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Modeling the Production Process of Fuel Gas, LPG, Propylene, and Polypropylene in a Petroleum Refinery Using Generalized Nets

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Abstract: The parallel processes involved in the production of refinery fuel gas, liquid petroleum gas (LPG), propylene, and polypropylene, occurring in thirteen refinery units, are modeled by the use of a Generalized Net (GN) apparatus. The modeling of the production of these products is important because they affect the energy balance of petroleum refinery and the associated emissions of greenhouse gases. For the first time, such a model is proposed and it is a continuation of the investigations of refinery process modelling by GNs. The model contains 17 transitions, 55 places, and 47 types of tokens, and considers the orders of fuel gas for the refinery power station, refinery process furnaces, LPG, liquid propylene, and 6 grades of polypropylene. This model is intended to be used as a more detailed lower-level GN model in a higher-level GN model that facilitates and optimizes the process of decision making in the petroleum refining industry.

Keywords: generalized net; fuel gas; LPG; propylene; polypropylene; petroleum refinery

MSC: 68Q85

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1. Introduction

Modeling the processes of refined products production in a petroleum refinery is a useful tool in production planning that allows for improvement in performance and refinery profitability.

In the literature, partial models of the diverse processes taking place in the chemical industry and in petroleum refining are made. For example, reference [1] deals with risk assessment in the chemical industry; reference [2] investigates the integration of engineering models with planning models; reference [3] focuses on the refinery-wide planning operations under uncertainties in product demand and price; reference [4] studies a nonlinear, multiperiod, industrial refinery problem extended to a two-stage stochastic problem, formulated as a mixed-integer nonlinear program; reference [5] employs a product trisection crude distillation unit model to build an accurate refinery model and determines the optimal crude selection using two-stage stochastic programming; reference [6] formulates a large-scale nonconvex mixed-integer nonlinear programming model and applies robust optimization for the multi-period operational planning of a real-world integrated refinery-petrochemical site in China under uncertain product demands and crude oil price; reference [7] deals with a refinery planning model that utilizes simplified empirical nonlinear process models with considerations for crude characteristics, product yields, and qualities, etc.; reference [8] makes an assessment of refinery efficiency using linear programming; references [9–14] present studies related to planning and scheduling; and reference [15] investigates the efficiency of the process of crude oil dewatering and desalting. Fuzzy modeling was another modeling technique employed in petroleum refining and renewable energy systems for the selection of the optimum working situations that produce a preferable efficiency with a very good veracity [16–29].

The approach to the modeling of the processes of manufacturing of petroleum refining products by the use of generalized nets (GN) is original and all publications to date are the work of the authors. To the extent that the GN is a process description tool with at least as much power as the Turing machine, it can describe processes in more detail than Petri nets, which are discussed, for example, in the works of Wu et al. [30–35] and Zhang et al. [36], as well as many other tools for modeling real-world processes, e.g., linear programming [37,38], transportation problems [39–41], neural networks [42–44], etc. For each of these, it is shown that their functions and results of their work are representable by a GN [45–47].

In linear programming, the algorithm is carried out step by step because it is sequential [48–52], whereas in all types of Petri nets, the processes run in parallel as in the real world. Section 2.2 of this article states that the GN includes, as a special case, the other types of Petri nets because of the presence of token characteristics and transition condition predicates. The entire analytics of any means of describing a real-world process (e.g., linear programming) can be described by the token characteristics in the GN model (see [45]), while the logic of the modeled process is represented by the predicates of the GN.

In our earlier studies [53,54] we demonstrated that the processes of automotive gasoline [53] and diesel fuel production [54] in a petroleum refinery can be modeled using generalized nets (GNs). The literature review indicates that there is a lack of models of all the processes taking place in a petroleum refinery that are related to the production of refined products and prepared with some mathematical instrumentation. Our aim is to prepare, in a series of papers, the description of the refining processes leading to the production of specific products using the GNs. The apparatus of GNs provides the capability of easier uniting of diverse GNs models. The GNs models already produced by us [53,54] can be transformed into the subnets of a general GN model. In the future, this will be realized using a software product developed in the Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences.

The use of GNs, appearing first as extensions of Petri nets [55] and their other extensions and modifications, was found to be a convenient methodology to model the complex parallel-sequential processes taking place in petroleum refinery during the production of finished refined products. They also allow the modeling of the production processes of different refined oil products using distinct GNs which can be further combined using a hierarchical approach. In order to obtain a complete refined oil products production modelling, all the processes involved in the production of all of the products are required. In this article, we focus on the modeling of the production of fuel gas, liquefied petroleum gas (LPG), propylene, and polypropylene, which are part of the production chain of hydrocarbon gas refinery, as a complement to the process modeling of automotive gasoline and diesel fuels production in petroleum refinery using generalized nets. The aim of this research is to investigate the process of production of hydrocarbon gas products: fuel gas, LPG, and propylene, produced from propylene polypropylene in a petroleum refinery, and model it using GNs.

2. Materials and Methods

2.1. Processing Scheme for Production of Fuel Gas, LPG, Propylene, and Polypropylene in a Petroleum Refinery

The processing technological chain employed in the LUKOIL Neftohim Burgas (LNB) refinery in order to produce the components and finished products of fuel gas, LPG, propylene, and polypropylene, the subject of this study, is presented in Figure 1.

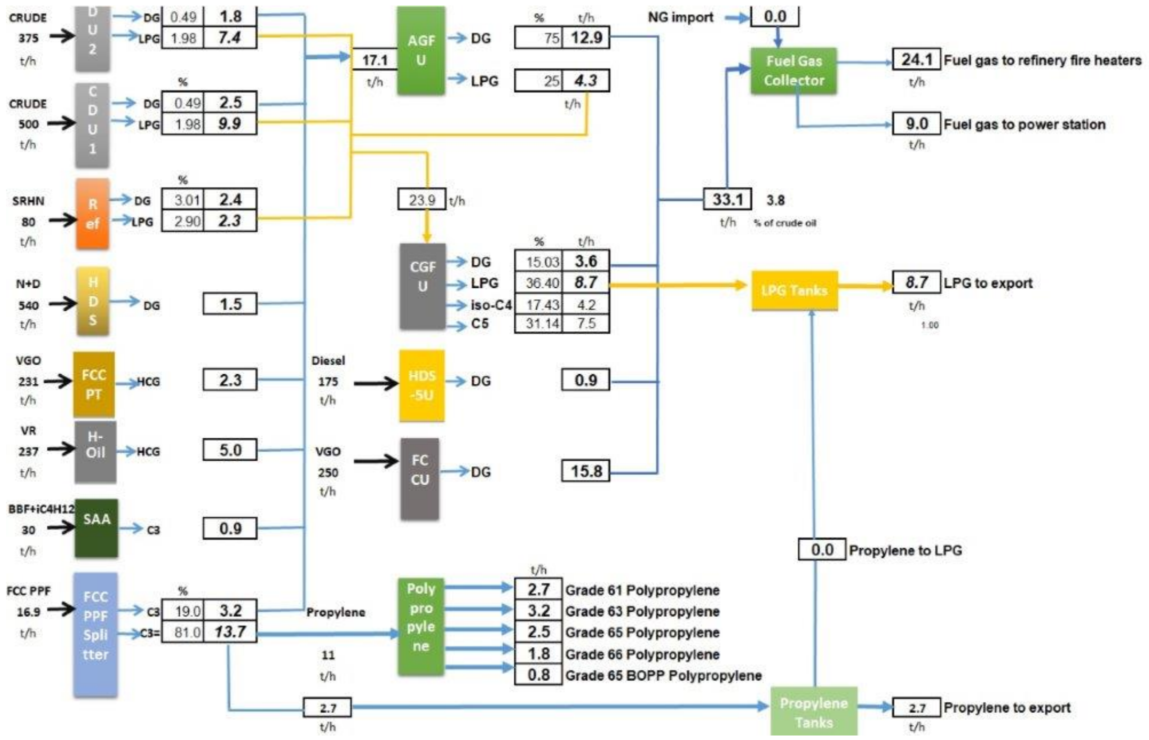

Figure 1. Processing scheme of fuel gas, LPG, propylene, and polypropylene production in a petroleum refinery. Note: CDU = Crude distillation unit; Ref = straight run heavy naphtha (SRHN) reformer; HDS = hydrodesulphurization units processing naphtha and diesel streams of primary origin (crude distillation units) and secondary origin (fluid catalytic cracking (FCC), FCC feed hydrotreater (FCCPT), and H-Oil ebullated bed vacuum residue hydrocracking); AGFU = Absorption gas fractionation unit; CGFU = Central gas fractionation unit; HDS-5 = primary and secondary diesel hydrotreating unit 5; SAA = Sulphuric acid alkylation; FCC PPF Splitter = fluid catalytic cracking propane-propylene fraction splitter; Polypropylene = Polypropylene production unit by polymerization of propylene.

Thirteen refinery process units are involved in the production process of fuel gas, LPG, propylene, and polypropylene. The amount of these vapour phase and liquid phase hydrocarbon gaseous products extracted from the crude oil in the process of atmospheric distillation and generated in the process units—the reformer, hydrotreating units, FCC, and H-Oil—as a result of chemical reactions, depends on the crude oil origin and on the operating conditions in the mentioned refinery units. For the case shown in Figure 1, the fuel gas production amounts to 3.8% of the crude oil quantity and more than half of this comes from the fluid catalytic cracking (51.8%). Another important contributor to fuel gas production is H-Oil ebullated bed vacuum residue hydrocracking. Thus, the production of fuel gas is strongly dependent on the severity of the operation conditions applied in both of the heavy oil conversion processes, FCC and H-Oil. The main contributor to LPG production, as evident from the data in Figure 1, is crude oil distillation. It provides 75% of the feed for the central Gas Fractionation Unit (CGFU), where the extraction of the propane and n-butane is carried out; their mixing forms the refined product, LPG. As apparent from the data in Figure 1, LPG production is about four times as low as that of the production of fuel gas. The data in Figure 1 indicates that, in the case that the amount of fuel gas is insufficient to meet the refinery energy needs, an option exists in importing natural gas to replenish the fuel gas availability. Depending on the market requirements, liquid propylene can be exported as a chemical grade for polymerization propylene, or as a component of the LPG product. Typically, six grades of polypropylene products are produced and exported from the polypropylene unit.

2.2. Short Remarks on Generalized Nets

GNs are an extension of the standard Petri nets [55] and the rest of their extensions and modifications. GNs are defined in a way that is principally different from the ways of defining other types of Petri nets (see references [45,46]).

When some of the GN components are omitted, the GN is called a reduced GN. For the needs of the model below, we describe the modeled process as a reduced GN.

Formally, every transition (see Figure 2) is described using a seven-tuple, but for our purposes, we use its following reduced form:

$$Z = \langle L', L'', r\rangle,$$

where:

- L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the transition in Figure 1, these are $L' = \{l'_1, l'_2, \ldots, l'_m\}$ and $L'' = \{l''_1, l''_2, \ldots, l''_n\}$;
- r is the transition's *condition* determining which tokens will pass (or *transfer*) from the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [56]):

$$r = \begin{array}{c|ccccc} & l''_1 & \ldots & l''_j & \ldots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & r_{i,j} & & \\ l'_m & & & & & \end{array};$$

  $r_{i,j}$ is the predicate that corresponds to the i-th input and j-th output place $(1 \le i \le m, 1 \le j \le n)$. When its truth value is "*true*", a token from the i-th input place transfers to the j-th output place; otherwise, this is not possible.

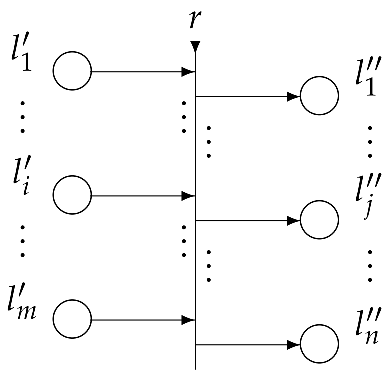

Figure 2. The form of a GN transition.

The formal definition of the reduced GN used in the present research has the following form:

$$E = \langle A, K, X, \Phi\rangle,$$

where:

- A is a set of transitions;
- K is the set of the GN's tokens;
- X is the set of all initial characteristics which the tokens can obtain on entering the net;
- Φ is the characteristic function that assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition.

Operations, relations, and operators are defined over GNs (see references [45,46]).

The operations defined over the GNs—"union", "intersection", "composition" and "iteration" (see references [45,46])—do not exist anywhere else in Petri net theory. These operations are useful for constructing GN models of real processes.

For example, when we have two GN models of real processes which we know flow in parallel, by operation union, defined over two GNs, we can construct a GN model of both processes.

The idea of defining operators over the set of GNs in the form suggested below dates back to 1982 (see references [45,46]). It is a proper extension of Valk's idea from [57] for self-modifying Petri nets, in which only the net structure can be changed.

Now, the operator aspect has an important place in the theory of GNs. Six types of operators are defined in its framework. Every operator assigns to a given GN a new GN with some desired properties.

The groups of operators comprise the following:

- Global operators (e.g., one of them changes the functions giving tokens characteristics);
- Local operators (e.g., one of then changes the transition condition predicates);
- Hierarchical operators (e.g., one of them replaces the GN with a whole new (sub)GN, another—a transition of the GN with a whole new (sub)GN, and two others do the opposite activity);
- Dynamic operators (e.g., operators that allow for the union or split of tokens)

and others.

We can mention that, for example, the colored Petri nets (see, e.g., reference [58]) can be represented by GNs in which the tokens have, as initial and current characteristics, the colors with which they are colored. It is important to mention that, when one token obtains a new colour, it does not "remember" its previous color. Meanwhile, interestingly, if modeled by a GN, the GN tokens can remember their own previous characteristics. For this reason, they can be interpreted as individuals with their own history.

3. Main Results: A GN Model

The GN (see Figure 3) contains 17 transitions, 55 places, and 47 types of tokens that correspond to the following feed, products, and processing units:

σ_1—crude oil processed in CDU-2, t/h
σ_2—crude oil processed in CDU-2, t/h
σ_3—straight run naphtha processed in the catalytic reformer, t/h
σ_4—naphtha and diesel fractions process in hydrotreating units, t/h
σ_5—vacuum gas oil processed in fluid catalytic cracking feed hydrotreater, t/h
σ_6—vacuum residue processed in H-Oil hydrocracker, t/h
σ_7—butane-bytelene fraction and isobutene processed in sulphuric acid alkylation, t/h
σ_8—propane-propylene fraction from fluid catalytic cracking to separate in propane, and propylene, t/h
σ_9—primary and secondary diesel to hydrotreat in HDS-5 unit, t/h
σ_{10}—hydrotreated and H-Oil vacuum gas oil to process in fluid catalytic cracking, t/h
σ_{11}—natural gas from importing to replenishing fuel gas in cases of high fuel gas demand, t/h.
ρ_1—crude distillation unit 2 (CDU-2)
ρ_2—crude distillation unit 1 (CDU-1)
ρ_3—catalytic reformer unit
ρ_4—naphtha and diesel hydrotreaters
ρ_5—fluid catalytic cracking feed hydrotreating unit
ρ_6—H-Oil vacuum residue hydrocracking unit
ρ_7—sulphuric acid alkylation unit
ρ_8—fluid catalytic cracking propane-propylene splitter unit (FCC PPF splitter)
ρ_9—absorption gas fractionation unit (AGFU)
ρ_{10}—LPG intermediary reservoir to collect feed for the central gas fractionation unit
ρ_{11}—HDS-5 diesel hydrotreating unit
ρ_{12}—fluid catalytic cracking unit (FCC)
ρ_{13}—central gas fractionation unit (CGFU)
ρ_{14}—liquid petroleum gas (LPG) tank farm

ρ_{15}—fuel gas tank farm
α_1—LPG product from CDU-2, t/h
α_2—LPG product from CDU-1, t/h
α_3—LPG product from naphtha reformer, t/h
α_4—LPG product from AGFU, t/h
α_5—CGFU feed, t/h
α_6—LPG product from CGFU to feed LPG tank farm,
β—fuel gas feed for AGFU, t/h
β_1—fuel gas product from CDU-2, t/h
β_2—fuel gas product from CDU-1, t/h
β_3—fuel gas product from naphtha reformer, t/h
β_4—fuel gas product from naphtha and diesel hydrotreaters, t/h
β_5—fuel gas product from FCC feed hydrotreater, t/h
β_6—fuel gas product from H-Oil vacuum residue hydrocrcker, t/h
β_7—propane fraction from sulphuric acid alkylation unit, t/h
β_8—dry fuel gas product from AGFU, t/h
β_9—dry fuel gas product from CGFU, t/h
β_{10}—dry fuel gas product from HDS-5 unit, t/h
β_{11}—dry fuel gas product from FCCU, t/h
γ_1—propane product from FCC PPF splitter, t/h
γ_2—propylene product from FCC PPF splitter to feed polypropylene unit, t/h
γ_3—propylene product from FCC PPF splitter, t/h
δ_1—high melting index grade 61 polypropylene product, t/h
δ_2—high melting index grade 63 polypropylene product, t/h
δ_3—high melting index grade 65 polypropylene product, t/h
δ_4—high melting index grade 66 polypropylene product, t/h
δ_5—high melting index grade 65 BOPP polypropylene product, t/h
ε_1—LPG product for export, t/h
ε_2—fuel gas product to feed the refinery power station, t/h
ε_3—fuel gas product to feed the refinery process furnaces, t/h

$$Z_1 = \langle \{l_1, l_{11}\}, \{l_9, l_{10}, l_{11}\}, \begin{array}{c|ccc} & l_9 & l_{10} & l_{11} \\ \hline l_1 & false & false & true \\ l_{11} & W_{11,9} & W_{11,10} & true \end{array} \rangle,$$

where
$W_{11,9} =$ "there is a request for LPG product from CDU-2";
$W_{11,10} =$ "there is a request for fuel gas product from CDU-2".

Token σ_1 enters place l_{11} and unites with token ρ_1 to obtain a characteristic:

"the current quantity of crude oil in CDU-2".

In the next time-step, token ρ_1 splits into three tokens—the same token ρ_1 that continues to stay in place l_{11}, α_1, and β_1.

The token α_1 obtains a characteristic:

"current quantity of LPG product from CDU-2"

in place l_9, token β_1 obtains a characteristic:

"current quantity of fuel gas product from CDU-2"

in place l_{10}, token ρ_1 obtains a characteristic:

"the current quantity of crude oil in CDU-2".

$$Z_2 = \langle \{l_2, l_{14}\}, \{l_{12}, l_{13}, l_{14}\}, \begin{array}{c|ccc} & l_{12} & l_{13} & l_{14} \\ \hline l_2 & false & false & true \\ l_{14} & W_{14,12} & W_{14,13} & true \end{array} \rangle,$$

where
$W_{14,12} =$ "there is a request for LPG product from CDU-1";
$W_{14,13} =$ "there is a request for fuel gas product from CDU-1".

Token σ_2 enters place l_{14} and unites with token ρ_2 to obtain a characteristic:

"the current quantity of crude oil in CDU-1".

In the next time-step, token ρ_2 splits into three tokens—the same token ρ_2 that continues to stay in place l_{14}, α_2, and β_2.

The token α_2 obtains a characteristic:

"current quantity of LPG product from CDU-1"

in place l_{12}, token β_2 obtains a characteristic:

"current quantity of fuel gas product from CDU-1"

in place l_{13}, token ρ_2 obtains a characteristic:

"the current quantity of the crude oil in CDU-1".

$$Z_3 = \langle \{l_3, l_{17}\}, \{l_{15}, l_{16}, l_{17}\}, \begin{array}{c|ccc} & l_{15} & l_{16} & l_{17} \\ \hline l_3 & false & false & true \\ l_{17} & W_{17,15} & W_{17,16} & true \end{array} \rangle,$$

where
$W_{17,15} =$ "there is a request for LPG product from naphtha catalytic reformer";
$W_{17,16} =$ "there is a request for fuel gas product from naphtha catalytic reformer".

Token σ_3 enters place l_{17} and unites with token ρ_3 to obtain a characteristic:

"the current quantity of naphtha in naphtha catalytic reformer".

In the next time-step, token ρ_3 splits into three tokens—the same token ρ_3 that continues to stay in place l_{17}, α_3, and β_3.

The token α_3 obtains a characteristic:

"current quantity of LPG product from naphtha catalytic reformer"

in place l_{15}, token β_3 obtains a characteristic:

"current quantity of fuel gas product from naphtha catalytic reformer"

in place l_{16}, token ρ_3 obtains a characteristic:

"current quantity of fuel gas product from naphtha catalytic reformer".

$$Z_4 = \langle \{l_4, l_{19}\}, \{l_{18}, l_{19}\}, \begin{array}{c|cc} & l_{18} & l_{19} \\ \hline l_4 & false & true \\ l_{19} & W_{19,18} & true \end{array} \rangle,$$

where
$W_{19,18} =$ "there is a request for fuel gas product from naphtha and diesel hydrotreaters".

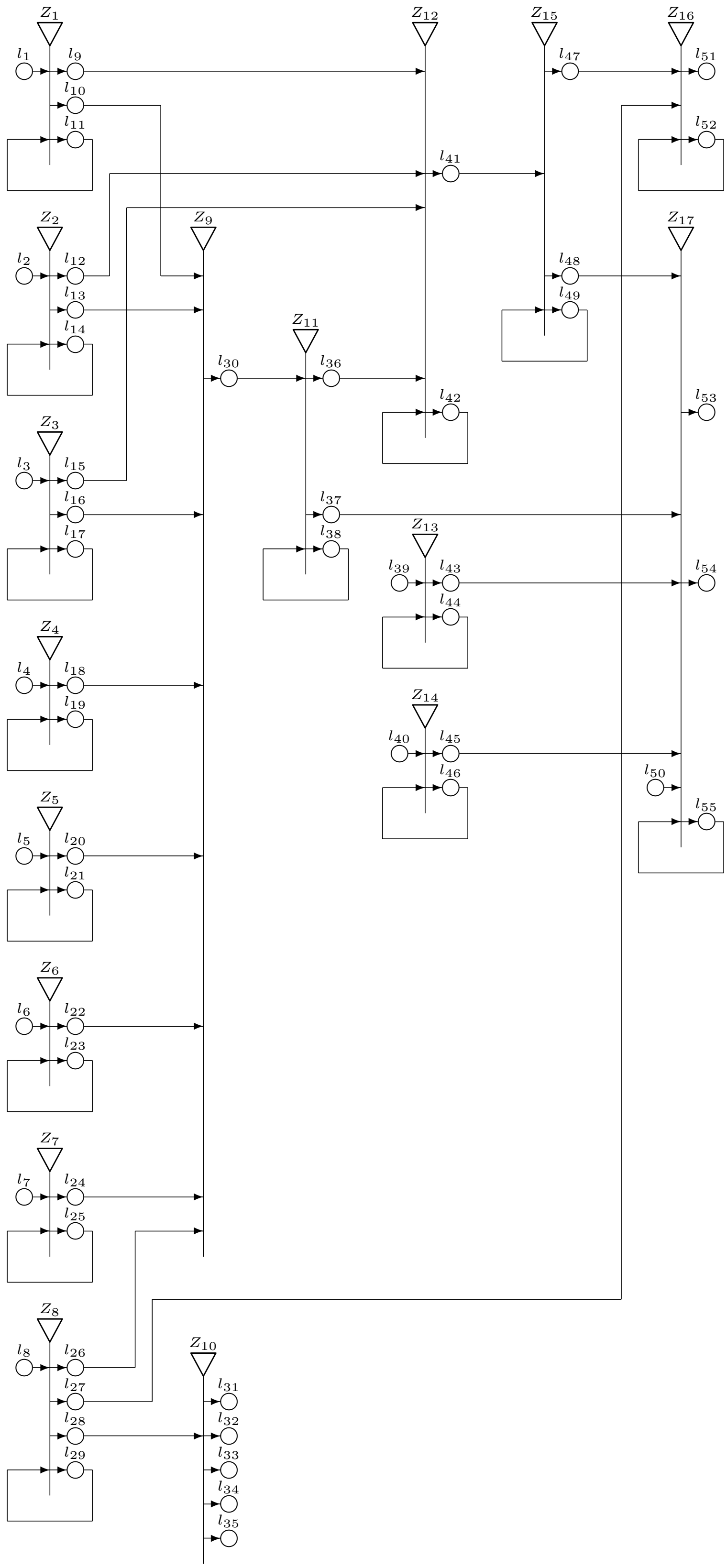

Figure 3. A GN model.

Token σ_4 enters place l_{19} and unites with token ρ_4 to obtain a characteristic:

"the current quantity of naphtha and diesel in naphtha and diesel hydrotreaters".

In the next time-step, token ρ_4 splits into two tokens—the same token ρ_4 that continues to stay in place l_{19} and β_4.

The token β_4 obtains a characteristic:

"current quantity of fuel gas product from naphtha and diesel hydrotreaters"

in place l_{18}, token ρ_4 obtains a characteristic:

"the current quantity of naphtha and diesel in naphtha and diesel hydrotreaters".

$$Z_5 = \langle \{l_5, l_{21}\}, \{l_{20}, l_{21}\}, \begin{array}{c|cc} & l_{20} & l_{21} \\ \hline l_5 & false & true \\ l_{21} & W_{21,20} & true \end{array} \rangle,$$

where

$W_{21,20} =$ "there is a request for fuel gas product from FCC feed hydrotreater".

Token σ_5 enters place l_{21} and unites with token ρ_5 to obtain a characteristic:

"the current quantity of vacuum gas oil in FCC feed hydrotreater".

In the next time-step, token ρ_5 splits into two tokens—the same token ρ_5 that continues to stay in place l_{21} and β_5.

The token β_5 obtains a characteristic:

"current quantity of fuel gas product in FCC feed hydrotreater"

in place l_{20}, token ρ_5 obtains a characteristic:

"the current quantity of vacuum gas oil in FCC feed hydrotreater".

$$Z_6 = \langle \{l_6, l_{23}\}, \{l_{22}, l_{23}\}, \begin{array}{c|cc} & l_{22} & l_{23} \\ \hline l_6 & false & true \\ l_{23} & W_{23,22} & true \end{array} \rangle,$$

where

$W_{23,22} =$ "there is a request for fuel gas product from H-Oil unit".

Token σ_6 enters place l_{23} and unites with token ρ_6 to obtain a characteristic:

"the current quantity of vacuum residue in H-Oil unit".

In the next time-step, token ρ_6 splits into two tokens—the same token ρ_6 that continues to stay in place l_{23} and β_6.

The token β_6 obtains a characteristic:

"the current quantity of fuel gas product from H-Oil unit"

in place l_{22}, token ρ_6 obtains a characteristic:

"the current quantity of vacuum residue in H-Oil unit".

$$Z_7 = \langle \{l_7, l_{25}\}, \{l_{24}, l_{25}\}, \begin{array}{c|cc} & l_{24} & l_{25} \\ \hline l_7 & false & true \\ l_{25} & W_{25,24} & true \end{array} \rangle,$$

where

$W_{25,24} =$ "there is a request for propane fraction product from sulphuric acid alkylation unit".

Token σ_7 enters place l_{25} and unites with token ρ_7 to obtain a characteristic:

"the current quantity of butane-butylene fraction and iso butane in sulphuric acid alkylation unit".

In the next time-step, token ρ_7 splits into two tokens—the same token ρ_7 that continues to stay in place l_{25} and β_7.

The token β_7 obtains a characteristic:

"current quantity of propane fraction product in sulphuric acid alkylation unit"

in place l_{24}, token ρ_7 obtains a characteristic:

"the current quantity of butane-butylene fraction and iso butane in sulphuric acid alkylation unit".

$$Z_8 = \langle \{l_8, l_{29}\}, \{l_{26}, l_{27}, l_{28}, l_{29}\}, \begin{array}{c|cccc} & l_{26} & l_{27} & l_{28} & l_{29} \\ \hline l_8 & false & true & & \\ l_{29} & W_{29,26} & W_{29,27} & W_{29,28} & true \end{array} \rangle,$$

where

$W_{29,26} =$ "there is a request for propane product from FCC PPF splitter for LPG production";

$W_{29,27} =$ "there is a request for propylene for polymerization from FCC PPF splitter";

$W_{29,28} =$ "there is a request for propylene product from FCC PPF splitter for export".

Token σ_8 enters place l_{29} and unites with token ρ_8 to obtain a characteristic:

"the current quantity of FCC PPF in FCC PPF splitter".

In the next time-step, token ρ_8 splits into four tokens—the same token ρ_8 that continues to stay in place l_{29} and tokens $\gamma_1, \gamma_2, \gamma_3$.

The tokens $\gamma_1, \gamma_2, \gamma_3$ obtain characteristics:

"current quantity of propane product in FCC PPF splitter"

in place l_{26},

"current quantity of propylene product for polymerization in FCC PPF splitter"

in place l_{27},

"current quantity of propylene product for export in FCC PPF splitter"

in place l_{28}, respectively. Token ρ_8 obtains a characteristic:

"the current quantity of FCC PPF in FCC PPF splitter".

$$Z_9 = \langle \{l_{10}, l_{13}, l_{16}, l_{18}, l_{20}, l_{22}, l_{24}, l_{28}\}, \{l_{30}\}, \begin{array}{c|c} & l_{30} \\ \hline l_{10} & true \\ l_{13} & true \\ l_{16} & true \\ l_{18} & true \\ l_{20} & true \\ l_{22} & true \\ l_{24} & true \\ l_{28} & true \end{array} \rangle.$$

All β-tokens unite in place l_{30}, with one token β with a characteristic:

"current quantity of fuel gas, feed for the AGFU".

$$Z_{10} = \langle \{l_{28}\}, \{l_{31}, l_{32}, l_{33}, l_{34}, l_{35}\}, \begin{array}{c|ccccc} & l_{31} & l_{32} & l_{33} & l_{34} & l_{35} \\ \hline l_{28} & W_{28,31} & W_{28,32} & W_{28,33} & W_{28,34} & W_{28,35} \end{array} \rangle,$$

where
$W_{27,31} =$ "there is a request for high melting index grade 61 polypropylene product from polypropylene unit";
$W_{27,32} =$ "there is a request for high melting index grade 63 polypropylene product from polypropylene unit";
$W_{27,33} =$ "there is a request for high melting index grade 65 polypropylene product from polypropylene unit";
$W_{27,34} =$ "there is a request for high melting index grade 66 polypropylene product from polypropylene unit";
$W_{27,35} =$ "there is a request for high melting index grade 66 BOPP polypropylene product from polypropylene unit".

Token γ_2 splits into five tokens $\delta_1, \ldots, \delta_5$ that obtain characteristics:

"current quantity of high melting index grade 61 polypropylene product from polypropylene"

in place l_{31},

"current quantity of high melting index grade 63 polypropylene product from polypropylene"

in place l_{32},

"current quantity of high melting index grade 65 polypropylene product from polypropylene"

in place l_{33},

"current quantity of high melting index grade 66 polypropylene product from polypropylene"

in place l_{34},

"current quantity of high melting index grade 65 BOPP polypropylene product from polypropylene unit"

in place l_{35}, respectively.

$$Z_{11} = \langle \{l_{30}, l_{38}\}, \{l_{36}, l_{37}, l_{38}\}, \begin{array}{c|ccc} & l_{36} & l_{37} & l_{38} \\ \hline l_{30} & false & false & true \\ l_{38} & W_{38,36} & W_{36,37} & true \end{array} \rangle,$$

where
$W_{38,36} =$ "there is a request for LPG product from AGFU";
$W_{38,37} =$ "there is a request for fuel gas product from AGFU".

Token β enters place l_{38} and unites with token ρ_9 to obtain a characteristic:

"the current quantity of fuel gas feed in AGFU".

In the next time-step, token ρ_9 splits into three tokens—the same token ρ_9 that continues to stay in place l_{38} and tokens α_4 and β_8.

The token α_4 obtains a characteristic:

"the current quantity of LPG product from AGFU"

in place l_{36}, token β_8 obtains a characteristic:

"current quantity of fuel gas product from AGFU"

in place l_{37}.

$$Z_{12} = \langle \{l_9, l_{12}, l_{15}, l_{36}, l_{42}\}, \{l_{41}, l_{42}\}, \begin{array}{c|cc} & l_{41} & l_{41} \\ \hline l_9 & false & true \\ l_{12} & false & true \\ l_{15} & false & true \\ l_{36} & false & true \\ l_{42} & true & true \end{array} \rangle.$$

All α-tokens ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$) unite in place l_{42} with token ρ_{10} and obtain the characteristic:

"current quantity of LPG feed stored in a reservoir for the CGFU".

In the next time-step, the token ρ_{10} splits into two tokens—the same token ρ_{10} that continues to stay in place l_{42} and token α_5 that enters place l_{41} with a characteristic:

"current quantity of LPG feed for the CGFU".

$$Z_{13} = \langle \{l_{39}, l_{44}\}, \{l_{43}, l_{44}\}, \begin{array}{c|cc} & l_{43} & l_{44} \\ \hline l_{39} & false & true \\ l_{44} & W_{44,43} & true \end{array} \rangle,$$

where

$W_{44,43} =$ "there is a request for fuel gas product from HDS-5 unit".

Token σ_9 enters place l_{44} and unites with token ρ_{11} to obtains a characteristic:

"the current quantity of primary and secondary diesel—feed for the HDS-5 unit".

In the next time-step, the token ρ_{11} splits into two tokens—the same token ρ_{11} that continues to stay in place l_{44} with a characteristic:

"current quantity of primary and secondary diesel—feed for the HDS-5 unit"

and token β_{10} that enters place l_{43} with a characteristic:

"current quantity of dry fuel gas product in HDS-5 unit".

$$Z_{14} = \langle \{l_{40}, l_{46}\}, \{l_{45}, l_{46}\}, \begin{array}{c|cc} & l_{45} & l_{46} \\ \hline l_{40} & false & true \\ l_{46} & W_{46,45} & true \end{array} \rangle,$$

where

$W_{46,45} =$ "there is a request for fuel gas product from FCCU".

Token σ_{10} enters place l_{46} and unites with token ρ_{12} to obtain a characteristic:

"the current quantity of hydrotreated vacuum gas oil-feed for the FCCU".

In the next time-step, the token ρ_{12} splits into two tokens—the same token ρ_{12} that continues to stay in place l_{46} with a characteristic:

"current quantity of hydrotreated vacuum gas oil—feed for the FCCU"

and token β_{11} that enters place l_{45} with a characteristic

"current quantity of dry fuel gas product in FCCU".

$$Z_{15} = \langle \{l_{41}, l_{49}\}, \{l_{47}, l_{48}, l_{49}\}, \begin{array}{c|ccc} & l_{47} & l_{48} & l_{49} \\ \hline l_{41} & false & false & true \\ l_{49} & W_{49,47} & W_{49,48} & true \end{array} \rangle,$$

where
$W_{49,47} =$ "there is a request for LPG product from CGFU";
$W_{49,48} =$ "there is a request for fuel gas product from CGFU".

Token α_5 enters place l_{49} and unites with token ρ_{13} to obtain a characteristic:

"the current quantity of LPG feed in CGFU".

In the next time-step, token ρ_{13} splits into three tokens—the same token ρ_{13} that continues to stay in place l_{49} and tokens α_6 and β_9.

The token α_6 obtains a characteristic:

"current quantity of LPG product from CGFU"

in place l_{47}, token β_9 obtains a characteristic:

"current quantity of fuel gas product from CGFU"

in place l_{48}.

$$Z_{16} = \langle \{l_{26}, l_{47}, l_{52}\}, \{l_{51}, l_{52}\}, \begin{array}{c|cc} & l_{51} & l_{52} \\ \hline l_{26} & false & true \\ l_{47} & false & true \\ l_{46} & true & true \end{array} \rangle.$$

Tokens α_6 and γ_2 enter place l_{52} and unite with token ρ_{14} to obtain a characteristic:

"the current quantity of LPG in LPG tank farm for export".

In the next time-step, the token ρ_{42} splits into two tokens—the same token ρ_{14} and token ε_1 that enters place l_{51} with a characteristic:

"the current quantity of LPG sent for export".

$$Z_{17} = \langle \{l_{37}, l_{43}, l_{45}, l_{48}, l_{50}, l_{55}\}, \{l_{53}, l_{54}, l_{55}\}, \begin{array}{c|ccc} & l_{53} & l_{54} & l_{55} \\ \hline l_{37} & false & false & true \\ l_{43} & false & false & true \\ l_{45} & false & false & true \\ l_{48} & false & false & true \\ l_{50} & false & false & true \\ l_{55} & W_{55,53} & W_{55,54} & false \end{array} \rangle,$$

where
$W_{55,53} =$ "there is a request for fuel gas product for the refinery power station";
$W_{55,54} =$ "there is a request for fuel gas product for the refinery process furnaces".

Tokens $\beta_8, \beta_9, \sigma_9, \sigma_{10}, \sigma_{11}$ enter place l_{55} and unite with token ρ_{15} to obtain a characteristic:

"current quantity of fuel gas".

The token ρ_{15} splits into three tokens—the same token ρ_{15} that continues to stay in place l_{55} and tokens ε_2 and ε_3.

The token ε_2 obtains a characteristic:

"current quantity of fuel gas product for the refinery power station"

in place l_{53}, token ε_3 obtains a characteristic:

"current quantity of fuel gas product for the refinery process furnaces"

in place l_{54}.

4. Discussion

The production of fuel gas, LPG, propylene, and polypropylene in a refinery is a complex parallel process involving many process units, which may deliver a variable amount of these oil refining products depending on the crude slate processed, the activity and selectivity of the catalyst employed, and the operating conditions in the process units. This complex process was found to be capable of modeling using generalized nets. The developed GN model can be used for the synchronization and optimization of these processes with the aim of finding and implementing the economically optimal mode of refinery operation. This is achieved by the presence of time parameters in GN and by the presence of token characteristics, which enable the collection of the whole information of the refinery processes; decisions about process optimization and, if necessary, process reorganization, should be made on this basis. For example, if a process unit is unexpectedly shut down due to an emergency, this would include how to organize the process performance in such a case. However, this will be discussed in our next article. Moreover, this model could enable the assessment of the efficiency of adding new process units into the refinery processing scheme before their construction. This paper is a continuation of a series of papers by the authors in which GN models of the production of automotive gasoline [53] and diesel [54] in a refinery are described. Modeling the processes of heavy oil products in a refinery using GN is the next study already submitted for publication. Then, a higher-level GN model could be created that encompasses the more detailed, already developed lower-level GN models. Based on the higher-level GN model that encompasses the more detailed lower-level GN models, the decision making process in petroleum refinery can be facilitated and optimized. The program realization of the higher-level GN model will be available on the internet and, as such, its vulnerability to stealthy attacks should be considered, as described in references [59,60].

5. Conclusions

The process of producing gaseous products in a petroleum refinery, such as fuel gas, propane-butane, and propylene, which can be exported as a final finished product or used as a raw material for polypropylene production, is a complex parallel process that is difficult to model using linear and even dynamic programming. The difficulty comes from the inability to reflect the logic of the cause and effect relationships therein which, as stated above, are easily interpreted by transition condition predicates. A visual means of representing the real processes are UML diagrams which, in reference [61], are shown to be representable by GN. This paper presents a GN model for the production of fuel gas, LPG, propylene, and polypropylene in a petroleum refinery, and a separate paper will discuss the software implemented with GN and the results of the present model.

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