education sciences

Article

Training in Mathematics Education from a Sustainability Perspective: A Case Study of University Teachers' Views

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Abstract: Humankind is faced with major global challenges to ensure the future of our planet. Target 4.7 of Sustainable Development Goal 4 (quality education) of the 2030 Agenda establishes the importance of ensuring that all learners acquire the theoretical and practical knowledge needed to promote sustainable development through education for sustainable development, human rights, gender equality, and global citizenship, amongst others. The research presented in this article is based on an exploratory study on the views of a group of university teachers from the area of Didactics of Mathematics in the Faculty of Education Sciences at Universidad de Cádiz with regard to Education for Sustainability and its integration in the field of training future teachers in mathematics education. It aims to analyse the perceptions teachers have regarding the subject of study, the work methodologies they use, etc. in greater depth. The authors of the research later interviewed three teachers in the area of Didactics of Mathematics who teach in different education degrees: bachelor's degree in early childhood education, bachelor's degree in primary education, and master's degree in teacher training for compulsory secondary education and baccalaureate. Three instruments were used to collect data: the syllabi of the subjects for which the teachers were responsible, an initial exploratory questionnaire, and a semi-structured personal interview. The instrument employed to analyse the information was the Tool for Methodological Analysis through Sustainability (Herramienta de Análisis Metodológico desde la Sostenibilidad, HAMS in Spanish). The results show the different views and positions the university teachers in the area of Didactics of Mathematics at Universidad de Cádiz have with respect to integrating sustainability into future teachers' training in mathematics education. It is concluded that the effective integration of sustainability into mathematics education requires university teachers to change and to work together from the same perspective—the one they intend to promote.

Keywords: higher education; curriculum for sustainability; critical mathematics education; teacher training; EDINSOST project; HAMS

Citation: Moreno-Pino, F.M.; Jiménez-Fontana, R.; Domingo, J.M.C.; Goded, P.A. Training in Mathematics Education from a Sustainability Perspective: A Case Study of University Teachers' Views. *Educ. Sci.* **2022**, *12*, 199. https://doi.org/10.3390/educsci12030199

Academic Editors: Athanasios Mogias, George Malandrakis and Panayota Koulouri

Received: 13 February 2022
Accepted: 8 March 2022
Published: 11 March 2022

1. Introduction

Education should aim to be a reflection of the social needs of a country, including the needs of the students who participate in it as learners if what is desired is to guide them and provide them with an opportunity for personal fulfilment to live in the 21st century [1]. However, the current educational system, conceived in the intellectual culture of the Enlightenment, and strongly promoted thanks to the improved economic circumstances of the Industrial Revolution [2], does not satisfy the needs demanded by many of today's societies, some of which are highly technological. In this context, the National Council of Teachers of Mathematics (NCTM) maintains that mathematics education should be reviewed in depth and claims that the new social objectives of education require the following: teachers with more mathematical culture, continuous learning, opportunities for all, and an informed electorate [1].

However, we live in an increasingly complex and globalised world [3], "a multi-dimensional social system in which there is a permanent phenomenal multi-causality" [4] (p. 34). The review that mathematics education should undergo must be in-depth if we want to equip it in its complex nature. The changes we refer to cannot be reduced to a set of objectives that enumerate social needs.

Generally speaking, a curriculum plans educational proposals, implements them, and assesses them within a specific social context. It is the social context that demands answers to at least three key questions: How is knowledge understood? How is learning interpreted? How is teaching being addressed? The answers to these questions are neither unique nor closed. However, any curriculum must offer specific answers that guide these questions. The choice of response to each of these questions will shape one curriculum structure or another.

The ecological, economic, educational, values, and thinking crises are the ones that question our knowledge of the world [5]. This leads us to the first question: How is knowledge understood? It is essential to reflect on this question because the conception of (mathematical or other) knowledge derives, to a large extent, from how its understanding is conceived.

Currently, the dominant neo-liberal economy in which we are immersed continues to produce individualistic models of thought in education. Knowledge is conceived as a (subjective) psychological entity [6–10]. However, the current planetary crisis [11] requires training critical, supportive, responsible, ethical human beings who are able to transform their reality [4]. Education cannot be limited to spreading knowledge; it must be an all-encompassing project [6–10]. Mathematics and mathematics education should be configured around the development of solutions to problems created by human beings and their way of life, which occur at a given time and in a given context [6–10].

Within the range of socio-cultural theories that seek to approach teaching and learning in terms of critical mathematics education, the theory of objectification (TO) can be found. The work carried out in this study is framed within the non-mentalist conception of thought assumed by this educational theory, which is distanced from those educational theories of an individualistic nature. In the TO, mathematical knowledge is not a psychological entity but is conceived as a historical–cultural entity [9]. The TO defines mathematics education as "a political, social, historical and cultural effort the purpose of which is the creation of ethical and reflective individuals who position themselves critically in historically and culturally constituted mathematical practices" [9] (p. 135). According to the TO, the learning of mathematics is defined based on two axes: the axis of "mathematical knowledge" and the axis of "mathematical being" that run through what this educational theory defines as "collective work". Sustainability in education emerges as an option for the construction of possible answers that should respond to some basic principles [5]. Numerous researchers have analysed the integration of sustainability principles into university curricula, and they agree that future graduates need to be equipped with competency-based training [12–14].

Integrating sustainability competencies into teacher training in mathematics education would allow creating an awareness of the conflicts and critical structures of society. It would also encourage the development of a reflective competency as an inherent part of mathematics literacy within the framework of a critical mathematics education [15].

In the international framework, a basic reference for incorporating sustainability into the curriculum is the ACES Network's Green Curriculum Network in Higher Education, which was created in 2000 within the Alfa programme of the European Union. One of the important results of this project was the definition of a series of criteria that allows identifying the suitability of a sustainable curriculum. Subsequent research [16] established an educational competency framework for sustainability in the training of future teachers based on these criteria.

In Spain, the Sectoral Commission of the Conference of Rectors of Spanish Universities, CRUE-Sustainability, approved the Guidelines for the Introduction of Sustainability in the Curriculum document [17] in 2005 and updated it in 2012. CRUE-Sustainability asked the university community for a comprehensive review of university curricula, ensuring the inclusion of the basic cross-curricular content in sustainability in all the degrees of the Spanish university system in agreement with the four cross-curricular competencies in sustainability (SUST) defined by the CRUE as follows:

- SUST1: Competency in the critical contextualisation of knowledge through interrelating social, economic, and environmental issues at a local and/or global level.
- SUST2: Competency in the sustainable use of resources and in the prevention of negative impacts on the natural and social environment.
- SUST3: Competency to participate in community processes that promote sustainability.
- SUST4: Competency to apply ethical principles related to sustainability values in personal and professional behaviour.

Integrating sustainability into university curricula means, on the one hand, that students develop competency in the critical contextualisation of knowledge. It thus concerns teaching in which the ways of producing knowledge and the circulation of ideas occur in a critical, reflective, and democratic manner. On the other hand, integrating sustainability into university curricula also means that students develop competency in applying ethical principles related to the values of sustainability. It thus regards teaching in which the forms of social cooperation are those in which the relationship to others is one of respect, responsibility, and solidarity—values that strengthen the principle of global perception. From this perspective, it is important to note how education for sustainability is a type of education in line with the postulates of the TO.

Incorporating sustainability into mathematics education requires an in-depth study of what elements would make it possible. The research presented in this article is based on an exploratory study regarding the view of a group of university teachers from the area of Didactics of Mathematics at Universidad de Cádiz (Spain) with respect to education for sustainability and its possible integration in the field of training future teachers in mathematics education. First of all, we wanted to know to what extent sustainability is part of the view this group of teachers has of the teaching and learning process they are in charge of planning themselves. Secondly, we intended to identify the elements necessary for teacher training in mathematics education when it is approached from the perspective of sustainability. This study is limited to the Professional Development of the Teacher Research Group-HUM462 of Universidad de Cádiz (Spain) within the framework of the EDINSOST [18] project in which the authors of this research participate.

2. Materials and Methods

This paper presents the partial results of a more general research study [19] within the EDINSOST research project: "Education and social innovation for sustainability. Training of professionals as agents of change in Spanish universities to address the challenges facing society"- (Reference: EDU2015-65574-R).

One of the limitations the authors found in a previous research study related to the documentary and curricular analysis of the presence of sustainability competencies in the syllabi in the area of Didactics of Mathematics was that the teachers could make a subjective interpretation of the syllabus they are responsible for [20]. It was therefore considered appropriate to carry out an exploratory study on the view university teachers in the area of Didactics of Mathematics have regarding education for sustainability and its integration into the teaching planning of their respective subjects. The survey technique was used to conduct the exploratory study.

The study here presented is not based on previous hypotheses but seeks to bring to light and explore the viewpoints of the teachers from the area of Didactics of Mathematics [21] in order to establish relationships and their possible motivations regarding the research topic. It is therefore a research study situated within the interpretive paradigm specified in an exploratory case study, which occurs at a specific moment in time and spatial location [22].

2.1. Sampling Method

Assuming that knowledge is a social construction made by humans and for humans [23], and considering that this research does not seek to generalise results but rather to approach the object of study to better understand it, and, in accordance with our research paradigm, we opted for a small and deliberate sample.

We started from the idea that any training proposal introduced in the field of mathematics education training seeking to integrate sustainability into the syllabi of the subjects of the area of Didactics of Mathematics should consider the views of the teachers entrusted with providing its training. Three teachers from the area of Didactics of Mathematics in the Faculty of Education Sciences at Universidad de Cádiz who voluntarily agreed to be recorded during the interviews were the main source of information.

The design of this research is therefore considered a multiple case study insofar as it encompasses different individual views. Our work is thus framed within the paradigm of interpretive research with a qualitative approach since it is a study that is oriented towards the understanding of socio-educational phenomena. As said earlier, the chosen sample was deliberate and small and was linked to the interest [24] in analysing the views of these three teachers regarding education for sustainability and its possible integration into the initial training of future teachers in mathematics education.

The three teachers participating in this study, who we will refer to as T1, T2, and T3, teach in three different education degrees: bachelor's degree in early childhood education (degree of a global character), master's degree in teacher training for compulsory secondary education and baccalaureate (degree in which a more professional focus is adopted, considering the students already have a bachelor's degree), and bachelor's degree in primary education (degree of a segmented nature, divided into different disciplines), respectively.

2.2. Research Objective and Questions

The general objective of this research was to study the views a group of teachers from the area of Didactics of Mathematics in the Faculty of Education Sciences at Universidad de Cádiz has regarding education for sustainability and its integration in the field of training future teachers in mathematics education.

To this aim, four specific goals formulated as four research questions were established in the study. The national and international review of the different theoretical frameworks related to mathematics education, education for sustainability and its possible integration into the initial training of future teachers in mathematics education, and in establishing the Tool for Methodological Analysis through Sustainability (HAMS), used as an analytical tool in this study (see Section 2.3.2), justifies the choice of these research questions.

- (Q1): What is the degree of integration of sustainability in the teaching planning of the subject in the area of Didactics of Mathematics for which teacher T1 is responsible?
- (Q2): What is the degree of integration of sustainability in the teaching planning of the subject in the area of Didactics of Mathematics for which teacher T2 is responsible?
- (Q3): What is the degree of integration of sustainability in the teaching planning of the subject in the area of Didactics of Mathematics for which teacher T3 is responsible?
- (Q4): Are there any notable differences between the three teachers from the area of Didactics of Mathematics regarding the degree of integration of sustainability in the teaching planning of the subject they each teach? If so, what information, through contrasting the three case studies, is relevant to advance towards integrating sustainability into teacher training in mathematics education?

2.3. Instruments

2.3.1. Data Collection Instruments

As mentioned earlier, in order to approach the object of study, this research mainly used the survey technique. The data collection instruments employed were an initial exploratory questionnaire and a semi-structured personal interview.

- Instrument 1 (I1). Initial exploratory questionnaire:
  The questionnaire is the basic instrument for obtaining data in research using the survey technique [25]. Izard [26] points out that a questionnaire should be created around the research objectives. Three scopes were therefore intentionally taken as references for the analysis. They complete and complement each other, and are in line with the general and specific objectives formulated in this study (see Section 2.2):
  1. Scope 1: The role of the university in general and of mathematics in particular regarding education for sustainability and its possible integration into higher education, especially with regard to integrating sustainability into the initial training of teachers in mathematics education.
  2. Scope 2: The learning model based on general sustainability competencies defined by the CRUE [17] within the framework of the European Higher Education Area. It refers to the opportunity this approach represents for teachers in the area of Didactics of Mathematics to integrate these sustainability competencies into the teaching planning of their respective subjects.
  3. Scope 3: The obstacles that, according to the teachers in the area, hinder incorporating sustainability into the syllabi of the area of Didactics of Mathematics in the Faculty of Education Sciences at Universidad de Cádiz.

Once these three scopes of analysis were established a priori, a questionnaire combining open questions (scope 1), multiple-choice questions (scope 2), and five-point Likert scale questions [27] (scope 3) was designed.

The questionnaire can be consulted in Appendix A. It was employed in a previous study [28] in which a group of seven teachers from the area of Didactics of Mathematics in the Faculty of Education Sciences at Universidad de Cádiz took part. Three of them participated in the present study.

One of the limitations encountered in the study mentioned above was the reduced number of answers provided by the seven teachers because the questionnaire was the only instrument used. The authors of this research therefore considered it relevant to include semi-structured personal interviews with teachers in order to analyse their views regarding the research topic in greater depth.

The objective of the questionnaire was twofold. On the one hand, its application allowed us to become familiar with the views of several teachers in the area of Didactics of Mathematics regarding education for sustainability and its possible integration into the initial training of teachers in mathematics education. On the other hand, the questionnaire was used as an additional criterion for choosing the three cases employed in the present study—teachers T1, T2, and T3—and to hold semi-structured personal interviews with them.

- Instrument 2 (I2). Semi-structured personal interview:

A flexible script of open questions around which the interview would revolve was developed in order to broaden and deepen the most relevant questions in the questionnaire. The questions of the personal interview were derived from the research questions [29] and the very design of the Tool for Methodological Analysis through Sustainability (HAMS). Likewise, the interview was designed based on a preliminary analysis of the questionnaires. The different questions of the semi-structured personal interview address aspects that turn around five different axes: the five dialogic axes around which the HAMS tool is set up. The script for the personal interview can be found in Appendix B.

Both the questions formulated for each of the three scopes of analysis that constitute the initial exploratory questionnaire and those that served as a guide for the semi-structured personal interview were directly selected from or inspired by important works in the field of mathematics education, education for sustainability, and its possible integration [17,30–34].

The items around which the questions formulated revolved were mainly from the teachers' perspective: What degree of participation does the teacher attribute to the students in the teaching–learning process, how does the teacher conceive the formulation of competencies, to what extent is the socio-environmental reality considered in their subjects, and what is the role of assessment in the teaching and learning process planned by the teacher?

- Instrument 3 (I3). Syllabi of the subjects:

As a complementary source of information, the syllabi of the subjects of each of the three teachers participating in this study were also considered. The syllabus is the official document prepared by university teachers in which the different elements of a subject are summarised: competencies, content, training activities, assessment system, etc. In this case, documentary analysis was used as the technique to study them.

2.3.2. Data Analysis Instrument

The data analysis instrument used was the Tool for Methodological Analysis through Sustainability (HAMS) by García-González et al. [33].

HAMS is a tool designed to analyse teaching praxis regarding education for sustainability from the teacher's perspective. The HAMS tool allows examining the role of the teacher within the teaching and learning process, the role assigned to the student, and the one assigned to the content, both in the "planning" of the subject put forward by the teacher, and in the "intervention" implemented by the teacher [32,33].

For each of the three agents that constitute the didactic system—the lecturer, the student, and the content—and for the two dimensions of "planning" and "intervention", HAMS is built around five dialogic axes. Each dialogic axis represents the dialectics between the two extremes of a binomial (methodological element), each of which is linked to the different parts that make up any teaching–learning process, namely:

- The relationship between the lecturer and the student (vertical vs. horizontal);
- The nature of the competencies (specific vs. cross-curricular);
- The socio-environmental reality (non-integrated vs. integrated);
- The nature of the resources (internal vs. external);
- The role of evaluation (summative vs. formative).

A series of indicators are defined for each of the five dialogic axes. The indicators express the degree of integration of sustainability in the study context in increasing order. Appendix C show how the HAMS tool was created for the "planning" dimension and for the three agents under analysis: the lecturer, the student, and the content. A complete version of the HAMS tool that includes the "intervention" dimension related to the three agents under analysis can be consulted in García-González [32].

2.4. Data Analysis

In qualitative research, the credibility of a study is used for its internal validity [35]. In this research, the phenomenon under study was described mainly by means of texts (the documents that make up the syllabi of the different subjects and the teachers' answers to the initial exploratory questionnaire and/or semi-structured personal interview). To ensure the credibility of our work, data triangulation through a plurality of techniques for data collection and analysis (described in Section 2.3) was considered appropriate. Researcher triangulation during the development and validation of the different data collection instruments, and during their analysis, was also considered relevant.

The documentary review of the syllabi of the different subjects was carried out by members of the EDINSOST project and by the Professional Development of the Teacher Research Group-HUM462, whose members are all familiar with the study context. The analysis system for the set of open questions in the initial exploratory questionnaire was based on the use of the content analysis technique [36]. The interviews (once recorded and transcribed) were also the subject of a first analysis that focused on their content [36].

The data analysis was planned in three different phases:

- In the first phase, once the data from the different sources were reduced to units of information, they were classified into different categories, each with its own meaning [37]. The categories were established deductively and correspond to the five methodological elements (dialogic axes) that constitute the analysis instrument described in Section 2.3.2 (HAMS). The flexibility of the HAMS tool enabled categorising the same unit of information in different dialogic axes.

- Once the units of information were categorised, assigning an indicator to each one of them made it possible, in the second phase of the work, to evaluate the degree of integration of sustainability into the teaching planning of the subjects. To this aim, the Transition towards Sustainability tool [32] was used. It classifies the indicators into five stages of transition towards sustainability—from basic to more complex stages (Table 1). Following García-González [32], three types of gradients were generated depending on whether the transition towards sustainability of a methodological element was described by three, four, or five indicators (see Appendix C).
- Due to the fact that the variables in this study were mostly qualitative, the relative frequencies for each of the HAMS indicators in the third phase of the study were calculated to facilitate the representation of the results. In other words, the relationship between the number of units of information that an indicator contained with respect to the total number of units of information contained in the dialogic axis to which said indicator belongs, was calculated. A map called the Transition to Sustainability Map could thus be defined in the teaching planning of the subjects that teachers T1, T2, and T3 are responsible for (see Table 2 in Section 3).

Table 1. Transition towards sustainability [32,33]. Reproduced with permission from Esther García-González, HAMS: A Tool for the Analyses of Methodological Activity of University Professors from the Principles of Sustainability and Complexity; published by Espacios, 2018.

Transition Towards Sustainability	Stages				
	Basic (Stage 1)	Elementary (Stage 2)	Intermediate (Stage 3)	Advanced (Stage 4)	Complex (Stage 5)
Gradient 1 (3 indicators)	1	-	2	-	3
Gradient 2 (4 indicators)	1	2	-	3	4
Gradient 3 (5 indicators)	1	2	3	4	5

2.5. Coding of Units of Information

An example of the coding used for the different units of information would be the following: T1_I2_L_RIE_1. This code returns a unit of information associated with teacher 1 (T1) and has been extracted from his personal interview (I2). It refers to the nature of the resources, internal vs. external (RIE), that the lecturer (L) considers. In the example, the stage of transition towards sustainability is one (1), which means that the unit of information is related to the "exclusive" use of internal context resources (see Appendix C).

3. Results and Discussion

Table 2 shows, expressed in percentages, the Map of Transition towards Sustainability resulting in the teaching planning of the subjects of the three teachers of the area of Didactics of Mathematics interviewed in agreement with the methodology described in Section 2. Said information is organised in accordance with the different dialogic axes for each of the three agents analysed: the lecturer (L), the student (S), and the content (C), as well as for each of the three teachers interviewed: T1, T2, T3 (rows), depending on the stage of transition towards sustainability in which each dialogic axis is situated: basic, elementary, intermediate, advanced, and complex (columns).

Table 2. Map of transition towards sustainability in the teaching planning of the subjects that teachers T1, T2, and T3 are responsible for.

Axis	Agent-T [1]	Stage				
		Basic	Elementary	Intermediate	Advanced	Complex
Relationship: Lecturer–Student Vertical vs. Horizontal	L-T1	-	-	1.00	-	-
	L-T2	-	-	1.00	-	-
	L-T3	-	-	0.71	-	0.29
	S-T1	-	-	1.00	-	-
	S-T2	-	-	1.00	-	-
	S-T3	-	-	0.58	-	0.42
	C-T1	0.67	0.33	-	-	-
	C-T2	0.25	0.50	0.25	-	-
	C-T3	-	-	0.33	-	0.67
Competencies Specific vs. Cross-curricular	L-T1	0.37	-	0.63	-	-
	L-T2	0.30	-	0.22	-	0.48
	L-T3	-	-	0.12	-	0.88
	S-T1	1.00	-	-	-	-
	S-T2	0.33	-	0.43	-	0.24
	S-T3	-	-	0.10	-	0.90
	C-T1	0.29	-	0.71	-	-
	C-T2	0.39	-	0.28	-	0.33
	C-T3	-	-	0.12	-	0.88
Socio-environmental reality Non-integrated vs. Integrated	L-T1	-	0.83	-	0.17	-
	L-T2	-	0.67	-	0.33	-
	L-T3	-	0.60	-	-	0.40
	S-T1	0.50	-	0.50	-	-
	S-T2	-	-	0.60	-	0.40
	S-T3	-	-	0.25	-	0.75
	C-T1	0.33	0.67	-	-	-
	C-T2	0.62	0.19	-	0.19	-
	C-T3	0.54	0.08	-	0.15	0.23
Resources Internal vs. External	L-T1	1.00	-	-	-	-
	L-T2	0.83	-	0.17	-	-
	L-T3	1.00	-	-	-	-
Evaluation Summative vs. Formative	L-T1	0.20	0.60	-	0.20	-
	L-T2	0.20	0.40	-	0.20	0.20
	L-T3	0.20	0.80	-	-	-
	C-T1	0.20	-	0.80	-	-
	C-T2	0.17	-	0.66	-	0.17
	C-T3	0.25	-	0.75	-	-

[1] L: Lecturer; S: student; C: content; Ti: teacher i, i = 1 (green), 2 (yellow), 3 (blue).

To answer the first three research questions (Q1, Q2, and Q3) in Sections 3.1–3.3, several figures are shown that represent the transition towards sustainability, expressed in percentages, in the teaching planning of each of the three teachers interviewed from the area of Didactics of Mathematics. The different figures represent the stage of transition towards sustainability in the teaching planning of their subject, analysed for each agent (lecturer, student, content) and in each of the dialogic axes in which they are organised. Their characterisation is justified through units of information that explain their meaning. The different figures illustrate the results summarised in Table 2.

To answer the fourth research question (Q4), Section 3.4 shows three graphs (spider charts) that allow us to obtain an image of the relationships between the different ways of working of the three teachers depending on how each of them understands the educational fact regarding sustainability and its possible integration into the training of teachers in mathematics education.

3.1. *Integration of Sustainability into the Teaching Planning of the Subject of Teacher T1*

Figures 1–3 allow answering the first research question (Q1): What is the degree of integration of sustainability in the teaching planning of the subject in the area of Didactics of Mathematics for which teacher T1 is responsible?

3.1.1. Lecturer Agent

Figure 1 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T1 is responsible. The stages are shown for each of the five dialogic axes that constitute the lecturer agent (L).

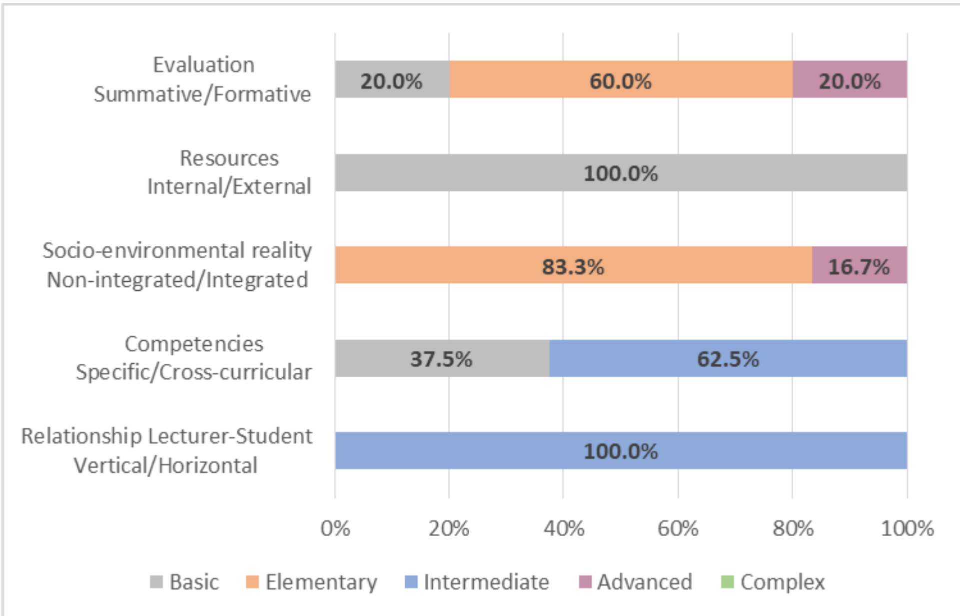

Figure 1. Stages of transition towards sustainability. Lecturer agent. Case T1.

As shown in Figure 1, most of the dialogic axes are represented by more than one indicator (stage). This explains that there are usually no unique or exclusive stages for each methodological element analysed. These results are in agreement with the study carried out by García-González [32]. The dialogic axes of the lecturer–student relationship and the resources are the only ones that have a single indicator identified in the intermediate and basic stages of transition towards sustainability. In his planning, T1 gives the students a certain role, but it is the teacher who leads the process. T1 stated that he uses active participatory methodologies in his classes when he says:

T1_I1_L_RVH_3: “Debate and dialogue between peers and with the teacher. Teamwork. These are methodologies used in the different subjects for the construction of mathematical knowledge”.

Likewise, in his planning, T1 referred only to internal contextual resources, although both in the initial questionnaire and in the personal interview he specified that they have been created with recycled material:

T1_I1_L_RIE_1: "One of the aspects used is the design of educational materials for the development of mathematical knowledge using recycled materials. Apart from giving them a new use, we work on mathematical knowledge".

With regard to the competencies, they appear explicitly in the syllabus of the subject. Some of them are formulated in specific terms linked to the subject, while others are of a more cross-curricular nature. This explains the different stages of transition towards sustainability for this methodological element: basic (37.5%) and intermediate (62.5%). Examples of these cases are:

T1_I3_L_CSC_1: "Know the scientific, mathematical and technological fundamentals of the curriculum of the early childhood education stage, as well as the theories on the acquisition and development of the corresponding learning".

T1_I3_L_CSC_3: "Promote interest and respect for the natural, social, and cultural environment through appropriate educational projects".

During the interview, T1 mentioned the difficulty of formulating the competencies of his subject in more global terms in order to consider sustainability in the syllabus when he said:

T1_I2_L_CSC_1: "It would mean to change everything stated in the curricula. It would indeed be possible, but with the necessary work of thinking about how to adapt them, how to include them in the planning of the subjects without causing a major upheaval among all the subjects that constitute a curriculum".

As far as the socio-environmental reality is concerned, 83.3% of the units of information, coming from the different sources of data collection and analysis (syllabus, questionnaire, and interview), is related only to determining some references to said reality in the teaching planning (elementary stage). To a lesser extent (16.7%), clearer statements were made regarding the socio-environmental reality (advanced stage). For instance, in the questionnaire, T1 considered mathematics as a discipline that should be structured around the development of solutions to problems related to reality and daily life:

T1_I1_L_RNI_4: "Mathematics education cannot turn its back on reality. In fact, if we look at the curricula for early childhood or primary education, they state mathematics should be oriented towards or aimed at activities related to the pupils' daily lives".

Finally, the evaluation provided by T1 was aimed mainly at the students. However, the variability in stages for this methodological element (basic → elementary → advanced) is attributed to the fact that the evaluation appears in the planning at different times and that different instruments and sources of information are used.

In addition to proposing the traditional written exam at the end of the process (the most basic stage), T1 also considers other aspects in the evaluation, such as participation and active engagement of the students in the classroom and in the group, the level of preparation of different theoretical reports, proper use of recycled material in resource design, and peer evaluation or co-evaluation (a more advanced stage).

T1_I1_L_ESF_2: "We assess different aspects, including the use of recycled material in the design of teaching material for future early childhood education teachers".

3.1.2. Student Agent

Figure 2 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T1 is responsible. The stages are shown for each of the three dialogic axes that constitute the student agent (S).

The socio-environmental reality is, in this case, the only methodological element represented by more than one indicator. This methodological element is characterised in two stages: basic (50%) and intermediate (50%). On the one hand, although the syllabus of the subject refers to certain socio-environmental issues, their relationship with the professional role of the students in the future (basic stage) is not explicitly expressed. On the other hand, in the personal interview, T1 did refer to certain aspects related to the socio-environmental reality and to the fact that the students should learn or become aware as future education professionals (intermediate stage) when he stated:

T1_I2_S_RNI_3: "Take advantage of this quality education by connecting it not only to logical mathematical knowledge, but also to other aspects, such as the use of (recycled) materials that would otherwise end up in the trash, this is the education our students have to learn and take into account".

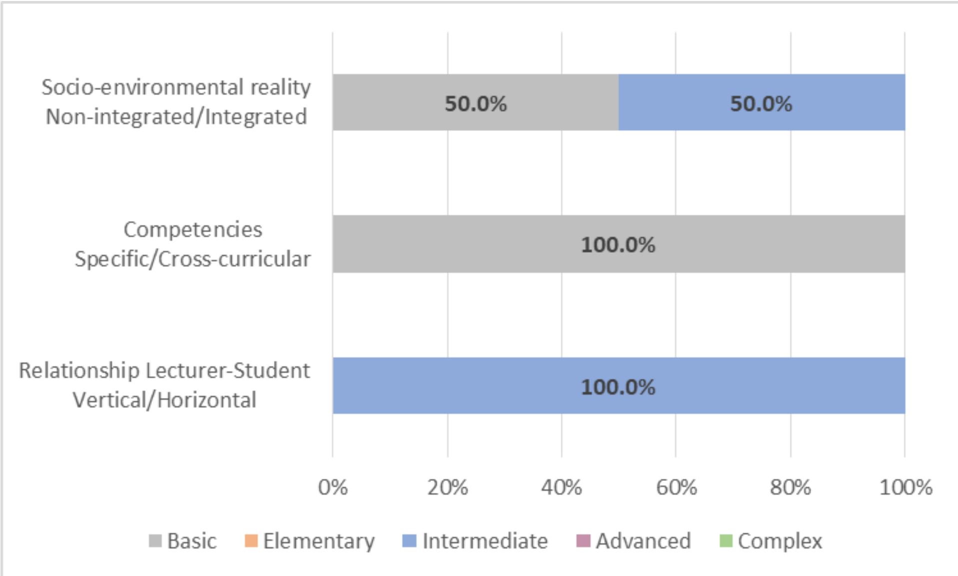

Figure 2. Stages of transition towards sustainability. Student agent. Case T1.

However, as observed in his statement, T1's view on sustainability is clearly biased since he puts special emphasis on the more environmental aspects (reducing, reusing, and recycling) to the detriment of other aspects of interest such as those of a social, economic, or cultural nature.

For this agent, the dialogic axes of the lecturer–student relationship and the competencies are represented by a single indicator (100%) in the intermediate and basic stages of transition, respectively. Regarding the first dialogic axis, the students and their interests appear as an element to be taken into account, but they are not the key players in the teaching and learning process, which is led by the teacher. T1 stresses carrying out activities related to the students' daily lives since he knows that the students show the most interest when relating the theory of the subject to the practical aspects:

T1_I1_S_RVH_3: "The main criterion to follow is to carry out activities related to the daily lives of both primary and early childhood education pupils".

T1_I2_S_RVH_3: "When I design my subjects, I consider content, how to put them into practice, especially from a practical point of view, so that they are useful in their daily lives. The relationship between life and content always occupies that practical part, what interests me is a practical part the students consider useful, that give meaning to what they are working on, that they do not see as something abstract that is useless".

With respect to the competencies, the basic stage of transition towards sustainability is attributed to not finding any references to the professional role of students in any of the units of information. From the different sources of information and the data analysis, competencies are merely perceived as the development of skills specific to the subject in the official curriculum of the degree.

3.1.3. Content Agent

Figure 3 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T1 is responsible. The stages are shown for each of the four dialogic axes that constitute the content agent (C).

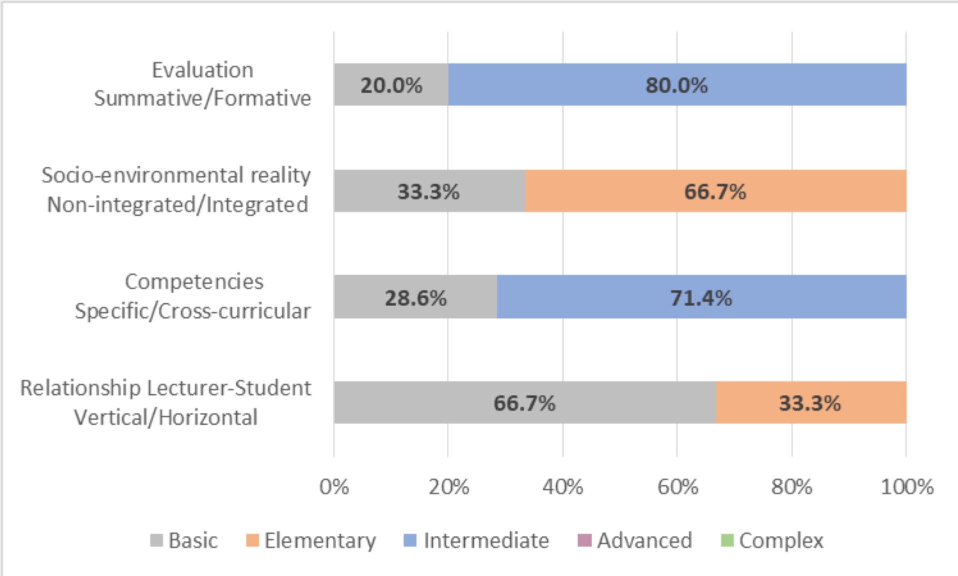

Figure 3. Stages of transition towards sustainability. Content agent. Case T1.

As shown in Figure 3, each of the four dialogic axes is represented by two indicators in different stages of integration of sustainability.

A total of 66.7% of the units of information refers to a basic stage of transition towards sustainability in the dialogic axis of the lecturer–student relationship for the content agent (C). This is because the content generally appears in a closed manner and is organised by topic. Likewise, it is inferred from the answers to the questionnaire that due to a lack of time, T1 finds it difficult to address content that does not correspond to the disciplinary organisation:

T1_I1_C_RVH_1: “I do not have enough time to cover all the topics of my subjects”.

In the interview, T1 admitted that he has only made small changes in his teaching planning, with mathematical knowledge being the main objective of the process:

T1_I2_C_RVH_2: “I try to make small changes, adjustments, which I think are good for the students to be able to work, in this case, on logical mathematical knowledge”.

While some of the specific competencies of the syllabus only refer to the use of knowledge and content specific to the subject, others interrelate different types of knowledge. This enables the dialogic axis of the competencies to move from a basic stage (28.6%) to an intermediate stage (71.4%). In the questionnaire, T1 considered the possibility of interrelating different types of knowledge to develop certain competencies. He thus referred to researching through ethnomathematics as a discipline that studies the relationships between mathematics and culture.

Ethnomathematics can indeed promote the development of cultural sustainability in teacher training in mathematics education—learning from what exists and recognising the cultural and identity values of each society:

T1_I1_C_CSC_3: “Conducting research through ethnomathematics, analysing what students from other cultures, or even our ancestors know about mathematics and its effect on society”.

The stage of transition towards sustainability for the socio-environmental reality axis is incipient. A total of 33.3% of the information units was categorised in a basic stage, while the rest (66.7%) was grouped in an elementary stage. The main reason that explains these stages of transition towards sustainability is that there is no constant presence of the socio-environmental reality in the content of the subject. However, in the interview, T1 was open to modifying the content of his subject to achieve the same objectives. This approach seems appropriate to us since the content of a syllabus must be understood as a means for the development of certain competencies and not as an end in itself:

T1_I2_C_RNI_2: "A situation to work on sustainability with the students could be used simply by changing the paradigm. In this situation, and using this example, the same contents could be worked on. In fact, I think it is convenient to constantly change situations and examples".

Finally, the evaluation provided by T1 was only aimed at the students (intermediate stage). In addition to focusing on conceptual knowledge by means of a written exam (basic stage), he also considers other aspects of the teaching and learning process. The following is stated in the syllabus:

T1_I3_C_ESF_3: "The evaluation will also take into account clarity when presenting, the quality of reasoning, and the complexity of the relationships the student establishes".

3.2. Integration of Sustainability into the Teaching Planning of the Subject of Teacher T2

Figures 4–6 allow answering the second research question (Q2): What is the degree of integration of sustainability in the teaching planning of the subject in the area of Didactics of Mathematics for which teacher T2 is responsible?

3.2.1. Lecturer Agent

Figure 4 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T2 is responsible. The stages are shown for each of the five dialogic axes that constitute the lecturer agent (L).

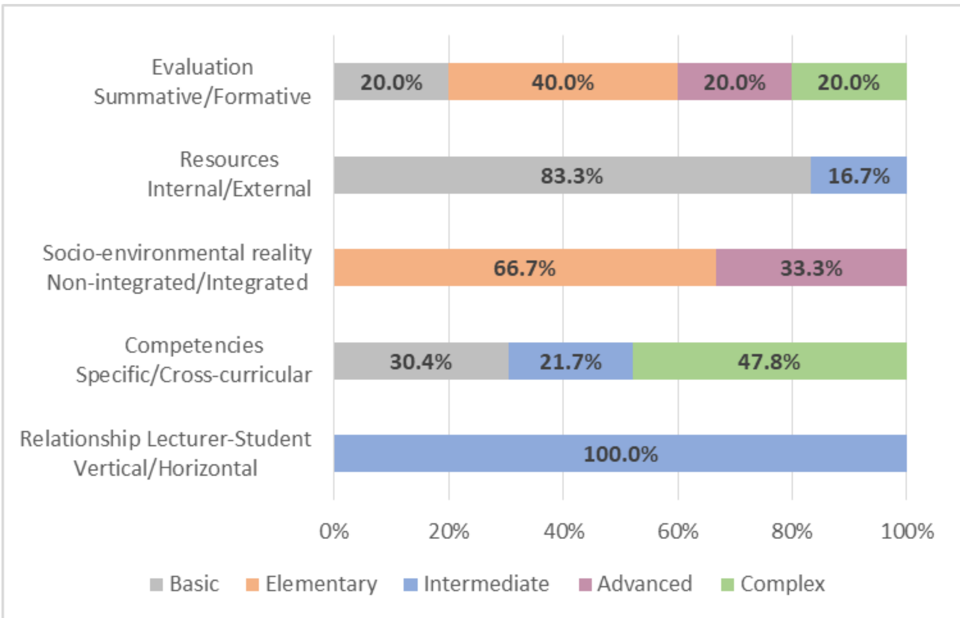

Figure 4. Stages of transition towards sustainability. Lecturer agent. Case T2.

Figure 4 shows, with the exception of the axis of the lecturer–student relationship, how all the methodological elements are represented by more than one indicator or stage.

The stage of transition towards sustainability for the axis that reports on the kind of lecturer–student relationship is intermediate. This means that, although the students are given a certain leading role, teacher T2 leads the process. T2 points out the following in the questionnaire:

T2_I1_L_RVH_3: "Methodologies that stimulate critical analysis based on everyday situations, as well as developing work together".

As far as the competencies are concerned, the teaching programme groups them into three types: specific, general, and cross-curricular. This allows a transition from more basic stages (30.4%) to more complex stages (47.8%) in this methodological element. In the questionnaire, T2 commented that he integrates the four sustainability competencies defined by the CRUE [17] into his teaching planning. In the personal interview, he mentioned that he is in favour of a more global formulation of competencies in order to promote mathematics education in agreement with education for sustainability. T2 stated:

T2_I2_L_CSC_5: "Everything that is related to sustainability is established in a cross-curricular manner, which means it is up to the teacher (. . .) If we change that cross-curricular nature and include it in the hard core of the competencies that must be developed, I believe it would help considerably to implement it in a more effective manner".

Regarding the socio-environmental reality, 66.7% of the units of information, coming from the different sources of data collection and analysis, is related to determining some references made to said reality in the teaching planning (elementary stage). T2 stated in the questionnaire:

T2_I1_L_RNI_2: "In the planning there are elements related to sustainability, but it does not appear as a section defined in it".

He then added certain difficulties found when planning his subject, as can be inferred from his answer to the questionnaire expressed using a Likert scale. He considers it to be a barrier that hinders incorporating sustainability into his subject:

T2_I1_L_RNI_2: "I do not know any methodologies that I could use to integrate sustainability into mathematics education".

However, after deepening the analysis, T2 offered a clear statement regarding the socio-environmental reality in the personal interview. This allows progressing towards a more advanced stage of transition towards sustainability (33.3%):

T2_I2_L_RNI_4: "Of course, right? When it comes to learning, for example, topics that I believe are very easy (to deal with) in mathematics, such as education for consumption (. . .), and, of course, equality, which is something that we address in the master's degree. We talk about consumption, about gender equality".

With respect to the nature of the resources, they are mainly internal (83.3%). In the planning, the activities considered are practicums, seminars, problems, tutoring, and evaluation. In the questionnaire, T2 stated:

T2_I1_L_RIE_1: "I find it difficult to find suitable materials to introduce these topics".

However, the syllabus does include visits to other centres, which allows us to move, though in an incipient way, towards an intermediate stage of transition towards sustainability for this methodological element (16.7%):

T2_I3_L_RIE_3: "Visit to specific contexts related to education".

Finally, regarding the evaluation, four different stages of transition towards sustainability were identified in the teaching planning of T2. The variability in indicators is due to the fact that for the evaluation, in addition to the final written exam, other follow-up instruments, such as an individual and group portfolio, are considered. They ensure an evaluation system that is more formative than summative. In the questionnaire, T2 claimed that he uses individualised evaluation instruments:

T2_I1_L_ESF_5: "I use individualised instruments that seek to reduce inequalities, in line with some of the objectives of the 2030 Agenda".

3.2.2. Student Agent

Figure 5 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T2 is responsible. The different stages are shown for each of the three dialogic axes that constitute the student agent (S).

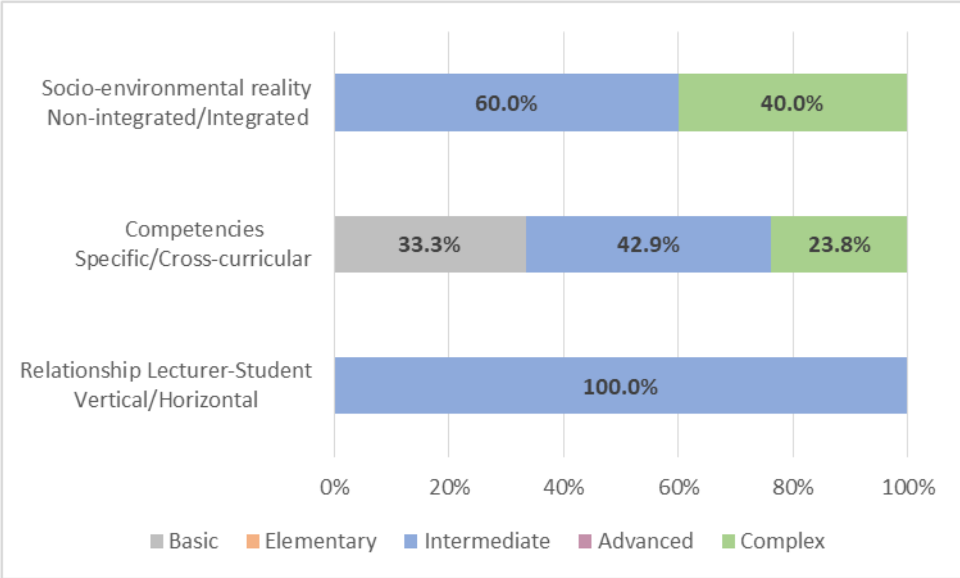

Figure 5. Stages of transition towards sustainability. Student agent. Case T2.

The lecturer–student relationship is the only methodological element represented by an indicator in an intermediate stage of transition towards sustainability in the teaching planning. Both in the questionnaire and in the personal interview, T2 commented that he takes the students and their interests into account, as well as the expressions the students use. However, he leads the process:

T2_I2_S_RVH_3: "When I design my subjects, even before I consider the contents, the first thing I think about is the human material I am going to work with, it is the starting point. Once I know that human material, I think about the contents established by law I will teach, and later, I think about how to make those contents match the human material I have in front of me".

T2_I2_S_RVH_3: "I believe that the language we use in the classroom has to be in line with what the students are already using. In that regard, using videos, digital experiences, any kind of communication in the media they use will benefit that what we want to achieve".

The reason why the dialogic axis of the competencies goes from basic (33.3%) to intermediate (42.9%) to complex (23.8%) is due to the fact that they do not only focus on the development of skills specific to the subject (basic stage) but also include the training of professionals committed to improving their environment and that of critical autonomous citizens (complex stage). For example, in the interview, T2 referred to mathematical literacy and linked it to the professional role of the students in the same way as the National Council of Teachers of Mathematics [1]:

T2_I2_S_CSC_3: "Mathematics is a very useful tool from a professional point of view. I believe that it helps students progress, enter the job market, and overcome inequalities. Encouraging both boys and girls to pursue their calling facilitates equality".

The syllabus of the subject T2 teaches also considers the development of these kinds of cross-curricular competencies:

T2_I3_S_CSC_5: "Promote and ensure respect for human rights and the principles of universal accessibility, equality, non-discrimination, democratic values, and the values of the culture of peace".

Finally, the socio-environmental reality is defined by two indicators in intermediate (60%) and complex (40%) stages of transition towards sustainability.

In the syllabus of the subject, some relationships are indeed established between the socio-environmental reality and the future professional role of the students. This occurs in an implicit manner in the learning outcomes and explicitly in the training activities planned in the subject. For example:

T2_I3_S_RNI_3: "Incorporate the science-mathematics and mathematics-society dimension into the teaching profession".

T2 commented in the questionnaire how the socio-environmental reality should be a pillar in the training of students when he said that mathematics must face interdisciplinarity. The fact that interdisciplinarity encourages the integration and relationship of mathematics with other disciplines in order to solve real problems should not be overlooked.

T2_I1_S_RNI_5: "I believe that mathematics should be strongly globalised and interdisciplinary. The main reason is that this is the society in which our students live, and in which they will work and grow as citizens. The school cannot be oblivious to this situation".

3.2.3. Content Agent

Figure 6 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T2 is responsible. The different stages are shown for each of the four dialogic axes that constitute the content agent (C).

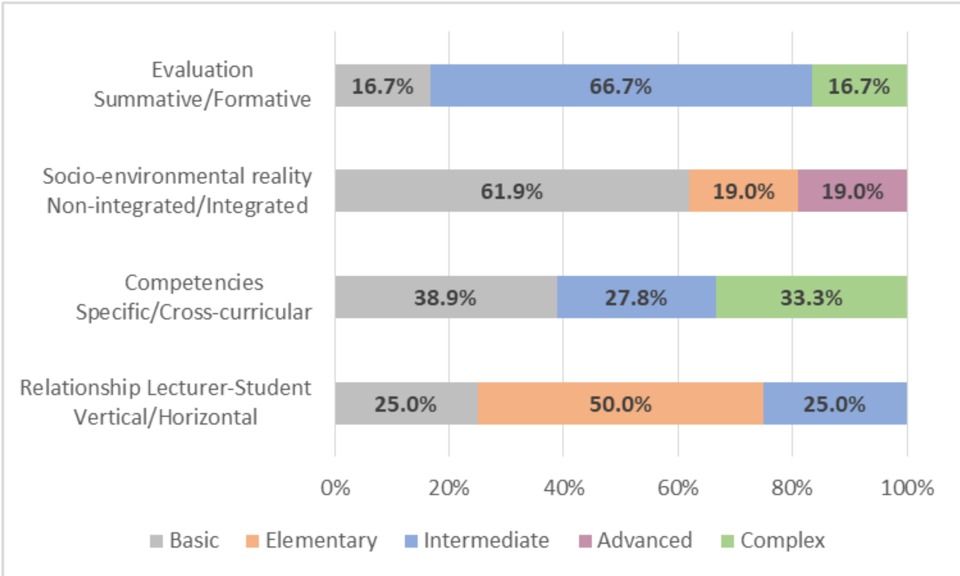

Figure 6. Stages of transition towards sustainability. Content agent. Case T2.

As shown in Figure 6, each of the four dialogic axes is represented by three indicators in different stages of transition towards sustainability.

Regarding the lecturer–student relationship, the syllabus of the subject T2 teaches is grouped by content blocks (not by topics), and, although the content is the main objective of the process, the blocks are sufficiently flexible to allow room to address other content of interest. In the interview, T2 mentioned that he works on his subject:

T2_I2_C_RVH_2: "The analysis, from a mathematical point of view, of news that appears in the media, related to many topics: related to the environment, the use and misuse of statistics to manipulate people (. . .) of how mathematical language is sometimes used as a barrier to hide reality, etc".

In the interview, when T2 was asked what he would like to change or introduce in the teaching planning of his subject, he answered:

T2_I2_C_RVH_3: "I would like us to not be so slaves to contents (. . .) I would change the fact that each subject is a sealed world that is not related to the other subjects because, I believe, you cannot look at the world critically when you are wearing glasses that only let you see a part: the glasses of mathematics, the glasses of language, the glasses of physics. I would change that, perhaps by areas, grouping subjects, and considering learning through different perspectives. We would contribute to training critical citizen, and that, right now, is essential".

As far as the competencies are concerned, the variability in stages of transition towards sustainability for this methodological element is explained, mainly, by the inclusion of specific (basic stage), general (intermediate stage), and cross-curricular (complex stage) competencies in the syllabus. In addition, the distribution of units of information that account for these three stages for this dialogic axis is practically homogeneous in percentage terms: 38.9% (basic stage), 27.8% (intermediate stage), and 33.3% (complex stage).

With regard to the socio-environmental reality, in the questionnaire, T2 showed how the mathematics content promoted the analysis of aspects related to said reality. He pointed out:

T2_I1_C_RNI_2: "Mathematics allows giving opinions based on data, not intuitions, it enables detecting fallacious reasoning, and develops the ability to make inferences that allow one to act knowingly".

It is inferred from the interview that the way T2 addresses the content is in agreement with the principles of sustainability, and it is beyond the purely disciplinary. We consider that the socio-environmental reality has a constant presence in the content of the subject T2 teaches when he commented:

T2_I2_C_RNI_4: "I think about the knowledge I transmit to my students. I want it to be a sustainable mathematics education in the sense that it generates equality, in the sense that the inequality that may exist among my students is reduced through educational action".

T2 clearly moves away from reductionist approaches in mathematics education and incorporates the mathematics-reality dimension when he says:

T2_I2_C_RNI_4: "Sometimes, we focus a lot on (mathematical) concepts and it seems that there are concepts that are alien to reality, but I think the opposite is true; there are few subjects in which we cannot give examples that come from reality. And if we bring the reality that we want to bring, the one that seems very important to us, it will be the starting point so that later on mathematical learning contributes to the student looking at that reality in a different way".

T2's evaluation focuses on conceptual knowledge through a written exam. However, he not only includes other elements of the teaching and learning process but also makes proposals for their improvement when, in the questionnaire, T2 states:

T2_I1_C_ESF_3: "I think the evaluation could be improved by including, as part of the evaluation process, projects that improve the way we consume in order to consume more responsibly".

3.3. Integration of Sustainability into the Teaching Planning of the Subject of Teacher T3

Figures 7–9 allow us to answer the third research question (Q3): What is the degree of integration of sustainability in the teaching planning of the subject in the area of Didactics of Mathematics for which teacher T3 is responsible?

3.3.1. Lecturer Agent

Figure 7 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T3 is responsible. The stages are shown for each of the five dialogic axes that constitute the lecturer agent (L).

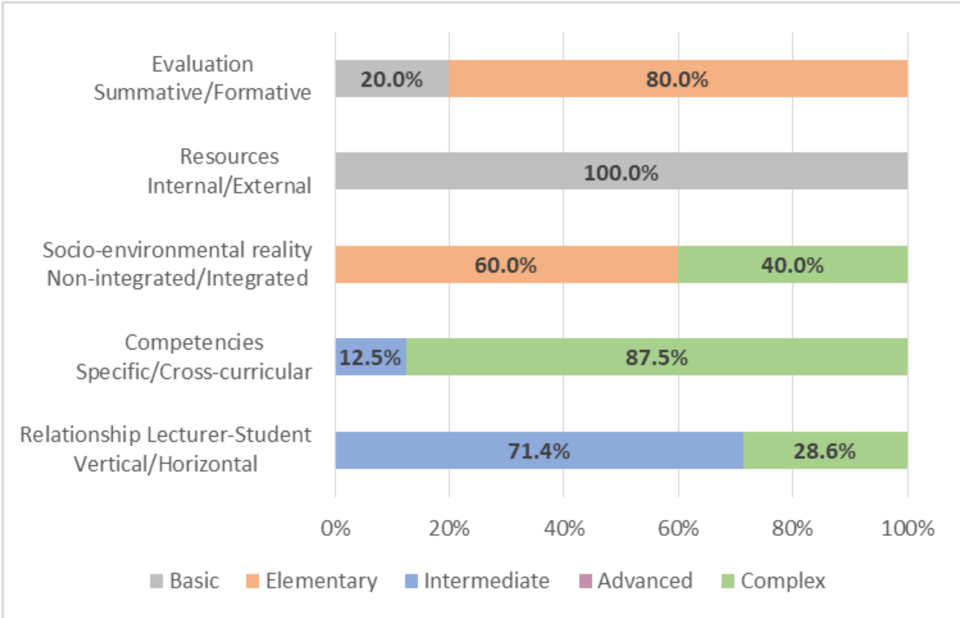

Figure 7. Stages of transition towards sustainability. Lecturer agent. Case T3.

In Figure 7, it is observed how the resources are the only methodological element represented by a single indicator in a basic stage of transition towards sustainability. In other words, in the teaching planning, only internal context resources are referred to. However, in the personal interview, T3 did consider that it is key to address aspects related to the environment through different types of materials in the teaching and learning process. Unfortunately, the current situation of the COVID-19 pandemic, together with a lack of time (subjects that have a low number of credits), makes it difficult to move towards stages of greater transition towards sustainability in this methodological element.

T3_I2_L_RIE_1: "There are a lot of resources that can be used. The problem is that there is not enough time".

T3_I2_L_RIE_1: "I work a lot with materials, group work, trying to solve problems (. . .) including environmental and social criteria in the purchasing criteria (. . .) The main problem is that there is very little time".

Regarding the lecturer–student relationship, the methodology provided by T3 reflects the participation of all agents in the teaching and learning process, which makes it possible to progress towards more complex stages of transition towards sustainability (28.6%). T3 does not only give the students a certain role but also includes them in his teaching planning, where they assume the role of apprentices. Excerpts related to these aspects are:

T3_I2_L_RVH_5: "A teaching–learning process means continuous listening of the students, continuous feedback they give you, which will help you learn as a teacher (. . .) All the inputs that you give have to be based on this continuous feedback".

T3_I3_L_RVH_5: "The percentages of the tests will be agreed upon with the students at the beginning of the course".

Regarding the competencies, it is worth pointing out that the very nature of the subject T3 teaches, framed in the specialisation of the integrated primary education curriculum, facilitates the achievement of more complex stages regarding the transition towards sustainability in the teaching planning (87.5%). In the syllabus, the competencies are formulated in global terms, and specific and cross-curricular competencies converge. In the personal interview, T3 commented that he works on cross-curricular competencies related to education for sustainability:

T3_I2_L_CSC_5: "I always consider developing competencies such as systemic view, a forward-looking approach, empathy, collaboration, problem- solving, because I think they are very important, I consider this to be essential".

However, T3 stated that the convergence of specific and cross-curricular competencies can help integrate sustainability into mathematics education but that it is not the main thing. He stated that the fundamental thing is for the teachers to acquire those competencies:

T3_I2_L_CSC_3: "The way competencies are formulated can help. I am more concerned about the fact that the person who has to help the students develop them has those competencies. They need to know what they are talking about and what methodologies they are using. That worries me more than how they are written although, obviously, the way they are written can help understand them. A person who knows what specific math skills and sustainability skills are, even if they are written down separately, has a very good chance of teaching them properly".

Addressing mathematics from a more integrated perspective, such as the one offered in the subject T3 teaches, allows the socio-environmental reality to become the axis from which the syllabus is built, thus advancing towards more complex stages of transition towards sustainability (40%). In the interview, T3 mentioned how the same socio-environmental reality allowed him to deal with scientific aspects. However, T3 does not so much focus on scientific knowledge, which he does consider, but on the training of competent professionals in its management:

T3_I2_L_RNI_5: "We started by addressing the management of the school structure, taking as a starting point the management of a house, a home: the consumption of water, energy, food, and addressing some scientific aspects of all this, but in a very basic way, because without having solid knowledge of science it is very difficult to get a good understanding of climate change, energy and all the relationships at stake. However, the students do understand that by getting information and researching, a lot can be done from the management of an organisation to reduce many impacts on climate change, water pollution".

Finally, the evaluation was represented by only two indicators in the basic (20%) and elementary (80%) stages, respectively. T3 plans more than one evaluation instrument and applies those instruments during or at the end of the process. They are only aimed at the students. Apart from a written exam, T3 provides other evaluation instruments, such as direct observation of the students in their involvement in the classroom, individual and group work through the analysis of experiences, problem-solving, reading articles, etc.

3.3.2. Student Agent

Figure 8 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T3 is responsible. The stages are shown for each of the three dialogic axes that constitute the student agent (S).

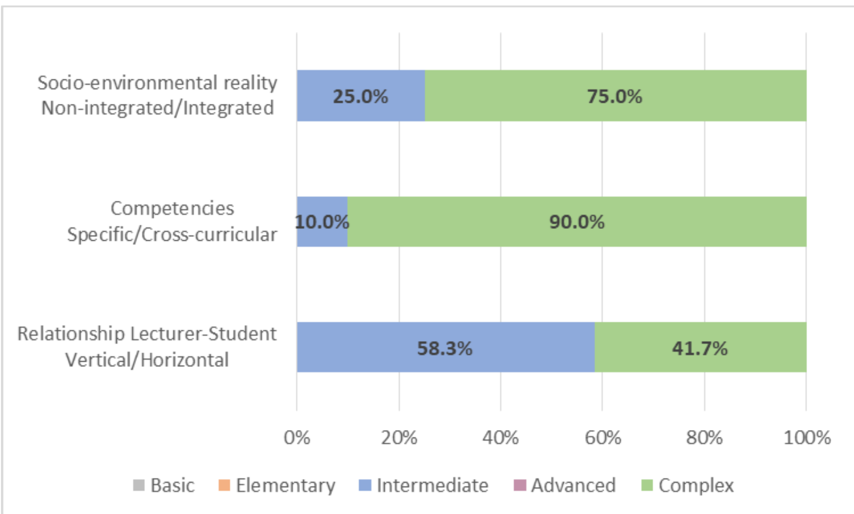

Figure 8. Stage of transition towards sustainability. Student agent. Case T3.

As shown in Figure 8, each of the three dialogic axes is represented by two indicators in intermediate and complex stages of transition towards sustainability.

Regarding the lecturer–student relationship, the teaching planning refers to the involvement and active participation of the students in the teaching–learning process, including the possibility of making decisions:

T3_I3_S_RVH_5: "Being able to manage a primary school classroom integrating different kinds of knowledge".

In the interview, T3 showed how the teaching–learning process is co-managed, takes the students into account, and involves them to an extent that goes beyond the mere discussion and debate of ideas among peers. The teaching and learning process is oriented in accordance with the concerns of the students, and is not led exclusively by the teacher:

T3_I2_S_RVH_5: "We performed an analysis with the students, not so much about preconceptions, but about their interests, their motivations in life, their socio-economic and environmental concerns. It was to get an idea of what direction to take, and I noticed that there was a lot of interest in sustainability issues. It was a quite fertile ground".

Likewise, T3 stated that he follows methodologies with characteristics of otherness. This is an inevitable aspect of a conception of education based on the "other". Implicitly, T3 shows how the actions and decisions that students make in their lives will be influenced by the level of development of skills such as empathy, a forward-looking approach, etc. throughout their training.

T3_I2_S_RVH_5: "I try to put myself in the position of others. I especially use exercises to develop empathy and help (the students) to imagine envisioning themselves in the role of other more unfortunate or more fortunate people. And help the students to feel their emotions. Emotions help because, if the students understand that it is something important to them, they will appreciate mathematics, physics, chemistry, and also sustainability, education for sustainability, more".

With respect to the competencies, 90.0% of the units of information, coming from the different sources of data collection and analysis, was related to training critical autonomous citizens committed to improving their environment (complex stage). In the interview, T3 stated that education should:

T3_I2_S_CSC_5: "Help people be motivated and empower them so that they can act in their day-to-day life, whether at work or in their family, in order to reduce their social and environmental impacts from shopping habits, energy consumption to food, as much as possible".

T3 also expressed the need to develop an appropriate mathematical competency to avoid expert-cracy, which is similar to what Skovsmose pointed out [15].

T3_I2_S_CSC_5: "For an expert society, I don't know how to put it, for a society with a modern level of awareness, everyone should have basic knowledge of mathematics to understand the world around them".

Likewise, T3 gave an account of his commitment to integrating sustainability into the syllabus of his subject. His reasons were related to training critical, literate, autonomous professionals in an integrating manner:

T3_I2_S_CSC_5: "I have always considered integrating sustainability into the curriculum of my subjects for many reasons. Since the idea of the mission of the Universidad de Ortega y Gasset, who said that the person had to be trained in an integrating talent. And this is now lost because the world has become more and more specialised".

Finally, 75% of the units of information referred to the socio-environmental reality as an essential pillar in student training. For example, from the reflection made by T3, it can be inferred how proper training in mathematics and sustainability would enable students to become aware of the human influence on trend changes from the analysis of real problems:

T3_I2_S_RNI_5: "Sustainability and mathematics are key to understand these things. The sustainability of the world cannot be understood without mathematics. And above all, we need to understand that we can change it, adopt a forward-looking approach. If you do not have the concepts of mathematics, you will not understand, you will not connect it to your actions, you will not understand what is important, you will not change your behaviour with respect to the use of energy, water, food, and everything that is useful for your health and that of the planet".

3.3.3. Content Agent

Figure 9 shows the different stages of transition towards sustainability, expressed in percentages, in the teaching planning of the subject of the area of Didactics of Mathematics for which teacher T3 is responsible. The stages are shown for each of the four dialogic axes that constitute the agent content (C).

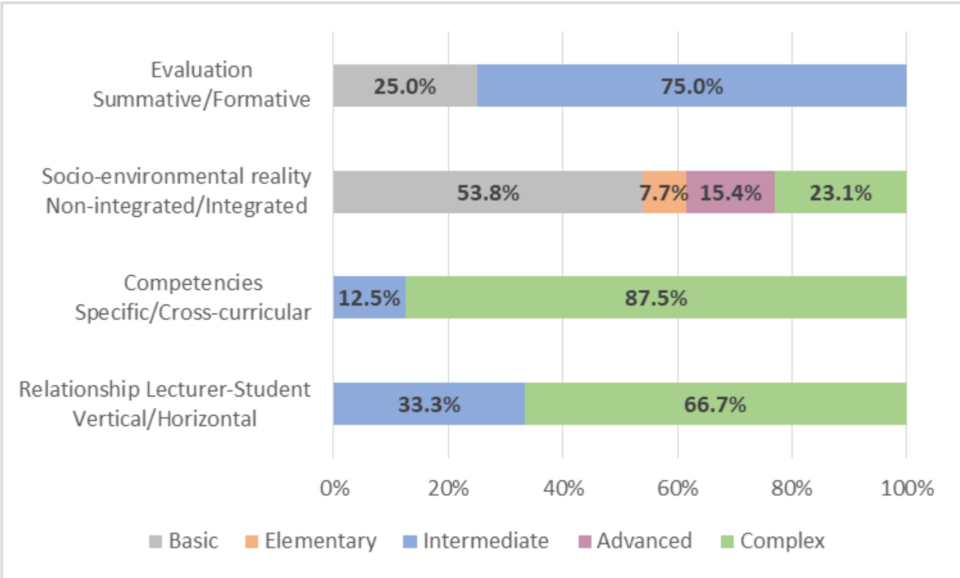

Figure 9. Stages of transition towards sustainability. Content agent. Case T3.

Figure 9 shows how the socio-environmental reality is the only methodological element defined by the four indicators that make up this dialogic axis. The rest of the methodological elements are represented by two indicators in different stages of transition towards sustainability.

The very nature of T3's syllabus, which is of a more integrated nature, enables the horizontality of his subject in an intermediate stage of transition towards sustainability (33.3%). However, 66.7% of the units of information refers to a complex stage in this dialogic axis because of T3's understanding of the content, which is based on problem-solving. In the interview, T3 stated:

T3_I2_C_RVH_5: "In the subject, we use the excuse of an umbrella fund for a project on the SDGs, because under this umbrella fund, each student that pretended to be a teacher developing an integrated project could implement their project on a topic, any topic they wanted. The SDGs were a great umbrella to fake financing adaptable to the needs of addressing the contents in different educational centres".

As regards the competencies, 87.5% of the units of information is grouped in a complex stage of transition towards sustainability. The same reasons as those stated above justify this fact. The syllabus includes the development of cross-curricular competencies as part of the knowledge:

T3_I3_C_CSC_5: "Analyse and critically incorporate the most relevant issues of today's society that affect family and school education: social and educational impact of audio-visual languages, screens, changes in gender and intergenerational relations, multiculturalism and interculturality, discrimination and social inclusion, and sustainable development".

In the interview, T3 commented on how knowledge alone does not enable students to become knowledgeable. He believes it is essential to work on the development of cross-curricular competencies as part of knowledge:

T3_I2_C_CSC_5: "I consider above all the aspect of developing competencies and working with emotions. As a competency, above all, the systemic view of observing the relationship between social, economic, environmental, and daily behaviour, as well as emotions, the interests of each one. I have always worked like this. Contents in themselves do not do much to improve the world, for people to act positively. I always keep this in mind".

Different indicators were assigned to the set of units of information that were categorised in the socio-environmental reality axis, which justifies the different stages of transition towards sustainability for this methodological element.

In the syllabus, both content specific to the subject (basic stage) and other content related to issues of the socio-environmental reality (elementary stage) were identified. However, it can be inferred that the socio-environmental reality has a constant presence in the content of the subject (advanced stage) when, in the questionnaire, T3 explicitly stated that he prioritises the NCTM [1] content standard "data analysis and probability" in his teaching planning. T3 considers it essential to develop a proper stochastic literacy in mathematics education to understand the problems of the world.

T3_I1_C_RNI_4: "The ability to read and compare numbers, data, data sets, probabilities, to understand graphs and trends, with a systemic view and a critical spirit is key to understand the condition of the world and how to act more responsibly as an individual or collective/organisation. Analysing data is essential to understand data on the pandemic, climate change, loss of biodiversity, poverty, or inequalities".

Furthermore, the integrative nature of his subject allows him to affirm how the socio-environmental reality can act as the axis from which to design it (complex stage).

T3_I1_C_RNI_5: "My subject allows me to integrate sustainability in a fairly cross-curricular manner. The subject has been planned to integrate, in addition to mathematics, the areas of language, social and environmental education".

T3_I2_C_RNI_5: "Using the excuse of the SDGs, we integrate it very well".

Concerning the evaluation, the units of information coming from the different sources of information and analysis are concentrated in indicators whose stage of transition towards sustainability is lower-intermediate. This was observed previously when we analysed this same methodological element regarding the role T3 assumes in his teaching planning (see Figure 7). As Jiménez-Fontana et al. [5] point out, evaluation is one of the methodological elements that has remained engraved in the ideas and actions of many teachers, and this seems to be the case for T3.

3.4. Contrasting Analysis

Figures 10–12 allow us to answer the fourth research question (Q4): Are there any notable differences between the three teachers from the area of Didactics of Mathematics regarding the degree of integration of sustainability in the teaching planning of the subject they each teach? If so, what information, through contrasting the three case studies, is relevant to advance towards integrating sustainability into teacher training in mathematics education?

In Figures 10–12, three spider charts represent the current stage of transition towards sustainability in the teaching planning of the subject of each of the three teachers. It is in accordance with the analysis carried out for each of the three agent-objects of this study: lecturer (Figure 10), student (Figure 11), and content (Figure 12).

In the spider charts, each of the (green, yellow, and blue) lines represents a different teacher, and each of the vertices of the different graphs represents a methodological element (dialogic axis), in accordance with the model by García-González et al. [33] described in Section 2 of this article.

The most central lines close to value "1" (basic stage) refer to stages of less integration of sustainability, while those farthest from the centre and close to value "5" (complex stage) suggest moving towards stages of greater integration of sustainability into the teaching planning of the subjects of the area of Didactics of Mathematics.

Figures 10–12 contain the same information as Table 2. However, the purpose of the spider charts is to complement that information in order to facilitate a contrast analysis. To perform this analysis, the methodology assumes that the "degree of global integration of sustainability" (GIGS in Spanish) in which a methodological element is found (dialogic axis) is given by:

$$GIGS = \begin{pmatrix} a & b & c & d & e \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = a + 2b + 3c + 4d + 5e \quad (1)$$

In which values $(a \quad b \quad c \quad d \quad e)$ represent, for each methodological element (dialogic axis), the relative frequencies corresponding to the basic, elementary, intermediate, advanced, and complex stages, respectively (this information can easily be found in the different rows of Table 2). Values $(1 \quad 2 \quad 3 \quad 4 \quad 5)^T$ represent the five values in accordance with the scale proposed in the Transition towards Sustainability tool by García-González [32]. It is evident that $1 \leq GIGS \leq 5$ in all cases.

Figures 1–9 allowed us to characterise the integration of sustainability into the teaching planning of the subject of each of the three teachers interviewed from the area of Didactics of Mathematics in a series of methodological elements (dialogic axes) and for each of the three agents that make up the didactic system: lecturer, student, and content. We agree with García-González [32] when she maintains that the model of the lecturer directly influences the role it grants to the students and the content of their subject. A careful examination of the charts in Figures 10–12 corroborates this idea, and, except for a few cases, it is generally true that $GIGS(T1) \leq GIGS(T2) \leq GIGS(T3)$ in each of the methodological elements (or dialogic axes) analysed.

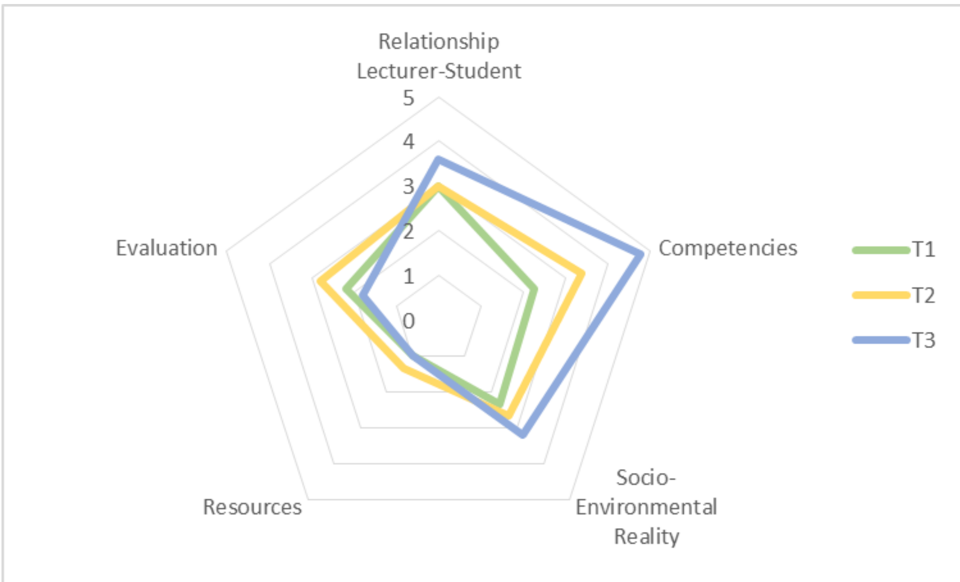

Figure 10. Contrast analysis. Lecturer agent.

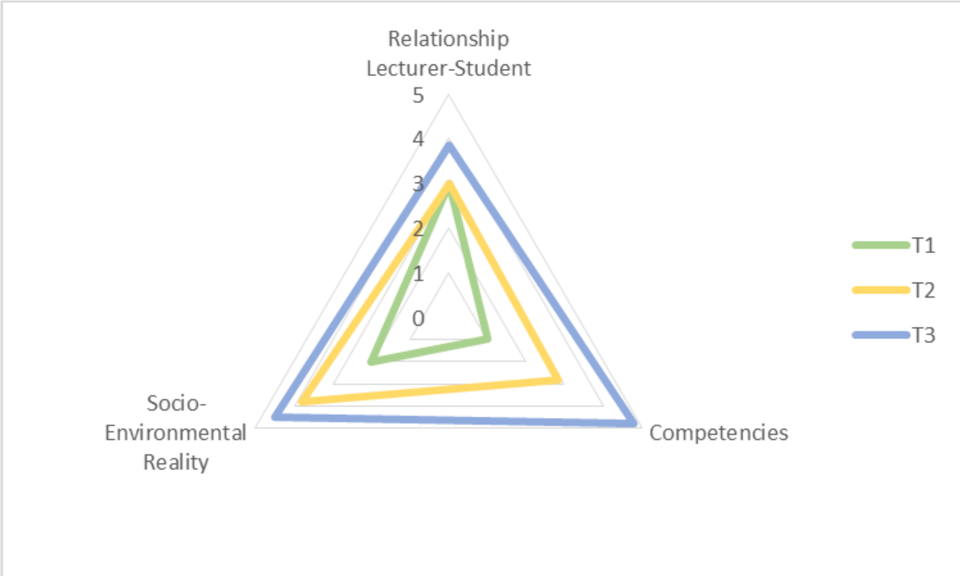

Figure 11. Contrast analysis. Student agent.

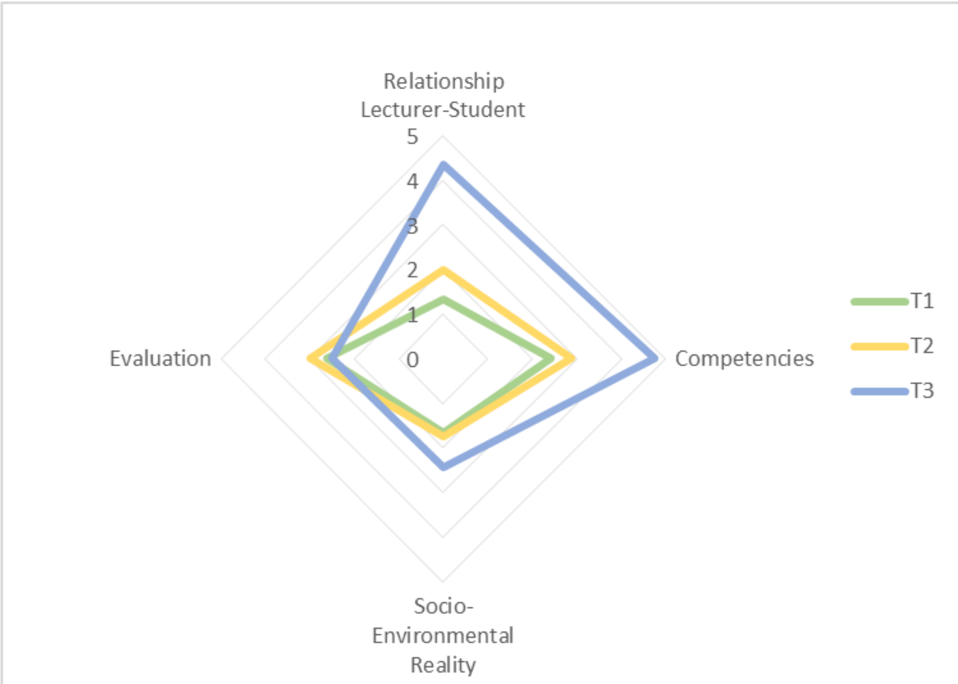

Figure 12. Contrast analysis. Content agent.

With respect to the lecturer–student relationship, $GIGS(T3) > GIGS(T1) = GIGS(T2) = 3$ in the lecturer and student agents (Figures 10 and 11). Assuming that learning mathematics is a relational and affective process [10] between students and teachers, this result explains how, in cases T1 and T2, the ethics provided in their classrooms in which the relationship to otherness is, to a certain extent, alienating. However, the responses that include characteristics of otherness provided by T3 in the interview guided our task. The Brundtland report [38], a founding reference of education for sustainability, defines sustainable development as that which meets the needs of the present without compromising the needs of future generations. Along these lines, we identify future generations as "the others" who challenge us and force us to take responsibility for future generations. We consider that training in mathematics education seeking to integrate sustainability into the syllabi of its subjects should look for mechanisms that enable developing ethics in which the relationship with intergenerational otherness is one of respect and solidarity. T3 is committed to the cross-curricular development of competencies, such as envisioning alternative future scenarios, clarifying values, and/or managing emotions, which is consistent with the competency framework proposed by Cebrián and Junyent [16] for integrating sustainability into higher education curricula. In mathematics education, a powerful option may also be to promote shared work in the classroom based on the study of different sources, such as those referring to scientific publications on mathematics education and sustainability [39] that allow students to become aware of the human influence on trend changes through analysing real problems.

For the content agent, $GIGS(T3) > GIGS(T2) > GIGS(T1)$ (Figure 12), there is, on the one hand, the more integrated view of T3 on content planning and, on the other hand, T2's commitment not to present content in a closed manner. This explains the greater degree of global integration of sustainability for this methodological element in this agent with respect to T1 (Figure 12). This means there is a tendency towards the training of a teacher profile in mathematics connected to the contextual environment [39]—in other words, a profile trained to understand the interwoven relationships that take place between the natural, social, economic, and cultural environments both locally and globally [20]. However, we consider that reorganising the curriculum established for training in mathematics education using holistic criteria is not enough. As García-González [32] points out, it is about promoting a change in the way of understanding knowledge in which mathematical knowledge is contextualised historically and culturally in order to provide it with meaning [6–10]. It is therefore necessary to rescue, within mathematics education, other relational concerns of a social, cultural, economic, political, philosophical, axiological, and democratic nature, in addition to those purely related to mathematical knowledge.

As to competencies, $GIGS(T3) > GIGS(T2) > GIGS(T1)$ in the three agents: lecturer, student, and content (Figures 10–12). The commitment T3 displays with regard to training a teaching profile that is "knowledgeable" in mathematics as a critical autonomous citizen explains the greater degree of global integration of sustainability for this methodological element, compared with T1 and T2. If good teachers are those who have an integrating talent, it is urgent to adopt a complex view of mathematics itself as an object of teaching and learning. Mathematics education cannot take the form of a disjointed and decontextualised delivery of information—even less so if the aim is to integrate sustainability competencies into the training of future teachers in mathematics education. "In the teaching and learning of mathematics, both the knowledge at stake and the training of the student as a human subject must be studied" [9] (p. 135). Mathematical literacy should be in keeping with this. It should be a competency that integrates not only mathematical knowledge but also reflective knowledge [15]. It would allow student teachers to critically encounter systems of ideas and actions that are culturally and historically constituted [6–10]. T3's statements go along these lines and are in tune with Giroux's formulations [40] that education should be defended as a public service that educates students to "be critical citizens who can think, challenge, take risks and believe that their actions will make a difference in society in general" [40] (p. 214). At the heart of critical thinking is the belief in possibilities and in the need to build alternative and better futures [41].

Regarding the socio-environmental reality, the fact that T1 does not consider it a pillar in the training of students for their professional development, together with T2's statements about his lack of knowledge of appropriate methodologies he could use to incorporate sustainability into the syllabus of his subject explains that $GIGS(T3) > GIGS(T2) > GIGS(T1)$ for this dialogic axis (Figures 10–12). However, in the three cases (T1, T2, and T3), there is a consensus regarding the need to incorporate the mathematics-reality dimension into mathematics education. T1 mentioned how mathematics education cannot turn its back on reality. T2 commented that mathematics should face interdisciplinarity. T3 even spoke of the need to develop a proper stochastic literacy in initial teacher training in mathematics education in order for future teachers to acquire a better understanding of the indeterministic reality that underlies the problems of sustainability. Quintanilla and Gallardo [42] point out that "mathematical formalisms make sense in real life situations" and add that "mathematical knowledge is built in processes of exchange and shared reflection mediated by language" [42] (p. 35).

Sales de Aguiar and Paterson [43] consider dialogic education that is linked to real-world experiences to be an optimal approach to teaching and learning about sustainability issues. Promoting dialogic mathematics in initial teacher training in mathematics education seems appropriate to us since it allows the construction of mathematical knowledge and at the same time encourages the development of people's identity in an ethical way [42]. Problem-based learning, project-based learning, or problem-solving are manifestations for a critical education [15] consistent with this approach that goes beyond the traditional approach of standard problems—closed (one-answer) problems. Mathematical modelling of problems is also a powerful tool for integrating sustainability principles into mathematics education training, especially when we involve the student teachers in the construction of normative systems that generate value judgments about a phenomenon or a fact of interest. From this perspective, incorporating the mathematical modelling of problems in mathematics education would facilitate coherence between ethical action and its mathematisation in a conscious (reflexive) manner. It would thus be possible for normative models built with the help of mathematics to be open to criticism, debate, and improvement [44]. The sustainable development goals of the 2030 Agenda are also a good resource for working with real situations in the subject of mathematics [45,46]. The methodologies here referred to break with the traditional pattern of the passive student in support of more interactive mathematical learning that is linked to the participation of the students in the training in a mathematics education based on dialogue directly connected to real-world problems.

The resources element is constituted as the methodological element whose transition towards sustainability is the lowest: $1 = GIGS(T1) = GIGS(T3) \approx GIGS(T2) = 1.34$ (Figure 10), in the only agent for which it is defined (lecturer). This explains why, in the teaching planning, the three teachers interviewed mainly referred to the use of internal context resources.

In a previous study [28], the answers provided to the questionnaire (Appendix A) by a group of teachers from the area of Didactics of Mathematics at our university allowed us to anticipate that the teachers do not disagree on the philosophy of sustainability, consider that they are important issues, and are not afraid of giving too negative a view of the future. However, they also commented that they find it difficult to integrate sustainability into mathematics education and do not know any appropriate materials they could use for including sustainability in the syllabus of their subject. In other words, the obstacles identified that hinder integrating sustainability into the syllabi in our area of knowledge are more related to teachers' aptitudes than to their attitudes. This result is in line with other similar studies [47]. However, it is clear that in order to improve the sustainability competencies of student teachers, it is first necessary to improve the sustainability competencies and aptitudes of the teachers who train them [48], as T3 commented in the interview.

Several studies [49,50] reveal that a significant proportion of university teaching staff does not feel capable of redesigning their subjects towards sustainability. We consider it necessary to plan training courses for all those university teachers interested in including sustainability in the syllabi of their subjects. They would be provided with training in the necessary content, which they could use to renew their syllabi. In any event, this is an aspect that requires further study. One of the objectives of the EDINSOST2-SDGs project, in which the first author of this study takes part, is the creation of a multidisciplinary free-access learning resource bank (Open Educational Resources, OERs) that helps teachers with the task of introducing education for sustainable development in their subjects.

Finally, the degree of global integration of sustainability identified for the evaluation, in the two agents in which it is defined (lecturer and content) follows a different pattern with respect to the rest of the methodological elements previously analysed: $GIGS(T2) > GIGS(T1) > GIGS(T3)$.

Traditionally, evaluation is limited to specific moments in time, usually at the end of a course. It focuses on the student, and its function is more summative than formative. Paradoxically, this is what seems to happen in the case of T3. However, if, as we said earlier, our aim is to promote dialogic mathematics in mathematics education training, evaluation then takes on a key role since it fulfils the function of regulating that dialogue, which is a determining factor in configuring the ethical dimension of sustainability in the students, who are future mathematics teachers [5]. The use T2 makes of individualised evaluation instruments that aim to reduce inequalities, or the proposal of including projects consistent with the principles of sustainability in said process, explains the greater degree of global integration of sustainability for this methodological element, in comparison with T1 and T3.

3.5. Limitations of the Study

This study has several limitations that should be taken into account when evaluating the results.

- In the first place, the analytical tool used in this study (HAMS) allows analysing the teaching praxis regarding sustainability and examining the role of the teacher within the teaching–learning process and the role the teacher assigns to the student and to the content, both in terms of the "planning" of their subject and the "intervention" implemented in their classroom. However, due to the COVID-19 pandemic, the teachers interviewed did not have access to the classroom. For this reason, only one of the two dimensions of analysis was addressed in this study: the teachers' "planning", but not their "intervention" in the classroom.
- Secondly, in this study we evaluated the degree of integration of sustainability in five variables (methodological elements or dialogic axes) using the Transition towards Sustainability tool described in Section 2.4. However, the configuration of this tool, which is based on the design of three different types of gradients (see Table 1), may slightly distort reality.
- This study is framed within the paradigm of interpretive research using a qualitative approach since it is a study that is oriented towards understanding socio-educational phenomena. One of the forms that qualitative research adopts is the case study [24]. Following Stake [24], the design of our research is hence considered to be multiple case study insofar as it encompasses different individual views (of three university teachers who train students in mathematics education). Since it is a qualitative study, specified in an exploratory case study, which occurs at a specific moment in time and in a specific spatial location, we are not concerned with the generalisation of the results, although the findings could be transferred to similar contexts [51].

4. Conclusions

This research analysed the degree of integration of sustainability in the teaching planning of the subject of three teachers from the area of Didactics of Mathematics in the Faculty of Education Sciences at Universidad de Cádiz (Spain) with a twofold objective:

In the first place, we wanted to know to what extent sustainability is part of the view the three teachers have of the teaching and learning process they plan.

Secondly, we set out to advance the identification of elements necessary for teacher training in mathematics education that consider education for sustainability as an advisable reference.

On the one hand, the analysis carried out gives rise to three different teaching profiles that contribute, in different ways, to integrating sustainability into the teaching planning of the subject. The degree of global integration of sustainability (GIGS) in each of the methodological elements or dialogic axes analysed was generally $GIGS(T3) \geq GIGS(T2) \geq GIGS(T1)$.

This result is in line with the profile of each of the three teachers interviewed. T3 has 20 years of experience working in sustainability and education for sustainability. On the contrary, T1 and T2 do not have any training in sustainability aspects. However, the professionalising nature of the degree in which T2 teaches (master's degree) may enable, in a certain manner, considering education for sustainability in the training of his students who already have a degree.

On the other hand, the analysis and contrast of the data inferred the identification of elements necessary to guide training in mathematics education when focused from the perspective of sustainability. They can be summarised as follows:

- Integral training. Encourage sustainable values and behaviour, which allow students to develop ethics in which the relationship with intergenerational otherness is based on respect and solidarity.
- Interdisciplinarity. Work in mathematics education from a global and interdisciplinary perspective in such a way that the (knowledgeable) teacher of mathematics, in addition to having proper training in mathematics didactics, establishes links with other disciplines such as history, philosophy, semiotics, ethnomathematics, sociology, or psychology.
- Critical and creative thinking. Teach future teachers to question what is being taught, promoting the critical encounter with systems of ideas and actions that are culturally and historically constituted.
- Reflective thinking. Training in reflective, dialogic methodologies as a strategy that involves student teachers in the construction of normative systems that generate value judgments about a phenomenon or fact of interest (project-based learning, problem-solving, mathematical modelling, etc.).
- Incorporating sustainability into the curriculum. Train current and future teachers in the selection, modification, and/or design of curricular elements (objectives, content, evaluation, etc.) using sustainability criteria, together with mathematics, to promote an education aimed at training critical, reflective, active, responsible, and democratic citizens.

The reader should not expect the elements cited here to close the discussion. Making mathematics education in higher education more sustainable requires an in-depth study that ensures explaining principles that guide educational practice in the initial training of mathematics teachers in accordance with the principles of sustainability. We consider that education for sustainable development plays a crucial role in higher education degrees, since it enables training professionals as agents of change and of social transformation.

The vertiginous technologisation and excessive growth in developed countries, which are the origin of unsustainable lifestyles, should not justify the abandonment or arithmetisation of mathematical knowledge that is difficult to turn into economic terms [9]. It is not a question of denying progress or arguing about abandoning the formal character of mathematics, but rather, as Radford proposes, of inviting the imagining of mathematics and “mathematics education as something that is more than a task centred on knowledge” [9] (p. 135). Mathematics education concerns training ethical and reflective citizens who position themselves critically and creatively in the face of world problems.

Author Contributions: F.M.M.-P.: Conceptualization; Data curation; Investigation; Formal analysis; Methodology; Software; Supervision; Validation; Visualization; Roles/Writing—original draft; review and editing. R.J.-F.: Conceptualization; Investigation; Methodology; Validation; Visualization; Roles/Writing—original draft. J.M.C.D.: Conceptualization; Investigation; Methodology; Validation; Visualization; Roles/Writing—original draft. P.A.G.: Conceptualization; Data curation; Funding acquisition; Investigation; Methodology; Resources; Supervision; Validation; Visualization; Roles/Writing—original draft. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Spanish Ministerio de Economía y Competitividad under grant number EDU2015-65574-R.

Institutional Review Board Statement: All the results presented in this work satisfy the “Ley Orgánica 3/2018, de 5 de diciembre, de Protección de Datos Personales y garantía de los derechos digitales” (LOPD) of Spain.

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: Not applicable.

Acknowledgments: The authors are grateful for the support of the members of the EDINSOST project and the Research Group "Professional Development of the Teacher-HUM462" of the Universidad de Cádiz, in which this study is framed. We also acknowledge all the teachers who have voluntarily participated in this study. We would also like to thank Ann Swinnen for her useful feedback and comments.

Conflicts of Interest: The authors declare that they have no conflict of interest.

Appendix A

The initial exploratory questionnaire answered by the teachers from the area of Didactics of Mathematics is shown below.

- Scope 1: The role of the university in general and of mathematics education in particular in higher education regarding education for sustainability.
    1. Do you think it is necessary for the university to train and educate for sustainability? Why? Why not?
    2. Give three reasons why knowing mathematics can help have an informed opinion and act knowingly.
    3. What methodologies do you consider the most appropriate to include sustainability in your subject? Do you normally use them?
    4. How do you think the socio-environmental reality can be used in the classroom in favour of mathematics education? Do you consider this aspect in the teaching planning of your subject?
    5. Do you think the way you evaluate is consistent with the inclusion of sustainability in your subject? Why? What could you change?
    6. The NCTM establishes five content standards that all children should have the opportunity to learn: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. Order from most important (1) to least important (5) the content that, in your opinion, should be covered to include sustainability in mathematics education. Why does the order you indicate seem to be the most appropriate?
    7. The NCTM establishes five process standards that all children should have the opportunity to acquire: Problem Solving, Reasoning and proof, Communication, Connections, and Representation. Order from most important (1) to least important (5) the processes that, in your opinion, should be addressed to include sustainability in mathematics education. Why does the order you indicate seem to be the most appropriate?
    8. Do you think mathematics education should be globalised and interdisciplinary at school? Why? Why not?
    9. What criteria do you have in mind when defining the content of your subject?
- Scope 2: The competency approach defined by CRUE-Sustainability [17]. CRUE-Sustainability requests a comprehensive review of the curricula to ensure the inclusion of sustainability in all the degrees of the Spanish university system in agreement with the following four competencies:
  SUST1: Competency in the critical contextualisation of knowledge through interrelating social, economic and environmental issues at a local and/or global level.
  SUST2: Competency in the sustainable use of resources and in the prevention of negative impacts on the natural and social environment.
  SUST3: Competency to participate in community processes that promote sustainability.
  SUST4: Competency to apply ethical principles related to sustainability values in personal and professional behaviour.

Indicate whether, in your teaching planning, you integrate some of the above competencies in some of the training activities of the subject you teach. If so, briefly say in which activity and how the competency is integrated.

- Scope 3: Obstacles that hinder incorporating sustainability into higher education. Rate the obstacles or barriers that, in your opinion, hinder the integration of sustainability in the curriculum of teacher training in Mathematics education on a scale of 1 to 5, with 1 being the most important indicator and 5 the least important indicator.
    1. I do not have enough time to cover all the topics of my subject.
    2. I find it difficult to integrate sustainability into the topics of my subject.
    3. I think those topics are not suitable for university teaching.
    4. I do not know the methodologies that I could use to include them.
    5. I think those topics are not important.
    6. I disagree on the philosophy of sustainability.
    7. I lack the sufficient knowledge about the environment.
    8. I find it difficult to find suitable materials to introduce these topics.
    9. Lack of commitment on behalf of the institution where I teach.
    10. I doubt they will be of use to my students in their professional future.
    11. I am afraid of giving too negative a view of the future.

Appendix B

Below is the script for the semi-structured personal interview.

1 What aspects, apart from the content, do you consider when you design your subjects?
2 What do you understand by education for sustainable development?
3 Have you ever considered integrating sustainability into your subjects? Why? Why not?
4 Do you feel ready to integrate education for sustainable development into your classes?
5 Do you consider that mathematics education is a discipline that can help incorporate education for sustainable development into the school classroom?
6 From what you already do in class, what do you think has a direct impact on the training of your students? How do you perceive it?
7 Do you consider that the competencies related to the subject should be reformulated in order to integrate sustainability into the syllabus of the subject, or would you leave them as they are?
8 How do you think you, as a teacher, could influence your students in your classes to become aware of socio-environmental issues; adopt a critical, creative, reflective stance; and assume the responsibilities they will have as future professionals?
9 What types of resources or materials do you think are the most appropriate for integrating sustainability into mathematics education training? Do you normally use them?
10 In order to promote the training of more critical, responsible citizens committed to the problems of the world in which we live, what would you like to change or introduce in the teaching planning of your subject and in your teaching?

Appendix C

Reproduced with permission from Esther García-González, HAMS: A Tool for the Analyses of Methodological Activity of University Professors from the Principles of Sustainability and Complexity; published by Espacios, 2018 [33].

Table A1. HAMS—Planning dimension—Lecturer agent (L) [32,33].

Dialogic Axis	Indicators
Relationship: Lecturer–Student Vertical vs. Horizontal (RVH)	1. In the planned work methodology, the teacher is the main agent in the process.
	2. Students are given a certain role in the planning, but it is the teacher who has the upper hand.
	3. The planned methodology reflects the participation of all agents in the teaching–learning process.
Competencies Specific vs. Cross-curricular (CSC)	1. All competencies are formulated in specific terms linked to the subject.
	2. The specific competencies linked to the subject are maintained and some additional ones of a cross-curricular nature are formulated.
	3. The competencies are formulated in global terms, and specific and cross-curricular competencies converge.
Socio-environmental reality Non-integrated vs. Integrated (RNI)	1. The socio-environmental reality does not appear in the planning of the subject.
	2. Some references are made to the socio-environmental reality.
	3. Clear indications to the socio-environmental reality are observed, although in parallel with the syllabus.
	4. The socio-environmental reality is the axis from which the subject is planned and designed.
Resources Internal vs. External (RIE)	1. In the planning, internal context resources: seminars, laboratories, ICTs, the library, etc. are referred to.
	2. In addition to the usual internal context resources, references are made to the use of other resources linked to the environment.
	3. The environment is considered a key resource in the teaching–learning process.
Evaluation Summative vs. Formative (ESF)	1. The planned evaluation focuses only on the student and uses tests or exams at the end of the process.
	2. More than one evaluation instrument is planned. They are implemented during or at the end of the process, but they are only aimed at the students.
	3. Evaluation appears in the planning at different times, uses different instruments and sources of information.
	4. The planning of the evaluation is reflected at different times, uses different instruments and sources of information, and regulates all the elements of the process.

Table A2. HAMS—Planning dimension—Student agent (S) [32,33].

Dialogic Axis	Indicators
Relationship: Lecturer–Student Vertical vs. Horizontal (RVH)	1. No room is provided for the students' contributions in the planning of the subject.
	2. The students and their interests appear as an element to be taken into account, but they are not considered key players in the teaching–learning process, which is led by the teacher.
	3. The planning refers to the involvement and active participation of the students in the teaching–learning process, including the possibility of making decisions.

Table A2. *Cont.*

Dialogic Axis	Indicators
Competencies Specific vs. Cross-curricular (CSC)	1. The competencies of the subject are focused on the development of competencies specific to the subject in the official curriculum of the degree.
	2. Some references are made to their role as professionals in the competencies of the subject, but without a clear link to the learning outcomes.
	3. In addition to the competencies specific to the subject, the competencies include the training of professionals committed to improving their environment and the training of critical autonomous citizens.
Socio-environmental reality Non-integrated vs. Integrated (RNI)	1. The socio-environmental reality is not linked to the professional role of the students in the planning.
	2. Some relations are established between the socio-environmental reality and the future professional role of the students.
	3. The socio-environmental reality is considered a pillar in the training of the students for their professional development.

Table A3. HAMS—Planning dimension—Content agent (C) [32,33].

Dialogic Axis	Indicators
Relationship: Lecturer–Student Vertical vs. Horizontal (RVH)	1. The content is presented in a closed manner and is organised by themes that respond to the disciplinary organisation.
	2. The content is grouped by thematic units and is the main objective of the process, but there is some room to address other content of interest.
	3. The content has a central theme, it are not closed, and it is receptive to incorporating new themes that arise in the teaching and learning process.
	4. The content has a central theme that responds to solving the problems posed, not being an end in itself, and it is open to incorporating new themes.
	5. The content, in addition to not being an end in itself, having a central theme, responding to problem solving, and being open to including new topics, incorporates as such the strategies developed in the teaching and learning process.
Competencies Specific vs. Cross-curricular (CSC)	1. The competencies only refer to the use of knowledge and content specific to the subject.
	2. The competencies interrelate different types of knowledge.
	3. The development of cross-curricular competencies is introduced in the planning as part of the knowledge.
Socio-environmental reality Non-integrated vs. Integrated (RNI)	1. The content is limited to that which is specific to the subject and appears in the official curriculum of the degree.
	2. The content of the subject includes some topics or aspects related to the socio-environmental reality.
	3. The socio-environmental reality is present at all times in the content of the subject.
	4. The socio-environmental reality acts as the axis from which the subject is designed.
Evaluation Summative vs. Formative (ESF)	1. The planned evaluation focuses on conceptual knowledge and on the student.
	2. The planned evaluation considers some aspects of the teaching–learning process, apart from knowledge, but it is only aimed at the students.
	3. The planned evaluation addresses all the elements of the teaching–learning process. Furthermore, the evaluation criteria are specified in the planning.

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