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Is There a Right Way? Productive Patterns of Interaction during Collaborative Problem Solving

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Abstract: Compelling research evidence shows benefits for student learning from explaining one's ideas and engaging with the ideas of others. However, whether certain patterns of group interaction may engender this productive student participation is unknown. Using data from two third grade mathematics classrooms, and over the course of six days during a five-month span, we investigated how students interacted with each other to solve problems when the teacher was not driving the interaction. We identified multiple profiles of group interaction that yielded highly-detailed participation for some or all students in the group. These profiles varied in terms of whether students interacted in an ongoing, sustained manner or interacted periodically but not continually, whether one or multiple students initiated problem-solving strategies, and whether group members worked jointly or largely separately on their strategies. No single profile of group interaction was either necessary or sufficient to lead to highly-detailed participation for all students in the group.

Keywords: cooperative/collaborative learning; small group learning; peer learning; peer collaboration; classroom-based talk; student interaction; mathematics; elementary schools; dialogue; problem-solving strategies

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1. Introduction

Cooperative learning researchers have made great strides in identifying student participation in peer-led groups that is linked to student learning outcomes. At the heart of many researchers' perspectives about interaction among students that is productive for learning are explaining one's own ideas and engaging in the ideas of others. Some perspectives include exploratory talk [1], accountable talk [2,3], transactive discussions or transactive dialogues [4,5]; see also [6], argumentation [7–10], collaborative reasoning [11], coordinated talk [12], co-construction [13–17], co-regulation [18], and shared regulation [19–21].

In exploratory talk, for example, students offer the relevant information they have and engage critically but constructively with each other's ideas by jointly considering, evaluating, challenging, and building upon each other's hypotheses [22]. In contrast, disputational talk, characterized by disagreements but little constructive criticism of suggestions, and cumulative talk, characterized by positive but uncritical building upon each other's suggestions, are seen as less constructive and involve less student engagement with each other's ideas.

Explaining one's own ideas and engaging with others' ideas can promote learning in multiple ways. Developing and offering ideas to others, being challenged or questioned by others, and attending to others' thinking all encourage students to rehearse information in their own minds, monitor their own thinking, reorganize and clarify material for themselves, recognize and rectify misconceptions and gaps in their understanding, make connections between new information and previously learned information, reconcile conflicting viewpoints, and acquire new strategies and knowledge and develop new perspectives [23–29].

Accumulating empirical evidence supports the hypothesized benefits of such participation for learning [30–33]. Studies comparing different approaches to training students on the use of productive dialogue, have shown positive effects on explaining their ideas and critically engaging with and evaluating each other's ideas on student mathematics and science achievement and reasoning ability [7,34–38]. Similarly, studies comparing different approaches to training teachers to implement instructional moves designed to promote student explaining and engagement with each other's thinking (e.g., asking students questions to clarify their thinking and give supporting reasons, and to discuss each other's ideas and predictions) have also shown positive effects on student achievement [39–50].

Correlational research also shows positive links between explaining and engaging with others' ideas and learning outcomes. Positive relationships with mathematics and science learning outcomes have been shown for providing explanations as part of arguments or justifications, and supporting, rebutting, and building on others' suggestions [37,51–58]. Furthermore, explaining and engaging with others' ideas (e.g., restating or paraphrasing another student's strategy and applying it) have been found to correlate with mathematics achievement among students who explicitly need help [59]. Recent studies show, moreover, that the relationships between participation and learning outcomes depend on the level of detail in explaining and engagement with others' ideas. For example, giving highly-detailed, fully complete, and correct explanations was more strongly related to achievement test scores than was giving incomplete or ambiguous explanations [60,61]. Similarly, students who engaged with others' ideas at a highly-detailed level (by adding details to other students' suggestions, or proposing alternatives), showed higher achievement than students who only repeated other students' suggestions, or who agreed or disagreed with others without providing reasons [62].

Close case-study analyses of student interaction reveal more details about the benefits for the participants in these conversations. Roscoe and Chi [54], for example, describe how students trying to explain to other students the structure and functioning of the human eye signaled that they did not fully understand (e.g., "This is something that I didn't really get before", p. 336), and revisited and reviewed material and rethought their ideas in the context of trying to provide more complete explanations, and arrived at a fuller understanding as a result. Brown, Campione, Webber, and McGilly [24] (pp. 177–178) illustrate how the group's challenge of an explainer's incomplete or incorrect ideas caused the explainer to re-examine her prior knowledge, to formulate and test predictions based on her incorrect mental model, and to use information provided by her peers in response to her predictions to revise her ideas (in this case, her ideas about animals' use of camouflage as a defense mechanism). Webb et al. [57] showed how, through the process of explaining their own thinking and engaging with others' ideas, students forged new connections between mathematical ideas and representations, and extended their problem-solving strategies.

While there is general agreement about the importance of explaining one's ideas and engaging with others' ideas, much less is known about whether there are certain patterns of group interaction that are more likely than others to produce this productive student participation for all students in the group. This paper explores how interaction in small groups solving mathematical problems unfolded, and examines how multiple features of the patterns of interaction corresponded to the emergence of highly-detailed participation in the group. In particular, we focus on two features of group interaction: (1) whether all students in the group have, and use, opportunities to lead the mathematics by contributing mathematical ideas; and (2) whether there is sustained interaction among the students. This paper then delves more deeply into the patterns of interaction in groups that are different or similar in respect to those features.

The importance of all students having opportunities to contribute ideas during group work finds support in previous research. Previous studies suggest that equitable opportunities to pose and develop ideas during group collaboration may prevent propagation of misleading or incomplete ideas, may prevent the development of negative self-perceptions that may arise when some students are marginalized or hindered from participating in group conversations, and may contribute to learning among all students in the group [63,64]. Inequitable participation opportunities within groups may arise when some members are positioned (by the teacher or by each other) as experts (or more capable) and others as novices (or less capable, the former having greater opportunities than the latter to contribute ideas, explain their thinking, and obtain feedback from other students about their ideas). In-depth studies of specific students within small groups have highlighted the dynamic, negotiated character of opportunities to contribute [65], and the ways that, for example, race, gender, and social status intersect with individuals' attempts to wield mathematical influence [66–69]. This work also documents how individuals embrace or resist particular positionings, and leverage different kinds of resources to reorganize access to the group's work. For example, Esmonde and Langer-Osuna [70] showed how taking on the role of critic enabled one student to position herself powerfully in relation to another member's attempts to lead the group's work.

The second feature of group interaction we examine is the degree of continuity of the interaction among group members. While previous research has not investigated whether interaction among group members must be sustained and ongoing for highly-detailed participation to take place, some hints have emerged. For example, Barron [12] described how group interaction that led to successful problem-solving work involved the rapid exchange of tightly coordinated conversational turns, frequent eye contact, and constant monitoring of each other's contributions. Group interaction that led to less successful outcomes had less eye contact, more self-directed talk, more interruptions, and more instances of ignoring students' suggestions, as well as episodes of silence [71].

Previous research suggests that these two features of group interaction may operate independently. Shah, Lewis, and Caires [72], for example, showed how ongoing interaction could be either equitable or inequitable. Dyads all exhibiting ongoing interaction varied in terms of the distribution of talk and the issuing of commands and directives, with consequences for individuals' opportunities to learn.

In this paper, we first examine the extent to which these two features of group interaction—all students taking the lead in contributing mathematical ideas in the context of solving problems, and sustained, continuous interaction among group members—are necessary for highly-detailed participation (explaining one's own ideas and engaging with others' ideas at a highly-detailed level) to emerge. We then analyze the patterns in how group interaction unfolded over time to better understand how highly-detailed participation emerged in groups that were similar or different in terms of whether all members of the group played an active role in contributing mathematical ideas, and the degree of continuity of interaction among group members.

2. Materials and Methods

2.1. Participants

The school district selected in this study (a large urban area in Southern California) expressed ongoing interest in improving mathematics instruction in alignment with the Common Core State Standards for Mathematical Practice [73]. The majority of students in this district are Hispanic or Latino students (greater than 75%) and a large proportion of students are classified as English Learners (over 40%) [74]. The demographics of students at the particular school used in this study were similar to the district. At this school, just over 80% of students identified as Hispanic or Latino with approximately 8%, 6%, and 2% identifying as White, African American, and Asian, respectively. Further, at the third grade level at the school (the grade level studied here), more than 84% of the students identified as Hispanic or Latino and approximately 7%, 6%, and 1% identified as White, African American, and Asian, respectively. Close to half of the students identified as female (in the district, school, and third grade at the school). In this school during that particular academic year, 22% of the third graders met or exceeded the standard for math achievement. This was comparable to the math achievement for the district, where 29% of the third graders met or exceeded the standard for math achievement.

Two third grade teachers within the same elementary school in this particular district were recruited to participate in this study due to their successful efforts to create learning environments where students generated their own strategies for solving problems and explained and responded to each other's mathematical ideas. Both teachers had extensive experience with professional learning opportunities related to Cognitively Guided Instruction [75] and were interested in learning how to better support their students to engage with each other's thinking. Within these two classrooms, 98% of the enrolled students consented to participate. The student who did not consent to participate was not videotaped during any of the lessons. The final sample analyzed here included 45 third graders.

2.2. Mathematics Lessons

The two teachers coordinated their lessons throughout the academic year. They selected similar tasks and generally organized and sequenced their instructional time similarly. Each lesson typically started with a 10–20 min whole-class warm-up where students solved and discussed activities or problems (e.g., 2 h and 45 mi = ___ min).

Next, the lesson included 25–35 min of small-group collaborative problem solving where students worked together (mostly in pairs but occasionally in groups of three students) to make sense of and solve one or more story problems involving multiplication or division. An example of a problem that both teachers used was: "(Name of student) is allowed to watch T.V. for ___ min each day. How many minutes of T.V. can they watch in ___ days?" Students selected from three different number sets: (15, 8), (16, 6), (26, 4). The teacher encouraged students to generate, and carry out, multiple strategies for solving each number set. During small-group work, students were not provided with specific instructions about how to explain or engage with each other, nor were they given specific roles to play, or particular protocols to follow. On only a few occasions did the teacher provide any direction to the class during the transition to small-group work (e.g., "I want you to work on this with your partner. Make sure you try and understand each other's strategies. Stop once in a while to explain what you're doing. And that's it. You can go get started with your partner.").

The lessons concluded with a 15–20 min whole-class wrap-up during which students (or pairs) shared the strategies they had developed for the problems during their small-group collaborative problem-solving time.

2.3. Data Collection

On six days (two in January, two in March, and two in May), we video recorded student and teacher interactions in each classroom. On each day in each classroom, we used six video-recording devices to capture the interaction of six pairs (or triads) of students during small-group work. The use of multiple recording devices made it possible to record the interaction for half of the students in the class (approximately 12 students) on each day. We rotated which students were recorded on each day so that we recorded all students in the class during an adjacent pair of lessons. This procedure resulted in 68 video recordings of small-groups: 63 pairs and 5 triads.

We collected all written student work on every day of observation. The written work showed students' problem-solving strategies for the problems assigned in the small-group collaborative problem-solving portion of the lessons.

2.4. Selection of Pairs for Analysis

For the analyses presented here, we examined pairs whose dialogue developed naturally and was not driven by the intervention of the teacher in any significant way. More specifically, we included pairs in which the teacher did not visit the pair or checked-in only briefly. If the teacher's intervention lasted longer than a brief check-in, we included the pair for analysis if the teacher did not steer or otherwise shape the trajectory of the conversation. We excluded pairs where the teacher played a large role in driving the pair's interaction, such as asking detailed questions that invited explanations, or inviting students to engage with each other's ideas. This resulted in the 55 videotapes of small-group work analyzed in this paper: 51 pairs and 4 triads. Because the overwhelming number of small-groups were pairs, we will use the term "pairs" to refer to both group sizes for convenience.

2.5. Coding of Student Participation

For each lesson in which a pair was videotaped, we coded each student according to whether they exhibited highly-detailed participation. Students were coded as exhibiting highly-detailed participation if they carried out highly-detailed explaining and/or highly-detailed engaging with others' ideas. Highly-detailed explaining consisted of explaining one's thinking with enough detail that most or all of their mathematical strategy was clear to the coders (e.g., For the problem, "___students in Room 18 raised ___ each for the jog-a-thon. How many dollars did those students raise in all? (6, 15)": "I did six boxes then I put 15 [in each box] and split it into 10 and 5. And then I did 10, 10, 10, 10, 10, 10, I put equal 60. Then 5 times, 5, 5, 5, 5, 5, 5. And then I put equals 30. And then I put 60 plus 30 equals 90. And then my answer was 90.") Ambiguous or incomplete explanations were not coded as highly-detailed explaining (e.g., Now I know what I did. So we have to draw, like, 15, 15, 15. We can count by fifteens. 15, 20, 25, 30. 5, 10, 15, 20, 25, 30, 35 . . . This is 140.").

Highly-detailed engagement with others' ideas consisted of explaining another student's strategy in detail, extending the details of their partner's idea(s), or suggesting detailed alternatives. In the following example, Melanie suggested an alternative way to combine numbers that she thought was more efficient than what Noemi suggested for the problem that required totaling seven fifteens:

Noemi: (Draws seven circles) Put 15 [in each circle].
Melanie: Why are we putting 15 in it?
Noemi: Because we are gonna split it in 5, 5, and 5.
Melanie: Why are we gonna split it?
Noemi: Because it's easier to do fives instead of fifteens.
Melanie: Isn't it more efficient to do fifteen?
Noemi: Well, in fives you can just keep on counting because fives are really easy.
Melanie: But don't you already know that 15 plus 15 is 30? So it's more easier than that way (nods at Noemi's paper).

Simply repeating details of their partner's suggestions, asking general questions, asking questions about elements in a student's work, or voicing agreement or disagreement without adding any details or reasons were not coded as highly-detailed engagement with others' ideas. In the following example, Mira repeated a step her partner articulated, and asked a question about another number her partner had written, but did not extend the mathematical work:

Mira: Can you explain what you are doing?
Donatello: I'm counting by fifteens, seven times.
Mira: So you are counting by fifteens?
Donatello: 45 . . .
Mira: How do you know the next number is 30? (points to Donatello's notebook)

We then classified each pair according to the number of students in the pair who exhibited highly-detailed participation (for pairs: two, one, none; for triads: three, fewer than three, none).

To check rater consistency, the research team selected a random sample of 10 students for a randomly selected lesson. Two raters independently coded student participation for this sample of students. Exact rater agreement was high (above 97%) for both highly-detailed explaining and highly-detailed engagement with others' ideas.

2.6. Coding of Group Interaction

2.6.1. Number of Students Leading the Math

The first group interaction feature coded was the number of students in the group who led the math in terms of contributing mathematical ideas. By leading the math, we mean that the students' efforts to contribute shaped the direction or nature of the mathematical work. This included making suggestions for alternative or more efficient problem-solving strategies, suggestions for how to begin or carry out an approach to solving the problem, making suggestions for alternative problem-solving strategies, suggesting new steps in a strategy, connecting a step with its relation to the story context, proposing a different way to carry out a particular step in a strategy, challenging the validity of others' problem-solving approaches or strategies, or exploring discrepancies that arose in problem-solving approaches, strategies, or results. Carrying out the steps of a strategy that another student proposed, repeating work that another student had described, asking questions of clarification, and confirming or disconfirming others' work (without providing justification or rationale), while important ways of contributing, were not, by themselves, bases for coding that a student led the mathematics in terms of contributing mathematical ideas.

As a preliminary step, we separated each video into segments corresponding to a solution strategy for a number set. We then assigned codes to each segment (as described below) and decided on the most representative code for the pair across all of the segments. Multiple members of the research team coded each video. Discrepancies were resolved through an iterative process of discussion in the whole research team and recoding, as described in a later section.

For each segment of a video, we assigned one of the following codes: (a) both students led the mathematics, (b) one student led the mathematics, or (c) no student led the mathematics (students were working on the problems but did not interact about the problems or their work). We then assigned one code that best represented the interaction in the pair across all segments. We selected a code as most representative by taking into account the number of segments with that code, the amount of interaction time with that code, and the centrality or importance of particular segments in terms of the pair's development of mathematical strategies and mathematical ideas (e.g., in some pairs, earlier segments showed the richest interaction around mathematical ideas; in other pairs, later segments involved in-depth interaction around discrepant strategies or answers). In addition, a pair was coded as both students leading the mathematics if both students led the math in most or all segments (or if both students participated but there was no clear leader), or if each student in the pair led the mathematics for different number sets and strategies (e.g., in some pairs, members of the pair alternated who led the mathematics).

2.6.2. Continuity of the Group's Interaction

The second group interaction feature coded was the degree to which pairs showed sustained interaction. For the continuity of the interaction during collaborative work, we assigned each pair one of the codes in Table 1.

Table 1. Continuity of interaction among students in the pair.

	Description
Ongoing	The pair showed sustained, regular back and forth engagement with each other that occurred continuously through small-group time.
Periodic	Students engaged with each other periodically, with long stretches of time between engagements during which they worked on their own papers without interacting with each other.
Negotiate and Move On	Students negotiated which strategy to use, selected one strategy to use, and moved on without further attempts to engage with each other.
1-Shot	The pair showed a limited, short-lived, burst of one-time engagement with each other. This interaction was over and done fairly quickly.
None	Students did not interact around the math.

2.6.3. Patterns of How Group Interaction Unfolded over Time

We next created descriptions of how the interaction unfolded within each pair, starting from the suggestions of strategies to pursue, and continuing through how the pair interacted as they carried them out. These descriptions provided additional details about the evolution of the interaction across the entire small-group collaborative problem-solving time. Three of the research team members first generated and discussed the descriptions. The other three members of the team then independently reviewed the descriptions. Finally, the whole research team discussed changes that needed to be made. The final ten interaction patterns are presented and discussed in the next section on results.

2.6.4. Coding Process for Group Interaction Variables

For the group interaction variables (number of students leading the math, continuity of interaction in the pair, and pattern of how group interaction unfolded over time), we conducted a team-based approach to make coding decisions [76]. The six members of the project's research team participated in the discussions around the development of the coding procedures and application of the codes. Multiple project members were responsible for coding different combinations of pairs. Refinement of codes and coding decisions occurred in an iterative fashion during weekly meetings over 19 months. During each meeting, project members raised questions and refined the coding scheme, compared coding results, and reviewed the coding of others. At every step, project members who had not initially performed the coding for a pair reviewed prior coding decisions, raised questions, and suggested alternatives. The purpose of this step was to check the dependability of applying the codes consistently across the entire data set and between the different coders [77]. Discrepancies between coders were discussed with the whole team and resolved. All project team members went back to the pairs they initially coded to ensure that the decisions about the discrepancies were consistently applied [78]. Discussions continued until consensus was reached for all codes for all pairs.

3. Results

In the following sections, we begin with a summary of the highly-detailed participation we observed from the pairs in our sample. Then, we describe the profiles of interaction in pairs and their relationship to highly-detailed participation. Next, we examine the patterns of interactions that underlie these profiles, and, lastly, we explore how highly-detailed participation emerged within these patterns.

3.1. Summary of Highly-Detailed Participation in the Pairs

Table 2 shows the breakdown of the pairs according to the number of students in each pair who participated at a highly-detailed level. In almost half of the pairs (24, 44%), both students exhibited highly-detailed participation. The majority of these pairs (19 of 24, 79%), showed both highly-detailed explaining and highly-detailed engagement with others' ideas. In the minority of pairs (5 of 24, 21%), students provided highly-detailed explanations of their own thinking without engaging at a highly-detailed level with their partner's ideas.

Table 2. Incidence of highly-detailed participation in the pair.

Students Participated at a Highly-Detailed Level	Number of Pairs
Both students	24 (44%)
One student	17 (31%)
No student	14 (25%)

3.2. Relationship between Leading the Math and Highly-Detailed Participation

Table 3 presents the distribution of pairs according to how many students in the pair led the mathematics and the number of students in the pair who participated at a highly-detailed level. As can be seen in Table 3, in the majority of pairs (36 of 55, 65%), both students in the pair were involved in leading the mathematics. In a minority of pairs (16 of 55, 29%), one student led the mathematics. In only a few pairs (3 of 55, or 5%) did no student lead the mathematics. When both students in the pair were involved in leading the math, it was very common for both students to participate at a highly-detailed level (22 of 36, 61%). When one student in the pair led the math, it was more common that one student participated at a highly-detailed level (10 of 16 pairs, 63%). Of these 10 pairs, most often (7 of 10, 70%), the student leading the math was also the student who participated at a highly-detailed level.

Table 3. Relationship between the number of students leading the math and the incidence of highly-detailed participation.

Number of Students Leading the Math	Students Participated at a Highly-Detailed Level			Total
	Both Students	One Student	No Student	
Both students led the math	22	7	7	36
One student led the math	2	10	4	16
No one led the math (no interaction around the math)	0	0	3	3
Total	24	17	14	55

The relationship between the number of students in the pair who led the mathematics and the number of students in the pair who participated at a highly-detailed level was statistically significant (Fisher–Freeman–Halton Exact Test = 18.40, $p < 0.001$), with a large effect size (Cohen's $W = 0.63$) [79]. Even omitting the three pairs in which there was no interaction around the math, the results are statistically significant (Fisher–Freeman–Halton Exact Test = 12.31, $p = 0.002$), with a large effect size (Cohen's $W = 0.48$). This result shows that whether both students or one student led the math significantly related to the number of students in the pair who participated at a highly-detailed level.

3.3. Relationship between Continuity of the Group's Interaction and Highly-Detailed Participation

Table 4 presents the distribution of pairs according to the degree of continuity of the pair's interaction and the number of students in the pair who participated at a highly-detailed level. As can be seen in Table 4, the majority of pairs had an interaction that was sustained and ongoing (34 of 55 pairs, 62%). A minority of pairs had an interaction that was periodic (18 of 55, 33%). In only a few pairs (3 of 55, or 5%) was there minimal interaction (1-shot or none).

Table 4. Relationship between the degree of continuity of a pair's interaction and the incidence of highly-detailed participation.

Degree of Continuity of the Pair's Interaction	Students Participated at a Highly-Detailed Level			Total
	Both Students	One Student	No Student	
Ongoing	16	11	7	34
Periodic	8	6	4	18
1-Shot/None	0	0	3	3
Total	24	17	14	55

While the degree of continuity in the pair's interaction was not significantly related to the number of students in the pair who participated at a highly-detailed level (Fisher–Freeman–Halton Exact Test = 6.24, $p = 0.134$), the effect size of the relationship was moderately large (Cohen's $W = 0.41$). This effect was due to the difference between the pairs with 1-shot or no interaction around the math and the pairs with ongoing or periodic interaction. Omitting the three pairs in which there was minimal interaction around the math produced both a statistically insignificant effect (Fisher–Freeman–Halton Exact Test = 0.15, p = 1.00) and a very small effect size (Cohen's W = 0.03). This result shows that whether students interacted in an ongoing, sustained interaction or interacted periodically (but not continuously) was not associated with the number of students in the pair who exhibited highly-detailed participation.

3.4. How Interaction Unfolded in the Pairs

As shown above, when both students participated at a high level, it could involve either one or both students leading the mathematics, or either ongoing or periodic interaction among members of the pair. The question then emerges about the particular ways in which the interaction in these pairs unfolded over the course of their problem solving that were most likely to give rise to highly-detailed participation by both members of the pair. To investigate this question, we examined more closely the pairs in the first column of Table 3 (both students exhibited highly-detailed participation). We also examined the other pairs in the first row of Table 3—where both students led the mathematics and highly-detailed participation was exhibited by one or no member of the pair—to see whether interaction in these pairs unfolded in ways that may have been different from the pairs in the first column of Table 3. The results are summarized in Table 5, which lists the multiple patterns of how interaction unfolded among these 38 pairs over the course of their work together.

Several findings are apparent in Table 5. First, the interaction in the pairs began and evolved in multiple, different ways. In some pairs, students jointly produced and jointly carried out their strategies. In other pairs, students alternated initiating strategies for how to solve the problem and then worked on carrying out those strategies. In still other pairs, students generated and carried out their own strategies. In some pairs, one member of the pair generated the strategies and the other member of the pair worked on those strategies. A few pairs did not interact at all until they had completed some strategies. There was also variation in how continuous the pair's interaction was as they carried out their strategies. In some cases, students conferred in an ongoing, sustained way as they worked, while in other cases, students only occasionally conferred or conferred only at certain well-defined points during the group's work (such as when they completed a strategy).

Second, Table 5 shows that all of the patterns of interaction led to instances in which both students in the pair exhibited highly-detailed participation. That is, there were many ways in which group interaction could begin and unfold that led to both students exhibiting highly-detailed participation. It was not necessary, for example, for pairs to be jointly involved in generating and carrying out strategies. It was not even necessary for both members of the pair to be involved in initiating or generating the strategies. It was not necessary for students to carry out the same strategies. Finally, it was not necessary for students to confer regularly as they worked.

Third, while all of the patterns of interaction led to instances in which both students in the pair exhibited highly-detailed participation, it was not the case that they guaranteed this outcome. For the pairs we analyzed here, many patterns of interaction had instances in which only one or no student exhibited highly-detailed participation. Importantly, then, patterns of how interaction unfolded in pairs in which both students exhibited highly-detailed participation were very similar to those for pairs in which only one or no student exhibited highly-detailed participation. Students found different ways of working together and sharing their mathematical work. For some pairs, these patterns of working together led to both students exhibiting highly-detailed participation; in other pairs, these same patterns did not.

Table 5. Patterns of how the interaction in pairs unfolded over time.

Pattern of How Interaction Unfolded in the Pair	Both Students in the Pair Participated at a Highly-Detailed Level: Yes	No
Students took turns initiating a strategy		
After one student suggested a strategy, they each worked on carrying it out, and they conferred in an ongoing, sustained way while they carried it out	7	4
After one student suggested a strategy, they each worked on carrying it out, and checked in from time to time with what the other was doing	2	2
Students each generated their own strategies		
They had ongoing interactions while they carried out their strategies	1	2
They occasionally checked in with what the other was doing while they carried out their strategies	2	1
They interacted only when they completed a strategy	2	0
One student generated the strategies		
While the first student generated the strategies and completed them, the other student interacted somewhat with the first student around what he/she had done	3	0
After the first student generated a strategy, the other interacted in a synchronous and sustained way with the first student while carrying it out	2	2
Students jointly produced the strategies and jointly carried them out	3	2
Students first solved the problem independently [1]		
Then students took turns sharing their strategies	1	0
Then students took turns sharing their strategies; after that, one student initiated further strategies, and the other student interacted in a synchronous and sustained way with the first student around those strategies	1	1

[1] On this particular day, unlike the other days, the teacher gave the class 15 min of independent time to work on the problems before having them gather in groups to discuss their strategies and work on further number sets.

3.5. Emergence of Highly-Detailed Participation during Pair Interaction

This section presents more details for each of the patterns of interaction identified in Table 5 to show how highly-detailed participation emerged in the pair. For each of the ten patterns identified in Table 5, Table 6 presents an example in which both students in the pair exhibited highly-detailed participation.

As can be seen in Table 6, highly-detailed participation emerged in pairs in multiple ways. One common feature of pair interaction across the group interaction profiles was for students to give unsolicited explanations of how they had solved the problem (sometimes in conjunction with explicitly stating that they should explain their strategies). Other common features of interaction that led to highly-detailed participation (fully explaining one's own approach and/or engaging with others' ideas at a highly-detailed level) included a student suggesting a strategy to pursue, students disagreeing about an answer, one student questioning another student's work, one student appropriating another student's strategy, and students seeking confirmation of their strategy. These features (and the highly-detailed participation that ensued) occurred regardless of which student or students generated the strategies, which student(s) led the execution of the strategies, and how continually the students conferred about the work.

Table 6. How highly-detailed participation emerged in pairs' interaction [1].

Pattern of How Interaction Unfolded in the Pair	Example of How Highly-Detailed Participation Emerged in the Pair
Students took turns initiating a strategy	
After one student suggested a strategy, they each worked on carrying it out, and they had ongoing, sustained interactions while they carried it out	While working to carry out the strategies, the students described steps of the strategy out loud, regularly asked and answered each other's questions, repeated steps that the other student had voiced, and expressed agreement or disagreement with what the other student had said. For each strategy, upon completion, the student who had initiated it offered an unsolicited fully-complete explanation of it. On one strategy, when the pair produced different final answers, the students carried out extensive discussion about their work, with one student challenging the other, asking her specific questions about particular steps she had performed, explaining why she thought her partner's work was incorrect, describing what her partner should do instead, and generating multiple ways to convince her partner of the correct solution.
After one student suggested a strategy, they each worked on carrying it out, and checked in from time to time with what the other was doing	While working on the strategies, the students occasionally asked questions or made comments about what their partner was doing, or briefly expressed agreement or disagreement with what their partner had written on their paper. For one number set, after they each had finished carrying out the strategy on their own, one student asked his partner how he got his answer, and his partner explained his strategy. For another number set, when the other student finished first, he turned his attention to making sure that his partner was carrying out the strategy correctly. In doing so, he told his partner what to write, and explained how to carry out those steps correctly.
Students each generated their own strategies	
They had sustained interaction while they carried out their strategies	After deciding on an approach, they used manipulative materials (blocks) to carry them out. As they worked, they monitored what the other was doing and frequently commented on each other's progress. The student who finished first gave a complete explanation of her strategy and the other student then added to her idea.
They occasionally checked in with what the other was doing while they carried out their strategies	They largely worked independently on their own strategies but occasionally reviewed and confirmed each other's ideas before moving on to the next strategy or number set. During one check in, one student started to question the other's nascent strategy and then added mathematical detail. Later, the student whose work was questioned gave a complete explanation of his strategy.
They interacted only when they completed a strategy	They worked on their own strategies independently and came together after completing each one to explain in detail what they each did. They explicitly voiced a norm that they should explain their strategies to each other (e.g., "Ok, so let's explain to each other our strategies."). While each student explained how she carried out her strategy, her partner asked a few questions to make sure she understood what her partner had done.
One student generated the strategies	
While the first student generated the strategies and completed them, the other student interacted periodically with the first student around what he/she had done	One student initiated the strategies and both students worked on them, with long pauses while they worked independently. Periodically, the second student asked questions of the first student to clarify and justify what he had done, challenged the first student about steps he had carried out, and suggested alternatives. The first student explained and justified his strategies in response to the second student's questions and challenges.
After the first student generated a strategy, the other student interacted in a synchronous and sustained way with the first student while carrying it out	At the beginning of the pair's interaction, one student gave a complete explanation of the strategy he intended to carry out. He continued to generate the strategies for the subsequent number sets while the other student made frequent comments and asks continual questions. In the discussion around one of those subsequent number sets, the other student noticed a mistake in his partner's work and offered an alternative path to solving the problem. That student went on to explain the strategy in thorough detail.
Students jointly produced the strategies and jointly carried them out	The pair consistently worked through the strategies jointly. For each number set, a student suggested a strategy that they could carry out together, one or both students suggested revisions to it, and they worked together to carry it out. Their interaction included voicing steps in unison, each student suggesting the next step in the strategy, each student suggesting alternatives, and jointly troubleshooting errors and discrepancies.

Table 6. *Cont.*

Pattern of How Interaction Unfolded in the Pair	Example of How Highly-Detailed Participation Emerged in the Pair
Students first solved the problem independently [2]	
Then students took turns sharing their strategies	Upon convening in groups, after working independently to develop multiple strategies, one group member requested that they take turns explaining their ideas. One student provided a fully-detailed explanation of how she had solved the first problem while the others listened. Then, the other two members of the group fully explained their strategies for subsequent problems.
Then students took turns sharing their strategies; after that, one student initiated further strategies, and the other student interacted in a synchronous and sustained way with the first student around those strategies	Upon convening in groups, one student immediately suggested that they "check with each other" about their strategies. This prompted her partner to explain her strategy in complete detail. After listening intently to her partner's explanation, the first student then provided a full explanation of her own approach to solving the problem.

[1] In these examples, both students in the pair exhibited highly-detailed participation. [2] On this particular day, unlike the other days, the teacher gave the class 15 min of independent time to work on the problems before having them gather in groups to discuss their strategies and work on further number sets.

4. Discussion

This paper investigated productive features of group interaction in pairs of students working to solve mathematical problems in which the students, and not the teacher, drove the interaction among students. Specifically, we examined patterns of group interaction that led to students giving highly-detailed explanations of their own ideas and engaging with others' ideas in highly-detailed ways, both of which have been found to be related to students' learning outcomes. Three major findings emerged. First, the number of group members taking the lead in contributing mathematical ideas about how to solve problems was significantly related to the number of students who exhibited highly-detailed participation. Contributing mathematical ideas did not only consist of making suggestions for how to begin or carry out problem-solving approaches. It also included reacting to, or building upon, others' ideas in ways that shaped the direction or nature of the work, such as suggesting extensions or alternatives. It did not include asking questions or disagreeing without offering new or alternative ideas.

The notion of taking the lead in contributing ideas is embedded in other researchers' perspectives about interaction among students that is productive for learning. For example, in exploratory talk [34–36], everyone "offers the relevant information they have" and "engages critically but constructively with each other's ideas" ([1], p. 187). In accountable talk ([2], p. 286), participants "build on each other's ideas . . . provide reasons when they disagree or agree with others" and "may extend or elaborate someone else's argument." In transactive discussions, students analyze and respond to their partners' ideas and assessments, rejecting ideas and proposing alternatives where relevant [5,6]. In argumentation, students present their ideas and listen to and criticize each other's explanations [7–10,30]. In collaborative reasoning, [40], after indicating their positions on an issue and offering reasons for them, students "either support and add to the reasons expressed or challenge and offer alternative reasons" ([11], p. 583). Co-construction [27] includes "additions (linking a new idea to someone else's idea or partial idea), corrections . . . or dialectical exchanges (disagreeing with the prior statement and offering a counterargument)" ([16], p. 394). The foregoing work shows that contributing ideas take many forms, and do not necessarily correspond with surface features of interaction such as which student talks the most or which student controls most of the conversation. What our findings make explicit is the importance of *every* student in a collaborative group having, and taking, opportunities to take the lead in contributing ideas that shape the group's work and move it forward.

Second, whether students interacted in an ongoing, sustained manner or interacted only periodically was not associated with the number of students in the pair who exhibited highly-detailed participation. While conferring in a sustained and ongoing fashion was one way in which students could converse productively, it was not the only way. Students could also interact with each other occasionally but not continuously, and still be engaged in ways that supported participation that led to highly-detailed participation.

Other researchers have also found that interaction around the task need not be continuous, and in particular that off-task interludes may be productive. For example, participation often characterized as off-task talk may serve to help groups ease into the task, may serve to help certain students gain access or re-access to the conversation, and can be used to resist the tendency of particular students to dominate groups' discussions [70,80].

Third, group interaction unfolded in many different ways over time, and multiple profiles of group interaction led to the emergence of highly-detailed participation for both students in the pair. In particular, highly-detailed participation among both members of the pair occurred whether both students or only one student initiated the problem-solving strategies, whether students worked on the same or different strategies, whether they worked through them jointly or separately, or whether they worked in an ongoing, sustained fashion or conferred only occasionally after periods of independent work. The multiplicity of productive group profiles is consistent with work by Shah and Lewis showing that collaboration that is equitable overall ("fair distribution of both participation opportunities and participation itself" ([65], p. 423) can have levels of equity and inequity that fluctuate over the course of collaboration, such as students alternating taking leadership roles [65,72].

The assortment of productive group interaction profiles we observed suggests that there is not a single "right way" for students to interact. An implication of these findings is that, when seeking to promote collaborative problem solving, teachers should not assume that particular patterns of interaction are "productive" or "unproductive." Rather, it is important to look beyond surface features to attend to the substance of students' participation: do students find particular moments to ask each other questions or make suggestions, do their contributions shape and add details to emergent strategies, and do they create openings for others to participate in collaborative work? These results are consistent with other research that highlights the importance of collaborations that involve students explaining and engaging with the substance of one another's ideas, while also showing that avenues to meaningful engagement can take varied forms [70,71,81]. Who initiates problem-solving strategies, for example, may be less important than whether and how students' questions and suggestions are considered by others in the group and incorporated into the group's work.

These findings highlight the importance of teachers monitoring the participation occurring in collaborative groups and then finding ways to interact with groups in the moment to encourage students to contribute new ideas. A number of researchers have identified ways in which teachers can do this. For example, Gillies [45,46] describes teachers' mediating behaviors that can encourage students to suggest new ideas, including providing hints to consider new information or perspectives, questioning how multiple ideas are the same and different, and asking groups to evaluate whether all necessary ideas have been covered. Many of the talk moves described by O'Connor and Michaels [47] explicitly call for students to contribute new thinking, such as asking students to add on to others' ideas and to explain why they agree or disagree. Multiple instructional moves used by the teacher in collaborative reasoning discussions [11,50] function similarly, including asking for alternative perspectives, and challenging students to consider counter-arguments.

While the group interaction profiles that we observed to be associated with both students in the pair exhibiting highly-detailed participation are similar to profiles observed in prior research [82], we do not claim that this is an exhaustive set of profiles nor that this set of profiles will necessarily generalize to all classrooms. The teachers included in this study had extensive experience using students' ideas as a basis for instruction, and were recruited based on their interest in furthering their expertise in supporting student participation. The profiles of group interaction that may emerge in classrooms of teachers without this level of experience will be a fruitful area for further research.

Similarly, as previous studies have suggested, the variety of group interaction profiles observed must be considered in relation to the nature of the group's task. The tasks teachers created in this study in some ways varied from those recommended by other scholars [83–85]; they were neither ill-structured nor procedural in nature. Tasks were open in the sense that they did not require students to use particular strategies, and instead asked for students to solve problems in ways that made sense to them. Yet in other ways tasks were somewhat closed in that the story problems themselves were relatively straightforward and had a single answer. The influence of the design and enactment of tasks on the variety of group interaction profiles that may emerge remains to be investigated. In addition, teachers in this study did not install specific structures for who participates, when, and how [43]. How such structures may impact the variety of productive group interaction patterns that emerge is another topic for further study.

Still unknown is whether the group interaction profiles we observed may vary according to the membership of the group, or over time even for groups with the same composition [55,56]. For example, students may make different choices on when and how they participate based on the synergy with their partners on that day. Finally, most of the groups we observed were pairs. Whether and how the size of the group influences the profiles of group interaction that emerge remains to be investigated [58].

In conclusion, we observed a wide variety of interaction patterns in collaborative groups that led to productive participation by all members of the group. These results suggest that teachers and researchers should not be concerned with legislating ways in which groups should interact. Instead, it may be more fruitful to work toward creating opportunities for students to engage with each other during collaborative work in ways that make sense to them at that moment.

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