

Article

Children's Informal Learning in Mathematics through Parental Involvement with Play-Based Activities: A Nonformal Training Program

Rita Panaoura *  and Chrisa Nitsiou

Department of Education, School of Educations and Social Sciences, Frederick University, Nicosia 1036, Cyprus; pre.nc@frederick.ac.cy

* Correspondence: r.panaoura@frederick.ac.cy

Abstract: The present study examines the role of everyday play-based activities, planned and implemented by parents, on the development of children's mathematical thinking at the age of up to 3 years old. Parents' knowledge and beliefs were measured by using an online questionnaire, while a nonformal training program was developed and implemented with the participation of a group of parents using technological tools. The aim was to improve their knowledge on how to use relevant play-based activities and the understanding of their role. Results indicated that parents asked for the contribution of experts in order to acquire the relevant knowledge and skills and develop positive self-efficacy beliefs about their role. They found the training practical, through the use of digital tools, as well as useful for collaboration and communication. The shared examples by them indicated that the training has to concentrate further on the use of daily activities outside of the school-based context, which is more known to the parents.

Keywords: early childhood mathematics; play-based activities; informal and nonformal education



Citation: Panaoura, R.; Nitsiou, C. Children's Informal Learning in Mathematics through Parental Involvement with Play-Based Activities: A Nonformal Training Program. *Educ. Sci.* **2023**, *13*, 870. <https://doi.org/10.3390/educsci13090870>

Academic Editor: John Barker

Received: 28 May 2023

Revised: 13 August 2023

Accepted: 23 August 2023

Published: 25 August 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Children's mathematical achievement is a vital educational goal with considerable public attention and discussions between policy makers, educators, and parents at different school ages. Parents are expected to be the collaborators who continue everyday what the school starts. What about the ages before school? What about the parents who do not have time, knowledge, and skills in order to be "educationally productive parents"?

Recognition of the importance of parental involvement in students' education is not new [1] and it is related directly or indirectly with the children's achievements. Usually, most governments' policies recognize that pupils need support from parents to ensure they reach their full potential [2]. According to [3], at the age of school education, teachers often count on parents to help them create a positive learning environment in their schools, by concentrating on children's behavior. Many studies have shown that children have higher achievement if their parents are involved in their education (e.g., [4]). Durisic and Bunijevac [5] focused on three forms of parental cognition: parents' aspirations concerning their children's future orientation, their self-efficacy in educating their children, and their perceptions of the school. Parents with high self-efficacy are more authoritative and consistent in their interactions with their children than those with lower parenting self-efficacy [6]. The vital issue is how we ensure high self-efficacy beliefs, especially at the early ages where parents seem to concentrate more on their children's development. One of the major concerns of parents at the age of pre-primary and primary education is about the difficulties their children may face in mathematics and thus, they try to introduce school-based activities of recognizing numerical symbols earlier. In previous years, most of the studies concentrated on formal education where parents get involved in their children's school homework without realizing the importance of the impact of informal learning.

More recently, Jay et al. [7] encouraged parents to work with their children to “find the math” in everyday life and activity.

The present study concentrates on the age of early childhood where the surrounding environment can offer opportunities for investigation and exploration of mathematical concepts through play-based activities. These activities are expected to be organized and conducted by the parents who need to have the appropriate skills, knowledge, and information. The use of technology and mainly social media enable us to share those ideas with parents as part of “friendly” non-formal training, with respect to the extremely demanding rhythm of life. While there is a general agreement that the everyday informal experiences affect the future learning processes and the skills which are developed, there is not any orientation of the planning and implementation of the relevant activities before the early nursery education, the first type of formal education. The present study aims to connect the daily activities, indoor or outdoor, with the future goals of nursery education in respect to mathematical thinking in order to bridge the gap between education and home activities. It is a connection of formal education with informal education, by using parents as the connectors. In order to have this role, parents need to have knowledge and skills. Technological tools are suggested and were used as a direct way of communication with parents, in the context of nonformal training. The closed groups through social media contribute to sharing the ideas more quickly and enable the direct interaction between the participants. We believe that children’s daily activities can be enriched in order to be an additional learning tool under an informal learning perspective. In the case of Cyprus, preschool education starts at the age of 3 years old.

The purpose of the present study was twofold, and as a consequence, two research questions were posed: (a) What are parents’ knowledge, beliefs, and self-efficacy beliefs about the development of their young children’s mathematical thinking through play-based activities and what are the respective interrelations? (b) What is the contribution of a non-formal training program by using technology on developing parents’ knowledge, disposition, and experiences on scheduling and implementing relevant play-based activities?

2. Theoretical Framework

2.1. Parental Involvement in Children’s Mathematical Achievements

Family and school are the two main factors playing a critical role in children’s overall development. The impact of parental involvement on children’s learning outcomes, in general and in the case of mathematics in particular, has a profound effect on achievement, motivation, and goals [8]. It is well-documented that when children enter kindergarten, they portray different levels of school readiness. The quality and amount of time that parents spend with their children at home, engaging in meaningful activities, contribute to different developmental trajectories in children. This type of parental involvement accounts for stronger literacy and mathematics performance when children enter kindergarten [9]. There are literally hundreds of publications on the subject of parental involvement in their children’s education. According to [10], it is relatively easy to describe what parents do in the name of “involvement”. It is much more difficult to decide whether the specific activity makes a difference to school achievements, as these are influenced by many other factors. As Cooper [11] claimed, regarding the “Equality of educational opportunity”, the child’s home conditions and socio-economic status have greater significance for learning achievements than all school-related variables.

According to [12], most obstacles to parental involvement have to do with the diverse backgrounds and socio-economic status in low-income families, where parents may not have resources or they may not have acquired a great deal of education. Studies about the home activities environment have in most cases found a positive relation between indices with the children’s skills [13]. However, the impact differs based on their socioeconomic background [14] due to the lack of knowledge or positive disposition. For example, in the case of mathematics, Levine et al. [15] indicated that parents with high social position and mainly those with high levels of educational qualifications provide more input about

estimation and number cardinality, while parents with low background concentrate only on simple counting. Additionally, parents tend to relate mathematics with numbers and numerosity, while shapes, figures, and images are related with spatial ability and geometric thinking. Any mathematical concept can be introduced under an interdisciplinary real-life scenario. Casey et al. [16] used blocks in relation to fairy tales for the introduction of geometrical concepts and spatial sense.

Epstein et al. [17] emphasize the relationship and collaboration between parents, school, and community. Sanders et al. [18] suggest that schools become successful when a strong and positive relationship among parents, students, teachers, and community has been established. Xiong et al. [4] found that parents as motivators, resource providers, and monitors are the most important predictors of students' mathematical achievement. Some indirect involvement actions include parents relaying the importance and value of education, maintaining high expectations of how far their child will go in school, and providing encouragement [19].

Parental involvement refers to the situation where parents are directly involved in the education of their children [20]. However, in most cases, studies concentrated on primary and secondary education and mainly in relation to the students' homework. At the same time, there are studies which examine the link between the family's orientation to mathematics and the related everyday activities with kids. Ref. [21] examined through observation the organized activities by two mothers of 30-month-old children. They highlighted the effect of engaged mathematical experiences on the development of mathematical learning. Huang et al. [22] examined the relations between parental involvement and student's mental health and mathematics self-efficacy in early adolescents in China. They indicated that different dimensions of parental involvement provided different beliefs on mathematics achievement. The influence of cognitive and behavioral involvement on mathematics achievement was partially mediated by students' mental health and mathematics self-efficacy.

Ref. [23] proposed a model on how parents can become more involved in schools: family obligations, communication with the school, parental involvement with school, parental involvement in learning activities at home, decision making in school, and cooperation with community. Panaoura [24] developed an intervention program that aimed to contribute to affecting parents' beliefs and their role on primary school children's homework in mathematics, by using the contribution of social media in order to affect indirectly the children's problem-solving behavior. In all cases, it seems that parents asked for training in order to have positive self-efficacy beliefs about their role. The barriers to parental involvement must always be taken into consideration. Parents from lower socioeconomic backgrounds face many more barriers to involvement, including nonflexible work schedules and lack of resources [5]. Ref. [25] identified four areas that are barriers to involvement: lack of access, time poverty, lack of financial resources, and lack of awareness.

2.2. *The Development of Mathematical Thinking in Early Childhood*

Children at all ages learn mathematics with different motivations and levels of achievement due to many known and unknown reasons [8]. Based on the findings of cognitive psychology, we know that children of all ages have some knowledge of mathematics, including infants [26]. Cognitive psychologists examine the cognitive structure and the activation of the relevant cognitive processes and how the people of the surrounding environment with important use can facilitate mental processes in order to develop skills and construct knowledge [27]. Undoubtedly, "parents are their children's first teachers and know more about their children than any teacher ever will" (p. 1, [19]).

The connection of mathematics with play-based activities and the real world directly fulfills the infants' tendency to investigate the surrounding world as well as their curiosity. At the same time, it fulfills mathematics' main goal to construct the first experiences of the mathematical concepts through real-life situations. Connecting mathematics to the real

world is thought to be beneficial, as it enhances children's understanding of fundamental mathematical ideas and it motivates mathematics learning [28].

The National Council of the Teachers of Mathematics and the National Association for the Education of Young Children underline that high-quality education at an early ages is a vital foundation for mathematical performance [29]. The starting age of formal preschool education is different for each educational system. The present study concentrates on the age before the formal entrance at the age of 3 years, which is the usual age in the case of Cyprus. The content of mathematical teaching and learning in the case of formal education is divided into five main domains: numbers and operations, geometry, estimation, patterns and relations, probabilities, and statistics.

The recognition of quantities and changes in them is presented soon after birth in the human infant [30]. Infants are able to discriminate between sets of objects of up to two items from the first months of their life [31] and they also recognize the basics of arithmetic operations, adding or removing objects from small sets [27]. They are able to estimate the magnitudes and the sizes by representing gross differences between sets of many vs. few toys on the floor, small vs. big animals. At the same time, infants have the requisite abilities to locate both themselves and objects in space [32]. Elia [33] studied geometric and spatial thinking in early childhood and offered further insights into the crucial role of the body and other semiotic resources by which they develop and express, in a communicative framework, geometric thinking. They learn to use visual information to reach places, people, and objects.

Mathematical educators and cognitive psychologists indicated that infancy and early childhood are the periods where the fastest cognitive development is observed [27]. The learning of mathematics should begin at birth and continue throughout the childhood years as children explore the world around them by themselves [20]. For example, infants can learn about patterns through songs and rhymes [34] and they can "learn" division through sharing things. Everyday activities provide the stimulus for informal contribution to mathematical development.

Everyday experiences, either structured or not structured, offer added value to the development of mathematical thinking. In order to face the inter-individual differences and the later performance difficulties, we need to study further the role and the impact of parental involvement and their contribution on the planning and implementation of play-based activities. One of the main aims of mathematical education is to ensure that mathematical learning is meaningful and enjoyable for children. Play and games are the leading tools for activities that children enjoy [35]. Everyday activities at home, such as cooking, sharing things, puzzling, and estimating, provide opportunities for young children to practice in relation to mathematical competencies. In general, the quality of the home learning environment, as well as mathematical activities such as counting and identifying shapes, have been found to account for differences in children's mathematical development [36].

2.3. Children's Informal Learning and Parents' Non-Formal Training

The concepts of formal, informal, and non-formal teaching and learning need to be defined, as there appears to be a consensus around the corresponding meaning. According to [37], the basic characteristics in each case are as follows:

- (a) Formal: Learning is structured, it is proposed through intended activities, it takes place in educational settings, and it tends to have a cognitive emphasis. It is related directly with the curriculum.
- (b) Non-formal: Learning may be structured, it involves cognitive, emotional, social, and behavioral elements, and it is promoted through indirect teaching behaviors.
- (c) Informal: Learning is not structured, it is not recognized by the learner, it can take place anywhere, and it involves cognitive and non-cognitive elements.

In the case of early childhood, the contact of children with any mathematical concept has to be pursued through play-based activities in order to be based on their nature and level

of cognitive development. Play is described as a context in which children can demonstrate their own learning. In many cases, they need the contribution of adults in order to scaffold the learning [35], and in the specific case, there are organizers of the play-based activities in order to enable the construction of the mathematical concepts. Farrugia [38] emphasized the concept of “construction” rather than “instruction”, as children need adult guidance in order to reach learning potential. As [26] mention, although it is clear that children learn through play, current research indicates a reliance on changes and they present challenges in order to promote the quality of the learning experiences. There is, for example, an emphasis on numeration and the recognition of mathematical symbols through activities not interesting for the children [39]. Levine et al. [15] examined how parental math-talk with 30-month-old children contributed to the acquisition of the cardinality principle. Similarly, Casey et al. [16] use block-building interventions to examine effects on the development of spatial skills. In all cases, the intervention condition included the use of mathematical play with rich language in the form of a story. Researchers concluded that storytelling provided an effective context for teaching spatial sense and other important mathematical concepts.

So far, attempts have been made to train parents, especially at the age of children’s school education, through meetings at the school, information on the school’s webpage, or the distribution of leaflets [40]. In many cases, the emphasis is on specific skills, such as the language skills of children from migrant families [41], physical health, mental health, or social skills [42]. In other cases, parental education has been related with family routines, such as mealtimes, bedtimes, and shared family activities [43]. Volodina et al. [44], in a study conducted in the UK, USA, and Germany, indicated that child outcomes vary by the family’s socioeconomic status even in the case of using training programs, which cannot be attended by all parents.

In all cases, we are talking about nonformal learning. In previous years, there have been attempts to communicate and interact with parents by using technological tools of communication such as viber groups, groups on messenger, and forums of discussion [24]. Research has shown that communication through the use of technology and mobile technology between parents and teachers play an important role in academic progress [45]. Social media as tools for communication, by creating closed groups which share ideas and experiences, enable fast information exchange at a minimal cost.

3. Methodology

In order to examine parents’ knowledge about mathematical thinking at the age of infancy and their conceptions about their role, we have constructed and used a questionnaire. The quantitative results, derived by the questionnaire, were used in order to design and implement an intervention program. The program was developed in order to train parents to plan and implement play-based activities which relate mathematical concepts with daily activities. The intervention program was based on three narrative presentations we created in order to explain to them the value of using play-based activities through the use of good practices and examples. For better and more direct communication, a closed group of the participants was created by using the platform messenger. The shared experiences, ideas, worries, and inquiries were analyzed qualitatively. Finally, a group discussion enabled them to express their thoughts and concerns.

3.1. Sample

The sample for the quantitative part of the study consisted of 153 parents with children up to the age of 3 years old (58 fathers and 95 mothers). The children’s mean age was 1 year and 11 months (SD = 5 months). The sample was not representative, as the participants voluntarily took part in the study after respecting strict rules of anonymity. Based on the rules of the Republic of Cyprus concerning ethical issues in research, when research offers anonymity for participating adults, it is not necessary to have any approval by the national ethics committee. However, all subjects gave their informed consent for inclusion before they participated in the study. The questionnaire was developed in a digital format, and it

was shared through the use of social media (open for 5 days). The sample for the qualitative part of the study consisted of 6 parents (4 mothers and 2 fathers) who expressed (on the last question of the questionnaire) their willingness to take part in the second part of the study.

3.2. Tools

The questionnaire was developed by the researchers in order to measure the parents' knowledge about the development of mathematical thinking at the early childhood age and their beliefs and self-efficacy beliefs about their respective role. We have not identified a questionnaire which was directed at the ages of early childhood in the framework of mathematics, and for this reason, we developed it. The questionnaire consisted of 20 Likert-type items (1 = strongly disagree and 5 = strongly agree). The items are presented in Table 1, as part of the exploratory factor analysis. The Cronbach α was 0.845, while the validity was examined in a pilot study where 5 other parents participated in order to express their opinion on what was not clear (two questions were rephrased). Additionally, the content validity of the questionnaire was examined by two researchers at a different university in our country with a special interest on play-based activities and preschool education.

Table 1. Factor loadings of the five factors against the items.

Items	F1	F2	F3	F4	F5	Mean
Children need to play for enjoyment.	0.817					4.57
Children can learn new things through play.	0.803					3.89
Children can learn new things through play with other children at the same age.	0.784					3.27
Children can learn new things through play with adults.	0.732					4.45
Children can learn new things through play with older children.	0.601					4.18
Outdoor activities can contribute to children's learning.	0.589					3.26
Indoor activities can contribute to children's learning.	0.574					4.07
All children can learn mathematics.		0.756				3.15
Children learn mathematics only at school.		0.713				3.88
Children need talent to learn mathematics.		0.685				4.25
Children need to learn mathematical symbols early.		0.601				4.12
Children learn mathematics through playing.		0.572				3.06
I can help my children learn mathematics			0.827			2.87
I can plan interesting learning activities for my children.			0.792			3.05
I can choose fairy tales which are related with mathematics.			0.673			3.24
Nursery teachers are responsible to teach mathematics to my child.				0.862		4.68
Nursery teachers are responsible to explain to me how to help my child learn things.				0.844		4.56

Table 1. *Cont.*

Items	F1	F2	F3	F4	F5	Mean
Nursery teachers are responsible to present to me how to plan play-based activities at home.				0.739		4.13
I know how to choose appropriate games for my children.					0.681	2.69
I know how to spend valuable time with my children.					0.576	3.05

F1 = parents' beliefs about the role of play-based activities; F2 = parents' beliefs about the development mathematical thinking; F3 = self-efficacy beliefs about parental contribution on developing math thinking; F4 = parents' expectations on teachers' contribution; F5 = parents' self-efficacy beliefs about their knowledge.

As we have already mentioned, the intervention program started with the sharing of three narrative presentations. Those presentations were developed by one of the main researchers of the study and had a duration of 10 min each. The first presentation explained in general the learning of mathematics (numbers, numerosity, cardinality, geometry, shapes, estimation, and patterns) and the development of mathematical thinking, mainly at the childhood ages. The second presentation explained the role of play-based activities at early ages and it included examples with indoor and outdoor activities and good practices. The third presentation gave examples of tools such as fairy tales, songs, and everyday home materials which could be used for indoor and outdoor activities (indicative examples in the images presented in Figure 1). Parents had a week in order to attend the presentations whenever they had the time and then we had a Zoom meeting (for half an hour) in order to explain to them further what they had to do for the following three weeks. We asked them to plan and implement at least eight activities with their children (indoor and outdoor). The main activities' characteristics needed to be the following: play-based, interesting and enjoyable for the children, related with any mathematical concept, and part of their everyday life. They had to share their ideas and experiences as a type of a diary in a closed messenger group (by using words and photos, after hiding the children's faces). A final group discussion by using Zoom was conducted two weeks after the program, where they were asked to express their thoughts, concerns, problems, and future suggestions.

**Figure 1.** Indicative examples included in the presentation.

3.3. Data Analysis

The quantitative data were analyzed using SPSS, with the aim to examine the first research question. Exploratory factor analysis was used in order to identify the major factors which constructed parents' knowledge, beliefs, and self-efficacy beliefs [46,47]. After the content analysis of the factors [48], we conducted descriptive statistics, correlation, and mean comparison in order to examine the interrelations. For example, a cluster analysis was used in order to divide the sample into groups [49] based on their self-efficacy beliefs and then examine further their knowledge and beliefs. Qualitative analysis of the data derived from their shared experiences and the final discussion was used by concentrating thematically [50] on two main dimensions: (a) the types of the shared examples with play-based activities and (b) the positive and negative beliefs and thoughts they expressed, their concerns, and their suggestions.

3.4. Limitations of the Present Study

The sample of the study was not representative, and parents who took part voluntarily probably had specific characteristics which were not examined by the present study (such as the parents' socioeconomic position and education or special interest in the domain). Additionally, we have to highlight that learning is a multidimensional phenomenon with complicated interrelations and the present study concentrated only on a few of the aspects.

4. Results

4.1. Parents' Beliefs and Self-Efficacy Beliefs about the Development of Mathematical Thinking

First, we subjected the parents' responses to exploratory factor analysis in order to examine the extent to which the statements reflected the main dimensions under examination. The analysis of the participants' responses to the items resulted in five factors with eigenvalues greater than 1 ($KMO = 0.846$, $p < 0.05$). Those five factors explained 76.346% of the total variance. The first factor consisted of seven items reflecting parents' beliefs about the role of play-based activities at early childhood ages. The second factor consisted of five items about their knowledge on mathematical thinking and their respective beliefs. The third factor consisted of three items expressing their self-efficacy beliefs about their contribution on developing children's mathematical thinking. The fourth factor consisted of three items about their expectations of teachers' contribution to their role, and the fifth factor consisted of two items about their self-efficacy beliefs about their respective knowledge.

As is obvious from the means which are presented at the final column of Table 1, parents recognize the vital role of play for their children, either for enjoyment or for learning. They recognize as more important the planning of play-based activities for enjoyment (mean = 4.57) and as more valuable for learning play-based activities with adults (mean = 4.45) and older children (mean = 4.18). They prefer indoor activities for learning (mean = 4.07) rather than outdoor activities (mean = 3.26). Parents do not have very high self-efficacy beliefs about their ability to contribute to the development of their children's mathematical thinking (with means near 3). Similarly, they do not have high self-efficacy beliefs about their knowledge of how to choose the appropriate games for children (mean = 2.69) and how to spend valuable time with them (mean = 3.05). They believe that nursery teachers have the main role for enabling children to learn mathematics and they expect to guide them on how to act at home.

Then, we used the means of the five factors in order to conduct further analyses (Table 2). There were, in all cases, statistically significant correlations between the factors ($p < 0.05$). The strongest correlation was between the third and fifth factors ($r = 0.815$), which indicated that parents with low beliefs about their contribution to developing math thinking are at the same time parents with low self-efficacy beliefs about their knowledge of how to choose the relevant activities. The weakest correlation was between the second factor and the first factor (0.576), indicating that there are parents with low beliefs about the role of play-based activities on learning and at the same time believe that mathematics as a language with symbols requires talent and activities at school. The correlation between the

parents' expectations of teachers' contribution (fourth factor) and self-efficacy beliefs about their knowledge of choosing the appropriate activities (fifth factor) was not too strong ($r = 0.632$), indicating that parents with low self-efficacy beliefs about their knowledge have high expectations of the contribution of the teachers.

Table 2. The means and standard deviations of the five factors.

Factor	Mean	Standard Deviation
1. Beliefs about the role of play-based activities	3.95	0.93
2. Beliefs about the development of mathematical thinking	3.69	0.86
3. Self-efficacy beliefs about parental contribution on developing math thinking	3.05	0.58
4. Expectations on teachers' contribution	4.45	1.01
5. Self-efficacy beliefs about their knowledge	2.87	0.62

The central interest of the present study concentrated on parental involvement in developing children's mathematical thinking. By using the cluster analysis, we divided the sample into three categories in respect to their self-efficacy beliefs about parental contribution to developing math thinking (third factor): 52 parents with low, 55 with medium, and 46 with high self-efficacy beliefs. It was interesting that 78% of the parents with high self-efficacy beliefs were mothers, and on the contrary, 60% of the parents with low self-efficacy beliefs were fathers. Analysis of variance was conducted in order to examine the statistically significant differences between the other four factors (F1, F2, F4, and F5) in respect to the three groups. Results showed that there were statistically significant differences between the parents with low self-efficacy beliefs and high self-efficacy beliefs in the case of the fourth factor, which was about their expectations of teachers' contribution ($F = 5.564, p < 0.05$). As was expected, parents with low self-efficacy beliefs had the highest beliefs (4.76) about the vital role of teachers in developing their children's mathematical thinking. Similarly, there were statistically significant differences between those two groups in respect to the second ($F = 3.708, p < 0.05$). Parents with low self-efficacy beliefs about their knowledge and abilities to choose relevant tools and activities have more formalistic conceptions about the learning of mathematics.

4.2. The Contribution of the Intervention Program

The contribution of the intervention program which was implemented through the use of digital tools was examined under two main perspectives: a) the types of the indoor and outdoor activities the group of parents planned, implemented, and shared with us and b) their conceptions about the strengths and limitations of the program. Parents were asked to describe and share at least eight activities. The number and the type of activities are presented in Table 3, using pseudonyms for parents (four mothers and two fathers).

Maximos and Christia shared eight activities, which was the minimum number that was asked. Maximos organized four indoor activities and four outdoor. He believed that only three of them were interesting for his child and he enjoyed mainly an outdoor activity at the park. "We found leaves and we grouped them into four different types. We counted many times the total of each type of leaves and then my daughter created a picture with them. She kept it in her bedroom and she is proud for it. I believe that she mainly enjoyed the creative time we spent together". His hesitation was that he did not feel capable, and for this reason, in all the other cases, he preferred to buy games with an indication that there were suitable for learning activities from 18 months up to 3 years. Christia preferred the outdoor activities. She played football with her son, and they counted the number of shots. She was extremely excited with an activity in which they used water. Actually, they used three boxes of different sizes in order to compare quantities.

Table 3. The type of activities shared by the six parents.

	Nicos (M)	Maximos (M)	Melina (F)	Corina (F)	Christia (F)	Eleni (F)
Total Number of activities	10	8	15	11	8	10
Number of indoor activities	4	4	6	5	3	4
Number of outdoor activities	6	4	9	6	5	6
Activities with numbers and operations	7	5	5	7	6	7
Activities with shapes and geometry	2	3	6	2	2	1
Activities with patterns	1	0	3	2	0	2
Activities with the use of fairy tales	2	0	3	1	2	1

Melina liked to share her activities and her experiences with the other parents, and she admitted that for the activity with the patterns, she copied the idea that Nicos presented earlier to the group with the colors of the balls. She used two colors of pegs with her daughter in order to put the clothes in the sun after the laundry. Similarly, she asked her daughter to follow a pattern for the six handkerchiefs which were used at night at their table. As she wrote in the group discussion, “although she insisted to put the red handkerchiefs for the kids and the blue for the adults, I am sure that she understood how she needed to act in order to have the right pattern”. Additionally, Melina believed that the use of fairy tales with concepts, such as the “triangle-fish” for the triangles, “fulfilled the goal of enriching the learning of language, the expression of emotions, and the acquaintance with mathematical words such as triangles, numbers and quantities”.

As parents said, in most cases, they found it easier to plan and implement outdoor activities, because they were more interesting for the kids. This result was probably due to the season in which the program was conducted, as it was spring and they were able to play outdoor. Eleni used the shapes which were part of the set for the beach in order to play with his son in the sandpit at the park. They made creative sand constructions and then they made cookies with sand. They counted those cookies. “I asked him to repeat the numbers after me like . . . we have one, two, three cookies, lets destroy one, we have two cookies, lets destroy another one, we have only one, lets destroy it, we have no cookies and he was laughing”.

During the group discussion, parents were asked to express their thoughts critically and without any hesitation towards the program. All of them agreed on the positive impact of the digital environment of the training; otherwise, they found it “too difficult to spend time late in the afternoon or evenings in order to attend face to face training”. Two of them had experiences with older children at the kindergarten and they were not able to find babysitting in order to go to the school’s afternoon meetings. However, they suggested adding a Zoom meeting at the end of each week in order to pose their questions and share their experiences with experts. Corina said, “I was feeling extremely anxious the first week, as I thought that my activities will not be interesting or related with mathematics. I think that I needed to feel support and to have guidance. At the same time, I have to admit that in order to feel confident on what I was doing I tried to find further examples from the webpages you suggest us. This was extremely useful”. All the parents agreed that one of the benefits of the program was the access they understood that they can have to many resources with ideas. They underlined the necessity to have that information translated into their native language in order to be appropriate for all the parents, independent of their knowledge of the English language. The follow-up part of the discussion is interesting:

- We are using our smart phones all day. I usually find information on google for medical or health issues. It is the first time I find information about my kid’s learning.
- What about the parents who do not have this access?
- Are there any parents without access? Especially after the pandemic, I do not believe it.

- The Ministry of Education needs to organize such trainings for all parents and the state has the responsibility to supply all the parents with the necessary tools.
- It is not so complicated. The first days I was not sure about my activities. Now I am thinking at the end of the day what I have done today with me kid and I realize that my behaviour has be affected by the program and unconsciously I use these types of activities.

Maximos posed a very interesting issue at the discussion: “Which is our role as parents and which is the role of the school? Now our kids are with their grandmother or the nanny, but next year they will be at the kindergarten. I would like to continue doing activities with my child, however I wonder whether her teacher will guide me with the appropriate activities which will prepare her for the primary education”. Eleni, who has a boy in primary education, claimed that they do not have very creative or interesting activities in preschool education. “They started having exercises in books and they need to know how to write symbols. My boy does not like painting and he found it too boring to do all those exercises of painting four apples in order to show the number of 4 etc.”. Although most of them did not have any previous experiences with the nursery teachers and the work which is done at nursery school, they expect them to have the relevant knowledge in order to organize creative mathematical activities with their kids and then guide them for the follow-up activities at home. They expected them to use outdoor math activities at school as “they must know about all those types of activities and how to organize them at the school yard”. Eleni and Nicos insisted that nursery teachers need training in order to organize innovative and enjoyable activities for the kids.

Parents admitted that they felt more comfortable organizing activities related with numbers and counting, as in this case, they are able to understand directly the results which are related with the development of mathematical thinking. They did not feel confident in understanding how the construction of a road for a toy would be useful for spatial orientation, which is related with geometrical thinking. The use of everyday tools and objects convinced them that could enable their children to get in touch with mathematics in real-life situations. Although kids found the outdoor activities more interesting, they believed that few of the indoor activities enabled kids to develop other important skills, such as “preparing the table, sharing things, putting their toys back into boxes”.

5. Discussion

Children explore their surrounding environment though structured experiences provided by their parents and through everyday non-scheduled activities [9]. Researchers have conceptualized parental involvement as a multifaceted construct, including cognitive, behavioral, and personal involvement [22]. The present study concentrated on the examination of parental involvement in planning and implementing play-based activities in order to develop mathematical thinking.

Results indicated the parents’ positive dispositions toward cooperating with experts in order to contribute to their children’s development in general and in the case of mathematics in particular. Their beliefs were stronger in the case of formal education and the vital role of the teacher. They had strong beliefs about their role in repeating school-based activities, even in early childhood, by using their experiences from their older children or their own experiences. In the case of mathematics, this type of behavior will probably accelerate the construction of negative beliefs towards mathematics. Previous research has identified that many children face psychological and social challenges, such as mathematical anxiety, which disrupts their mathematical learning and performance in school [46]. Additionally, parents who participated in our study recognized the important role of play-based activities [1]; however, they did not have high self-efficacy beliefs on planning and implementing relevant activities. Parents’ examples indicated their persistence in numbers, cardinality, and knowledge of symbols. It was important that a few examples included other concepts which were related to spatial abilities, orientation, and estimation. It was expected to be affected by the school-based activities, which were nearest to their own experiences, and

the other activities proved the influence of the intervention program. It was extremely important that they recognized the value of introducing mathematical concepts through a daily framework, as a part of family routines [28].

Parental involvement has always been an essential component of every teacher [1], school, student, and academic endeavor [51]. Most researchers have agreed that parental involvement is of critical importance for the academic success of children regardless of age [12]. In middle school, [1] revealed that the students with the most supportive parents not only have higher proficiency levels but also more positive attitudes toward mathematics than those students with the least supportive parents. While a large body of literature on parental involvement exists, few efforts have particularly looked into early childhood years where the strong foundations are constructed. Especially in the domain of mathematical competence, it has been well-documented that early mathematical competence has long-term effects for children's future mathematics development and functional skills [52]. Undoubtedly, parental involvement in early childhood does not necessarily guarantee positive effects, as the learning process is too complicated, with the involvement of many interrelated factors [10]. Thus, we believe that the lack of knowledge, education, skills, and tools on the part of the parents can be partially addressed by conducting training programs which can be attended by all parents. Digital tools and social media fulfill this goal by taking advantage of using the nonformal educational setting, through delivering the training in respect to the parents' practical and occupational barriers [5]. At the same time, parents become acquainted with the presence of the valuable sources of examples, tools, and interventions which can be found through digital tools. We have to underline that we do not suggest the transfer of the teaching responsibility to the parents. We suggest the enrichment of the repertoire of activities with their children in order to enable them to construct children's first mathematical concepts within a context of interdisciplinary, playful, daily, and enjoyable activities.

As we have already mentioned, the present study could not examine the diverse backgrounds and the socioeconomic status of the families. We hypothesized that the use of social media could enable parents from lower socioeconomic backgrounds to face many barriers to their involvement, such as nonflexible work schedules [5], without being able to generalize this finding. A future study could examine this aspect in order to emphasize further the accountability of the formal or nonformal training programs. Additionally, the present study highlighted the parents' needs in order to develop further their knowledge, beliefs, and self-efficacy beliefs on the role of play-based activities in the development of young children's mathematical thinking. A future longitudinal study could examine the impact of this type of parental involvement on children's math achievements during the following school life.

Author Contributions: Conceptualization, R.P.; methodology, R.P. and C.N.; analysis R.P.; writing—original draft preparation, R.P., writing-review and editing, C.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: The study was conducted in accordance with the Declaration of Helsinki and approved by the Ethics Committees for Bioethical Evaluation of Scientific Research in Cyprus.

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study. Before participation in the study, all the parents read a statement of commitment to respecting their anonymity and a declaration that the research is not related with any biological intervention.

Data Availability Statement: Data can be shared directly by the authors if they obtain before in each case the permission of the University Council.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Cai, J.; Moyer, J.; Wang, N. Parental roles in students' learning of mathematics: An exploratory study, research in middle level education. *Quarterly* **1999**, *22*, 1–18.
2. Williams, B.; Williams, J.; Ullman, A. *Parental Involvement in Education*; Research Report 332; BMRB Social Research: Florence, Italy, 2002.
3. OECD. Parental involvement in school activities. In *What School Life Means for Students' Lives*; PISA 2018 Results; OECD Publishing: Paris, France, 2020; Volume III.
4. Xiong, Y.; Qin, X.; Wang, Q.; Ren, P. Parental involvement in adolescents' learning and academic achievement: Cross—Lagged effect and mediation of academic engagement. *J. Youth Adolesc.* **2021**, *50*, 1811–1823. [CrossRef] [PubMed]
5. Durisic, M.; Bunijevac, M. Parental involvement as an important factor for successful education. *CEPS J.* **2017**, *7*, 137–153. [CrossRef]
6. Ardel, M.; Eccles, J. Effects of mothers' parental efficacy beliefs and promotive parenting strategies on inner—City youth. *J. Fam. Issues* **2001**, *22*, 944–972. [CrossRef]
7. Jay, T.; Rose, J.; Simmons, B. Finding “Mathematics”. Parents questioning school-centered approaches to involvement in children's mathematics learning. *Sch. Community J.* **2017**, *27*, 201–230.
8. Fiskerstrand, A. Literature review—Parent involvement and mathematics outcome. *Educ. Res. Rev.* **2022**, *37*, 100458. [CrossRef]
9. Van Voorhis, F.L.; Maier, M.F.; Epstein, J.L.; Lloyd, C.M. *The Impact of Family Involvement on the Education of Children Ages 3 to 8: A Focus on Literacy and Math Achievement Outcomes and Social-Emotional Skills*; MDRC: New York, NY, USA, 2013; Available online: http://arxiv.org/abs//dev.mdrc.org/sites/default/files/The_Impact_of_Family_Involvement_FR.pdf (accessed on 13 January 2018).
10. Desforges, C.; Abouchaar, A. *The Impact of Parental Involvement, Parental Support and Family Education on Pupil Achievement and Adjustment: A Literature Review*; Research Report 433; Department of Education and Skills: London, UK, 2003.
11. Cooper, H.M. *Synthesizing Research: A Guide for Literature Reviews*, 3rd ed.; Sage Publications: Thousand Oaks, CA, USA, 1998; Volume 2.
12. Lin, Q.; Gao, M. Obstacles to parental involvement in children's education: New teachers' perceptions and strategies. *Int. J. Innov. Educ. Res.* **2022**, *10*, 139–148.
13. Soto-Calvo, E.; Simmons, F.; Adams, A.; Francis, H.; Giofre, D. Pre-schoolers' home numeracy and home literacy experiences and their relationships with early number skills. Evidence from a UK study. *Early Educ. Dev.* **2019**, *31*, 113–136. [CrossRef]
14. Nguyen, T.; Watts, T.; Duncan, G.; Clements, D.; Sarama, J.; Wolfe, C.; Spitler, N. Which preschool mathematics competencies are most predictive of 5th grade achievement? *Early Child. Res. Q.* **2016**, *36*, 550–560. [CrossRef]
15. Levine, S.C.; Suriyakham, L.W.; Rowe, M.L.; Huttenlocher, J.; Gunderson, E.A. What counts in the development of young children's number knowledge? *Dev. Psychol.* **2010**, *46*, 1309–1319. [CrossRef]
16. Casey, B.; Andrews, N.; Schindler, H.; Kersh, J.; Sampler, A.; Copley, A. The development of spatial skills through interventions involving block building activities. *Cogn. Instr.* **2008**, *26*, 269–309. [CrossRef]
17. Epstein, J.L.; Sanders, M.G.; Simon, B.S.; Salinas, K.C.; Jansorn, N.R.; Van Voorhis, F.L. *School, Family and Community Partnerships: Your Handbook for Action*; Sage Publications: Thousand Oaks, CA, USA, 2002.
18. Sanders, M.; Sheldon, B. *Principles Matter: A Guide to School, Family and Community Partnerships*; Sage Publications: Thousand Oaks, CA, USA; Corwin: Newbury Park, CA, USA, 2014.
19. Sarah, A.M. Parental involvement and student achievement: Effects of indirect parent involvement on student achievement in mathematics. In *Electronic Theses, Projects and Dissertations*; CSUSB Scholarworks: San Bernadino, CA, USA, 2021.
20. Panaoura, R.; Petrou, P. Parental role on using activities for the development of mathematical thinking at the age of infancy. *Mediterr. J. Res. Math. Educ.* **2022**, *19*, 73–81.
21. Aubrey, C.; Durmaz, D. Policy—To practice contexts for early childhood mathematics in England. *Int. J. Early Years Educ.* **2012**, *20*, 59–77. [CrossRef]
22. Huang, F.; Huang, Z.; Li, Z.; Zhang, M. Relationship between parental involvement and mathematics achievement in Chinese early adolescents: Multiple mediating role of mental health and mathematics self-efficacy. *Int. J. Environ. Res. Public Health* **2021**, *18*, 9565. [CrossRef] [PubMed]
23. Epstein, J.L. *School, Family and Community Partnerships: Preparing Educators and Improving Schools*; Westview Press: Boulder, CO, USA, 2011.
24. Panaoura, A. Parental involvement in developing students' perseverance in solving mathematical problem through the use of social media. *Int. J. Technol. Educ. Sci.* **2017**, *1*, 36–47.
25. Williams, T.; Sanchez, B. Identifying and decreasing barriers to parent involvement for inner—City parents. *Youth Soc.* **2011**, *45*, 54–74. [CrossRef]
26. Bruce, C.; Flynn, T.; Moss, J. *Mathematics for Young Children: Literature Review*; Comprehensive literature review submitted to the literacy and numeracy secretariat; Ontario Ministry of Education: Toronto, ON, Canada, 2012.
27. Demetriou, A.; Greiff, S.; Makris, N.; Spanoudis, G.C.; Panaoura, R.; Kazi, S. *Bridging Educational Priorities with Developmental Priorities: Towards a Developmental Theory of Instruction*; University of Nicosia: Nicosia, Cyprus, 2020. [CrossRef]
28. Popovic, G.; Lederman, J. Implication of informal education experiences for mathematics teachers' ability to make connections beyond formal classroom. *Sch. Sci. Math.* **2015**, *115*, 129–140. [CrossRef]

29. Cerezci, B. The impact of the quality of early mathematics instruction on mathematics achievement outcomes. *J. Child. Educ. Soc.* **2020**, *1*, 216–228. [\[CrossRef\]](#)
30. Carey, S. *The origin of Concepts*; Oxford University Press: New York, NY, USA, 2009.
31. Chesney, D.; Gelman, R. What counts? Visual and verbal cues interact to influence what is considered a countable thing. *Mem. Cogn.* **2015**, *43*, 798–810. [\[CrossRef\]](#)
32. Clements, D.; Samara, J. Effects of a preschool mathematics curriculum. Summative research on the Building Blocks Project. *J. Res. Math. Educ.* **2007**, *38*, 136–163. [\[CrossRef\]](#)
33. Elia, I. Observing the use of gestures in young children's geometric thinking. In *Contemporary Research and Perspectives on Early Childhood Mathematics Education*; ICME-13, Monographs; Elia, I., Mulligan, J., Anderson, A., Baccaglini-Frank, A., Benz, C., Eds.; Springer: Cham, Switzerland, 2018. [\[CrossRef\]](#)
34. Anthony, G.; Walshaw, M. Characteristics of effective teaching of mathematics: A view of the west. *J. Math. Educ.* **2009**, *2*, 147–164.
35. Dockett, S.; Perry, B. What makes mathematics play. In *MERGA 33: Shaping the Future of Mathematics Education*; Mathematics Education Research Group of Australasia: Fremantle, Australia, 2010; pp. 715–718.
36. Anders, Y.; Rossbach, H.-G.; Weinert, S.; Ebert, S.; Kuger, S.; Lehl, S.; von Maurice, J. Home and preschool learning environments and their relations to the development of early numeracy skills. *Early Child. Res. Q.* **2012**, *27*, 231–244. [\[CrossRef\]](#)
37. Johnson, M.; Majewska, D. *Formal, Nonformal and Informal Learning: What Are They, and How Can We Research Them? Research Report*; Cambridge University Press: Cambridge, UK, 2022.
38. Farrugia, B. Wasp: Sampling in qualitative research. *Early Hum. Dev.* **2019**, *133*, 7–8. [\[CrossRef\]](#)
39. Petrou, P. The Role of Play-Based Activities on the Development of Mathematical Thinking at the Age of Infancy (11 months to 2 years). Ph.D. Thesis, Frederick University, Nicosia, Cyprus, 2022. *Unpublished*.
40. Afxord, N.; Berry, V.; Lloyd, J.; Moore, D.; Rogers, M.; Hurst, A.; Blockley, K.; Durkin, H.; Minton, J. *How Can Schools Support Parents' Engagement in Their Children's Learning? Evidence from Research and Practice*; Education Endowment Foundation: London, UK, 2019.
41. Clair, L.; Jackson, B. Effects of family involvement training on the language skills of young elementary children from migrant families. *Sch. Community J.* **2006**, *16*, 31–42.
42. Froiland, J.; Peterson, A.; Davidson, M. The long-term effects of early parent involvement and parent expectation in the USA. *Sch. Psychol. Int.* **2013**, *34*, 33–50. [\[CrossRef\]](#)
43. Martinez, N.; Xerxa, Y.; Law, J.; Serdarevic, F.; Jansen, P.; Tiemeier, H. Double advantage of parental education for child educational achievement: The role of parenting and child intelligence. *Eur. J. Public Health* **2022**, *32*, 690–695. [\[CrossRef\]](#)
44. Volodina, A.; Weinert, S.; Washbrook, E.; Waldfogel, J.; Kwon, S.; Wang, Y.; Casoni, V. Explaining gaps by parental education in children's early language and social outcomes at age 3–4 years: Evidence from harmonised data from three counties. *Curr. Psychol.* **2022**, 1–20. [\[CrossRef\]](#)
45. Palts, K.; Kalmus, V. Digital channels in teacher-parent communication: The case of Estonia. *Int. J. Educ. Dev. Using Inf. Technol.* **2015**, *11*, 65–81.
46. Dowker, A.; Sarkar, A.; Looi, C.Y. Mathematics anxiety: What have we learned in 60 Years? *Front. Psychol.* **2016**, *7*, 508. [\[CrossRef\]](#)
47. Fontaine, J. Equivalence. In *Encyclopaedia of Social Measurement*; Ghent University: Gent, Belgium, 2005; Volume 1, pp. 803–813.
48. Chiu, C.; Chia, S.; Wan, W. Measures of cross-cultural values, personality and beliefs. *Meas. Personal. Soc. Psychol. Constr.* **2015**, *1*, 621–651.
49. Vasconcelos, R.; Baptista, M. Cluster analysis, a powerful tool for data analysis in education. In *Proceedings of the 56th Session*; International Statistical Institute: Voorburg, The Netherlands, 2007.
50. Thomas, J.; Harden, A. Methods for the thematic synthesis of qualitative research in systematic reviews. *BMC Med. Res. Methodol.* **2008**, *8*, 45. [\[CrossRef\]](#) [\[PubMed\]](#)
51. Sapungan, G.; Sapungan, R. Parental involvement in child's education: Importance barriers and benefits. *Asian J. Manag. Sci. Educ.* **2009**, *3*, 42–48.
52. Geary, D.C. Early foundations for mathematics learning and their relations to learning disabilities. *Curr. Dir. Psychol. Sci.* **2013**, *22*, 23–27. [\[CrossRef\]](#) [\[PubMed\]](#)

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.