mathematics

Article

Optimal Circular Economy and Process Maintenance Strategies for an Imperfect Production–Inventory Model with Scrap Returns

Rung-Hung Su [1], Ming-Wei Weng [2,*], Chih-Te Yang [3,*] and Chia-Hsuan Hsu [1]

[1] Department of Statistics and Information Science, Fu Jen Catholic University, New Taipei City 242062, Taiwan; 141637@mail.fju.edu.tw (R.-H.S.); 408336190@mail.fju.edu.tw (C.-H.H.)
[2] Department of Marketing and Logistics Management, Chaoyang University of Technology, Taichung 413310, Taiwan
[3] Department of Business Administration, Tamkang University, New Taipei City 251301, Taiwan
* Correspondence: mwweng@cyut.edu.tw (M.-W.W.); 122300@mail.tku.edu.tw (C.-T.Y.)

Abstract: To protect our environment, current firms are committed to the circular economy and process maintenance strategies to reduce the waste of resources. In this way, they can also save costs and create an enterprise image and value. Therefore, this study explores an imperfect production system with a circular economy and process maintenance activities, wherein the defective products can be converted into scrap returns (i.e., secondary raw materials) and products can be manufactured using mixed materials containing scrap returns. The proposed system considers multiple products with varying feed rates of scrap returns. According to the scenario of the aforementioned production system, this paper develops a production–inventory model aimed at cost minimization, in which the production run time, purchased quantity of material, number of maintenance times, and recovery rate are decision variables. Furthermore, we also develop a computational algorithm to obtain these optimal solutions efficiently. Finally, the numerical and sensitivity analyses based on a practical case are presented to illustrate the applicability of our method and some managerial implications. For example, both strategies efficiently reduce the total cost per unit time in the proposed numerical example. The sensitivity results can be used to determine the optimal combination of two strategies and the execution moment under various changes in cost parameters.

Keywords: inventory; circular economy; imperfect processes; maintenance; scrap returns

MSC: 90B05

Citation: Su, R.-H.; Weng, M.-W.; Yang, C.-T.; Hsu, C.-H. Optimal Circular Economy and Process Maintenance Strategies for an Imperfect Production–Inventory Model with Scrap Returns. *Mathematics* **2023**, *11*, 3041. https://doi.org/10.3390/math11143041

Academic Editors: Ripon Kumar Chakrabortty and Tiwari Sunil

Received: 24 May 2023
Revised: 20 June 2023
Accepted: 5 July 2023
Published: 8 July 2023

1. Introduction

The economic production quantity (EPQ) model is common in research involving production–inventory problems. The primary objective of EPQ is to identify the optimal production quantity or production run time, with the aim of minimizing costs or maximizing profits. Drawing from the EPQ model's structure, several researchers have incorporated diverse manufacturing scenarios into their models, enhancing their practicality in real-world applications. Multiple-stage production is a common system among them. For instance, products typically undergo arduous processing procedures, including molding, cutting, grinding, painting, assembling, and more. Inventory-related costs arise from the temporary storage of work-in-process goods between stations as a consequence of the varying production speeds at each processing station. Therefore, inventory management in a multi-stage production system is challenging from a practical and academic perspective.

The traditional linear economy pattern follows the take–make–dispose scheme [1], which leads to the irrational use of available resources. To protect our environment, current firms have committed to the circular economy for scrap recycling and low waste during the manufacturing process. According to the Ellen MacArthur Foundation [2], the most recent schematic diagram comparing the linear economy and the circular economy. highlights the recovery process taking place within the factory, which involves the conversion of defective products (or waste) into scrap returns. This process represents an inner circular economy and serves as a manifestation of green manufacturing, with the objective of reducing the usage of natural resources, pollution, and waste, promoting recycling and material reuse, and regulating emissions in various processes. Nowadays, numerous green manufacturing industries have adopted circular economy measures and achieved significant outcomes. One notable example is the utilization of sulfuric acid as a crucial raw material in the semiconductor etching process. Over the past few years, semiconductor manufacturers have dedicated themselves to investing in waste sulfuric acid purification technology to reduce the reliance on natural sulfur mining. In the manufacturing sectors of metal and plastic products, defective items can be repurposed into secondary raw materials by utilizing the melting process. The scrap metal industry is growing in the Sunyani Municipality, and a similar trend is likely to be observed in other cities in Ghana as well [3]. Paletta et al. [4] proposed accelerating the circularity of plastic-based material systems for radical innovations in Italy. Considering the principles of the circular economy, it becomes imperative to address significant aspects of circular systems, including reuse, sharing, repair, refurbishment, remanufacturing, and recycling. By doing so, a closed-loop system can be created, thereby effectively reducing the utilization of resources and carbon emissions. Accordingly, current firms are constantly developing reusable materials and processing equipment such that defective items and scrap can be converted into scrap returns (i.e., secondary raw materials). Not only can this help control costs by reducing waste, but it also serves as one of the implements of the circular economy, thereby improving the financial performance of enterprises. On the other hand, process maintenance activity can also be part of green manufacturing. Process maintenance is a common activity in current firms. It can avoid more defective products caused by equipment degradation through regular or irregular maintenance such that material is not wasted.

Based on the above-mentioned strategies involving low waste, this study would develop a multiple-stage and imperfect production–inventory model incorporating scrap returns and a controllable defective rate. Furthermore, the finished products are manufactured using mixed materials that include scrap returns. To avoid a lack or an unlimited accumulation of scrap returns, we consider two products with different feed rates of scrap returns in this production system. The joint economic order quantity (EOQ) model and the EPQ model are established, wherein the production run time, purchase quantity of material, number of maintenance times, and recovery rate are decision variables. The rest of this paper is organized as follows. Section 2 focuses on the review of the related literature and compares the differences between this study and prior research to highlight the contributions of our study. In Section 3, the notation and assumptions employed in the entire paper are established. Section 4 presents the mathematical formulations and theoretical results. Section 5 provides the solution procedure for this inventory model. Section 6 presents a numerical example to illustrate a solution procedure, followed by a sensitivity analysis based on this example in Section 7, which provides managerial insights and references for decision making. Finally, Section 8 concludes this paper and offers directions for future research.

2. Literature Review

For the production system regarded as multiple-stage, most studies have considered it in their production–inventory models. For instance, a two-stage production inventory system, considering imperfect production processes, preventive maintenance, and inspection, was modeled by Darwish and Ben-Daya [5]. Pearn et al. [6] developed a multiple-stage EPQ with an investment in quality improvement. Then, Chang et al. [7] further considered the variable assembly rate in the manual process. Sarkar and Shewchuk [8] proposed a production system with advanced demand information and early order fulfillment. Paul et al. [9]

developed a disruption recovery model in a three-stage production inventory system. Wang and Chan [10] discussed a robust production problem involving inventory inaccuracies and time delays. The innovative maintenance problem of complement replacement in a multiple-stage production system with imperfect processes was presented by Su et al. [1]. In a recent study conducted by Su et al. [11], the impact of corporate social responsibility (CSR) activities in a two-stage assembly production system was investigated. The authors also presented a conceptual framework that links the appropriate timing and extent of CSR execution to the outcomes of marketing strategies. Other similar multi-stage production systems are discussed by Tayyab et al. [12], Wang and Shi [13], and Cui et al. [14].

As far as we know, it is difficult to achieve a perfect process in the actual manufacturing environment. To enhance the practicality of the production–inventory model in the real world, many scholars have established various assumptions for the imperfect process, such as constant defective rates, random defective rates, random machine breakdowns, and random state shifts (i.e., in-control stage to out-of-control stage). Chang et al. [7] considered a two-stage assembly system (automatic stage and manual stage) with constant defective rates. Pearn et al. [6] explored a two-stage imperfect production system with completely backlogged and constant defective rates. Paul et al. [9] proposed a recovery strategy for maintaining disruptions with random defective rates in a mixed production system. Sarkar and Shewchuk [8] proposed a new approach for production replenishment and order fulfillment in a mixed production system with random defective rates. Öztürk [15] considered a random machine breakdown in an imperfect production system. In addition, Öztürk [16] examined an imperfect production inventory system that involved rework, random breakdowns, and inspection costs. To perform an error-free inspection, Dey et al. [17] examined the effect of random defective rates (that obey a certain distribution) in an imperfect production system with random shifts. Dey et al. [18] developed a sustainable imperfect system with random shifts and selling-price demand. Huang et al. [19] proposed a coordination supply chain model in which the supplier's system had random shifts and price-sensitive demand. Manna et al. [20] addressed important claims regarding the effect of carbon emission on an imperfect system with random shifts and no advance payment base. For the disposal of defective products, in addition to discarding or selling at a discount, most scholars have assumed that the defective products can be completely or partially reworked. In the real world, there exists another disposal method capable of creating value for defective products, known as scrap returns. Su et al. [21] proposed an imperfect system where defective products are transformed into scrap returns. However, the issue of scrap returns is rarely discussed in the literature involving multi-stage production–inventory systems. For convenience, Table 1 presents a brief comparison of the above-mentioned literature by types of stages, imperfect process, and disposal. From Table 1, it is easy to see that there is a research gap involving an imperfect production–inventory model with multiple stages, scrap returns, and maintenance. Therefore, we take this research gap as our research direction.

Table 1. Comparison of the major issues of existing EPQ models with the present paper.

References	Single/Multiple Stage	Imperfect Process	Rework Process	Scrap Returns	Maintenance
Darwish and Ben-Daya [5]	Multiple	RS			V
Pearn et al. [6]	Multiple	CD	V		
Chang et al. [7]	Multiple	CD	V		
Sarker and Shewchuk [22]	Multiple	RD	V		
Paul et al. [9]	Multiple	RD	V		

Table 1. *Cont.*

References	Single/Multiple Stage	Imperfect Process	Rework Process	Scrap Returns	Maintenance
Su et al. [1]	Multiple	CD	V		
Su et al. [11]	Multiple	CD	V		
Tayyab et al. [12]	Multiple	RS	V		
Öztürk [15]	Single	RB			
Öztürk [16]	Single	RB	V		V
Dey et al. [17]	Single	RD/RS	V		
Dey et al. [18]	Single	RS			
Huang et al. [19]	Single	RS			V
Manna et al. [20]	Single	RS	V		
Mahata [23]	Single	RS	V		
Su et al. [21]	Multiple	RS	V	V	
This paper	Multiple	RS	V	V	V

Note: constant defective rate (CD), random defective rate (RD), random machine breakdowns (RB), random state shift (RS).

3. Notation and Assumptions

To facilitate the explanation of the research questions and the model establishment of this paper, the notation and assumptions used throughout this study are summarized as follows.

3.1. Notation

In order to develop the model, we present the following notation, which includes parameters and decision variables.

3.2. Assumptions

1. Consider a production line that is not perfect and that produces multiple products, with the availability of process maintenance. For simplicity, we first establish a production–inventory model with two products. The production cost and materials used for the two products are the same, and the materials used are a mixture of new materials and scrap returns. Figure 1 illustrates a schematic process of the production line.
2. The defective products can be processed to scrap returns after passing the conversion stage. The conversion stage is used to process defective products into usable secondary raw materials through some special equipment, such as a furnace, blender, and extractor. However, a portion of defective products cannot be processed to scrap returns due to the imperfect conversion process. This study considers that capital investment in recovery capacity improvement is available, assuming it increases with the recovery rate.
3. The process is not always in the in-control stage due to degraded equipment; it randomly shifts from the in-control stage to the out-of-control stage after a while, t_1. In this study, t_1 is assumed to be a random variable with a normal distribution. That is, $t_1 \sim N(\mu, \sigma)$, where μ and σ are the mean and standard deviation, respectively (please refer to Su et al. [12]).
4. The defective rate during the in-control stage is constant and lower than or equal to that during the out-of-control stage, i.e., $\theta_1 \leq \theta_2 < 1$. In contrast, the defective rate during the out-of-control stage increases over time, but it can be returned to θ_1 after process maintenance. Note that process maintenance needs to be performed once for the next cycle after the end of the production system. However, if the maintenance cost

is affordable, increasing the number of maintenance times during the out-of-control stage can reduce the waste of resources and indirectly increase production run time to retard the growth of the production cost. At this time, the number of maintenance times during the production cycle will be greater than one. Consequently, we will establish a generalized model for finding the optimal number of maintenance times. In this study, the defective rate during the out-of-control stage is set as the following linear function of time (denoted by $g(t)$):

$$\theta_2 = g(t) = \theta_1 + k\left[t - \left(t_1 + \frac{(i-1)t_2}{n}\right)\right] < 1,$$

where $t_1 + (i-1)t_2/n < t \leq t_1 + it_2/n$, $i = 1, 2, ..., n$ and $k > 0$. Note that the maximum defective rate is $\theta_1 + kt_2/n$. Because the defective rate must be less than 1, there is a condition for a combination of decision variables, i.e., $t_2/n < (1-\theta_1)/k$. Figure 2 shows the change in the defective rate over time during a cycle.

5. It is assumed that the feed rates of scrap returns for two products are different and that they satisfy the inequality, $f_{r,\,1} < f_{r,\,2}$. Generally, product 1 and product 2 are arranged for manufacture in the in-control and out-of-control stages, respectively. In the in-control stage, the feed rate of scrap returns must be less than the recovery rate to avoid a lack of scrap returns, i.e., $f_{r,1} < p_f\theta_1 r$. On the contrary, the feed rate of scrap returns should exceed the recovery rate to avoid an unrestricted accumulation of scrap returns in the out-of-control stage, i.e., $f_{r,2} > p_f\theta_2 r$. Consequently, the inequality, $f_{r,1} < p_f\theta_1 r \leq p_f\theta_2 r < f_{r,\,2}$, is held, which implies the recovery rate, $r \in (f_{r,1}/p_f\theta_1,\ f_{r,\,2}/p_f\theta_2)$.
6. To avoid shortages in sales, the production rate of the finished product is assumed to exceed the demand rate, i.e., $p_f(1-\theta_1) > d_1$ and $p_f(1-\theta_2) > d_2$.

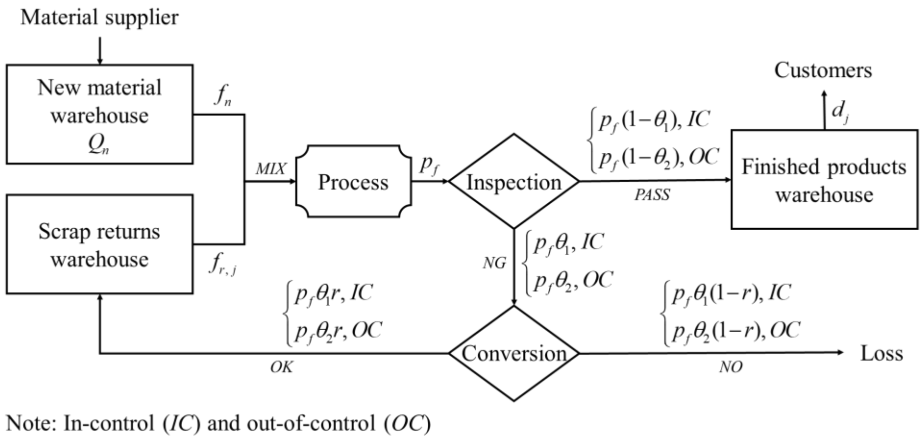

Figure 1. Imperfect production system with the conversion process.

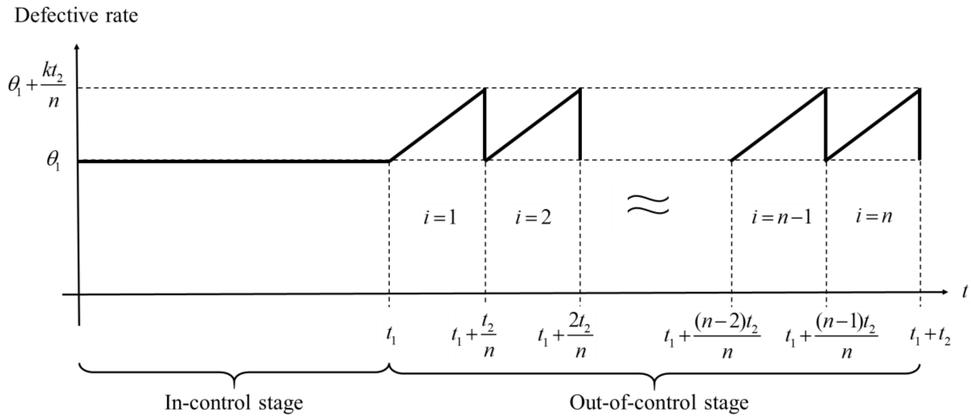

Figure 2. The change in defective rate during a cycle.

4. Model Formulation

Based on the notation and assumptions listed in the previous section, an EPQ model describing the imperfect production–inventory system with recovery and maintenance activities can be established. The main objective is to find the optimal production run time, the number of maintenance times, the purchasing quantity of new material per cycle, and the recovery rate, while minimizing the total cost per unit time. First, Figure 3 presents the inventory levels of scrap returns, new material, and finished products throughout a cycle. As for the inventory level of scrap returns, it increases during the in-control stage (i.e., $p_f\theta_1 r - f_{r,1} > 0$) while it decreases during the out-of-control stage (i.e., $p_f\theta_2 r - f_{r,2} < 0$) based on Assumption (4). For the inventory level of new materials, the amount of Q_n is purchased from the supplier at the beginning of the cycle, and then it gradually depletes to zero over time with the rate $-f_n < 0$. For the inventory level of finished products, according to Assumption (5), the inventory level of product 1 increases during the in-control stage with the rate $p_f(1-\theta_1) - d_1 > 0$, and then it depletes to zero over time with the rate d_1. The inventory level of product 2 increases during the out-of-control stage with the rate $p_f(1-\theta_2) - d_2 > 0$, and then it depletes to zero over time with the rate d_2. Now, we can obtain the following relationships by observing Figure 3:

1. During the in-control stage, the maximum stock of scrap returns can be obtained by multiplying growth rate and time length, i.e., $Q_{rt} = (p_f\theta_1 r - f_{r,1})t_1$. During the out-of-control stage, the inventory level of scrap returns (denoted by $IS_i(t)$) can be governed by the following differential equations:

$$\begin{aligned}\frac{dIS_i(t)}{dt} &= P_f\theta_2 r - f_{r,2} \\ &= P_f r\left\{\theta_1 + k\left[t - \left(t_1 + \frac{(i-1)t_2}{n}\right)\right]\right\} - f_{r,2},\ t_1 + \frac{(i-1)t_2}{n} \le t \le t_1 + \frac{it_2}{n},\end{aligned} \tag{1}$$

with boundary conditions $IS_1(t_1) = Q_{rt}$ and $IS_i(t_1 + it_2/n) = IS_{i+1}(t_1 + it_2/n)$, where $i = 1,\ 2.....,\ n$. Solving these differential equations, a general formulation for $IS_i(t)$ can be obtained as follows:

$$\begin{aligned}IS_i(t) = \quad &(p_f\theta_1 r - f_{r,2})t + (f_{r,2} - f_{r,1})t_1 + p_f kr\left\{\frac{t_1^2 + t^2}{2} - \left[t_1 + \frac{(i-1)t_2}{n}\right]t\right. \\ &\left. + \frac{(i-1)t_2}{n}\left(t_1 + \frac{it_2}{2n}\right)\right\},\ t_1 + \frac{(i-1)}{n}t_2 \le t \le t_1 + \frac{i}{n}t_2.\end{aligned} \tag{2}$$

Based on $IS_n(t_1 + t_2) = 0$, a closed form of t_2 can be derived as follows:

$$t_2 = \frac{f_{r,2} - p_f\theta_1 r + \sqrt{(f_{r,2} - p_f\theta_1 r)^2 + 2p_f kr(f_{r,1} - p_f\theta_1 r)t_1/n}}{p_f kr/n}. \tag{3}$$

2. The maximum purchasing quantity for new material can be obtained by multiplying feed rate and total production run time, i.e., $Q_n = f_n(t_1 + t_2)$;
3. The maximum stock of product 1 can be obtained by multiplying the growth rate and time length during the in-control stage. Furthermore, it can also be obtained by multiplying the demand rate and the period during which the stock depletes, i.e.,

$$Q_{f,1} = \left[p_f(1-\theta_1) - d_1\right]t_1 = d_1 t_{1,d}. \tag{4}$$

After rearranging Equation (4), we can formulate the period during which the stock of product 1 depletes; that is:

$$t_{1,d} = \frac{\left[p_f(1-\theta_1) - d_1\right]t_1}{d_1}. \tag{5}$$

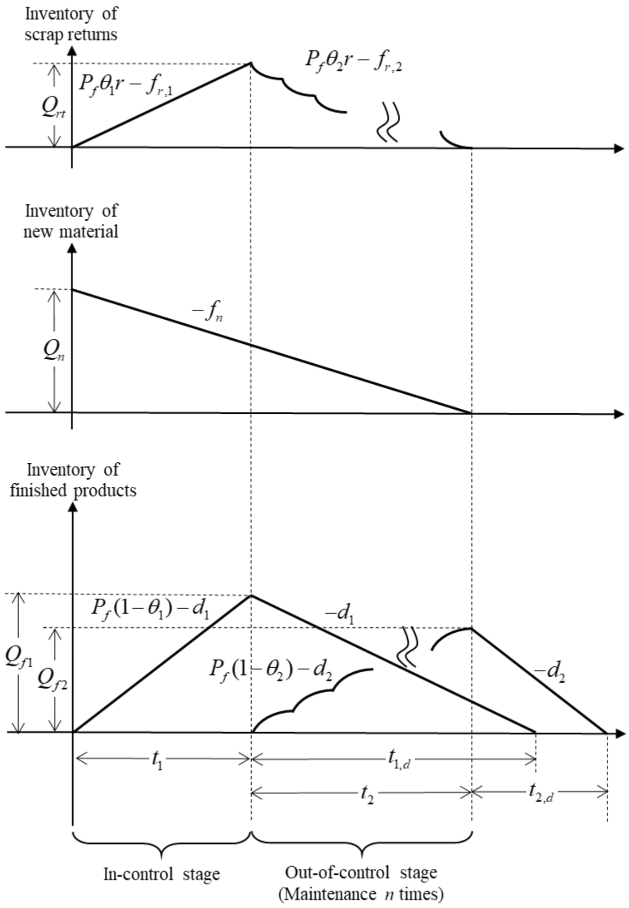

Figure 3. The patterns of inventory levels.

4. The following differential equations can govern the inventory level of product 2 during the out-of-control stage (denoted by $IP_i(t)$):

$$\begin{aligned}\frac{dIP_i(t)}{dt} &= P_f(1-\theta_2)-d_2 \\ &= P_f\left\{1-\theta_1-k\left[t-\left(t_1+\frac{(i-1)t_2}{n}\right)\right]\right\}-d_2,\ t_1+\frac{(i-1)t_2}{n}\le t\le t_1+\frac{it_2}{n}.\end{aligned} \tag{6}$$

with boundary conditions $IP_1(t_1) = 0$ and $IP_i(t_1+it_2/n) = IP_{i+1}(t_1+it_2/n)$, where $i = 1,\ 2.....,\ n$. Similarly, by solving these differential equations, $IP_i(t)$ can be obtained as follows:

$$\begin{aligned}IP_i(t) = \ & (p_f-p_f\theta_1-d_2)(t-t_1)+\frac{p_f k}{2}\left[(t_1^2-t^2)+2(i-1)\frac{(t-t_1)t_2}{n}-i(i-1)\frac{t_2^2}{n^2}\right], \\ & t_1+\frac{(i-1)}{n}t_2\le t\le t_1+\frac{i}{n}t_2.\end{aligned} \tag{7}$$

5. Because the maximum stock of product 2 is $IP_n(t_1+t_2)$, it can be obtained by multiplying the demand rate and the period during which the stock depletes, i.e.,

$$Q_{f,2} = IP_n(t_1+t_2) = (p_f-p_f\theta_1-d_2)t_2+\frac{p_f k}{2}\left(-2t_1-\frac{t_2}{n}\right)t_2 = d_2 t_{2,d}. \tag{8}$$

After rearranging Equation (8), we can formulate the period during which the stock of product 2 depletes; that is:

$$t_{2,d} = \frac{(p_f-p_f\theta_1-d_2)t_2+\frac{p_f k}{2}\left(-2t_1-\frac{t_2}{n}\right)t_2}{d_2}. \tag{9}$$

In this paper, we consider the total cost, including purchasing, setup, production, holding, recovery, maintenance, and opportunity costs. Next, we establish the following elements of the total cost per cycle:

(a) Ordering and set-up costs (denoted by OSC): Both ordering and set-up costs are fixed costs, which implies that $OSC = K_c$. These costs are associated with the delivery, layout, inspection, adjustment, and preparation.

(b) Holding cost: The holding cost includes the storage space and preservation for three stock types: material, scrap returns, and finished products. By examining the pattern in Figure 3, we have separately compiled the holding cost per cycle for the three types, as shown below:

(b-1) Cost of holding material (denoted by HCm):

$$HCm = S_n\left\{\frac{1}{2}Q_n(t_1+t_2)\right\} = \frac{1}{2}S_n f_n (t_1+t_2)^2.$$

(b-2) Cost of holding scrap returns (denoted by HCr):

$$\begin{aligned} HCr &= S_r\left\{\tfrac{1}{2}\left(P_f\theta_1 r - f_{r,1}\right){t_1}^2 + \sum_{i=1}^{n}\int_{t_1+\frac{i-1}{n}t_2}^{t_1+\frac{i}{n}t_2} IS_i(t)dt\right\} \\ &= S_r\left\{\tfrac{1}{2}\left(P_f\theta_1 r - f_{r,1}\right){t_1}^2 - \tfrac{p_f k r t_2^2(n+1)(2n+1)}{12n^2}\right. \\ &\quad + \tfrac{1}{2}\left(p_f\theta_1 r - f_{r,2} - p_f k r t_1 + \tfrac{p_f k r}{n}\right)(t_1+t_2) + \tfrac{p_f k r(n+1)(2nt_1t_2-{t_2}^2)}{4n^2} \\ &\quad \left. + (f_{r,2}-f_{r,1})t_1 + \tfrac{p_f k r {t_1}^2}{2} - \tfrac{p_f k r t_1 t_2}{n} + \tfrac{p_f k r(3n^2{t_1}^2-3nt_1t_2-2{t_2}^2)}{6n^2}\right\}. \end{aligned}$$

(b-3) Cost of holding product 1 (denoted by HCf_1):

$$\begin{aligned} HCf_1 &= S_{f,1}\left\{\tfrac{1}{2}Q_{f1}(t_1+t_{1,d})\right\} \\ &= \tfrac{1}{2}S_{f,1}\left[P_f(1-\theta_1)-d_1\right]t_1(t_1+t_{1,d}). \end{aligned}$$

(b-4) Cost of holding product 2 (denoted by HCf_2):

$$\begin{aligned} HCf_2 &= S_{f,2}\left\{\tfrac{1}{2}Q_{f2}t_{2,d} + \sum_{i=1}^{n}\int_{t_1+\frac{i-1}{n}t_2}^{t_1+\frac{i}{n}t_2} IP_i(t)dt\right\} \\ &= S_{f,2}\left\{\tfrac{1}{2}d_2{t_{2,d}}^2 + \tfrac{p_f k t_2^3(n+1)(2n+1)}{12n^2} + \tfrac{(n+1)t_2^2}{2n}\left(p_f - p_f\theta_1 - d_2 - \tfrac{p_f k t_2 + 2p_f k n t_1}{2n}\right)\right. \\ &\quad \left. + \tfrac{p_f k(t_1^2t_2n^2+t_2^3)}{2n^2} + \tfrac{(p_f-p_f\theta_1-d_2)(4nt_1t_2-t_2^2)}{2n} - \tfrac{p_f k t_2(3n^2t_1^2-3nt_1t_2-2t_2^2)}{6n^2}\right\}. \end{aligned}$$

(c) Production cost (denoted by PCf): Because the total yield in a cycle is $P_f(t_1+t_2)$, the production cost per cycle is obtained by multiplying the unit production cost by the total yield; that is: $PCf = CP_f(t_1+t_2)$.

(d) Recovery cost (denoted by RC): The total quantity of the defective products in a cycle is $p_f\theta_1 t_1 + p_f\theta_2 t_2$. After multiplying it by the unit processing cost, the recovery cost per cycle can be obtained as follows:

$$RC = C_r\left\{p_f\theta_1 t_1 + p_f\theta_2 t_2\right\} = C_r\left\{p_f\theta_1 t_1 + \left[p_f t_2 - (t_2+t_{2,d})d_2\right]\right\},$$

wherein the term of $p_f t_2 - (t_2+t_{2,d})d_2$ is the difference between the total yield and the demand of product 2 in a cycle. It can also be presented as the total quantity of defective items for product 2, i.e., $p_f\theta_2 t_2 = p_f t_2 - (t_2+t_{2,d})d_2$.

(e) The opportunity cost for recovery loss (denoted by OC): Because the total unrecovered quantity of defective products is $p_f\theta_1 t_1(1-r) + p_f\theta_2 t_2(1-r)$, the opportunity cost for recovery loss per cycle can be obtained as follows:

$$\begin{aligned} OC &= C_{l,1}\Big\{p_f\theta_1 t_1(1-r)\Big\} + C_{l,2}\Big\{p_f\theta_2 t_2(1-r)\Big\} \\ &= C_{l,1}\Big\{p_f\theta_1 t_1(1-r)\Big\} + C_{l,2}\Big\{\Big[p_f t_2 - (t_2+t_{2,d})d_2\Big](1-r)\Big\}. \end{aligned}$$

(f) Purchasing cost (denoted by PC): The purchasing cost is the result of multiplying the unit purchasing cost by the ordering quantity of material, i.e.,

$$PC = C_p Q_n = C_p f_n(t_1+t_2)$$

(g) Maintenance cost for production line (denoted by MC): This cost is the maintenance cost multiplied by the number of maintenance times; that is:

$$MC = C_m n$$

(h) Investment cost in the improvement of recovery equipment (denoted by IC): This cost is usually formulated as an increasing function of the recovery rate. Imitating the investment function proposed by Priyan et al. [8], we establish an exponential investment function of recovery rate, which is $IC = \lambda[e^{(r-r_0)/r_0} - 1]$, in which r_0 is an initial rate of recovery and λ represents the percentage increase in r per dollar increase in IC. Because the recovery rate must fall in $r \in (f_{r,1}/p_f\theta_1,\ f_{r,\,2}/p_f\theta_2)$, this study considers an initial rate with the worst state of equipment, such that the initial recovery rate r_0 is larger and close to $f_{r,1}/p_f\theta_1$. Consequently, the feasible solution interval of the recovery rate is $r \in [r_0,\ f_{r,\,2}/p_f\theta_2)$.

By calculating the sum of the above elements and dividing by the cycle time, $t_1 + t_2 + t_{2,d}$, we can obtain the following total cost per unit time (denoted by $AC(t_2,\ n,\ r)$):

$$AC(t_2,\ n,\ r) = \frac{1}{t_1+t_2+t_{2,d}} \times \left\{ \begin{array}{c} OSC + HCm + HCr + HCf_1 + HCf_2 \\ +PCf + RC + OC + PC + MC + IC \end{array} \right\}, \quad (10)$$

in which t_2, n, and r are decision variables.

5. Model Solution

The primary objective of this study is to determine the optimal production, maintenance, and investment strategies to achieve minimal total cost per unit. Equation (3) shows that the value of the t_2 depends on n and r; hence, the total cost function can be reduced to $AC(n,\ r)$. Because n is a positive integer, we first explore the objective under the given number of maintenance times, i.e., Min $AC(r|n)$. According to the first-derivative test, we observe the pattern of $AC(r|n)$ during $r \in [r_0,\ f_{r,\,2}/p_f\theta_2)$. Under the $AC(r|n)$ is a convex function of r; if $dAC(r|n)/dr|_{r=r_0} \geq 0$, $AC(r|n)$ increases during $r \in [r_0,\ f_{r,\,2}/p_f\theta_2)$. This implies that the investment of recovery equipment would not be beneficial because the larger value for r results in a higher value of $AC(r|n)$. Therefore, the optimal recovery rate is $r^* = r_0$. If $dAC(r|n)/dr|_{r=r_0} < 0$, and the recovery rate can be increased to r^*, which satisfies $dAC(r|n)/dr|_{r=r^*} = 0$. For the number of maintenance times, we can observe the effect of increasing the times on the value of $AC(r^*|n)$ to determine the optimal times. Based on the above results, Figure 4 plots a simple algorithm procedure for finding the optimal solution $(r^*,\ n^*,\ t_2^*)$ in which the $\varepsilon > 0$ is an accuracy of solution t_2^*. In summary, because the proposed model is a nonlinear integer programming problem with the high-power expression of the polynomial function, it is difficult to provide a theoretical proof to verify the optimal solution. Instead, this study tries to use numerical analysis to verify the convexity of the total cost function.

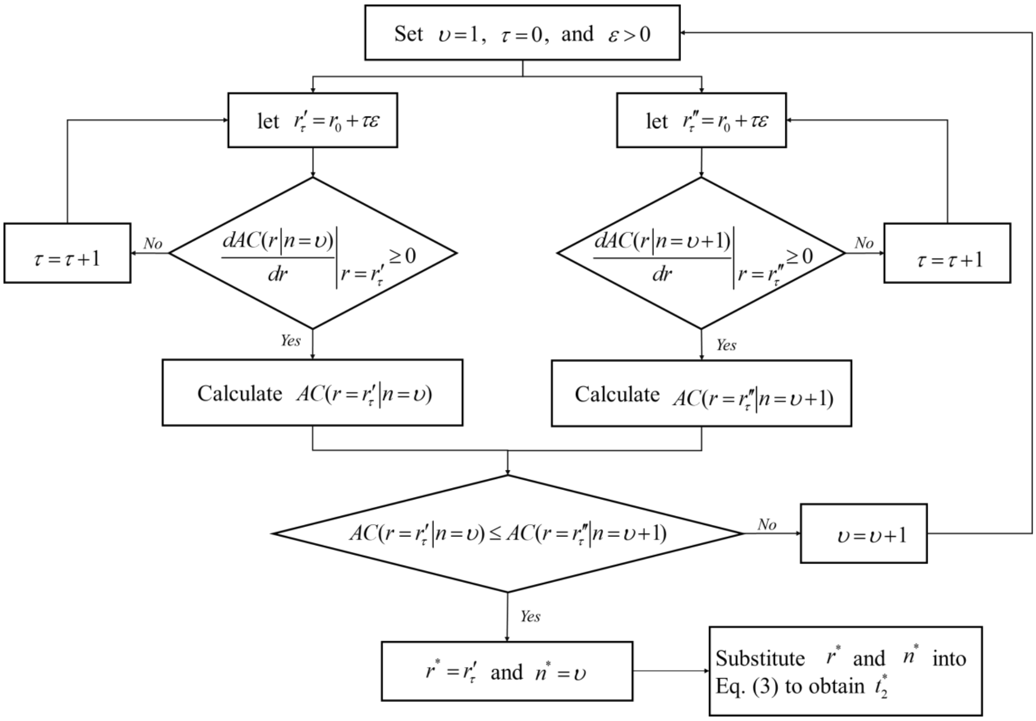

Figure 4. The procedure of the proposed algorithm.

6. Numerical Example

To exhibit the usability of the proposed model, we quote an application example proposed by Su et al. [12]. They investigated an imperfect production–inventory system involving a pulp and paper manufacturing industry with an inner circular economy, wherein defective products and waste can become scrap returns after passing specific processing in converted equipment. This system manufactures two products with different feed rates of scrap returns, and the lower feed rate takes precedence to enter the production line. Based on Su et al., this study extends their work to survey the maintenance strategy during the out-of-control stage. The values of the parameters pertaining to the imperfect production–inventory system are listed as follows:

- Demand parameters: $d_1 = 400$ kg/day and $d_2 = 300$ kg/day;
- Production parameters: $f_n = 100$ kg/day, $f_{r,1} = 2$ kg/day, $f_{r,2} = 40$ kg/day, $p_f = 600$ kg/day, and $t_1 = 3.4799$ days. Note that the value of t_1 adopts the estimated results from Su et al. [12];
- Cost parameters: $K_c = \$1000$, $C = \$10$/kg, $C_p = \$5$/kg, $C_r = \$2$/kg, $C_{l,1} = \$0.15$/kg, $C_{l,2} = \$0.10$/kg, $C_m = 50$/time, $S_r = \$2$ kg/day, $S_n = \$3$ kg/day, $S_{f,1} = \$6$ kg/day, $S_{f,2} = \$4$ kg/day, and $\lambda = \$100$;
- Imperfect parameters: $\theta_1 = 0.1$, $k = 0.01$, and $r_0 = 0.1$;
- Accuracy parameter: $\varepsilon = 10^{-4}$.

After implementing the proposed algorithm, the optimal solutions can be obtained, i.e., $r^* = 0.1950$, $n^* = 2$, $t_2^* = 1.2081$, and $AC(t_2^*, n^*, r^*) = 9036.76$. Furthermore, the curves of the total cost function with respect to r under different values of n are plotted in Figure 5. Note that these curves are plotted in the feasible solution interval of the recovery rate, $r \in [r_0, f_{r,2}/p_f\theta_2)$. Figure 6 also shows the graphical representation of $AC(n|r^*)$. From Figures 5 and 6, it is found that the convexity of the total cost function can be verified numerically, which implies that the optimal solutions obtained through the algorithm can ensure the global optimum. In other words, implementing both a circular economy and process maintenance strategies can promote cost control in this numerical example. From Figure 5, it is found that the total cost per unit time is 9182.02 when both circular economy and process maintenance strategies are unimplemented (i.e., our model becomes the traditional EPQ model when $n = 1$ and $r = r_0 = 0.1$). If the capital investment in recovery capacity improvement is implemented, the total cost per unit time will be saved by 138.40 (i.e., cost from 9182.02 to 9043.62). However, the total cost per unit time can be saved by 145.26 (i.e., cost from 9182.02 to 9036.76) if both capital investment and maintenance activity are implemented. This result is partly similar to that of Darwish and Ben-Day [5]. The difference is that we further take the rework process and scrap returns into consideration at the same time.

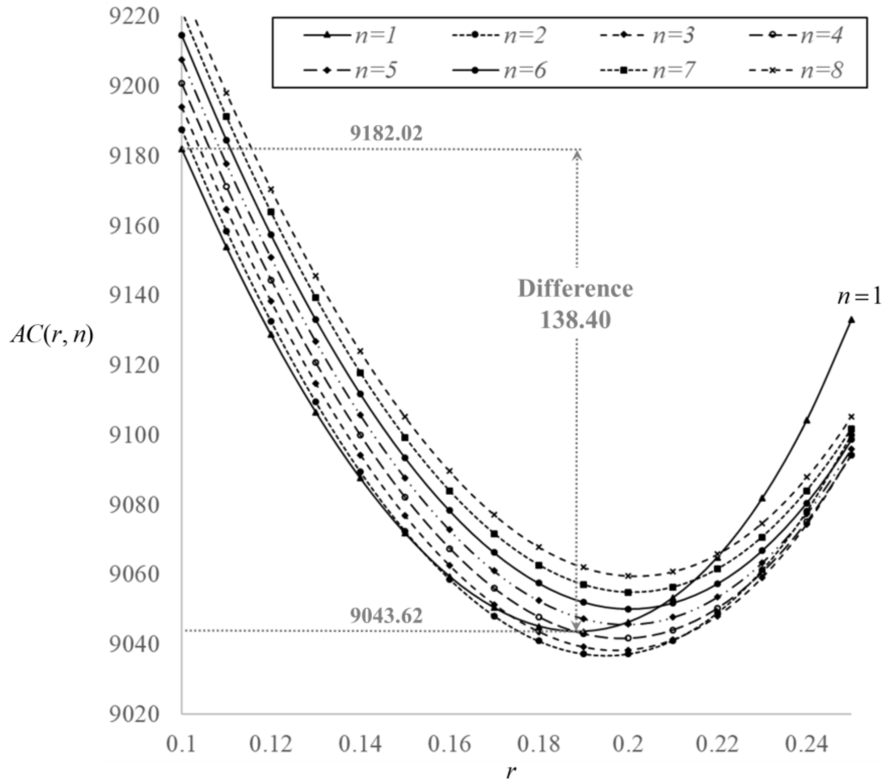

Figure 5. The curves of the total cost per unit time with r for $n = 1(1)8$.

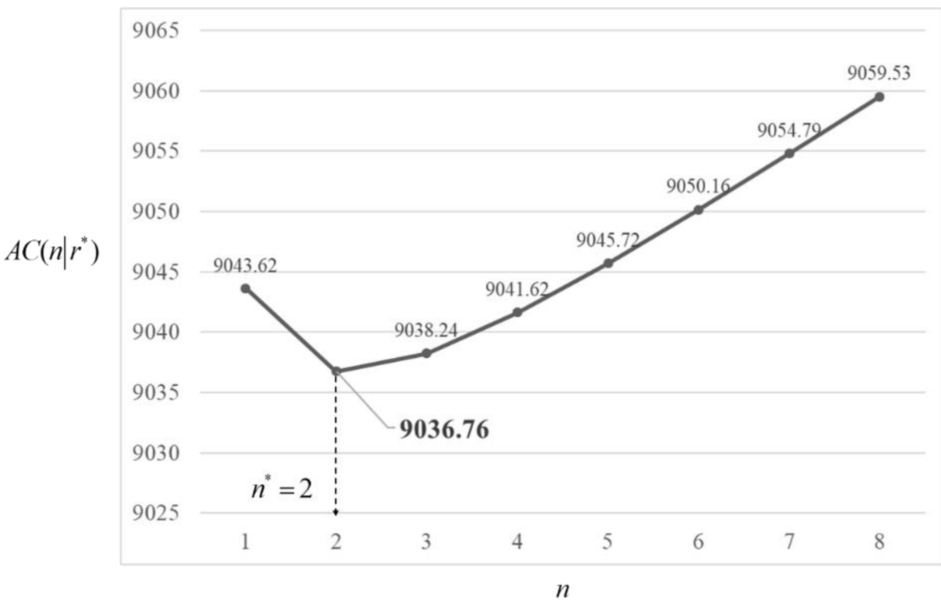

Figure 6. The graphical representation of $AC(n|r^*)$.

7. Sensitivity Analysis and Managerial Insights

To obtain some managerial insights for decision making, this section further performs a sensitivity analysis involving costs based on the above numerical example. That is, we investigate the effects of changes to cost parameters, including K_c, C, $C_{l,1}$, $C_{l,2}$, C_p, C_r, $S_{f,1}$, $S_{f,2}$, S_n, S_r, λ, and C_m, on the values of n^*, r^*, t_2^*, Q_n^*, and $AC(t_2^*,\ n^*,\ r^*)$. This type of parameter is directly related to costs, and, logically, the increase in each cost directly leads to an increase in the total cost. Therefore, this study aims to analyze the impacts of these parameters on the optimal solutions through sensitivity analysis. The goal is to understand the contribution and influence of these parameters on the overall system under their changes. The results of the sensitivity analysis are tabulated, as shown in Table 2. Note that each parameter is changed separately by +20%, +10%, −20%, or −10%. By observing the data trend in Table 2, we can derive the following managerial insights, which could be a guide for decision making:

1. Though the result that increasing the cost parameters will lead to an increase in the total cost is in line with practical intuition, it is found that the total cost per unit time is relatively sensitive to the production and holding costs (i.e., C, $S_{f,1}$, $S_{f,2}$, and S_n). From a business operations perspective, if companies aim to effectively reduce total costs, they should prioritize parameters that exhibit relatively high sensitivity.
2. When considering production-related cost parameters (i.e., K_c, C, C_p, C_r, λ, and C_m), an increase in the cost parameter value leads to higher optimal values of r^* and t_2^*. This implies that the manager could lengthen the production cycle to allocate the growth of these costs. Furthermore, the number of maintenance times could simultaneously increase when these costs increase excessively. From Table 2, the number of maintenance times changes from two to three as the production cost increases by 20%. More precisely, the number of maintenance times starts to increase when the production cost increases by 12%. Table 3 also provides information about the other cost parameters related to the upper boundaries of change in the number of maintenance times. The results show that a change in production-related cost has a higher impact on the number of maintenance times. On the other hand, the total cost per unit time is 9182.02 when both strategies are unimplemented. If both strategies are implemented, the total cost per unit can be saved by 150.84, as the parameter λ decreases by 20% (i.e., cost from 9182.02 to 9031.18; the reduction rate is 16.4%). If the parameter C_m decreases by 20%, the total cost per unit can be saved by 147.35 (i.e., cost from 9182.02 to 9034.67; the reduction rate is 16.0%). From the above results, although both reduction rates are close, reducing parameter λ may be relatively efficient.
3. Regarding holding cost parameters (i.e., $S_{f,1}$, $S_{f,2}$, S_n, and S_r), with the increase in the value of $S_{f,1}$ or S_r, the optimal values of r^* and t_2^* also increase. This implies that the manager could extend the production cycle to control the growth of these kinds of holding costs. For the holding cost of product 1, the upper boundary of change in the number of maintenance times is also shown in Table 3. Note that the effect of the change to a parameter S_r on the number of maintenance times is not significant. However, the holding costs of product 2 ($S_{f,2}$) and the new material (S_n) yield opposite results, and even the number of maintenance times decreases. The main reason is that the production of product 2 is in the out-of-control stage involving decision making, and increasing the recovery rate and production time would lead to an increase in inventories of product 2 and new material requirements. This results in the manager having to reduce the length of the production cycle and the number of maintenance times to avoid the excessive growth of holding costs when the values of $S_{f,2}$ and S_n increase.
4. As for opportunity cost parameters (i.e., $C_{l,1}$ and $C_{l,2}$), the effects of the changes to opportunity costs on the values of optimal solutions and total cost are not significant. This implies that the manager could focus on other high-impact parameters related to production or investment control.

Table 2. Sensitivity analysis for cost parameters.

Parameter	Change (%)	Change in				
		n^*	r^*	t_2^*	Q_n^*	$AC(t_2^*, n^*, r^*)$
K_c	20	2	0.1981	1.2402	472.0100	9071.29 (+0.38%)
	10	2	0.1966	1.2242	470.4100	9054.07 (+0.19%)
	−10	2	0.1935	1.1919	467.1800	9019.35 (−0.19%)
	−20	2	0.1918	1.1757	465.5600	9001.85 (−0.39%)
C	20	3	0.2256	1.5375	501.7400	9995.42 (+10.61%)
	10	2	0.2098	1.3657	484.5600	9520.64 (+5.35%)
	−10	2	0.1783	1.0426	452.2500	8543.59 (−5.46%)
	−20	2	0.1590	0.8673	434.7200	8039.53 (−11.04%)
$C_{l,1}$	20	2	0.1951	1.2092	468.9140	9037.63 (+0.01%)
	10	2	0.1951	1.2087	468.8590	9037.19 (+0.00%)
	−10	2	0.1950	1.2076	468.7470	9036.32 (−0.00%)
	−20	2	0.1949	1.2070	468.6920	9035.88 (−0.01%)
$C_{l,2}$	20	2	0.1950	1.2081	468.7950	9036.87 (+0.01%)
	10	2	0.1950	1.2081	468.7990	9036.81 (+0.01%)
	−10	2	0.1950	1.2082	468.8070	9036.70 (−0.00%)
	−20	2	0.1950	1.2082	468.8110	9036.64 (−0.00%)
C_p	20	2	0.1976	1.2349	471.4810	9118.01 (+0.90%)
	10	2	0.1963	1.2216	470.1450	9077.42 (+0.45%)
	−10	2	0.1937	1.1947	467.4560	8996.03 (−0.45%)
	−20	2	0.1924	1.1811	466.1030	8955.25 (−0.90%)
C_r	20	2	0.1961	1.2187	469.8630	9054.06 (+0.19%)
	10	2	0.1955	1.2134	469.3330	9045.41 (+0.10%)
	−10	2	0.1945	1.2028	468.2730	9028.09 (−0.10%)
	−20	2	0.1940	1.1975	467.7430	9019.41 (−0.90%)
$S_{f,1}$	20	2	0.2148	1.4216	490.1450	9267.00 (+2.55%)
	10	2	0.2053	1.3168	479.6730	9153.84 (+1.30%)
	−10	2	0.1837	1.0950	457.4850	8915.29 (−1.34%)
	−20	2	0.1712	0.9767	445.6579	8788.90 (−2.74%)
$S_{f,2}$	20	1	0.1315	0.6431	412.3048	9303.05 (+2.95%)
	10	2	0.1664	0.9332	441.3092	9185.70 (+1.65%)
	−10	3	0.2245	1.5246	500.4450	8858.75 (−1.97%)
	−20	3	0.2507	1.8530	533.2880	8650.57 (−4.27%)
S_n	20	2	0.1921	1.1780	465.7870	9151.05 (+1.26%)
	10	2	0.1935	1.1929	467.2790	9093.94 (+0.63%)
	−10	2	0.1965	1.2237	470.3600	8979.49 (−0.63%)
	−20	2	0.1981	1.2396	471.9520	8922.13 (−1.27%)
S_r	20	2	0.1952	1.2099	468.9770	9045.76 (+0.10%)
	10	2	0.1951	1.2090	468.8900	9041.26 (+0.05%)
	−10	2	0.1949	1.2073	468.7160	9032.26 (−0.05%)
	−20	2	0.1949	1.2064	468.6300	9027.75 (−0.10%)
λ	20	2	0.1931	1.1889	466.8750	9042.20 (+0.06%)
	10	2	0.1941	1.1985	467.8350	9039.49 (+0.03%)
	−10	2	0.1960	1.2179	469.7790	9033.99 (−0.03%)
	−20	2	0.1969	1.2277	470.7640	9031.18 (−0.06%)
C_m	20	2	0.1952	1.2101	468.9970	9038.84 (+0.02%)
	10	2	0.1951	1.2091	468.9000	9037.80 (+0.01%)
	−10	2	0.1949	1.2072	468.7060	9035.72 (−0.01%)
	−20	2	0.1948	1.2062	468.6090	9034.67 (−0.02%)

Table 3. Boundaries of change in the number of maintenance times.

Parameter	K_c	C	C_p	C_r	λ	C_m	$S_{f,1}$
Boundary	139.5%	12.0%	143.6%	220.5%	499.7%	160.9%	20.3%

8. Conclusions

Based on Su et al. [12], this paper explored a production–inventory scenario involving an imperfect production system and developed a more general model with both the circular economy and process maintenance activities, which is not presented in previous studies. This system was formulated as a joint EOQ and EPQ model aimed at determining the optimal production run time, purchase quantity of material, number of maintenance times, and recovery rate that would minimize the total cost per unit time. Furthermore, a computational algorithm was provided to effectively find the optimal solutions. Though it is challenging to mathematically prove the existence and uniqueness of the optimal solution, this study utilized numerical and graphical techniques to ensure the optimal solutions obtained through the algorithm were the local optimum. This study also performed numerical and sensitivity analyses on a practical case to provide some managerial insights pertaining to the reference for decision making. First, an optimal combination of circular economy and process maintenance strategies was determined by observing the impact of parameter changes on optimal decisions. In the proposed case, the total cost per unit time can be saved by 145.26 (i.e., reducing the cost from 9182.02 to 9036.76) when both capital investment and process maintenance strategies are implemented in the production system. Second, when the cost-related parameter increases, the capital investment in recovery capacity improvement would be increased first, and the number of maintenance times starts to increase when it increases to a boundary (please see Table 3). Third, enhancing the production cost can significantly decrease the overall cost per unit time. The total cost per unit time can be saved by 997.23 if the unit production cost of the finished product is decreased by 20%. Finally, in terms of holding cost parameters, the total cost per unit time can be effectively reduced by improving the holding cost of product 2 (i.e., a reduction of 4.27% in the total cost can be achieved by reducing the holding cost of product 2 by 20%).

Some research limitations and suggestions for future research can be considered to extend our model. Initially, one can take into account the deteriorating properties of items, such as raw materials and finished products. Second, the maintenance process may be imperfect in practice; that is, the defective rate in the out-of-control stage cannot be adjusted to the in-control stage at each maintenance time. Third, multi-products, multi-materials, and multi-stages can be explored to make the proposed model more applicable to production situations. Fourth, we can consider the impacts of external factors, such as government policy, COVID-19, and international influences, on demand patterns, material procurement, labor, and shipment. Finally, implementing CSR activities (e.g., social donations, environmental protection, green industrial development, etc.) has already been an important topic at present, and it has been confirmed that the financial performance of firms can be improved. Therefore, the effects of CSR activities in the proposed production system can be explored in future research.

Author Contributions: Conceptualization, R.-H.S. and M.-W.W.; methodology, M.-W.W.; software, R.-H.S.; validation, R.-H.S., M.-W.W. and C.-T.Y.; formal analysis, R.-H.S., M.-W.W. and C.-T.Y.; investigation, R.-H.S. and C.-H.H.; resources, R.-H.S. and M.-W.W.; data curation, R.-H.S. and C.-H.H.; writing—original draft preparation, R.-H.S., M.-W.W., C.-T.Y. and C.-H.H.; writing—review and editing, R.-H.S., M.-W.W. and C.-T.Y.; project administration, R.-H.S. and C.-T.Y.; visualization, R.-H.S., M.-W.W. and C.-T.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This work is partially supported by the National Science and Technology Council, Taiwan, under grant numbers NSC 110-2410-H-231-001-MY2 and NSC 111-2410-H-030-027.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare that there are no conflict of interest.

Abbreviations

Parameters

K_c	The ordering and set-up costs per cycle;
f_n	The feed rate of new materials;
$f_{r,j}$	The feed rate of scrap returns for a product j, where $j = 1, 2$;
p_f	The production rate of the finished product;
d_j	The demand rate of the product j, where $j = 1, 2$;
θ_1	The defective rate during the in-control stage;
θ_2	The defective rate during the out-of-control stage;
Q_{rt}	The maximum stock of scrap returns;
$Q_{f,j}$	The maximum stock of the product j, where $j = 1, 2$;
s_r	The holding cost of scrap returns per unit/per unit time;
s_n	The holding cost of new material per unit/per unit time;
$S_{f,j}$	The holding cost of product j per unit/per unit time, where $j = 1, 2$;
C	The production cost of a finished product per unit;
C_r	The processing cost for converting a defective product to scrap returns;
$C_{l,j}$	The opportunity cost of a defective product j that cannot be converted to scrap returns, where $j = 1, 2$;
C_p	The purchasing cost of new material per unit;
C_m	The maintenance cost of equipment for recovering an out-of-control stage to an in-control stage in the production line;
$t_{j,d}$	The period during which the stock is depleted for the product j, where $j = 1, 2$;
t_1	The production run time during the in-control stage; random variable.

Decision variables

t_2	The production run time during the out-of-control stage;
Q_n	The purchasing quantity of new material per cycle;
r	The recovery rate of scrap returns;
n	The number of maintenance times with $n \geq 1$.

References

1. Su, R.H.; Weng, M.W.; Huang, Y.F. Innovative maintenance problem in a two-stage production-inventory system with imperfect processes. *Ann. Oper. Res.* **2019**, *287*, 379–401. [CrossRef]
2. Ellen MacArthur Foundation. Towards the Circular Economy: Economic and Business Rationale for an Accelerated Rransition. 2012. Available online: https://ellenmacarthurfoundation.org/topics/circular-economy-introduction/overview (accessed on 11 May 2023).
3. Nkansah, A.; Attiogbe, F.; Kumi, E. Scrap metals' role in circular economy in Ghana, using Sunyani as a case study. *Afr. J. Environ. Sci. Technol.* **2015**, *9*, 793–799.
4. Paletta, A.; Filho, W.L.; Balogun, A.L.; Foschi, E.; Bonoli, A. Barriers and challenges to plastics valorization in the context of a circular economy: Case studies from Italy. *J. Clean. Prod.* **2019**, *241*, 118149. [CrossRef]
5. Darwish, M.A.; Ben-Daya, M. Effect of inspection errors and preventive maintenance on a two-stage production inventory system. *Int. J. Prod. Econ.* **2007**, *107*, 301–313. [CrossRef]
6. Pearn, W.L.; Su, R.H.; Weng, M.W.; Hsu, C.H. Optimal production run time for two-stage production system with imperfect processes and allowable shortages. *Cent. Eur. J. Oper. Res.* **2010**, *19*, 533–545. [CrossRef]
7. Chang, H.J.; Su, R.H.; Yang, C.T.; Weng, M.W. An economic manufacturing quantity model for a two-stage assembly system with imperfect processes and variable production rate. *Comput. Ind. Eng.* **2012**, *63*, 285–293. [CrossRef]
8. Priyan, S.; Mala, P.; Palanivel, M. A cleaner EPQ inventory model involving synchronous and asynchronous rework process with green technology investment. *Clean. Logist. Supply Chain* **2022**, *4*, 100056. [CrossRef]
9. Paul, S.K.; Sarker, R.; Essam, D. A disruption recovery plan in a three-stage production-inventory system. *Comput. Oper. Res.* **2015**, *57*, 60–72. [CrossRef]
10. Wang, Z.; Chan, F.T.S. A Robust Production Control Policy for a Multiple-Stage Production System with Inventory Inaccuracy and Time-Delay. In Proceedings of the 2013 IEEE International Conference on Automation Science and Engineering (CASE 2013), Madison, WI, USA, 17–20 August 2013.
11. Su, R.H.; Weng, M.W.; Yang, C.T. Effects of corporate social responsibility activities in a two-stage assembly production system with multiple components and imperfect processes. *Eur. J. Oper. Res.* **2020**, *293*, 469–480. [CrossRef]
12. Tayyab, M.; Sarkar, B.; Yahya, B.N. Imperfect multi-stage lean manufacturing system with rework under fuzzy demand. *Mathematics* **2019**, *7*, 13. [CrossRef]

13. Wang, A.; Shi, J. Holistic modeling and analysis of multistage manufacturing processes with sparse effective inputs and mixed profile outputs. *IISE Trans.* **2020**, *53*, 582–596. [CrossRef]
14. Cui, P.H.; Wang, J.Q.; Li, Y. Data-driven modeling, analysis and improvement of multistage production systems with predictive maintenance and product quality. *Int. J. Prod. Res.* **2021**, *22*, 6848–6865.
15. Öztürk, H. Modeling an inventory problem with random supply, inspection and machine breakdown. *Opsearch* **2019**, *56*, 497–527. [CrossRef]
16. Öztürk, H. Optimal production run time for an imperfect production inventory system with rework, random breakdowns and inspection costs. *Oper. Res.* **2021**, *21*, 167–204. [CrossRef]
17. Dey, B.K.; Pareek, S.; Tayyab, M.; Sarkar, B. Autonomation policy to control work-in-process inventory in a smart production system. *Int. J. Prod. Res.* **2021**, *59*, 1258–1280. [CrossRef]
18. Dey, B.K.; Sarkar, B.; Sarkar, M.; Pareek, S. An integrated inventory model involving discrete setup cost reduction, variable safety factor, selling price dependent demand, and investment. *RAIRO Oper. Res.* **2019**, *53*, 39–57. [CrossRef]
19. Huang, H.; He, Y.; Li, D. Coordination of pricing, inventory, and production reliability decisions in deteriorating product supply chains. *Int. J. Prod. Res.* **2018**, *56*, 6201–6224. [CrossRef]
20. Manna, A.K.; Das, B.; Tiwari, S. Impact of carbon emission on imperfect production inventory system with advance payment base free transportation. *RAIRO Oper. Res.* **2020**, *54*, 1103–1117. [CrossRef]
21. Su, R.H.; Weng, M.W.; Yang, C.T.; Li, H.T. An imperfect production–inventory model with mixed materials containing scrap returns based on a circular economy. *Processes* **2021**, *9*, 1275. [CrossRef]
22. Sarkar, S.; Shewchuk, J.P. Use of advance Demand Information in multi-stage production-inventory systems with multiple demand classes. *Int. J. Prod. Res.* **2013**, *51*, 57–68. [CrossRef]
23. Mahata, G.C. A production-inventory model with imperfect production process and partial backlogging under learning considerations in fuzzy random environments. *J. Intell. Manuf.* **2014**, *28*, 883–897. [CrossRef]