36 odd4
                                                     3p^2 !P\,3d^3 !D_{2} }^0
25 3
      3 -
          -1177.5443213 1068287.93 odd4
                                                     $3p~^2\!P\,3d~^1\!F_{ 3 }^o$
      1 -
            -1177.4837515 1081581.45 odd4
26 5
                                                     3p^2 \ln 3d^1 \ln 3
```

The LaTeX strings in the column to the right will be used as labels in the transition table and in the lifetime table. The user might want to edit the strings by, for example, removing unnecessary quantum labels. In the case above, we do the global substitutions:  $3s^22!S, \rightarrow 3s$  and  $3p^2!N, \rightarrow 3p$ .

#### >>rtabtrans2

#### RTABTRANS2

This program reads energy label data and transition data and creates transition and lifetime tables in LaTeX or ASCII format. An Octave file with a scatterplot of dT and 10log(A) is also produced

Energy label data are given in the file energylabel created by the rtabtrans1 program

Transition data file can be concatenated \*.t or \*.ct files.

Give the name of the transition data file
>>even4.odd4.ct
Energy label file in LaTeX or ASCII format (0/1)?
>>0
Give cut-off for printing A values
>>1e4
Give fraction of accumulated A value for upper level
for printing A value of a transition
>>1e-4
Transition data wavelength sorted?
>>y
Give number of decimals for wavelength (1,...6)

Mean dT 5.5603015391651460E-002

>>3

Program finished. The transition tables in latex have been written to file

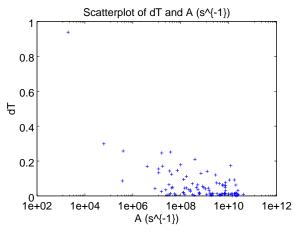
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Observe that there are two available criteria for selecting transitions to be printed: one criterion that is based on the value of A itself and one criterion that is related to the accumulated A value for the upper level. If a transition satisfies one of the two criteria, the A value is printed. The second criterion makes it possible to select important decay channels from metastable states without having to print weak transitions from all states. A value greater than 1 for the second criterion means that printing will be based only on the first criterion, i.e., transitions will be printed if they have A values larger than the cut-off. When processing the file transitiontable.tex we get Table 4 below. Please note that we edited the file energylabel.latex before running rtabtrans2. Transition rates and gf values are given in length gauge for electric transitions. dT is a measure of the uncertainty of the electric transitions given by  $dT = \frac{|A_C - A_B|}{\max(A_C, A_B)}$ , see TP Section 3.5.

Upper	Lower	EM	$\Delta E \ (\text{cm}^{-1})$	λ (Å)	$A (s^{-1})$	gf	dT
$3p3d\ ^{1}P_{1}^{o}$	$3s^2  {}^1S_0$	E1	1081581	92.457	1.717E+08	6.602E-04	0.112
$3p3d  {}^{3}P_{1}^{0}$	$3s^2  {}^1S_0$	E1	997715	100.229	6.158E+04	2.782E-07	0.299
$3s3d\ ^{1}D_{2}$	$3s3p  ^3P_1^o$	E1	526554	189.914	3.634E+08	9.826E-03	0.003
$3p3d\ ^{1}P_{1}^{o}$	$3p^{2}(_{2}^{3}P)^{3}P_{0}$	E1	526531	189.922	2.515E+08	4.080E-03	0.034
$3p3d\ ^{1}P_{1}^{0}$	$3p^2(\frac{1}{2}D) {}^1D_2$	E1	521353	191.808	3.915E+08	6.478E-03	0.210
$3p3d\ ^{1}P_{1}^{o}$	$3p^2(\frac{3}{2}P)^{3}P_1$	E1	516577	193.582	6.170E+07	1.040E-03	0.005
$3s3d\ ^{1}D_{2}$	$3s3p^{-3}P_2^0$	E1	512401	195.160	1.565E+07	4.469E-04	0.025
$3p3d\ ^{1}F_{3}^{o}$	$3p^{2}(^{1}_{2}D)^{1}D_{2}$	E1	508060	196.827	1.994E+10	8.106E-01	0.063
	- · · · ·		:				
$3p3d\ ^{1}D_{2}^{o}$	$3s3d\ ^{1}D_{2}$	E1	183057	546.277	6.962E+08	1.557E-01	0.130
$3p3d  {}^{3}F_{3}^{\bar{o}}$	$3s3d\ ^{1}D_{2}$	E1	172951	578.198	1.172E+07	4.112E-03	0.155
3p3d <sup>3</sup> F <sub>2</sub> <sup>o</sup>	$3s3d\ ^{1}D_{2}$	E1	163018	613.428	7.152E+07	2.017E-02	0.146

Table 4. Transition data.

When processing the file lifetimetable.tex we get Table 5. Please note that we edited the file energylabel before running rtabtrans2. Transition rates for electric transitions that enter the calculation of the lifetimes are in Babushkin (length) and Coulomb (velocity) gauges. The rtabtrans2 produces also an M-file scatterplot.m that, when executed under GNU Octave or Matlab, produces a dT and A scatter plot. The scatter plot is shown in Figure 6 and indicates that dT is smaller, on the average, for the stronger transitions than for the weaker ones.



**Figure 6.** Scatterplot of dT and  $A(s^{-1})$ .

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**Table 5.** Lifetimes in s.

State	$\tau_l$ (s)	$ au_v$ (s)
3s3p <sup>3</sup> P <sub>1</sub> <sup>0</sup>	2.3678E-08	2.2603E-08
$3s3p^{3}P_{2}^{0}$	2.9430E-01	2.9430E-01
$3s3p^{-1}P_{1}^{o}$	4.6050E-11	4.6043E-11
$3p^{2}  {}^{3}P_{0}^{1}$	5.7890E-11	5.7208E-11
$3p^2  {}^1\!D_2$	2.0563E-10	2.0099E-10
$3p^2  ^3P_1$	5.4422E-11	5.3844E-11
$3p^2  ^3P_2$	5.9938E-11	5.9389E-11
$3p^2  {}^1S_0$	5.1955E-11	5.1832E-11
	<b>:</b>	
$3p3d\ ^{3}P_{2}^{o}$	3.0579E-11	3.1347E-11
$3p3d  {}^{3}D_{3}^{0}$	2.5051E-11	2.5304E-11
$3p3d\ ^{3}P_{0}^{o}$	3.2589E-11	3.4062E-11
$3p3d  {}^{3}P_{1}^{o}$	3.0190E-11	3.1215E-11
$3p3d  {}^{3}D_{2}^{0}$	2.8117E-11	2.8766E-11
$^{3}p3d\ ^{1}F_{3}^{\tilde{o}}$	2.3319E-11	2.3243E-11
$3p3d  {}^{1}P_{1}^{o}$	2.6514E-11	2.8629E-11

# 7.10. Producing Energy Tables with LS-Composition in LaTeX

The PERL script lscomp.pl creates a LaTeX file lscomp.tex which contains level information with the dominating LS-component and up to two extra LS-components if their contributions to the total wave function exceed 0.02 along with energies and, optionally, Landé  $g_J$ -factors. In addition, a file energylabel is created, which may be used together with rtabtrans2 for the creation of a LaTeX file with transition data.

To be able to run the script, PERL has to be installed, see <a href="https://www.perl.org/get.html">https://www.perl.org/get.html</a> (accessed on 4 November 2022)

To run the script copy lscomp.pl to the working directory and type:

```
>>perl lscomp.pl
```

However, assuming that the script lscomp.pl is located in the \$HOME/GRASP2018/bin directory, the following line may be added to the .profile or .bashrc file:

alias perl\_lscomp='perl \$HOME/GRASP2018/bin/lscomp.pl'

In this case, to run the script from any working directory simply type:

>>perl\_lscomp

As an example we make a table of the LS-compositions, energies and Landé  $g_J$ -factors in the fourth example, see Section 6.4. The table is obtained by processing the even4 and odd4 files.

! For even 4

>>rhfs

RHFS

This is the hyperfine structure program

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```
Input files: isodata, name.c, name.(c)m, name.w
 Output files: name.(c)h, name.(c)hoffd
 Default settings?
>>y
 Name of state
>>even4
 Mixing coefficients from a CI calc.?
>>y
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
 There are 5712 relativistic CSFs;
  ... load complete;
 Loading Radial WaveFunction File ...
   nelec =
                      12
   ncftot =
                    5712
                     16
   nw
   nblock =
                       4
   block
           ncf
                    nev
                           2j+1 parity
            556
      1
                      3
                              1
                                      1
       2
          1448
                      2
                             3
                                      1
                              5
       3
           1898
                      4
                                      1
       4
           1810
                      1
                              7
                                      1
 Column 100 complete;
 Column 200 complete;
 Column 300 complete;
Column 5600 complete;
 Column 5700 complete;
 RHFS: Execution complete.
! For odd4
>>rhfs
 RHFS
 This is the hyperfine structure program
 Input files: isodata, name.c, name.(c)m, name.w
 Output files: name.(c)h, name.(c)hoffd
 Default settings?
>>y
Name of state
>>odd4
 Mixing coefficients from a CI calc.?
>>y
```

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```
Loading Configuration Symmetry List File ...
There are 16 relativistic subshells;
There are 7100 relativistic CSFs;
  ... load complete;
Loading Radial WaveFunction File ...
   nelec =
                  12
                 7100
   ncftot =
                  16
   ทพ =
   nblock =
                    5
                       2j+1 parity
  block
          ncf nev
                  2
      1
          546
                          1
                                 -1
                          3
      2
         1456
                   5
                                 -1
                          5
                                 -1
                   5
      3
        1891
      4 1814
                   3
                          7
                                -1
      5
         1393
                   1
                          9
                                -1
Column 100 complete;
Column 200 complete;
Column 300 complete;
Column 7000 complete;
Column 7100 complete;
RHFS: Execution complete.
        RUN LSCOMP.PL TO PRODUCE A LATEX FILE 1scomp.tex WITH ENERGY LEVEL *
        INFORMATION WITH DOMINATING LS-COMPONENT AND UP TO TWO EXTRA
        LS-COMPONENTS. IN ADDITION THE FILE energylabel.latex IS PRODUCED
        WHICH CAN BE USED FOR FURTHER PROCESSING WITH RTABTRANS2
        INPUT FILES: even4.lsj.lbl, even4.ch
                    odd4.lsj.lbl, odd4.ch
        OUTPUT FILES: lscomp.tex, energylabel.latex
>>perl_lscomp
  LSCOMP.PL
  This PERL script creates files lscomp.tex and energylabel.latex
  File lscomp.tex contains energy level data with up to
  three LS components with a contribution > 0.02 of the
  total wave function.
  File energylabel.latex may be used by RTABTRANS2 to produce
  LaTeX tables of transition data.
  Input files : state1.lsj.lbl and state2.lsj.lbl
               state1.ch and state2.ch (optional for gJ-factors)
  Output files: lscomp.tex and energylabel.latex
```

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```
State 1?
>>even4
       State 2?
>>odd4
       Necessary input file(s) exist!
       Do you want to include Lande g_J factors in the energy table? (y/n)
>>y
       Lande g_J factors from a CI calculation? (y/n)
       File(s) with g_J factors exist!
       Do you want an extra empty column for e_obs in the energy table? (y/n)
>>y
        Inspect the labels of the states and
        determine how many positions should be skipped in
        the string that determines the label. For example
        if all the states have a common core 1s(2) in the
       label then 6 positions should be skipped
       How many positions should be skipped?
>>12
       Files lscomp.tex and energylabel.latex written to disc.
The produced file lscomp.tex, slightly edited, is shown below.
\documentclass[12pt]{article}
\usepackage{longtable}
\usepackage[cm]{fullpage}
\thispagestyle{empty}
\begin{document}
{\c criptsize}
\begin{longtable}{@{}rllrrr}
\color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \color= \col
\midrule
No. & State & $LS$-composition & $E(CI) $ & $E(OBS) $ & $g_J $ \
\midrule
\endfirsthead
\caption{Continued.}\\
\midrule
No. & State & $LS$-composition & $E(CI) $ & $E(OBS) $ & $g_J $ \
\midrule
\endhead
\midrule
\endfoot
1 & $3s^{2}~^{1}\!S_{0}$
                                                                                   & 0.97 + 0.02^{3}p^{2}(^{1}_{0}S)^{^{1}S}
                                                                                                                                                                          & 0
                                                                                                                                                                                                                             11
      & $3s~^{2}S\,3p~^{3}P_{0}^{o}$ & 1.00
                                                                                                                                                              & 233~924
                                                                                                                                                                                     & &
3
       & $3s^^{2}S\,3p^^{3}P_{1}^{o}$ & 0.99
                                                                                                                                                              & 239<sup>~</sup>772
                                                                                                                                                                                      & & 1.49513
                                                                                                                                                                                                                 11
        & 3s^{2}S\,3p^{3}P_{2}^{0} & 1.00
                                                                                                                                                              & 253<sup>~</sup>925
                                                                                                                                                                                     &
                                                                                                                                                                                           & 1.49886
      & $3s~^{2}S\,3p~^{1}P_{1}^{0}$ & 0.97 + 0.02~$3p~^{2}P\,3d~^{1}P^{0}$ & 353~195 & & 1.00254 \\
                      23 & 3p^{2}P\3d^{3}P_{1}^{0} & 0.75 + 0.243p^{2}P\3d^{3}D^{0} & 997~715 & 4.25688 \
24 & 3p^{2}P\3d^{3}D_{2}^{0} & 0.54 + 0.45 $3p^{2}P\,3d^{3}P^{0}$ & 998 005 & 1.31616 \
25 & 3p^{2}P\,3d^{1}F_{3}^{o} & 0.99
                                                                                                                                                              & 1~068~288 & & 1.00120 \\
26 \& \$3p^{2}P\3d^{1}P_{1}^{o}\$ \& 0.96 + 0.02^{\$3s^{2}S\3p^{1}P^{o}\$ \& 1^{0}81^{5}81 \& \& 0.9972 \setminus 10^{1}P^{0}\$ \& 1^{0}81^{1}P^{0}\$ \& 1^{0}81^{1}P^{0} \& 0.9972 \setminus 10^{1}P^{0}
```

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\midrule
\end{longtable}
}
\end{document}

After manually including observed energies from the NIST tables and editing the caption, we obtain Table 6.

**Table 6.** Energies in cm<sup>-1</sup> and Landé  $g_I$ -factors for states in Mg-like iron.

No.	State	LS-Composition	E(CI)	E(OBS)	
1	$3s^2  {}^1S_0$	$0.97 + 0.02 \ 3p^2 \binom{1}{0}S) \ ^1S$	0		
2	$3s^{2}S 3p^{3}P_{0}^{o}$	1.00	233 924	233 842	
3	$3s {}^{2}S 3p {}^{3}P_{1}^{0}$	0.99	239 772	239 660	1.49513
4	$3s^{2}S 3p^{3}P_{2}^{0}$	1.00	253 925	253 820	1.49886
5	$3s {}^{2}S 3p {}^{1}P_{1}^{0}$	$0.97 + 0.02  3p^{2}P  3d^{1}P^{o}$	353 195	351 911	1.00254
6	$3p^2({}_{2}^{3}P) {}^{3}P_{0}$	$0.96 + 0.03 \ 3p^2({}_{0}^{1}S) \ {}^{1}S$	555 050	554 524	
7	$3p^2({}_2^1D) {}^1D_2$	$0.66 + 0.18 3p^{2}({}_{2}^{3}P)^{3}P + 0.16 3s^{2}S 3d^{1}D$	560 227	559 600	1.09056
8	$3p^2(^3_2P) ^3P_1$	1.00	565 003	564 602	1.49901
9	$3p^2({}_2^3P) {}^3P_2$	$0.81 + 0.14  3p^2 ({}^1_2D)  {}^1\!D + 0.04  3s  {}^2\!S  3d  {}^1\!D$	582 272	581 803	1.40706
10	$3p^2({}^1_0S)\ {}^1\!S_0$	$0.93 + 0.03 3p^{2}({}_{2}^{3}P) {}^{3}P + 0.02 3d^{2}({}_{0}^{1}S) {}^{1}S$	662 423	659 627	
11	$3s  {}^{2}\!S  3d  {}^{3}\!D_{1}$	1.00	679 493	678 772	0.49922
12	$3s {}^{2}S 3d {}^{3}D_{2}$	1.00	680 516	679 785	1.16567
13	$3s  {}^{2}S  3d  {}^{3}D_{3}$	1.00	682 119	681 416	1.33238
14	$3s  {}^{2}\!S  3d  {}^{1}\!D_{2}$	$0.79 + 0.20 \ 3p^2({}_2^1D) \ {}^1D$	766 326	762 093	0.99940
15	3p <sup>2</sup> P 3d <sup>3</sup> F <sub>2</sub> °	$0.87 + 0.13  3p^{2}P^{3}d^{1}D^{o}$	929 344	928 241	0.71134
16	$3p^{2}P 3d^{3}F_{3}^{0}$	0.98	939 277	938 126	1.08503
17	$3p^{2}P3d^{1}D_{2}^{o}$	$0.83 + 0.12  3p^2 P  3d^3 F^o + 0.03  3p^2 P  3d^3 P^o$	949 383	948 513	0.97509
18	3p <sup>2</sup> P 3d <sup>3</sup> F <sub>4</sub> <sup>o</sup>	1.00	950 771	949 658	1.24906
19	$3p^{2}P 3d^{3}D_{1}^{o}$	$0.75 + 0.24  3p^2 P  3d^3 P^o$	984 230	982 868	0.74338
20	$3p^{2}P 3d^{3}P_{2}^{0}$	$0.51 + 0.45  3p^2 P  3d^3 D^o + 0.03  3p^2 P  3d^1 D^o$	984 864	983 514	1.32709
21	$3p^{2}P3d^{3}D_{3}^{o}$	0.98	996 147	994 852	1.32759
22	$3p^{2}P 3d^{3}P_{0}^{o}$	1.00	997 424	995 889	
23	$3p^{2}P 3d^{3}P_{1}^{0}$	$0.75 + 0.24  3p^2 P  3d^3 D^o$	997 715	996 243	1.25688
24	$3p^{2}P3d^{3}D_{2}^{10}$	$0.54 + 0.45  3p^2 P  3d^3 P^o$	998 005	996 623	1.31616
25	$3p^{2}P 3d^{1}F_{3}^{o}$	0.99	1 068 288	1 062 515	1.00120
26	$3p^{2}P3d^{1}P_{1}^{o}$	$0.96 + 0.02  3s^{2}S  3p^{1}P^{o}$	1 081 581	1 074 887	0.99722

Some states are strongly mixed in LS-coupling. For example, the states 20 and 24 are an almost equal mix of  ${}^{3}P_{2}^{o}$  and  ${}^{3}D_{2}^{o}$ . The mixing is also reflected in the Landé  $g_{J}$ -factors which for these states are far from their pure LS values. If desired, one can apply global substitutions in the LaTeX file to get the quantum labels in the desired form.

# 7.11. Using rasfsplit to Split Files Defining ASFs in Symmetry Blocks

The program rasfsplit splits the files defining a number of ASFs of different blocks (J and parity) into groups of files, one for each symmetry block. Such a splitting would make it possible to distribute computation (of transition properties, for instance) on different computer systems. Calculations of transition rates for one combination of J and parity may be performed on one computer system (perhaps using MPI codes), while calculations of transition rates for another combination of J and parity may be performed on another computer system.

#### Overview

- 1. Split the ASFs defined by the files 2s22p3\_2p5\_3.c, 2s22p3\_2p5\_3.w, 2s22p3\_2p5\_3.m, 2s22p3\_2p5\_3.cm of the third example, see Section 6.3
- 2. Display the energies with J = 3/2.

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# Program Input

# >>rasfsplit

#### RASFSPLIT

Splits an Atomic State Function made up by the files name.c, name.(c)m, name.w into the corresponding files for each parity and J block. If only the name.c file is available this file will be split

Input files: name.c,name.(c)m, name.w

Output files: name\_even1.c, name\_even1.(c)m. name\_even1.w name\_odd1.c, name\_odd1.(c)m. name\_odd1.w ...

Name of the state >>2s22p3\_2p5\_3

Each of the blocks must be built from the same orbital set
This may not be true for MR expansions, but is normally true
for SD-MR expansions

Is the above condition fullfilled? (y,n)

>>y

nblock3nblockodd3nblockeven0

File 2s22p3\_2p5\_3.m available

nelec = 7
ncftot = 1165
nw = 9
nvectot = 7
nvecsize = 3212
nblock = 3

Block data read from mixing file

block	ncf	nev	2j+1	parity
1	274	2	2	1
2	591	4	4	1
3	300	1	6	1

File 2s22p3\_2p5\_3.cm available

nelec = 7 ncftot = 1165 Atoms 2023, 11, 68 238 of 369

nw	=	9
nvectot	=	7
${\tt nvecsize}$	=	3212
nblock	=	3

Block data read from mixing file

	bloc	:k	ncf		nev		2j+1	parity	7
	1		274		2		2	-1	
	2		591		4		4	-1	
	3		300		1		6	-1	
Exit	status	of the	name.w	file	copying	for	block	1 was	3

Exit	status	of	the	name.w	file	copying	for	block	1	was	0
Exit	status	of	the	${\tt name.w}$	${\tt file}$	copying	for	block	2	was	0
Exit	status	of	the	name.w	file	copying	for	block	3	was	0

There are three blocks of odd parity and program has produced the files:

```
2s22p3_2p5_3_odd1.c, 2s22p3_2p5_3_odd1.w, 2s22p3_2p5_3_odd1.m, 2s22p3_2p5_3_odd1.cm
2s22p3_2p5_3_odd2.c, 2s22p3_2p5_3_odd2.w, 2s22p3_2p5_3_odd2.m, 2s22p3_2p5_3_odd2.cm
2s22p3_2p5_3_odd3.c, 2s22p3_2p5_3_odd3.w, 2s22p3_2p5_3_odd3.m, 2s22p3_2p5_3_odd3.cm
```

The files with the extension odd1 define the ASFs with J = 1/2 and the files with the extension odd2 define the ASFs with J = 3/2 etc. To see the energies of the ASFs produced by the rci program with J = 3/2 odd parity we give the command

```
rlevels 2s22p3_2p5_3_odd2.cm
```

and get the result

```
nblock = 1 ncftot = 591 nw = 9 nelec = 7
```

Energy levels for ...

Rydberg constant is 109737.31569

No - Serial number of the state; Pos - Position of the state within the J/P block; Splitting is the energy difference with the lower neighbor

.-----

No	Pos	J Parit	y Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	
1	1	3/2 -	-263.2797841			
2	2	3/2 -	-262.9550555	71269.67	71269.67	
3	3	3/2 -	-262.7882742	107873.94	36604.27	
4	4	3/2 -	-259.5241179	824273.45	716399.51	

# 8. Interpreting the Output Files

In this section, we describe in detail what information can be found in the different output files and how this information should be interpreted

# 8.1. Output Files from the First Example

The Isodata File

Below is the isodata file for the Li example, Section 6.1.

Atomic number:

3.0000000000000000

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The calculation was for  ${}^7\text{Li}$  with Z=3 and A=7. The nuclear charge distribution  $\rho(r)$  was modelled as an extended Fermi distribution with

 $\rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}} \tag{5}$ 

The parameters a and c are computed from the root mean squared radius and the skin thickness. The root mean squared radius is taken from the tables of Angeli and Marinova [Atomic Data and Nuclear Data Tables Volume 99, Issue 1, 69-95, (2013)]. This gives a = 0.52338755531043146 fm and c = 1.2520789669753825 fm. On the lines following these quantities, the nuclear mass and nuclear spin I are given, along with the nuclear magnetic dipole moment  $\mu$  in nuclear magnetons and the nuclear quadrupole moment Q in barns.

The rcsfgenerate Log-File

After each rcsfgenerate run, there is a log-file displaying the response to the different questions. Below is the rcsfgenerate.log file from the n=3 complete active space expansion for  $1s^22p$   $^2P^o_{1/2,3/2}$ .

The log-file is a copy of the input data. By executing the command

```
rcsfgenerate < rcsfgenerate.log
```

the rcsf.out file will be reproduced. The rcsfgenerate.log file can easily be edited to give a new list of CSFs. For example

```
*! Orbital order

0! Selected core

1s(2,*)2p(1,*)

*

4s,4p,4d,4f

1 3! Lower and higher 2*J

3! Number of excitations
```

will give a file rcsf.out with CSFs corresponding to an active set n = 4.

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#### The CSF File

The rcsfgenerate program produces an rcsf.out file. The file has a header with information about the radial orbitals and the closed shells (core shells). After this information, there is a list of CSFs. The CSFs are ordered in blocks with specified value of *J*. Each block is separated by a line with an asterisk. Below is the file 2p\_3.c.

Core subshells:

The line with core subshells is empty, and in this case we have no closed core. The radial orbitals are 1s, 2s, 2p-, 2p, 3s, 3p-, 3p, 3d-, 3d. After the radial orbitals, there are lists of CSFs arranged in blocks. The first block of CSFs has J = 1/2. The second block has J = 3/2. An asterisk is separating the blocks. In the file, each CSF occupies three lines. On the first line the subshells and their occupations are listed in a linear form where, for example,  $1s^2$  becomes 1s (2). The second line shows the coupling of each subshell to a J quantum number, and the third line shows how the J quantum numbers of each subshell are coupled from left to right to a final J, see TP Section 2.4.

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The rangular Log-File

The program rangular produces a log-file displaying the response to the different questions. After running rsave this file is saved in name.alog. Below is the log-file 2p\_3.alog

```
y ! Full interaction
```

The log-file is a copy of the input data. We see that angular data were computed for all interactions. The log-file is more useful in cases where we do not have full interaction, see Section 14.1. In these cases, the file contains information about the zero-order space.

The rmcdhf Log-File

The SCF program rmcdhf produces a log-file displaying the response to the different questions. After running rsave this file is saved in name.log. Below is the log-file 2p\_3.log from the run on weighted average of the  $1s^22p$   $^2P_{1/2,3/2}^o$  states.

```
y ! Default settings

1

5 ! level weights

3*
```

The log-file is a copy of the input data. We see that the run was with default settings (there will be no log-file for non-default settings). It was a calculation targeting the first levels of the two blocks (ASF serial numbers 1 for each of the two blocks). The level weight is 5 (default option), meaning that the levels, in the energy functional, are weighted according to the statistical weight 2J + 1, see TP Section 2.7, e.g., (44). On the line that follows, 3\* means that orbitals with principal quantum numbers 3 are optimized. The blank line indicates that none of the optimized orbitals are spectroscopic. 100 SCF cycles were requested. By executing the command

```
rmcdhf < rmcdhf.log (or rmcdhf < name.log)</pre>
```

the rmcdhf run will be executed again with the settings in rmcdhf.log. The log-file can easily be edited and used as an input also to other runs.

The rmcdhf Summary File

The SCF program rmcdhf produces a summary file. After running rsave this file is saved in name.sum. Below is the summary file  $2p_3$ .sum from the run on weighted average of the  $1s^22p^2P_{1/2,3/2}^0$  states.

```
There are 3 electrons in the cloud
  in 186 relativistic CSFs
  based on 9 relativistic subshells.
The atomic number is
                     3.000000000;
the mass of the nucleus is 1.264966898269D+04 electron masses;
 Fermi nucleus:
  c = 3.612753059646D-05 Bohr radii,
  a = 9.890591370096D-06 Bohr radii;
  there are 82 tabulation points in the nucleus.
Speed of light = 137.0359991390D+00 atomic units.
Radial grid: R(I) = RNT*(exp((I-1)*H)-1), I = 1, ..., N;
    = 6.66666666667D-07 Bohr radii;
RNT
        5.00000000000D-02 Bohr radii;
N
        590:
```

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```
R(2) = 3.418073091735D-08 Bohr radii;
R(N) = 4.110372988964D+06 Bohr radii.

EOL calculation.
2 levels will be optimised;
their indices are: 1, 1.
Each is assigned its statistical weight;
```

# Radial wavefunction summary:

Subshe	ll e	p0 g	amma	P(2)	Q(2)	Self Consistency	MTP	
1s	2.5177395314D+00	9.280D+00	1.00	3.172D-07	-3.513D-12	0.000D+00	355	
2s	1.9634308400D-01	1.452D+00	1.00	4.965D-08	-5.499D-13	0.000D+00	361	
2p-	1.2867248397D-01	5.116D-05	1.00	2.518D-15	1.598D-10	0.000D+00	366	
2p	1.2866992757D-01	4.265D-01	2.00	4.983D-16	-5.519D-21	0.000D+00	366	
3s	8.0600844816D+00	1.181D+01	1.00	5.027D-07	-7.981D-13	8.965D-08	360	
3p-	8.7786093395D+00	2.853D-03	1.00	1.994D-13	8.921D-09	8.940D-07	364	
3р	8.7823537644D+00	2.381D+01	2.00	2.786D-14	-1.173D-18	1.303D-06	364	
3d-	1.6298092328D+01	8.146D-03	2.00	2.740D-20	1.739D-15	6.785D-06	358	
3d	1.6306599649D+01	8.169D+01	3.00	3.262D-21	-3.617D-26	8.697D-06	358	
	-3	-1				2	4	Generalised
Subshe	11 < r >	< r >		< r >	< r	> <	r >	occupation
1s	0.0000D+00	2.68556D+00	5	.73199D-01	4.47081	0-01 5.33	751D-01	1.99386D+00
2s	0.0000D+00	3.45596D-01	. 3	.87317D+00	1.77347	0+01 5.650	669D+02	2.01584D-04
2p-	0.0000D+00	2.65023D-01	4	.79564D+00	2.782651	0+01 1.47	509D+03	3.33335D-01
2p	5.85643D-02	2.65011D-01	4	.79578D+00	2.782801	0+01 1.47	522D+03	6.66669D-01
3s	0.0000D+00	3.09463D+00	8	.79428D-01	1.760191	0+00 3.83	411D+01	2.52035D-03
3p-	0.0000D+00	1.94750D+00	6	.74894D-01	8.409001	0-01 2.20	221D+01	1.08534D-03
3р	1.72790D+01	1.94712D+00	6	.74750D-01	8.40046	0-01 2.19	775D+01	2.17087D-03
3d-	1.01792D+01	1.82956D+00	6	.31485D-01	4.579701	0-01 4.14	736D-01	6.48046D-05
3d	1.01575D+01	1.82935D+00	6	.31441D-01	4.578521	0-01 4.14	179D-01	9.72851D-05

# Eigenenergies:

Level	J Parity	Hartrees	Kaysers	eV
1	1/2 -	-7.404576963163D+00	-1.625116799442D+06	-2.014888020446D+02
1	3/2 -	-7.404574103806D+00	-1.625116171886D+06	-2.014887242376D+02

Weights of major contributors to ASF:

Block	Level	J Parity	CSF co	ntributions			
1	1	1/2 -	0.9985	-0.0343		0.0230	
			1	56	58	60	30
2	1	3/2 -	0.9985	-0.0343	0.0326	0.0230	0.0099
			1	62	67	71	32

The first lines of the file tell us that the calculation was for a three electron system and that there were in total 186 CSFs built on 9 relativistic radial orbitals. After this, there is information about the nucleus. In this case, the nucleus has Z=3 and a mass of 1.264966898269  $\times$  10<sup>4</sup> electron masses. The nuclear charge distribution is modelled by a Fermi distribution

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with  $c = 3.612753059646 \times 10^{-5}$  Bohr radii, and  $a = 9.89059137009 \times 10^{-6}$  Bohr radii. There is information about the radial grid used in the calculation. The grid is given by

$$R(I) = RNT(\exp((I-1)H) - 1), I = 1,...,N$$

with  $RNT = 6.66666666667 \times 10^{-7}$  Bohr radii and  $H = 5 \times 10^{-2}$  Bohr radii. In the current implementation of GRASP, RNT = A/Z with  $A = 2 \times 10^{-6}$ , see TP Section 2.2, Equation (8). There are N = 590 grid points.

We see that it is an EOL calculation and that the calculation was on the lowest state (first eigenvalue) of each block (J = 1/2 and J = 3/2). In the optimization, each state is weighted according to the statistical weight 2J + 1, see TP Section 2.7, Equation (44). The information on optimization is followed by a radial orbital summary. Important characteristics of a radial orbital are the orbital energy eigenvalue and parameters that determine the behavior near r = 0. The radial amplitudes

$$u(r) = \left(\begin{array}{c} P(r) \\ Q(r) \end{array}\right),$$

can be expanded in power series

$$u(r) = r^{\gamma}[u_0 + u_1r + u_2r^2 + \ldots], \quad u_k = \begin{pmatrix} p_k \\ q_k \end{pmatrix}$$

near the origin where the index  $\gamma$ ,  $p_k$ , and  $q_k$  are constants that depend on the nuclear potential model. In the radial orbital summary e is orbital energy eigenvalue, p0 is a parameter related to the leading expansion coefficients of the radial amplitudes and gamma is the exponent  $\gamma$ , for details see I.P. Grant, Relativistic Quantum Theory of Atoms and Molecules, Springer 2007, p 272–273 and also the subroutine start in 1ib92. P(2) and Q(2) are the values of the radial amplitudes at the first grid point R(2) away from zero. Then the self-consistency (weighted change of an orbital during an iteration) is given for each orbital. In this case, the orbitals 1s, 2s, 2p-, 2p were kept frozen, and they thus have a self-consistency of zero. The orbitals 3s, 3p-, 3p, 3d-, 3d were optimized and the self-consistency is between  $8.697 \times 10^{-6}$  for 3d and  $8.965 \times 10^{-8}$  for 3s. Finally, the value MTP gives the number of the outermost grid point used for representing the radial amplitudes of the orbital. At the remaining grid points, the radial amplitudes of the orbital are set to zero. Around 360 of the available 590 grid points are utilized.

Different radial expectation values

$$\langle r^k \rangle = \langle nlj | r^k | nlj \rangle$$

of the orbitals are given along with the generalized occupation numbers. The generalized occupation number  $\bar{q}(nlj)$  of an orbital nlj is defined as

$$\overline{q}(nlj) = \sum_{r=1}^{NCSF} d_r^2 \ q_r(nlj),$$

where  $q_r(nlj)$  is the number of electrons in subshell nlj in CSF r and  $d_r^2$  is the generalized weight

$$d_r^2 = \frac{\sum_{i=1}^{n_L} (2J_i + 1)c_{ri}^2}{\sum_{i=1}^{n_L} (2J_i + 1)}.$$

In the expression for the generalized weight the sum is over all levels in the EOL calculation.  $c_{ri}$ , r = 1,...,NCSF are the mixing coefficients of level i in the basis of the CSFs. An orbital with a small generalized occupation number is associated with CSFs that have small expansion coefficients.

At the end of the summary file, the eigenenergies for the states are displayed in different energy units, where Kayser is the synonym of cm<sup>-1</sup>. The weights of the major CSF contributors are also given. Please note that the CSFs in this case are counted block wise.

The rci Log-File

The relativistic configuration interaction program rci produces a log-file displaying the response to the different questions. This file is saved in name.clog. Below is the log-file  $2p_3$ .clog from the run of the  $1s^22p^2P_{1/2,3/2}^0$  states.

y ! Default settings 2p\_3

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```
! Contribution of H Transverse?
У
             ! Modify photon frequencies?
У
   9.99999999999995E-007 ! Scale factor
             ! Vacuum polarization?
У
             ! Normal mass shift?
n
             ! Specific mass shift?
n
             ! Self energy?
у
           3 ! Max n for including self energy
1
1
```

The log-file is a copy of the input data. The name of the state is  $2p_3$  and full interaction was included. Contributions from the transverse interaction (Breit) were added, where the photon frequencies were multiplied with a factor  $10^{-6}$ , see TP Section 2.3, Equation (11). Vacuum polarization as well as self-energy corrections were added. The self-energy corrections are based on estimations for the orbitals. For correlation orbitals with high principal quantum numbers, these estimations may fail. In this case, self-energy corrections were based on orbitals with principal quantum numbers smaller than 3. Finally, we see that the calculation determined the first levels of the two blocks (ASF serial numbers 1 for each of the two blocks). By executing the command

```
rci < 2p_3.clog
```

the rci run will be executed again with the settings in 2p\_3.clog. The log-file can easily be edited and used as an input also to other runs.

The rci Summary File

The rci program produces a summary file name.csum. Below is the summary file  $2p_3$ .csum from the run on weighted average of the  $1s^22p$   $^2P_{1/2,3/2}^o$  states

```
There are 3 electrons in the cloud
  in 186 relativistic CSFs
 based on 9 relativistic subshells.
The atomic number is
                      3.000000000;
the mass of the nucleus is 1.264966898269D+04 electron masses;
 Fermi nucleus:
  c = 3.612753059646D-05 Bohr radii,
  a = 9.890591370096D-06 Bohr radii;
  there are 82 tabulation points in the nucleus.
Speed of light = 1.370359991390D+02 atomic units.
To H (Dirac Coulomb) is added
 H (Transverse) --- factor multiplying the photon frequency: 1.0000000D-06;
 H (Vacuum Polarisation);
  the total will be diagonalised.
Diagonal contributions from H (Self Energy) will be estimated
  from a screened hydrogenic approximation.
Radial grid: R(I) = RNT*(exp((I-1)*H)-1), I = 1, ..., N;
        6.6666666667D-07 Bohr radii;
     = 5.00000000000D-02 Bohr radii;
     = 590;
R(2) = 3.418073091735D-08 Bohr radii;
```

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#### R(N) = 4.110372988964D+06 Bohr radii.

Subshell radial wavefunction summary:

Subshe	11 e	p0	gamma	P(2)	Q(2)	MTP
1s 2s 2p- 2p 3s 3p- 3p	2.5177395314D+00 1.9634308400D-01 1.2867248397D-01 1.2866992757D-01 8.0600844816D+00 8.7786093395D+00 8.7823537644D+00	9.280D+00 1.452D+00 5.116D-05 4.265D-01 1.181D+01 2.853D-03 2.381D+01	1.00 1.00 1.00 2.00 1.00 2.00	4.965D-08 2.518D-15 4.983D-16 5.027D-07 1.994D-13	-3.513D-12 -5.499D-13 1.598D-10 -5.519D-21 -7.981D-13 8.921D-09 -1.173D-18	355 361 366 366 360 364 364
3d- 3d	1.6298092328D+01 1.6306599649D+01	8.146D-03 8.169D+01	2.00	2.740D-20	1.739D-15 -3.617D-26	358 358

. . . . . . . . . . .

Information about number of radial integrals, density of the Hamiltonian matrix etc, the energies and the leading CSFs for each level etc.

From the summary file, we again see what operators were included in the Hamiltonian. Information about the grid and the orbitals, same as in the name. sum file is also available.

# The Hyperfine Structure Files

The rhfs program computes hyperfine structure data. In addition, the Landé  $g_J$ -factor is computed. Below is the output file 2p\_3.ch, edited to fit the page, from the rhfs run for the rci wave function, given in the 2p\_3.c, 2p\_3.w and 2p\_3.cm files, of the  $1s^22p$   $^2P_{1/2,3/2}^o$  states.

```
      Nuclear spin
      1.500000000000000D+00 au

      Nuclear magnetic dipole moment
      3.25642680000000D+00 n.m.

      Nuclear electric quadrupole moment
      -4.0000000000000D-02 barns
```

#### Interaction constants:

Level1	J Parity	A (MHz)	B (MHz)	total $g_J$
1	1/2 -	4.4821853986D+01	-0.00000000D+00	6.6588395646D-01
1	3/2 -	-3.5378452915D+00	-1.7729096327D-01	1.3340987050D+00

At the top, the nuclear spin and moments are displayed. Then, for each level, the A and B hyperfine interaction constants, see TP Section 3.1 Equations (59)–(60) are given in MHz. In addition, the Landé  $g_J$ -factors, TP Section 3.2 Equation (66), are given.

The rhfs program gives another file 2p\_3.choffd, which contains off-diagonal hyperfine data

```
      Nuclear spin
      1.500000000000000D+00 au

      Nuclear magnetic dipole moment
      3.25642680000000D+00 n.m.

      Nuclear electric quadrupole moment
      -4.0000000000000D-02 barns
```

#### Interaction constants:

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Level1	J Parity	Level2	J Parity	A	(MHz)	B (MHz)
1	3/2 -	1	1/2 -	1.176	8857887D+01	-0.000000000D+00 -3.8388220012D-02 -1.7729096288D-01
Matrix	elements:					
Level1	J Parity	Level2	J Parity	F	Matrix eleme	nt (a.u.)
1 1	1/2 - 1/2 -	1 1	1/2 - 1/2 -	1 2	-8.5152606 5.1091563	438D-09 863D-09
Matrix	elements:					
Level1	J Parity	Level2	J Parity	F	Matrix eleme	nt (a.u.)
					4.0297075 5.3525245	
Matrix	elements:					
Level1	J Parity	Level2	J Parity	F	Matrix eleme	nt (a.u.)
1 1	3/2 - 3/2 -	1 1	3/2 - 3/2 -	1 2	1.9828496 1.4720532 4.2351513	074D-09 722D-10
1	3/2 -	1	3/2 -	3	-1.2166549	923D-09

Given are diagonal and off-diagonal hyperfine interaction constants A and B in MHz and the F dependent hyperfine matrix elements in atomic units. The above quantities are defined in [8], Equations (13)–(17) and Equations (7)–(8).

# The Isotope Shift Files

The ris4 program computes mass- and field shift isotope data. Below is the output file  $2p_3.ci$ , edited to fit the page, from the ris4 run for the rci wave function, given in the  $2p_3.c$ ,  $2p_3.w$  and  $2p_3.cm$  files, of the  $1s^22p^2P_{1/2.3/2}^o$  states.

-0.6657064450D-02

0.7402945843D+01 (a.u.)

Number of eigenvalues: 2

3/2 -

```
Level J Parity Energy
                     -0.7404260995D+01 (a.u.)
  1
         1/2 -
  1
         3/2 -
                     -0.7404259683D+01 (a.u.)
Level J Parity Normal mass shift parameter
                            <K^1>
                                              <K^2+K^3>
                                                                <K^1+K^2+K^3>
         1/2 -
                                        -0.6671237484D-02
                                                               0.7402940590D+01
  1
                       0.7409611828D+01
                                                                                 (a.u.)
                       0.2674486353D+05 -0.2407971433D+02
                                                               0.2672078382D+05
                                                                                 (GHz u)
                                                                <K^1+K^2+K^3>
                            <K^1>
                                              <K^2+K^3>
```

0.7409602908D+01

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```
0.2674483134D+05
                                            -0.2402855701D+02
                                                                   0.2672080278D+05
                                                                                      (GHz u)
Level J Parity Specific mass shift parameter
                             <K^1>
                                                 <K^2+K^3>
                                                                    <K^1+K^2+K^3>
  1
         1/2 -
                        0.2425644688D+00
                                            -0.1746264308D-03
                                                                   0.2423898424D+00
                                                                                      (a.u.)
                        0.8755321826D+03
                                            -0.6303110296D+00
                                                                                      (GHz u)
                                                                   0.8749018716D+03
                             <K^1>
                                                 <K^2+K^3>
                                                                    <K^1+K^2+K^3>
         3/2 -
  1
                        0.2425741100D+00
                                            -0.1915018511D-03
                                                                   0.2423826081D+00
                                                                                      (a.u.)
                        0.8755669823D+03
                                            -0.6912225626D+00
                                                                                      (GHz u)
                                                                   0.8748757597D+03
Level J Parity Electron density in atomic units
                        Dens. (a.u.)
         1/2 -
                        0.1372240739D+02
  1
         3/2 -
  1
                        0.1372240990D+02
Level J Parity Field shift electronic factors and average point discrepancy in fit
                        FO (GHz/fm<sup>2</sup>)
                                             F2 (GHz/fm^4)
                                                                   F4 (GHz/fm<sup>6</sup>)
         1/2 -
  1
                        0.2025876387D+00
                                            -0.3303847114D-05
                                                                   0.5227748000D-07
  1
         3/2 -
                        0.2025876757D+00
                                            -0.3303847831D-05
                                                                   0.5227749057D-07
                        F6 (GHz/fm^8)
                                             Disc. (per mille)
  1
         1/2 -
                        -0.6985943239D-09
                                              0.0000
         3/2 -
                                              0.0000
  1
                        -0.6985944586D-09
Level J Parity Field shift electronic factors (corrected for varying density inside nucleus)
                        FOVEDO (GHz/fm^2)
                                             FOVED1 (GHz/fm<sup>4</sup>)
  1
         1/2 -
                        0.2025433326D+00
                                            -0.2805899138D-05
  1
         3/2 -
                        0.2025433696D+00
                                            -0.2805899756D-05
```

We see that there are two eigenvalues for which the energies are printed. After that, for each level, the normal mass shift parameters, decomposed in three parts, see [12] Section 3.2, Equation (41) and TP Section 3.3, Equation (73), are given in (a.u.) and (GHz u). After the normal mass shift parameters, the specific mass shift parameters, decomposed in three parts, see [12] Section 3.2, Equation (41) and TP Section 3.3, Equation (74), are given in (a.u.) and (GHz u). Next, the electron density at the origin, r=0, is given in a.u. After that follow the field shift electronic factors,  $F_0, F_2, \ldots, F_6$ , as defined in [12], Section 3.3, Equation (18), see also TP Section 3.3, Equation (79). To estimate the effect on the field shift from the varying electronic density (ved) inside the nuclear volume, the quantity  $F_{i,0}^{(0)\text{ved}}$  is introduced, see [12] Section 4, Equation (39). The latter can be expressed in terms of  $F_{i,0}^{(0)\text{ved}}$  and  $F_{i,0}^{(1)\text{ved}}$ , see [12] Section 4, Equations (47) and (48). These parameters are displayed at the end of the output file.

#### The Transition File

The rtransition program computes transition data. Below is the output file 2s\_3.2p\_3.ct from the rtransition electric dipole E1 run for rci wave functions given in the 2s\_3.c, 2s\_3.w, 2s\_3.cm and 2p\_3.c, 2p\_3.w and 2p\_3.cm files.

```
Transition between files:
f1 = 2s_3
f2 = 2p_3
```

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# Electric 2\*\*( 1)-pole transitions

```
Upper Lower

File Lev J P File Lev J P E (Kays) A (s-1) gf S

f2 1 1/2 - f1 1 1/2 + 14861.28 C 3.81311D+07 5.17671D-01 1.14676D+01

B 3.74756D+07 5.08773D-01 1.12705D+01

f2 1 3/2 - f1 1 1/2 + 14861.57 C 3.81334D+07 1.03537D+00 2.29353D+01

B 3.74782D+07 1.01758D+00 2.25413D+01
```

The first lines of the file give the name of the files defining the wave functions. Then data are given for the electric dipole transition E1. The first transition is from the upper level 1 with J = 1/2 and negative parity in file f2, i.e.,  $1s^22p \,^2P_{1/2}^o$  to the lower level 1 with J = 1/2 and positive parity in file f1, i.e.,  $1s^22s \,^2S_{1/2}$ . The second transition is from the upper level 1 with J = 3/2 and negative parity in file f2, i.e.,  $1s^22p \,^2P_{3/2}^o$  to the lower level 1 with J = 1/2 and positive parity in file f1, i.e.,  $1s^22s \,^2S_{1/2}$ . For each transition the transition energy E is given in Kaysers (cm<sup>-1</sup>). Additionally, the transition rate E in emission (Einstein E-coefficient), the weighted oscillator strength E and the line strength E are given in Coulomb (velocity) and Babushkin (length) gauge.

If the 2s\_3.1sj.1bl and 2p\_3.1sj.1bl files produced by jj21sj are available at the run of rtransition an additional output file 2s\_3.2p\_3.ct.1sj is produced. This file is shown below

```
Transition between files:
2s_3
2p_3
     -7.47197402 1s(2).2s_2S
     -7.40426103 1s(2).2p_2P
  14861.28 CM-1
                 6728.89 ANGS(VAC) 6728.20 ANGS(AIR)
E1 S = 1.12705D+01 GF = 5.08773D-01 AKI = 3.74756D+07
                                                           dT = 0.01719
        1.14676D+01
                           5.17671D-01
                                              3.81311D+07
     -7.47197402 1s(2).2s_2S
    -7.40425972 1s(2).2p_2P
  14861.57 CM-1
                 6728.76 ANGS(VAC)
                                         6728.06 ANGS(AIR)
E1 S = 2.25413D+01 GF = 1.01758D+00
                                        AKI = 3.74782D+07
                                                           dT = 0.01718
        2.29353D+01
                           1.03537D+00
                                               3.81334D+07
```

The 2s\_3.2p\_3.ct.1sj file has a different format. Here, the labels of the upper and lower states in the transition are in LSJ-notation. The J quantum number (multiplied by 2) is written to the left. The transition energy is given in cm<sup>-1</sup> and the wavelengths (vacuum and air) in angstrom (ANGS) where  $1 \text{ ANGS} = 10^{-10} \text{ m}$ . The line strength S, the weighted oscillator strength S and the transition rates in emission S (AKI) are given on two lines, where the upper line corresponds to the Babushkin (length) gauge and the lower line to the Coulomb (velocity) gauge. Finally,

$$dT = \frac{|A_C - A_B|}{\max(A_C, A_B)}$$

is a parameter that can be related to the estimated uncertainty of the transition rates [43].

# 8.2. Output Files from the Third Example

The third example case, see Section 6.3, was calculations of the states belonging to the  $1s^22s^22p^3$  and  $1s^22p^5$  configurations in Si VIII.

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# The jj2lsj File

The jj2lsj program transforms from jj to LSJ coupling and gives the LSJ composition of the states. Below is the output file 2s22p3\_2p5\_3.lsj.lbl from the jj2lsj run of the rci wave functions given in the 2s22p3\_2p5\_3.c, 2s22p3\_2p5\_3.w, 2s22p3\_2p5\_3.cm files. For each case, the first line gives the position (number) of the eigenstate in the interaction matrix, parity, total energy and the percentage of the ASF that has been transformed. Thus, 99.907 % implies that 0.093 % has not been transformed.

```
Comp. of ASF
Pos
      J
          Parity
                      Energy Total
 1 1/2
                       -262.790633876
                                           99.907%
        0.98656029
                      0.97330122
                                   1s(2).2s(2).2p(3)2P1_2P
        0.15010905
                      0.02253273
                                   1s(2).2p(5)_2P
                                   1s(2).2s_2S.2p(3)2P1_1P.3d_2P
       -0.03364614
                      0.00113206
 2
   1/2
                       -259.497939898
                                           99.123%
        0.98251140
                      0.96532866
                                   1s(2).2p(5)_2P
       -0.14989835
                      0.02246951
                                   1s(2).2s(2).2p(3)2P1_2P
       -0.03674439
                      0.00135015
                                   1s(2).2s_2S.2p(3)2D3_1D.3d_2P
        0.03527443
                      0.00124429
                                   1s(2).2s_2S.2p(3)2P1_3P.3d_2P
                       -263.279784072
                                           99.550%
   3/2
        0.99652486
                                   1s(2).2s(2).2p(3)4S3_4S
                      0.99306180
        0.03703202
                      0.00137137
                                   1s(2).2s(2).2p(3)2P1_2P
                                           99.670%
   3/2
                       -262.955055547
 2
        0.98954100
                      0.97919139
                                   1s(2).2s(2).2p(3)2D3_2D
       -0.12139907
                      0.01473773
                                   1s(2).2s(2).2p(3)2P1_2P
       -0.03690740
                      0.00136216
                                   1s(2).2s_2S.2p(3)4S3_3S.3d_2D
 3
   3/2
                       -262.788274233
                                           99.912%
        0.97818840
                      0.95685254
                                   1s(2).2s(2).2p(3)2P1_2P
        0.15010551
                      0.02253166
                                   1s(2).2p(5)_2P
                      0.01507118
                                   1s(2).2s(2).2p(3)2D3_2D
        0.12276473
       -0.03672205
                      0.00134851
                                   1s(2).2s(2).2p(3)4S3_4S
       -0.03335975
                      0.00111287
                                   1s(2).2s_2S.2p(3)2P1_1P.3d_2P
   3/2
                       -259.524117905
                                           99.004%
                      0.96499455
        0.98234136
                                   1s(2).2p(5)_2P
       -0.15102023
                      0.02280711
                                   1s(2).2s(2).2p(3)2P1_2P
        0.03537700
                      0.00125153
                                   1s(2).2s_2S.2p(3)2P1_3P.3d_2P
   5/2
                       -262.953820595
                                           99.429%
        0.99713868
                      0.99428554
                                   1s(2).2s(2).2p(3)2D3_2D
```

There is a total of seven states. For each state, the file gives the LSJ-expansion. The lowest J = 1/2 state (pos 1) with negative parity and energy -262.790633876 a.u. has the LSJ-expansion

```
0.98656029 0.97330122 1s(2).2s(2).2p(3)2P1_2P

0.15010905 0.02253273 1s(2).2p(5)_2P

-0.03364614 0.00113206 1s(2).2s_2S.2p(3)2P1_1P.3d_2P
```

The second-lowest J = 1/2 state (pos 2) with negative parity and energy -259.497939898 a.u. has the LSJ-expansion

```
0.98251140 0.96532866 1s(2).2p(5)_2P

-0.14989835 0.02246951 1s(2).2s(2).2p(3)2P1_2P

-0.03674439 0.00135015 1s(2).2s_2S.2p(3)2D3_1D.3d_2P

0.03527443 0.00124429 1s(2).2s_2S.2p(3)2P1_3P.3d_2P
```

We see that the states are close to pure *LSJ*-coupling and the file provides meaningful labels that match labels given in, for example, the NIST data tables. The second column in the table gives the *LSJ*-composition, i.e., the squared expansion coefficients.

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Finally, a few words about how to interpret the notation in the composition of the ASF.

Each subshell in the configuration is given with occupation, *LS* term designation and seniority. When the subshell is singly or fully occupied, the term designation and seniority are not written out. The *LS* term for each subshell is the coupled from left to right. Intermediate couplings are given after the underscore sign \_.

In Table 7 below, there is a list of possible terms and their seniority for commonly occurring subshells.

**Table 7.** List of possible terms and their seniority for commonly occurring subshells.

Subshell	Terms (2S+1, L, Seniority)
s(1)	2S1
s(2)	1S0
p(1)	2P1
p(2)	1SO 1D2 3P2
p(3)	2P1 2D3 4S3
d(1)	2D1
d(2)	1SO 1D2 1G2 3P2 3F2
d(3)	2D1 2P3 2D3 2F3 2G3 2H3 4P3 4F3
d(4)	1SO 1D2 1G2 3P2 3F2 1S4 1D4 1F4 1G4 1I4 3P4 3D4 3F4 3G4 3H4 5D4
d(5)	2D1 2P3 2D3 2F3 2G3 2H3 4P3 4F3 2S5 2D5 2F5 2G5 2I5 4D5 4G5 6S5
f(1)	2F1
f(2)	1SO 1D2 1G2 1I2 3P2 3F2 3H2

As a specific example of how to interpret the notation, we look at

```
1s(2).2s_2S.2p(3)2P1_3P.3d_2P
```

The first subshell 1s(2) is fully occupied and have only one *LS* term 1S0 that is not written out explicitly. The second subshell 2s is singly occupied and has only one *LS* term 2S1 that is not written out explicitly. The third subshell 2p(3) is coupled to an *LS* term 2P1. The fourth subshell 3d is singly occupied and has only one *LS* term 2D1 that is not written out explicitly. Coupling 1S0 and 2S1 of subshells one and two leads to an intermediate term \_2S. Coupling \_2S with 2P1 of the third subshell leads to the intermediate term \_3P. Finally, coupling \_3P with 2D1 of the fourth subshells gives the final *LS* term 2P.

The programs rtablevels and rtabtransE1 implement a LaTeX translation of the ASCII notation. In the LaTeX translation, the LS term and seniority of a subshell are given in parenthesis just after the subshell. For the intermediate terms, the underscore of the ASCII notation has been replaced by a space. In the LaTeX translation, the user also has a choice to omit the closed core. Translating the above example to LaTeX and omitting the 1s(2) we get

$$2s^2S 2p^3(^2P)^3P 3d^2P^o$$
.

Please note how the seniority enters as a subscript.

The Transition File in LSJ-Coupling

The rtransition program computes transition data. Below is the output file

2s22p3\_2p5\_3.2s22p3\_2p5\_3.ct from the rtransition magnetic dipole M1 run of the rci wave functions given in the 2s22p3\_2p5\_3.c, 2s22p3\_2p5\_3.w, 2s22p3\_2p5\_3.cm files giving the states belonging to the  $1s^22s^22p^3$  and  $1s^22p^5$  configurations

```
Transition in file:
f = 2s22p3_2p5_3

Magnetic 2**( 1)-pole transitions
```

\_\_\_\_\_

Upper Lower

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```
E (Kays)
    J P
           Lev J P
                                          A (s-1)
                                                           gf
                                                                         S
    2 \frac{1}{2} - f
                1 1/2 -
                               722662.80 M 6.00621D-05 3.44839D-16 1.18001D-11
f
    3 \ 3/2 - f
                  1 1/2 -
                                  517.88 M 1.22716D-03 2.74383D-08 1.31018D+00
    4 \ 3/2 - f
                  1 1/2 -
f
                               716917.39 M 4.29992D+00 5.01695D-11 1.73052D-06
f
    1 1/2 -
              f
                  1 3/2 -
                               107356.06 M 3.11300D+01 8.09867D-09 1.86549D-03
    2 \frac{1}{2} - f
                1 3/2 -
                               830018.86 M 4.56677D+00 1.98756D-11 5.92158D-07
f
f
    1 \ 1/2 - f
                2 3/2 -
                               36086.39 M 1.27613D+01 2.93830D-08 2.01353D-02
    2 \frac{1}{2} - f
                 2 3/2 -
                               758749.18 M 4.24418D+00 2.21047D-11 7.20430D-07
f
f
    2 1/2 -
              f
                  3 3/2 -
                               722144.92 M 1.09837D+01 6.31521D-11 2.16256D-06
    2 \frac{1}{2} - f
                4 3/2 -
f
                                5745.41 M 3.40764D+00 3.09528D-07 1.33224D+00
    2 \frac{3}{2} - f
                1 3/2 -
f
                                71269.67 M 1.41118D+00 1.66606D-09 5.78084D-04
    3 \ 3/2 - f
                1 3/2 -
                               107873.94 M 7.52312D+01 3.87688D-08 8.88733D-03
f
    4 3/2 -
              f
                  1 3/2 -
f
                               824273.45 M 1.25260D+01 1.10557D-10
                                                                      3.31682D-06
                2 3/2 -
    3 \ 3/2 - f
f
                               36604.27 M 2.11594D+01 9.47019D-08 6.39782D-02
f
    4 \ 3/2 - f
                2 3/2 -
                               753003.77 M 9.62931D+00 1.01840D-10 3.34446D-06
    4 \ 3/2 - f
                3 3/2 -
                              716399.51 M 7.21346D-02 8.42851D-13 2.90938D-08
f
    1 \ 5/2 - f
                 1 3/2 -
                                71540.71 M 1.99970D-02
f
                                                         3.51453D-11 1.21484D-05
    1 \ 5/2 - f
                2 3/2 -
                                  271.04 M 2.11526D-04 2.59002D-08 2.36306D+00
f
                1 5/2 -
f
    3 \ 3/2 - f
                                36333.23 M 1.17593D+01 5.34186D-08 3.63575D-02
f
    4 \ 3/2 - f
                  1 5/2 -
                               752732.73 M 5.41422D+00 5.73023D-11 1.88251D-06
If the information of LSJ-coupling is available from a jj21sj run, rtransition also produces a file
2s22p3_2p5_3.2s22p3_2p5_3.ct.lsj
Transition between files:
2s22p3_2p5_3
2s22p3_2p5_3
  1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
  1 -259.49793990 1s(2).2p(5)_2P
 722662.80 CM-1
                      138.38 ANGS(VAC)
                                            138.38 ANGS(AIR)
M1 S = 1.18001D-11
                       GF = 3.44839D-16
                                          AKI = 6.00621D-05
   1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
  3 -262.78827423 1s(2).2s(2).2p(3)2P1_2P
                   193094.26 ANGS(VAC)
    517.88 CM-1
                                        193074.30 ANGS(AIR)
                      GF = 2.74383D-08
M1 S = 1.31018D+00
                                         AKI = 1.22716D-03
  1 -262.79063388 1s(2).2s(2).2p(3)2P1_2P
  3 -259.52411791 1s(2).2p(5)_2P
 716917.39 CM-1
                      139.49 ANGS(VAC)
                                            139.49 ANGS(AIR)
M1 S = 1.73052D-06
                     GF = 5.01695D-11
                                          AKI = 4.29992D+00
  3 -263.27978407 1s(2).2s(2).2p(3)4S3_4S
  1 -262.79063388
                  1s(2).2s(2).2p(3)2P1_2P
 107356.06 CM-1
                      931.48 ANGS(VAC)
                                            931.48 ANGS(AIR)
M1 S = 1.86549D-03
                     GF = 8.09867D-09 AKI = 3.11300D+01
  3 -263.27978407 1s(2).2s(2).2p(3)4S3_4S
   1 -259.49793990 1s(2).2p(5)_2P
```

830018.86 CM-1

120.48 ANGS(VAC)

120.48 ANGS(AIR)

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M1 S = 5.92158D-07 GF = 1.98756D-11 AKI = 4.56677D+003 -262.95505555 1s(2).2s(2).2p(3)2D3\_2D 1 -262.79063388 1s(2).2s(2).2p(3)2P1\_2P 36086.39 CM-1 2771.13 ANGS(VAC) 2770.83 ANGS(AIR) M1 S = 2.01353D-02 GF = 2.93830D-08 AKI = 1.27613D+013 -262.95505555 1s(2).2s(2).2p(3)2D3\_2D 1 -259.49793990 1s(2).2p(5)\_2P 758749.18 CM-1 131.80 ANGS(VAC) 131.80 ANGS(AIR) M1 S = 7.20430D-07 GF = 2.21047D-11 AKI = 4.24418D+003 - 262.78827423  $1s(2).2s(2).2p(3)2P1_2P$ 1 -259.49793990 1s(2).2p(5)\_2P 722144.92 CM-1 138.48 ANGS(VAC) 138.48 ANGS(AIR) M1 S = 2.16256D-06 GF = 6.31521D-11 AKI = 1.09837D+013 -259.52411791 1s(2).2p(5)\_2P 1 -259.49793990 1s(2).2p(5)\_2P 17405.20 ANGS(VAC) 17403.40 ANGS(AIR) 5745.41 CM-1 M1 S = 1.33224D+00 GF = 3.09528D-07 AKI = 3.40764D+00 $3 \ -263.27978407 \ 1s(2).2s(2).2p(3)4S3\_4S \\$ 3 -262.95505555 1s(2).2s(2).2p(3)2D3\_2D 71269.67 CM-1 1403.12 ANGS(VAC) 1403.12 ANGS(AIR) M1 S = 5.78084D-04 GF = 1.66606D-09 AKI = 1.41118D+003 -263.27978407 1s(2).2s(2).2p(3)4S3\_4S 3 -262.78827423 1s(2).2s(2).2p(3)2P1\_2P 107873.94 CM-1 927.01 ANGS(VAC) 927.01 ANGS(AIR) M1 S = 8.88733D-03 GF = 3.87688D-08 AKI = 7.52312D+013 -263.27978407 1s(2).2s(2).2p(3)4S3\_4S 3 -259.52411791 1s(2).2p(5)\_2P 121.32 ANGS(VAC) 824273.45 CM-1 121.32 ANGS(AIR) M1 S = 3.31682D-06 GF = 1.10557D-10 AKI = 1.25260D+013 -262.95505555 1s(2).2s(2).2p(3)2D3\_2D 3 -262.78827423 1s(2).2s(2).2p(3)2P1\_2P 36604.27 CM-1 2731.92 ANGS(VAC) 2731.63 ANGS(AIR) M1 S = 6.39782D-02 GF = 9.47019D-08 AKI = 2.11594D+013 -262.95505555 1s(2).2s(2).2p(3)2D3\_2D 3 -259.52411791 1s(2).2p(5)\_2P 753003.77 CM-1 132.80 ANGS(VAC) 132.80 ANGS(AIR)

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```
M1 S = 3.34446D-06 GF = 1.01840D-10
                                        AKI = 9.62931D+00
 3 -262.78827423 1s(2).2s(2).2p(3)2P1_2P
 3 -259.52411791 1s(2).2p(5)_2P
                                          139.59 ANGS(AIR)
716399.51 CM-1
                     139.59 ANGS(VAC)
M1 S = 2.90938D-08
                     GF = 8.42851D-13
                                        AKI = 7.21346D-02
 3 -263.27978407 1s(2).2s(2).2p(3)4S3_4S
 5 -262.95382060 1s(2).2s(2).2p(3)2D3_2D
 71540.71 CM-1
                   1397.81 ANGS(VAC)
                                         1397.81 ANGS(AIR)
M1 S = 1.21484D-05 GF = 3.51453D-11
                                        AKI = 1.99970D-02
 3 - 262.95505555 1s(2).2s(2).2p(3)2D3_2D
 5 -262.95382060 1s(2).2s(2).2p(3)2D3_2D
   271.04 CM-1
                 368948.37 ANGS(VAC) 368910.23 ANGS(AIR)
M1 S = 2.36306D+00 GF = 2.59002D-08 AKI = 2.11526D-04
 5 -262.95382060 1s(2).2s(2).2p(3)2D3_2D
 3 -262.78827423 1s(2).2s(2).2p(3)2P1_2P
 36333.23 CM-1
                   2752.30 ANGS(VAC)
                                         2752.01 ANGS(AIR)
M1 S = 3.63575D-02 GF = 5.34186D-08
                                        AKI = 1.17593D+01
 5 -262.95382060 1s(2).2s(2).2p(3)2D3_2D
 3 -259.52411791 1s(2).2p(5)_2P
 752732.73 CM-1
                    132.85 ANGS(VAC)
                                          132.85 ANGS(AIR)
M1 S = 1.88251D-06 GF = 5.73023D-11
                                        AKI = 5.41422D+00
```

Here, labels of the upper and lower states in the transition are in LSJ-notation. In addition to transition energies in cm<sup>-1</sup> also the wavelengths (vacuum and air) are given in angstrom (ANGS). On the next line the line strength S, the weighted oscillator strength S and the transition rate S (AKI) are given. The format is the same as the one produced by the transition program of ATSP2K [1]

# The Coupling Files

In this section, we discuss contents of the files:

```
2s2p_DF.coup3.LK3.lbl
2s2p_DF.coup3.JK3.lbl
2s2p_DF.coup3.LS.lbl
2s2p_DF.coup3.LS3.lbl
2s2p_DF.coup3.LSJ3.lbl
2s2p_DF.coup3.jj.lbl
2s2p_DF.coup3.cLSJ3.lbl
```

These files are from the Coupling run of the rci/rmcdhf programs. The input files 2s2p\_DF.lsj.c and 2s2p\_DF.lsj.j were created by the program jj2lsj in non-default mode.

The 2s2p\_DF.coup3.LK3.1b1 file

The Coupling program transforms from *LSJ* to *LK3* coupling and gives the *LK3* composition of the states.

```
Pos J Parity Energy Total Comp. of ASF
1 0 -24.127087737 100.000%
-1.00000000 1.00000000 1s2_ 2s_2p_(3P) P_3[1]<0>
```

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```
100.000%
                       -24.127040409
1
    1
       0.9999995
                    0.99999990 1s2_2s_2p_(3P) P_3[1]<1>
       0.00030652
                    0.00000009 1s2_ 2s_2p_(1P) P_1[1]<1>
2
    1
                       -23.915406084
                                         100.000%
       0.9999995
                    0.99999990
                                 1s2_ 2s_2p_(1P) P_1[1]<1>
      -0.00030652
                    0.00000009
                                  1s2_ 2s_2p_(3P) P_3[1]<1>
1
    2
                       -24.126945696
                                         100.000%
       1.0000000
                    1.00000000
                                 1s2_ 2s_2p_(3P) P_3[1]<2>
```

Let us explain how to interpret the notation of the following ASF

```
1s2_ 2s_2p_(3P) P_3[1]<0>
```

First, it should be noted that the spin multiplicity, M=2S+1, is used to represent the spin of individual and coupled shells in the output files. The first 1s(2) subshell is fully occupied, whereas the second 2s and the third 2p subshells are singly occupied. Therefore, they have only one LS term,  $^1S$ ,  $^2S$ , and  $^2P$  respectively, that are not written out explicitly. The result of the coupling of the second and third subshells, 2s and 2p , is written in parentheses, i.e.,  $(M_{23}L_{23})=(3P)$ . Following the LK3 coupling scheme, the total orbital angular momentum L is obtained by coupling the 1s shell angular momentum,  $L_1=0$ , with  $L_{23}=1$ . This momentum appears as the first spectroscopic symbol, P, of the final term construction  $P_3[1]<0>$ . The number 3 preceding the "[1]" symbol represents  $M_{23}$ . Coupling L=1 with the spin  $S_1=0$  leads to the term L=1 which is written in square brackets [ and ]. Coupling L=1 with the spin

The <name>.coup3.JK3.1b1 file

The Coupling program transforms from *LSJ* to *JK*3 coupling and gives the *JK*3 composition of the states.

```
Pos
                      Energy Total
                                        Comp. of ASF
      J
          Parity
                        -24.127087737
                                          100.000%
 1
      0
                                   1s2_<0>2s_2p_(3P) 3[1]<0>
       -1.00000000
                      1.00000000
                        -24.127040409
                                          100.000%
 1
        0.9999995
                      0.99999990
                                   1s2_<0>2s_2p_(3P) 3[1]<1>
        0.00030652
                      0.0000009
                                   1s2_<0>2s_2p_(1P) 1[1]<1>
 2
                                          100.000%
      1
                        -23.915406084
        0.9999995
                      0.99999990 1s2_<0>2s_2p_(1P) 1[1]<1>
       -0.00030652
                      0.00000009 1s2_<0>2s_2p_(3P) 3[1]<1>
 1
      2
                        -24.126945696
                                          100.000%
        1.00000000
                      1.00000000
                                   1s2_<0>2s_2p_(3P) 3[1]<2>
```

The notation for the following ASF

```
1s2_<0>2s_2p_(3P) 3[1]<0>
```

can be understood as follows. The first subshell 1s(2) is fully occupied and has only one LSJ term  $^1S_0$ . This part of the term ( $^1S$ ) is not mentioned and only  $J_1$  is written in the first angle brackets < and >. The second subshell 2s is singly occupied and has only one LS term,  $^2S$ , that is not written out explicitly. The third subshell 2p is singly occupied and has the term  $^2P$ , also omitted in the notation. The result of the coupling of the second and third subshells, 2s and 2p, is written in parentheses, i.e., ( $M_{23}L_{23}$ ) = (3P) . The first number, 3, appearing in the final term construction 3[1] <0> represents  $M_{23}$ . Coupling  $J_1$ =0 with the orbital angular momentum  $L_{23}$ =1 leads to the term K=1 that is written in square brackets [ and ]. Coupling K=1 with the spin  $S_{23}$ =1 leads to the final J term J=0 that is written in the final angle brackets < and >.

The <name>.coup3.LS.1b1 file

The Coupling program transforms from *LSJ* to *LS* coupling and gives the *LS* composition of the states.

```
Pos
      J
          Parity
                      Energy Total
                                         Comp. of ASF
 1
                         -24.127087737
                                           100.000%
       -1.00000000
                      1.00000000
                                  1s2_.2s_2S.2p_ 3P<0>
                                           100.000%
 1
      1
                        -24.127040409
        0.9999995
                      0.99999990
                                   1s2_.2s_2S.2p_ 3P<1>
```

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```
0.00030652
                    0.0000009
                                 1s2_.2s_2S.2p_ 1P<1>
2
    1
                      -23.915406084
                                        100.000%
      0.9999995
                    0.99999990
                                 1s2_.2s_2S.2p_ 1P<1>
      -0.00030652
                                 1s2_.2s_2S.2p_ 3P<1>
                    0.00000009
                       -24.126945696
                                        100.000%
1
       1.0000000
                    1.00000000
                                 1s2_.2s_2S.2p_ 3P<2>
```

The following ASF notation

```
1s2_.2s_2S.2p_ 3P<0>
```

can be interpreted as follows.

The first subshell 1s(2) is fully occupied and has only one LS term,  $^1S$ , that is not written out explicitly. The second and third subshells 2s and 2p are singly occupied and have only one LS terms,  $^2S$  and  $^2P$  respectively, not displayed in the notation. The result of the coupling of the first and second subshells,  $M_{12}L_{12} = 2S$ , is written without parentheses. Coupling the spins  $S_{12}$  and  $S_3$  leads to the total spin S. A similar coupling is done with the orbital angular momenta, leading to the total orbital angular momentum, L, that is written as P, adopting the spectroscopic notation. Coupling the orbital angular momentum L=1 with the spin S=1 leads to the final J term J=0 that is written in angle brackets < and > and presented in the final term 3P<0>.

The <name>.coup3.LS3.1b1 file

The Coupling program transforms from *LSJ* to *LS3* coupling and gives the *LS3* composition of the states.

Pos	J Parity	Energy Total	Comp. of ASF
1	0	-24.127087737	100.000%
	-1.00000000	1.00000000 1s2_	2s_2p_(3P) 3P<0>
1	1	-24.127040409	100.000%
	0.9999995	0.99999990 1s2_	2s_2p_(3P) 3P<1>
	0.00030652	0.00000009 1s2_	2s_2p_(1P) 1P<1>
2	1	-23.915406084	100.000%
	0.9999995	0.99999990 1s2_	2s_2p_(1P) 1P<1>
	-0.00030652	0.00000009 1s2_	2s_2p_(3P) 3P<1>
1	2	-24.126945696	100.000%
	1.00000000	1.00000000 1s2_	2s_2p_(3P) 3P<2>

The following ASF

```
1s2_ 2s_2p_(3P) 3P<0>
```

should be read as follows. The first subshell 1s(2) is fully occupied and has only one LS term,  $^1S$ , that is not written out explicitly. The second and third subshells 2s and 2p are singly occupied and have only one LS terms,  $^2S$  and  $^2P$  respectively, not shown in the notation. The result of the coupling of the second and third subshells, 2s and 2p, is written in parentheses, i.e., ( $M_{23}L_{23}$ ) = (3P). Coupling the spin of the first shell 1s(2)  $M_{23}=0$ 0 with the spin  $M_{23}=0$ 1 leads to the total spin multiplicity  $M_{23}=0$ 2, which is the first number of the final term 3P<0>. P is the total orbital angular momentum, obtained by coupling  $M_{23}=0$ 2, Coupling the latter,  $M_{23}=0$ 3, with the total spin  $M_{23}=0$ 4 that is written in angle brackets < and >.

The <name>.coup3.LSJ3.1b1 file

The Coupling program transforms from *LSJ* to *LSJ*3 coupling and gives the *LSJ*3 composition of the states.

```
Energy Total
                                        Comp. of ASF
Pos
         Parity
                                          100.000%
 1
      0
                        -24.127087737
       -1.0000000
                      1.00000000
                                   1s2_ 2s_2p_(3P) (0,0)<0>
                        -24.127040409
                                          100.000%
 1
      1
        0.9999995
                      0.99999990
                                   1s2_2s_2p_(3P)(0,1)<1>
        0.00030652
                      0.00000009
                                  1s2_ 2s_2p_(1P) (0,1)<1>
 2
                        -23.915406084
                                          100.000%
      1
        0.9999995
                      0.99999990
                                 1s2_ 2s_2p_(1P) (0,1)<1>
```

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In this coupling scheme, the ASF

```
1s2_2 2s_2p_(3P) (0,0)<0>
```

is built as follows. The first subshell 1s (2) is fully occupied and has only one LS term  $^1S$  that is not written out explicitly. The second and third subshells 2s and 2p are singly occupied and have only one LS terms,  $^2S$  and  $^2P$  respectively, not displayed in the notation. The result of the coupling of the second and third subshells, 2s and 2p, is written in parentheses, i.e., ( $M_{23}L_{23}$ ) = (3P). Coupling  $S_{23}$ =1 with the angular momentum  $L_{23}$ =1 leads to angular momentum  $L_{23}$ =0. The  $L_{23}$ =0 of the first subshell 1s (2) and  $L_{23}$ =0 can be found in the round brackets (0,0) of the final term (0,0)<0>. Coupling the angular momentum  $L_{23}$ =0 leads to the final  $L_{23}$ =0 that is written in angle brackets < and >.

The <name>.coup3.jj.lbl file

The Coupling program transforms from LSJ to jj3 coupling and gives the jj composition of the states.

Pos	J Parity	Energy Total Comp. of ASF
1	0	-24.127087737 100.000%
	1.00000000	1.00000000 1s+2_2s+_<1/2>.2p(1/2) <0>
1	1	-24.127040409 100.000%
	0.81667351	0.66695562 1s+2_2s+_<1/2>.2p(1/2) <1>
	-0.57709997	$0.33304437$ $1s+2_2s+_<1/2>.2p+_(3/2) <1>$
2	1	-23.915406084 100.000%
	0.81667351	0.66695562 1s+2_2s+_<1/2>.2p+_(3/2) <1>
	0.57709997	0.33304437 1s+2_2s+_<1/2>.2p(1/2) <1>
1	2	-24.126945696 100.000%
	1.00000000	1.00000000 1s+2_2s+_<1/2>.2p+_ <2>

Let us illustrate the notation for the following ASF

```
1s+2_2s+_<1/2>.2p-_(1/2) <0>
```

The first subshell 1s+(2) is fully occupied and has only one jj term,  $J_1 = 0$ , that is not written out explicitly. The second subshell 2s+ is singly occupied and has only one jj term,  $J_2 = 1/2$ , not reported in the notation. Coupling the angular momenta J<sub>1</sub>=0 and J<sub>2</sub>=1/2 leads to the  $J_{12}$  term J=1/2 that is written in angle brackets < and >. Coupling the latter, J<sub>12</sub>=1/2, with the angular momentum J<sub>3</sub>=1/2 (written in parentheses) leads to the final J term J=0 that is written in angle brackets < and >.

The <name>.coup3.cLSJ3.1b1 file

The Coupling program transforms from *LSJ* to *cLSJ*3 coupling and gives the *cLSJ*3 composition of the states.

```
Pos
      J
          Parity
                       Energy Total
                                          Comp. of ASF
                         -24.127087737
 1
      0
                                            100.000%
       -1.00000000
                       1.00000000
                                     1s+2_{(0,0)<0>} 2s_2p_{(3P)<0>} (0,0)<0>
                         -24.127040409
                                            100.000%
 1
        0.9999995
                                     1s+2_{(0,0)<0>} 2s_2p_{(3P)<1>} (0,1)<1>
                       0.99999990
        0.00030652
                       0.00000009
                                     1s+2_{(0,0)<0>} 2s_2p_{(1P)<1>} (0,1)<1>
 2
                         -23.915406084
                                            100.000%
                                     1s+2_{0,0}<0> 2s_{2p_{1}}(1P)<1> (0,1)<1>
        0.9999995
                       0.99999990
       -0.00030652
                       0.0000009
                                     1s+2_{(0,0)<0>} 2s_2p_{(3P)<1>} (0,1)<1>
                         -24.126945696
                                            100.000%
 1
        1.0000000
                       1.0000000
                                     1s+2_{(0,0)<0>} 2s_2p_{(3P)<2>} (0,2)<2>
```

Lastly, let us consider the following ASF

```
1s+2_{(0,0)<0} 2s_2p_{(3P)<0} (0,0)<0
```

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that should be read as follows. In the three shells coupling scheme cLSJ3, the first non relativistic subshell,  $n_1l_1^{N_1}$ , is split into two relativistic subshells,  $j_-^{N_-}$  and  $j_+^{N_+}$ , with  $j_\pm=l\pm1/2$  and  $N=N_++N_-$ , while the two others are expressed in LSJ coupling (see Equation (26) of [14]). In the corresponding jj notation of this first shell, the coupling is written as  $(J_{1-},J_{1+})$   $< J_1 >$ . In the present case, the first subshell is a closed subshell 1s for which  $j_+=1/2$  is the only j-value allowed in the relativistic splitting  $j_\pm=l\pm1/2$ . However, for coding commodity, the s subshells are treated as all the others  $(l\neq0)$ , keeping an artificial jj coupling notation  $(0,J_{1+})$   $< J_1 >$ , with  $N_-=0$  and  $N=N_+$ . Reading the ASF from left to right, the two next shells, 2s and 2p , are singly occupied and have only one LS terms,  ${}^2S$  and  ${}^2P$  respectively, that are not displayed in the notation. The result of the coupling of the second and third subshells, 2s and 2p , is written in parentheses, i.e.,  $(M_{23}L_{23})=(3P)$ . The following angle brackets  $< J_{23} >$  contain the result of the coupling between  $S_{23}$  and  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the result of the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  contain the coupling of  $S_{23} >$  cont

# 8.3. Output Files from the Fifth Example

The fifth example, see Section 6.5, was the study of energy spectra for Ni XIV, giving the unique labels.

The unique label summary file

Using the unique option of the jj21sj program produces a summary name.uni.lsj.sum. Below is the summary file Ni\_even\_n4.uni.lsj.sum. In name.uni.lsj.sum information for the levels is given: Pos, composition of the level, serial number of composition, and the label of the level. From the Ni\_even\_n4.uni.lsj.sum file we see that the level with J = 1/2 and Pos = 2 has serial No of composition = 2 and the level with J = 1/2 and Pos = 5 has serial No of composition = 4. These levels were thus re-identified.

```
Composition Serial No.
                                      Coupling
                    of compos.
    1/2
        0.941868580
                      1 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
Pos
    1 0.860336790 1 2s(2).2p(6).3s_2S.3p(4)3P2_4P
Pos
Pos
     8 0.664884270 1 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
     7 0.554223930 1 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
Pos
       0.550189830
     6
                          2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
Pos
     3 0.487994450 1
                          2s(2).2p(6).3s_2S.3p(4)1S0_2S
Pos
     2 0.301420530 2 2s(2).2p(6).3s_2S.3p(4)3P2_2P
     5 0.112794340 4 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
Pos
```

. . . . . . . . . . .

```
Composition Serial No. Coupling of compos.

J = 9/2

Pos 1 0.937469590 1 2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4F
Pos 2 0.936205640 1 2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2G
```

The unique label composition file

The jj2lsj program also produces name.uni.lsj.lbl. Below is the output file Ni\_even\_n4.uni.lsj.lbl from jj2lsj with the unique option. This file has the same format as Ni\_even\_n4.lsj.lbl, except that the levels with the same labels were re-identified. As seen from the output file, for the level with J = 1/2 and Pos = 2 the largest expansion coefficient does not appear on the first line. This level was re-identified. The users should use name.lsj.uni.lbl file in further calculations (rtransition, rhfs, etc.) to obtain output with unique labels.

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```
Pos
      J
          Parity
                      Energy Total
                                         Comp. of ASF
 1
   1/2
                      -1441.689593921
                                            99.941%
            +
       -0.92754342
                      0.86033679
                                    2s(2).2p(6).3s_2S.3p(4)3P2_4P
       -0.31644623
                      0.10013822
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
       -0.13107223
                      0.01717993
                                    2s(2).2p(6).3s_2S.3p(4)1S0_2S
       -0.06808224
                      0.00463519
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)1S0_4P
                      0.00397659
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)1D2_4P
       -0.06306024
       -0.06139607
                      0.00376948
                                    2s(2).2p(6).3p(4)3P2_3P.3d_4P
       -0.04384478
                      0.00192236
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3P2_4P
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
        0.04315453
                      0.00186231
        0.04160917
                      0.00173132
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_4P
   1/2
 2
                      -1441.146026942
                                            99.870%
            +
        0.54901778
                      0.30142053
                                    2s(2).2p(6).3s_2S.3p(4)3P2_2P
                      0.30510158
                                    2s(2).2p(6).3s_2S.3p(4)1S0_2S
        0.55236001
       -0.51850029
                      0.26884256
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
       -0.25241177
                      0.06371170
        0.14974129
                      0.02242245
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
                      0.00782060
                                    2s(2).2p(6).3p(4)1D2_1D.3d_2P
        0.08843416
       -0.07913818
                      0.00626285
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
        0.06957348
                      0.00484047
                                    2s(2).2p(6).3s_2S.3p(2)1S0_2S.3d(2)1S0_2S
       -0.06792804
                      0.00461422
                                    2s(2).2p(6).3s_2S.3p(4)3P2_4P
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2P
       -0.04635416
                      0.00214871
       -0.04439733
                      0.00197112
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2P
                      0.00144056
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3P2_2P
        0.03795472
       -0.03450153
                      0.00119036
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2S
       -0.03371402
                      0.00113663
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2S
       -0.03274764
                      0.00107241
        0.03171981
                      0.00100615
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)3F2_2P
 3
   1/2
                      -1441.041027919
                                            99.883%
            +
        0.69856599
                      0.48799445
                                    2s(2).2p(6).3s_2S.3p(4)1S0_2S
        0.44943909
                      0.20199550
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_2P
       -0.37641525
                      0.14168844
                                    2s(2).2p(6).3s_2S.3p(4)3P2_2P
       -0.31029154
                      0.09628084
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2S
       -0.14516094
                      0.02107170
                                    2s(2).2p(6).3s(2).3p(2)1D2_1D.3d_2P
        0.11017096
                      0.01213764
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4D
                                    2s(2).2p(6).3s_2S.3p(4)3P2_4P
       -0.10592606
                      0.01122033
        0.08894930
                      0.00791198
                                    2s(2).2p(6).3s_2S.3p(2)1S0_2S.3d(2)1S0_2S
       -0.06646514
                      0.00441762
                                    2s(2).2p(6).3p(4)1D2_1D.3d_2P
                                    2s(2).2p(6).3s(2).3p(2)3P2_3P.3d_4P
       -0.04537257
                      0.00205867
       -0.04336944
                      0.00188091
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2S
       -0.04274245
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2S
                      0.00182692
        0.04115897
                      0.00169406
                                    2s(2).2p(6).3s_2S.3p(2)1D2_2D.3d(2)1D2_2P
        0.03553871
                      0.00126300
                                    2s(2).2p(6).3s_2S.3p(2)3P2_4P.3d(2)3P2_2P
```

# 8.4. Output Files from the Eighth Example

. . . . . . . . . . .

The eighth example, see Section 6.8, was the calculation of the radial density distribution D(r) for Be ground state and the transformation to natural orbitals. The file n4.cd contains three columns with the radial grid, the radial density distribution D(r) and the spherical electron density function  $\rho(r)$  as shown below. Using Matlab, GNU Octave or Python the distribution is readily plotted.

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```
D(r)=4\pi^2*rho(r)
  r [au]
                                              rho(r)
1
        0 +
0.00000000D+00
                     0.00000000D+00
                                         3.5595248135D+01
2.5635548188D-08
                     2.9395876524D-13
                                         3.5595191429D+01
5.2585459038D-08
                     1.2368931242D-12
                                         3.5595161894D+01
8.0917121364D-08
                     2.9287460232D-12
                                         3.5595164086D+01
1.1070137908D-07
                     5.4815947105D-12
                                         3.5595163799D+01
1.4201270834D-07
                     9.0210123946D-12
                                         3.5595163778D+01
1.7492940379D-07
                     1.3687574330D-11
                                         3.5595163713D+01
2.0953377430D-07
                     1.9638527873D-11
                                         3.5595163617D+01
2.4591234882D-07
                     2.7049642650D-11
                                         3.5595163498D+01
2.8415609275D-07
                     3.6117260296D-11
                                         3.5595163346D+01
                                         3.5595163160D+01
3.2436063535D-07
                     4.7060564994D-11
3.6662650893D-07
                     6.0124098290D-11
                                         3.5595162934D+01
4.1105940020D-07
                     7.5580544151D-11
                                         3.5595162664D+01
 . . . . . .
2.2006596267D+01
                     1.4378899592D-12
                                         2.3627079091D-16
2.3134898611D+01
                     2.4605656113D-13
                                         3.6583890201D-17
2.4321050253D+01
                     3.8221221504D-14
                                         5.1419760355D-18
2.5568017190D+01
                     5.3624070241D-15
                                         6.5276342428D-19
2.6878917489D+01
                     6.7510077496D-16
                                         7.4359283127D-20
2.8257029084D+01
                     7.6386795748D-17
                                         7.6129930176D-21
2.9705797971D+01
                     5.9008393899D-18
                                         5.3213458250D-22
3.1228846828D+01
                     3.8138158242D-19
                                         3.1119883125D-23
3.2829984069D+01
                     1.9628254786D-22
                                         1.4492071857D-26
```

## 8.5. Output Files from the Ninth Example

The ninth example was for the unexpected transition 2s2p  $^3P_0^o - 2s^2$   $^1S_0$  in Ni XXV, see Section 6.9. The file odd\_n3.cgjhfs is shown below. First, the J quantum numbers, the parities, and the energies are shown for the computed states. Next come the reduced matrix elements

$$\langle \Gamma J || \mathbf{N}^{(1)} + \Delta \mathbf{N}^{(1)} || \Gamma' J' \rangle$$

for the magnetic (Zeeman) interaction, see [11] Equations (34), (35), (44) and (45). This is followed by the reduced electronic matrix elements

$$\langle \Gamma J \| \mathbf{T}^{(1)} \| \Gamma' J' \rangle$$

for the magnetic dipole interaction, see [11] Equations (13) and (15). Finally, the reduced electronic matrix elements

$$\langle \Gamma J \| \mathbf{T}^{(2)} \| \Gamma' J' \rangle$$

for the electric quadrupole interaction, see [11] Equations (14) and (16). The reduced matrix elements adhere to the Brink and Satchler definition of the Wigner-Eckart theorem and they are not symmetric, see [11] Equation (57).

```
Number of relativistic eigenvalues
 4
Lev.
       J Parity
                       F.
 1
      2.0 -
                 -944.099455445
                 -944.694852121
 1
      1.0
      1.0
                 -942.723282825
      0.0
                 -944.877056498
Zeeman interaction matrix
0.18322E+01 -0.34691E+00 0.68227E-01 0.00000E+00
0.44786E+00 0.10439E+01 -0.67174E-01 0.40125E+00
```

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```
-0.88081E-01 -0.67174E-01 0.71718E+00 -0.78350E-01 0.00000E+00 -0.69499E+00 0.13571E+00 0.00000E+00 HFI-matrix for the magnetic dipole operator 0.36369E+02 -0.10509E+02 0.27002E+02 0.00000E+00 0.13567E+02 0.36113E+02 0.22641E+02 0.15293E+02 -0.34859E+02 0.22641E+02 -0.18146E+01 0.81436E+01 0.00000E+00 -0.26488E+02 -0.14105E+02 0.00000E+00 HFI-matrix for the electric quadrupole operator 0.28620E+03 0.32475E+03 -0.59600E+02 -0.22196E+03 -0.41925E+03 -0.22145E+03 0.14396E+03 0.00000E+00 0.76944E+02 0.14396E+03 0.46833E+03 0.00000E+00 -0.49633E+03 -0.00000E+00 -0.00000E+00 0.00000E+00 0.00000E+00
```

# 9. Case Study I: $2s^22p$ , $2s2p^2$ in Mo XXXVIII Using Scripts

In this case study, we use script files to perform systematic calculations for all the states of the  $2s^22p$  and  $2s2p^2$  configurations in Mo XXXVIII. The 10 states are as follows:

odd: 
$$2s^22p \,^2P^o_{1/2,3/2}$$
 even:  $2s2p^2 \,^4P_{1/2,3/2,5/2}$ ,  $2s2p^2 \,^2D_{3/2,5/2}$ ,  $2s2p^2 \,^2P_{1/2,3/2}$ ,  $2s2p^2 \,^2S_{1/2}$ 

The script files can be found in grasptest/case1/script.

In a real application a correlation model should be defined, i.e., some rule to generate the CSFs from an orbital set. The convergence of computed properties is then monitored as the orbital set is increased. For the odd state, a reasonable correlation model is to start from the  $\{1s^22s^22p, 1s^22p^3\}$  MR and then generate all CSFs that can be obtained by single and double excitations from the MR to an active set of orbitals. The active set of orbitals is then systematically increased. Following the normal conventions, the orbital set is denoted by the highest principal quantum number. For example, n=3 means the orbital set  $\{1s, 2s, 2p, 3s, 3p, 3d\}$ . In this study, we increase the active set of orbitals up to n=6. For the even states, we start from the  $1s^22s2p^2$  reference and generate all CSFs that can be obtained by single and double excitations from the MR to the active sets of orbitals. The correlation model can be easily extended by adding CSFs to the MR.

## 9.1. Running Script Files

To automate the calculations, we use script files. For convenience, we have a main script that calls subscripts to perform different tasks. The construction of the scripts is greatly simplified if the names of the files are chosen in a simple and systematic way. In the case study we use the names odd2, odd3, odd4, odd5, odd6 and even2, even3, even4, even5, even6 to denote files for the odd and even parity states, respectively. The digit indicates which orbital set has been used to generate the expansion.

Before starting, please make sure that the GRASP executables are on the path.

The main script sh\_case1 is shown below. This script controls the computational flow and calls several subscripts.

```
#!/bin/sh
```

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Each of the subscripts is given below together with some comments.

If all script files are available with execute permission (use the command chmod +x) we start the computation by typing the name of the main script

```
./sh_case1
```

Please note that these calculations will take several hours!

# 1. Generate Expansions

The expansions are generated by the script sh\_files\_c. This is by far the most complicated script. It is simplified by generating lists for large active sets and then using rcsfsplit, see Section 7.1.

```
#!/bin/sh
set -x
     Generate CSF expansions
       1.1 MR for 2s(2)2p, 2p(3)
rcsfgenerate <<EOF1
0
1s(2,i)2s(2,i)2p(1,i)
1s(2,i)2p(3,i)
2s,2p
1,3
0
n
FOF1
cp rcsf.out odd2.c
         1.2 SD-MR for n=6
rcsfgenerate <<EOF3
0
1s(2,*)2s(2,*)2p(1,*)
1s(2,*)2p(3,*)
6s,6p,6d,6f,6g,6h
```

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```
n
EOF3
cp rcsf.out odd.c
       Split into odd3.c, odd4.c, odd5.c, odd6.c
rcsfsplit <<EOF5
odd
3s,3p,3d
4s, 4p, 4d, 4f
5s,5p,5d,5f,5g
6s,6p,6d,6f,6g,6h
EOF5
2. Generate CSF expansions
      2.1 for 2s2p(2)
rcsfgenerate <<EOF1
1s(2,i)2s(1,i)2p(2,i)
2s,2p
1,5
0
n
EOF1
cp rcsf.out even2.c
        2.2 SD for n=6
rcsfgenerate <<EOF3
1s(2,*)2s(1,*)2p(2,*)
6s,6p,6d,6f,6g,6h
1,5
n
EOF3
cp rcsf.out even.c
        Split into even3.c, even4.c, even5.c, even6.c
```

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```
rcsfsplit <<EOF5
even
4
3s,3p,3d
3
4s,4p,4d,4f
4
5s,5p,5d,5f,5g
5
6s,6p,6d,6f,6g,6h
6
EOF5
```

## 2. Get Nuclear Data

Nuclear data are defined by the script sh\_nuc. Since we are not interested in hyperfine structure, the nuclear spin and moments have all been set to 1.

```
#!/bin/sh
set -x

# 2. Get nucleardata
rnucleus <<S1
42
96
n
96
1
1
1
1
S1</pre>
```

# 3. Get Initial Estimates

cat isodata

The script sh\_initial performs angular integration, gets initial estimates and performs rmcdhf calculations for the odd and even reference states (odd2 and even2). As initial estimates, we use screened hydrogenic functions. For the reference states, all orbitals are required to be spectroscopic, i.e., they should have the correct number of nodes, see Section 7.1

```
#!/bin/sh
set -x

# 3. For n=2, Get initial estimates for odd.

cp odd2.c rcsf.inp
rangular <<S4
y
S4

# Get initial estimates of wave functions
rwfnestimate <<S5
y
3
*
S5</pre>
```

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```
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf_initial <<S6</pre>
1
1
5
100
S6
  Save the result to odd2
rsave odd2
# 3. For n=2, Get initial estimates for even
cp even2.c rcsf.inp
rangular <<S4
S4
   Get initial estimates of wave functions
rwfnestimate <<S5
3
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_initial <<S6</pre>
у
1,2,3
1,2,3
1,2
5
100
S6
   Save the result to even2
rsave even2
```

## 4. rmcdhf and rci Calculations

The script  $sh\_scf$  performs angular integration, estimates the new radial functions and performs rmcdhf for the odd and even states up to n=6. At the end, rci calculations are performed for the largest expansions. The rci calculations include Breit interaction and QED corrections. All results are transformed to LSJ-coupling. Please note how we loop in the script over the digit n that indicates the size of the orbital set.

```
#!/bin/sh
set -x
# 4. Get results for odd n=3,4,5,6
for n in 3 4 5 6
```

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```
do
   (cp odd${n}.c rcsf.inp
# Get angular data
rangular <<S4
S4
# Get initial estimates of wave functions
m='expr $n - 1'
echo m=m = n
rwfnestimate <<S5
1
odd${m}.w
3
S5
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf_${n} <<S6</pre>
У
1
1
5
${n}*
100
rsave odd${n}
# transform to LSJ-coupling
jj2lsj <<S1
odd{n}
У
у
S1
   echo)
done
   Perform Breit-correction using RCI for n=6. First copy to other file names
n=6
cp odd${n}.c oddCI${n}.c
cp odd${n}.w oddCI${n}.w
rci > outodd_rci <<S7</pre>
oddCI${n}
```

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```
у
у
1.d-6
У
n
n
у
4
1
1
S7
# transform to LSJ-coupling
jj2lsj <<S1
oddCI${n}
у
У
S1
    4. Get results for even n=3,4,5,6
for n in 3 4 5 6
do
   (cp even${n}.c rcsf.inp
# Get angular data
rangular <<$4
У
S4
# Get initial estimates of wave functions
m='expr $n - 1'
echo m=m = n
rwfnestimate <<S5
у
1
even${m}.w
3
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_${n} <<S6</pre>
У
1,2,3
1,2,3
1,2
5
${n}*
```

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```
100
S6
\verb"rsave even$\{n\}
# transform to LSJ-coupling
jj2lsj <<S1
even${n}
у
S1
   echo)
done
   Perform Breit-correction using RCI for n=6
cp even${n}.c evenCI${n}.c
cp even${n}.w evenCI${n}.w
rci > outeven_rci <<S7</pre>
evenCI${n}
У
у
1.d-6
У
n
n
У
4
1,2,3
1,2,3
1,2
S7
# transform to LSJ-coupling
jj2lsj <<S1
evenCI${n}
у
У
S1
```

## 5. Transition calculation

The script sh\_tr computes the E1 transition rates between the odd and even states. First we perform a biorthonormal transformation, and then we perform the transition calculation itself.

```
#!/bin/sh
set -x
```

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```
6. Perform transition calculation for the n=6 CI results
n=6
  First the biorthonormal transformations
rbiotransform > out_rbiotransform <<EOF</pre>
у
У
oddCI$n
evenCI$n
EOF
# Then the transition calculations
rtransition > out_transition <<EOF
у
oddCI$n
evenCI$n
E1
EOF
9.2. Comparison with Experiment
To display the computed energies we give the command
rlevels oddCI6.cm evenCI6.cm
The computer returns the energies together with labels in LSJ-coupling for all the states.
                          ncftot =
                                           20641
                                                                    36
 nblock =
                                                    nw =
                                                                         nelec =
                                                                                              5
 nblock =
                          ncftot =
                                           36290
                                                                         nelec =
                                                    nw =
 Energy levels for ...
 Rydberg constant is 109737.31569
 Splitting is the energy difference with the lower neighbor
```

No	Pos	J Pari	ty Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	Configuration
1	1	1/2 -	-2386.3200262	0.00	0.00	1s(2).2s(2).2p_2P
2	1	1/2 +	-2382.2391997	895637.89	895637.89	1s(2).2s_2S.2p(2)3P2_4P
3	1	3/2 -	-2381.9242100	964770.13	69132.25	1s(2).2s(2).2p_2P
4	1	3/2 +	-2378.9986341	1606859.82	642089.69	1s(2).2s_2S.2p(2)3P2_4P
5	1	5/2 +	-2378.1476302	1793633.59	186773.77	1s(2).2s_2S.2p(2)3P2_4P
6	2	3/2 +	-2376.7269966	2105426.64	311793.05	1s(2).2s_2S.2p(2)1D2_2D
7	2	1/2 +	-2376.5312708	2148383.48	42956.84	1s(2).2s_2S.2p(2)3P2_2P
8	2	5/2 +	-2373.9013617	2725581.82	577198.34	1s(2).2s_2S.2p(2)1D2_2D
9	3	1/2 +	-2371.9005560	3164707.90	439126.08	1s(2).2s_2S.2p(2)1S0_2S
10	3	3/2 +	-2371.8561115	3174462.35	9754.45	1s(2).2s_2S.2p(2)3P2_2P

The Mo XXXVIII transitions have been observed in the JET Tokamak, Myrnäs et al. [44]. In the Table 8, the experimental transition energies are compared with the calculated energies. Please note that the quantum labels for the  $2s2p^2$   $^2P_{1/2}$  and

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 $2s2p^2$   $^2S_{1/2}$  seem to have been swapped in the experimental paper, i.e., the highest J=1/2 state should be  $2s2p^2$   $^2S_{1/2}$  and not  $2s2p^2$   $^2P_{1/2}$ . We see that the odd states are somewhat too high. This is due to an imbalance in the MR. As discussed in the beginning of the section the correlation model can be refined by extending the MR. Adopting the MRs  $\{2s^22p, 2p^3, 2s2p3d, 2p^23d\}$  and  $\{2s2p^2, 2p^23d, 2s^23d, 2s^3d^2\}$  for, respectively, the odd and even parity states improves the energy separations considerably [45]. A careful investigation of the effects of increasing the MR is part of any systematic calculation (see Section 6.6).

State (Exp. Label)	$\Delta E^a$ (cm <sup>-1</sup> )	$\Delta E^b$ (cm <sup>-1</sup> )
$2s^22p ^2P_{1/2}^o$	0	0
$2s2p^2 {}^4P_{1/2}$	$894\ 050 \pm 400$	895 848
$2s^22p ^2P_{3/2}^{0}$	$964\ 050 \pm 90$	964 715
$2s2p^2  ^4P_{3/2}$		1 606 863
$2s2p^2 {}^4P_{5/2}$	$1790130\pm200$	1 793 682
$2s2p^2 {}^2D_{3/2}$	$2\ 102\ 900\pm 900$	2 106 354
$2s2p^2 {}^2S_{1/2}$	$2147300\pm900$	2 149 456
$2s2p^2 {}^2D_{5/2}$		2 725 586
$2s2p^2 {}^2P_{1/2}$	$3\ 164\ 770\pm1500$	3 166 168
$2s2p^2 {}^2P_{3/2}$	$3\ 171\ 300\pm1500$	3 175 559

Table 8. Experimental and computed excitation energies for Mo XXXVIII.

#### 9.3. Transition Rates

Below are the transition parameters as given in the file oddCI6.evenCI6.ct.lsj. The agreement between calculated values in the two gauges (the first line gives values in length gauge and the second line gives values in velocity gauge) is quite good, especially for the strong transitions. Expansions based on a larger MR set will further improve the agreement. The quantity dT is defined as

$$dT = \frac{|A_C - A_B|}{\max(A_C, A_B)},$$

where  $A_B$  and  $A_C$  are the transition rates in length and velocity gauge. dT is a measure of the uncertainty of the computed transition rates [43].

```
Transition between files: oddCI6 evenCI6
```

```
1-2386.32002619 1s(2).2s(2).2p_2P
 1-2382.23919972 1s(2).2s_2S.2p(2)3P2_4P
895637.89 CM-1 111.65 ANGS(VAC) 111.65 ANGS(AIR)
E1 S = 4.42135D-03 GF = 1.20285D-02 AKI = 3.21802D+09 dT = 0.05152
       4.66152D-03
                        1.26819D-02
                                          3.39282D+09
 1-2386.32002619 1s(2).2s(2).2p_2P
 1-2376.53127082 1s(2).2s_2S.2p(2)3P2_2P
2148383.48 CM-1 46.55 ANGS(VAC)
                                      46.55 ANGS(AIR)
E1 S = 1.87910D-02 GF = 1.22627D-01 AKI = 1.88765D+11 dT = 0.00415
       1.88692D-02
                       1.23137D-01
                                          1.89551D+11
 1-2386.32002619 1s(2).2s(2).2p_2P
 1-2371.90055603 1s(2).2s_2S.2p(2)1S0_2S
31.60 ANGS(VAC) 31.60 ANGS(AIR)
E1 S = 1.51571D-05 GF = 1.45705D-04 AKI = 4.86690D+08
                                                     dT = 0.06893
```

 $<sup>\</sup>overline{}^a$  Exp. (Myrnäs et al. [44]),  $\overline{}^b$  Calc. (Rynkun et al. [45].)

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1.62792D-05 1.56492D-04 5.22723D+08 1-2386.32002619 1s(2).2s(2).2p\_2P 3-2378.99863414 1s(2).2s\_2S.2p(2)3P2\_4P 1606859.82 CM-1 62.23 ANGS(VAC) 62.23 ANGS(AIR) E1 S = 1.11563D-04 GF = 5.44533D-04 AKI = 2.34457D+08 dT = 0.016901.13481D-04 5.53892D-04 2.38486D+08 1-2386.32002619 1s(2).2s(2).2p\_2P 3-2376.72699656 1s(2).2s\_2S.2p(2)1D2\_2D 2105426.64 CM-1 47.50 ANGS(VAC) 47.50 ANGS(AIR) E1 S = 2.45759D-02 GF = 1.57172D-01 AKI = 1.16181D+11 dT = 0.006442.47352D-02 1.58190D-01 1.16934D+11 1-2386.32002619 1s(2).2s(2).2p\_2P 3-2371.85611149 1s(2).2s\_2S.2p(2)3P2\_2P 3174462.35 CM-1 31.50 ANGS(VAC) 31.50 ANGS(AIR) E1 S = 6.34294D-04 GF = 6.11625D-03 AKI = 1.02780D+10 dT = 0.005296.30940D-04 6.08391D-03 1.02236D+10 1-2382.23919972 1s(2).2s\_2S.2p(2)3P2\_4P 3-2381.92421002 1s(2).2s(2).2p\_2P 69132.25 CM-1 1446.50 ANGS(VAC) 1446.50 ANGS(AIR) E1 S = 5.06265D-04 GF = 1.06312D-04 AKI = 8.47280D+04 dT = 0.188438.62797D-05 4.10869D-04 6.87626D+04 3-2381.92421002 1s(2).2s(2).2p\_2P 1-2376.53127082 1s(2).2s\_2S.2p(2)3P2\_2P 1183613.34 CM-1 84.49 ANGS(VAC) 84.49 ANGS(AIR) E1 S = 2.36464D-03 GF = 8.50157D-03 AKI = 3.97220D+09 dT = 0.052272.49506D-03 8.97046D-03 4.19128D+09 3-2381.92421002 1s(2).2s(2).2p\_2P 1-2371.90055603 1s(2).2s\_2S.2p(2)1S0\_2S 2199937.76 CM-1 45.46 ANGS(VAC) 45.46 ANGS(AIR) E1 S = 1.48021D-02 GF = 9.89140D-02 AKI = 1.59658D+11 dT = 0.002609.86566D-02 1.47636D-02 1.59243D+11 3-2381.92421002 1s(2).2s(2).2p\_2P 3-2378.99863414  $1s(2).2s_2S.2p(2)3P2_4P$ 642089.69 CM-1 155.74 ANGS(VAC) 155.74 ANGS(AIR) E1 S = 1.00858D-03 GF = 1.96712D-03 AKI = 1.35240D+08 dT = 0.115312.22352D-03 1.14004D-03 1.52867D+08 3-2381.92421002 1s(2).2s(2).2p\_2P

3-2376.72699656 1s(2).2s\_2S.2p(2)1D2\_2D

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```
1140656.51 CM-1
                    87.67 ANGS(VAC)
                                        87.67 ANGS(AIR)
E1 S = 2.73257D-03 GF = 9.46784D-03 AKI = 2.05420D+09
                                                          dT = 0.04984
        2.87592D-03
                          9.96450D-03
                                             2.16196D+09
 3-2381.92421002 1s(2).2s(2).2p_2P
 3-2371.85611149 1s(2).2s_2S.2p(2)3P2_2P
2209692.21 CM-1
                    45.26 ANGS(VAC)
                                         45.26 ANGS(AIR)
E1 S = 4.49452D-02 GF = 3.01675D-01 AKI = 2.45632D+11
                                                          dT = 0.00389
                          3.02854D-01
        4.51208D-02
                                             2.46592D+11
 3-2381.92421002 1s(2).2s(2).2p_2P
 5-2378.14763025 1s(2).2s_2S.2p(2)3P2_4P
828863.45 CM-1
                   120.65 ANGS(VAC)
                                         120.65 ANGS(AIR)
E1 S = 9.97378D-03 GF = 2.51112D-02 AKI = 1.91789D+09 dT = 0.08159
        1.08599D-02
                          2.73422D-02
                                             2.08828D+09
 3-2381.92421002 1s(2).2s(2).2p_2P
 5-2373.90136166 1s(2).2s_2S.2p(2)1D2_2D
1760811.69 CM-1
                     56.79 ANGS(VAC)
                                         56.79 ANGS(AIR)
E1 S = 1.52580D-02 GF = 8.16083D-02
                                       AKI = 2.81288D+10 dT = 0.01679
        1.55185D-02
                          8.30017D-02
                                             2.86091D+10
```

## 9.4. LaTeX Table with Energies as Functions of the Active Set

The script sh\_scf is written in such a way that all results are transformed to *LSJ*-coupling using jj21sj. The output from rlevels will then contain quantum labels of the states in *LSJ*-coupling. By saving the output from rlevels we can generate a LaTeX table showing the convergence of the energies as the active set is increased. If we include also the output from the final rci calculation, we can see the effect of the Breit interaction and QED.

Issuing the commands below saves the output from rlevels corresponding to the increasing active set of orbitals in files energy3, energy4, energy5, energy6, energyCI6

```
>>rlevels even3.m odd3.m > energy3
>>rlevels even4.m odd4.m > energy4
>>rlevels even5.m odd5.m > energy5
>>rlevels even6.m odd6.m > energy6
>>rlevels evenCI6.cm oddCI6.cm > energyCI6
```

We now call rtablevels to produce a LaTeX table, see Section 7.6

#### >>rtablevels

## RTABLEVELS

```
Makes LaTeX and ASCII tables of energy files produced by rlevels (in ljs format)
Multiple energy files can be used as input
Energies from file 1 fills column 1, energies from file 2 fills column 2 etc. Checks are done to see if the labels if the labels in the files are consistent
Input file: name1, name2, ...
Output files: energylabellatex.tex, energylabelascii.txt
```

Inspect energy files and determine how many positions should be skipped in the string that determines the label

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```
e.g., if the string is 1s(2).2s_2S.2p(2)3P2_4P and 1s(2) is a core
 then you would like to skip 1s(2). i.e., 6 positions and determine
 the label from 2s_2S.2p(2)3P2_4P
 How many positions should be skipped?
>>6
 Give the number of energy files from rlevels
 Name of file 1
>>energy3
 Name of file 2
>>energy4
 Name of file 3
>>energy5
 Name of file 4
>>energy6
 Name of file 5
>>energyCI6
```

The generated LaTeX table is named energytablelatex.tex. After processing we obtain Table 9.

**Table 9.** Energies in  $cm^{-1}$  from the files energy3, energy4, energy5, energy6, energyCI6.

$2s^2 2p^2 P_{1/2}^0$	0	0	0	0	0
$2s^{2}S 2p^{2}(^{3}P) ^{4}P_{1/2}$	894248	894018	893855	893920	895637
$2s^2 2p^2 P_{3/2}^{o^2}$	982556	982796	982807	982793	964770
$2s^{2}S 2p^{2}(^{3}P) ^{4}P_{3/2}$	1620240	1621107	1621267	1621540	1606859
$2s^{2}S 2p^{2}(\frac{3}{2}P) {}^{4}P_{5/2}$	1823232	1823052	1822927	1823033	1793633
$2s^{2}S 2p^{2}(\bar{1}D)^{2}D_{3/2}$	2130522	2128609	2128002	2127792	2105426
$2s^{2}S 2p^{2}(\frac{3}{2}P)^{2}P_{1/2}$	2163550	2161641	2160998	2160705	2148383
$2s  {}^{2}S  2p^{2}(\overline{\smash{\frac{1}{2}}}D)  {}^{2}D_{5/2}$	2764810	2764403	2764217	2764261	2725581
$2s^{2}S 2p^{2}(_{0}^{1}S)^{2}S_{1/2}$	3193694	3191337	3190537	3190063	3164707
$2s  {}^{2}S  2p^{2} ({}^{3}_{2}P)  {}^{2}P_{3/2}$	3213578	3211491	3210805	3210469	3174462

From the table, we see that the energies seem to be reasonably converged when the active orbital set has been increased to n = 6. We also see that the Breit interaction and QED, as included in the final rci calculation, change the energies substantially.

At this point, it is appropriate to comment on the labels. The *LS* term for electrons with occupation one is not written out. The *LS* terms for equivalent electrons with occupation two or more are written in parentheses together with the seniority number. The angular momenta are coupled from left to right and written in a linear fashion. As an example, we look at

$$2s \, {}^{2}S \, 2p^{2} ({}^{3}_{2}P) \, {}^{4}P_{5/2}$$

The 2s electron has the LS term  ${}^2S$ , but this is not written out explicitly. The  $2p^2$  has the LS term  ${}^3P$  with seniority 2. This is written as  $2p^2({}^3_2P)$ . Coupling the  ${}^1S$  term of the  $1s^2$  core (not included in the label) with the  ${}^2S$  term of 2s results in  ${}^2S$ . The  ${}^2S$  term in turn is coupled with the  ${}^3P$  term to yield  ${}^4P$ . The final L and S are then coupled to J = 5/2. See also section 9.2 for a more detailed account of the labels.

## 9.5. LaTeX Table with Transition Data

There is also a program rtabtransE1, see Section 7.7, that produces a transition file. There are different options for the table. Below, we choose to display gf and A in the length gauge together with dT

#### >>rtabtransE1

## RTABTRANSE1

Makes LaTeX tables of transition data from transition files

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```
name1.name2.ct.lsj
 Input file: name1.name2.ct.lsj
 Output file: transitiontable.tex
 Specify table format
 (1). Lower & Upper & Energy diff. & wavelength & S & gf & A & dT
 (2). Lower & Upper & Energy diff. & wavelength & gf & A & dT
 (3). Lower & Upper & Energy diff. & wavelength & gf & A
 (4). Lower & Upper & Energy diff. & S & gf & A & dT
 (5). Lower & Upper & Energy diff. & gf & A & dT
 (6). Lower & Upper & Energy diff. & gf & A
>>5
 Inspect the name1.name2.ct.lsj file and determine how many positions
 should be skipped in the string that determines the label
 e.g., if the string is 1s(2).2s_2S.2p(2)3P2_4P and 1s(2) is a core
 then you would like to skip 1s(2). i.e., 6 positions and determine
 the label from 2s_2S.2p(2)3P2_4P
 How many positions should be skipped?
>>6
Name of file
>>oddCI6.evenCI6.ct.lsj
```

The generated LaTeX table is named transitiontable.tex. After processing we obtain Table 10.

**Table 10.** Transition data from the file oddCI6.evenCI6.ct.lsj.

Lower State	Upper State	$\Delta E  (\mathrm{cm}^{-1})$	gf	$A (s^{-1})$	dT
$2s^2 2p^2 P_{1/2}$	$2s^{2}S 2p^{2}(_{2}^{3}P)^{4}P_{1/2}$	895637	1.202D-02	3.218D+09	0.051
$2s^2 2p^2 P_{1/2}$	$2s^{2}S 2p^{2}(\frac{3}{2}P)^{2}P_{1/2}$	2148383	1.226D-01	1.887D+11	0.004
$2s^2 2p^2 P_{1/2}$	$2s^{2}S 2p^{2}(\overline{0}S)^{2}S_{1/2}$	3164707	1.457D-04	4.866D+08	0.068
$2s^2 2p^2 P_{1/2}$	$2s^{2}S 2p^{2}({}_{2}^{3}P) {}^{4}P_{3/2}$	1606859	5.445D-04	2.344D+08	0.016
$2s^2 2p^2 P_{1/2}$	$2s^2S 2p^2({}_{2}^{\dagger}D)^2D_{3/2}$	2105426	1.571D-01	1.161D+11	0.006
$2s^2 2p^2 P_{1/2}$	$2s^{2}S 2p^{2}({}_{2}^{3}P)^{2}P_{3/2}$	3174462	6.116D-03	1.027D+10	0.005
$2s^2S 2p^2({}_{2}^{3}P)^4P_{1/2}$	$2s^2 2p^2 P_{3/2}$	69132	1.063D-04	8.472D+04	0.188
$2s^2 2p {}^{2}P_{3/2}$	$2s^{2}S 2p^{2}({}_{2}^{3}P)^{2}P_{1/2}$	1183613	8.501D-03	3.972D+09	0.052
$2s^2 2p^2 P_{3/2}$	$2s^{2}S 2p^{2}(\overline{0}S)^{2}S_{1/2}$	2199937	9.891D-02	1.596D+11	0.002
$2s^2 2p^2 P_{3/2}$	$2s^{2}S 2p^{2}(^{3}_{2}P)^{4}P_{3/2}$	642089	1.967D-03	1.352D+08	0.115
$2s^2 2p^2 P_{3/2}$	$2s^2S 2p^2({}_{2}^{\dagger}D)^2D_{3/2}$	1140656	9.467D-03	2.054D+09	0.049
$2s^2 2p^2 P_{3/2}$	$2s^{2}S 2p^{2}({}_{2}^{3}P)^{2}P_{3/2}$	2209692	3.016D-01	2.456D+11	0.003
$2s^2 2p^2 P_{3/2}$	$2s^{2}S 2p^{2}(\frac{3}{2}P)^{4}P_{5/2}$	828863	2.511D-02	1.917D+09	0.081
$2s^2 2p^2 P_{3/2}$	$2s^2S 2p^2({}_2^{\dagger}D)^2D_{5/2}$	1760811	8.160D-02	2.812D+10	0.016

## 9.6. Editing the LaTeX Table

The table programs translate the *LSJ*-notation from jj21sj to LaTeX notation. If the user wants to simplify or change the LaTeX notation, this is easily done. For example, the global substitution  $2s^2 \le 1$ ,  $\rightarrow 2s$  in the LaTeX file produces Table 11.

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Lower State	Upper State	$\Delta E  (\mathrm{cm}^{-1})$	gf	$A (s^{-1})$	dT
$2s^2 2p^2 P_{1/2}$	$2s2p^2(^3_2P) ^4P_{1/2}$	895637	1.202D-02	3.218D+09	0.051
$2s^2 2p^2 P_{1/2}$	$2s2p^2(\frac{3}{2}P)^2P_{1/2}$	2148383	1.226D-01	1.887D+11	0.004
$2s^2 2p^2 P_{1/2}$	$2s2p^2(\bar{1}S)^2S_{1/2}$	3164707	1.457D-04	4.866D+08	0.068
$2s^2 2p^2 P_{1/2}$	$2s2p^2({}_{2}^{3}P) {}^{4}P_{3/2}$	1606859	5.445D-04	2.344D+08	0.016
$2s^2 2p^2 P_{1/2}$	$2s2p^2(\bar{1}D)^2D_{3/2}$	2105426	1.571D-01	1.161D+11	0.006
$2s^2 2p {}^2P_{1/2}$	$2s2p^2(\frac{3}{2}P)^2P_{3/2}$	3174462	6.116D-03	1.027D+10	0.005
$2s2p^2({}_{2}^{3}P) {}^{4}P_{1/2}$	$2s^2 2p^{\frac{7}{2}}P_{3/2}$	69132	1.063D-04	8.472D+04	0.188
$2s^2 2p^2 P_{3/2}$	$2s2p^2({}_{2}^{3}P)^{2}P_{1/2}$	1183613	8.501D-03	3.972D+09	0.052
$2s^2 2p^2 P_{3/2}$	$2s2p^2(\bar{1}S)^2S_{1/2}$	2199937	9.891D-02	1.596D+11	0.002
$2s^2 2p^2 P_{3/2}$	$2s2p^2(^3_2P) {}^4P_{3/2}$	642089	1.967D-03	1.352D+08	0.115
$2s^2 2p^2 P_{3/2}$	$2s2p^2(\frac{1}{2}D)^2D_{3/2}$	1140656	9.467D-03	2.054D+09	0.049
$2s^2 2p^2 P_{3/2}$	$2s2p^2(\frac{3}{2}P)^2P_{3/2}$	2209692	3.016D-01	2.456D+11	0.003
$2s^2 2p^2 P_{3/2}$	$2s2p^2(\frac{3}{2}P) {}^4P_{5/2}$	828863	2.511D-02	1.917D+09	0.081
$2s^2 2p ^2P_{3/2}$	$2s2p^2(\bar{1}D)^2D_{5/2}$	1760811	8.160D-02	2.812D+10	0.016

**Table 11.** Transition data from the file oddCI6.evenCI6.ct.lsj.

## 9.7. Scripts for MPI Codes

The scripts above can, with very small modifications, also be used for performing the runs using the MPI codes. Before running the scripts for the MPI code, the file disks with paths to the working directory and to the directory containing temporary data must be created, see Section 6.4. In the different scripts, the calls to the MPI programs amount to changes of the type

```
rangular --> mpirun -np 8 rangular_mpi
rmcdhf --> mpirun -np 8 rmcdhf_mpi
rci --> mpirun -np 8 rci_mpi
```

etc. Scripts for the MPI cases are included under case1\_mpi in the test data set. Consult the README file in the working directory for more details on setting up the file disks.

## 10. Case study II: The Li Iso-Electronic Sequence Using Scripts

In this case study, we use script files to perform systematic calculations for the  $1s^22s$   $^2S_{1/2}$  ground state and the  $1s^22p$   $^2P_{1/2,3/2}^o$  excited states in the Li iso-electronic sequence. Computing data for an iso-electronic sequence, angular data can be reused and need not be recomputed for each member of the sequence. The script files can be found in grasptest/case2/script.

We start with a single calculation of the three reference states. After that, separate calculations are done for the two parities. Correlation is then included by allowing single, double, and triple (SDT) excitations from the reference to active sets up to n = 5 (complete active space calculations). Calculations including hyperfine structures and transition rates are performed from Z = 6 to Z = 12.

It is convenient to save the results for the different ions in directories Z6, Z7, Z8, ..., Z12.

#### 10.1. Running Script Files

The main script sh\_case2 is shown below. This script controls the computational flow and calls several subscripts.

#!/bin/sh

```
set -x
```

- # Main script for iso-electronic sequence
- # 1. Generate directories Z6, Z7, .. for the elements
- # Define nuclear data for each element

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```
./sh_nuc_seq
# 2. Generate lists of CSFs in main directory
./sh_files_c
```

# 3. Start by performing rmcdhf calculations for the 1s(2)2s, 1s(2)2p

# reference states

./sh\_DF

# 4. Perform rmcdhf calculations for all the even expansions

# Angular data computed only once and then moved to different directories

./sh\_even

# 5. Perform rmcdhf calculations for all the odd expansions

Angular data computed only once and then moved to different directories

./sh\_odd

# 6. Perform rci calculations for the even5 and odd5 expansions

Perform rhfs and transition calculations.

# Angular data computed only once and then moved to different directories

./sh\_even\_odd

Each of the subscripts is given below together with some comments.

If all script files are available with execute permission (use the command chmod +x) we start the computation by typing the name of the main script

```
./sh_case2
```

Please note that these calculations will take several hours!

1. Generate Directories and Define Nuclear Data

The script  $sh_nuc_seq$  produces nuclear data for Z = 6,7,...,12 in the directories Z6, Z7 ,..., Z12. By modifying the script, we can produce nuclear data for any sequence of charges.

#!/bin/sh

do

```
set -x
# Full loop over all Z
#
   for z in 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 \
#
             26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 \
             48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 \
#
#
             70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 \
#
             92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109
#
             110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 \
             126 127 128 129 130 131 132 133 134 135 136 137 138
# We select Z from 6 to 12
  for z in 6 7 8 9 10 11 12
```

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```
# Data from Jefferson Lab (http://education.jlab.org/itselemental)
case $z in
   1) m=0; MM=1.0794;;
                          # Need to use point nucleus
  2) m=4; MM=4.002602;;
  3) m=7; MM=6.941;;
  4) m=9; MM=9.012182;;
  5) m=11; MM=10.811;;
  6) m=12; MM=12.0107;;
  7) m=14; MM=14.0067;;
  8) m=16; MM=15.9994;;
  9) m=19; MM=18.9984032;;
  10) m=20; MM=20.1797;;
  11) m=23; MM=22.98976928;;
  12) m=24; MM=24.3050;;
  13) m=27; MM=26.9815386;;
  14) m=28; MM=29.0855;;
  15) m=31; MM=30.973762;;
  16) m=32; MM=32.065;;
  17) m=35; MM=35.453;;
  18) m=40; MM=39.948;;
  19) m=39; MM=39.0938;;
  20) m=40; MM=40.078;;
  21) m=45; MM=44.955912;;
  22) m=48; MM=47.867;;
  23) m=51; MM=50.9415;;
  24) m=52; MM=51.9961;;
  25) m=55; MM=54.938045;;
  26) m=56; MM=55.845;;
  27) m=59; MM=58.933195;;
  28) m=59; MM=58.6934;;
  29) m=64; MM=63.546;;
  30) m=65; MM=65.409;;
  31) m=70; MM=69.723;;
  32) m=73; MM=72.64;;
  33) m=75; MM=74.92160;;
  34) m=79; MM=78.96;;
  35) m=80; MM=79.904;;
  36) m=84; MM=83.798;;
  37) m=85; MM=85.4678;;
  38) m=88; MM=87.62;;
  39) m=89; MM=88.90585;;
  40) m=91; MM=91.224;;
  41) m=93; MM=92.90638;;
  42) m=96; MM=95.94;;
  43) m=98; MM=98;;
  44) m=101; MM=10.07;;
  45) m=103; MM=102.90550;;
  46) m=106; MM=106.42;;
  47) m=108; MM=107.8682;;
  48) m=112; MM=112.411;;
  49) m=115; MM=114.818;;
  50) m=119; MM=118.710;;
  51) m=122; MM=121.760;;
```

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```
52) m=128; MM=127.60;;
 53) m=127; MM=126.90447;;
 54) m=131; MM=131.293;;
 55) m=133; MM=132.9054519;;
 56) m=137; MM=137.327;;
 57) m=139; MM=138.90547;;
 58) m=140; MM=140.116;;
 59) m=141; MM=140.90765;;
 60) m=144; MM=144.242;;
 61) m=145; MM=145;;
 62) m=150; MM=150.36;;
 63) m=152; MM=151.964;;
 64) m=157; MM=157.25;;
 65) m=159; MM=158.92535;;
 66) m=163; MM=162.5;;
 67) m=165; MM=164.93032;;
 68) m=167; MM=167.259;;
 69) m=169; MM=168.93421;;
 70) m=173; MM=173.04;;
 71) m=175; MM=174.967;;
 72) m=178; MM=178.49;;
 73) m=181; MM=180.94788;;
 74) m=184; MM=183.84;;
 75) m=186; MM=186.207;;
 76) m=190; MM=190.23;;
 77) m=192; MM=192.217;;
 78) m=195; MM=195.084;;
 79) m=197; MM=196.966569;;
 80) m=201; MM=200.59;;
 81) m=204; MM=204.3833;;
 82) m=207; MM=207.2;;
 83) m=209; MM=208.9804;;
 84) m=209; MM=209;;
 85) m=210; MM=210;;
 86) m=222; MM=222;;
 87) m=223; MM=223;;
 88) m=226; MM=226;;
 89) m=227; MM=227;;
 90) m=232; MM=232.03806;;
 91) m=231; MM=231.03588;;
 92) m=238; MM=238.02891;;
 93) m=237; MM=237;;
 94) m=244; MM=244;;
 95) m=243; MM=243;;
 96) m=247; MM=247;;
 97) m=247; MM=247;;
 98) m=251; MM=251;;
 99) m=252; MM=252;;
100) m=257; MM=257;;
101) m=258; MM=258;;
102) m=259; MM=259;;
103) m=262; MM=262;;
104) m=267; MM=267;;
105) m=268; MM=268;;
```

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```
106) m=271; MM=271;;
 107) m=272; MM=272;;
 108) m=277; MM=277;;
 109) m=276; MM=276;;
 110) m=281; MM=281;;
 111) m=280; MM=280;;
 112) m=285; MM=285;;
 113) m=284; MM=284;;
 114) m=289; MM=289;;
 115) m=288; MM=288;;
 116) m=291; MM=291;;
 117) m=293; MM=293;;
                         #Estimated
 118) m=294; MM=294;;
 119) m=316; MM=316;;
 120) m=318; MM=318;;
 121) m=322; MM=322;;
 122) m=324; MM=324;;
 123) m=326; MM=326;;
 124) m=330; MM=330;;
 125) m=332; MM=332;;
 126) m=334; MM=334;;
 127) m=338; MM=338;;
 128) m=340; MM=340;;
 129) m=342; MM=342;;
 130) m=346; MM=346;;
 131) m=348; MM=348;;
 132) m=350; MM=350;;
 133) m=354; MM=354;;
 134) m=356; MM=356;;
 135) m=358; MM=358;;
 136) m=362; MM=362;;
 137) m=364; MM=364;;
 138) m=366; MM=366;;
esac
echo "Starting: Z::"${z}, "ZZ::"$ZZ, "mass::"${m}, "Weight::"${MM}
rm -r Z$z
mkdir Z$z
cd Z$z
rnucleus <<EOF
$z
$m
$MM
1
1
1
EOF
cd ..
```

done

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# 2. Generate Expansions

```
The expansions are generated by the script {\tt sh\_files\_c}.
```

```
rcsfgenerate << EOF
1s(2,i)2s(1,i)
2s,2p
1,1
1s(2,i)2p(1,i)
2s,2p
1,3
0
n
EOF
cp rcsf.out DF.c
rcsfgenerate << EOF
1s(2,*)2s(1,*)
5s,5p,5d,5f,5g
1,1
3
n
EOF
cp rcsf.out even.c
rcsfsplit << EOF
even
3
3s,3p,3d
4s, 4p, 4d, 4f
5s,5p,5d,5f,5g
EOF
##################################
rcsfgenerate << EOF
```

1s(2,\*)2p(1,\*)

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```
5s,5p,5d,5f,5g
1,3
3
n
EOF

cp rcsf.out odd.c

rcsfsplit << EOF
odd
3
3s,3p,3d
3
4s,4p,4d,4f
4
5s,5p,5d,5f,5g
5
EOF</pre>
```

# 3. Ground and Excited Reference States

The script  ${\rm sh\_DF}$  performs angular integration, gets initial estimates and performs SCF calculations for the  $1s^22s$ ,  $1s^22p$  reference states. The angular integration is done only once and the mcp.30, mcp.31 ... files are moved between the directories.

```
for z in 6 7 8 9 10 11 12
do
   (if test $z -lt 7
    then
cd Z${z}
cp ../DF.c rcsf.inp
# Get angular data
rangular <<S4
S4
#Get initial estimates of wave functions
rwfnestimate <<S5</pre>
У
2
S5
# Perform self-consistent field calculations
rmcdhf > out_rmcdhf <<S6</pre>
у
1
1
1
5
100
S6
```

rsave DF

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```
cp DF.w even2.w
cp DF.w odd2.w
cd ..
    else
cd Z${z}
cp ../DF.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
mv .../Z${m}/mcp*.
#Get initial estimates of wave functions
rwfnestimate <<S5
2
S5
# Perform self-consistent field calculations
rmcdhf > out_rmcdhf <<S6
у
1
1
1
5
100
S6
rsave DF
cp DF.w even2.w
cp DF.w odd2.w
cd ..
    fi
    echo)
done
```

# 4. Perform Calculations for the Even States

The script sh\_even performs angular integration, gets initial estimates and performs rmcdhf calculations for the even states. The script loops over both the active set and the atomic number *Z*. Angular files are reused and moved between the directories.

```
for n in 3 4 5
do
    (
for z in 6 7 8 9 10 11 12
do
    (if test $z -lt 7
        then
cd Z${z}
```

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```
cp ../even${n}.c rcsf.inp
# Get angular data
rangular <<S4
S4
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5
1
even${k}.w
2
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_${n} <<S6</pre>
У
1
${n}*
100
S6
\texttt{rsave even}\$\{n\}
cd ..
    else
cd Z${z}
cp ../even${n}.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
mv .../Z${m}/mcp* .
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5
1
even${k}.w
2
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_${n} <<S6</pre>
У
${n}*
```

100

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```
s6
rsave even${n}

cd ..
   fi
   echo)

done
)
done
```

## 5. Perform Calculations for the Odd States

The script sh\_odd performs angular integration, gets initial estimates and performs rmcdhf calculations for the odd states. The script loops over both the active set and the atomic number Z. Angular files are reused and moved between the directories.

```
for n in 3 4 5
do
for z in 6 7 8 9 10 11 12
   (if test $z -lt 7
    then
cd Z${z}
cp ../odd${n}.c rcsf.inp
  Get angular data
rangular <<S4
S4
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5
1
odd${k}.w
2
S5
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf_${n} <<S6</pre>
У
1
1
${n}*
100
S6
rsave odd${n}
cd ..
```

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```
else
cd Z${z}
cp ../odd${n}.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
mv .../Z${m}/mcp*.
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5
1
odd{k}.w
2
S5
# Perform self-consistent field calculations
rmcdhf > out_rmcdhf_${n} <<S6</pre>
1
1
${n}*
100
S6
rsave odd${n}
cd ..
    fi
    echo)
done
)
done
```

# 6. Configuration Interaction and Transition Calculations

The script sh\_even\_odd performs configuration interaction and transition calculations for even5 and odd5. Angular files are reused and moved between the directories.

```
for z in 6 7 8 9 10 11 12
do
(cd Z${z}
# RCI calculations for even5
rci > outeven_rci <<S6
y
even5
y
1.d-6</pre>
```

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```
n
n
3
1
S6
# RCI calculations for odd5
rci > outodd_rci <<S6</pre>
odd5
У
у
1.d-6
у
n
n
У
3
1
1
S6
    if test $z -lt 7
    then
# Run rbiotransform and save angular data
rbiotransform <<S4
у
even5
odd5
S4
# Run rtransition save angular data
rtransition <<$4
у
у
even5
odd5
E1
S4
    else
#Move angular files from previous directory
m='expr $z - 1'
mv .../Z${m}/even5.TB .
mv .../Z${m}/odd5.TB.
mv ../Z${m}/even5.odd5.-1T .
# Run rbiotransform using available angular data
rbiotransform <<S4
```

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```
У
У
even5
odd5
S4
   Run rtransition using available angular data
rtransition <<S4
у
even5
odd5
E1
S4
    fi
cd ..
echo)
done
```

# 10.2. Comparison with Experiment

To display the computed energies for Z = 6 we enter the Z6 directory, and we give the command

```
rlevels even5.cm odd5.cm
```

The computer returns the energies together with labels in *LSJ*-coupling for all the states.

```
Energy levels for ...

Rydberg constant is 109737.31569

No - Serial number of the state; Pos - Position of the state within the J/P block; Splitting is the energy difference with the lower neighbor
```

```
No Pos J Parity Energy Total Levels Splitting
(a.u.) (cm^-1) (cm^-1)

1 1 1/2 + -34.7859395
2 1 1/2 - -34.4919396 64525.53 64525.53
3 1 3/2 - -34.4914500 64632.98 107.45
```

These energies should be compared with NIST that give 64484.0 cm<sup>-1</sup>, 64591.7 cm<sup>-1</sup>. Increasing the active set further will improve the agreement with experiment.

The transition parameters are given in even5.odd5.ct. There is a good agreement between length (B) and velocity (C) forms of the parameters. The gf values in the length form are in good agreement with the values 0.1895 and 0.3789 from large-scale MCHF calculations [46]. Again, an increased active set will improve the agreement.

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```
B 2.64473D+08 1.90461D-01 9.71741D-01 f2 1 3/2 - f1 1 1/2 + 64632.98 C 2.69981D+08 3.87564D-01 1.97408D+00 B 2.65907D+08 3.81715D-01 1.94429D+00
```

## 10.3. Scripts for MPI Codes

The scripts above can, with very small modifications, also be used for performing the calculations using the MPI codes. The most important change is that the user needs to prepare the files disks6, disks7, ..., disks12 with paths to the working directory and to the directory containing temporary data. The disks files are copied to the Z6, Z7, ..., Z12 directories by sh\_nuc\_seq. In the different scripts, the calls to the MPI programs amount to changes of the type

```
rangular --> mpirun -np 8 rangular_mpi
rmcdhf --> mpirun -np 8 rmcdhf_mpi
rci --> mpirun -np 8 rci_mpi
```

etc. For the MPI runs, the saved angular files reside in the tmp\_mpi directory, and thus they need not be copied from Z6 to Z7 etc. Scripts for the MPI cases are included under case2\_mpi in the test data set. Consult the README file in the working directory for more details on setting up the file disks.

# 11. Case Study III: Graphical Analysis of the Mg Iso-Electronic Sequence

In this case study, we use script files to perform systematic calculations for states belonging to the  $3s^2$ ,  $3p^2$ ,  $3d^2$ , 3s3d even configurations and to the 3s3p, 3p3d odd configurations in the Mg iso-electronic sequence. Angular data are reused from one ion to another. The script files can be found in grasptest/case3/script.

Calculations are done by parity and valence–valence correlation is accounted for by allowing for SD excitations from the valence orbitals to active sets up to n = 5. Calculations for transition rates are performed for Z = 26, 27, ..., 60.

It is convenient to save the results for the different ions in directories named Z26, Z27, ..., Z60. After all calculations are finished the energies from rlevels, the hyperfine data and the transition data are collected from the different directories and saved in files energy26, energy27, ..., energy60, hfs26, hfs27, ..., hfs60, trans26, trans27, ..., trans60. These files are read by the rsequency and rseqtrans programs to produce GNU Octave/Matlab M-files that plot computed properties as functions of the nuclear charge Z of the ions. The M-files include some fitting capabilities as well.

## 11.1. Iso-Electronic Sequences

Properties of states, as specified by parity, J quantum number and order number within the symmetry (e.g., the second eigenvalue), are smoothly varying functions of the nuclear charge Z along the iso-electronic sequence. Based on hydrogenic approximations, scaling with Z can be derived for different properties (see for example [47], chapter 19). Using spline methods or least-squares fits to scaling expressions, atomic data along a sequence can be reconstructed with high accuracy from a limited set of calculations. When reconstructing data, attention must be paid to label changes. These changes are consequences of the transition from LSJ to jj-coupling, which introduces a label change between the low Z and high Z regions. In the Mg-sequence, a label change occurs for the 3l3l', J=2 even parity states. At low Z the ordering is  $3p^2 \,^1\!D_2$ ,  $3p^2 \,^3\!P_2$ ,  $3s3d \,^3\!P_2$ ,  $3s3d \,^1\!D_2$ . When the spin-orbit coupling becomes dominant, the ordering in jj-coupling is (1/2,1/2), (1/2,3/2), (1/2,5/2), (3/2,3/2). Since in the high-Z limit the  $3s_{1/2}3d_{3/2}$  state is lower than the  $3p_{1/2}^2$  state, there must be a label change for some Z. A label change corresponds to an energy level anti-crossing, where two energy levels with the same *J* and parity will be very close to each other and there will, in the multiconfiguration approximation, be strong interactions between CSFs over a range of Z values. These interactions may result in a decrease or increase of transition probabilities due to negative or positive interference between terms in the expressions for the transition matrix element. For Mg, such interference effects can be seen around Z = 45. In Section 11.2 we will generate atomic data for the 3l3l' states in the Mg iso-electronic sequence. In Section 11.3 we will explore the energy level anti-crossings and the interference effects using the graphical tools.

## 11.2. Running Script Files

The main script sh\_case3 is shown below. This script controls the computational flow and calls several subscripts.

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```
#!/bin/sh
set -x
     Main script for iso-electronic sequence
       Generate directories for the elements and nuclear data
# 1.
        ./sh_nuc_seq
       Generate lists of CSFs in main directory
# 2.
        ./sh_files_c
# 3.
       Perform MCDHF calculations for the even reference states
       ./sh_DF_even
# 4.
       Perform MCDHF calculations for the odd reference states
       ./sh_DF_odd
# 5.
       Perform MCDHF calculations for even states
       ./sh_even
# 6.
       Perform MCDHF calculations for odd states
       ./sh_odd
# 7.
       Perform RCI, transition calculations
       ./sh_even_odd
# 8.
       Transformation to LSJ, run rlevels and pipe to energyZ
       ./sh_rlevels
# 9.
       Collect all data files and copy to the main directory
       ./sh_collect
1. Generate Directories and Define Nuclear Data
The script sh_nuc_seq produces nuclear data for Z=26,27,\ldots,60 in the directories Z26, Z27,..., Z60. By modifying the
script, we can produce nuclear data for any sequence of charges.
#!/bin/sh
set -x
# Full loop over all Z
    for z in 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 \
#
             26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 \
```

48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 \
70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 \
92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 \

#

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```
110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 \
             126 127 128 129 130 131 132 133 134 135 136 137 138
# We select Z from 26 to 60
   for z in \phantom{0} 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 \backslash
             49 50 51 52 53 54 55 56 57 58 59 60
# Data from Jefferson Lab (http://education.jlab.org/itselemental)
case $z in
   1) m=0; MM=1.0794;;
                           # Need to use point nucleus
   2) m=4; MM=4.002602;;
   3) m=7; MM=6.941;;
   4) m=9; MM=9.012182;;
   5) m=11; MM=10.811;;
   6) m=12; MM=12.0107;;
   7) m=14; MM=14.0067;;
   8) m=16; MM=15.9994;;
  9) m=19; MM=18.9984032;;
  10) m=20; MM=20.1797;;
  11) m=23; MM=22.98976928;;
  12) m=24; MM=24.3050;;
  13) m=27; MM=26.9815386;;
  14) m=28; MM=29.0855;;
  15) m=31; MM=30.973762;;
  16) m=32; MM=32.065;;
  17) m=35; MM=35.453;;
  18) m=40; MM=39.948;;
  19) m=39; MM=39.0938;;
  20) m=40; MM=40.078;;
  21) m=45; MM=44.955912;;
  22) m=48; MM=47.867;;
  23) m=51; MM=50.9415;;
  24) m=52; MM=51.9961;;
  25) m=55; MM=54.938045;;
  26) m=56; MM=55.845;;
  27) m=59; MM=58.933195;;
  28) m=59; MM=58.6934;;
  29) m=64; MM=63.546;;
  30) m=65; MM=65.409;;
  31) m=70; MM=69.723;;
  32) m=73; MM=72.64;;
  33) m=75; MM=74.92160;;
  34) m=79; MM=78.96;;
  35) m=80; MM=79.904;;
  36) m=84; MM=83.798;;
  37) m=85; MM=85.4678;;
  38) m=88; MM=87.62;;
  39) m=89; MM=88.90585;;
  40) m=91; MM=91.224;;
  41) m=93; MM=92.90638;;
  42) m=96; MM=95.94;;
  43) m=98; MM=98;;
  44) m=101; MM=10.07;;
```

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```
45) m=103; MM=102.90550;;
46) m=106; MM=106.42;;
47) m=108; MM=107.8682;;
48) m=112; MM=112.411;;
49) m=115; MM=114.818;;
50) m=119; MM=118.710;;
51) m=122; MM=121.760;;
52) m=128; MM=127.60;;
53) m=127; MM=126.90447;;
54) m=131; MM=131.293;;
55) m=133; MM=132.9054519;;
56) m=137; MM=137.327;;
57) m=139; MM=138.90547;;
58) m=140; MM=140.116;;
59) m=141; MM=140.90765;;
60) m=144; MM=144.242;;
61) m=145; MM=145;;
62) m=150; MM=150.36;;
63) m=152; MM=151.964;;
64) m=157; MM=157.25;;
65) m=159; MM=158.92535;;
66) m=163; MM=162.5;;
67) m=165; MM=164.93032;;
68) m=167; MM=167.259;;
69) m=169; MM=168.93421;;
70) m=173; MM=173.04;;
71) m=175; MM=174.967;;
72) m=178; MM=178.49;;
73) m=181; MM=180.94788;;
74) m=184; MM=183.84;;
75) m=186; MM=186.207;;
76) m=190; MM=190.23;;
77) m=192; MM=192.217;;
78) m=195; MM=195.084;;
79) m=197; MM=196.966569;;
80) m=201; MM=200.59;;
81) m=204; MM=204.3833;;
82) m=207; MM=207.2;;
83) m=209; MM=208.9804;;
84) m=209; MM=209;;
85) m=210; MM=210;;
86) m=222; MM=222;;
87) m=223; MM=223;;
88) m=226; MM=226;;
89) m=227; MM=227;;
90) m=232; MM=232.03806;;
91) m=231; MM=231.03588;;
92) m=238; MM=238.02891;;
93) m=237; MM=237;;
94) m=244; MM=244;;
95) m=243; MM=243;;
96) m=247; MM=247;;
97) m=247; MM=247;;
98) m=251; MM=251;;
```

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```
99) m=252; MM=252;;
 100) m=257; MM=257;;
 101) m=258; MM=258;;
 102) m=259; MM=259;;
 103) m=262; MM=262;;
 104) m=267; MM=267;;
 105) m=268; MM=268;;
 106) m=271; MM=271;;
 107) m=272; MM=272;;
 108) m=277; MM=277;;
 109) m=276; MM=276;;
 110) m=281; MM=281;;
 111) m=280; MM=280;;
 112) m=285; MM=285;;
 113) m=284; MM=284;;
 114) m=289; MM=289;;
 115) m=288; MM=288;;
 116) m=291; MM=291;;
 117) m=293; MM=293;;
                         #Estimated
 118) m=294; MM=294;;
 119) m=316; MM=316;;
 120) m=318; MM=318;;
 121) m=322; MM=322;;
 122) m=324; MM=324;;
 123) m=326; MM=326;;
 124) m=330; MM=330;;
 125) m=332; MM=332;;
 126) m=334; MM=334;;
 127) m=338; MM=338;;
 128) m=340; MM=340;;
 129) m=342; MM=342;;
 130) m=346; MM=346;;
 131) m=348; MM=348;;
 132) m=350; MM=350;;
 133) m=354; MM=354;;
 134) m=356; MM=356;;
 135) m=358; MM=358;;
 136) m=362; MM=362;;
 137) m=364; MM=364;;
 138) m=366; MM=366;;
esac
echo "Starting: Z::"${z}, "ZZ::"$ZZ, "mass::"${m}, "Weight::"${MM}
rm -r Z$z
mkdir Z$z
cd Z$z
rnucleus <<EOF
$z
$m
n
$MM
1
```

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```
1
1
EOF
cd ..
done
2. Generate Expansions
The expansions are generated by the script {\tt sh\_files\_c}.
rcsfgenerate << EOF
2
3s(2,i)
3p(2,i)
3d(2,i)
3s(1,i)3d(1,i)
3s,3p,3d
0,8
0
n
EOF
cp rcsf.out DFeven.c
rcsfgenerate << EOF
3s(1,i)3p(1,i)
3p(1,i)3d(1,i)
3s,3p,3d
0,8
0
n
EOF
cp rcsf.out DFodd.c
rcsfgenerate << EOF
2
3s(2,*)
5s, 5p, 5d, 5f, 5g
0,8
2
n
```

EOF

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```
cp rcsf.out even.c
rcsfsplit << EOF
even
4s,4p,4d,4f
5s,5p,5d,5f,5g
EOF
###################################
rcsfgenerate << EOF
3s(1,*)3p(1,*)
5s,5p,5d,5f,5g
0,8
2
n
EOF
cp rcsf.out odd.c
rcsfsplit << EOF
odd
2
4s, 4p, 4d, 4f
5s,5p,5d,5f,5g
EOF
```

## 3. Even Parity Reference States

The script sh\_DF\_even performs angular integration, gets initial estimates and performs rmcdhf calculations for the even reference states. The angular integration is done only once and the mcp.30, mcp.31 ... files are moved between the directories.

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```
2
*
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf <<S6</pre>
1-5
1-3
1-7
1-2
1-2
100
S6
rsave DFeven
cp DFeven.w even3.w
cd ..
    else
cd Z${z}
cp ../DFeven.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
mv .../Z${m}/mcp* .
#Get initial estimates of wave functions
rwfnestimate <<S5
2
*
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf <<S6</pre>
у
1-5
1-3
1-7
1-2
1-2
5
100
S6
rsave DFeven
cp DFeven.w even3.w
```

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```
fi echo)
```

#### 4. Odd Parity Reference States

The script sh\_DF\_odd performs angular integration, gets initial estimates and performs rmcdhf calculations for the odd reference states. The angular integration is done only once and the mcp.30, mcp.31 ... files are moved between the directories.

```
for z in 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 \
         49 50 51 52 53 54 55 56 57 58 59 60
do
   (if test $z -lt 27
    then
cd Z${z}
cp ../DFodd.c rcsf.inp
# Get angular data
rangular <<S4
S4
#Get initial estimates of wave functions
rwfnestimate <<S5
у
2
*
S5
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf <<S6</pre>
1-2
1-5
1 - 5
1-3
1
5
*
100
S6
rsave DFodd
cp DFodd.w odd3.w
cd ..
    else
cd Z${z}
cp ../DFodd.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
```

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```
mv .../Z${m}/mcp* .
#Get initial estimates of wave functions
rwfnestimate <<S5
2
S5
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf <<S6</pre>
У
1-2
1-5
1-5
1-3
1
5
100
S6
rsave DFodd
cp DFodd.w odd3.w
cd ..
    echo)
done
```

# 5. Perform Calculations for Even States

The script sh\_even performs angular integration, gets initial estimates and performs rmcdhf calculations for the odd states. The script loops over both the active set and the atomic number *Z*. Angular files are reused and moved between the directories.

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```
rwfnestimate <<S5</pre>
у
1
even${k}.w
2
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_${n} <<S6</pre>
1-5
1-3
1-7
1-2
1-2
${n}*
100
S6
rsave even${n}
cd ..
    else
cd Z${z}
cp ../even${n}.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
mv .../Z${m}/mcp* .
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5
1
even${k}.w
2
S5
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_${n} <<S6</pre>
У
1-5
1-3
1-7
1-2
1-2
5
${n}*
```

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```
100
S6
rsave even${n}
cd ..
   fi
   echo)
done
)
done
```

## 6. Perform Calculations for Odd States

The script  $sh\_odd$  performs angular integration, gets initial estimates and performs rmcdhf calculations for the odd states. The script loops over both the active set and the atomic number Z. Angular files are reused and moved between the directories.

```
for n in 4 5
do
for z in 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 \
         49 50 51 52 53 54 55 56 57 58 59 60
do
   (if test $z -lt 27
    then
cd Z${z}
cp ../odd${n}.c rcsf.inp
# Get angular data
rangular <<S4
S4
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5</pre>
У
1
odd${k}.w
2
*
S5
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf_${n} <<S6</pre>
1-2
1-5
1-5
1-3
1
5
${n}*
```

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```
100
S6
rsave odd${n}
cd ..
    else
cd Z${z}
cp ../odd${n}.c rcsf.inp
#Move mcp files from previous directory
m='expr $z - 1'
mv .../Z${m}/mcp* .
k='expr $n - 1'
#Get initial estimates of wave functions
rwfnestimate <<S5
У
1
odd${k}.w
2
*
S5
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf_${n} <<S6</pre>
1-2
1-5
1-5
1-3
1
${n}*
100
S6
rsave odd${n}
cd ..
    fi
    echo)
done
)
done
```

## 7. Perform rci and Transition Calculations

The script sh\_even\_odd performs configuration interaction and transition calculations for even5 and odd5. Angular files are reused and moved between the directories.

```
for z in 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 \setminus 49 50 51 52 53 54 55 56 57 58 59 60
```

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```
do
(cd Z${z}
# RCI calculations for even5
rci > outeven_rci <<S6
even5
у
у
1.d-6
У
n
n
У
3
1-5
1-3
1-7
1-2
1-2
S6
\# RCI calculations for odd5
rci > outodd_rci <<S6</pre>
odd5
У
У
1.d-6
у
n
n
у
3
1-2
1-5
1-5
1-3
1
S6
    if test $z -lt 27
    then
# Run rbiotransform and save angular data
rbiotransform <<S4
у
у
even5
odd5
У
S4
# Run rtransition save angular data
```

rtransition <<\$4

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```
у
у
even5
odd5
E1
S4
    else
#Move angular files from previous directory
m='expr $z - 1'
mv .../Z${m}/even5.TB.
mv .../Z${m}/odd5.TB.
mv .../Z{m}/even5.odd5.-1T .
   Run rbiotransform using available angular data
rbiotransform <<S4
У
у
even5
odd5
у
S4
   Run rtransition using available angular data
rtransition <<$4
у
even5
odd5
E1
S4
    fi
cd ..
echo)
done
Transformation to LSJ, Run rlevels and Pipe to energyZ
This script runs jj21sj to transform to LSJ-coupling. The energy files energyZ are created by redirecting the output from
rlevels.
for z in 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 \
         49 50 51 52 53 54 55 56 57 58 59 60
do
(cd Z${z}
jj2lsj <<S1
even5
У
у
у
S1
```

jj21sj <<S2

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```
odd5
y
y
y
S2
rlevels even5.cm odd5.cm > energy${z}
cd ..
echo)
done
```

Collect Data to Prepare for the Runs of the ISO-Electronic Plotting Tools

This script collects, in one directory, all the energy, hfs and transition files that are needed to run the tools that create the iso-electronic plots.

# 11.3. Generating Plots of Properties along the Sequence

After the script sh\_case3 has been executed the energy files energy26, energy27, ..., energy60, as obtained from rlevels, the hyperfine structure files hfs26, hfs27,..., hfs60 and the transition files trans26, trans27,..., trans60 all reside in one directory. The energy file energy26 is shown below

```
      nblock =
      5 ncftot =
      327 nw =
      25 nelec =
      12

      nblock =
      5 ncftot =
      320 nw =
      25 nelec =
      12
```

Energy levels for ...

Rydberg constant is 109737.31569

Splitting is the energy difference with the lower neighbor

\_\_\_\_\_\_ No Pos J Parity Energy Total Levels Splitting Configuration (a.u.)  $(cm^-1)$  $(cm^-1)$ 0 + -1182.3727764  $0.00 3s(2)_1S0$ 1 1 0.00 2 0 -1181.3113834 232948.84 232948.84 3s.3p\_3P 238799.35 3 1 1 -1181.2847265 5850.52 3s.3p\_3P 4 1 2 --1181.2205551 252883.36 14084.01 3s.3p\_3P 5 2 -1180.7554700 354957.74 102074.38 3s.3p\_1P 1 -6 2 0 -1179.8373301 201508.42 3p(2)\_3P2 556466.16 7 2 + 1 -1179.8217549 559884.50 3418.34 3p(2)\_1D2 8 1 1 + -1179.7920926 566394.64 6510.13 3p(2)\_3P2 9 2 2 + -1179.7154124 583224.00 16829.37  $3p(2)_3P2$ 10 3 0 + -1179.3515752 663077.02 79853.02  $3p(2)_1S0$ 2 1 + -1179.2726721 680394.26 17317.24 3s.3d\_3D 11 12 3 -1179.2681277 681391.63 997.37 3s.3d\_3D

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```
13
   1
        3
           +
               -1179.2610200
                                682951.61
                                              1559.97
                                                       3s.3d_3D
        2
14
               -1178.8798082
                               766617.92
                                             83666.31
                                                       3s.3d_1D
15
    2
        2
               -1178.1400307
                               928980.31
                                            162362.40
                                                       3p.3d_3F
16
    1
        3
               -1178.0956514
                               938720.44
                                              9740.12
                                                       3p.3d_3F
17
    3
        2
               -1178.0442947
                               949991.94
                                             11271.50
                                                       3p.3d_1D
18
    1
        4
               -1178.0436090
                               950142.42
                                               150.48
                                                       3p.3d_3F
19
    3
        1
               -1177.8805639
                               985926.68
                                             35784.26
                                                       3p.3d_3D
        2
20
    4
               -1177.8791774
                               986230.98
                                               304.30
                                                       3p.3d_3P
21
    2
        3
               -1177.8255556
                               997999.61
                                             11768.62
                                                       3p.3d_3D
    2
        0
22
                                               383.51
                                                       3p.3d_3P
               -1177.8238082
                               998383.12
23
        1
               -1177.8213613
                               998920.17
                                               537.05
                                                       3p.3d_3P
    5
        2
                               999474.72
24
               -1177.8188346
                                                       3p.3d_3D
                                               554.55
    3
        3
25
               -1177.5116442
                              1066895.20
                                             67420.48
                                                       3p.3d_1F
26
    5
        1
               -1177.4560254
                              1079102.13
                                             12206.93
                                                       3p.3d_1P
27
    5
        2
               -1176.1165400
                                            293983.04
                                                       3d(2)_3F2
                              1373085.18
    2
        3
                                                       3d(2)_3F2
28
               -1176.1091750
                              1374701.61
                                              1616.44
29
    1
        4
               -1176.1001735
                              1376677.21
                                              1975.59
                                                       3d(2)_3F2
30
    6
        2
               -1175.9683503
                              1405609.08
                                             28931.87
                                                       3d(2)_1D2
31
   4
        0
               -1175.9539314 1408773.66
                                              3164.58 3d(2)_3P2
32
   3
        1 +
               -1175.9511232 1409389.97
                                               616.31
                                                       3d(2)_3P2
33
    2
        4
               -1175.9496890
                              1409704.75
                                               314.78
                                                       3d(2)_1G2
34
   7
        2
               -1175.9446852 1410802.96
                                              1098.21
                                                       3d(2)_3P2
35
        0
               -1175.5824952 1490294.48
                                             79491.51 3d(2)_1S0
```

-----

Each state is specified by the position within the symmetry, the J quantum number and the parity. For example, the four states  $3p^2$   $^1D_2$ ,  $3p^2$   $^3P_2$ , 3s3d  $^3P_2$ , 3s3d  $^1D_2$  with even parity and J=2 are specified as 1 2 +, 2 2 +, 3 2 +, 4 2 +. These specifications remain valid over the iso-electronic sequence, although the LSJ designation may change. Thus, to follow states along the iso-electronic sequence, the above specifications should be used. To generate a GNU Octave/Matlab M-file that plots the  $3p^2$   $^1D_2$ ,  $3p^2$   $^3P_2$ , 3s3d  $^3P_2$ , 3s3d  $^1D_2$  states along the sequence, the rseqenergy program should be run. The program looks for all energy files in a given range of Z. Then, after having specified the states to be plotted, there is an option to perform least squares fits to obtain analytical expressions of the trends. If no fits are done, the data are instead interpolated using cubic splines. The rseqenergy program outputs an M-file with name seqenergyplot.m. The M-file contains all data needed for the plot, and the file can also very easily be edited to comply with the desires of the user. The input session for rseqenergy is shown below. Please note that you should input 2J and not J and the sequence 1 2 +, 2 2 +, 3 2 +, 4 2 + above should thus be inserted in the program as 1 4 +, 2 4 +, 3 4 +, 4 4 +.

## >>rseqenergy

RSEQENERGY

```
This program reads output from rlevels for several ions and produces a Matlab/Octave file that plots energy as a function of Z
Input files: energyZ1, energyZ2, .., energyZn
Output file: seqenergyplot.m

Give the first Z and last Z of the sequence
>>26,60
How many states do you want to plot?
>>4
Give number within symmetry,2*J and parity (+/-)
>>1,4,+
Give number within symmetry,2*J and parity (+/-)
>>2 4 +
```

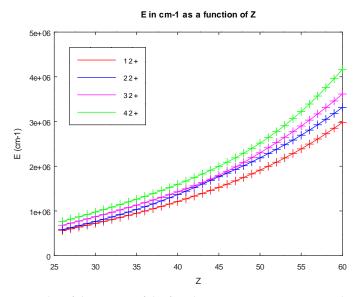
Give number within symmetry, 2\*J and parity (+/-)

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```
>>3,4,+
Give number within symmetry,2*J and parity (+/-)
>>4,4,+
Least-squares fit (y/n) ?
>>n
```

rsequence produces the file sequence results. To run this file, open GNU Octave (or Matlab) and issue the command octave: 1>sequence results.

On the GNU Octave command line, and the plot shown in Figure 7 will appear. There is an energy level anti-crossing around Z = 44.



**Figure 7.** Plot of the energy of the four lowest even parity states with J = 2 as function of the nuclear charge Z. There is an energy level anti-crossing around Z = 44. The M-file was edited, and we added the legend, see Section 11.5.

We now turn to the hyperfine structure. The hyperfine structure file hfs26 is shown below

#### Interaction constants:

Level1	J Parity	A (MHz)	B (MHz)	$g_{-}J$
1	1 +	-3.4234867098D+02	4.4613609212D+03	1.5001883281D+00
2	1 +	-1.9244748754D+04	6.2528206291D+02	4.9806452460D-01
3	1 +	-1.1351236243D+01	6.2649343224D+02	1.5002882611D+00
1	2 +	8.7390089731D+03	1.1076017221D+04	1.0772838047D+00
2	2 +	4.4795474977D+03	-5.4746961897D+03	1.4215362332D+00
3	2 +	8.8230369547D+03	8.9310422936D+02	1.1660563087D+00
4	2 +	2.4077816233D+03	5.0250957600D+03	9.9939681308D-01
5	2 +	1.8500821856D+03	5.8565536591D+02	6.6595057454D-01
6	2 +	1.2714709459D+03	-7.1108884710D+02	1.0473515636D+00
7	2 +	7.3078965369D+02	-1.2063797242D+03	1.4511063589D+00
1	3 +	1.5225771095D+04	1.7639256187D+03	1.3331477513D+00
2	3 +	1.1924106175D+03	6.4316600861D+02	1.0826447519D+00
1	4 +	9.0356371406D+02	8.6788620599D+02	1.2493914010D+00

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```
2 4 + 1.2426420132D+03 3.5362685668D+03 9.9942996698D-01
```

States are specified in the same way as in the energy file by giving position (level1) within the symmetry, the J quantum number and the parity. To plot the hyperfine interaction constants or the Landé  $g_J$  factor as a function of the nuclear charge, we use the program rseqhfs. The input session for plotting the magnetic dipole interaction constant for the states 2 2 + and 3 2 + is shown below (again please note that you should input 2J and not J)

```
>>rseqhfs
```

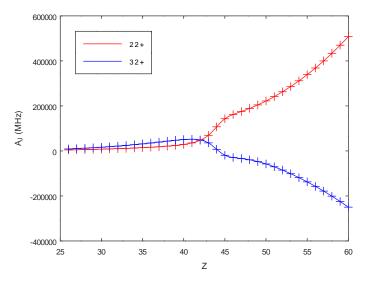
```
RSEQHFS
This program reads output from rhfs for several ions and produces a Matlab/Octave file that plots hfs parameters as functions of Z
Input files: hfsZ1, hfsZ2, .., hfsZn or Output file: seqhfsplot.m

Give the first Z and last Z of the sequence
>>26,60
How many states do you want to plot?
>>2
Give number within symmetry,2*J and parity (+/-)
>>2,4,+
Give number within symmetry,2*J and parity (+/-)
>>3,4,+
Plot A (1), B (2) or gJ (3) ?
>>1
Least-squares fit (y/n) ?
>>n
```

rseqhfs produces the file seqhfsplot.m. To run this file open GNU Octave (or Matlab) and issue the command

```
octave:1>seqhfsplot
```

at the GNU Octave command line and the plot in Figure 8 will now be displayed. The strong mixing of the CSFs around the level anti-crossing at Z=44 causes interference effects that have large influence on the hyperfine structure constants of the two states.



**Figure 8.** Plot of the hyperfine interaction constants  $A_J$  for the two interfering even parity states with J = 2 as function of the nuclear charge Z. The M-file was edited, and we added the legend, see Section 11.5.

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The transition file trans26 is shown below. A transition is specified by giving the multipolarity along with the position (Lev) within the symmetry, the *J* quantum number and the parity for the upper and lower states.

```
Transition between files:
f1 = even5
f2 = odd5
```

# Electric 2\*\*( 1)-pole transitions

```
Upper
           Lower
f2 1
        1 - f1 1
                              238799.35 C 4.30268D+07 3.39353D-03 4.67836D-03
                                        B 4.08059D+07 3.21837D-03 4.43688D-03
f2 2
        1 - f1 1
                              354957.74 C 2.33297D+10 8.32789D-01 7.72385D-01
                                        B 2.28391D+10 8.15275D-01 7.56142D-01
f2
  3
        1 - f1 1
                              985926.68 C 2.38998D+05 1.10582D-06 3.69245D-07
                                        B 9.80884D+04 4.53845D-07 1.51544D-07
f2 4
        1 - f1 1
                              998920.17 C 6.50658D+04 2.93272D-07 9.66530D-08
                                        B 4.71469D+04 2.12506D-07 7.00351D-08
                             1079102.13 C 3.67086D+08 1.41782D-03 4.32549D-04
f2 5
        1 - f1 1
                                        B 3.05391D+08 1.17953D-03 3.59851D-04
f1 2
        0 + f2 1
                      1 -
                              317666.81 C 1.88797D+10 2.80485D-01 2.90679D-01
                                        B 1.82388D+10 2.70963D-01 2.80811D-01
                     . . . . . . . . . . . . . . . .
f1 2
                              342809.55 C 2.26092D+10 2.59586D+00
        4 + f2 3
                      3 -
                                                                   2.49289D+00
                                        B 2.24521D+10 2.57782D+00
                                                                   2.47557D+00
        4 + f2 1
                              426534.79 C 1.66046D+10 1.23146D+00
                                                                   9.50477D-01
                                        B 1.52594D+10 1.13169D+00 8.73472D-01
   2
                              459562.33 C 2.30387D+07 1.47187D-03 1.05439D-03
                                        B 2.11352D+07 1.35026D-03 9.67271D-04
```

To plot A, gf or S as a function of the nuclear charge, we use the program rseqtrans. The input session for plotting the transition rate A from the states 2 2 + and 3 2 + to 1 1 - is shown below. Please observe that we should input 2J.

#### >>rseqtrans

```
RSEQTRANS
 This program reads output from rtransition for several
 ions and produces a Matlab/Octave file that plots
 A, gf, or S as a function of Z
 Input files: transZ1, transZ2, .., transZn
 Output file: seqtransplot.m
 Give the first Z and last Z of the sequence
>>26,60
 Give multipolarity of transition: E1, M1, E2, M2
>>E1
 How many transitions do you want to plot?
>>2
 Give number within symmetry, 2*J and parity (+/-)
 for upper and lower state
>>2,4,+,1,2,-
 Give number within symmetry, 2*J and parity (+/-)
```

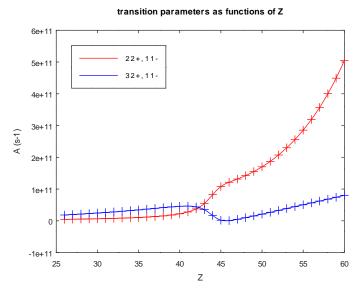
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```
for upper and lower state
>>3,4,+,1,2,-
Plot A (1), gf (2) or S (3) ?
>>1
Least-squares fit (y/n) ?
>>n
```

The rseqtrans program produces the file seqtransplot.m. To run this file, open GNU Octave or Matlab and issue the command

```
octave:1>seqtransplot
```

and the plot in Figure 9 will now be shown. The strong mixing of the CSFs around the level anti-crossing at Z=44 causes interference effects that influence the rates.



**Figure 9.** Plot of transition rates involving two interfering states. The M-file was edited, and we added the legend, see Section 11.5.

## 11.4. Least-Squares Fits to Data

If deemed important, least-squares fits can be done for atomic data that are not affected by interference effects from level anti-crossings. Below we fit a polynomial to the energies for the 1 0 -, 1 1 -, 2 1 -, and 1 2 - states.

# >>rseqenergy

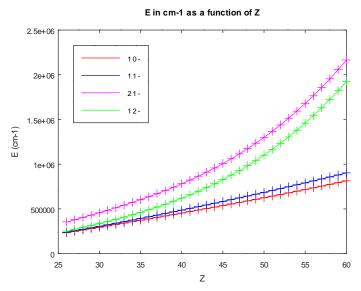
```
RSEQENERGY
This program reads output from rlevels for several ions and produces a Matlab/Octave file that plots energy as a function of Z
Input files: energyZ1, energyZ2, .., energyZn
Output file: seqenergyplot.m

Give the first Z and last Z of the sequence
>>26,60
How many states do you want to plot?
>>4
Give number within symmetry,2*J and parity (+/-)
>>1,0,-
Give number within symmetry,2*J and parity (+/-)
>>1,2,-
Give number within symmetry,2*J and parity (+/-)
```

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```
>>2,2,-
 Give number within symmetry, 2*J and parity (+/-)
>>1,4,-
 Least-squares fit (y/n) ?
>>y
 Type of fitting: a1 Z^-2 + a2 Z^-1 + ... + a6 Z^3 (1)
                   a1 + a2 Z + a3 Z^2 + a4 Z^3
                                                      (2)
>>2
Starting GNU Octave (or Matlab) and giving the command
octave:1>seqenergyplot
at the GNU Octave command line gives the fitting coefficients for the four states
  -3.8990e+00
   9.3243e-02
  -3.2068e-04
   5.5596e-06
a =
  -3.3884e+00
   5.7768e-02
   4.6734e-04
  -7.9666e-08
a =
  -3.4271e+00
   1.4637e-01
  -3.6443e-03
   4.5791e-05
  -3.1674e+00
   1.3634e-01
  -3.6161e-03
   4.7051e-05
    along with the plot in Figure 10.
```

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**Figure 10.** Polynomial fitted to the energies of the 10-, 11-, 21-, and 12- states. The m-file was edited, and we added the legend, see Section 11.5.

We can do fits to transition data as well. Below we fit a Laurent series to the line strength S for the transition from 1 1 -, 2 1 - down to 1 0 +.

```
>>rseqtrans
```

```
RSEQTRANS
 This program reads output from rtransition for several
 ions and produces a Matlab/Octave file that plots
 A, gf, or S as a function of \boldsymbol{Z}
 Input files: transZ1, transZ2, .., transZn
 Output file: seqtransplot.m
 Give the first Z and last Z of the sequence
>>26,60
 Give multipolarity of transition: E1, M1, E2, M2
>>E1
 How many transitions do you want to plot?
>>2
 Give number within symmetry, 2*J and parity (+/-)
 for upper and lower state
>>1,2,-,1,0,+
 Give number within symmetry, 2*J and parity (+/-)
 for upper and lower state
>>2,2,-,1,0,+
 Plot A (1), gf (2) or S (3) ?
>>3
 Least-squares fit (y/n) ?
>>y
 Type of fitting: a1 Z^-2 + a2 Z^-1 + ... + a6 Z^3
                                                     (1)
                   a1 + a2 Z + a3 Z^2 + a4 Z^3
                                                     (2)
>>1
```

Starting GNUOctave (or Matlab) and giving the command

octave:1>seqtransplot

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at the GNU Octave command line gives the fitting coefficients

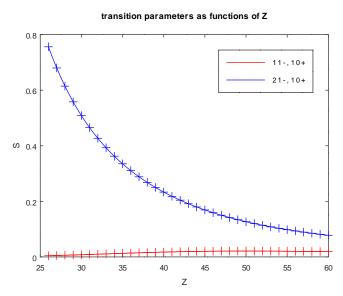
a =

- -2.9422e+04
- 4.7527e+03
- -2.9700e+02
- 8.5307e+00
- -1.1226e-01
- 5.4992e-04

a =

- 1.4276e+04
- -1.1873e+03
- 4.8383e+01
- -1.0910e+00
- 1.2034e-02
- -5.2794e-05

The produced plot is displayed in Figure 11. The fitted function describes the data very well.



**Figure 11.** Fitted function to the line strength S for the transitions of 1 1 -, 2 1 - down to the 1 0 + groundstate. The m-file was edited, and we added the legend, see Section 11.5.

#### 11.5. Modifying the GNU Octave/MATLAB M-Files

The M-files produced by rseqenergy, rseqhfs and rseqtrans are very easy to modify to include legends, change captions etc. Additionally, other types of modifications should be considered. If, for example, calculations are done for even Z in an iso-electronic sequence then the user can easily modify seqenergyplot.m to output interpolated values of the energies for odd Z. Away from level anti-crossings the accuracy of the interpolated values should be quite high. In many cases data for a full iso-electronic sequence can be interpolated from a comparatively small number of ions. The M-files can be concatenated (some minor editing is needed) and it is then possible to overlay several plots.

The seqtransplot.m file from the last run is shown below. The data are organized in a matrix A where the first column contains the nuclear charge Z. The atomic data are stored in columns 2 and 3. Standard commands are used for plotting and least-squares fits.

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```
A = \Gamma
         26
              4.436879999999998E-003 0.7561419999999998
         27
              5.194630000000004E-003 0.6800840000000002
         28
              29
              6.895980000000003E-003
                                      0.55813599999999997
         30
              7.826949999999992E-003 0.5087380000000002
         31
              8.800749999999995E-003 0.4653439999999999
         32
              9.805019999999994E-003
                                      0.42698700000000001
              1.082830000000001E-002
                                      0.39289900000000000
         34
              1.185750000000000E-002 0.36245699999999997
         35
              1.287870000000000E-002 0.3351529999999999
         36
              1.387840000000001E-002 0.3105669999999998
              1.48433000000000E-002
         37
                                      0.28835200000000000
         38
              1.576110000000000E-002 0.2682209999999999
         39
              1.662110000000000E-002 0.2499290000000001
         40
              1.74140999999998E-002 0.2332690000000000
         41
              1.813300000000000E-002 0.2180620000000001
         42
              1.877270000000000E-002 0.2041590000000001
         43
              1.933000000000000E-002 0.1914260000000001
         44
              1.980370000000000E-002 0.1797459999999999
         45
              2.019419999999999E-002
                                      0.16901600000000000
         46
              2.050360000000000E-002
                                      0.15914600000000001
         47
              2.073490000000001E-002 0.1500559999999999
         48
              2.089229999999999E-002 0.1416719999999999
         49
              2.098059999999999E-002 0.1339299999999999
         50
              2.100480000000001E-002 0.1267709999999999
         51
              2.097069999999998E-002 0.1201430000000000
         52
              2.088380000000001E-002 0.1139980000000000
         53
              2.07496999999999E-002 0.1082940000000000
                                      0.10299200000000000
         54
              2.057370000000000E-002
         55
              2.036079999999998E-002 9.805650000000005E-002
         56
              2.011610000000001E-002 9.345680000000007E-002
         57
              1.98441000000000E-002 8.916380000000001E-002
         58
              1.954880000000002E-002 8.515169999999997E-002
              1.923410000000001E-002 8.139720000000003E-002
         59
              1.890330000000001E-002 7.787910000000007E-002
 ];
 clf, hold on
zip = linspace(26, 60);
title('transition parameters as functions of Z')
xlabel('Z')
ylabel('S')
plot(A(:,1),A(:, 2),'+')
z = A(:,1);
AD = [z.^{(-2)} z.^{(-1)} z.^{0} z.^{1} z.^{2} z.^{3}];
y = A(:, 2);
m = mean(y); s = std(y);
a = AD \setminus (y-m)/s
aiplsq = a(1)./zip.^2 + a(2)./zip + a(3) + a(4)*zip + a(5)*zip.^2 + a(6)*zip.^3;
aiplsq = s*aiplsq + m;
plot(zip,aiplsq,'r')
```

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```
plot(A(:,1),A(:, 3),'+')

z = A(:,1);
AD = [z.^(-2) z.^(-1) z.^0 z.^1 z.^2 z.^3];
y = A(:, 3);
m = mean(y); s = std(y);
a = AD\(y-m)/s
aiplsq = a(1)./zip.^2 + a(2)./zip + a(3) + a(4)*zip + a(5)*zip.^2 + a(6)*zip.^3;
aiplsq = s*aiplsq + m;
plot(zip,aiplsq,'r')
```

## 12. Case Study IV: Isotope Shift in Li-like Nd and the Effect of Nuclear Deformation Using fical

In this case study, we use script files to generate wave functions for the  $1s^22s$   $^2S_{1/2}$  and  $1s^22p$   $^2P^o_{1/2,3/2}$  states in Li-like Nd from which the isotope shift parameters are computed. The isotope shift parameters are used as input for the fical program that computes the frequency isotope shift based on that 1) the  $^{150}$ Nd nucleus is assumed to be spherical and 2) the  $^{150}$ Nd nucleus is assumed to be deformed with a deformation parameter  $\beta_{20}=0.28$  [48]. The  $^{142}$ Nd nucleus is assumed to be spherical.

We start with a single calculation for the  $1s^22s$   $^2S_{1/2}$  and  $1s^22p$   $^2P_{1/2,3/2}^o$  states. After that, separate calculations are done for the even and odd parities. Correlation is included by allowing single, double, and triple (SDT) excitations from the reference to active sets up to n=4 (complete active space calculations). The Breit interactions and QED effects are included in the rci calculations. At the end, the ris4 program is run to produce isotope data for the even and odd states. The script files can be found in grasptest/case2/script.

## 12.1. Running Script Files

The main script sh\_case4 is shown below. This script controls the computational flow and calls several subscripts.

#!/bin/sh

```
set -x
     Main script for 1s(2)2s and 1s(2)2p
# 1.
      Generate the expansions
        ./sh_files_c
# 2.
      Get the nuclear data
        ./sh_nuc
# 3.
       Get screened hydrogenic orbitals as initial estimates
        ./sh_initial
# 4.
      Perform scf calculations and a final rci calculation that
       includes the Breit correction and QED. Perform ris4
       calculations for the rci wave functions
        ./sh_scf
```

Each of the subscripts is given below together with some comments.

If all script files are available with execute permission (use the command chmod +x) we start the computation by typing the name of the main script

```
./sh_case4
```

#### 1. Generate Expansions

The expansions are generated by the script sh\_files\_c. The script is simplified by generating lists for large active sets and then using rcsfsplit, see Section 7.1.

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```
#!/bin/sh
set -x
# 1. Generate CSF expansions
       1.1 DF for 1s(2)2s and 1s(2)2p
rcsfgenerate <<EOF1
0
1s(2,i)2s(1,i)
2s
1,1
0
1s(2,i)2p(1,i)
1s,2p
1,3
n
EOF1
cp rcsf.out DF.c
        1.2 SDT even for n=4
rcsfgenerate <<EOF3
1s(2,*)2s(1,*)
4s,4p,4d,4f
1,1
3
n
EOF3
cp rcsf.out even.c
        Split into even3.c, even4.c
rcsfsplit <<EOF5
even
2
3s,3p,3d
4s,4p,4d,4f
EOF5
         1.3 SDT odd for n=4
```

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```
rcsfgenerate <<EOF3
*
0
1s(2,*)2p(1,*)

4s,4p,4d,4f
1,3
3
n
EOF3

cp rcsf.out odd.c

# Split into odd3.c, odd4.c

rcsfsplit <<EOF5
odd
2
3s,3p,3d
3
4s,4p,4d,4f
4
EOF5</pre>
```

#### 2. Get Nuclear Data

Nuclear data are defined by the script sh\_nuc. Since we are not interested in hyperfine structure, the nuclear spin and moments have all been set to 1.

```
#!/bin/sh
set -x

# 2. Get nuclear data for 150Nd
rnucleus <<S1
60
150
n
150
1
1
1
1
1</pre>
```

## 3. Get Initial Estimates

cat isodata

The script sh\_initial performs angular integration, gets initial estimates and performs rmcdhf calculations for the  $1s^22s$   $^2S_{1/2}$  and  $1s^22p$   $^2P^o_{1/2,3/2}$  reference states. As initial estimates, we use screened hydrogenic functions. For the reference states, all orbitals are required to be spectroscopic, i.e., they should have the correct number of nodes, see Section 7.1. Please note how we (to simplify the scripts for the rmcdhf calculations for n=3 and n=4) copy the radial wave functions to two files even 2. w and odd 2. w

```
#!/bin/sh
set -x
```

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```
# 3. Get initial estimates for DF
cp DF.c rcsf.inp
rangular <<S4
S4
  Get initial estimates of wave functions
rwfnestimate <<S5
3
S5
# Perform self-consistent field calculations
rmcdhf > DF <<S6
1
1
1
5
*
100
S6
  Save the result to DF
rsave DF
# For convenience in the scf script, and to avoid if statements in the latter,
# copy DF.w to even2.w and odd2.w
cp DF.w even2.w
cp DF.w odd2.w
```

## 4. rmcdhf, rci and ris4 Calculations

The script  $sh\_scf$  performs angular integration, estimates the new radial functions and performs rmcdhf for the odd and even states up to n=4. At the end, rci calculations are performed for the largest expansions. The rci calculations include Breit interaction and QED corrections. All results are transformed to LSJ-coupling. Please note how we loop in the script over the digit n that indicates the size of the orbital set.

```
#!/bin/sh
set -x

# 4. Get results for even n=3,4
# Please note that we copied DF.w to even2.w so this is available

for n in 3 4
do
    (cp even${n}.c rcsf.inp

# Get angular data
rangular <<S1
y
S1</pre>
```

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```
# Get initial estimates of wave functions
m='expr $n - 1'
echo m=$m n=$n
rwfnestimate <<S2
1
even${m}.w
3
S2
# Perform self-consistent field calculations
rmcdhf > outeven_rmcdhf_${n} <<S3</pre>
1
${n}*
100
S3
rsave even${n}
   echo)
done
\# Perform Breit-correction using CI for n=4
cp even${n}.c evenCI${n}.c
cp even${n}.w evenCI${n}.w
rci > outeven_rci <<$4</pre>
evenCI${n}
у
у
1.d-6
у
n
n
У
4
1
S4
# RIS4 calculation using CI for n=4
ris4 > outeven_ris4 <<S5
evenCI${n}
у
у
```

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```
n
S5
    Get results for odd n=3,4
# Please note that we copied DF.w to odd2.w so this is available
for n in 34
do
   (cp odd${n}.c rcsf.inp
# Get angular data
rangular <<S6
S6
# Get initial estimates of wave functions
m='expr $n - 1'
echo m=$m n=$n
rwfnestimate <<S7
1
odd${m}.w
3
# Perform self-consistent field calculations
rmcdhf > outodd_rmcdhf_${n} <<S8</pre>
у
1
1
5
${n}*
100
S8
rsave odd${n}
   echo)
done
  Perform Breit-correction using CI for n=4. First copy to other file names
cp odd${n}.c oddCI${n}.c
cp odd${n}.w oddCI${n}.w
rci > outodd_rci <<S9
oddCI${n}
у
у
```

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```
1.d-6
У
n
n
У
4
1
1
S9
   RIS4 calculation using CI for n=4
ris4 > outodd_ris4 <<S10
oddCI${n}
У
У
n
S10
```

## 12.2. Evaluating the Isotope Shift Using fical

Given the isotope shift parameters in evenCI4.ci and oddCI4.ci we use fical (frequency isotope calculation) to compute the  $^{150,142}$ Nd isotope shift. In fical, a five-parameter Fermi distribution is used to compute the nuclear radial moments, which together with the electronic factors is used to compute the line frequency field shift. To study the effect of deformation in  $^{150}$ Nd we do two calculations, where we in the first case assume a spherical nucleus for  $^{150}$ Nd ( $\beta_{20}=0$ ) and in the second case a deformed nucleus for  $^{150}$ Nd with  $\beta_{20}=0.28$ . In all cases, we use skin diffuseness parameter t=2.3 fm ,  $\beta_{40}=0$  and  $\omega=0$ , where the latter is a parameter describing the nuclear interior. The rest of the used nuclear parameters are collected in Table 12.

**Table 12.** Nuclear parameters for Nd isotopes.

	Mass (amu)	$r_{rms}$ (fm)	$eta_{20}$
<sup>142</sup> Nd	141.907719	4.9123	0
<sup>150</sup> Nd: spherical	149.920887	5.0400	0
<sup>150</sup> Nd: deformed	149.920887	5.0400	0.28

Input to fical in the first case with spherical nucleus for <sup>150</sup>Nd.

#### >>fical

```
WELCOME TO PROGRAM FICAL

Computes line frequency isotope shift parameters and/or energies using output files from ris4

Input files: <state1>.(c)i, <state2>.(c)i
Output file: <state1>.<state2>.(c)fi

Default settings (y/n)?:
>>n
Give name of state 1:
>>evenCI4
Give name of state 2:
>>oddCI4
Resulting isotope shifts from CI calculations (y/n)?:
```

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```
>>y
 Have electronic factors been calculated (y/n)?:
 Compute IS parameters (para), IS energies (ener) or both (both)?:
both
 Units (GHz, MHz or meV)?:
>>meV
 Use relativistically corrected mass shift parameters (y/n)?:
 Use sophisticated model for radial moments (y/n)?:
>>7
 Data for isotope 1
 Enter mass(amu), rms radius, t, omega, b20, b40:
>>141.907719d0,4.9123d0,2.3d0,0.d0,0.d0,0.d0
 Data for isotope 2
 Enter mass(amu), rms radius, t, omega, b20, b40:
>>149.920887d0,5.0400d0,2.3d0,0.d0,0.d0,0.d0
 program FICAL finished ...
 Isotope shift parameters/energies written to file evenCI4.oddCI4.cfi
    Copy output file
cp evenCI4.oddCI4.cfi spherical_150Nd.cfi
Input to fical in the second case with deformed nucleus for ^{150}Nd with deformation parameter \beta = 0.28.
>>fical
 WELCOME TO PROGRAM FICAL
 Computes line frequency isotope shift parameters and/or energies using
 output files from ris4
 Input files: <state1>.(c)i, <state2>.(c)i
 Output file: <state1>.<state2>.(c)fi
 Default settings (y/n)?:
>>n
 Give name of state 1:
>>evenCT4
 Give name of state 2:
>>oddCI4
 Resulting isotope shifts from CI calculations (y/n)?:
 Have electronic factors been calculated (y/n)?:
 Compute IS parameters (para), IS energies (ener) or both (both)?:
>>both
 Units (GHz, MHz or meV)?:
>>meV
 Use relativistically corrected mass shift parameters (y/n)?:
>>y
 Use sophisticated model for radial moments (y/n)?:
>>y
 Data for isotope 1
```

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Enter mass(amu), rms radius, t, omega, b20, b40:

```
>>141.907719d0,4.9123d0,2.3d0,0.d0,0.d0,0.d0
 Data for isotope 2
 Enter mass(amu), rms radius, t, omega, b20, b40:
>>149.920887d0,5.0400d0,2.3d0,0.d0,0.28d0,0.d0
 program FICAL finished ...
 Isotope shift parameters/energies written to file evenCI4.oddCI4.cfi
Copy output file
cp evenCI4.oddCI4.cfi deformed_150Nd.cfi
The output file from fical in the spherical case is shown below
 REFERENCE ISOTOPE DATA FROM ISODATA
 Atomic number: 60.000000
 Fermi nucleus:
                6.002295 fm
           c:
        r_rms: 5.040000 fm
            t: 2.300000 fm
 INPUT ISOTOPE DATA
              Isotope 1
                           Isotope 2
   Mass [amu]: 141.907719
                           149.920887
   r_rms [fm]:
                4.912300
                             5.040000
       t [fm]: 2.300000
                             2.300000
        omega: 0.000000
                             0.000000
         b_20: 0.000000
                             0.000000
         b_40: 0.000000
                             0.000000
 NUCLEAR RADIAL MOMENTS
               Isotope 1
                           Isotope 2
                                         Isotope 1 - Isotope 2
  <r^2> [fm^2]: 2.41307D+01
                                        -1.27091D+00
                           2.54016D+01
  r^4> [fm^4]: 7.60906D+02
                           8.39120D+02
                                         -7.82141D+01
  <r^6> [fm^6]: 2.88468D+04 3.31530D+04
                                         -4.30613D+03
                                        -2.44823D+05
  <r^8> [fm^8]: 1.28426D+06 1.52909D+06
 LINE MASS SHIFT PARAMETERS
 Upper level Lower level
                          Energy (cm-1)
                                          NMS-S (meV u)
                                                           NMS (meV u)
                                                                          SMS (meV u)
                                                                                           MS (meV u)
  1 1/2 -
               1 1/2 +
                           1125499.71
                                         -7.6551133D+01
                                                         -6.3724766D+01
                                                                          -3.3925317D+03
                                                                                          -3.4562565D+03
  1 3/2 -
               1 1/2 +
                           5888985.01
                                         -4.0054073D+02
                                                         -3.5384665D+02
                                                                          -3.6362230D+03
                                                                                         -3.9900697D+03
 LINE FIELD SHIFT PARAMETERS
 Upper level Lower level
                          Energy (cm-1)
                                                                          F4 (meV fm-6)
                                                                                           F6 (meV fm-8)
                                         FO (meV fm-2)
                                                          F2 (meV fm-4)
  1 1/2 -
              1 1/2 +
                           1125499.71
                                                          2.5299266D-02
                                                                          -7.6869849D-05
                                                                                           1.4267634D-07
                                                                                                           0.0000000
                                         -3.2436717D+01
  1 3/2 -
              1 1/2 +
                           5888985.01
                                         -3.3575752D+01
                                                          2.6140884D-02
                                                                          -7.9461478D-05
                                                                                           1.4748393D-07
                                                                                                           1.7895846
 Upper level Lower level
                          Energy (cm-1) FOVEDO (meV fm-2) FOVED1 (meV fm-4)
  1 1/2 -
               1 1/2 +
                            1125499.71
                                         -3.1084870D+01
                                                          2.1805779D-02
                                                                         0.0000000
  1 3/2 -
               1 1/2 +
                           5888985.01
                                         -3.2179044D+01
                                                          2.2527429D-02
                                                                         2.0376932
 LINE ISOTOPE SHIFT ENERGIES
 Upper level Lower level
                          Energy (cm-1)
                                             MS (meV)
                                                            FS (meV)
                                                                            IS (meV)
  1 1/2 -
              1 1/2 +
                           1125499.71
                                          1.3017934D+00
                                                         -3.9541428D+01
                                                                         -3.8239635D+01
  1 3/2 -
              1 1/2 +
                           5888985.01
                                          1.5028532D+00
                                                         -4.0933194D+01
                                                                         -3.9430341D+01
```

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The output file from fical in the deformed case is shown below

```
REFERENCE ISOTOPE DATA FROM ISODATA
Atomic number: 60.000000
Fermi nucleus:
                 6.002295 fm
           c:
                 5.040000 fm
        r_rms:
                 2.300000 fm
            t:
INPUT ISOTOPE DATA
               Isotope 1
                            Isotope 2
   Mass [amu]: 141.907719
                            149.920887
   r_rms [fm]:
                 4.912300
                              5.040000
       t [fm]:
                 2.300000
                              2.300000
                 0.000000
                              0.000000
        omega:
         b_20:
                 0.000000
                              0.280000
                0.000000
                              0.000000
         b 40:
NUCLEAR RADIAL MOMENTS
                                           Isotope 1 - Isotope 2
               Isotope 1
                             Isotope 2
 <r^2> [fm^2]: 2.41307D+01
                                           -1.27091D+00
                            2.54016D+01
 <r^4> [fm^4]: 7.60906D+02
                            8.54142D+02
                                           -9.32367D+01
 <r^6> [fm^6]: 2.88468D+04
                                           -5.94381D+03
                            3.47906D+04
 <r^8> [fm^8]: 1.28426D+06
                            1.66639D+06
                                           -3.82129D+05
LINE MASS SHIFT PARAMETERS
Upper level Lower level
                           Energy (cm-1)
                                            NMS-S (meV u)
                                                              NMS (meV u)
                                                                               SMS (meV u)
                                                                                                 MS (meV u)
  1 1/2 -
               1 1/2 +
                             1125499.71
                                           -7.6551133D+01
                                                             -6.3724766D+01
                                                                              -3.3925317D+03
                                                                                               -3.4562565D+03
  1 3/2 -
               1 1/2 +
                             5888985.01
                                           -4.0054073D+02
                                                             -3.5384665D+02
                                                                              -3.6362230D+03
                                                                                               -3.9900697D+03
LINE FIELD SHIFT PARAMETERS
Upper level Lower level
                           Energy (cm-1)
                                            FO (meV fm-2)
                                                             F2 (meV fm-4)
                                                                               F4 (meV fm-6)
                                                                                                F6 (meV fm-8)
  1 1/2 -
               1 1/2 +
                             1125499.71
                                           -3.2436717D+01
                                                             2.5299266D-02
                                                                              -7.6869849D-05
                                                                                                1.4267634D-07
                                                                                                                  0.0000000
  1 3/2 -
               1 \frac{1}{2} +
                            5888985.01
                                           -3.3575752D+01
                                                             2.6140884D-02
                                                                              -7.9461478D-05
                                                                                                1.4748393D-07
                                                                                                                  1.7895846
Upper level Lower level
                           Energy (cm-1) FOVEDO (meV fm-2) FOVED1 (meV fm-4)
  1 1/2 -
               1 1/2 +
                             1125499.71
                                           -3.1084870D+01
                                                             2.1805779D-02
                                                                               0.000000
  1 3/2 -
               1 1/2 +
                            5888985.01
                                                             2.2527429D-02
                                                                               2.0376932
                                           -3.2179044D+01
LINE ISOTOPE SHIFT ENERGIES
                           Energy (cm-1)
                                                                                 IS (meV)
Upper level Lower level
                                               MS (meV)
                                                               FS (meV)
  1 1/2 -
               1 \frac{1}{2} +
                            1125499.71
                                            1.3017934D+00
                                                             -3.9267665D+01
                                                                              -3.7965872D+01
  1 3/2 -
               1 1/2 +
                            5888985.01
                                            1.5028532D+00
                                                            -4.0650373D+01
                                                                              -3.9147519D+01
```

In Table 13, the resulting line frequency mass shifts (MS), field shifts (MS) and isotope shifts (IS) in units of meV for the  $^2P_{1/2}^o - ^2S_{1/2}$  and  $^2P_{3/2}^o - ^2S_{1/2}$  transitions are collected. The field shifts labeled  $^{150}$ Nd ( $\beta = 0.28$ ) and  $^{150}$ Nd (spherical) have been computed using the full set of line field shift factors (F0, F2, F4, F6) and radial moments (<rN>) given in the fical output files (see TP Section 3.3). It is seen that with the onset of deformation in  $^{150}$ Nd, the resulting field shifts increase with 0.27 meV and 0.28 meV for the  $^2P_{1/2}^o - ^2S_{1/2}$  and  $^2P_{3/2}^o - ^2S_{1/2}$  transitions, respectively. This is in very good agreement with other calculations using alternative methods [49,50]. In the table, as  $^{150}$ Nd (spherical, ved), we also show the field shifts computed with the reduced electronic factors  $\delta F_{k,0}^{(0)}$  (FOVED0 in output) and  $\delta F_{k,0}^{(1)}$  (FOVED1 in output) as (see TP Section 3.3)

$$\delta v_{k,FS}^{150,142} = \delta F_{k,0}^{(0)\text{ved}} \cdot \delta \langle r^2 \rangle^{150,142} + \delta F_{k,0}^{(1)\text{ved}} \cdot \left[ \delta \langle r^2 \rangle^{150,142} \right]^2, \tag{6}$$

where  $\delta \langle r^2 \rangle^{150,142} = 1.2709$  fm<sup>2</sup> has been used. As seen in the table, these approximate values, in units of meV, agree well with field shifts computed with the full set of electronic factors, assuming the <sup>150</sup>Nd nucleus to be spherical.

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Table 13. Line frequency isotope shifts in units of meV for	or the ${}^{2}P_{1/2}^{o}$ –	${}^{2}S_{1/2}$ and ${}^{2}P_{3/2}^{o}$	$_{2} - {}^{2}S_{1/2}$
transitions in Nd.		-,	

	$^{2}P_{1/2}^{o}-^{2}S_{1/2}$		${}^{2}P_{3/2}^{o} - {}^{2}S_{1/2}$			
	MS	FS	IS	MS	FS	IS
<sup>150</sup> Nd ( $\beta$ = 0.28) <sup>150</sup> Nd (spherical)	1.30 1.30	-39.27 -39.54	-37.97 $-38.24$	1.50 1.50	-40.65 $-40.93$	-39.15 -39.43
difference	0.00	0.27	0.27	0.00	0.28	0.28
<sup>150</sup> Nd (spherical, ved)	1.30	-39.47	-38.17	1.50	-40.86	-39.36

## 13. Methods to Ensure Convergence

In this section, we will try to give some practical advice on how to handle cases when the rmcdhf calculations for the MR do not converge. In the calculations for the MR the orbitals are spectroscopic and are required to have the correct number of nodes. Once the MR is in place, the remaining rmcdhf calculations for layers of correlation orbitals (no node counting required) are unproblematic.

## 13.1. Start with the Core and Gradually Build the Orbitals

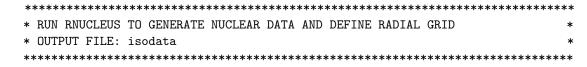
Advice is to start from the inner part of the core and gradually include more and more core orbitals until simultaneous convergence of all core orbitals has been achieved. Once the core orbitals are in place, gradually start to build more and more valence orbitals. If needed, keep all previous orbitals fixed the first time a layer of new orbitals is introduced. Once the new layer of orbitals is converged, optimize all orbitals together. This somewhat tedious multistep procedure is in general the preferred way to achieve convergence. If not working, the methods suggested below can be used.

## 13.2. Using Converted Hartree–Fock Orbitals

It can be difficult to achieve convergence when the MR consists of many configurations. In these cases, advice is to do a sequence of average HF calculations for the different configurations. The wave functions for each HF run are saved and at the end all wave functions are concatenated and then converted to relativistic wave functions that are used as starting estimates. As a practical example, we will perform a calculation for all states belonging to  $3s^2$ , 3s3p, 3s4s, 3s3d, 3s4p in Mg I using converted HF orbitals as starting estimates.

## Overview

- 1. Define nuclear data.
- 2. Generate list of CSFs for the  $\{3s^2, 3s3p, 3s4s, 3s3d, 3s4p\}$  MR set.
- 3. Perform angular integration.
- 4. Generate initial estimates of radial orbitals.
- 5. Perform SCF calculation on the weighted average of the states (this will fail).
- 6. Perform average HF calculations for the 3s3p, 3s4s, 3s3d, 3s4p configurations. Save the wave functions for each run.
- 7. Concatenate the HF wave functions to a file wfn.inp.
- 8. Use rwfnmchfmcdf to convert the wfn.inp to rwfn.out.
- 9. Copy rwfn.out to rwfn.inp and run rmcdhf (this will converge).
- 10. Run rsave.



#### >>rnucleus

## RNUCLEUS

This program defines nuclear data and the radial grid Outputfile: isodata

Enter the atomic number:

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```
>>12
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
 The default root mean squared radius is
                                          3.0569999217987061
                                                                 fm;
                                                                      (Angeli)
   the default nuclear skin thickness is
                                          2.299999999999998
                                                                 fm:
 Revise these values?
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
 Enter the nuclear dipole moment (in nuclear magnetons):
 Enter the nuclear quadrupole moment (in barns):
************************************
         RUN RCSFGENERATE TO GENERATE LIST OF CSFs
         FOR THE MULTIREFERNCE
         OUTPUT FILES: rcsfgenerate.log, rcsf.out
************************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
       0: No core
        1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                           = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                           = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                           = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>2
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
                             1
>>3s(2,i)
 Give configuration
                             2
>>3s(1,i)3p(1,i)
 Give configuration
                             3
>>3s(1,i)4s(1,i)
 Give configuration
                             4
>>3s(1,i)3d(1,i)
```

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```
Give configuration
                         5
>>3s(1,i)4p(1,i)
Give configuration
>>
Give set of active orbitals, as defined by the highest principal quantum number
per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,3d
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,6
Number of excitations (if negative number e.g., -2, correlation
orbitals will always be doubly occupied)
>>0
Generate more lists ? (y/n)
>>n
7 blocks were created
      block J/P
                       NCSF
         1 0+
                          2
         2
             0-
                           2
                           2
         3 1+
         4 1-
                           4
                           2
         5
            2+
         6
             2-
                           2
             3+
************************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
***********************************
>>cp rcsf.out rcsf.inp
************************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
**********************************
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
           rangular.log
Full interaction? (y/n)
>>y
```

RANGULAR: Execution complete.

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```
***********************************
        RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp, rwfnestimate.log
***********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                     12 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
                             gamma <r> MTP SRC
Shell
         е
                    р0
     0.4601D+02  0.8063D+02  0.1000D+01  0.1332D+00  331  T-F
  2s
     0.3992D+01 0.2155D+02 0.1000D+01 0.6753D+00 355 T-F
  2p- 0.2856D+01 0.2719D-01 0.1000D+01 0.6377D+00 358 T-F
     0.2845D+01 0.5572D+02 0.2000D+01 0.6395D+00 358 T-F
  2p
     0.4681D+00 0.6648D+01 0.1000D+01 0.2415D+01 378
  3s
                                                     T-F
  3p- 0.2743D+00 0.7261D-02 0.1000D+01 0.2954D+01 383 T-F
     0.2736D+00 0.1488D+02 0.2000D+01 0.2960D+01 383 T-F
  3d- 0.8141D-01 0.4498D-03 0.2000D+01 0.6214D+01 397 T-F
     0.8141D-01 0.1119D+01 0.3000D+01 0.6215D+01 397
  3d
                                                     T-F
  4s
     0.1150D+00 0.2438D+01 0.1000D+01 0.7036D+01 394
                                                     T-F
  4p- 0.8009D-01 0.2721D-02 0.1000D+01 0.8957D+01 398 T-F
     0.7998D-01 0.5583D+01 0.2000D+01 0.8969D+01 398 T-F
  4p
 RWFNESTIMATE: Execution complete.
   RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
        INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
        OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
        NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
```

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```
THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC *
         ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p, 3s, 3p, 3d, 4s, 4p
         AND THEY ARE ALL SPECTROSCOPIC.
***********************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                       12 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                    15 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                     7 blocks (block
                                        J/Parity
                                                   NCF):
  1
      0+
             2
                     2
                          0-
                                2
                                        3
                                             1+
                                                    2
                                                                 1-
  5
                     6
                          2-
                                                    1
      2+
 Enter ASF serial numbers for each block
                    ncf =
                                      2 id =
 Block
                 1
                                                 0+
>>1,2
 Block
                 2
                      ncf =
                                      2 id =
                                                 0-
>>1,2
Block
                 3
                     ncf =
                                      2 id =
                                                 1+
>>1,2
                 4
                                      4 id =
Block
                      ncf =
                                                 1_
>>1-4
                 5
                      ncf =
                                      2 id =
                                                 2+
Block
>>1,2
Block
                 6
                      ncf =
                                      2 id =
                                                 2-
>>1,2
Block
                 7
                      ncf =
                                      1 id =
>>1
level weights (1 equal; 5 standard; 9 user)
>>5
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p
 Enter orbitals to be varied (Updating order)
>>*
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
```

Method 2 unable to solve for 4s orbital Iteration number: 12, limit: 12 Present estimate of PO; 0.35970148280037D+01 Present estimate of E(J): 0.67584059942824D-01, DELEPS: -0.56222533604713D-02 Lower bound on energy: 0.41905738989649D-01, upper bound: 0.58877041760686D+01 Join point: 366, Maximum tabulation point: 404 Number of nodes counted: 3, Correct number: 3 Sign of P at first oscillation: -1. Failure; equation for orbital 4s could not be solved using method 2 \*\*\*\*\* Error in SUBROUTINE IMPROV \*\*\*\*\* Convergence not obtained \* RMCDHF CALCULATION DOES NOT CONVERGE. WE PERFORM HF CALCULATIONS FOR 3s3p, 3s4s, 3s3d, 3s4p TO OBTAIN BETTER ESTIMATES OF THE WAVE FUNCTION \* HF CALCULATION FOR 3s3p >>hf \_\_\_\_\_ HARTREE-FOCK.86

START OF CASE

Enter ATOM, TERM, Z

Examples: 0,3P,8. or Oxygen,AV,8.

>>Mg, AV, 12.

List the CLOSED shells in the fields indicated (blank line if none)

... ... ... ... ... ... etc.

>> 1s 2s 2p (! NOTE That shells occupy three positions and are right-justified)

Enter electrons outside CLOSED shells (blank line if none)

Example: 2s(1)2p(3)

>>3s(1)3p(1)

There are 5 orbitals as follows:

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1s 2s 2p 3s 3p Orbitals to be varied: ALL/NONE/=i (last i)/comma delimited list/H >>all Default electron parameters ? (Y/N/H) >>y 1.00 0.000 76.282 SCREENED HYDROGENIC 1s 3.00 0.000 22.274 SCREENED HYDROGENIC 7.00 0.000 32.977 2p SCREENED HYDROGENIC 10.00 0.000 3.810 SCREENED HYDROGENIC 3s SCREENED HYDROGENIC 11.00 0.000 5.111 Зр Default values for remaining parameters? (Y/N/H)>>y TOTAL ENERGY (a.u.) -----Non-Relativistic -199.52165286 Kinetic 199.52163203 -0.29267673 Relativistic Shift Potential -399.04328489 Relativistic -199.81432960 Ratio -2.00000104 Additional parameters ? (Y/N/H) Do you wish to continue along the sequence ? >>n END OF CASE \* COPY FILE WFN.OUT TO WFN3S3P >>cp wfn.out wfn3s3p \* HF CALCULATION FOR 3s4s COPY wfn.out TO wfn3s4s \* >>hf

HARTREE - FOCK . 86

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```
START OF CASE
  -----
  Enter ATOM, TERM, Z
  Examples: 0,3P,8. or Oxygen,AV,8.
>>Mg,AV,12.
  List the CLOSED shells in the fields indicated (blank line if none)
  ... ... ... ... ... ... etc.
>> 1s 2s 2p
  Enter electrons outside CLOSED shells (blank line if none)
  Example: 2s(1)2p(3)
>>3s(1)4s(1)
  There are 5 orbitals as follows:
    1s 2s 2p 3s 4s
  Orbitals to be varied: ALL/NONE/=i (last i)/comma delimited list/H
>>all
 Default electron parameters ? (Y/N/H)
>>y
         1s
                1.00
                         0.000
                                  76.282
                                            SCREENED HYDROGENIC
                         0.000 22.274
         2s
               3.00
                                            SCREENED HYDROGENIC
               7.00
                         0.000 32.977
         2p
                                            SCREENED HYDROGENIC
                                  3.810
         3s 10.00
                         0.000
                                            SCREENED HYDROGENIC
              11.00
                          0.000
                                  1.625
                                            SCREENED HYDROGENIC
 Default values for remaining parameters? (Y/N/H)
>>y
. . . . . . . . . . . . . . . .
    TOTAL ENERGY (a.u.)
          Non-Relativistic -199.45963917
                                               Kinetic 199.45962102
          Relativistic Shift
                               -0.29285961
                                               Potential -398.91926019
          Relativistic
                             -199.75249878
                                               Ratio
                                                           -2.000000091
  Additional parameters ? (Y/N/H)
>>n
  Do you wish to continue along the sequence ?
>>n
  END OF CASE
  ========
```

COPY FILES WFN.OUT TO WFN3S4S

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\* >>cp wfn.out wfn3s4s HF CALCULATION FOR 3s3d \* >>hf \_\_\_\_\_ H A R T R E E - F O C K . 86 THE DIMENSIONS FOR THE CURRENT VERSION ARE: NWF= 20 NO=220 START OF CASE -----Enter ATOM, TERM, Z Examples: 0,3P,8. or Oxygen,AV,8. >>Mg,AV,12. List the CLOSED shells in the fields indicated (blank line if none) ... ... ... ... ... ... etc. >> 1s 2s 2p Enter electrons outside CLOSED shells (blank line if none) Example: 2s(1)2p(3)>>3s(1)3d(1)There are 5 orbitals as follows: 1s 2s 2p 3s 3d Orbitals to be varied: ALL/NONE/=i (last i)/comma delimited list/H >>all Default electron parameters ? (Y/N/H) >>y 1.00 0.000 76.282 SCREENED HYDROGENIC 1s 2s 3.00 0.000 22.274 SCREENED HYDROGENIC 7.00 0.000 32.977 SCREENED HYDROGENIC 2p 10.00 3s 0.000 3.810 SCREENED HYDROGENIC 3d 11.00 0.000 2.476 SCREENED HYDROGENIC Default values for remaining parameters? (Y/N/H) >>y

. . . . . . . . . . . . .

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TOTAL ENERGY (a.u.) Non-Relativistic -199.42914481 Kinetic 199.42914489 Potential -398.85828970 Relativistic Shift -0.29277506 Relativistic -199.72191987 Ratio -2.000000000 Additional parameters ? (Y/N/H) >>n Do you wish to continue along the sequence ? >>n END OF CASE \* COPY FILE WFN.OUT TO WFN3S3D >>cp wfn.out wfn3s3d HF CALCULATION FOR 3s4p \* >>hf H A R T R E E - F O C K . 86 THE DIMENSIONS FOR THE CURRENT VERSION ARE: NWF= 20 NO=220 START OF CASE

=========

Enter ATOM, TERM, Z

Examples: 0,3P,8. or Oxygen,AV,8.

>>Mg,AV,12.

... ... etc.

>> 1s 2s 2p

 ${\tt Enter\ electrons\ outside\ CLOSED\ shells\ (blank\ line\ if\ none)}$ 

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```
Example: 2s(1)2p(3)
>>3s(1)4p(1)
 There are 5 orbitals as follows:
   1s 2s 2p 3s 4p
Orbitals to be varied: ALL/NONE/=i (last i)/comma delimited list/H
>>all
Default electron parameters ? (Y/N/H)
>>y
             1.00
                     0.000
                            76.282
                                    SCREENED HYDROGENIC
        1s
        2s
             3.00
                     0.000
                            22.274
                                    SCREENED HYDROGENIC
            7.00
        2p
                     0.000
                            32.977 SCREENED HYDROGENIC
           10.00
                     0.000
                           3.810 SCREENED HYDROGENIC
           11.00
                     0.000
                            3.409
                                    SCREENED HYDROGENIC
        4p
Default values for remaining parameters? (Y/N/H)
. . . . . . . . .
    TOTAL ENERGY (a.u.)
                        -199.43021560
        Non-Relativistic
                                      Kinetic
                                               199.43022615
                                      Potential -398.86044175
        Relativistic Shift
                          -0.29277432
        Relativistic
                       -199.72298992
                                                -1.999999947
                                      Ratio
Additional parameters ? (Y/N/H)
>>n
Do you wish to continue along the sequence ?
>>n
END OF CASE
 _____
********************************
       COPY WFN.OUT TO WFN3S4P
******************************
>>cp wfn.out wfn3s4p
*********************************
       CONCATENATE HF WAVE FUNCTION FILES
**************************************
>>cat wfn3s3p wfn3s4s wfn3s3d wfn3s4p > wfn.inp
RUN RWFNMCHFMCDF TO CONVERT NON-RELATIVISTIC RADIAL ORBITALS TO
       RELATIVISTIC ONES
```

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```
INPUT FILE: wfn.inp
        OUTPUT FILE: rwfn.out
******************************
>>rwfnmchmcdf
RWFNMCHFMCDF
This program converts non-relativistic radial
orbitals to relativistic ones in GRASP format
Input file: wfn.inp
Output file: rwfn.out
********************************
        COPY FILES
        WE DONT NEED TO INVOKE RWFNESTIMATE SINCE ALL ORBITALS HAVE
        BEEN ESTIMATED THROUGH THE MCHF MCDF CONVERSION
***********************************
>>cp rwfn.out rwfn.inp
***********************************
        RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
        INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
        OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
        NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
        THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
        ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p, 3s, 3p, 3d, 4s, 4p AND
        THEY ARE ALL SPECTROSCOPIC.
*************************************
>>rmcdhf
RMCDHF
This program determines the radial orbitals
and the expansion coefficients of the CSFs
in a self-onsistent field proceedure
Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
>>y
Loading CSF file ... Header only
There are/is
                    12 relativistic subshells;
Loading CSF File for ALL blocks
                  15 relativistic CSFs... load complete;
There are
Loading Radial WaveFunction File ...
There are
                   7 blocks (block
                                    J/Parity
                                              NCF):
      0+
                   2
                       0-
                             2
                                      1+
                                               2
                                                          1-
      2+
            2
                       2-
                             2
                                    7
                   6
                                        3+
                                               1
Enter ASF serial numbers for each block
Block
              1 ncf =
>>1,2
```

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```
Block
                2
                    ncf =
                                    2 id =
                                              0-
>>1,2
Block
                    ncf =
                                    2 id =
                                              1+
>>1,2
 Block
                4
                     ncf =
                                       id =
                                              1_
>>1-4
Block
                5
                     ncf =
                                       id =
                                              2+
>>1,2
 Block
                     ncf =
                                      id =
                                              2-
>>1,2
                     ncf =
                                      id =
Block
>>1
 level weights (1 equal; 5 standard; 9 user)
>>5
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p
 Enter orbitals to be varied (Updating order)
>>*
 Which of these are spectroscopic orbitals?
>>*
 Enter the maximum number of SCF cycles:
>>100
. . . . . .
 Wall time:
      46 seconds
 Finish Date and Time:
  Date (Yr/Mon/Day): 2014/09/05
  Time (Hr/Min/Sec): 13/51/24.996
  Zone: +0200
 RMCDHF: Execution complete.
********************************
         THIS TIME IT CONVERGED! RUN RSAVE
************************************
>>rsave mr
```

# 13.3. Decrease Nuclear Charge in Small Steps

Created mr.w, mr.c, mr.m, mr.sum, mr.alog and mr.log

Convergence can sometimes be difficult to achieve for large systems that are neutral or near neutral. In these cases, one advice is to edit isodata and increase the nuclear charge. If the rmcdhf run is converged for the increased charge then copy rwfn.out to rwfn.inp, decrease the nuclear charge by a small amount and run rmcdhf that hopefully will converge. Repeat the procedure until you are down to the correct charge. To illustrate the technique, we will perform a calculation for the ground state [Rn]  $5f^{14}7s^2$  of No I (Z=102).

### Overview

- 1. Define nuclear data.
- 2. Generate list of CSFs.
- 3. Perform angular integration.
- 4. Generate initial estimates of radial orbitals.

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```
5.
    Perform SCF calculation on the weighted average of the states (this will not converge)
    Edit isodata and increase nuclear charge to Z = 105.
6.
7.
    Generate initial estimates of radial orbitals.
8.
    Perform SCF calculation on the weighted average of the states (this will converge).
9.
    Copy rwfn.out to rwfn.inp. Decrease nuclear charge to Z = 104.
10.
    Perform SCF calculation on the weighted average of the states (this will converge).
    Copy rwfn.out to rwfn.inp. Decrease nuclear charge to Z = 103.
11.
12.
    Perform SCF calculation on the weighted average of the states (this will converge).
13.
    Copy rwfn.out to rwfn.inp. Decrease nuclear charge to Z = 102.5.
14.
   Perform SCF calculation on the weighted average of the states (this will converge).
    Copy rwfn.out to rwfn.inp. Decrease nuclear charge to Z = 102.
    Perform SCF calculation on the weighted average of the states (this will converge).
16.
17.
    Run rsave.
* RUN RNUCLEUS TO GENERATE NUCLEAR DATA
* OUTPUT FILE: isodata
***********************************
>>rnucleus
 RNUCLEUS
 This program defines nuclear data and the radial grid
 Outputfile: isodata
 Enter the atomic number:
>>102
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>259
 The default root mean squared radius is
                                            5.8989242234501091
                                                                          (default)
                                                                     fm:
   the default nuclear skin thickness is
                                            2.29999999999998
                                                                     fm;
 Revise these values?
>>n
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
 Enter the nuclear dipole moment (in nuclear magnetons):
>>1
 Enter the nuclear quadrupole moment (in barns):
>>1
************************************
          RUN RCSFGENERATE TO GENERATE LIST OF CSFs
          OUTPUT FILES: rcsfgenerate.log, rcsf.out
**********************************
>>rcsfgenerate
 RCSFGENERATE
 This program creates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
```

occupation e.g., 1s(2,1)2s(2,\*)

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```
Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
       0: No core
       1: He (
                  1s(2)
                                        = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                        = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                        = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                        = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                        = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>6
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>5f(14,i)7s(2,i)
 Give configuration
                           2
>>
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>7s,6p,5d,5f
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,0
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
 1 blocks were created
      block J/P
                          NCSF
          1 1/2+
*************************************
        COPY FILES
        IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
        RECORD ON HOW THE LIST OF CSFs WAS CREATED
*******************************
>>cp rcsfgenerate.log mr.exc
>>cp rcsf.out rcsf.inp
************************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
*************************************
```

>>rangular

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```
RANGULAR
 This program performs angular integration
 Input file: rcsf.inp
 Outputfiles: mcp.30, mcp.31, ....
             rangular.log
 Full interaction? (y/n)
>>y
    . . . . . .
 RANGULAR: Execution complete.
********************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         INPUT FILES: isodata, rcsf.inp
         OUTPUT FILE: rwfn.inp, rwfnestimate.log
*********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                      27 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-
 6p 5f- 5f 7s
 Read subshell radial wavefunctions. Choose one below
    1--GRASP92 File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
                                          <r> MTP SRC
Shell
          е
                     р0
                              gamma
     0.5535D+04  0.8367D+04  0.1000D+01  0.1161D-01  328
  1s
                                                       T-F
  2s
     0.1090D+04 0.3673D+04 0.1000D+01 0.4758D-01 344
                                                       T-F
  2p- 0.1062D+04 0.5365D+03 0.1000D+01 0.3770D-01
                                                  344 T-F
     0.8187D+03 0.4714D+05 0.2000D+01 0.4932D-01
  2p
                                                  347
                                                       T-F
      0.2880D+03 0.1820D+04 0.1000D+01 0.1242D+00
                                                  358
                                                       T-F
  3p- 0.2739D+03 0.2826D+03 0.1000D+01 0.1154D+00 358 T-F
     0.2159D+03 0.2629D+05 0.2000D+01 0.1361D+00 360 T-F
  Зр
  3d- 0.1922D+03 0.6033D+03 0.2000D+01 0.1163D+00 361 T-F
```

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```
3d
      0.1810D+03  0.1243D+06  0.3000D+01  0.1223D+00  362
                                                        T-F
      0.7989D+02 0.9782D+03 0.1000D+01 0.2658D+00
                                                  370
                                                        T-F
  4p- 0.7297D+02 0.1523D+03 0.1000D+01 0.2614D+00
                                                   371
                                                        T-F
  4p
      0.5672D+02 0.1435D+05 0.2000D+01 0.2997D+00
                                                   374
                                                        T-F
  4d- 0.4539D+02 0.3560D+03 0.2000D+01 0.2913D+00
                                                   375
                                                        T-F
  4d
      0.4246D+02 0.7373D+05 0.3000D+01 0.3022D+00
                                                   376
                                                        T-F
  4f- 0.2737D+02 0.4518D+03 0.3000D+01 0.2818D+00
                                                   379
                                                        T-F
  4f
      0.2652D+02 0.1174D+06 0.4000D+01 0.2869D+00
                                                   379
                                                        T-F
      0.1976D+02 0.5211D+03 0.1000D+01 0.5350D+00
                                                   384
                                                        T-F
  5p- 0.1675D+02 0.7958D+02 0.1000D+01 0.5469D+00
                                                   385
                                                        T-F
  5p
      0.1243D+02 0.7413D+04 0.2000D+01 0.6265D+00
                                                   388
  5d- 0.7880D+01 0.1754D+03 0.2000D+01 0.6730D+00
                                                   392
                                                        T-F
  5d
      0.7220D+01 0.3613D+05 0.3000D+01 0.6986D+00
                                                   393
                                                        T-F
  6s
      0.3744D+01 0.2510D+03 0.1000D+01 0.1117D+01
                                                   400
                                                        T-F
  6p- 0.2724D+01 0.3631D+02 0.1000D+01 0.1204D+01
                                                   403
                                                        T-F
      0.1859D+01 0.3246D+04 0.2000D+01 0.1414D+01
                                                   406
  6p
                                                        T-F
  5f- 0.2174D-01 0.1722D+01 0.3000D+01 0.2805D+02
                                                   457
                                                        T-F
  5f
      0.2173D-01 0.4523D+03 0.4000D+01 0.2807D+02
                                                   457
                                                        T-F
      0.5319D+00 0.9661D+02 0.1000D+01 0.2704D+01
                                                       T-F
 RWFNESTIMATE: Execution complete.
*************************************
         RUN RMCDHF FOR Z = 102 (WILL NOT CONVERGE)
*************************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                      27 relativistic subshells;
 Loading CSF File for ALL blocks
                    1 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    1 blocks (block
                                       J/Parity
                                                  NCF):
      0+
 Enter ASF serial numbers for each block
                     ncf =
 Block
                 1
                                     1 \quad id =
                                                0+
>>1
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-
 6p 5f-5f 7s
 Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
```

>>\*

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Enter the maximum number of SCF cycles: >>100

Iteration number 2

						Self-	-			Dampin	ıg			
Subshell	Energy	Met	hod	PO	CO	nsister	ісу	Norm-	-1	factor	JP	MTP	INV	NNP
1s	5.5386641D+03	1	8.3	70D+03	1.3	36D-03	1	.62D-0	5 0	.000	274	377	0	0
2s	1.0949842D+03	1	3.6	49D+03	4.	51D-03	-1	.88D-04	1 0	.000	302	382	0	1
2p-	1.0594702D+03	1	5.2	67D+02	4.	11D-03	-2	.43D-0	5 0	.000	302	384	0	0
2p	8.2094271D+02	1	4.6	36D+04	7.	53D-03	-1	.04D-0	5 0	.000	306	385	0	0
3s	2.9768023D+02	1	1.8	00D+03	1.	17D-02	-7	.39D-04	1 0	.000	320	385	0	2
3p-	2.8135963D+02	1	2.7	60D+02	1.	16D-02	-6	.04D-04	1 0	.000	321	388	0	1
3р	2.2441931D+02	1	2.5	76D+04	1.9	98D-02	-7	.16D-04	1 0	.000	324	389	0	1
3d-	1.9934224D+02	1	5.7	73D+02	1.8	36D-02	-2	.83D-04	1 0	.000	325	411	0	0
3d	1.8864448D+02	1	1.1	94D+05	2.5	21D-02	-2	.02D-04	1 0	.000	326	411	0	0
4s	9.0606030D+01	1	9.5	51D+02	1.	73D-02	-1	.59D-03	3 0	.100	335	393	0	3
4p-	8.2942623D+01	1	1.4	68D+02	1.	77D-02	-1	.56D-03	3 0	.100	336	404	0	2
4p	6.7111537D+01	1	1.3	95D+04	2.	78D-02	-2	.27D-03	3 0	. 100	339	408	0	2
4d-	5.5119356D+01	1	3.3	69D+02	2.9	90D-02	-2	.26D-03	3 0	.100	341	434	0	1
4d	5.2313964D+01	1	7.0	11D+04	2.9	95D-02	-1	.95D-03	3 0	.100	342	434	0	1
4f-	3.6521991D+01	1	4.1	26D+02	2.	19D-03	1	.73D-04	1 0	.050	346	408	0	0
4f	3.5747535D+01	1	1.0	84D+05	6.0	02D-03	-2	.02D-04	1 0	.100	346	408	0	0
5s	3.0255721D+01	1	5.0	92D+02	1.8	33D-02	3	.85D-03	3 0	.100	348	407	0	4
5p-	2.6881305D+01	1	7.6	99D+01	2.3	36D-02	5	.21D-03	3 0	.100	350	428	0	3
5p	2.2514178D+01	1	7.3	71D+03	3.9	91D-02	6	.74D-03	3 0	.100	352	429	0	3
5d-	1.7326694D+01	1	1.7	29D+02	5.	78D-02	1	.18D-02	2 0	.100	356	440	0	2
5d	1.6650195D+01	1	3.6	11D+04	6.	18D-02	1	.05D-02	2 0	.100	356	440	0	2
6s	1.2077305D+01	1	2.8	10D+02	4.0	61D-03	-1	.36D-03	3 0	.050	361	429	0	5
6p-	1.0606432D+01	1	4.1	87D+01	7.	79D-03	-2	.88D-03	3 0	.050	363	436	0	4
6p	9.2631692D+00	1	3.9	69D+03	4.	70D-02	-1	.32D-02	2 0	.190	365	438	0	4
Method	2 unable to so	olve	for	5f- o	rbi	tal								
Iteration number: 15, limit: 15														
Present estimate of P0; 0.19118742906131D+01														
Present estimate of E(J): 0.98080405136720D+02, DELEPS: -0.60275500767866D+02														
Lower bound on energy: 0.13875636426564D+02, upper bound: 0.26234163356640D+03														
Join point: 339, Maximum tabulation point: 457														

Failure; equation for orbital  $\,$  5f- could not be solved using method 2

\*\*\*\*\* Error in SUBROUTINE IMPROV \*\*\*\*\*\*
Convergence not obtained

Sign of P at first oscillation: -1.

Number of nodes counted: 2, Correct number: 1

>>open editor and change  ${\tt Z}$  to 105 in isodata

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```
************************************
         RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
         INPUT FILES: isodata, rcsf.inp, previous rwfn files
         OUTPUT FILE: rwfn.inp, rwfnestimate.log
***********************************
>>rwfnestimate
RWFNESTIMATE
This program estimates radial wave functions
for orbitals
Input files: isodata, rcsf.inp, optional rwfn file
Output file: rwfn.inp
Default settings ?
>>y
Loading CSF file ... Header only
There are/is
                      27 relativistic subshells;
The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-
6p 5f - 5f 7s
Read subshell radial wavefunctions. Choose one below
    1--GRASP92 File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
Enter the list of relativistic subshells:
All required subshell radial wavefunctions have been estimated:
                               gamma
Shell
          е
                     p0
                                           <r>
                                                   MTP SRC
      0.5997D+04 0.9512D+04 0.1000D+01 0.1105D-01
                                                   328
                                                        T-F
  2s
      0.1211D+04 0.4259D+04 0.1000D+01 0.4510D-01
                                                   344
                                                        T-F
  2p- 0.1185D+04 0.6741D+03 0.1000D+01 0.3545D-01
  2p
     0.8980D+03 0.5365D+05 0.2000D+01 0.4752D-01
                                                   347
                                                        T-F
      0.3318D+03 0.2128D+04 0.1000D+01 0.1176D+00
                                                   357
                                                        T-F
  3p- 0.3177D+03 0.3582D+03 0.1000D+01 0.1088D+00
                                                   357
                                                        T-F
     0.2480D+03  0.3038D+05  0.2000D+01  0.1300D+00
                                                   360
                                                        T-F
  3d- 0.2242D+03 0.7505D+03 0.2000D+01 0.1107D+00
                                                   360
                                                        T-F
  3d
      0.2109D+03 0.1473D+06 0.3000D+01 0.1167D+00
                                                   361
                                                        T-F
  4s
      0.9893D+02 0.1162D+04 0.1000D+01 0.2491D+00
                                                   369
                                                        T-F
  4p- 0.9179D+02 0.1966D+03 0.1000D+01 0.2437D+00
                                                   370
                                                        T-F
  4p
      0.7154D+02 0.1696D+05 0.2000D+01 0.2817D+00
                                                   372
  4d- 0.5977D+02 0.4572D+03 0.2000D+01 0.2713D+00
                                                   374
                                                        T-F
  4d
      0.5611D+02 0.9032D+05 0.3000D+01 0.2818D+00
                                                   374
                                                        T-F
  4f- 0.4043D+02 0.6235D+03 0.3000D+01 0.2580D+00
                                                   377
                                                        T-F
  4f
      0.3931D+02 0.1559D+06 0.4000D+01 0.2630D+00
                                                   377
                                                        T-F
      0.2861D+02 0.6415D+03 0.1000D+01 0.4882D+00
                                                   382
                                                        T-F
  5s
  5p- 0.2533D+02 0.1071D+03 0.1000D+01 0.4944D+00
                                                   383
                                                        T-F
                                                   385
     0.1943D+02 0.9208D+04 0.2000D+01 0.5672D+00
                                                        T-F
  5p
  5d- 0.1433D+02 0.2428D+03 0.2000D+01 0.5924D+00
                                                   388
                                                        T-F
```

0.1337D+02 0.4789D+05 0.3000D+01 0.6140D+00

389 T-F

5d

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```
0.7815D+01 0.3387D+03 0.1000D+01 0.9513D+00 395
  6p- 0.6526D+01 0.5491D+02 0.1000D+01 0.9968D+00 396 T-F
  6p
     0.5004D+01 0.4650D+04 0.2000D+01 0.1150D+01
                                                    399
                                                         T-F
 5f- 0.7111D+01 0.3258D+03 0.3000D+01 0.6774D+00
                                                    394
                                                         T-F
      0.6883D+01 0.8130D+05 0.4000D+01 0.6891D+00 395
                                                         T-F
      0.2311D+01 0.1686D+03 0.1000D+01 0.1898D+01 408 T-F
  7s
RWFNESTIMATE: Execution complete.
*******************************
         RUN RMCDHF FOR Z = 105 (WILL CONVERGE)
>>rmcdhf
RMCDHF
This program determines the radial orbitals
and the expansion coefficients of the CSFs
in a self-onsistent field proceedure
Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
Default settings? (y/n)
Loading CSF file ... Header only
                       27 relativistic subshells;
There are/is
Loading CSF File for ALL blocks
There are
                    1 relativistic CSFs... load complete;
Loading Radial WaveFunction File ...
There are
                     1 blocks (block J/Parity
                                                   NCF):
    0+
Enter ASF serial numbers for each block
                                      1 id =
                1 	 ncf =
                                                 0+
>>1
Radial functions
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-
6p 5f- 5f 7s
Enter orbitals to be varied (Updating order)
>>*
Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
. . . . . . . . . . . . . . . . . . . .
Wall time:
      75 seconds
Finish Date and Time:
  Date (Yr/Mon/Day): 2018/11/26
  Time (Hr/Min/Sec): 12/58/42.303
   Zone: +0100
```

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RMCDHF: Execution complete. RMCDHF IS CONVERGING EDIT ISODATA AND DECREASE Z TO Z = 104 (SMALLER STEPS MAY BE NEEDED)\* >>open editor and change Z to 104 in isodata \* COPY RWFN.OUT TO RWFN.INP THUS WE USE THE PREVIOUS OUTPUT AS INPUT FOR THE NEW RUN \* >>cp rwfn.out rwfn.inp \* RUN RMCDHF FOR Z = 104 (WILL CONVERGE) \* >>rmcdhf RMCDHF This program determines the radial orbitals and the expansion coefficients of the CSFs in a self-onsistent field proceedure Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ... Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log Default settings? (y/n)>>y Loading CSF file ... Header only 27 relativistic subshells; There are/is Loading CSF File for ALL blocks There are 1 relativistic CSFs... load complete; Loading Radial WaveFunction File ... 1 blocks (block J/Parity There are NCF): 1 0+ Enter ASF serial numbers for each block Block 1 ncf =1 id = 0+>>1 Radial functions 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-6p 5f-5f 7s Enter orbitals to be varied (Updating order) Which of these are spectroscopic orbitals? Enter the maximum number of SCF cycles:

. . . . . . . . . . . . . . . . . . .

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Wall time:

75 seconds

Finish Date and Time:

Date (Yr/Mon/Day): 2018/11/26 Time (Hr/Min/Sec): 12/58/42.303

Zone: +0100

RMCDHF: Execution complete.

### 13.4. Using Non-Default Options

If combining HF estimates and decreasing the nuclear charge in small steps does not ensure convergence, then the remaining alternative is to use the non-default options in rmcdhf. The user may play around with the threshold for node counting. An oscillation in the large-component of the radial wavefunction is disregarded for the purposes of node counting if its amplitude is less than 1/20 the maximum amplitude. The user may change this value. If convergence is achieved, then it is required that all the spectroscopic orbitals are plotted and inspected so that they have the correct node structure. The user may also want to set accelerating parameters odamp for subshell radial wavefunctions. Setting odamp to a value close to 1 damps the changes in the radial wave functions at each iteration. This may sometimes help.

### 13.5. Changing the Grid

For neutral or near neutral super heavy systems, it is sometimes desirable to increase the number of grid points and change the grid parameters. To install the program with the extended grid, follow the instructions in Section 1.4. Go to GRASP2018/src/lib/libmod, open the file parameter\_def\_M.f90 and change the NNNP and NNN1 variables to, respectively, 1990 and 2000 and recompile the full package. Below we perform a calculation for all states of the U I ground configuration  $5f^36d7s^2$  where the grid parameters have been changed to smaller values and the number of grid points has been set to 1990.

#### Overview

- 1. Define nuclear data.
- 2. Generate list of CSFs.
- 3. Perform angular integration.
- 4. Generate initial estimates of radial orbitals. Override default options and change grid parameters.
- 5. Perform SCF calculation on the weighted average of the states. Override default options and change grid parameters.

```
* RUN RNUCLEUS TO GENERATE NUCLEAR DATA
* OUTPUT FILE: isodata
*****************************
>>rnucleus
 Enter the atomic number:
>>92
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>238
 The default root mean squared radius is
                                       5.8571000099182129
                                                                  (Angeli)
                                                             fm:
  the default nuclear skin thickness is
                                       2.299999999999998
                                                             fm:
 Revise these values?
>>n
```

\*

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```
Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
>>238
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
 Enter the nuclear dipole moment (in nuclear magnetons):
>>1
 Enter the nuclear quadrupole moment (in barns):
>>1
*************************************
         RUN RCSFGENERATE TO GENERATE LIST OF CSFs
         OUTPUT FILES: rcsfgenerate.log, rcsf.out
**********************************
>>rcsfgenerate
 RCSFGENERATE
 This program generates a list of CSFs
 Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
 Select core
        0: No core
       1: He (
                     1s(2)
                                           = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                           = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                           = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                           = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                           = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>6
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
                             1
>>5f(3,*)6d(1,*)7s(2,*)
 Give configuration
                             2
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>7s,6d,5f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
```

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### 12 blocks were created

block	J/P	NCSF
1	0-	13
2	1-	35
3	2-	51
4	3-	61
5	4-	61
6	5-	54
7	6-	44
8	7-	31
9	8-	19
10	9-	11
11	10-	5
12	11-	1

### >>rangular

#### RANGULAR

This program performs angular integration

Input file: rcsf.inp

Outputfiles: mcp.30, mcp.31, ....

rangular.log

Full interaction? (y/n)

>>y

. . . . .

RANGULAR: Execution complete.

#### >>rwfnestimate

#### RWFNESTIMATE

This program estimates radial wave functions

for orbitals

Input files: isodata, rcsf.inp, optional rwfn file

Output file: rwfn.inp Default settings ?

>>n

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```
Generate debug printout?
 File erwf.sum will be created as the ERWF SUMmary File;
 enter another file name if this is not acceptable; null otherwise:
 Loading CSF file ... Header only
                       29 relativistic subshells;
 There are/is
 Change the default speed of light or radial grid parameters?
 The physical speed of light in atomic units is
                                              137.03599913900001
 revise this value?
>>n
 The default radial grid parameters for this case are:
          2.1739130434782606E-008;
        5.00000000000003E-002;
         0.000000000000000
 HP =
 N =
             1990 ;
 revise these values?
 Enter RNT:
>>2.17d-08
 Enter H:
>>1.5d-02
Enter HP:
>>0.0d0
 Enter N:
>>1990
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-
 6p 5f- 5f 6d- 6d 7s
 Read subshell radial wavefunctions. Choose one below
     1--GRASP92 File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                     p0
                                                   MTP SRC
                                           <r>
          е
                               gamma
  1s
      0.4269D+04 0.5532D+04 0.1000D+01 0.1367D-01 1111
     0.8051D+03 0.2294D+04 0.1000D+01 0.5637D-01 1166
  2s
  2p- 0.7805D+03 0.2568D+03 0.1000D+01 0.4550D-01 1166
      0.6379D+03 0.3086D+05 0.2000D+01 0.5568D-01 1172
  2p
  3s
      0.2059D+03 0.1122D+04 0.1000D+01 0.1464D+00 1211
  3p- 0.1939D+03 0.1335D+03 0.1000D+01 0.1371D+00 1213
      0.1609D+03 0.1675D+05 0.2000D+01 0.1554D+00 1219
  3d- 0.1407D+03 0.3040D+03 0.2000D+01 0.1337D+00 1222
                                                        T-F
  3d
      0.1339D+03 0.7409D+05 0.3000D+01 0.1391D+00 1223
     0.5410D+02 0.5921D+03 0.1000D+01 0.3157D+00 1255
  4s
                                                        T_F
  4p- 0.4849D+02 0.7042D+02 0.1000D+01 0.3128D+00 1258
  4p
```

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```
4d- 0.3040D+02 0.1726D+03 0.2000D+01 0.3423D+00 1272
      0.2873D+02  0.4221D+05  0.3000D+01  0.3528D+00  1274
  4f- 0.1647D+02 0.1933D+03 0.3000D+01 0.3375D+00 1289
     0.1602D+02 0.5762D+05 0.4000D+01 0.3425D+00 1290
  4f
      0.1225D+02  0.3046D+03  0.1000D+01  0.6505D+00  1304
                                                        T-F
  5s
  5p- 0.9992D+01 0.3527D+02 0.1000D+01 0.6732D+00 1309
      0.7827D+01 0.4401D+04 0.2000D+01 0.7505D+00 1317
  5p
  5d- 0.4418D+01 0.7930D+02 0.2000D+01 0.8322D+00 1334
                                                        T-F
  5d
      0.4088D+01 0.1926D+05 0.3000D+01 0.8594D+00 1337
  6s
     0.2056D+01 0.1373D+03 0.1000D+01 0.1427D+01 1362 T-F
  6p- 0.1383D+01 0.1474D+02 0.1000D+01 0.1583D+01 1374
      0.1019D+01 0.1768D+04 0.2000D+01 0.1811D+01 1384
  6p
     0.4670D+00 0.6493D+02 0.3000D+01 0.1279D+01 1405
  5f
      0.4268D+00 0.1908D+05 0.4000D+01 0.1313D+01 1408
                                                       T-F
  6d- 0.2724D+00 0.2243D+02 0.2000D+01 0.2720D+01 1428
  6d
      0.2470D+00 0.5293D+04 0.3000D+01 0.2869D+01 1432 T-F
  7s
      0.2991D+00 0.4866D+02 0.1000D+01 0.3695D+01 1428 T-F
 Revise any of these estimates?
 RWFNESTIMATE: Execution complete.
*******************************
         RUN RMCDHF WITH NON-DEFAULT OPTIONS FOR GRID PARAMETERS
*************************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
 Generate debug output? (y/n)
>>n
 Loading CSF file ... Header only
 There are/is
                      29 relativistic subshells;
 Loading CSF File for ALL blocks
 There are
                  385 relativistic CSFs... load complete;
 Change the default speed of
 light or radial grid parameters? (y/n)
>>v
 Speed of light =
                   137.03599913900001
                                          ; revise ?
>>n
 The default radial grid parameters for this case are:
          2.1739130434782606E-008
        5.00000000000003E-002
  HP =
         0.0000000000000000
             1990
  revise these values?
>>y
```

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```
Enter RNT:
>>2.17d-08
 Enter H:
>>1.5d-02
 Enter HP:
>>0.0d0
 Enter N:
>>1990
 Revised RNT =
                  2.16999999999999E-008
                  1.4999999999999E-002
 Revised H =
 Revised HP =
                  0.0000000000000000
 Revised N
                       1990
 Revise the default ACCY =
                              1.5625000000000006E-008
>>n
 Loading Radial WaveFunction File ...
 There are
                     11 blocks (block
                                          J/Parity NCF):
                        2
                                                   2-
                                                                          3-
  1
       0-
               13
                             1-
                                     35
                                              3
                                                           51
                                                                                  61
  5
       4-
               61
                        6
                             5-
                                     54
                                              7
                                                   6-
                                                           44
                                                                     8
                                                                          7-
                                                                                  31
       8-
               19
                       10
                             9-
                                                   10-
                                     11
                                             11
                                                            5
 Enter ASF serial numbers for each block
 Block
                  1
                       ncf =
                                       13 id =
                                                   0-
>>1-13
                                       35 id =
Block
                  2
                       ncf =
                                                   1-
>>1-35
 Block
                  3
                       ncf =
                                       51 id =
                                                   2-
>>1-51
                       ncf =
                                       61 id =
                                                   3-
Block
>>1-61
 Block
                  5
                       ncf =
                                       61 id =
>>1-61
Block
                  6
                       ncf =
                                       54 id =
>>1-54
 Block
                  7
                       ncf =
                                       44 id =
                                                   6-
>>1-44
 Block
                  8
                       ncf =
                                       31 id =
                                                   7-
>>1-31
 Block
                  9
                       ncf =
                                       19
                                          id =
                                                   8-
>>1-19
Block
                 10
                       ncf =
                                       11 id =
                                                   9-
>>1-11
                 11
                                        5
 Block
                       ncf =
                                          id =
                                                   10-
>>1-5
 level weights (1 equal; 5 standard; 9 user)
>>5
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f 5s 5p- 5p 5d- 5d 6s 6p-
 6p 5f- 5f 6d- 6d 7s
 Enter orbitals to be varied (Updating order)
>>*
 Which of these are spectroscopic orbitals?
 Enter the maximum number of SCF cycles:
>>100
```

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```
Modify other defaults? (y/n)
>>n
Orthonomalization order?
1--Update order
2--Self consistency connected
>>1

......

Wall time:
98 seconds

Finish Date and Time:
Date (Yr/Mon/Day): 2018/11/26
Time (Hr/Min/Sec): 23/43/02.595
Zone: +0100

RMCDHF: Execution complete.
```

# 13.6. Correlation Orbitals Not Converging

For large orbital sets it may happen that the higher layers of correlation orbitals, not spectroscopic orbitals, do not converge. This is often due to the fact that the initial orbital estimates, either Thomas-Fermi or screened hydrogenic, options 2 and 3 for rwfnestimate, are localized too far out in relation to the region where the spectroscopic orbitals reside. In these cases, the user can use option 4 in rwfnestimate and increase Z so that the orbitals are contracted and overlap the desired region, see Section 6.8 for an example of the use of the option.

## 14. Managing Large Expansions

14.1. Rearrange Lists of CSFs into Zero- and First-Order Spaces

Sometimes the CSF expansions get so large that they cannot be handled by the normal SCF procedure in the rmcdhf program. In these cases, an approximate optimization scheme can be employed in which the CSF list is rearranged into zero- and a first-order spaces:

$$\underbrace{\Phi(\gamma_1^0 PJ), \Phi(\gamma_2^0 PJ), \dots, \Phi(\gamma_M^0 PJ)}_{\text{zero-order space}}, \underbrace{\Phi(\gamma_1^1 PJ), \Phi(\gamma_2^1 PJ), \dots, \Phi(\gamma_N^1 PJ)}_{\text{first-order space}}$$

where M+N is the total number of CSFs in the original list. The zero-order space contains the most important CSFs, while the first-order space contain less important CSFs that can be regarded as minor corrections. Normally  $M \ll N$ . Associated with the rearrangement of the CSFs is a decomposition of the Hamiltonian interaction matrix in submatrices

$$\left(\begin{array}{cc} H^{(PP)} & H^{(PQ)} \\ H^{(QP)} & H^{(QQ)} \end{array}\right),$$

The energy expression, on which to optimize, is now obtained from the limited interaction matrix where the full  $H^{(PP)}$ ,  $H^{(QP)}$ ,  $H^{(QP)}$  submatrices are included (interactions within the zero-order space and between the zero- and first-order spaces) but only the diagonal part of  $H^{(QQ)}$ . The rearrangement of the list of CSFs in zero- and first-order spaces is done by the program rcsfzerofirst.

As an example, we use zero- and first-order spaces for a calculation of the states belonging to the  $3s^23p^2$  configuration in Si-like iron. The calculation accounts for valence–valence and core–valence correlation and is based on a MR of the form  $\{3s^23p^2, 3s3p^23d, 3p^4\}$ .

#### Overview

Define nuclear data.

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- 2. Obtain common orbitals for the  $\{3s^23p^2, 3s3p^23d, 3p^4\}$  MR set from DHF
  - (a) Generate list of CSFs for MR
  - (b) Perform angular integration.
  - (c) Generate initial estimates of radial orbitals.
  - (d) Perform SCF calculation on the weighted average of the  $3s^23p^2$  states.
  - (e) Save output to mr.
- 3. Improve the states using the zero- and first-order method where only part of the interactions are retained
  - (a) Generate n = 4 valence–valence and core–valence CSF expansion from the MR
  - (b) Rearrange CSFs in zero- and first-order spaces using rcsfzerofirst
  - (c) Perform angular integration.
  - (d) Generate initial estimates of radial orbitals.
  - (e) Perform SCF calculation on the weighted average of the  $3s^23p^2$  states.
  - (f) Save output to zerofirst\_n4

This program creates a list of CSFs

(g) Perform rci calculations in which the transverse photon interaction (Breit) and vacuum polarization and self-energy (QED) corrections are added.

```
***************************
         RUN RNUCLEUS TO GENERATE NUCLEAR DATA AND DEFINE RADIAL GRID
         OUTPUT FILE: isodata
*************************************
>>rnucleus
 RNUCLEUS
 This program defines nuclear data and the radial grid
 Outputfile: isodata
 Enter the atomic number:
>>26
 Enter the mass number (0 if the nucleus is to be modelled as a point source:
>>56
 The default root mean squared radius is
                                         3.7376999855041504
                                                                    (Angeli)
                                                               fm;
   the default nuclear skin thickness is
                                         2.299999999999998
                                                               fm:
 Revise these values?
>>n
 Enter the mass of the neutral atom (in amu) (0 if the nucleus is to be static):
 Enter the nuclear spin quantum number (I) (in units of h / 2 pi):
>>1
 Enter the nuclear dipole moment (in nuclear magnetons):
>>1
 Enter the nuclear quadrupole moment (in barns):
>>1
         RUN RCSFGENERATE TO GENERATE LIST OF CSFs
         FOR THE MULTIREFERNCE
         OUTPUT FILES: rcsfgenerate.log, rcsf.out
>>rcsfgenerate
 RCSFGENERATE
```

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```
Configurations should be entered in spectroscopic notation
 with occupation numbers and indications if orbitals are
 closed (c), inactive (i), active (*) or has a minimal
 occupation e.g., 1s(2,1)2s(2,*)
 Outputfiles: rcsf.out, rcsfgenerate.log
 Default, reverse, symmetry or user specified ordering? (*/r/s/u)
 Select core
       0: No core
       1: He (
                   1s(2)
                                          = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                          = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                          = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                         = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                         = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
 Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
 Give configuration
>>2s(2,i)2p(6,i)3s(2,i)3p(2,i)
 Give configuration
>>2s(2,i)2p(6,i)3s(1,i)3p(2,i)3d(1,i)
 Give configuration
>>2s(2,i)2p(6,i)3p(4,i)
 Give configuration
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>3s,3p,3d
Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
      block J/P
                           NCSF
              0+
                              9
          2
                             15
             1+
          3
               2+
**********************************
         COPY FILES
         IT IS ADVISABLE TO SAVE THE rcsfgenerate.log FILE TO HAVE A
         RECORD ON HOW THE LIST OF CSFs WAS CREATED
*************************************
```

>>cp rcsfgenerate.log mr.exc

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>>cp rcsf.out rcsf.inp

```
*********************************
        RUN RANGULAR TO GENERATE ENERGY EXPRESSION
        INPUT FILE : rcsf.inp
        OUTPUT FILES: rangular.alog, mcp.30, mcp.31,....
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
            rangular.log
Full interaction? (y/n)
>>y
 . . . . .
RANGULAR: Execution complete.
*********************************
        RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS
        INPUT FILES: isodata, rcsf.inp, previous rwfn files
        OUTPUT FILE: rwfn.inp, rwfnestimate.log
***********************************
>>rwfnestimate
RWFNESTIMATE
This program estimates radial wave functions
for orbitals
Input files: isodata, rcsf.inp, optional rwfn file
Output file: rwfn.inp
Default settings ?
>>y
Loading CSF file ... Header only
There are/is
                     9 relativistic subshells;
The following subshell radial wavefunctions remain to be estimated:
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
    2--Thomas-Fermi
    3--Screened Hydrogenic
    4--Screened Hydrogenic [custom Z]
>>2
Enter the list of relativistic subshells:
>>*
All required subshell radial wavefunctions have been estimated:
```

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```
Shell
                     p0
                               gamma
                                           <r>
                                                   MTP
                                                        SRC
  1s
      0.3024D+03 0.2944D+03 0.1000D+01 0.5776D-01
                                                   329
                                                        T-F
  2s
      0.5961D+02 0.1001D+03 0.1000D+01 0.2416D+00
                                                   346
                                                        T-F
  2p- 0.5782D+02 0.7556D+00 0.1000D+01 0.2037D+00
                                                   346
                                                        T-F
      0.5718D+02 0.6771D+03 0.2000D+01 0.2062D+00
                                                   346
                                                        T-F
      0.2068D+02 0.4969D+02 0.1000D+01 0.5885D+00
                                                   359
                                                        T-F
  3p- 0.1992D+02 0.4010D+00 0.1000D+01 0.5568D+00
                                                   359
                                                        T-F
      0.1976D+02 0.3607D+03 0.2000D+01 0.5608D+00
                                                   359
                                                        T-F
  ďΣ
  3d- 0.1847D+02 0.5063D+00 0.2000D+01 0.4838D+00
                                                   359
                                                        T-F
      0.1842D+02 0.5695D+03 0.3000D+01 0.4852D+00
 RWFNESTIMATE: Execution complete.
***********************************
         RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 1s, 2s, 2p, 3s, 3p, 3d AND THEY ARE
         ALL SPECTROSCOPIC. WE CAN USE WILD CARDS FOR SPECIFYING ORBITALS
         NOTE WE HAVE ASKED FOR 900 ITERATIONS
***********************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
                       9 relativistic subshells;
 There are/is
 Loading CSF File for ALL blocks
                   44 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    3 blocks (block
                                       J/Parity
                                                  NCF):
      0+
                    2
                               15
                         1+
                                            2+
                                                  20
 Enter ASF serial numbers for each block
 Block
                     ncf =
                                     9 id =
>>1,2
Block
                     ncf =
                                    15 id =
>>1
Block
                 3
                     ncf =
                                    20 id =
                                                2+
>>1,2
 level weights (1 equal; 5 standard; 9 user)
>>5
```

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```
Radial functions
1s 2s 2p- 2p 3s 3p- 3p 3d- 3d
Enter orbitals to be varied (Updating order)
>>*
Which of these are spectroscopic orbitals?
>>*
Enter the maximum number of SCF cycles:
>>900
RMCDHF: Execution complete.
**********************************
       RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum
                                   name.alog, name.log
**********************************
>>rsave mr
Created mr.w, mr.c, mr.m, mr.sum, mr.alog and mr.log
RUN RCSFGENERATE TO GENERATE LIST OF CSFs
        ACCOUNTING FOR VALENCE-VALENCE AND CORE-VALENCE CORRELATION
        OUTPUT FILES: rcsfgenerate.log, rcsf.out
>>rcsfgenerate
RCSFGENERATE
This program creates a list of CSFs
Configurations should be entered in spectroscopic notation
with occupation numbers and indications if orbitals are
closed (c), inactive (i), active (*) or has a minimal
occupation e.g., 1s(2,1)2s(2,*)
Outputfiles: rcsf.out, rcsfgenerate.log
Default, reverse, symmetry or user specified ordering? (*/r/s/u)
>>*
Select core
       0: No core
       1: He (
                  1s(2)
                                      = 2 electrons)
       2: Ne ([He] + 2s(2)2p(6)
                                      = 10 electrons)
       3: Ar ([Ne] + 3s(2)3p(6)
                                      = 18 electrons)
       4: Kr ([Ar] + 3d(10)4s(2)4p(6)
                                      = 36 electrons)
       5: Xe ([Kr] + 4d(10)5s(2)5p(6)
                                      = 54 electrons)
       6: Rn ([Xe] + 4f(14)5d(10)6s(2)6p(6) = 86 electrons)
>>1
Enter list of (maximum 100) configurations. End list with a blank line or an asterisk (*)
Give configuration
>>2s(2,i)2p(6,5)3s(2,*)3p(2,*)
Give configuration
```

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```
>>2s(2,i)2p(6,5)3s(1,*)3p(2,*)3d(1,*)
 Give configuration
>>2s(2,i)2p(6,5)3p(4,*)
 Give configuration
                          4
 Give set of active orbitals, as defined by the highest principal quantum number
 per 1-symmetry, in a comma delimited list in s,p,d etc order, e.g., 5s,4p,3d
>>4s,4p,4d,4f
 Resulting 2*J-number? lower, higher (J=1 -> 2*J=2 etc.)
>>0,4
 Number of excitations (if negative number e.g., -2, correlation
 orbitals will always be doubly occupied)
>>2
 Generate more lists ? (y/n)
>>n
       . . . . . . . . .
  3 blocks were created
      block J/P
                         NCSF
         1
             0+
                         4720
         2
              1+
                        12774
              2+
                        17554
**************************************
************************************
>>cp rcsf.out rcsf.inp
 ***************************
        RUN RCSFZEROFIRST TO ARRANGE LIST
  **************************
>>rcsfzerofirst
 RCSFzerofirst: Takes a list of CSFs and partitions each symmetry
              block into a zero- and first-order CSF space from
              a zero-order list.
                  Copyright by G. Gaigalas and Ch. F. Fischer
              (Fortran 95 version)
                                              NIST (2017).
              Input files:
                             list with CSFs to be partitioned
                             list with CSFs defining
                                        the zero-order space
              Output file:
                             rcsf.out
 Give the full name of the list that contains the zero-order space
mr.c
 Give the full name of the list that should be partitioned
rcsf.inp
```

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```
Loading Configuration Symmetry List File ...
There are 16 relativistic subshells;
  Block
         Zero-order Space
                        Complete Space
   1
                  9
                               4720
                               12774
   2
                 15
   3
                 20
                              17554
RCSFzerofirst: Execution complete.
**************************************
       COPY FILES
>>cp rcsf.out rcsf.inp
RUN RANGULAR TO GENERATE ENERGY EXPRESSION
       INPUT FILE : rcsf.inp
       OUTPUT FILES: rangular.alog, mcp.30, mcp.31,...
       NOTE EXECUTION VERY FAST SINCE WE DO NOT INCLUDE ALL INTERACTIONS
*********************************
>>rangular
RANGULAR
This program performs angular integration
Input file: rcsf.inp
Outputfiles: mcp.30, mcp.31, ....
          rangular.log
Full interaction? (y/n)
>>n
Block
             1 , ncf =
                            4720
             2 , ncf =
Block
                           12774
             3 , ncf =
                           17554
Loading CSF file ... Header only
There are/is
                  16 relativistic subshells;
The contribution of CSFs 1--ICCUT will be treated variationally;
the remainder perturbatively; enter ICCUT:
Give ICCUT for block
>>9
Give ICCUT for block
>>15
Give ICCUT for block
>>20
RANGULAR: Execution complete.
RUN RWFNESTIMATE TO GENERATE INITIAL ESTIMATES FOR RADIAL ORBITALS *
```

\* INPUT FILES: isodata, rcsf.inp, previous rwfn files

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```
OUTPUT FILE: rwfn.inp, rwfnestimate.log
**********************************
>>rwfnestimate
 RWFNESTIMATE
 This program estimates radial wave functions
 for orbitals
 Input files: isodata, rcsf.inp, optional rwfn file
 Output file: rwfn.inp
 Default settings ?
>>y
 Loading CSF file ... Header only
 There are/is
                       16 relativistic subshells;
 The following subshell radial wavefunctions remain to be estimated:
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
    1--GRASP2K File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>1
 Enter the file name (Null then "rwfn.out")
 Enter the list of relativistic subshells:
 The following subshell radial wavefunctions remain to be estimated:
 4s 4p- 4p 4d- 4d 4f- 4f
 Read subshell radial wavefunctions. Choose one below
     1--GRASP92 File
     2--Thomas-Fermi
     3--Screened Hydrogenic
     4--Screened Hydrogenic [custom Z]
>>2
 Enter the list of relativistic subshells:
 All required subshell radial wavefunctions have been estimated:
Shell
                                                    MTP SRC
                      p0
          е
                               gamma
                                            <r>
  1s
      0.2768D+03 0.2922D+03 0.1000D+01 0.5839D-01 358
                                                         rwf
  2s
     0.4499D+02 0.9142D+02 0.1000D+01 0.2600D+00
                                                    361
                                                         rwf
  2p- 0.4040D+02 0.6302D+00 0.1000D+01 0.2307D+00
                                                     359
                                                         rwf
      0.3993D+02 0.5641D+03 0.2000D+01 0.2336D+00
  2p
                                                     359
                                                         rwf
  3s
      0.1448D+02 0.3728D+02 0.1000D+01 0.7068D+00
                                                    364
                                                         rwf
  3p- 0.1327D+02 0.2857D+00 0.1000D+01 0.6992D+00
                                                    364
                                                         rwf
  Зр
      0.1318D+02 0.2569D+03 0.2000D+01 0.7038D+00
                                                    364
                                                         rwf
  3d- 0.1475D+02 0.1697D+00 0.2000D+01 0.6708D+00 364
                                                         rwf
  3d
      0.1477D+02 0.1889D+03 0.3000D+01 0.6732D+00
                                                    364
     0.9572D+01 0.2915D+02 0.1000D+01 0.1142D+01
  4s
                                                    368
                                                         T-F
  4p- 0.9220D+01 0.2388D+00 0.1000D+01 0.1123D+01
                                                    368 T-F
      0.9167D+01 0.2151D+03 0.2000D+01 0.1129D+01 368 T-F
  4p
```

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```
4d- 0.8577D+01 0.3386D+00 0.2000D+01 0.1074D+01 369
     0.8563D+01 0.3811D+03 0.3000D+01 0.1076D+01 369
                                                        T-F
  4f- 0.7863D+01 0.1681D+00 0.3000D+01 0.9589D+00 369 T-F
      0.7857D+01 0.2052D+03 0.4000D+01 0.9598D+00 369 T-F
 RWFNESTIMATE: Execution complete.
***********************************
         RUN RMCDHF TO OBTAIN SELF CONSISTENT SOLUTIONS
         INPUT FILES: isodata, rcsf.inp, rwfn.inp, mcp.30, mcp.31,...
         OUTPUT FILES: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
         NOTE: ORBITALS BUILDING REFERENCE STATES ARE REQUIRED TO HAVE
         THE CORRECT NUMBER OF NODES. THEY ARE REFERRED TO AS SPECTROSCOPIC
         ORBITALS. IN THIS RUN WE VARY 4s,4p,4d,4f AND THEY ARE ALL
         CORRELATION ORBITALS WITH NO NODE COUNTING
***********************************
>>rmcdhf
 RMCDHF
 This program determines the radial orbitals
 and the expansion coefficients of the CSFs
 in a self-onsistent field proceedure
 Input file: isodata, rcsf.inp, rwfn.inp, mcp.30, ...
 Outputfiles: rwfn.out, rmix.out, rmcdhf.sum, rmcdhf.log
 Default settings? (y/n)
>>y
 Loading CSF file ... Header only
 There are/is
                      16 relativistic subshells;
 Loading CSF File for ALL blocks
                35048 relativistic CSFs... load complete;
 Loading Radial WaveFunction File ...
 There are
                    3 blocks (block
                                       J/Parity NCF):
      0+ 4720
                    2
                         1+ 12774
                                            2+ 17554
 Enter ASF serial numbers for each block
 Block
                1 	 ncf =
                                 4720 \text{ id} =
>>1,2
 Block
                2 	 ncf =
                                 12774 id =
>>1
                3
 Block
                     ncf =
                                 17554 id =
                                                2+
>>1,2
level weights (1 equal; 5 standard; 9 user)
>>5
 Radial functions
 1s 2s 2p- 2p 3s 3p- 3p 3d- 3d 4s 4p- 4p 4d- 4d 4f- 4f
 Enter orbitals to be varied (Updating order)
 Which of these are spectroscopic orbitals?
Enter the maximum number of SCF cycles:
>>100
```

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. . . . . . . \* RUN RSAVE TO SAVE OUTPUT FILES: name.c, name.w, name.m, name.sum name.alog, name.log \* >>rsave zerofirst\_n4 Created zerofirst\_n4.w, zerofirst\_n4.c, zerofirst\_n4.m, zerofirst\_n4.sum zerofirst\_n4.alog and zerofirst\_n4.log \* RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS OUTPUT FILE: zerofirst\_n4.cm, zerofirst\_n4.csum, ...., rci.res THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH HTGH N. NOTE THAT THIS IS VERY FAST SINCE WE DO NOT INCLUDE ALL INTERACTIONS\* >>rci RCI This is the configuration interaction program Input file: isodata, name.c, name.w Outputfiles: name.cm, name.csum, name.clog, rci.res rci.res (can be used for restart) Default settings? >>n Name of state: >>zerofirst\_n4 1 , ncf =Block 4720 Block 2 , ncf = 12774 Block 3 , ncf =17554 Loading CSF file ... Header only There are/is 16 relativistic subshells; Restarting RCI ? >>n Revise the physical speed of light ( 137.03599913900001 in a.u.)? Treat contributions of some CSFs as first-order perturbations? >>y 3 blocks. They are: There are block J Parity No of CSFs 1 0+ 4720 12774 2 1+ 3 2+ 17554

4720 id =

0+

Enter iccut for each block

1

ncf =

Block

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```
>>9
                  2
                     ncf =
 Block
                                    12774 id =
                                                   1+
>>15
                  3
                       ncf =
                                    17554 id =
 Block
                                                   2+
>>20
 Include contribution of H (Transverse)?
>>y
 Modify all transverse photon frequencies?
 Include H (Vacuum Polarisation)?
>>y
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
 Estimate self-energy?
>>y
 Largest n quantum number for including self-energy for orbital
 n should be less or equal 8
>>3
 Loading Radial WaveFunction File ...
 There are
                      3 blocks (block
                                          J/Parity
                                                     NCF):
       0+
             4720
                        2 1+
                                 12774
                                              3
                                                   2+
                                                        17554
 Enter ASF serial numbers for each block
 Block
                  1 \quad \text{ncf} =
                                     4720 id =
>>1,2
 Block
                  2 ncf =
                                    12774 id =
                                                   1+
>>1
                  3 ncf =
                                    17554 id =
 Block
>>1,2
```

RCI: Execution complete.

Below we compare the energies from calculations with zero- and first-order spaces and limited interactions and calculations with full interaction in different combinations. In addition we show the energies from the rci run of the MR and the experimental energies from NIST.

Energies from the rci run with zero- and first-order spaces and wave functions from an rmcdhf calculation with zero- and first-order spaces:

No Po	s ,	J F	ari	ty Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	
1 1	. (	0	+	-1210.0270500			
2 1	. :	1	+	-1209.9855970	9097.89	9097.89	
3 1	. :	2	+	-1209.9431699	18409.56	9311.67	
4 2	: :	2	+	-1209.8062337	48463.58	30054.02	
5 2	: (	0	+	-1209.6073434	92114.95	43651.37	

Energies from the rci run with the full interaction and radial wave functions from an rmcdhf calculation with zero- and first-order spaces:

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No	Pos	J	Par	ity Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	
1	1	0	+	-1210.0256925			
2	1	1	+	-1209.9837680	9201.37	9201.37	
3	1	2	+	-1209.9412180	18540.02	9338.65	
4	2	2	+	-1209.8027763	48924.45	30384.43	
5	2	0	+	-1209.6000618	93415.14	44490.69	

Energies from an rci run with the full interaction and radial wave functions from an rmcdhf calculation with the full interaction:

No	Pos	J	Par	ity Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	
1	1	0	+	-1210.0257424			
2	1	1	+	-1209.9838159	9201.80	9201.80	
3	1	2	+	-1209.9412660	18540.41	9338.61	
4	2	2	+	-1209.8028394	48921.54	30381.14	
5	2	0	+	-1209.6001081	93415.93	44494.39	

Energies from an rci run for only the MR:

No Pe	os	J :	Par	ity Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	
1	1	0	+	-1209.9394925			
2	1	1	+	-1209.8995764	8760.57	8760.57	
3	1	2	+	-1209.8558763	18351.63	9591.06	
4	2	2	+	-1209.7026579	51979.19	33627.56	
5	2	0	+	-1209.4545733	106427.47	54448.28	

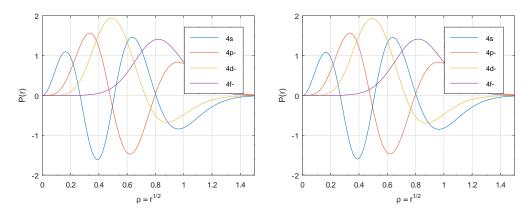
Experimental energies from NIST:

Configuration	Term	1	J	Level	.1
					-
	1				
3s2.3p2	3P	1	0	0.0	-
		1	1	9302.5	-
	1	1	2	18561.0	-
					-
3s2.3p2	1D	1	2	48068	-
		1	1		-
3s2.3p2	1S	1	0	91508	-

In Figure 12 we compare the 4s, 4p, 4d, 4f correlation orbitals from rmcdhf calculations with limited and full interactions, respectively. The differences between the orbitals are very small.

The conclusion of all this, energy tables and shapes of radial orbitals, is that a limited interaction rmcdhf calculation combined with full interaction rci recovers almost perfectly the result of a full interaction rmcdhf combined with full interaction rci.

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**Figure 12.** Plot of orbitals from an rmcdhf calculation using the full interaction matrix and an rmcdhf calculation with only part of the interaction.

## 14.2. Accumulating the Wave Function to a Specified Fraction

A very good way of selecting the zero-order space is to accumulate the wave function to a specified fraction of the squared weights. This is done by the following procedure:

1. Start from a calculation targeting one or more states, thus start from a number of ASFs

$$ASF_1: \Psi(\gamma_1 PJ) = \sum_{i=1}^{N} c_i^1 \Phi(\gamma_i PJ)$$

$$ASF_{M}: \Psi(\gamma_{M}PJ) = \sum_{i=1}^{N} c_{i}^{M} \Phi(\gamma_{i}PJ)$$

built from a set of CSFs.

2. For *i* from 1 to *N* compute

$$s_i = (c_i^1)^2 + (c_i^2)^2 + \ldots + (c_i^M)^2.$$

- 3. Sort  $s_1, s_2, \ldots, s_N$  in descending order.
- 4. Accumulate terms of *s* until a specified fraction of the total squared weight

$$M = s_1 + \ldots + s_M = \sum_{i,j} (c_i^j)^2$$

is attained.

The CSFs that are associated with the accumulated fraction can then be taken as the zero-order space. Alternatively, and dependent on the fraction, the method can be used to condense the list of CSFs.

Below are some different scenarios:

- 1. Perform some initial calculations. Use accumulation to a specified fraction to select the CSFs (configurations) in the MR. The selected CSFs can then also be used by rcsfinteract, see Section 5.5.
- 2. Perform large-scale calculations. To further push the calculations, use accumulation to a specified fraction to select the zero-order space.
- Perform large-scale calculations. Use accumulation to a specified fraction to condense the list of CSFs.

The accumulation to a specified fraction is done with the program rmixaccumulate.

As an example, we apply the accumulation to a specified fraction (0.9999 in this case) to the states defined in zerofirst\_n4. We use the accumulated list as the zero-order space and redo the rci calculation to see how big is the difference between the obtained energies and the energies from the full interaction calculation.

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\* RMIXACCUMULATE TO ACCUMULATE TO A SPECIFIED FRACTION INPUT FILE: zerofirst\_n4.c, zerofirst\_n4.cm OUTPUT FILE: rcsf.out \* WELCOME TO PROGRAM RMIXACCUMULATE Input files: <state>.(c)m, <state>.c Reduced CSF list is written to rcsf.out Give name of the state: >>zerofirst\_n4 Expansion coefficients resulting from CI calculation (y/n)? Fraction of total wave function [0-1] to be included in reduced list: >>0.9999 CSFs in output file sorted by mixing coefficients (y/n)? >>y Block data read from mixing file block 2j+1 parity ncf nev 1 4720 2 1 1 12774 1 3 3 17554 2 5 Number of CSF:s written to rcsf.out block ncf 1 188 2 396 712 For the different blocks we see that 188, 396, and 712 CSFs, respectively, contribute to 99.99% of the total squared weight. \* COPY FILES \* >>cp rcsf.out zero\_order\_0.9999.c \* RUN RCSFZEROFIRST TO ARRANGE LIST \* >>rcsfzerofirst RCSFzerofirst: Takes a list of CSFs and partitions each symmetry block into a zero- and first-order CSF space from a zero-order list. Copyright by G. Gaigalas and Ch. F. Fischer (Fortran 95 version) NIST (2017). Input files: list with CSFs to be partitioned

list with CSFs defining

rcsf.out

Output file:

the zero-order space

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```
Give the full name of the list that contains the zero-order space
>>zero_order_0.9999.c
 Give the full name of the list that should be partitioned
>>rcsf.inp
 Loading Configuration Symmetry List File ...
 There are 16 relativistic subshells;
  Block
          Zero-order Space
                          Complete Space
   1
                   188
                                    4720
   2
                   391
                                   12774
   3
                                   17554
                   712
 Wall time:
      11 seconds
 Finish Date and Time:
  Date (Yr/Mon/Day): 2018/07/31
  Time (Hr/Min/Sec): 11/56/50.926
  Zone: +0200
 RCSFzerofirst: Execution complete.
 ************************************
        COPY FILES
**********************************
>>cp rcsf.out zerofirst_0.9999.c
>>cp zerofirst_n4.w zerofirst_0.9999.w
RUN RCI TO INCLUDE TRANSVERSE PHOTON INTERACTION AND QED EFFECTS
        OUTPUT FILE: zerofirst_0.9999.cm, ..., rci.res
        THE TRANSVERSE PHOTON FREQUENCIES CAN BE SET TO THE LOW FREQUENCY
        LIMIT. RECOMMENDED IN CASES WHERE YOU HAVE CORRELATION ORBITALS
        THE SELF ENERGY CORRECTION MAY FAIL FOR CORRELATION ORBITALS WITH
        HIGH N.
        NOTE THAT THIS IS VERY FAST SINCE WE DO NOT INCLUDE ALL INTERACTIONS*
*************************************
 RCI
 This is the configuration interaction program
 Input file: isodata, name.c, name.w
 Outputfiles: name.cm, name.csum, name.clog
            rci.res (can be used for restart)
 Default settings?
>>n
 Name of state:
>>zerofirst_0.9999
 Block
               1 , ncf =
                                4720
 Block
               2 , ncf =
                               12774
               3 , ncf =
                               17554
 Loading CSF file ... Header only
```

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```
There are/is
                      16 relativistic subshells;
 Restarting RCI90 ?
 Revise the physical speed of light ( 137.03599913900001
                                                        in a.u.) ?
 Treat contributions of some CSFs as first-order perturbations?
>>y
 There are
                    3 blocks. They are:
           J Parity
  block
                       No of CSFs
              0+
                        4720
          1
          2
               1+
                      12774
          3
               2+
                      17554
 Enter iccut for each block
 Block
          1 	 ncf =
                                 4720 id =
                                                0+
>>188
                2 	 ncf =
                                  12774 id =
 Block
                                                1+
>>396
                 3 ncf =
                                  17554 id =
Block
                                                2+
>>712
 Include contribution of H (Transverse)?
>>y
Modify all transverse photon frequencies?
 Include H (Vacuum Polarisation)?
 Include H (Normal Mass Shift)?
 Include H (Specific Mass Shift)?
>>n
Estimate self-energy?
Largest n quantum number for including self-energy for orbital
n should be less or equal 8
Loading Radial WaveFunction File ...
                    3 blocks (block
                                        J/Parity NCF):
 There are
      0+
            4720
                       2
                               12774
                                         3
                                                2+ 17554
 1
                           1+
 Enter ASF serial numbers for each block
Block
                1 	 ncf =
                               4720 id =
>>1,2
Block
               2 	 ncf =
                                 12774 id =
>>1
Block
                 3 	 ncf =
                                  17554 id =
>>1,2
 Finish time, Statistics
 Wall time:
```

95 seconds

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Finish Date and Time:

Date (Yr/Mon/Day): 2018/07/31 Time (Hr/Min/Sec): 12/03/43.197

Zone: +0200

RCI: Execution complete.

Below we display the energies from the rci run with the zero-order space from an accumulation to 0.9999.

35048 16 nblock =ncftot = nw = nelec = 14

Energy levels for ...

Rydberg constant is 109737.31569

No - Serial number of the state; Pos - Position of the state within the J/P block; Splitting is the energy difference with the lower neighbor

No	Pos	J	Par	ity Energy Total (a.u.)	Levels (cm^-1)	Splitting (cm^-1)	
1	1	0	+	-1210.0257297			
2	1	1	+	-1209.9838005	9202.40	9202.40	
3	1	2	+	-1209.9412600	18538.95	9336.55	
4	2	2	+	-1209.8028337	48920.02	30381.07	

93408.50

We see that with a larger zero-order space, we now have energies in very good agreement with the ones from an rci calculation with full interaction. In this example, we did two rci calculations. The first was with a very small zero-order space in terms of the MR. We then used this calculation to accumulate to a defined fraction. By redoing the rci with the new zero-order space, we get energies that are very close to the ones from a full interaction calculation. For large expansions, two calculations with limited interaction are much faster than one calculation with full interaction.

44488.48

## 14.3. Computational Strategies Using Zero- and First-Order

-1209.6001292

Based on the experience from a number of studies, we suggest the following computational strategy for large cases:

- The MR is always generated using full interaction 1.
- 2. To run rmcdhf for an expansion that is large:
  - Start by running rmixaccumulate with 0.99 or something similar on an expansion you have that is not too large, e.g., an expansion based on just one or two orbital layers.
  - (b) Generate your large expansion and run rcsfinteract to make sure you only retain CSFs that interact with the CSFs of the MR.
  - (c) Run rcsfzerofirst zero-order—output from rmixaccumulate with 0.99 (or something similar) list to be partitioned—output from rcsfgenerate (step above)
  - (d) Run rangular with ICCUT values for the size of the zero-order expansion from rmixaccumulate
  - (e) Run rmcdhf in the usual way. Due to the fact that limited interaction is included in the angular integration, the rmcdhf calculation will be fast.
- Run rci for the large expansion with full interaction. 3.
- For very large expansions, consider performing the rci calculation with the expansion from the previous layer as a zero-order space or the expansion from the previous layer accumulated to a high fraction, say 0.99999999, as the zero-order space. Alternatively, run rci with a small zero-order space and accumulate to some fraction and use this list as a new zero-order space and redo the rci calculation.

Please remember that all strategies are dependent on the atomic system at hand, and that some explorations of the fractions used for rmixaccumulate are needed. See [28] for one application of the zero- and first-order strategy.

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## 15. Learn More about Computational Atomic Structure and GRASP

To learn more about computational atomic structure and the use of GRASP, the reader is encouraged to consult <a href="https://github.com/compas">https://github.com/compas</a>, accessed on 5 November 2022. Here, in addition to the GRASP code, there is an extensive list of books and articles that provide the theoretical background to multiconfiguration methods, electron correlation, and the systematic computation of different atomic properties with GRASP in real- and large-scale applications. The list is constantly updated to cover the latest studies.

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Conflicts of Interest: The authors declare no conflict of interest.

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