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Assessment of Enhanced Dempster-Shafer Theory for Uncertainty Modeling in a GIS-Based Seismic Vulnerability Assessment Model, Case Study—Tabriz City

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Abstract: Earthquake is one of the natural disasters which threaten many lives every year. It is impossible to prevent earthquakes from occurring; however, it is possible to predict the building damage, human and property losses in advance to mitigate the adverse effects of the catastrophe. Seismic vulnerability assessment is a complex uncertain spatial decision making problem due to intrinsic uncertainties such as lack of complete data, vagueness in experts' comments and uncertainties in the numerical data/relations. It is important to identify and model the incorporated uncertainties of seismic vulnerability assessment in order to obtain realistic predictions. Fuzzy sets theory can model the vagueness in weights of the selected criteria and relationships of the criteria with building damage. Dempster's combination rule is useful for fusion of information on the vulnerability of the buildings which leads to decreased uncertainty of the results. However, when there is a conflict among information sources, classical Dempster rule of combination is not efficient. This paper analyses the uncertainty sources in a geospatial information system (GIS)-based seismic vulnerability assessment of buildings and then focuses on assessing the efficiency of Dempster rule of combination in the fusion of the information sources for the seismic vulnerability assessment. Tabriz, a historical and earthquake prone city in the north west of Iran was selected as the study area. The results verified that some inconsistencies among information sources exist which are important to be considered while proposing a method for the fusion of the information in order to obtain vulnerability assessments with less uncertainty. Based on the assessed building damage, the number of probable victims was estimated. The produced physical and social seismic vulnerability maps provide the required information for urban planners and administrators to reduce property and human losses through pre-earthquake mitigation and preparedness plans efficiently.

Keywords: Uncertainty; Dempster-Shafer Theory; Fuzzy sets; Seismic vulnerability assessment; Population loss; GIS

1. Introduction

Earthquake is one of the natural disasters that causes severe physical, social and financial damages around the world every year. Seismic vulnerability assessment is used to determine the likely effects of the hazards on human beings and property within a particular area [1]. If the catastrophic effects of earthquakes are calculated in advance, the human and property losses can be reduced through suitable and timely planning in mitigation and preparedness stage. However, it is crucial to consider the incorporated uncertainties in seismic vulnerability assessment to obtain realistic information and thus efficiently reduce potential future losses [2]. This research firstly studies the knowledge-based uncertainties associated with a geographic information system (GIS)-based model for assessing building damage (also called physical seismic vulnerability assessment (PSVA)) emphasizing on the involved inconsistency. After modelling buildings seismic vulnerability emphasizing on considering and subsuming the incorporated epistemic uncertainties, the population loss, that is, human beings exposed to be killed or injured, considering the assessed building damage are estimated.

There are many areas having high seismic risk in the world that lack predefined reliable seismic damage relations and/or vulnerability codes for existing structures. It is very important to propose simple, efficient and easily applicable methods for physical and social seismic vulnerability assessment as a decision support tool in such earthquake prone areas considering and subsuming the incorporated uncertainties [3]. To handle the incorporated knowledge-based uncertainties in seismic vulnerability assessment efficiently and realistically, integration of fuzzy sets theory (FST) and Dempster-Shafer theory (DST) has been introduced [2]. However, considering that Dempster combination rule is not efficient when evidences conflict, Shafer discounting rule was used through information fusion. This paper contributes to investigating the role of Shafer discounting rule in combining information sources in a seismic vulnerability assessment model using Dempster combination rule. The innovation of this research comparing to other research is that it considers and classifies the uncertainty types involved in a GIS-based multi-criteria decision making model for PSVA. It focuses on assessing the role of inconsistency in a proposed PSVA model that uses integration of Fuzzy sets and DS theories for handling the uncertainties. In this paper by comparing the results of combining the discounted beliefs using enhanced Dempster combination rule (i.e., using Shafer discounting coefficients) to the results of combining the information sources without the coefficients, the impact of inconsistency in the fusion process would be specified.

The scenario in this study follows; firstly severity of damages to the buildings using Dempster's combination rule are estimated. Second, the results are compared to the outputs of enhanced Dempster combination rule using Shafer discounted beliefs. As a result, the importance of using discounting coefficients in the seismic vulnerability assessment model is specified. Afterwards the estimates of the number of injured/ killed people based on the possible building damage (also called social vulnerability) is assessed.

There are studies on estimating physical and social seismic vulnerabilities in urban areas in the literature ([e.g., References [3–9]). Some of the studies in the literature use existing analytical relations or vulnerability indices in a spatial decision support system (SDSS) framework where the pre-defined vulnerability codes for buildings are available for estimating buildings' damages ([e.g., References [8,10–17]). However, a few studies have considered estimating building damage as a decision making problem concerning the necessity of PSVA in earthquake prone areas that lack required data for defining mathematical vulnerability relations/ indices (the mentioned data include details of structural properties of existing buildings and/or the statistical data of building damage from previous earthquakes) [18–21]. However, decision making for estimating building damage involves some uncertainties [22]. Limited number of studies have focused on dealing with intrinsic uncertainties of multiple decision making for assessing seismic vulnerability of buildings ([e.g., References [2,23–26]). This paper focuses on studying one of the epistemic uncertainties in assessing seismic vulnerability of buildings namely inconsistency in a multi criteria decision making framework using fuzzy sets and Dempster-Shafer theories implemented on vector datasets.

The remaining parts of this paper are organized as follows—Section 2 describes the proposed methods; Section 3 discusses the implementation and results and Section 4 provides concluding remarks.

2. Materials and Methods

Based on the nature of uncertainties they are categorized into two main categories; aleatory and epistemic uncertainties [15,27–30]. Aleatory uncertainty is related to the intrinsic randomness of phenomena while epistemic uncertainty is related to the lack of knowledge about a system. This paper aims to investigate the epistemic uncertainties in the seismic vulnerability assessment of buildings with the purpose of reducing the subsumed uncertainties.

Considering the sources of uncertainty, different classifications are introduced (e.g., References [31–33]). Reference [31] Categorized epistemic uncertainties into two main classes namely fuzziness and ambiguity, where the ambiguity is then re-classified into non-specificity and discord. Also different aspects of each type of uncertainty are determined, for example, *vagueness* is introduced as one of the aspects of fuzziness and *conflict* is considered as an aspect of discord uncertainty. In this paper the involved epistemic uncertainties of seismic vulnerability assessment in areas where there is incomplete data are considered based on the classification determined in Reference [31]. Figure 1 shows the involved epistemic uncertainties and the solutions proposed by the authors for reducing the uncertainties.

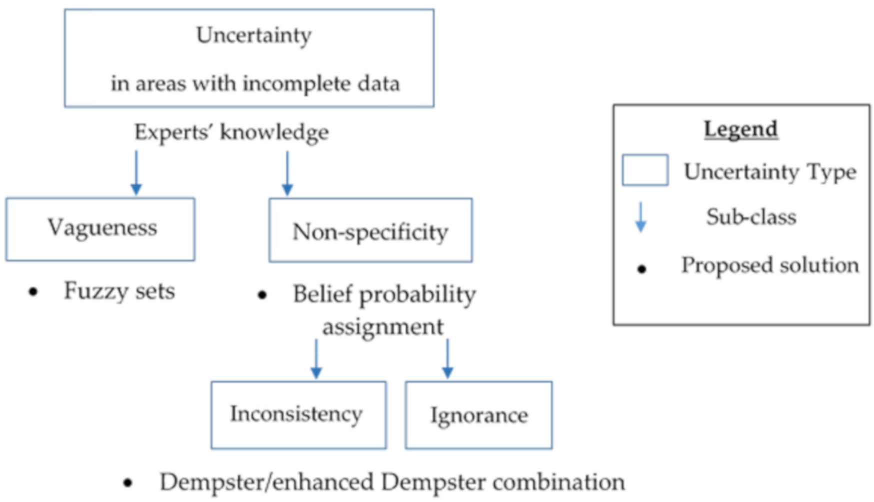

Figure 1. Analysis of incorporated epistemic uncertainties and the methods proposed for modelling them [34].

This paper assesses the influence of modelling inconsistency on seismic vulnerability assessment using fuzzy sets and Dempster-Shafer theory.

2.1. Fuzzy Sets Theory

The Fuzzy sets theory [35] focuses on representing and managing the vague information. A fuzzy set is essentially a set whose members have degrees of membership between 0 and 1, opposing to a crisp set in which each member must have either the membership degree of 0 or 1. A fuzzy set definition follows. Let X be a universe of discourse, A is a fuzzy subset of X if for all $x \in X$ there is a number $\mu_A(x) \in [0,1]$ assigned to represent the membership of x to A, $\mu_A(x)$ is called the membership of A [36].

A triangular fuzzy number A can be defined by a triplet (a, b, c) where a, b, c are real numbers (Figure 2) or a trapezoid. The membership function for Figure 2 is defined as Equation (1).

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{b-a} & a \leq x \leq b \\ \frac{(x-c)}{b-c} & b \leq x \leq c \\ 0 & otherwise \end{cases} \quad (1)$$

Linguistic variables represented as words or sentences are useful to describe the ill-defined situations that cannot be reasonably described quantitatively. The variables can be modelled by triangular or trapezoidal fuzzy numbers.

In this study fuzzy sets theory is used due to the vagueness or imprecision of severity of damages to the buildings caused by the influencing criteria, given by the experts using linguistic variables.

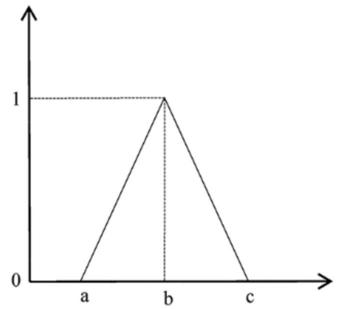

Figure 2. A triangular fuzzy number.

2.2. Dempster- Shafer Theory

Dempster Shafer theory of evidence (DST) [37,38] is a theory of uncertainty management. Dempster-Shafer Theory (DST) is the mathematical theory of evidence introduced by Dempster [37] which is extended by Shafer [38]. It is a powerful tool for data integration and management of uncertainty [29]. DST can be regarded as a general extension of Bayesian theory that is used for integrating data acquired from independent sources as well as dealing with incomplete data [39]. Fusion is a solution to obtain more reliable results in case we have uncertain and incomplete data.

DST or evidence theory, first considers frame of discernment defined as a finite set of hypotheses. It is composed of N exhaustive and exclusive hypotheses defined as follows (Equation (2)).

$$\Theta = \{H_1, \mathrm{H}_2, \ldots, H_N\}, \tag{2}$$

where Θ is a set of hypotheses called frame of discernment. A key point of evidence theory is the basic probability assignment (BPA). For any subset of frame of discernment $\mathrm{H} = \{\mathrm{H}_1, \mathrm{H}_2, \ldots, \mathrm{H}_\mathrm{N}\}$, a BPA (also called mass function) could be defined as a function from $\mathrm{P}(\Theta)$ to [0,1] having the following properties [40] (Equation (3)).

$$m\text{: } \mathrm{P}(\Theta) \rightarrow [0,1]$$

$$\Sigma_{A\in \mathrm{P}(\Theta)}\, \mathrm{m(A)} = 1,\ \mathrm{m}(\varnothing) = 0. \tag{3}$$

Each subset $\mathrm{A}\subset\Theta$ such as $\mathrm{m}_\mathrm{j}\,(\mathrm{A}) > 0$ is called a focal element of m. The $\mathrm{m}_\mathrm{j}(\mathrm{A})$ shows how strongly the evidence supports A. DST provides a function to combine two Basic Probability Assignments (BPAs also called mass functions) m_1 and m_2 to yield a new BPA with decreased uncertainty [37] (Equation (4)).

$$m(A) = \frac{\sum_{B\cap C=A} m_1\,(B) m_2(C)}{1-k} \quad when\ A \neq \varnothing \qquad k = \sum\nolimits_{B\cap C=\varnothing} m_1(B) m_2(C), \tag{4}$$

where B and C are two focal elements of m_1 and m_2. K in Equation (4) is a normalization constant, k = 0 corresponds to the absence of conflict between m_1 and m_2 and k = 1 implies complete inconsistency between m_1 and m_2.

In situations where knowledge of experts (or subjective information) is used, the Dempster-Shafer (DS) belief structures are useful to represent uncertainty [41]. It is possible to use a combination rule to provide combined masses synthesizing the knowledge of different j information sources. Supposing $\mathrm{m}_1, \mathrm{m}_2 \ldots \mathrm{m}_\mathrm{j}$ as mass functions their combination using Dempster rule is defined as Equation (5) [37] where $\oplus$ represents the operator of combination. (Equation (5) is the extension of Equation (4) where there are more than two mass functions to be combined).

$$\mathrm{m} = \mathrm{m}_1 \oplus \mathrm{m}_2 \ldots \oplus \mathrm{m}_{\mathrm{j}.} \tag{5}$$

In this study the information sources considering the indicators of the vulnerability of buildings are combined using Dempster combination rule (see Section 3.3 for details).

2.3. Discounting Rule

It should be noted that when evidence highly conflicts with each other, the classical Dempster rule of combination is not efficient [42,43]. Different methods are proposed to deal with the combination problem when evidences conflict [42,44–47].

One of the efficient methods to handle conflict between information sources is using discounting rule introduced by Shafer [38] (see Equation (6)). Discounting coefficients are used to determine the strength of the reliability of the evidences. Let $Bel^{\alpha} : 2^{\Theta} \to [0,1]$ be a belief function and α $(0 \le \alpha \le 1)$ a discounting coefficient which represents the strength of reliability of the evidence, the discounted belief $Bel^{\alpha}(A)$ is defined as Equation (6).

$$\begin{gathered} Bel^{\alpha} : 2^{\Theta} \to [0,1]\,(0 \le \alpha \le 1) \\ Bel^{\alpha}(\Theta) = 1 \\ Bel^{\alpha}(A) = (1-\alpha).\,Bel(A), \forall A \subset \Theta \text{ and } A \ne \varnothing. \end{gathered} \tag{6}$$

The discounted beliefs are incorporated into the Dempster's rule of combination. In this paper, the influence of discounting coefficients on assessing seismic vulnerability is investigated.

2.4. The Proposed Method

In this paper two important aspects of seismic vulnerability assessment are considered; firstly the seismic vulnerability of buildings called physical seismic vulnerability assessment and second the population loss caused by damages to the buildings or social vulnerability assessment. The flowchart of the proposed method is shown in Figure 3.

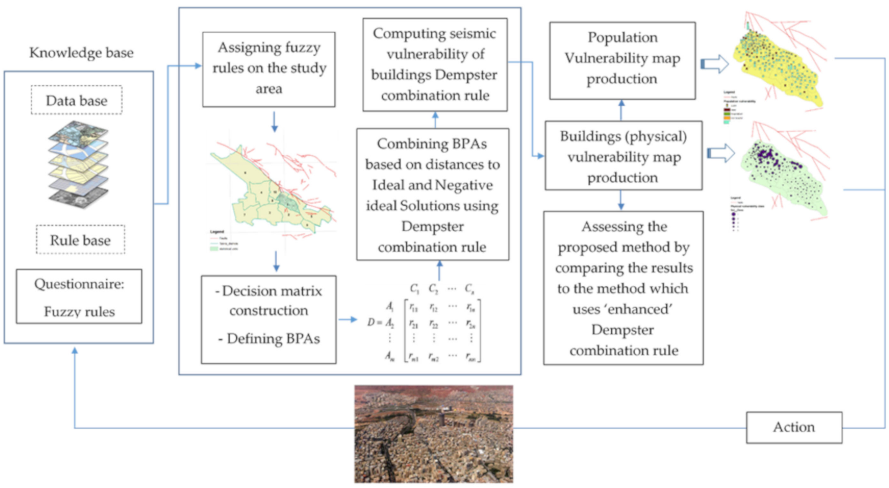

Figure 3. The flowchart of the proposed method.

As the first step, the main indicators of building damage are defined concerning the literature and experts' knowledge. In this regard seven influencing criteria are selected, including four criteria as ground motion intensity affecting indicators; namely *lithology*, *distance to faults*, *slope* and *ground water level in coarse loose soils* and three criteria as structural damages indicators including the *structure construction year*, *number of floors* and the *structural type* [22]. The impact of the selected main indicators (criteria) on the severity of the damages to the buildings are identified by the experts.

The knowledge base depicted in Figure 3 is developed containing a database and a rule base. The database contains the necessary spatial data for estimating the building damage. The rule base contains a set of rules defined by the experts on the effects of influencing criteria on the severity of the damages to the buildings as fuzzy rules using linguistic variables.

The next part of the proposed method is the inference engine (see Figure 3) in which the associated rules are assigned to each urban statistical unit's buildings considering the attributes of that unit. Based on the activated rules, a decision matrix is constructed in which each row is corresponding to one of the statistical units and each column is representing one of the influencing criteria. Each element of the matrix indicates seismic vulnerability of the buildings of the relevant unit (corresponding row) considering one of the criteria (corresponding column).

Considering the incompleteness of data in the study area, the DS combination rule is utilized to calculate the overall physical seismic vulnerability of the urban statistical units. For this purpose Ideal Solution (IS) and Negative ideal Solution (NS) are defined for each column of the decision matrix. In this context the statistical unit with *the lowest seismic vulnerability* is called '*IS*' (or the statistical unit which does not have priority in mitigation activities) and the statistical unit with *the highest vulnerability* in that column is called '*NS*.' The BPAs for the statistical units in the decision matrix are defined based on each unit's Euclidean distance to the IS and NS of the related column [48] (see Equation (7)). More detailed information on the calculation steps is given in Section 3.3.

$$\begin{aligned} m(NS) &= \frac{d\ (IS)}{d\ (IS) + d\ (NS) + d(ISNS)} \\ m(IS) &= \frac{d\ (NS)}{d\ (IS) + d\ (NS) + d(ISNS)} \\ m(ISNS) &= \frac{d\ (ISNS)}{d\ (IS) + d\ (NS) + d(ISNS)} \end{aligned} \tag{7}$$

Dempster combination rule using discounting coefficients [38] has been previously used to combine BPAs namely m (NS)s (or beliefs), m (IS)s (or disbeliefs) and m (ISNS)s (or ignorance uncertainties) [2]. In order to study the influence of *conflicts between evidences* in the combination process on the outputs of PSVA, in this paper the Shafer discounting rule coefficients are eliminated and the results of PSVA are compared to the outputs of using enhanced Dempster combination rule using Shafer discounting coefficients.

Considering that over 75% of fatalities in large-scale earthquakes are caused by building collapse (and excluding secondary disasters, approximately 90% of earthquake-related deaths are caused by building collapse) [49] in the next step of this study, the number of probable injured and/or killed people caused by the predicted damages to the buildings are estimated.

After the physical and social seismic vulnerabilities are assessed the produced information on estimated damages/losses are ready to be investigated by administrative managers in mitigation/preparedness phase. The required actions for reducing losses are planned in the action stage. These actions cause changes to the data which should be updated in the database. Therefore, seismic vulnerability for modified data can be assessed using proposed method. The outputs will represent both the influence of executed plans and the current vulnerable areas for further actions by urban planners and administrators.

2.5. Study Area

The study area selected for this research is Tabriz city, a seismic hazard prone metropolis in the north west of Iran. Tabriz is a historical city which is in need of seismic vulnerability assessment studies but lack predefined reliable seismic damage relations and/or vulnerability codes for existing structures. In this paper the proposed methods for studying the influence of conflicts between evidences in PSVA is successfully implemented in urban statistical units (identified by Iran Statistical Center (ISC)) of District One in Tabriz. Social seismic vulnerability assessment is then implemented to estimate population loss based on probable building damage. The outputs of seismic vulnerability assessment enable disaster managers to plan for population settlements, retrofit and strengthening of the buildings and other essential pre-earthquake mitigation programs.

3. Results and Discussion

The study area is a municipality district in Tabriz city. Tabriz population is around 1.6 million people and the city has a number of historical and cultural buildings. There is a high seismic risk in Tabriz due to its adjacency to the North Tabriz active faults (NTF). However, there are some limitations in the availability of data for assessing physical and social seismic vulnerability assessment in Tabriz. For example, there is lack of pre-defined seismic vulnerability codes/indices or proper region-specific fragility curves for the existing structures. Therefore, it is crucial to propose a quick and realistic seismic vulnerability assessment method which considers the subsumed uncertainties.

The study area in this research is Tabriz District One with an area of approximately 1900 hectares which is one of the districts adjacent to the active part of the NTF (Figure 4). The statistical data referred to and used in this study are based on 2016 census data (the latest census data announced officially in Iran) conducted by Iran Statistical Centre (ISC). The worst earthquake scenario among Tabriz historical earthquakes with maximum magnitude (Ms~7.7), which occurred due to the NTF movement, is hypothesized as the earthquake scenario of the present study.

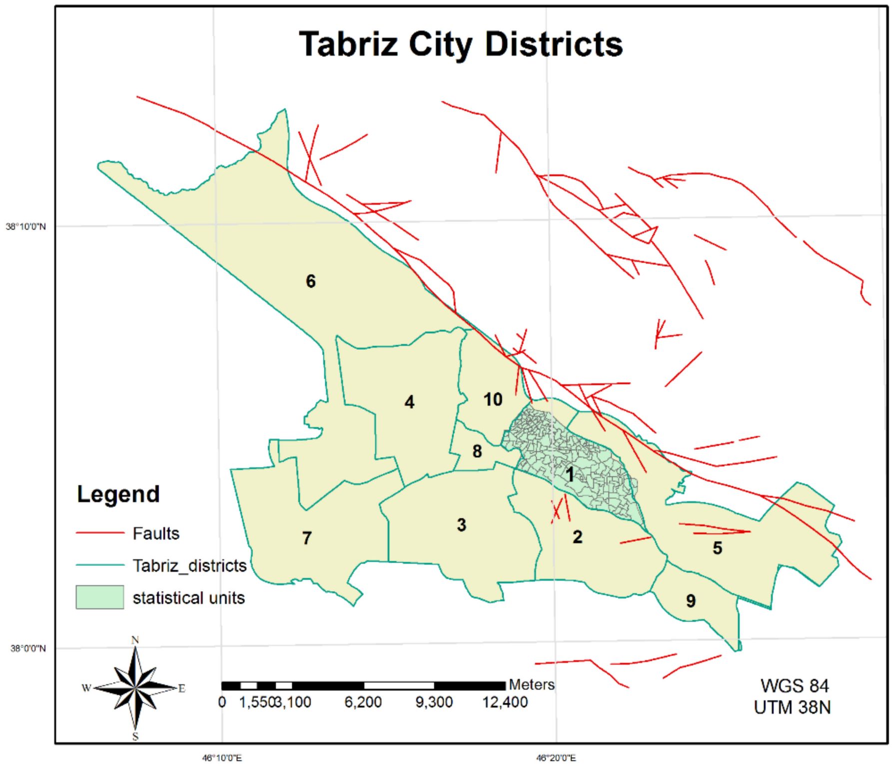

Figure 4. Study area.

The data sources used in this study are displayed in Table 1. The set of criteria/sub-criteria, selected to represent the problem domain in this paper is described in Table 2 [2]. Linguistic variables are used by the seismic experts to give their opinions on the impacts of the adopted criteria in causing damages to the buildings.

Table 1. The used data and the providing organizations.

Prepared Data	Source Data	Source Scale	Organization
Tabriz Districts layer	Tabriz county map	1:2000	Statistics and IT organization/ Municipality of Tabriz
Slope layer	Topographical map	1:2000	Iran National Cartographic Center (NCC)
Lithology layer	Geological map	1:2000	Geological Survey of Iran (GSI)
Faults layer	Active faults of Iran map	1:2,500,000	International Institute of Earthquake Eng. and Seismology (IIEES)
Statistical units layer	District One statistical units map	1:2000	Iran Statistical Center (ISC)
Age of the buildings	Year of the construction (Excel worksheet)	-	Iran Statistical Center (ISC)
Structural types	Frame types of the buildings (Excel worksheet)	-	Iran Statistical Center (ISC)
Ground water layer	(UTM) X,Y,Z of wells of East Azerbaijan (Excel worksheet)	-	Water Resources Management Company/Ministry of Energy
Number of floors	Land use map (attribute table)	1:2000	Ministry of Roads and Urban
Population statistics	District One statistical data (Excel worksheet)	-	East Azarbaijan Province Management and Planning Organization

Table 2. The influencing criteria/sub-criteria and used weights based on the experts' opinions.

Criteria/ Sub-criteria Influencing macro-seismic intensity (hazard parameters)	Relative importance/ Aggregated weights	Criteria/ Sub-criteria Pertinent to structural properties (physical vulnerability parameters)	Relative importance/ Aggregated weights
Slope (%)	**[0.623]**	**Height of the Building**	**[0.678]**
0-3	(0.03,0.16,0.36)		
3-7	(0.08,0.27,0.47)	Density of 1-2 story buildings (%)	
7-15	(0.24,0.44,0.64)	10 >	(0.08,0.22,0.42)
15-20	(0.47,0.67,0.86)	10-40	(0.24,0.42,0.62)
20 <	(0.64,0.84,0.97)	40-70	(0.26,0.46,0.66)
		70-100	(0.32,0.5,0.68)
Ground water level (m)	**[0.521]**	Density of 3-4 story buildings (%)	
0-3	(0.53,0.73,0.9)	10 >	(0.14,0.3,0.5)
3-7	(0.4,0.6,0.78)	10-40	(0.3,0.5,0.7)
7-10	(0.23,0.43,0.63)	40-70	(0.52,0.62,0.82)
10-15	(0.3,0.5,0.68)	70-100	(0.58,0.78,0.92)
15-20	(0.1,0.26,0.46)	Density of (> 5) story buildings (%)	
20-25	(0.02,0.13,0.33)	10 >	(0.38,0.58,0.76)
25<	(0,0.1,0.3)	10-40	(0.46,0.66,0.84)
		40-70	(0.5,0.7,0.88)
Dist. to faults (km)	**[1.000]**	70-100	(0.58,0.7,0.86)
1>	(0.7,0.9,1)		
1-2	(0.64,0.84,0.97)	**Age of the Building**	**[0.918]**
2-3	(0.61,0.81,0.96)		
3-4	(0.56,0.76,0.91)	Age of the buildings (years)	
4-5	(0.5,0.7,0.87)	10>	(0.15,0.33,.53)
5-6	(0.56,0.61,0.78)	10-20	(0.3,0.5,0.7)
6<	(0.23,0.43,0.63)	20-30	(0.47,0.66,0.87)
		30-40	(0.66,0.86,0.98)
Lithology	**[0.827]**	40-50	(0.7,0.9,1)
Q^{f} Quaternary (Pleistocene)	(0.7,0.9,1)	50<	(0.7,0.9,1)
Q^{fl} Quaternary (Pleistocene)	(0.7,0.9,1)		
$M\text{-}Pl^{t}$ Neogene (Pilocene)	(0.5,0.7,0.9)	**Structural type of the Building**	**[0.770]**
M^{sm} Neogene (Miocene)	(0.5,0.7,0.9)		
M^{m} Neogene (Miocene)	(0.6,0.8,0.95)	Structural type of the buildings	
PLQc Quaternary (Pleistocene)	(0.5,0.7,0.85)	Steel frame	(0.18,0.38,0.58)
Q^{al} Quaternary	(0.63,0.83,0.96)	RC frame	(0.14,0.34,0.54)
Q^{t} Quaternary	(0.63,0.83,0.96)	Brick and steel/ Stone and steel	(0.5,0.7,0.9)
M^{m} Oligocene-Miocene	(0.3,0.5,0.7)	Brick and wood/ Stone and wood	(0.58,0.78,0.94)
(10) QPL^{c} Quaternary	(0.2,0.4,0.6)	Timber	(0.22,0.42,0.62)
(11) Ku^{m} Cretaceous	(0.15,0.3,0.5)	Brick and wood	(0.62,0.82,0.96)
(12) K^{m} Cretaceous	(0.15,0.3,0.5)	Brick and clay	(0.7,0.9,1)

3.1. Efficiency of Dempster Combination Rule in Seismic Vulnerability Assessment of Buildings

The proposed method rates the statistical units due to the adopted sub-criteria influences on building damage using fuzzy sets theory and DS combination rule. Dempster combination rule efficiency is affected by high conflicts between evidences (or BPAs). To study the influence of the conflicts in the seismic vulnerability assessment model proposed in this paper, it is suggested that the discounted BPAs using Shafer discounting rule are replaced with BPAs for fusion of the information. Discounting coefficients can be seen as reliability coefficients that determine the strength of the reliability of the evidence. Discounting coefficients were introduced based on the relative importance of the criteria explained in Reference [2]. The more the discounting coefficient of the criterion, the more the effect by the criterion.

The seismic vulnerability of the statistical units are estimated based on Dempster's combination rule. Then, the calculated physical seismic vulnerability classes of the two methods are compared. The results showed that vulnerability classes for seven statistical units out of the total 156 units (4%) were changed. Figure 5 demonstrates the number of statistical units categorized in each vulnerability class using the two methods. Figure 6 shows the percentage of changed units in each vulnerability class.

Figure 5. Number of units categorized in each vulnerability class.

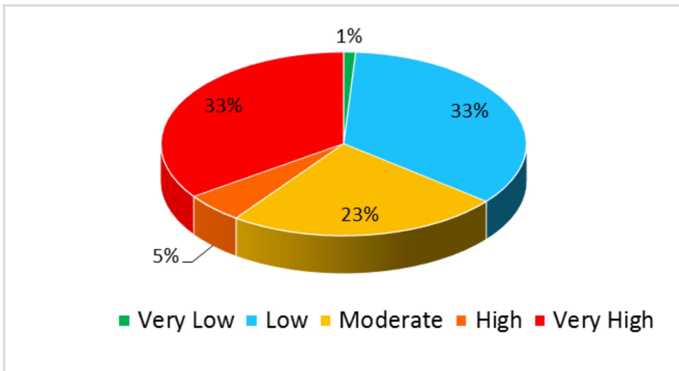

Figure 6. The changes occurred in classification of statistical units to vulnerability classes by removing the discounting coefficients.

To investigate the impact of using Shafer discounting rule on the seismic vulnerability assessment of buildings closely, a map representing the distribution and IDs of statistical units which were categorized into *different* vulnerability classes is shown in Figure 7. The legend of Figure 7 describes that changes has occurred between two adjacent classes, for example, the unit in vulnerability class of *high vulnerability* 'H' is categorized to vulnerability class of *very high vulnerability* 'VH' or the units in class of *very low vulnerability* 'VL' are categorized to *low vulnerability* 'L' class.

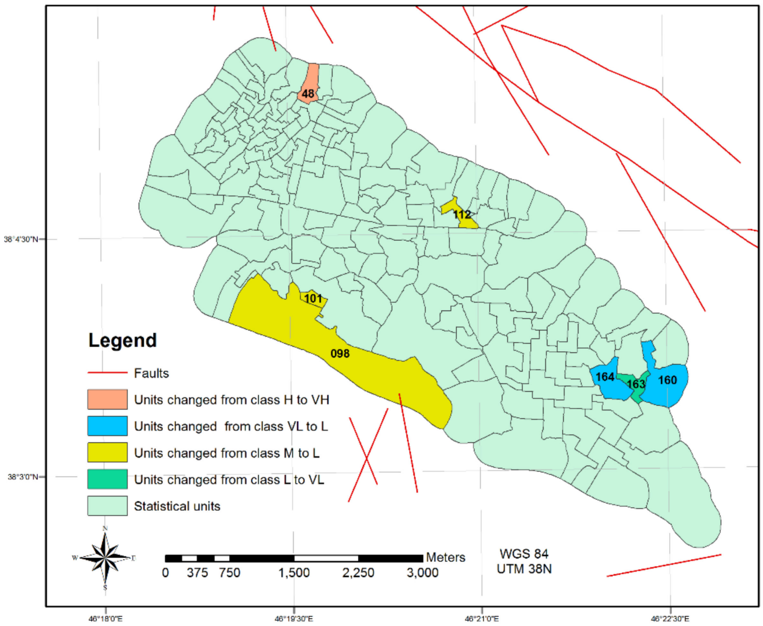

Figure 7. The statistical units which their vulnerability classes were changed.

The results of comparing the two models demonstrate that although stability of the whole model is seen but there are some variations in the estimated vulnerability classes. The changes in vulnerability classes verify that in the proposed method using FST and DST, the inconsistency of information sources exist. In other words the beliefs used for estimating seismic vulnerability of buildings were not based on entirely independent evidences [50]. The changes in vulnerability classes is the result of the discounting coefficients not applied in the fusion process, demonstrating that these coefficients affect the predictions. Therefore it can be concluded that it is required to consider and model the inconsistencies before fusion of information sources.

The uncertain measure decreases after the information fusion process using Dempster combination rule and by applying discounting coefficients the effect of conflicts are considered and thus decreased uncertainty and more reliable results are provided by the proposed PSVA method.

3.2. Producing Physical Seismic Vulnerability Maps

Using the proposed model, three maps are produced including M (NS), M (IS) and M (ISNS). M (NS) demonstrates the belief supporting the hypothesis that the unit has priority in mitigation activities (see Figure 8). In other words belief or support map (Figure 8) represents the degree to which the evidence provides support for the *vulnerability* of the statistical units buildings. For example, if the m (NS) for one statistical unit is 0.7 that is, the BPA supports the hypothesis 'the unit has priority in mitigation activities with belief degree 0.7.'

In the belief map, the obtained values (ranging 0.00–0.889) were classified into five equal intervals: 0-0.178, 0.179-0.356, 0.357-0.533, 0.534-0.711 and 0.712-0.889 where higher belief ranks mean that the relevant units have higher physical seismic vulnerability beliefs. The units with higher vulnerability beliefs need to have priority in pre-earthquake mitigation programs for reducing the destructions.

M (IS) demonstrates the belief supporting the hypothesis that the unit does not have priority in mitigation activities (or disbelief). M (ISNS) illustrates the uncertainty of having priority or not, in mitigation plans called ignorance (Figure 9). In other words M (ISNS) demonstrates the total belief supporting the hypothesis that there is no knowledge if the unit has priority in mitigation activities. In this study the obtained values for M (ISNS) ranging 0.007–0.200 were classified into five equal intervals as 0.007-0.045, 0.046-0.084, 0.085-0.123, 0.124-0.161 and 0.162-0.200 shown in the legend of Figure 9.

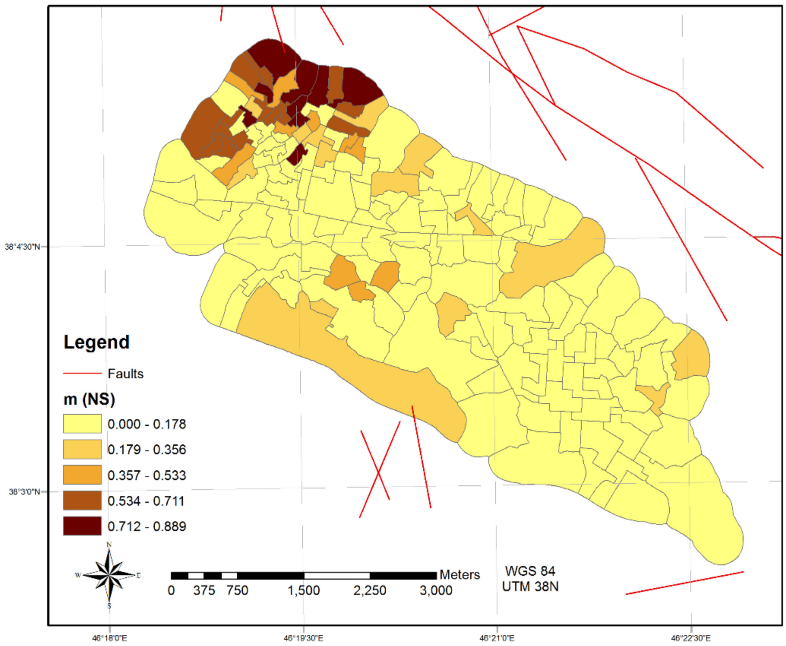

Figure 8. Map of belief.

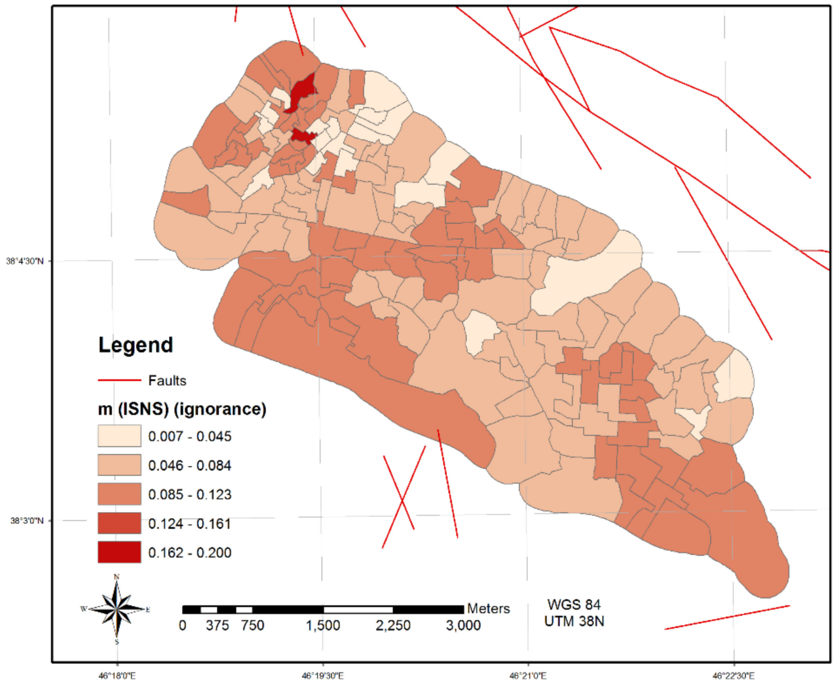

Figure 9. Map of uncertainty.

The vulnerability map representing the physical seismic vulnerability of urban statistical units is produced based on the calculated beliefs and uncertainty (see Figure 10).

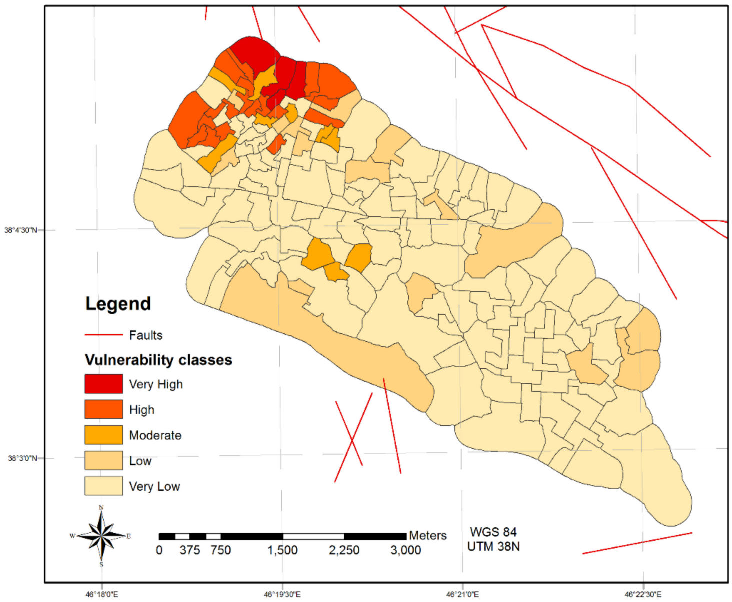

Figure 10. Map of seismic vulnerability of buildings.

In order to determine the vulnerability classes of the statistical units, Pignistic Probability Transformation (PPT) is used (Equation (8)) [51] where Mj (NS) and Mj (ISNS) represent the combined beliefs and ignorance accordingly for jth statistical unit.

$$\text{Bet j (NS)} = \text{Mj (NS)} + (\text{Mj (ISNS)}/2) \quad (8)$$

Bet (NS) illustrates the jth statistical units' priority for mitigation activities representing a value between 0 and 1. Table 3 illustrates how the physical seismic vulnerability of the statistical units are classified based on the Bet (NS).

Table 3. The physical vulnerability classes based on Bet (NS).

Bet (NS)	Vul. Class
0-0.2	Very High (VH)
0.2-0.4	High (H)
0.4-0.6	Moderate (M)
0.6-0.8	Low (L)
0.8-1	Very Low (VL)

In the vulnerability map, the units which are categorized into vulnerability class Very High have the highest priority in mitigation activities due to the very high destruction of buildings and so on. According to the proposed model, the northern and western parts of District One are expected to have the highest physical seismic vulnerability.

The vulnerability classes obtained using the proposed method are compared to the previous study of the authors [22], which assessed physical seismic vulnerability of Tabriz District One statistical units using Analytical Hierarchy Process (AHP) and Fuzzy sets theory. The study considered and modeled the *vagueness* uncertainty. The results of comparison showed that 50% (2 out of 4) of the statistical units which are classified into VH vulnerability class in the present study were categorized in VH and H classes in the previous study. Among the statistical units which are classified into H vulnerability class, 12 out of 19 units (63%) were classified into H and VH classes previously. Considering vulnerability class M, all of the 10 units (100%) were classified into L class. Among the 16 units in L class in the present study 6 units (37%) were classified into L and VL vulnerability classes. Finally considering the 107 units classified into VL class, 38 units (35%) were classified into L and VL classes in the previous study. The comparison results represent averagely 57% agreement between vulnerability classes of the total 156 units in District One when *non-specificity* uncertainty is modeled in the present study using combination of DS and fuzzy sets theories besides *vagueness* uncertainty modeled in the previous study.

The same comparison was conducted between the vulnerability classes assessed in the above mentioned study [22] and the previous study of the authors [2] which considered and modelled *inconsistency* besides *non-specificity* and *vagueness* uncertainties. The results showed 67%, 60%, 77%, 33% and 35% agreement for VH, H, M, L and VL vulnerability classes accordingly. The comparison showed an average of 54.4% agreement between the outputs of the two methods for PSVA (one method includes *vagueness* uncertainty modelling and the other includes *vagueness*, *non-specificity* and *inconsistency* uncertainties) in the 156 statistical units.

The above mentioned comparisons between different proposed PSVA methods represent the influence of utilizing different methods for subsuming different types of incorporated epistemic uncertainties on the outputs.

3.3. An Overview of the Calculations for the Proposed Physical Seismic Vulnerability Assessment

In this section a brief review of the algorithm programmed with Matlab, proposed for assessing physical seismic vulnerability of 156 statistical units in Tabriz District One considering the seven influencing criteria is explained.

A. Creating a matrix of the criteria weights (using upper and lower bounds given by the experts) considering the 7 influencing criteria.
B. Normalizing the matrix of Step A and obtaining a 7-by-7 matrix. Each row corresponds to one of the experts and each column corresponds to one of the criteria.
C. Calculating mean of the criteria weights given by the experts (a 1-dimentional matrix with 7 rows/columns is constructed where each element represents one criterion mean weight).
D. Constructing a parametric Data Matrix for the statistical units based on their attributes' classes (see Table 2) (The matrix dimension is 156×7).
E. Assigning fuzzy rules of vulnerability (given by the experts) to matrix of Step D and defuzzification of the elements of the matrix (The matrix dimension is 156×7).
F. Determining ideal solutions and negative ideal solutions for each column considering maximum and minimum values in the column. Calculating ignorance (ISNS) for each column.
G. Constructing Euclidean distances matrices from IS, NS and ISNS (defined as (IS+NS)/2). Three matrices are constructed accordingly (Each matrix dimension is 156×7).
H. Constructing three mass functions matrices using Equation (7) (Each matrix dimension is 156×7).
I. Calculating total vulnerability of each statistical unit. Considering each row of matrices in Step H, Dempster combination rule (using $\oplus$ operator between the first two elements of that row and then using the same operator between the combination result and 3rd element and so on) is applied. For example, for combination of m (NS)s considering the seven influencing criteria, $M(NS) = m_1(NS) \oplus m_2(NS) \oplus \ldots \oplus m_7(NS)$ where M(NS) demonstrates the total belief supporting the hypothesis that the unit has priority in mitigation activities. The calculations using $\oplus$ operator are shown in Equation (9) (see also Equation (4)).

$$
\begin{aligned}
\mathrm{M}_{1,2}(\mathrm{NS}) &= \mathrm{m}_1(\mathrm{NS}) \oplus \mathrm{m}_2(\mathrm{NS}) = \frac{[m_1(NS) * (m_2(NS) + m_2(ISNS))] + [(m_1(ISNS) * m_2(NS)]}{1 - [(m_1(IS) * m_2(NS)) + (m_1(NS) * m_2(IS))]} \\
\mathrm{M}_{1,2}(\mathrm{IS}) &= \mathrm{m}_1(\mathrm{IS}) \oplus \mathrm{m}_2(\mathrm{IS}) = \frac{[m_1(IS) * (m_2(IS) + m_2(ISNS))] + [(m_1(ISNS) * m_2(IS)]}{1 - [(m_1(IS) * m_2(NS)) + (m_1(NS) * m_2(IS))]} \\
\mathrm{M}_{1,2}(\mathrm{ISNS}) &= \mathrm{m}_1(\mathrm{ISNS}) \oplus \mathrm{m}_2(\mathrm{ISNS}) = \frac{m_1(ISNS) * m_2(ISNS)}{1 - [(m_1(IS) * m_2(NS)) + (m_1(NS) * m_2(IS))]}
\end{aligned}
\tag{9}
$$

Corresponding to the units' obtained BPAs of NS, IS and ISNS considering the first two criteria using Equation (9) the combined BPAs of belief, disbelief and uncertainty are calculated according to Equation (10). The subscripts in Equation (10) show the criteria No. s.

$$\begin{gathered} M_{1,2,3}\ (NS) = m_{1,2}\ (NS) \oplus m_3\ (NS) \\ M_{1,2,3}\ (IS) = m_{1,2}\ (IS) \oplus m_3\ (IS) \\ M_{1,2,3}\ (ISNS) = m_{1,2}\ (ISNS) \oplus m_3\ (ISNS) \end{gathered} \tag{10}$$

J. Three 156×1 matrices will be constructed as $M_{1,\ldots,7}$ (NS) (or belief), $M_{1,\ldots,7}$ (IS) (or disbelief) and $M_{1,\ldots,7}$ (ISNS) (or ignorance). It should be noted that the sum of corresponding elements in M (IS), M(NS) and M(ISNS) must equal to '1' (see Equation (3)).
K. Calculating Bet (NS)s using Equation (8) from M (NS)s and M (ISNS)s for the statistical units and then classifying the outputs to vulnerability classes (using Table 3).
L. Representing the outputs of Step I as separate maps of belief, disbelief and ignorance.
M. Representing the outputs of Step J as physical seismic vulnerability map.

3.4. Producing Social Seismic Vulnerability Maps

The next step of this study is estimating the population loss. Based on the estimated building damage and the population statistics of each statistical unit, the population vulnerability (i.e., number of deaths, hospitalized, injured and non-hospitalized people) can be estimated. Different methods have been used for estimating the injuries to the human beings some of which were introduced in the Introduction Section. In this study the population losses have been calculated using the human vulnerability function given by Reference [8] which is based on local questionnaire surveys and reports of previous earthquakes in the area. Due to the estimated seismic damages to the buildings and population statistics, the human vulnerability is estimated using Equation (11) [8].

$$H = \sum_{i=1}^{n} BP_i \times PK_i, \tag{11}$$

where, H is the number of people in each population vulnerability class,

BP_i is the number of people in each statistical unit and

PK_i is the injury probability state percentages (i.e., not injured, injured/not hospitalized, hospitalized and dead).

The statistics of population loss estimated for District One statistical units is shown in Figure 11.

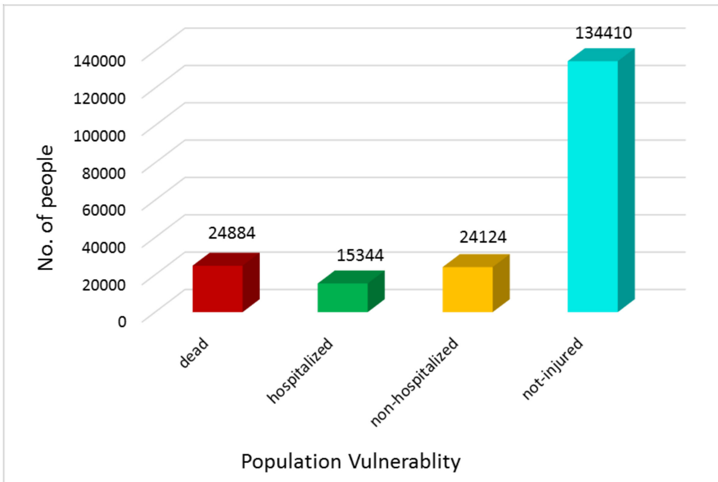

Figure 11. The estimated population loss.

A population vulnerability map depicting the distribution of population losses is presented in Figure 12. The results showed that out of the total 196,620 people in District One statistical units, the percentage of dead, hospitalized, injured/ not hospitalized and not-injured people are 12%, 8%, 12% and 68%, respectively.

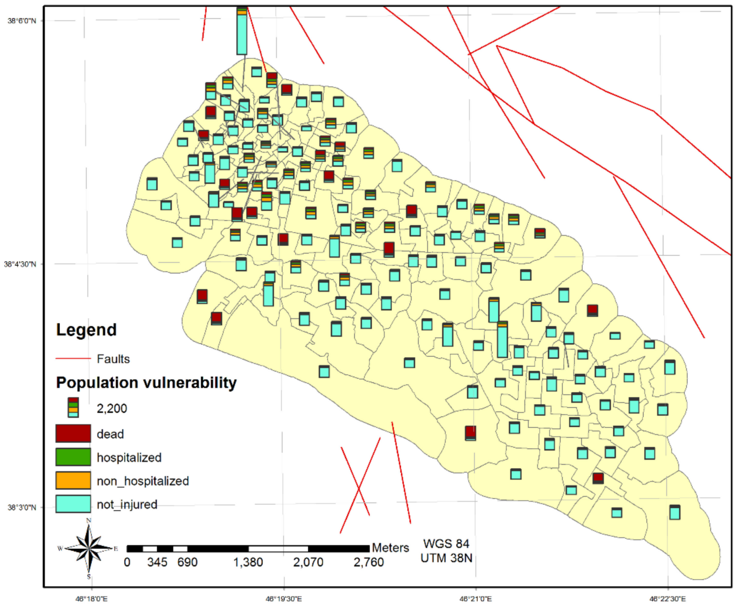

Figure 12. Map of social vulnerability.

4. Conclusions

In this paper a pragmatic and rational method for seismic vulnerability assessment is proposed. The paper aims to study the role of inconsistency in the PSVA model using fuzzy sets and D-S theory. The Dempster rule of combination assumes equivalent importance for evidences while Shafer discounting rule is suggested to be used for enhancing Dempster combination rule when evidences highly conflict. In this paper by comparing two models_ one of them considers Shafer discounting coefficients and the other does not apply these coefficients_ the incorporated inconsistency of the model is evaluated. The results confirmed that although the selected influencing criteria were independent however, it is necessary to consider the inconsistency between evidences in the proposed model.

The method could improve the accuracy and efficiency of estimating *buildings* and *population losses* in high seismic risk areas with data scarcity by dealing with and decreasing incorporated epistemic uncertainties and thus providing realistic estimates. Against the related works, this paper analyzed different kinds of involved uncertainties and focused on investigating the role of inconsistency in the approximate reasoning approach for seismic vulnerability assessment using Fuzzy Set (FS) and Dempster-Shafer (DS) theories. The main contribution of this study was specifying the deal of inconsistency in seismic vulnerability assessment model in Tabriz District One statistical units. The results confirmed that the proposed model is stable to some extent using the selected criteria and experienced experts' opinions. However, the efficiency of enhanced Dempster's combination rule using Shafer discounting coefficients for the fusion of information sources was confirmed compared to Dempster's combination rule. This concluding remark was perceived since the results demonstrated that the outputs of the combination of multiple evidences were affected by the strength of the reliability of evidences, which implies some inconsistencies among information sources.

A strength point of the present model is that besides considering knowledge-based uncertainties (vagueness, non-specificity and inconsistency) it is able to model ignorance which is of significant importance in approximate reasoning approaches.

Real earthquake data is not available for the study area. However, the efficiency and accuracy of the model was tested by implementing the proposed model in a nearby area by the authors [2]. The results of comparing the outputs of the model to the real building damage caused by a real earthquake confirmed that the model provides realistic assessment of building damage compared to the real world damages.

Considering the model of social vulnerability utilized in this paper, it can be concluded that since the uncertainties of physical seismic vulnerability model used for estimating the population loss were handled, the assessed social vulnerability would be realistic which is of importance in succeeding the pre-earthquake mitigation programs. This point has been ignored in the previous social vulnerability studies.

Physical seismic vulnerability maps representing the distribution and severity of the estimated damages to the existing buildings based on the proposed method were produced for the statistical units in Tabriz District One. The outcomes showed that the physical seismic vulnerability in the North-West parts of the study area is very high. Upon the assessed physical seismic vulnerability the human losses were estimated as well and related map was provided.

The produced information are critical for pre-earthquake programs to reduce future earthquake losses in seismic prone urban areas and accordingly assist the sustainable development, for example, planning for population propagation and settlement, urban expansions, retrofit and strengthening solutions and insurance scheme for areas with higher seismic risk. The proposed model is independent of studied area and can be utilized in other areas in need of realistic predictions but lack sufficient data for seismic vulnerability assessment to help reduce human and economic losses.

For future works, it is suggested that the proposed seismic assessment method be carried out at urban parcel level as well. Future studies may also consider more influencing criteria than the selected criteria in this paper and study the effect of inconsistency in the assessed seismic vulnerability of buildings using Shafer discounting rule. The efficiency of other methods developed for improving Dempster combination rule, other than the Shafer discounting rule, may also be studied in seismic vulnerability assessment by authors in future.

Author Contributions: Mahmoud Reza Delavar and Mansoureh Sadrykia conceived and designed the initial idea of the research; Mansoureh Sadrykia developed the methodology; Mansoureh Sadrykia gathered and analyzed the data; Mansoureh Sadrykia wrote the draft paper. Mahmoud Reza Delavar critically reviewed and finalized the paper. All authors have read and agreed to the published version of the manuscript.

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