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The Differential Game of a Closed-Loop Supply Chain with Manufacturer Competition Considering Goodwill

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Abstract: The dynamic optimization of the closed-loop supply chain (CLSC) is a hot research topic. Members' competitive behavior and product goodwill play an important role in the decision making of CLSC members. In this paper, a closed-loop supply chain (CLSC) with competitive manufacturers and single retailers is studied, in which the manufacturer produces and recycles the products, and the retailer is responsible for the sales of the products. On this basis, a dynamic linear differential equation of product goodwill is constructed, the optimal dynamic path of each decision variable is found, and the influence of price competition among manufacturers on the decision making of members in a dynamic closed-loop supply chain is studied. The conclusion is verified by an example. The results show that goodwill directly affects the wholesale price, the retail price, the recovery price, and the profit of supply chain members. The wholesale price and the retail price of products are not only positively affected by their own goodwill, but also by the goodwill of competing products. The manufacturer competition intensity will affect the product price and the supply chain member profit. To a certain extent, the more intense the manufacturer's competition is, the higher the wholesale price and the retail price, and the greater the profit of the supply chain members.

Keywords: closed-loop supply chain; marginal profit fluctuates; product goodwill; manufacturer competition; differential game

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1. Introduction

In recent years, resource consumption, ecological degradation, and other environmental issues have become increasingly prominent and are becoming a hot topic of concern for all mankind. The closed-loop supply chain, through the recycling of used products, effectively realizes resource conservation and recycling, which is an important way to realize the low-carbon cycle development of a green economy. In recent years, recycling and re-manufacturing have been vigorously implemented in many industries, including automotive parts, power batteries, tires, mobile phones, computers, and so on. How to improve the efficiency of closed-loop supply chain management is, therefore, still a hot issue in current studies.

Real-life manufacturer competition is commonplace, with cases like Mengniu and Yili, P&G and Unilever, Apple and Huawei, and other manufacturers competing against each other. Competitive behavior between companies directly affects the decisions of supply chain participants. In an increasingly competitive market environment, consumers increasingly value product goodwill and, therefore, the impact of product goodwill on the supply chain has attracted the attention of many researchers.

The closed-loop supply chain is formed on the basis of an open-loop supply chain, including product recycling and reverse logistics. In order to obtain the residual value of waste products, reduce production costs, and reduce the negative impact on the environment, the supply chain members recycle and reuse the waste products, so that the resources can achieve a circular flow, thus forming a closed-loop supply chain. The optimal coordination of closed-loop supply chains has been studied by many experts [1]. Taking into account the quality of recycled waste products, Masoudipour [2] developed a bi-objective model to study its impact on supply chain members' profits. Hong [3] studied the impact of advertising on supply chain members' pricing decisions, reverse channel performance, and profits, demonstrating that local advertising strongly influences members' pricing and recycling strategies, as well as profits. The effect of customers' intuitive impressions on the recycling efficiency of a closed-loop supply chain of competitive recycling was studied by He [4], who concluded that the entry of retailers into recycling competition can reduce costs but not increase recycling rates, but that the establishment of contractual and empowerment mechanisms can be effective in improving recycling efficiency. Zhen [5] modeled a bi-objective optimization with CO_2 emissions and total costs under the premise of demand uncertainty, and researched the problem of a green closed-loop supply chain network integration using stochastic programming theory.

Sun [6] studied the optimal green investment strategy for green closed-loop supply chains under government subsidy policies given government involvement. Considering the risk attitudes of supply chain members, Li [7] studied the impact of equity concerns on pricing decisions in a dual-channel, closed-loop supply chain, using a supply chain consisting of risk-averse retailers and risk-neutral manufacturers. Zheng [8] further analyzed the impact of retailers' equity concerns on channel profits in a three-tier, closed-loop supply chain, and coordinated the decisions of supply chain members and profit allocation using cooperative game theory.

Panda [9] discussed the optimal decision-making in closed-loop supply chains in terms of profit maximization and member social responsibility, with consideration of corporate social responsibility. On this basis, He [10] further delved into the dual-channel closed-loop supply chain where manufacturers sell new products through retailers, and he also studied the channel structure and pricing decisions of manufacturers when selling remanufactured products through third parties or platforms, as well as government subsidy policies for competing new and remanufactured products.

However, all of the above studies looked at the simplest of secondary supply chains. None of them considered the impact of inter-firm competitive behavior on closed-loop supply chains. Savaskan et al. [11] were the first to introduce retailer competition into closed-loop supply chains and researched the impact of the intensity of price competition between retailers on pricing and recovery decisions in closed-loop supply chains. By introducing CSR, three models were developed by Liu [12] to compare and contrast the impact of whether members were CSR or not on the decisions of two competing retailers, summarizing that those retailers with CSR behaviors gained more profit than those without CSR investment. Huang [13] constructed three re-manufacturing models with trade-in strategies and explored the impact of "trade-in" strategies on the equilibrium decisions and benefits of supply chain members in the context of retailer competition.

Hong [14] studied the impact of technology licensing on a two-cycle closed-loop supply chain of manufacturers and re-manufacturers competing to recycle used products by considering manufacturer competition. By comparing two licensing models, fixed fee, and commission, the work showed that the optimal licensing strategy for manufacturers is influenced by the fixed fee. Zhang [15], on the other hand, researched the impact of competitive sales between the original manufacturer and a third-party re-manufacturer licensed under a patent on product prices and supply chain members' profits, and designed revenue-sharing contracts to determine a range of apportionment factors to achieve supply chain coordination. Further, Zhang [16] conducted a study of a competitive closed-loop supply chain consisting of two leading OEMs and two third-party re-manufacturers. He also investigated the optimal choice between two third-party re-manufacturing models, outsourcing, and licensing. The study revealed that the duopoly manufacturer firms always preferred the outsourcing strategy, while the choice preferences of the duopoly re-manufacturers were diverse.

The above models of closed-loop supply chains are all-time-static models, but the establishment of product goodwill is often a dynamic process. As a result, closed-loop supply chains often exhibit long-term and dynamic characteristics continuously, and the traditional closed-loop supply chain decision-making approach of finding an extreme value point in the short term is no longer applicable. Therefore, closed-loop supply chains have changed from static to dynamic decision making, and the focus of research has shifted from finding the extreme value point in the short term to finding the optimal decision path in the long time, and from finding the optimal "point" to find the optimal "line".

For the dynamic optimization of closed-loop supply chains, Giovanni [17,18] et al. explored the problem of optimal decision making in closed-loop supply chains when the recycling rate changes dynamically in the case of joint recycling by manufacturers and retailers, and compared and analyzed the impact of different incentive mechanisms on closed-loop supply chain decisions. The decision on price and recovery rates for closed-loop supply chain members was explored by Wen [19] under retailer-differentiated pricing and average pricing conditions, respectively, considering heterogeneous consumers with environmental responsibility.

Others have further considered the impact of member competition on dynamic closed-loop supply chain decisions. For example, Giovanni [20] took a further investigation of a supply chain for recycling used batteries and explored the impact of joint incentives of two competing retailers on manufacturers' recycling input decisions in a closed-loop supply chain where manufacturers are responsible for recycling. Wu [21] worked on a closed-loop supply chain with one manufacturer and one re-manufacturer competing. The impact of government policy on supply chain members' profits, consumer surplus, and social welfare is explored by introducing government intervention and studying government taxation or government subsidies.

The advertising and promotion are also a hot topic of concern in supply chain dynamic optimization, and the advertising investment of supply chain members can increase product goodwill and thus expand product demand [22], so some scholars also introduce goodwill into supply chain dynamic optimization research to analyze its influence on members' decision making. Taboubi [23] investigates how incentives can be designed to motivate retailers to invest more in advertising in a manufacturer-driven scenario, considering the need for brand goodwill influence. Zhang [24] investigated the impact of the consumer reference low-carbon effect and product low-carbon goodwill on reduction decisions and profits of dual-channel supply chain members, and designed a low-carbon publicity cost-sharing contract to achieve supply chain coordination.

Giovanni [25] hypothesized that closed-loop supply chain members engage in green advertising to build goodwill, which not only expands demand but also increases recycling rates, and proposed a new revenue-sharing contractual agreement. Based on this, Jena [26] further investigated the impact of advertising cost sharing on the resupply chain in the case of product demand and revenue uncertainty. Xiang [27] investigated the closed-loop supply chain operation of remanufacturing in the presence of a third-party web-based recycling platform, also considering that goodwill can increase demand and recycling rates, and explored the effects of corporate goodwill, technological innovation, and overconfidence on supply chain members' decisions. The study showed that firms can improve goodwill by cooperating with Internet service platforms, and that in the manufacturer cost-sharing scenario, the Internet service platforms invest more in big data marketing, and retailers have a tendency to "piggyback", which further encourages firms to improve goodwill by cooperating with Internet service platforms.

As shown in Table 1. The above studies on the dynamic optimization of closed-loop supply chains either failed to consider the behavior of market competition or only considered the dynamic changes in product recovery rates under market competition. No study has yet considered the impact of manufacturer competition and goodwill dynamics on the optimal decision paths of closed-loop supply chain members. No settlement has been made under the scenario of random fluctuations in marginal profits. However, these issues are commonplace in the reality of supply chain management.

Table 1. Some literature most relevant to this paper.

Author	Close-Loop Supply Chain	Manufacturer	Goodwill
Giovanni [20]	√	√	
Wu [21]	√	√	
Giovanni [25]	√		√
Jena [26]	√		√
This paper	√	√	√

In summary, based on the study in the literature [25], this paper further considers the influence of manufacturer competition on the optimal decision of supply chain members by taking a closed-loop supply chain in which manufacturers are responsible for recycling scrap products as the research object. Under the premise of price competition among manufacturers, the dynamic linear differential equation of product goodwill is constructed, and it is assumed that product goodwill affects the quantity of product demand and the quantity of recycling. On this basis, centralized and decentralized decision models are established respectively to study the optimal dynamic decision paths of supply chain members under the dual effects of manufacturer competition and product goodwill, and to explore the strength of competition among manufacturers and the effect of product goodwill on the decision and profit of a closed-loop. The impact of competition among manufacturers and product goodwill on the decision making and profitability of members in the closed-loop supply chain is investigated.

There are three main innovations in our paper. (1) Unlike most static studies of closed-loop supply chains, in this paper, the dynamic optimization of closed-loop supply chains is investigated and based on the literature [25], the manufacturer competition scenario is further considered to explore the impact of manufacturer competition, and product goodwill on the optimal decision of the supply chain is measured, assuming that competing manufacturers have different product goodwill. (2) While the existing literature on dynamic optimization of closed-loop supply chains considering either goodwill or member competition has been conducted under the assumption of fixed marginal profits, this paper breaks away from this restriction. We assumed that marginal profits are random, and under this premise, we investigated the effects of goodwill and manufacturer competition on the wholesale, retail, and recycling prices of products. (3) The study not only found that goodwill affects the wholesale, retail, and recall prices of products as well as the profits of supply chain members, but also that the wholesale and retail prices of products are positively influenced not only by their goodwill but also by the goodwill of competing products. In addition, the study found that product goodwill plays a role in protecting product prices in the context of manufacturer competition.

2. Results

In this paper, a two-tier closed-loop supply chain consisting of two competing manufacturers and a single retailer is studied, where the manufacturer, as the supply chain leader, is responsible for the production and recycling of the new product and its advertising; the retailer is responsible only for the sale of the product. This paper assumes that there is no difference in appearance and performance between the new and remanufactured products, which are regarded as homogeneous products. The flow of operation of the supply chain is shown below.

We assume that both the manufacturer and the retailer are risk-neutral and use their own profit, dynamically optimized over a defined period as their decision criterion. The manufacturer produces a single, short life cycle product, sells it to the retailer at the wholesale price, $w_i(t)$, and recovers it from the consumer at the recovery price, b_i. The retailer sells to the consumer at the selling price $p(t)$.

Hypothesis 1 (H1). *Let the manufacturer's advertising input level be $A_{mi}(t)$ and the manufacturer's advertising input cost, $C_{ami}(t)$, be a convex function of the advertising input level, i.e.: $C_{ami}(t) = \frac{1}{2}\mu_m A_{mi}(t)$.*

Here, μ_m represents the cost factor for the manufacturer's level of advertising input.

Hypothesis 2 (H2). *The manufacturer's advertising will enhance the product's brand image and affect the goodwill of the product. The process of change in the goodwill of the product is assumed to be*

$$\dot{G}_i(t) = \alpha_i A_{mi}(t) - \beta G_i(t) \tag{1}$$

where $\alpha > 0$ is the coefficient of the impact of the manufacturer's level of advertising investment on product goodwill and $\beta > 0$ is the natural rate of goodwill decay.

Hypothesis 3 (H3). *It is assumed that the quantity of recycled products is not related to market demand and is mainly influenced by its own recycling price. In addition, the goodwill of the product also affects the quantity of the product recycled; the higher the goodwill of the product, the greater the willingness of consumers to recycle the goods. The quantity of product recycled can therefore be expressed as:*

$$Q_i(t) = kG_i(t) + hb_i(t) \tag{2}$$

Hypothesis 4 (H4). *In addition to price, product goodwill also helps to increase consumers' willingness to buy and thus expand product demand [26]. The market demand for the product is negatively related to the market price and, because of the substitutability of the products produced by the two manufacturers, a competitive relationship is constituted between them in the sales process, which can be described by function:*

$$\begin{aligned} D_1(t) = \eta G_1 + \phi d - a_1 p_1(t) + \gamma p_2(t) \\ D_2(t) = \eta G_2 + (1-\phi) d - a_2 p_2(t) + \gamma p_1(t) \end{aligned} \tag{3}$$

where d denotes basic market demand, ϕ stands for Manufacturer 1's market share, a_i represents the product's sensitivity to product price, and γ is the price competition factor between the two manufacturers.

Hypothesis 5 (H5). *When a manufacturer makes a new product, it will first make use of recycled waste products for production, and only when the amount of recycled waste products is not sufficient to meet the market demand for the product will it make use of new materials for production of the product. This paper assumes that there is a situation where products are produced from new materials, i.e., that the amount of recycled waste products is not sufficient to meet the market demand for the products. Therefore, there are two main sources of revenue for manufacturers, namely wholesale product revenue $w(t)$ and recycling and re-manufacturing revenue, where c_i represents the manufacturer's production costs, Δ_i is the manufacturer's revenue from recycling and re-manufacturing a product, and b_i denotes the manufacturer's recycling price to consumers. In terms of retailers, the main source of revenue for retailers is sales revenue.*

Hypothesis 6 (H6). *The manufacturer and retailer have the same discount factor at any point in time over an infinite time horizon. The manufacturer's advertising input level, $A_{mi}(t)$, the manufacturer's wholesale price, $w_i(t)$, the recycling price, $b_i(t)$, and the retailer's selling price,*

$p_i(t)$ are the decision variables, and goodwill, $G_i(t)$, is the state variable. Therefore, under the above assumptions, the manufacturer, i's, long-run profit function is:

$$
\begin{aligned}
&J_{m1} = \int_0^{\infty}[(w_1 - c_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + \\
&(\Delta_1 - b_1)(kG_1 + hb_1) - \tfrac{1}{2}\mu_m A_{m1}(t)^2]dt \\
&J_{\mathrm{m}}^2 = \int_0^{\infty}[(w_2 - c_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1) + \\
&(\Delta_2 - b_2)(kG_2 + hb_2) - \tfrac{1}{2}\mu_m A_{m2}^2]dt
\end{aligned}
\tag{4}
$$

The retailer's long-term profit function is:

$$
\begin{aligned}
&J_{ri} = \int_0^{\infty}[(p_1 - w_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + \\
&(p_2 - w_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1)]dt
\end{aligned}
\tag{5}
$$

The overall long-term profit function for the supply chain is as follows:

$$
\begin{aligned}
&J_h = \int_0^{\infty}[(p_1 - c_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + (p_2 - \\
&c_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1) + (\Delta_1 - b_1)(kG_1 + \\
&hb_1) + (\Delta_2 - b_2)(kG_2 + hb_2) - \tfrac{1}{2}\mu_m A_{m1}(t)^2 - \\
&\tfrac{1}{2}\mu_m A_{m2}(t)^2]dt
\end{aligned}
\tag{6}
$$

3. Model Analysis

This section is devoted to two aspects of this differential game model, the centralized decision-making case and the decentralized decision-making case. In the centralized decision, the two manufacturers and one retailer are considered as a whole, and the objective is to find the optimal retail price, the recycling price, and the optimal level of advertising input to optimize the overall profitability of the supply chain. In the decentralized decision-making scenario, the manufacturer and the retailer are considered as independent individuals whose decision criteria are to maximize their own profits respectively.

3.1. Centralized Decision-Making Situations (C)

In the centralized decision situation, both the competing manufacturer and the individual retailer play a cooperative game to maximize the overall profit of the supply chain, finding the optimal path of sales price and the optimal level of advertising input. In this case, the objective function of the supply chain as a whole is:

$$
\begin{aligned}
&max J_h^c = \int_0^{\infty}[(p_1 - c_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + \\
&(p_2 - c_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1) + \\
&(\Delta_1 - b_1)(kG_1 + hb_1) + (\Delta_2 - b_2)(kG_2 + hb_2) - \\
&\tfrac{1}{2}\mu_m A_{m1}^2(t) - \tfrac{1}{2}\mu_m A_{m2}^2(t)]dt
\end{aligned}
\tag{7}
$$

Proposition 1. *In the case of centralized decision-making:*
(1) Optimal retail price path for retailer 1:

$$
p_1^C(t) = \frac{d\gamma - d\gamma\phi + d\phi a_2 - \gamma^2 c_1 + a_1 a_2 c_1 + \eta a_2 G_1 + \gamma\eta G_2}{2(-\gamma^2 + a_1 a_2)}
$$

(2) Retailer 2's optimal retail price path:

$$p_2^C(t) = \frac{d\gamma\phi + da_1 - d\phi a_1 - \gamma^2 c_2 + a_1 a_2 c_2 + \gamma\eta G_1 + \eta a_1 G_2}{2(-\gamma^2 + a_1 a_2)}$$

(3) The optimal path for the recovery price of the manufacturer's product is:

$$b_i^C(t) = \frac{-kG_i + h\Delta_i}{2h}$$

(4) The optimal level of advertising investment for Manufacturer 1 is:

$$A_{m1}^C(t) = \frac{(2g_1^C G_1^C + g_3^C G_2^C + g_4^C)\alpha_1}{\mu_m}$$

(5) The optimal level of advertising investment for Manufacturer 2 is:

$$A_{m2}^C(t) = \frac{(2g_2^C G_2^C + g_3^C G_1^C + g_5^C)\alpha_2}{\mu_m}$$

(6) The optimal path for product goodwill:

$$G_1^C(t) = e^{-t\beta + \frac{2tg_1\alpha_1^2}{\mu_m}} C_1 + \frac{g_4\alpha_1^2}{-2g_1\alpha_1^2 + \beta\mu_m} + \frac{g_3 G_2 \alpha_1^2}{-2g_1\alpha_1^2 + \beta\mu_m}$$

$$G_2^C(t) = e^{-t\beta + \frac{2tg_2\alpha_2^2}{\mu_m}} C_2 + \frac{g_5\alpha_2^2}{-2g_2\alpha_2^2 + \beta\mu_m} + \frac{g_3 G_1 \alpha_2^2}{-2g_2\alpha_2^2 + \beta\mu_m}$$

(7) The overall optimal profit path for the supply chain:

$$V_h^C(t) = g_1^C G_1^{C2} + g_2^C G_2^{C2} + g_3^C G_1^C G_2^C + g_4^C G_1^C + g_5^C G_2^C + g_6^C$$

If $A = (\gamma^2 - a_1 a_2)$, $B = -4hAg_3^2 a_1^2 a_2^2$,
Then:

$$g_1^C = \frac{h(2\beta+\rho)A\mu_m - \sqrt{B + \mu_m(2(-k^2\gamma^2 + (h\eta^2 + k^2 a_1)a_2)\alpha_1^2 + h(2\beta+\rho)^2 A\mu_m))}}{4hA\alpha_1^2}$$

$$g_2^C = \frac{h(2\beta+\rho)A\mu_m - \sqrt{B + \mu_m(2(-k^2\gamma^2 + a_1(h\eta^2 + k^2 a_2))\alpha_2^2 + h(2\beta+\rho)^2 A\mu_m))}}{4hA\alpha_2^2}$$

$$g_3^C = \frac{\gamma\eta^2\mu_m}{2A(2g_1\alpha_1^2 + 2g_2\alpha_2^2 - (2\beta+\rho)\mu_m)}$$

$$g_4^C = \frac{-2Ag_3g_5\alpha_2^2 + (\gamma(d\eta - d\eta\phi + \gamma\eta c_1 - k\gamma\Delta_1) + a_2(d\eta\phi + a_1(-\eta c_1 + k\Delta_1)))\mu_m}{2A(2g_1\alpha_1^2 - (\beta+\rho)\mu_m)}$$

$$g_5^C = \frac{-2(\gamma^2 - a_1a_2)g_3g_4\alpha_1^2 + (\gamma(d\eta\phi + \gamma\eta c_2 - k\gamma\Delta_2) + a_1(-d\eta(\phi-1) + a_2(-\eta c_2 + k\Delta_2)))\mu_m}{2(\gamma^2 - a_1a_2)(2g_2\alpha_2^2 - (\beta+\rho)\mu_\mathrm{m})}$$

$$g_6^C = \tfrac{1}{4\rho A\mu_m}(2Ag_4^2\alpha_1^2 + 2Ag_5^2\alpha_2^2 + (-a_1^2a_2c_1^2 + a_2(-d^2\phi^2 + \gamma^2c_2^2) + \gamma(-2d^2\phi + 2d^2\phi^2 + 2d\gamma(-1+\phi)c_2 - 2\gamma c_1(d\phi + \gamma c_2) + h\gamma\Delta_1^2 + h\gamma\Delta_2^2) - a_1(d^2(-1+\phi)^2 - \gamma^2c_1^2 + a_2^2c_2^2 + a_2(2d(-1+\phi)c_2 - 2c_1(d\phi + \gamma c_2) + h(\Delta_1^2 + \Delta_2^2))))\mu_m)$$

Proof. In order to find the optimal decision path of the supply chain when the decision is centralized, note J_h^c as the profit function of the supply chain as a whole, which is the first-order derivative of the profit function of the supply chain as a whole, with respect to the recovery rate. From continuous dynamic programming theory, it is known that the HJB equation for the overall profit function of the supply chain is satisfied as

$$\begin{aligned}&\rho V_h^c = (p_1 - c_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + \\ &(p_2 - c_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1) + \\ &(\Delta_1 - b_1)(kG_1 + hb_1) + (\Delta_2 - b_2)(kG_2 + hb_2) - \\ &\tfrac{1}{2}\mu_m A_{m1}^2(t) - \tfrac{1}{2}\mu_m A_{m2}^2(t) + V_{hG1}^{c'}(\alpha_1 A_{m1} - \\ &\beta G_1) + V_{hG2}^{c'}(\alpha_2 A_{m2} - \beta G_2)\end{aligned} \tag{8}$$

Equation (8) takes the first derivative of price p_1^C, p_2^C, and makes the first derivative equal to 0, so

$$d\phi + \eta G_1 - a_1 p_1 - a_1(-c_1 + p_1) + \gamma p_2 + \gamma(-c_2 + p_2) = 0$$

$$d(1-\phi) + \eta G_2 + \gamma p_1 + \gamma(-c_1 + p_1) - a_2 p_2 - a_2(-c_2 + p_2) = 0$$

$$p_1^C = \frac{d\gamma - d\gamma\phi + d\phi a_2 - \gamma^2 c_1 + a_1 a_2 c_1 + \eta a_2 G_1 + \gamma\eta G_2}{2(-\gamma^2 + a_1 a_2)}$$

$$p_2^C = \frac{d\gamma\phi + da_1 - d\phi a_1 - \gamma^2 c_2 + a_1 a_2 c_2 + \gamma\eta G_1 + \eta a_1 G_2}{2(-\gamma^2 + a_1 a_2)} \tag{9}$$

Equation (8) derives the first-order derivative for price b_1^C, b_2^C and makes the first-order derivative equal to 0, giving

$$-hb_1 - kG_1 + h(-b_1 + \Delta_1) = 0$$

$$-hb_2 - kG_2 + h(-b_2 + \Delta_2) = 0$$

$$b_1^C = \frac{-kG_1 + h\Delta_1}{2h}$$

$$b_2^C = \frac{-kG_2 + h\Delta_2}{2h} \tag{10}$$

Equation (8) solves for the first-order derivative of the manufacturer's advertising input level, A_{mi}^C, and makes the first-order derivative equal to 0 to obtain

$$-\mu_m A_{m1} + V_{hG1}^{C'}\alpha_1 = 0$$

$$A_{m1}^C = \frac{V_{hG1}^{C'}\alpha_1}{\mu_m}$$

$$-\mu_m A_{m2} + V_{hG2}^{C'}\alpha_2 = 0$$

$$A_{m2}^C = \frac{V_{hG2}^{C'}\alpha_2}{\mu_m} \tag{11}$$

Substituting Equations (9)–(11) back into Equation (8), and observing the structure of the equation, set

$$V_h^C = g_1^C G_1^{C2} + g_2^C G_2^{C2} + g_3^C G_1^C G_2^C + g_4^C g_1^C + g_5^C G_2^C + g_6^C$$

then, $V_{hG1}^{C'} = 2g_1^C G_1^C + g_3^C G_2^C + g_4^C$, $V_{hG2}^{C'} = 2g_2^C G_2^C + g_3^C G_1^C + g_5^C$.

Substitute the above equation into Equation (8) for collation and observe both sides of the equation. According to the undetermined coefficient method, the following equation can be obtained:

$$\rho g_1 = \frac{8hAg_1^2\alpha_1^2 + 2hAg_3^2\alpha_2^2 + (k^2\gamma^2 - (h\eta^2 + k^2a_1)a_2)\mu_m - 8h\beta Ag_1\mu_m}{4hA\mu_m}$$

$$\begin{aligned}
\rho g_2^C &= \tfrac{k^2}{4h} + \tfrac{\gamma^2\eta^2 a_1}{4A^2} - \tfrac{\eta^2 a_1^2 a_2}{4A^2} + \tfrac{\eta^2 a_1}{2A} - 2\beta g_2 + \tfrac{g_3^2\alpha_1^2}{2\mu_m} + \tfrac{2g_2^2\alpha_2^2}{\mu_m} \\
\rho g_3^C &= \tfrac{\gamma^3\eta^2}{2A^2} - \tfrac{\gamma\eta^2 a_1 a_2}{2A^2} + \tfrac{\gamma\eta^2}{A} - 2\beta g_3 + \tfrac{2g_1g_3\alpha_1^2}{\mu_m} + \tfrac{2g_2g_3\alpha_2^2}{\mu_m} \\
\rho g_4^C &= \tfrac{4Ag_1g_4\alpha_1^2+2Ag_3g_5\alpha_2^2+(\gamma(-d\eta+d\eta\phi-\gamma\eta c_1-2\beta\gamma g_4+k\gamma\Delta_1)}{2A\mu_m} \\
&+ \tfrac{a_2(-d\eta\phi+a_1(\eta c_1+2\beta g_4-k\Delta_1)))\mu_m}{2A\mu_m} \\
\rho g_5^C &= \tfrac{2Ag_3g_4\alpha_1^2+4Ag_2g_5\alpha_2^2-(\gamma(d\eta\phi+\gamma\eta c_2+2\beta\gamma g_5-k\gamma\Delta_2)}{2A\mu_m} \\
&+ \tfrac{a_1(-d\eta(-1+\phi)+a_2(-\eta c_2-2\beta g_5+k\Delta_2)))\mu_m}{2A\mu_m} \\
\rho g_6^C &= -\tfrac{a_1^2a_2c_1^2-(d\phi+\gamma c_2)(-2\gamma(d-d\phi+\gamma c_1)+a_2(-d\phi+\gamma c_2))}{4A} \\
&+ \tfrac{a_1(d^2(-1+\phi)^2-\gamma^2c_1^2+a_2^2c_2^2-2a_2(-d(-1+\phi)c_2+c_1(d\phi+\gamma c_2)))}{4A} \\
&+ \tfrac{2g_4^2\alpha_1^2+2g_5^2\alpha_2^2+h\left(\Delta_1^2+\Delta_2^2\right)\mu_m}{4\mu_m}
\end{aligned} \tag{12}$$

Substitute the results of the solution into Equation (11) to find the manufacturer's optimal level of advertising input, and then substitute the results of the optimal level of advertising input into Equation (1) to determine the optimal goodwill path for the product under the centralized decision, and then the overall profit of the supply chain can be determined. □

3.2. Decentralized Decision-Making Situations (D)

In a decentralized decision-making situation, the supply chain members play a master–slave game in which the manufacturer is the leader, and the retailer is the follower, with both the manufacturer and the retailer making decisions based on maximizing their own revenue. The decision sequence of the supply chain members is as follows: the two manufacturers first determine the wholesale price of the product and the level of advertising input based on factors such as production costs, and then the retailer determines the selling price of the product based on the wholesale price provided by the manufacturer.

The HJB equation for each of the two manufacturers is:

$$\begin{aligned}
&max J_{m1}^D = \int_0^\infty [(w_1 - c_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + \\
&(\Delta_1 - b_1)(kG_1 + hb_1) - \tfrac{1}{2}\mu_m A_{m1}(t)^2]dt \\
&max J_{m2}^D = \int_0^\infty [(w_2 - c_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1) + \\
&(\Delta_2 - b_2)(kG_2 + hb_2) - \tfrac{1}{2}\mu_m A_{m2}(t)^2]dt
\end{aligned} \tag{13}$$

The retailer's objective function is:

$$\begin{aligned}
&max J_r^D = \int_0^\infty [(p_1 - w_1)(\eta G_1 + \phi d - a_1 p_1 + \gamma p_2) + \\
&(p_2 - w_2)(\eta G_2 + (1-\phi)d - a_2 p_2 + \gamma p_1)]dt
\end{aligned} \tag{14}$$

Proposition 2. *In the case of decentralized decision making,*
(1) Manufacturer 1's best wholesale prices:

$$w_1^D = \frac{-a_2(2d\phi + 2a_1c_1 + \gamma c_2 + 2\eta G_1) + \gamma(d(-1+\phi) - \eta G_2)}{\gamma^2 - 4a_1a_2}$$

(2) Manufacturer 2's best wholesale prices:

$$w_2^D = \frac{-\gamma(d\phi + \eta G_1) + a_1(-2d + 2d\phi - \gamma c_1 - 2a_2c_2 - 2\eta G_2)}{\gamma^2 - 4a_1a_2}$$

(3) Retailer's optimal retail price for Product 1:

$$p_1^D = a_1a_2^2(6d\phi + 2a_1c_1 + \gamma c_2 + 6\eta G_1) + 2\gamma^3(d(\phi - 1) - \eta G_2) - \gamma a_2(\gamma(3d\phi + \gamma c_2 + 3\eta G_1) + a_1(5d(\phi - 1) + 2\gamma c_1 - 5\eta G_2)))/2(\gamma^2 - 4a_1a_2)(\gamma^2 - a_1a_2)$$

(4) Retailer's optimal retail price for Product 2:

$$p_2^D = (-2\gamma^3(d\phi + \eta G_1) + a_1^2a_2(6d - 6d\phi + \gamma c_1 + 2a_2c_2 + 6\eta G_2) - \gamma a_1(a_2(-5d\phi + 2\gamma c_2 - 5\eta G_1) + \gamma(3d - 3d\phi + \gamma c_1 + 3\eta G_2)))/(2(\gamma^2 - 4a_1a_2)(\gamma^2 - a_1a_2))$$

(5) Optimal path to recovery price for manufacturer's products:

$$b_i^C(t) = \frac{-kG_i + h\Delta_i}{2h}$$

(6) Optimal level of advertising investment by manufacturers:

$$A_{m1}^D = \frac{(2g_{11}G_1^D + g_{31}G_2^D + g_{41})\alpha_1}{\mu_m}$$

$$A_{m2}^D = \frac{(2g_{22}G_2^D + g_{32}G_1^D + g_{52}))\alpha_2}{\mu_m}$$

(7) The optimal path for product goodwill is:

$$G_1^D(t) = e^{-t\beta + \frac{2tg_{11}\alpha_1^2}{\mu_m}}C_1 + \frac{g_{41}\alpha_1^2}{-2g_{11}\alpha_1^2 + \beta\mu_m} + \frac{g_{31}G_2\alpha_1^2}{-2g_{11}\alpha_1^2 + \beta\mu_m}$$

$$G_2^D(t) = e^{-t\beta + \frac{2tg_{22}\alpha_2^2}{\mu_m}}C_2 + \frac{g_{52}\alpha_2^2}{-2g_{22}\alpha_2^2 + \beta\mu_m} + \frac{g_{32}G_1\alpha_2^2}{-2g_{22}\alpha_2^2 + \beta\mu_m}$$

(8) The manufacturer's optimal profit is respectively:

$$V_{m1}^D = g_{11}G_1^{D2} + g_{21}G_2^{D2} + g_{31}G_1^DG_2^D + g_{41}G_1^D + g_{51}G_2^D + g_{61}$$

$$V_{m2}^D = g_{12}G_1^{D2} + g_{22}G_2^{D2} + g_{32}G_1^DG_2^D + g_{42}G_2^D + g_{52}G_2^D + g_{62}$$

(9) The optimal profit for the retailer is:

$$V_r^D = g_1^DG_1^{D2} + g_2^DG_2^{D2} + g_3^DG_1^DG_2^D + g_4^DG_1^D + g_5^DG_2^D + g_6^D$$

The proof is the same as Proposition 1 and is omitted here.

Further comparative analysis of the results of the above propositions leads to the following inferences.

Corollary 1. *The retail price of a product in a concentrated decision situation is influenced not only by its goodwill but also by the goodwill of competing products. The retail price of the product* *is proportional to both the product's goodwill and the goodwill of the competing product, i.e., the greater the product's own goodwill and the greater the goodwill of the competing product, the higher the retail price of the product.*

Corollary 2. *The recovery price of a product is inversely proportional to the goodwill of the product itself in both centralized and decentralized decision making. The impact of goodwill on the product's recovery price is the same for both decision scenarios.*

Corollary 3. *The wholesale and retail prices of a competing manufacturer's product in a decentralized decision situation are also influenced by the product's own goodwill and the goodwill of the competing product, and both wholesale and retail prices are proportional to the product's own goodwill and the goodwill of the competing product.*

4. Example Analysis

In order to verify the previous conclusions, this section is conducted to compare and analyze the optimal decision paths of the closed-loop supply chain under different decision scenarios using numerical simulations of the previous model with the software Mathematica, and further studied the effects of different parameters on the decision variables in the closed-loop supply chain and the benefits of the supply chain and the chain members. The parameter assignment draws on the values selected from previous studies in the literature and is selected in the context of objective reality. The specific parameters are assigned as follows:

$$\alpha_1 = 1; \alpha_2 = 1; \beta = 0.5; a_1 = 1; a_2 = 0.8; \gamma = 0.2; \eta = 2; \Delta_1 = 5; \Delta_2 = 7;$$
$$c_1 = 10; c_2 = 15; \mu_m = 10; \rho = 0.2; \phi = 0.6; d = 30; h = 10; k = 1$$

The results (1): Table 2 shows that the advertising level of the two competing manufacturers was 10.12 and 12.05, and the product goodwill was 20.25 and 24.1, respectively, in the centralized decision, while the advertising level of the manufacturers was significantly reduced to 1.78 and 1.37, and the product goodwill was 3.57 and 2.75, respectively, in the decentralized decision. The level of advertising and product goodwill of competing manufacturers is significantly lower in decentralized decision making than in centralized decision-making.

Table 2. Comparison of optimal decisions for closed-loop supply chains under steady-state conditions.

Variable	Centralized Decision	Decentralized Decision
Manufacturer 1 Ad. input	10.12	1.78
Manufacturer 2 Ad. input	12.05	1.37
Manufacturer 1 wholesale price	-	19.66
Manufacturer 2 wholesale price	-	20.89
Product 1 retail price	43.71	25.36
Product 2 retail price	54.8	25.27
Product 1 recycle price	1.49	2.32
Product 2 recycle price	2.3	3.36
Product 1 reputation	20.25	3.57
Product 2 reputation	24.1	2.75
Manufacturer 1 profits	-	512.31
Manufacturer 2 profits	-	683.79
Retailer profits	-	189.29
Supply chain total profits	4865.94	1385.39

Result (2): The retail prices of Product 1 and Product 2 at centralized decision making were 43.71 and 54.8, respectively, and the retail price of Product 2 was higher than that of Product 1; whereas the retail prices of the manufacturer's products at decentralized decision making were significantly lower than those at centralized decision making–25.36 and 25.27, respectively–and the difference between the retail prices of the two products was significantly reduced.

Result (3): The recovery price of Product 1 at the time of the centralized decision is 1.49, which is smaller than the recovery price of Product 1 at the time of decentralized decision, which is 2.32. The recovery price of Product 2 at the time of the centralized decision is 2.3, which is also smaller than the recovery price of the Product 2 at the time of decentralized decision, which is 3.36. In summary, the recovery price of the product at the time of the decentralized decision is larger than the recovery price of products at the time of the centralized decision.

Result (4): The profits of competing manufacturers at decentralized decision making are 512.31 and 683.79, respectively, and the profits of retailers are 189.29, for a total supply chain profit of 1385.39, which is much smaller than the total supply chain profit of 4865.94 at centralized decision making.

Result (5): For the two competing manufacturers, the level of advertising input of Manufacturer 1 is 10.12 smaller than that of Manufacturer 2 at 12.05 when the decision is centralized, but the level of advertising input of Manufacturer 1 is 1.78 larger than that of Manufacturer 2 at 1.37 when the decision is decentralized.

Further comparing the goodwill of the different products, it is clear from Figure 1 that Product 1 goodwill is initially higher than Product 2 goodwill at the time of centralized decision making, but over time, Product 2 goodwill gradually begins to be greater than Product 1 goodwill. In the case of decentralized decisions, the goodwill curve for Product 1 is always above and greater than that of Product 2.

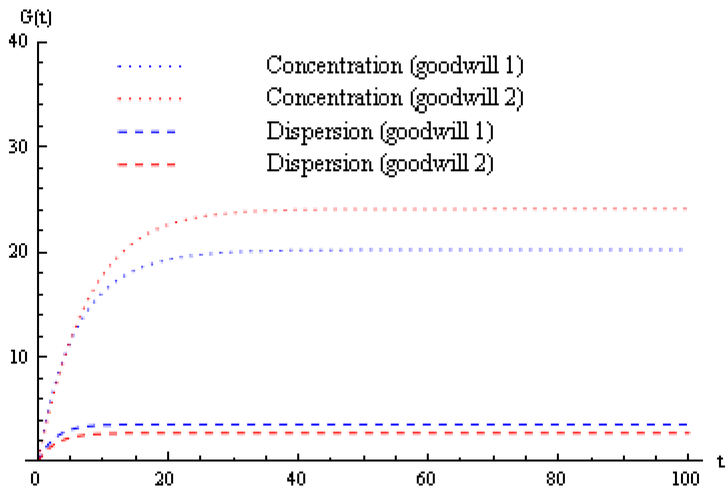

Figure 1. Product goodwill over time for different decision scenarios.

Result (6): Figure 2 reveals that Manufacturer 2 produces a product with a higher wholesale price compared to Manufacturer 1, and its price curve is significantly higher than the wholesale price curve of Product 1. Comparing the retail prices of competing manufacturers' products, it is found that Product 1 has a higher retail price, but the retail price of Product 2 is only slightly lower than that of Product 1, and the difference is not significant, with the two retail price curves approximately overlapping.

Result (7): For a better verification of Corollary 3, the impact of competing product goodwill on the decision variable product's retail price is investigated. Figure 3 clearly illustrates that the goodwill of both Product 1 and Product 2 positively affects the retail price of Product 1, and the impact of the product's goodwill will be greater, while the impact of Product 2 goodwill on the retail price of Product 1 is slightly smaller. Similarly, Figure 4 shows that an increase in the goodwill of both Product 1 and Product 2 will increase the retail price of Product 2, but the retail price of Product 2 is mainly influenced by its own goodwill, and the goodwill of Product 1 has a smaller impact on the retail price of Product 2.

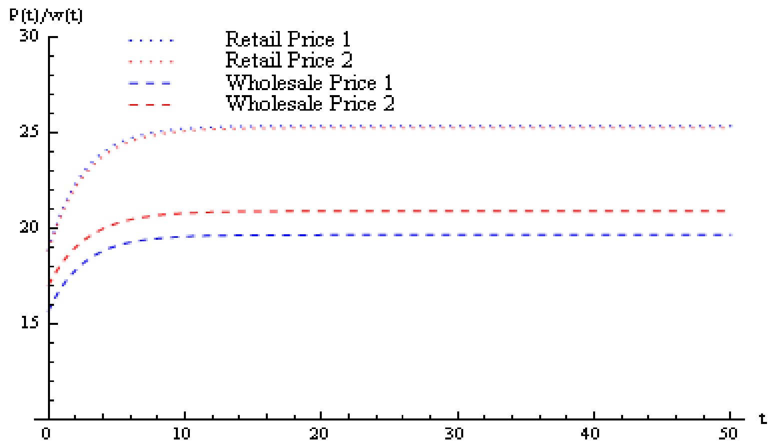

Figure 2. Product optimal price curve over time, under decentralized decision making.

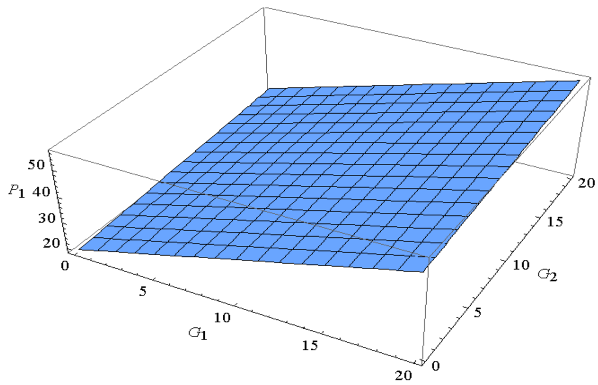

Figure 3. The impact of competitive product goodwill on the retail price of Product 1.

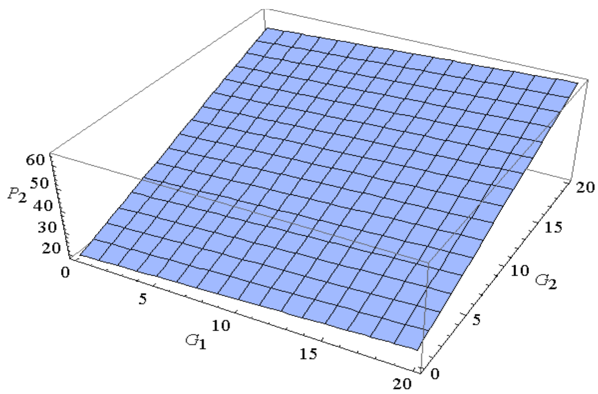

Figure 4. The impact of competitive product goodwill on the retail price of Product 2.

As can be seen from Figure 5, the greater the goodwill of Product 1, the greater the wholesale price of Product 1; the greater the goodwill of Product 2, the greater the wholesale price of Product 1, but the increase in Product 1 goodwill has a greater impact on the wholesale price of Product 1. Similarly for Product 2, Figure 6 illustrates that the goodwill of both Product 1 and Product 2 will have a positive impact on the wholesale price of Product 2, but will be more affected by changes in its own goodwill.

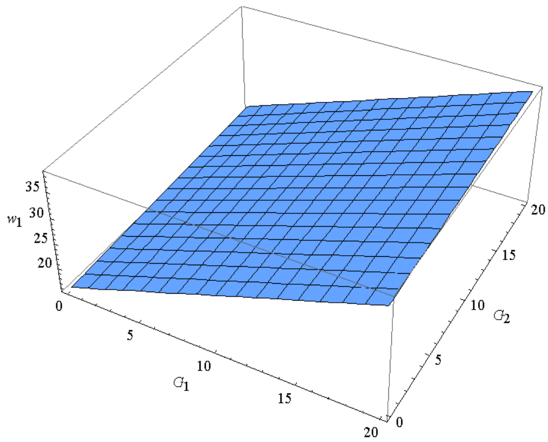

Figure 5. The impact of competitive product goodwill on the wholesale price of Product 1.

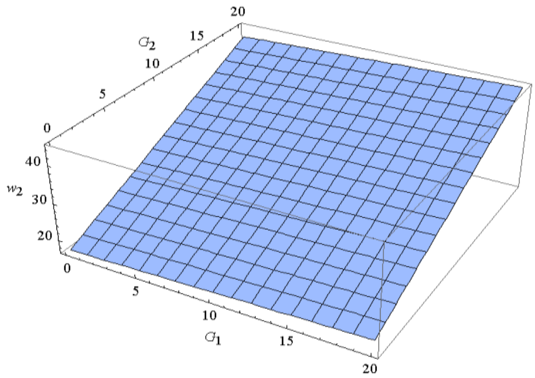

Figure 6. The impact of competitive product goodwill on the wholesale price of Product 2.

In summary, it can be concluded that the wholesale and retail prices of a competing manufacturer's products are influenced not only by the product's own goodwill, but also by the goodwill of the competing product, and that both have a positive effect on the wholesale and retail prices of the product. Further comparison also shows that the wholesale and retail prices of the product are more affected by its own goodwill and less affected by the goodwill of the competing product on its own wholesale and retail prices.

In order to further investigate the effect of the intensity of manufacturer price competition on the equilibrium solution of the closed-loop supply chain differential game, a sensitivity analysis is next performed on the relevant covariates.

Results (8): Since the price demand coefficient of Manufacturer 2 is 0.8, considering that product demand should be mainly influenced by its own price, the impact of competitive product price on its demand should be less than the impact of its own price on demand, so the price competition intensity should be less than 0.8. The sensitivity analysis experiment in this paper sets the upper limit of the value taken at 0.7. As can be seen from Table 3, compared to the initial assumption of price competition intensity (i.e., when the price competition intensity is 0.2), when the manufacturer's price competition intensity increases to 0.3, the manufacturer's product goodwill increases to 4.28 and 3.6, respectively, while the wholesale price of the product increases to 21.82 and 23.59, respectively, and the retail price increases to 29.93 and 30.93, respectively. The recycling price of the manufacturer's product decreased to 2.29 and 3.32, respectively. Manufacturers' profits increased to 602.91 and 743.87, respectively, and retailers' profits increased to 365.74. Similarly, when the intensity of competition among manufacturers increased further, the goodwill, wholesale price, and retail price of the products increased, and manufacturers' and retailers' profits became larger.

Table 3. Impact of price competition intensity on optimal decision making in closed-loop supply chains.

γ	Reputation1	Reputation2	Wholesale Price 1	Wholesale Price 2	Retail Price 1	Retail Price 2	Recycle Price 1	Recycle Price 2	Manufactuer1 Profits	Manufactuer2 Profits	Retailer Profits
0	2.66	1.59	16.66	16.99	19.99	17.98	2.37	3.42	413.33	632.74	59.39
0.1	3.05	2.11	17.99	18.76	22.22	21.17	2.35	3.39	453.15	650.47	102.71
0.2	3.57	2.75	19.66	20.89	25.36	25.27	2.32	3.36	512.31	683.79	189.29
0.3	4.28	3.6	21.82	23.59	29.93	30.93	2.29	3.32	602.91	743.87	365.74
0.4	5.26	4.73	24.68	27.08	36.87	39.22	2.24	3.26	747.35	849.99	740.66
0.5	6.77	6.5	28.8	32.13	48.7	53.13	2.16	3.18	999.72	1054.37	1655.09
0.6	9.29	9.44	35.3	40.04	71.96	80.05	2.04	3.03	1499.37	1486.52	4322.65
0.7	14.48	15.4	47.78	55.16	132.81	149.63	1.78	2.73	2778.05	2653.43	15,619.7

In comparison to the initial assumption of price competition intensity (i.e., when the price competition intensity is 0.2), when the manufacturer price competition intensity decreases to 0.1, the goodwill of the two products decreases to 3.05 and 2.11, respectively, and the wholesale price of the products decreases to 17.99 and 18.76, and the retail price decreases to 22.22 and 21.17. Conversely, the recycling price of the products increases to 2.35 and 2.39, respectively. The profit of the two competing manufacturers was reduced to 453.15 and 650.47, and the profit of the retailers was reduced to 102.71.

In particular, when the intensity of price competition is further reduced to zero, i.e., there is no price competition between the two manufacturers, product goodwill is further reduced to 2.66 and 1.59, product wholesale prices are reduced to 16.66 and 16.99, retail prices are reduced to 19.99 and 17.98, and product recall prices are increased to 2.37 and 3.42. The profits of the two manufacturers are reduced to 413.33 and 632.74, and the retailers' profit decreased by 59.39.

In summary, the greater the intensity of price competition, i.e., the more intense the competition between manufacturers, the more the two manufacturers will spend on advertising and the greater the goodwill of the product. As a result, the wholesale and retail prices of the product will be higher, the recycling price of the product will decrease as the price competition increases, and the profits of the supply chain members will increase as the price competition increases.

Result (9): The effect of the intensity of price competition on the retail price of the products was further investigated. We can see from Figure 7 that as the intensity of price competition increases, the retail prices of both products gradually increase. However, when the degree of price competition is small, the retail price of Product 1 is greater than that of Product 2, but as the intensity of manufacturer competition increases, the retail price of Product 2 begins to be higher than that of Product 1 when the intensity of price competition reaches 0.3.

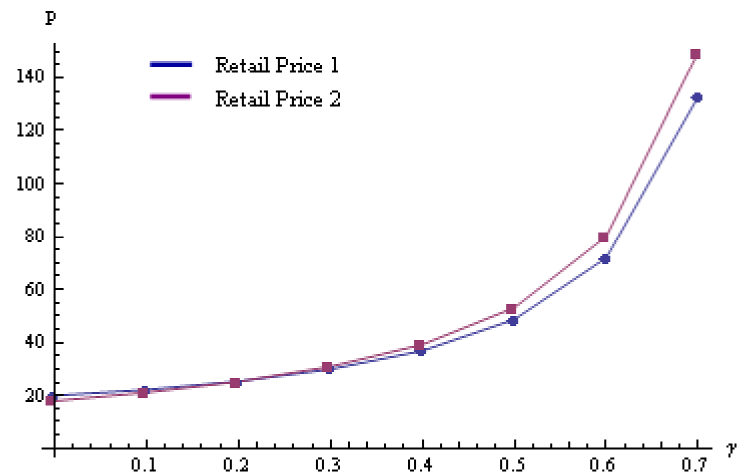

Figure 7. The impact of the intensity of price competition on retail prices.

5. Conclusions and Outlook

On the premise of random fluctuations in marginal profits of members in the supply chain and manufacturer price competition, a dynamic closed-loop supply chain differential game model in which manufacturers' goodwill for different products affects product demand and recall is developed to investigate the optimal decision paths of members in the closed-loop supply chain under centralized and decentralized decision making, and to further explore the impact of manufacturer competition on the decisions of members in the closed-loop supply chain. The main findings of this paper are summarized as follows:

Conclusion (1): By comparing the centralized decision scenario with the decentralized decision scenario, the study found that the level of advertising investment and product goodwill of competing manufacturers was significantly higher in the centralized decision than in the decentralized decision. The overall profitability of the supply chain was significantly higher in the centralized decision than in the decentralized decision. The recall price of the manufacturer's products was significantly higher in the decentralized decision than in the centralized decision, but the difference was that the retail price of the products was less than the retail price of the products in the centralized decision.

Conclusion (2): Through the study, we found that the wholesale, retail, and recycling prices of competing manufacturers' products are all related to product goodwill, with the wholesale and retail prices of products being positively related to product goodwill and the recycling price of products being inversely related to product goodwill. The greater the product goodwill, the greater the profitability of the supply chain members. This means that companies must pay attention to maintaining product goodwill, which can bring them greater profits. Retailers should encourage manufacturers to increase their product goodwill, as the manufacturer's investment in advertising will increase the profitability of the retailer.

Conclusion (3): Unlike the findings of the paper [17], the study also found that the wholesale and retail prices of a product are affected not only by its goodwill, but also by the goodwill of competing products, and both have a positive impact on the wholesale and retail prices of the product. That is, the greater the product's goodwill and the greater the goodwill of competing products, the higher the wholesale and retail prices of the product.

Conclusion (4): The study finds that manufacturer competition leads to higher wholesale and retail prices of competing products, which also increases supply chain members' profits. This is different from the conclusion of previous studies that the higher the intensity of manufacturer competition, the lower the product price, mainly because the higher the intensity of price competition, the greater the goodwill of the product will be, the higher the wholesale and retail prices of the product will be, and the greater the profits of the supply chain members. This indicates that product goodwill plays a protective role on product prices in the case of manufacturer competition. In addition, compared with the literature [19], this paper further investigates the effect of manufacturer competition on the recycling price, and finds that the recycling price is not directly affected by the intensity of retailer competition, but the intensity of retailer competition affects the recycling price by influencing the product goodwill.

The mathematical model of this study is more comprehensive and more consistent with objective reality, and to a certain extent, it expands the depth and breadth of research on closed-loop supply chain management. The research in this paper is in line with the current business competition model in the context of economic globalization, and provides a theoretical reference for enterprises' closed-loop supply chain decisions. Specifically, the key management insights from this paper are as follows:

(1) The manufacturer's advertising investment can effectively increase product goodwill, and the increase of product goodwill will further increase the level of product demand, because the product's advertising goodwill will attract some price-sensitive customers, who will expand the product demand, and the manufacturer and the retailer can obtain greater profits. Thus, the retailer should also support the manu-

facturer's advertising investment and help the manufacturer to further increase the product goodwill.

(2) Manufacturers' competition will lead to the improvement of product goodwill, product price will be further increased by the influence of competitive product goodwill, and the profit of supply chain members will be increased, so manufacturers' competition is beneficial to supply chain members. Thus, proper competition is necessary for a supply chain, and manufacturers should strive to continuously improve their competitiveness and gain competitive advantage in the competition.

(3) The competition among manufacturers is mostly price competition, i.e., common price war. In the process of manufacturer competition, manufacturers will choose to reduce prices in order to obtain greater profits to expand demand, but the constant price cuts are obviously detrimental to the long-term development of manufacturers. Goodwill can protect the demand for products, so that manufacturers do not have to lower prices in order to further expand demand in the competition, which plays a certain role in price protection. Therefore, manufacturers should strengthen the advertising of goodwill for their products and attract customers with the goodwill of the products themselves to achieve long-term development.

This paper investigates a differential game for a second-level, dynamic closed-loop supply chain with complete information; however, in real life, the structure of supply chains and the external environment they face are often quite complex and can be influenced by many factors. Therefore, the influencing factors and the premises assumed in this study are still somewhat limited and do not consider more complex situations. In subsequent studies, the impact of information asymmetry and the risk aversion and equity concerns of supply chain members on the dynamic optimization of closed-loop supply chains can be considered, and the two-tier supply chain in this study can be expanded to a three-tier or even multi-level supply chain, or a two-channel supply chain can be studied to further explore the dynamic optimization of more complex closed-loop supply chains.

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