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On Coefficient Estimates for a Certain Class of Analytic Functions

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Abstract: In this paper, we consider a subclass $\mathcal{SQ}$ of normalized analytic functions f satisfying $\Re\sqrt{f'(z)} > 1/2$. For the functions in the class $\mathcal{SQ}$, we determine upper bounds for a number of coefficient estimates, among which are initial coefficients, the second Hankel determinant, and the Zalcman functional. Upper estimates for higher-order Schwarzian derivatives are also obtained.

Keywords: analytic functions; coefficient estimates; Hankel determinant; Schwarzian derivative

MSC: 30C45; 30C50

1. Introduction

Let $\mathcal{A}$ be the family of all analytic and normalized functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

defined on the open unit disk $\mathcal{U} = \{z \in \mathbb{C} : |z| < 1\}$.

The class of Schwarz functions ω, which are analytic in $\mathcal{U}$ and satisfy $|\omega(z)| < 1$, $\omega(0) = 0$, is denoted by $\mathcal{B}$. If $\omega \in \mathcal{B}$, then its power series expansion is given by

$$\omega(z) = \sum_{n=1}^{\infty} c_n z^n. \tag{2}$$

For two functions f and g analytic in $\mathcal{U}$, we say that f is subordinate to g, written $f \prec g$, if $\omega \in \mathcal{B}$ exists such that

$$f(z) = g(\omega(z)), \;\; z \in \mathcal{U}.$$

If, in particular, g is univalent in $\mathcal{U}$, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$.

Let $f \in \mathcal{A}$ given by (1). The Hankel determinant $H_2(1) = a_3 - a_2^2$ is the well known Fekete-Szegö functional, which is also a particular case of the Zalcman functional $a_{n+m-1} - a_n a_m$ [1]. The second Hankel determinant is given by $H_2(2) = a_2 a_4 - a_3^2$. For related results to upper bounds of the Hankel determinant and the Zalcman functional, see for example [2–9].

The Schwarzian derivative for $f \in \mathcal{A}$ is defined by

$$S(f)(z) = \left(\frac{f''(z)}{f'(z)}\right)' - \frac{1}{2}\left(\frac{f''(z)}{f'(z)}\right)^2, \;\; z \in \mathcal{U}.$$

The higher-order Schwarzian derivatives are defined inductively (see [10,11]) as follows:

$$\sigma_3(f) = S(f) \tag{3}$$

$$\sigma_{n+1}(f) = (\sigma_n(f))' - (n-1)\sigma_n(f)\frac{f''}{f'}, \;\; n \geq 4. \tag{4}$$

Let $S_n = \sigma_n(f)(0)$. If $f \in \mathcal{A}$ is of the form (1), then

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$$\begin{aligned} S_3 &= 6(a_3 - a_2^2) \\ S_4 &= 24(a_4 - 3a_2a_3 + 2a_2^3) \\ S_5 &= 24(5a_5 - 20a_2a_4 - 9a_3^2 + 48a_3a_2^2 - 24a_2^4). \end{aligned} \tag{5}$$

The related results for higher-order Schwarzian derivatives may be found in [12,13].

Denote by $\mathcal{SQ}$ the class of analytic functions f satisfying

$$\Re\sqrt{f'(z)} > \frac{1}{2}, \;\; z \in \mathcal{U} \tag{6}$$

or in terms of subordination

$$f'(z) \prec \frac{1}{(1-z)^2}, \;\; z \in \mathcal{U}. \tag{7}$$

Recently, several authors have investigated various coefficient estimates for functions belonging to different subclasses of univalent functions (see, for example [14–19], to mention only a few).

Based on the results obtained in previous research, in this paper, we investigate the initial coefficient bounds, the Zalcman functional, and the second Hankel determinant for functions in the class $\mathcal{SQ}$. Bounds for the higher-order Schwarzian derivatives for the class $\mathcal{SQ}$ are also obtained.

In order to prove our results, the next lemmas for Schwarz functions will be used.

Lemma 1 ([20])**.** *Let $\omega(z) = c_1z + c_2z^2 + \ldots$ be a Schwarz function. Then, for any real numbers α, β such that*

$$(\alpha, \beta) \in \left\{|\alpha| \le \frac{1}{2}, -1 \le \beta \le 1\right\} \cup \left\{\frac{1}{2} \le |\alpha| \le 2, \frac{4}{27}(|\alpha|+1)^3 - (|\alpha|+1) \le \beta \le 1\right\}$$

the following estimate holds:

$$|c_3 + \alpha c_1c_2 + \beta c_1^3| \le 1.$$

Lemma 2 ([21])**.** *Let $\omega(z) = c_1z + c_2z^2 + \ldots$ be a function in the class $\mathcal{B}$. Then, the next estimates hold*

$$\begin{gathered} |c_1| \le 1, \;\; |c_2| \le 1 - |c_1|^2 \\ |c_3| \le 1 - |c_1|^2 - \frac{|c_2|^2}{1+|c_1|} \\ |c_4| \le 1 - |c_1|^2 - |c_2|^2. \end{gathered}$$

The next result obtained by Efraimidis will be also needed.

Lemma 3 ([22])**.** *Let $\omega(z) = c_1z + c_2z^2 + \ldots$ be a Schwarz function. Then, for any complex number λ, the following estimates hold:*

$$|c_2 + \lambda c_1^2| \le \max\{1, |\lambda|\} \tag{8}$$

$$|c_4 + (1+\lambda)c_1c_3 + c_2^2 + (1+2\lambda)c_1^2c_2 + \lambda c_1^4| \le \max\{1, |\lambda|\}. \tag{9}$$

2. Coefficient Estimates

In this section, we obtain sharp bounds for the first five Taylor coefficients for functions in the class $\mathcal{SQ}$.

Theorem 1. *Let $f \in \mathcal{SQ}$ be of the form (1). Then, the first five initial coefficients of f are bounded by one. The estimates are sharp.*

Proof. Assume that f is in $\mathcal{SQ}$. Then, from (7), we obtain that there exists a Schwarz function ω of the form (2) such that

$$f'(z) = \frac{1}{(1-\omega(z))^2}, \quad z \in \mathcal{U}. \tag{10}$$

Making use of the series (1) and (2) into (10) and equating the coefficients, we obtain

$$\begin{aligned} a_2 &= c_1 \\ a_3 &= \frac{1}{3}(2c_2 + 3c_1^2) \\ a_4 &= \frac{1}{2}(c_3 + 3c_1c_2 + 2c_1^3) \\ a_5 &= \frac{1}{5}(2c_4 + 6c_1c_3 + 3c_2^2 + 12c_1^2c_2 + 5c_1^4). \end{aligned} \tag{11}$$

It is obvious that $|a_2| \leq 1$. Since

$$a_3 = \frac{2}{3}(c_2 + \frac{3}{2}c_1).$$

the bound $|a_3| \leq 1$ follows easily from (8) with $\lambda = 3/2$. For the fourth coefficient, we have

$$|a_4| = \frac{1}{2}\Big|(c_3 + 2c_1c_2 + c_1^3) + (c_1c_2 + c_1^3)\Big| \leq \frac{1}{2}\Big(|c_3 + 2c_1c_2 + c_1^3| + |c_1c_2 + c_1^3|\Big).$$

The inequality $|c_3 + 2c_1c_2 + c_1^3| \leq 1$ follows from Lemma 1 with $\alpha = 2$ and $\beta = 1$. Applying (8) with $\lambda = 1$, we obtain

$$|c_1c_2 + c_1^3| = |c_1||c_2 + c_1^2| \leq 1.$$

Finally, we have $|a_4| \leq 1$. Observe that

$$|a_5| = \frac{2}{5}\left|(c_4 + 3c_1c_3 + c_2^2 + 5c_1^2c_2 + 2c_1^4) + \frac{1}{2}(c_2^2 + 2c_1^2c_2 + c_1^4)\right|$$

$$\leq \frac{2}{5}|c_4 + 3c_1c_3 + c_2^2 + 5c_1^2c_2 + 2c_1^4| + \frac{1}{5}|c_2^2 + 2c_1^2c_2 + c_1^4|.$$

From (9) with $\lambda = 2$, we immediately obtain $|c_4 + 3c_1c_3 + c_2^2 + 5c_1^2c_2 + 2c_1^4| \leq 2$. For the bound of the second term, the triangle inequality and the inequality of $|c_2|$ in Lemma 2 give

$$|c_2^2 + 2c_1^2c_2 + c_1^4| \leq |c_2|^2 + 2|c_1|^2|c_2| + |c_1|^4 \leq (1 - |c_1|^2)^2 + 2|c_1|^2(1 - |c_1|^2) + |c_1|^4 = 1$$

and therefore $|a_5| \leq 1$.

The estimates for all five coefficients are sharp for the function $f(z) = \frac{z}{1-z}$. $\square$

3. Bounds for Hankel Determinant and Zalcman Functional

In this section, the bounds for Hankel determinants $H_2(1)$, $H_2(2)$, and the Zalcman functional $a_4 - a_2a_3$ and $a_5 - a_3^2$ are obtained.

Theorem 2. *Let $f \in \mathcal{SQ}$ be of the form (1). Then,*

$$|H_2(1)| \leq \frac{2}{3} \text{ and } |H_2(2)| \leq \frac{4}{9}.$$

The bounds are sharp.

Proof. Suppose that $f \in \mathcal{SQ}$ has the form (1). The first inequality follows easily:

$$|H_2(1)| = |a_3 - a_2^2| = \frac{2}{3}|c_2| \leq \frac{2}{3}.$$

Making use of (11), we have

$$|H_2(2)| = |a_2a_4 - a_3^2| = \frac{1}{18}|9c_1c_3 + 3c_1^2c_2 - 8c_2^2|.$$

By triangle inequality, we obtain

$$|H_2(2)| \leq \frac{1}{18}\Big(9|c_1||c_3| + 3|c_1|^2|c_2| + 8|c_2|^2\Big).$$

Applying the inequalities for $|c_2|$ and $|c_3|$ in Lemma 2, we receive

$$\begin{aligned}
|H_2(2)| &\leq \frac{1}{18}\left[9|c_1|\left(1-|c_1|^2-\frac{|c_2|^2}{1+|c_1|}\right) + 3|c_1|^2|c_2| + 8|c_2|^2\right] \\
&= \frac{1}{18}\left[9|c_1|(1-|c_1|^2) + |c_2|^2\frac{8-|c_1|}{1+|c_1|} + 3|c_1|^2|c_2|\right] \\
&\leq \frac{1}{18}\left[9|c_1|(1-|c_1|^2) + (1-|c_1|^2)^2\frac{8-|c_1|}{1+|c_1|} + 3|c_1|^2(1-|c_1|^2)\right] \\
&= \frac{2}{9}(-|c_1|^4 - |c_1|^2 + 2) \leq \frac{4}{9}.
\end{aligned}$$

If $c_2 = 1$ and $c_k = 0, k \neq 2$, then $a_2 = 0, a_3 = 2/3$, and $a_4 = 0$. This shows that the equality in the assertion of our theorem holds for the function given by (10) with $\omega(z) = z^2$. $\square$

Theorem 3. *If $f \in \mathcal{SQ}$ is of the form (1), then the next inequalities hold*

$$|a_4 - a_2a_3| \leq \frac{8\sqrt{3}}{27} \quad \textit{and} \quad |a_5 - a_3^2| \leq 0.7789\ldots$$

Proof. Assume that $f \in \mathcal{SQ}$. From (11), we obtain

$$|a_4 - a_2a_3| = \frac{1}{6}|3c_3 + 5c_1c_2|.$$

Then, by triangle inequality, we have

$$|a_4 - a_2a_3| \leq \frac{1}{6}(3|c_3| + 5|c_1||c_2|).$$

In view of Lemma 2, we obtain

$$\begin{aligned}
|a_4 - a_2a_3| &\leq \frac{1}{6}\left[3\left(1-|c_1|^2-\frac{|c_2|^2}{1+|c_1|}\right) + 5|c_1||c_2|\right] \\
&= \frac{1}{6}\left(3 - 3|c_1|^2 - \frac{3|c_2|^2}{1+|c_1|} + 5|c_1||c_2|\right). \qquad (12)
\end{aligned}$$

Writing $|c_1| = x$ and $|c_2| = y$ in (12), we obtain

$$|a_4 - a_2a_3| \leq g(x,y)$$

where

$$g(x,y) = \frac{1}{6}\left(3 - 3x^2 - \frac{3y^2}{1+x} + 5xy\right).$$

Since $|c_2| \leq 1 - |c_1|^2$, the region of variability of (x,y) coincides with

$$D = \Big\{(x,y) : 0 \leq x \leq 1, 0 \leq y \leq 1 - x^2\Big\}.$$

Therefore, we need to find the maximum value of $g(x,y)$ over the region D. The critical points of $g(x,y)$, given by the system

$$\begin{cases} -6x + \dfrac{3y^2}{(1+x)^2} + 5y = 0 \\ -\dfrac{6y}{1+x} + 5x = 0 \end{cases}$$

are $(0,0)$ and $(\frac{22}{75}, \frac{1067}{3375})$. Elementary calculations show that $(0,0)$ is a maximum point and $g(0,0) = 1/2$. On the boundary of D, we have

$$\begin{aligned} g(x,0) &= \frac{1}{6}(3-3x^2) = \frac{1}{2}(1-x^2) \le \frac{1}{2} \\ g(0,y) &= \frac{1}{6}(3-3y^2) = \frac{1}{2}(1-y^2) \le \frac{1}{2} \\ g(x,1-x^2) &= \frac{4}{3}(x-x^3) \le \frac{8\sqrt{3}}{27} = 0.5132\ldots, \text{ for } x = \frac{1}{\sqrt{3}}. \end{aligned}$$

From all these inequalities, we obtain

$$g(x,y) \le \frac{8\sqrt{3}}{27}, \text{ for all } (x,y) \in D$$

which is the desired bound for $|a_4 - a_2 a_3|$. The Schwarz function

$$\omega(z) = z\frac{z+c_1}{1+c_1 z} = c_1 z + (1-c_1^2)z^2 - c_1(1-c_1^2)z^3 + \ldots,$$

where $c_1 = \frac{1}{\sqrt{3}}$, $c_2 = \frac{2}{3}$, and $c_3 = -\frac{2}{3\sqrt{3}}$, which shows that this inequality is sharp.

Now, we continue with the estimate of $|a_5 - a_3^2|$. From (11), we obtain

$$|a_5 - a_3^2| = \frac{1}{5}\left|2c_4 + 6c_1 c_3 + \frac{16}{3}c_1^2 c_2 + \frac{7}{9}c_2^2\right|.$$

Using the triangle inequality and the inequalities for $|c_4|$, $|c_3|$ in Lemma 2, we obtain

$$\begin{aligned} |a_5 - a_3^2| &\le \frac{1}{5}\left[2(1-|c_1|^2-|c_2|^2) + 6|c_1|\left(1-|c_1|^2 - \frac{|c_2|^2}{1+|c_1|}\right) + \frac{16}{3}|c_1|^2|c_2| + \frac{7}{9}|c_2|^2\right] \\ &= \frac{1}{5}\left[2 + 6|c_1| - 2|c_1|^2 - 6|c_1|^3 - \frac{6|c_1||c_2|^2}{1+|c_1|} - \frac{11}{9}|c_2|^2 + \frac{16}{3}|c_1|^2|c_2|\right]. \end{aligned} \tag{13}$$

Writing $|c_1| = x$ and $|c_2| = y$ in (13), we have $|a_5 - a_3^2| \le h(x,y)$, where

$$h(x,y) = \frac{1}{5}\left(2 + 6x - 2x^2 - 6x^3 - \frac{6xy^2}{1+x} - \frac{11}{9}y^2 + \frac{16}{3}x^2 y\right).$$

Taking into account the inequality $|c_2| \le 1 - |c_1|^2$, the region of variability of (x,y) coincides with

$$D = \left\{(x,y) : 0 \le x \le 1, 0 \le y \le 1 - x^2\right\}.$$

Thus, we we need to find the maximum value of $h(x,y)$ over the region D. The solutions of the system

$$\begin{cases} 6 - 4x - 18x^2 - \dfrac{6y^2}{(1+x)^2} + \dfrac{32}{3}xy = 0 \\ -\dfrac{12xy}{1+x} - \dfrac{22}{9}y + \dfrac{16}{3}x^2 = 0 \end{cases}$$

are the critical points of $h(x,y)$. The maximum of $h(x,y)$ is attained in $(0.5285\ldots, 0.2259\ldots)$, and its value is $h(0.5285\ldots, 0.2259\ldots) = 0.7789\ldots$ On the boundary of the region D, we have

$$\begin{aligned} h(x,0) &= \frac{1}{5}(2-2x^2-6x^3+6x) \le \frac{32(10+7\sqrt{7})}{1215} = 0.7511\ldots \\ h(0,y) &= \frac{1}{5}(2-\frac{11}{9}y^2) \le \frac{2}{5} = 0.4 \\ h(x,1-x^2) &= \frac{1}{45}(7+106x^2-113x^4) \le \frac{80}{113} = 0.7079\ldots \end{aligned}$$

It follows that $h(x, y) \leq 0.7789\ldots$ for $(x, y) \in D$, which is the desired bound for $|a_5 - a_3^2|$. $\square$

4. Bounds for Higher-Order Schwarzian Derivatives

In this section, we investigate the upper bounds of $|S_3|$, $|S_4|$, and $|S_5|$, where S_3, S_4, S_5 are given by (5).

Theorem 4. *Let $f \in \mathcal{SQ}$ be given by (1). Then, the following estimates hold:*

$$|S_3| \leq 4, \quad |S_4| \leq 12, \quad |S_5| \leq 73.176\ldots$$

Proof. Let $f \in \mathcal{SQ}$. From (11), we have

$$|S_3| = 6|a_3 - a_2^2| = 4|c_2| \leq 4$$

For $c_2 = 1$ and $c_k = 0, c_k \neq 2$, we obtain $a_2 = 0$ and $a_3 = 2/3$. This shows that the equality holds for the function given by (10) with $\omega(z) = z^2$. Further

$$|S_4| = 24|a_4 - 3a_2a_3 + 2a_2^3| = 12|c_3 - c_1c_2|.$$

The inequality $|c_3 - c_1c_2| \leq 1$ follows from Lemma 1 with $\alpha = -1$ and $\beta = 0$. Hence, $|S_4| \leq 12$. If $c_3 = 1$ and $c_k = 0, k \neq 3$, then $a_2 = 0, a_3 = 0$ and $a_4 = 1/2$. This means that equality holds for the function given by (10) with $\omega(z) = z^3$.

We continue with the estimate for $|S_5|$. Taking into account (11), we obtain

$$|S_5| = 24|2c_4 - 4c_1c_3 - c_2^2 + 2c_1^2c_2|.$$

By the triangle inequality, we have

$$|S_5| \leq 24(2|c_4| + 4|c_1||c_3| + |c_2|^2 + 2|c_1|^2|c_2|).$$

Applying the inequalities for $|c_3|$ and $|c_4|$ in Lemma 2, we obtain

$$\begin{aligned} |S_5| &\leq 24\left[2(1 - |c_1|^2 - |c_2|^2) + 4|c_1|\left(1 - |c_1|^2 - \frac{|c_2|^2}{1 + |c_1|}\right) + |c_2|^2 + 2|c_1|^2|c_2|\right] \\ &= 24\left(2 - 2|c_1|^2 + 4|c_1| - 4|c_1|^3 - |c_2|^2 - \frac{4|c_1||c_2|^2}{1 + |c_1|} + 2|c_1|^2|c_2|\right). \end{aligned} \tag{14}$$

If we replace $|c_1| = x$ and $|c_2| = y$ in (14), then $|S_5| \leq k(x, y)$ where

$$k(x, y) = 24\left(2 - 4x^3 - 2x^2 + 4x - y^2 - \frac{4xy^2}{1 + x} + 2x^2y\right).$$

Since $|c_2| \leq 1 - |c_1|^2$, the region of variability of (x, y) coincides with

$$D = \left\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1 - x^2\right\}.$$

In order to obtain the upper bound of $|S_5|$, we need to find the maximum value of $k(x, y)$ over the region D. The critical points of $k(x, y)$ are the solution of the system

$$\begin{cases} 1 - x - 3x^2 - \dfrac{y^2}{(1 + x)^2} + xy = 0 \\ -\dfrac{4xy}{1 + x} - y + x^2 = 0 \end{cases}$$

The maximum value of $k(x, y)$ is attained in $(0.44402\ldots, 0.088414\ldots)$. In this case, $k(0.44402\ldots, 0.088414\ldots) = 73.176\ldots$ Next, we verify the behaviour of the function $k(x, y)$ on the boundary of D:

$$
\begin{aligned}
k(x,0) &= 24(2-2x^2-4x^3+4x) \le \frac{8(35+13\sqrt{13})}{9} = 72.7752\ldots \\
k(0,y) &= 24(2-y^2) \le 48 \\
k(x,1-x^2) &= 24(1+6x^2-7x^4) \le \frac{384}{7} = 54.8571\ldots
\end{aligned}
$$

In view of the above inequalities, we obtain $k(x,y) \le 73.176\ldots$ Finally, the proof of the theorem is completed. □

5. Conclusions

In this paper, we investigate a number of coefficient problems for the class $\mathcal{SQ}$. The upper bounds for the initial coefficients, the second Hankel determinant, the Zalcman functional, and the higher-order Schwarzian derivatives have been derived. In our research, we have used the relationship between the coefficients of functions in the considered class $\mathcal{SQ}$ and the coefficients for the corresponding Schwarz functions. The results obtained in this note could be the subject of further investigation related with the Fekete–Szegö type functional such as $a_3 - \mu a_2^2$, $a_2a_4 - \mu a_3^2$ or $a_4 - \mu a_2 a_3$.

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References

1. Brown, J.E.; Tsao, A. On the Zalcman conjecture for starlike and typically real functions. *Math. Z.* **1986**, *191*, 467–474. [CrossRef]
2. Allu, V.; Pandey, A. On the generalized Zalcman comjecture for the class of univalent functionsusing variational method. *arXiv* **2022**, arXiv:2209.10595.
3. Allu, V.; Arora, V.; Shaji, A. On the second Hankel determinant of logarithmic coefficients for certain univalent functions. *arXiv* **2021**, arXiv:2112.03067.
4. Ma, W. Generalized Zalcman conjecture for starlike and typically real functions. *J. Math. Anal. Appl.* **1999**, *234*, 328–339. [CrossRef]
5. Obradović, M.; Tuneski, N. On third order Hankel determinant for inverse functions of certain classes of univalent functions. *Eur. J. Math. Appl.* **2022**, *2*, 7.
6. Obradović, M.; Tuneski, N. Zalcman and generalized Zalcman conjecture for a subclass of univalent functions. *Novi Sad J. Math.* **2022**, *52*, 185–190.
7. Raducanu, D.R.; Zaprawa, P. Second Hankel determinant for close-to-convex functions. *C. R. Math.* **2017**, *355*, 1063–1071. [CrossRef]
8. Sim, Y.J.; Thomas, D.K.; Zaprawa, P. The second Hankel determinant for starlike and convex functions of order α. *Complex Var. Elliptic Equ.* **2022**, *67*, 2433–2443. [CrossRef]
9. Zaprawa, P.; Obradović, M.; Tuneski, N. Third Hankel determinant for univalent starlike functions. *Rev. Real Acad. Cienc. Exactas Fís. Nat. Ser. A Mat.* **2021**, *115*, 49. [CrossRef]
10. Dorff, M.; Szynal, J. Remark on the higher-order Schwarzian derivatives for convex univalent functions. *Tr. Petrozavodsk. Gos. Univ. Ser. Mat.* **2009**, *15*, 7–11.
11. Schippers, E. Distortion theorems for higher-order Schwarzian derivatives of univalent functions. *Proc. Am. Math. Soc.* **2000**, *128*, 3241–3249. [CrossRef]
12. Cho, N.E.; Kumar, V.; Ravichandran, V. Sharp bounds on the higher-order Schwarzian derivatives for Janowski classes. *Symmetry* **2018**, *10*, 348. [CrossRef]
13. Hu, Z.; Wang, X.; Fan, J. Estimate for Schwarzian derivative of certain close-to-convex functions. *AIMS Math.* **2021**, *6*, 10778–10788. [CrossRef]
14. Răducanu, D. Coefficient estimatesfor a subclass of starlike functions. *Mathematics* **2020**, *8*, 1646. [CrossRef]
15. Obradović, M.; Tuneski, N. The third logarithmic coefficient for the class $\mathcal{S}$. *Turk. J. Math.* **2020**, *44*, 1950–1954. [CrossRef]
16. Obradović, M.; Tuneski, N. Coefficients of the inverse of functions for the subclass $\mathcal{U}(\lambda)$. *J. Anal.* **2022**, *30*, 399–404. [CrossRef]
17. Srivastava, H.M.; Hussain, S.; Raziq, A.; Raza, M. The Fekete-Szegö functional for a subclass of analytic functions associated with quasi-subordination. *Carpathian J. Math.* **2018**, *34*, 103–113. [CrossRef]
18. Lee, S.K.; Ravichandran, V.; Supramanian, S. Bounds for the second order Hankel determinant of certain univalent functions. *J. Inequal. Appl.* **2013**, *2013* , 281. [CrossRef]

19. Zaprawa, P. On coefficient problems for functions starlike with respect to symmetric points. *Bol. Soc. Mat. Mex.* **2022**, *28*, 17. [CrossRef]
20. Prokhorov, D.V.; Szynal, J. Inverse coefficients for (α, β)-convex functions. *Ann. Univ. Mariae Curie Sklodowaka A* **1981**, *35*, 125–143.
21. Carlson, F. Sur les coefficients d'une fonction bornée dans le cercle unité. *Ark. Mat. Astr. Fys.* **1940**, *27A*, 8.
22. Efraimidis, I. A generalization of Livingston's coefficient inequalities for functions with positive real part. *J. Math. Anal. Appl.* **2016**, *435*, 369–379. [CrossRef]