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Sample Size Determination for Two-Stage Multiple Comparisons for Exponential Location Parameters with the Average

Shu-Fei Wu

Department of Statistics, Tamkang University, Tamsui, Taipei 251301, Taiwan; 100665@mail.tku.edu.tw

Abstract: In this paper, the design constant is determined based on a criterion of probability of correct detection for a given deviation of k location parameters from the average being at least γ for the two-stage multiple comparisons for location parameters of exponential distributions when scale parameters are unequal. All required values for determining the design constant and then the total sample size for a two-stage procedure are listed in tables for practical use. For illustrative purposes, two examples are given to demonstrate the sample size determination for this two-stage procedure.

Keywords: two-stage procedure; multiple comparisons with the average; exponential distribution

MSC: 62F07

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1. Introduction

In the field of ranking and selection, Bechhofer [1] and Gupta [2] are two pioneers of normal distribution. Other than normal distribution, the lifetime of products may follow an exponential distribution, gamma distribution or Weibull distribution, etc. (see Bain [3], Johnson et al. [4], Lawless [5], Wu et al. [6] and Wu and Lin [7]). Since Balakrishnan and Basu [8] stated that the exponential distribution is one of the most significant and widely used distributions in statistical practice, we focus our research on exponential distribution in this paper. For this distribution, Lam and Ng [9] proposed a multiple comparison procedure for exponential location parameters with a control based on a two-stage design. Unlike a one-stage procedure, a two-stage procedure is design-oriented. Maurya et al. [10] modified the procedure proposed by Lam and Ng [9] to get a new procedure with smaller confidence width. They also discussed the criteria to find the design constant required for the correct detection for the given deviation of k location parameters from the control. When the comparison target is changing from the control population to the average of all populations, Wu and Wu [11] proposed a two-stage procedure for comparing several exponential location parameters with the average under heteroscedasticity based on the techniques given in Lam ([12,13]). To get better coverage probabilities, in order to be closer to the nominal confidence coefficient, Wu [14] proposed a new two-stage procedure.

In this research, we investigate the sample size determination based on the two-stage multiple comparison procedures with the average in Wu and Wu [11], under the criterion of the probability of correct detection for a given deviation of k location parameters from the average being at least γ. Our approach for the sample size determination for the two-stage multiple comparison procedures in Wu and Wu [11] is elaborated in Section 2. The required values to determine the control constant c are also tabulated for practical use. In Section 3, two examples are provided to illustrate the proposed detection procedures. Finally, our conclusions are summarized in Section 4.

2. Sample Size Determination for the Two-Stage Multiple Comparisons with the Average

We consider k (≥ 2) independent exponential populations with exponential distributions denoted by $E(\theta_i, \sigma_i)$, $i = 1, \ldots, k$, where $\theta_1, \cdots, \theta_k$ are unknown location parameters and $\sigma_1, \cdots, \sigma_k$ are unknown and possibly unequal scale parameters. In reliability and engineering, the location parameters are regarded as the threshold values or guaranteed time, and the scale parameters are regarded as the mean lifetime in addition to the guaranteed time. The two-stage sampling procedure is depicted as follows: In the first stage with initial sample size m, we take a random sample of size m denoted as $X_{i,1}, \cdots, X_{i,m}$ from the ith population $E(\theta_i, \sigma_i)$. The maximum likelihood estimator of θ_i is $Y_i = \min(X_{i,1}, \cdots, X_{i,m})$ and the maximum likelihood estimator of σ_i is $S_i = \sum_{j=1}^{m} (X_{ij} - Y_i)/(m-1)$, based on the initial sample data $i = 1, \ldots, k$. The design constant $c > 0$ will later be chosen in Theorem 1 in order to assure the probability of correct detection being at least γ. The total sample size N_i for the ith population is determined by

$$N_i = \max\left\{ m, \left[\frac{S_i}{c}\right] + 1 \right\}, \tag{1}$$

where the notation $[x]$ represents the largest integer less than or equal to x, $i = 1, \ldots, k$. When $N_i > m$, we need to take additional N_i-m observations from the ith population at the second stage denoted as $X_{i,m+1}, \cdots, X_{i,N_i}$. Based on the overall N_i observations $X_{i1}, \cdots, X_{im}, X_{i,m+1}, \cdots, X_{i,N_i}$, we can find the maximum likelihood estimator of θ_i as $\widetilde{X}_i = \min(Y_i, X_{i,m+1}, \cdots, X_{i,N_i})$, $i = 1, \ldots, k$. The mean of these k estimators is obtained as $\widetilde{X} = \sum_{i=1}^{k} \widetilde{X}_i / k$. Observe that $2(m-1)S_i/\sigma_i$, $i = 1, \cdots, k$ follows a chi-squared distribution with $2m-2$ degrees of freedom (df), and $W_i^* = N_i(\widetilde{X}_i - \theta_i)/S_i$, $i = 1, \cdots, k$ is distributed as an F distribution with (2,2m $-$ 2) df.

In Wu and Wu [11], they proposed a simultaneous confidence interval for $\theta_i - \overline{\theta}$ with confidence length of $L = 2c\ h_t$, $i = 1, \ldots, k$ where the critical value is $h_t = (k-1)/kF^{-1}_{2,2m-2}(P^{*1/k})$ and $F^{-1}_{2,2m-2}(P^{*1/k})$ is the $100P^{*1/k}$th percentile of F distribution with (2,2m $-$ 2) df. One criterion to choose the design constant c is to reach a pre-assigned confidence length L. Then, we can determine the value of c as $c = L/(2h_t)$. In this paper, we consider the other criterion from Liu [15]. Liu [15] provided a sample size formula for Dunnett's ([16,17]) procedure of comparing several treatments with a control for the guarantee of the probability of correctly detecting each treatment with minimum guaranteed lifetime θ_i sufficiently different from the control one. Based on Liu [15], the design constant c is chosen to guarantee that the probability of all minimum guarantee lifetimes θ_i satisfying $\left|\theta_i - \overline{\theta}\right| \geq \delta$, $i = 1, \ldots, k$ being detected is at least γ, where δ is a pre-assigned number of minimum absolute effect from the average which is the quantity that users want to detect. The probability of correct detection depends on the parameter configuration $\widetilde{\theta} = (\theta_1, \ldots, \theta_k) \in R^k$. Let the index sets be defined as $A_U(\widetilde{\theta}, \delta) = \{i : \theta_i - \overline{\theta} \geq \delta, i = 1, \ldots, k\} \subseteq \{i = 1, \ldots, k\}$ and $A_L(\widetilde{\theta}, \delta) = \{i : \theta_i - \overline{\theta} \leq -\delta, i = 1, \ldots, k\} \subseteq \{i = 1, \ldots, k\}$ and $A_U(\widetilde{\theta}, \delta) \cap A_L(\widetilde{\theta}, \delta) = \varphi$.

For $\widetilde{\theta}$ under $\left|\theta_i - \overline{\theta}\right| \leq \delta, i = 1, \ldots, k$, the probability of correct detection of $\left|\theta_i - \overline{\theta}\right| \geq \delta, i = 1, \ldots, k$ is 1. Under any $\widetilde{\theta} = (\theta_1, \ldots, \theta_k) \in R^k$, we desire the probability of correct detection $g(\widetilde{\theta}, \delta)$ = P(correct detection of $\left|\theta_i - \overline{\theta}\right| \geq \delta$, $i = 1, \ldots, k) \geq \gamma$. From Theorem 1 (c) in Wu and Wu [11], it can be inferred that $\theta_i \geq \overline{\theta}$ if $\widetilde{X}_i - \widetilde{X} \geq ch_t$, $h_t = (k-1)/kF^{-1}_{2,2m-2}(P^{*1/k})$. Likewise, it can be inferred that $\theta_i \leq \overline{\theta}$ if $\widetilde{X}_i - \widetilde{X} \leq -ch_t$. Thus, the probability of correct detection is given by $g(\widetilde{\theta}, \delta)$ = P(correct detection of $\left|\theta_i - \overline{\theta}\right| \geq \delta$, $i = 1, \ldots, k) = P(\widetilde{X}_i - \widetilde{X} \geq ch_t\, for\, i \in A_U(\widetilde{\theta}, \delta) \cap \widetilde{X}_i - \widetilde{X} \leq -ch_t\, for\, i \in A_L(\widetilde{\theta}, \delta))$.

When $A_U(\widetilde{\theta}, \delta) = A_L(\widetilde{\theta}, \delta) = \varphi$, we have $g(\widetilde{\theta}, \delta) = 1$. The lower bound of the probability of correct detection $g(\widetilde{\theta}, \delta)$ is proposed in the following Theorem.

Theorem 1. *For a given* $0 < P^* < 1$*, letting the index set* $A_L(\tilde{\theta}, \delta) = \{i : \theta_i - \bar{\theta} \le -\delta, i = 1, \ldots, k\}$ *with cardinality given by l and* $A_U(\tilde{\theta}, \delta) = \{i : \theta_i - \bar{\theta} \ge \delta, i = 1, \ldots, k\}$ *with cardinality given by* l^**. Then we have the probability of correct detection of* $|\theta_i - \bar{\theta}| \ge \delta,\ i = 1, \ldots, k$ *given by* $g(\tilde{\theta}, \delta) =$ P(correct detection of $|\theta_i - \bar{\theta}| \ge \delta,\ i = 1, \ldots, k) \ge P(F_{2,2m-2} \le \frac{k}{k-1}(\frac{\delta}{c} - h_t))^k$.

Proof of Theorem 1. From Wu and Wu [11], we have $g(\tilde{\theta}, \delta)$ = P(correct detection of $|\theta_i - \bar{\theta}| \ge \delta,\ i = 1, \ldots, k) = \mathrm{P}(\tilde{X}_i - \tilde{X} \ge ch_t for\ i \in A_U(\tilde{\theta}, \delta) \cap\ \tilde{X}_{i^*} - \tilde{X} \le -ch_t\ for\ i^* \in A_L(\tilde{\theta}, \delta))$
$= \mathrm{P}(\tilde{X}_i - \theta_i - \tilde{X} + \bar{\theta} + \theta_i - \bar{\theta} \ge ch_t for\ i \in A_U(\tilde{\theta}, \delta) \cap \tilde{X}_{i^*} - \theta_{i^*} - \tilde{X} + \bar{\theta} + \theta_{i^*} - \bar{\theta} \le -ch_t\ for\ i^* \in \mathrm{A}_L(\tilde{\theta}, \delta))$
$= \mathrm{P}(\frac{S_i}{N_i}W_i^* - \sum_{i=1}^{k} \frac{S_i W_i^*}{kN_i} + \theta_i - \bar{\theta} \ge ch_t for\ i \in A_U(\tilde{\theta}, \delta) \cap \frac{S_{i^*}}{N_{i^*}}W_{i^*}^* - \sum_{i^*=1}^{k} \frac{S_{i^*} W_{i^*}^*}{kN_{i^*}} + \theta_{i^*} - \bar{\theta} \le -ch_t for\ i^* \in A_L(\tilde{\theta}, \delta))$
$\ge \mathrm{P}(\frac{S_i}{N_i}W_i^* - \sum_{i=1}^{k} \frac{S_i}{kN_i}W_i^* + \delta \ge ch_t for\ i \in A_U(\tilde{\theta}, \delta) \cap\ \frac{S_{i^*}}{N_{i^*}}W_{i^*}^* - \sum_{i^*=1}^{k} \frac{S_{i^*} W_{i^*}^*}{kN_{i^*}} - \delta \le -ch_t for\ i^* \in A_L(\tilde{\theta}, \delta))$
$\ge \mathrm{P}(\frac{S_i}{N_i}W_i^* - \sum_{l \ne i}^{k} \frac{S_l W_l^*}{N_l(k-1)} \ge -\frac{k}{k-1}(\delta - ch_t) for\ i \in A_U(\tilde{\theta}, \delta) \cap \frac{S_{i^*}}{N_{i^*}}W_{i^*}^* - \sum_{l \ne i^*}^{k} \frac{S_l W_l^*}{N_l(k-1)} \le \frac{k}{k-1}(\delta - ch_t) for\ i^* \in A_L(\tilde{\theta}, \delta))$
$\ge \mathrm{P}(\frac{S_i}{N_i}W_i^* - c\sum_{l \ne i}^{k} \frac{W_l^*}{(k-1)} \ge -\frac{k}{k-1}(\delta - ch_t) for\ i \in A_U(\tilde{\theta}, \delta) \cap cW_{i^*}^* \le \sum_{l \ne i^*}^{k} \frac{S_l W_l^*}{N_l(k-1)} + \frac{k}{k-1}(\delta - ch_t) for\ i^* \in A_L(\tilde{\theta}, \delta))$
$\ge \mathrm{P}(0 \ge c \max_{l \ne i} W_l^* - c\frac{k}{k-1}(\frac{\delta}{c} - h_t) for\ i \in A_U(\tilde{\theta}, \delta) \cap\ cW_i^* - c\frac{k}{k-1}(\frac{\delta}{c} - h_t) \le 0\ for\ i^* \in A_L(\tilde{\theta}, \delta))$
$\ge \mathrm{P}(0 \ge \max_{l \ne i} W_l^* - \frac{k}{k-1}(\frac{\delta}{c} - h_t) for\ i \in A_U(\tilde{\theta}, \delta) \cap\ W_i^* - \frac{k}{k-1}(\frac{\delta}{c} - h_t) \le 0\ for\ i^* \in A_L(\tilde{\theta}, \delta)\ )$
(after the cancellation of c) $\ge \mathrm{P}(\frac{k}{k-1}(\frac{\delta}{c} - h_t) \ge W_i^* for\ i = 1, \ldots, k\ \cap\ W_{i^*}^* \le \frac{k}{k-1}(\frac{\delta}{c} - h_t)\ for\ i^* \in A_L(\tilde{\theta}, \delta)\ )$
$\ge \mathrm{P}(\frac{k}{k-1}(\frac{\delta}{c} - h_t) \ge W_i^* for\ i = 1, \ldots, k\ ) = \mathrm{P}(F_{2,2m-2} \le \frac{k}{k-1}(\frac{\delta}{c} - h_t))^k$. The proof is thus obtained. □

Suppose that the users desire to set the probability of correct detection as γ. The design constant c can be determined as a solution of the following equation by equating the above lower bound as γ:

$$P(F_{2,2m-2} \le \frac{k}{k-1}(\frac{\delta}{c} - h_t))^k = \gamma.$$

That is $\frac{k}{k-1}(\frac{\delta}{c} - h_t) = F_{2,2m-2}^{-1}(\gamma^{1/k})$ which is the $100\gamma^{1/k}$th percentile of F distribution with (2,2m − 2) df. Let $g_\gamma = \frac{k-1}{k}F_{2,2m-2}^{-1}(\gamma^{1/k})$. Thus, we have $\frac{\delta}{c} - h_t = g_\gamma$ and then the design constant c can be determined as $c = \delta\varphi(m, P^*, \gamma)$, where $\varphi(m, P^*, \gamma) = \frac{1}{h_t + g_\gamma}$. Once the design constant is determined, the total sample size $N_i,\ i = 1, \cdots, k$ for a two-stage procedure can be determined by Equation (1).

For example, suppose an experimenter would like to detect a difference of at least δ = 0.3 with γ = 0.90, P^* = 0.90, k = 4, m = 15, then we find $g_\gamma = h_t$ = 3.1273. Using these values, we yield $\varphi(m, P^*, \gamma) = \frac{1}{h_t + g_\gamma} = \frac{1}{3.1273 + 3.1273}$ = 0.1599 and then the design constant is determined as $c = \delta\varphi(m, P^*, \gamma)$ = 0.3(0.1599) = 0.04797. The values of h_t and g_γ in Maurya et al. [10] are $h_t = F_{2,2m-2}^{-1}(P*^{1/k+1})$ and $g_\gamma = F_{2,2m-2}^{-1}(\gamma^{1/k})$, which are different from the one given in Theorem 1 so that the design constant in Maurya et al. [10] is different from ours.

Under P^* = 0.80(0.05)0.95,0.975, γ = 0.80(0.05)0.95,0.975 and m = 2(1)10(5)30,60, the values of $\varphi(m, P^*, \gamma)$ are listed in Table A1. Once the value of δ is specified, the design constant c can be computed and then the total sample size $N_i,\ i = 1, \cdots, k$ can be determined.

3. Example

Example 1: The data of duration of remission of 20 patients treated by four drugs given in Table 1 of Wu and Wu [11] is used for the purpose of illustration; the duration time is assumed to have exponential distribution. Set the initial sample size for the first stage to be 20. The maximum likelihood estimators for scale parameters of four populations are (S_1, S_2, S_3, S_4) = (1.238, 1.530, 3.233, 4.075). In this example, we have k = 4 and m = 20. Suppose that the users want to detect a difference of at least δ = 0.82. Under γ = 0.90 and P^* = 0.90, we can find $\varphi(m, P^*, \gamma)$ = 0.1657 in Table 1 and then $c = \delta\varphi(m, P^*, \gamma)$ = 0.82 * 0.1657 = 0.136.

If an experimenter chose to detect a difference of at least δ = 1.0 with γ = 0.95, P^* = 0.95, we can find $\varphi(m, P^*, \gamma)$ = 0.1359 in Table 1, resulting in the same c. The total sample size for the two-stage procedure for four drugs are (N_1, N_2, N_3, N_4) = (20, 20, 24, 30). Then, we can obtain the maximum likelihood estimators for four location parameters as ($\widetilde{X}_1$, $\widetilde{X}_2$, $\widetilde{X}_3$, $\widetilde{X}_4$) = (1.013, 2.214, 3.064, 4.498) and the average these four estimators is $\widetilde{X}$ = 2.697.

Table 1. The 90% and 95% one-sided confidence intervals and two-sided confidence interval for four drugs compared with average.

P^*	$\theta_1 - \overline{\theta}$	$\theta_2 - \overline{\theta}$	$\theta_3 - \overline{\theta}$	$\theta_4 - \overline{\theta}$
90%	(−2.1091, −1.2589)	(−0.9081, −0.0579)	(−0.0581,0.7921)	(1.3759, 2.2261)
95%	(−2.2020, −1.1660)	(−1.0010,0.0350)	(−0.1510,0.8850)	(1.2830, 2.3190)

Using part (c) of Theorem 1 in Wu and Wu [11], the two-sided confidence interval for $\theta_{\mathrm{i}} - \overline{\theta}$, i = 1,2, … ,k are $(\widetilde{X}_i - \widetilde{X} \pm ch_t)$. When k = 4, the critical values are $h_t = (k-1)/kF^{-1}_{2,2m-2}(P^{*1/k})$ = 3.018 and 3.678 for P^* = 0.90 and 0.95. Based on these estimations, we can build the two-sided confidence interval for $\theta_{\mathrm{i}} - \overline{\theta}$, i = 1,2.3,4 under confidence coefficient 0.90 and 0.95 in Table 1. The confidence lengths are $2ch_t$ = 0.8501, 1.0361 for P^* = 0.90 and 0.95. From Table 1, we observe that only the lower limit of drug 4 is positive for P^* = 0.90 and 0.95. Thus, we conclude that this drug is detected to be greater than the average. For P^* = 0.90, we observe that the upper limits of drug 1 and drug 2 are negative. Thus, we conclude that drugs 1 and 2 are detected to be smaller than the average. Furthermore, we can say that drug 4 is superior to drugs 1, 2 and 3 compared with the average for P^* = 0.90. But for P^* = 0.95, we only observe that the upper limits of drug 1 is negative. Thus, we conclude that only drug 1 is detected to be smaller than the average. To sum up, we can say that drug 4 compared with the average is superior to drugs 1, 2 and 3 for P^* = 0.95.

Example 2: Referring to Wu [18], the data of survival days of patients with four types of lung cancer in weeks is used to determine the design constant c. In this case, we consider m = 5 and k = 4. The maximum likelihood estimators for scale parameters of four populations are (S_1, S_2, S_3, S_4) = (11.536, 1.357, 12.286, 16.714). Suppose that the users want to detect a difference of at least δ = 26. Under γ = 0.80 and P^* = 0.80, we can find $\varphi(m, P^*, \gamma)$ = 0.1555 in Table 1 and then $c = \delta\varphi(m, P^*, \gamma)$ = 26 × 0.1555 = 4.043. The total sample sizes for the two-stage procedure for four populations are N_1 = N_2 = N_3 = N_4 = 5. Then, we can obtain that the consistent estimator for the location parameters for four populations are ($\widetilde{X}_1$, $\widetilde{X}_2$, $\widetilde{X}_3$, $\widetilde{X}_4$) = (1.429, 1.857, 1.143, 22.286) and the average is $\widetilde{X}$ = 6.679. Using part (c) of Theorem 1 in Wu and Wu [11], the critical value is $h_t = (k-1)/kF^{-1}_{2,2m-2}(P^{*1/k})$ = 3.216 for P^* = 0.80. Based on these estimations, we can build the two-sided confidence interval for $\theta_{\mathrm{i}} - \overline{\theta}$, i = 1,2.3,4 under confidence coefficient 0.80 in Table 2. From Table 2, we observe that only the lower limit of the fourth type of lung cancer has guaranteed survival time greater than the average for P^* = 0.8. But the other three types are concluded not to be different from the average. Thus, we concluded that the fourth type of lung cancer compared with the average is superior to the other three types.

Table 2. The 80% two-sided confidence interval for four types of lung cancer compared with average.

P^*	$\theta_1 - \overline{\theta}$	$\theta_2 - \overline{\theta}$	$\theta_3 - \overline{\theta}$	$\theta_4 - \overline{\theta}$
80%	(−18.252, 7.752)	(−17.824, 8.180)	(−18.538, 7.466)	(2.605, 28.609)

4. Conclusions

The two-stage sampling procedure is a design-oriented procedure. In this research, we present a sampling design to determine the control constant c and then the total sample size for a two-stage procedure to detect a difference of at least δ under the confidence coefficient P^* and the probability of correct detection being at least γ. Finally, two examples are given to illustrate this sampling design and to conduct the two-stage multiple comparisons with the average. There is no work related to this research topic in the literatures so that our research results cannot be compared with any existing research methods. The limitation of this research is that fact that it only focuses on the exponential distributions. In the future, we hope to extend these research results to other distributions, such as Pareto distribution, Weibull distribution or any other lifetime distributions.

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Data Availability Statement: Data available in a publicly accessible repository The data presented in this study is openly available in Wu and Wu [11] and Wu [18].

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. The values of $\varphi(m, P^*, \gamma)$ under P^* = 0.80(0.05)0.95,0.975, γ = 0.80(0.05)0.95,0.975 and m = 2(1)10(5)30,60.

			$\varphi(m,P^*,\gamma)$								
m	P^*	γ	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	0.80	0.80	0.1180	0.0579	0.0382	0.0285	0.0227	0.0189	0.0162	0.0141	0.0125
		0.85	0.0986	0.0485	0.0321	0.0240	0.0191	0.0159	0.0136	0.0119	0.0105
		0.90	0.0742	0.0367	0.0243	0.0182	0.0145	0.0121	0.0103	0.0090	0.0080
		0.95	0.0426	0.0211	0.0140	0.0105	0.0084	0.0070	0.0060	0.0052	0.0047
		0.975	0.0230	0.0115	0.0076	0.0057	0.0046	0.0038	0.0033	0.0028	0.0025
	0.85	0.80	0.0986	0.0485	0.0321	0.0240	0.0191	0.0159	0.0136	0.0119	0.0105
		0.85	0.0847	0.0418	0.0276	0.0206	0.0165	0.0137	0.0117	0.0102	0.0091
		0.90	0.0660	0.0327	0.0217	0.0162	0.0129	0.0108	0.0092	0.0080	0.0071
		0.95	0.0398	0.0197	0.0131	0.0098	0.0078	0.0065	0.0056	0.0049	0.0043
		0.975	0.0221	0.0110	0.0073	0.0055	0.0044	0.0037	0.0031	0.0027	0.0024
2	0.90	0.80	0.0742	0.0367	0.0243	0.0182	0.0145	0.0121	0.0103	0.0090	0.0080
		0.85	0.0660	0.0327	0.0217	0.0162	0.0129	0.0108	0.0092	0.0080	0.0071
		0.90	0.0541	0.0268	0.0178	0.0133	0.0106	0.0088	0.0076	0.0066	0.0059
		0.95	0.0351	0.0174	0.0116	0.0087	0.0069	0.0058	0.0049	0.0043	0.0038
		0.975	0.0206	0.0103	0.0068	0.0051	0.0041	0.0034	0.0029	0.0026	0.0023
	0.95	0.80	0.0426	0.0211	0.0140	0.0105	0.0084	0.0070	0.0060	0.0052	0.0047
		0.85	0.0398	0.0197	0.0131	0.0098	0.0078	0.0065	0.0056	0.0049	0.0043
		0.90	0.0351	0.0174	0.0116	0.0087	0.0069	0.0058	0.0049	0.0043	0.0038
		0.95	0.0260	0.0129	0.0086	0.0064	0.0052	0.0043	0.0037	0.0032	0.0029
		0.975	0.0171	0.0085	0.0057	0.0043	0.0034	0.0028	0.0024	0.0021	0.0019
2	0.975	0.80	0.0230	0.0115	0.0076	0.0057	0.0046	0.0038	0.0033	0.0028	0.0025
		0.85	0.0221	0.0110	0.0073	0.0055	0.0044	0.0037	0.0031	0.0027	0.0024
		0.90	0.0206	0.0103	0.0068	0.0051	0.0041	0.0034	0.0029	0.0026	0.0023
		0.95	0.0171	0.0085	0.0057	0.0043	0.0034	0.0028	0.0024	0.0021	0.0019
		0.975	0.0127	0.0064	0.0042	0.0032	0.0025	0.0021	0.0018	0.0016	0.0014
3	0.80	0.80	0.2407	0.1371	0.1012	0.0825	0.0709	0.0628	0.0568	0.0522	0.0485
		0.85	0.2147	0.1232	0.0913	0.0746	0.0642	0.0569	0.0516	0.0474	0.0441
		0.90	0.1821	0.1054	0.0785	0.0644	0.0555	0.0493	0.0447	0.0412	0.0383
		0.95	0.1358	0.0797	0.0598	0.0493	0.0426	0.0380	0.0346	0.0319	0.0297
		0.975	0.1001	0.0593	0.0448	0.0371	0.0322	0.0288	0.0262	0.0242	0.0226
	0.85	0.80	0.2147	0.1232	0.0913	0.0746	0.0642	0.0569	0.0516	0.0474	0.0441
		0.85	0.1938	0.1118	0.0831	0.0681	0.0586	0.0521	0.0472	0.0434	0.0404
		0.90	0.1668	0.0969	0.0724	0.0594	0.0513	0.0456	0.0414	0.0381	0.0355
		0.95	0.1272	0.0747	0.0562	0.0463	0.0401	0.0358	0.0325	0.0300	0.0280
		0.975	0.0953	0.0565	0.0427	0.0354	0.0307	0.0275	0.0250	0.0231	0.0216
3	0.90	0.80	0.1821	0.1054	0.0785	0.0644	0.0555	0.0493	0.0447	0.0412	0.0383
		0.85	0.1668	0.0969	0.0724	0.0594	0.0513	0.0456	0.0414	0.0381	0.0355

Table A1. *Cont.*

			$\varphi(m, P^*, \gamma)$								
m	P^*	γ	2	3	4	5	6	7	8	9	10
		0.90	0.1464	0.0856	0.0641	0.0527	0.0456	0.0406	0.0369	0.0340	0.0317
		0.95	0.1150	0.0678	0.0510	0.0422	0.0365	0.0326	0.0297	0.0274	0.0255
		0.975	0.0883	0.0525	0.0397	0.0329	0.0286	0.0256	0.0233	0.0215	0.0201
	0.95	0.80	0.1358	0.0797	0.0598	0.0493	0.0426	0.0380	0.0346	0.0319	0.0297
		0.85	0.1272	0.0747	0.0562	0.0463	0.0401	0.0358	0.0325	0.0300	0.0280
		0.90	0.1150	0.0678	0.0510	0.0422	0.0365	0.0326	0.0297	0.0274	0.0255
		0.95	0.0946	0.0561	0.0424	0.0351	0.0305	0.0272	0.0248	0.0229	0.0214
		0.975	0.0758	0.0452	0.0343	0.0284	0.0247	0.0221	0.0202	0.0187	0.0174
3	0.975	0.80	0.1001	0.0593	0.0448	0.0371	0.0322	0.0288	0.0262	0.0242	0.0226
		0.85	0.0953	0.0565	0.0427	0.0354	0.0307	0.0275	0.0250	0.0231	0.0216
		0.90	0.0883	0.0525	0.0397	0.0329	0.0286	0.0256	0.0233	0.0215	0.0201
		0.95	0.0758	0.0452	0.0343	0.0284	0.0247	0.0221	0.0202	0.0187	0.0174
		0.975	0.0632	0.0378	0.0288	0.0239	0.0208	0.0186	0.0170	0.0157	0.0147
4	0.80	0.80	0.2987	0.1776	0.1354	0.1132	0.0993	0.0896	0.0824	0.0767	0.0722
		0.85	0.2715	0.1627	0.1245	0.1044	0.0918	0.0829	0.0763	0.0712	0.0671
		0.90	0.2375	0.1437	0.1106	0.0931	0.0821	0.0743	0.0686	0.0640	0.0604
		0.95	0.1893	0.1163	0.0903	0.0764	0.0677	0.0615	0.0568	0.0532	0.0503
		0.975	0.1510	0.0939	0.0734	0.0625	0.0555	0.0506	0.0469	0.0440	0.0416
	0.85	0.80	0.2715	0.1627	0.1245	0.1044	0.0918	0.0829	0.0763	0.0712	0.0671
		0.85	0.2488	0.1500	0.1152	0.0969	0.0853	0.0772	0.0711	0.0664	0.0626
		0.90	0.2199	0.1338	0.1033	0.0871	0.0769	0.0697	0.0643	0.0601	0.0567
		0.95	0.1780	0.1097	0.0853	0.0723	0.0641	0.0583	0.0539	0.0505	0.0477
		0.975	0.1437	0.0895	0.0701	0.0597	0.0531	0.0484	0.0449	0.0421	0.0398
4	0.90	0.80	0.2375	0.1437	0.1106	0.0931	0.0821	0.0743	0.0686	0.0640	0.0604
		0.85	0.2199	0.1338	0.1033	0.0871	0.0769	0.0697	0.0643	0.0601	0.0567
		0.90	0.1971	0.1207	0.0935	0.0791	0.0700	0.0635	0.0587	0.0549	0.0519
		0.95	0.1627	0.1007	0.0786	0.0667	0.0592	0.0539	0.0499	0.0468	0.0442
		0.975	0.1336	0.0835	0.0655	0.0558	0.0497	0.0453	0.0421	0.0395	0.0374
	0.95	0.80	0.1893	0.1163	0.0903	0.0764	0.0677	0.0615	0.0568	0.0532	0.0503
		0.85	0.1780	0.1097	0.0853	0.0723	0.0641	0.0583	0.0539	0.0505	0.0477
		0.90	0.1627	0.1007	0.0786	0.0667	0.0592	0.0539	0.0499	0.0468	0.0442
		0.95	0.1386	0.0864	0.0677	0.0577	0.0513	0.0468	0.0434	0.0407	0.0386
		0.975	0.1169	0.0734	0.0578	0.0494	0.0440	0.0402	0.0373	0.0351	0.0332
4	0.975	0.80	0.1510	0.0939	0.0734	0.0625	0.0555	0.0506	0.0469	0.0440	0.0416
		0.85	0.1437	0.0895	0.0701	0.0597	0.0531	0.0484	0.0449	0.0421	0.0398
		0.90	0.1336	0.0835	0.0655	0.0558	0.0497	0.0453	0.0421	0.0395	0.0374
		0.95	0.1169	0.0734	0.0578	0.0494	0.0440	0.0402	0.0373	0.0351	0.0332
		0.975	0.1010	0.0638	0.0504	0.0431	0.0385	0.0352	0.0328	0.0308	0.0292
5	0.80	0.80	0.3314	0.2010	0.1555	0.1315	0.1165	0.1060	0.0982	0.0920	0.0871
		0.85	0.3037	0.1857	0.1443	0.1224	0.1087	0.0990	0.0918	0.0862	0.0817
		0.90	0.2695	0.1665	0.1301	0.1108	0.0987	0.0901	0.0837	0.0787	0.0747
		0.95	0.2211	0.1387	0.1093	0.0937	0.0838	0.0768	0.0716	0.0674	0.0641
		0.975	0.1825	0.1159	0.0921	0.0793	0.0712	0.0655	0.0611	0.0577	0.0550
	0.85	0.80	0.3037	0.1857	0.1443	0.1224	0.1087	0.0990	0.0918	0.0862	0.0817
		0.85	0.2803	0.1725	0.1346	0.1145	0.1018	0.0929	0.0863	0.0811	0.0769
		0.90	0.2509	0.1558	0.1221	0.1043	0.0930	0.0850	0.0791	0.0744	0.0706
		0.95	0.2084	0.1312	0.1037	0.0890	0.0797	0.0731	0.0682	0.0643	0.0611
		0.975	0.1737	0.1106	0.0880	0.0759	0.0682	0.0627	0.0586	0.0554	0.0528
5	0.90	0.80	0.2695	0.1665	0.1301	0.1108	0.0987	0.0901	0.0837	0.0787	0.0747
		0.85	0.2509	0.1558	0.1221	0.1043	0.0930	0.0850	0.0791	0.0744	0.0706
		0.90	0.2271	0.1420	0.1118	0.0958	0.0856	0.0784	0.0730	0.0688	0.0653
		0.95	0.1917	0.1213	0.0961	0.0827	0.0742	0.0681	0.0636	0.0600	0.0571
		0.975	0.1620	0.1035	0.0825	0.0713	0.0641	0.0590	0.0552	0.0522	0.0497
	0.95	0.80	0.2211	0.1387	0.1093	0.0937	0.0838	0.0768	0.0716	0.0674	0.0641

Table A1. *Cont.*

			$\varphi(m,P^*,\gamma)$								
m	***P****	γ	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
		0.85	0.2084	0.1312	0.1037	0.0890	0.0797	0.0731	0.0682	0.0643	0.0611
		0.90	0.1917	0.1213	0.0961	0.0827	0.0742	0.0681	0.0636	0.0600	0.0571
		0.95	0.1659	0.1059	0.0843	0.0728	0.0654	0.0602	0.0563	0.0532	0.0507
		0.975	0.1431	0.0920	0.0737	0.0638	0.0575	0.0530	0.0496	0.0470	0.0448
5	0.975	0.80	0.1825	0.1159	0.0921	0.0793	0.0712	0.0655	0.0611	0.0577	0.0550
		0.85	0.1737	0.1106	0.0880	0.0759	0.0682	0.0627	0.0586	0.0554	0.0528
		0.90	0.1620	0.1035	0.0825	0.0713	0.0641	0.0590	0.0552	0.0522	0.0497
		0.95	0.1431	0.0920	0.0737	0.0638	0.0575	0.0530	0.0496	0.0470	0.0448
		0.975	0.1259	0.0814	0.0654	0.0568	0.0513	0.0474	0.0444	0.0421	0.0401
6	0.80	0.80	0.3522	0.2161	0.1686	0.1436	0.1278	0.1169	0.1087	0.1023	0.0971
		0.85	0.3243	0.2006	0.1572	0.1343	0.1199	0.1098	0.1022	0.0963	0.0916
		0.90	0.2901	0.1813	0.1429	0.1226	0.1097	0.1007	0.0940	0.0887	0.0844
		0.95	0.2419	0.1535	0.1221	0.1054	0.0948	0.0873	0.0817	0.0773	0.0737
		0.975	0.2034	0.1307	0.1048	0.0909	0.0821	0.0758	0.0711	0.0674	0.0644
	0.85	0.80	0.3243	0.2006	0.1572	0.1343	0.1199	0.1098	0.1022	0.0963	0.0916
		0.85	0.3006	0.1872	0.1473	0.1262	0.1128	0.1035	0.0965	0.0910	0.0866
		0.90	0.2709	0.1703	0.1347	0.1158	0.1038	0.0954	0.0891	0.0842	0.0802
		0.95	0.2284	0.1455	0.1160	0.1003	0.0903	0.0833	0.0780	0.0738	0.0704
		0.975	0.1937	0.1249	0.1003	0.0871	0.0787	0.0728	0.0683	0.0648	0.0619
6	0.90	0.80	0.2901	0.1813	0.1429	0.1226	0.1097	0.1007	0.0940	0.0887	0.0844
		0.85	0.2709	0.1703	0.1347	0.1158	0.1038	0.0954	0.0891	0.0842	0.0802
		0.90	0.2466	0.1561	0.1240	0.1070	0.0961	0.0885	0.0828	0.0783	0.0747
		0.95	0.2109	0.1351	0.1080	0.0936	0.0844	0.0780	0.0731	0.0692	0.0661
		0.975	0.1810	0.1171	0.0943	0.0820	0.0742	0.0687	0.0645	0.0612	0.0586
	0.95	0.80	0.2419	0.1535	0.1221	0.1054	0.0948	0.0873	0.0817	0.0773	0.0737
		0.85	0.2284	0.1455	0.1160	0.1003	0.0903	0.0833	0.0780	0.0738	0.0704
		0.90	0.2109	0.1351	0.1080	0.0936	0.0844	0.0780	0.0731	0.0692	0.0661
		0.95	0.1842	0.1190	0.0957	0.0832	0.0753	0.0697	0.0654	0.0621	0.0593
		0.975	0.1610	0.1049	0.0847	0.0739	0.0670	0.0621	0.0585	0.0555	0.0532
6	0.975	0.80	0.2034	0.1307	0.1048	0.0909	0.0821	0.0758	0.0711	0.0674	0.0644
		0.85	0.1937	0.1249	0.1003	0.0871	0.0787	0.0728	0.0683	0.0648	0.0619
		0.90	0.1810	0.1171	0.0943	0.0820	0.0742	0.0687	0.0645	0.0612	0.0586
		0.95	0.1610	0.1049	0.0847	0.0739	0.0670	0.0621	0.0585	0.0555	0.0532
		0.975	0.1429	0.0937	0.0760	0.0665	0.0604	0.0561	0.0528	0.0503	0.0482
7	0.80	0.80	0.3666	0.2266	0.1777	0.1520	0.1358	0.1245	0.1161	0.1096	0.1043
		0.85	0.3386	0.2111	0.1663	0.1427	0.1278	0.1174	0.1096	0.1035	0.0986
		0.90	0.3044	0.1917	0.1519	0.1309	0.1176	0.1083	0.1013	0.0958	0.0914
		0.95	0.2564	0.1640	0.1311	0.1137	0.1026	0.0948	0.0889	0.0843	0.0806
		0.975	0.2181	0.1413	0.1139	0.0992	0.0899	0.0833	0.0784	0.0745	0.0713
	0.85	0.80	0.3386	0.2111	0.1663	0.1427	0.1278	0.1174	0.1096	0.1035	0.0986
		0.85	0.3146	0.1975	0.1562	0.1344	0.1206	0.1110	0.1038	0.0981	0.0936
		0.90	0.2849	0.1804	0.1435	0.1239	0.1115	0.1028	0.0963	0.0912	0.0871
		0.95	0.2424	0.1557	0.1248	0.1084	0.0979	0.0906	0.0851	0.0807	0.0772
		0.975	0.2079	0.1351	0.1091	0.0952	0.0863	0.0801	0.0753	0.0716	0.0686
7	0.90	0.80	0.3044	0.1917	0.1519	0.1309	0.1176	0.1083	0.1013	0.0958	0.0914
		0.85	0.2849	0.1804	0.1435	0.1239	0.1115	0.1028	0.0963	0.0912	0.0871
		0.90	0.2602	0.1661	0.1327	0.1150	0.1037	0.0958	0.0898	0.0852	0.0814
		0.95	0.2243	0.1449	0.1166	0.1015	0.0919	0.0851	0.0800	0.0760	0.0727
		0.975	0.1945	0.1269	0.1027	0.0898	0.0815	0.0757	0.0713	0.0679	0.0650
	0.95	0.80	0.2564	0.1640	0.1311	0.1137	0.1026	0.0948	0.0889	0.0843	0.0806
		0.85	0.2424	0.1557	0.1248	0.1084	0.0979	0.0906	0.0851	0.0807	0.0772
		0.90	0.2243	0.1449	0.1166	0.1015	0.0919	0.0851	0.0800	0.0760	0.0727
		0.95	0.1971	0.1285	0.1039	0.0908	0.0824	0.0765	0.0721	0.0686	0.0657
		0.975	0.1737	0.1141	0.0928	0.0813	0.0740	0.0689	0.0650	0.0619	0.0594
7	0.975	0.80	0.2181	0.1413	0.1139	0.0992	0.0899	0.0833	0.0784	0.0745	0.0713

Table A1. *Cont.*

			$\varphi(m,P^*,\gamma)$								
m	P^*	γ	2	3	4	5	6	7	8	9	10
		0.85	0.2079	0.1351	0.1091	0.0952	0.0863	0.0801	0.0753	0.0716	0.0686
		0.90	0.1945	0.1269	0.1027	0.0898	0.0815	0.0757	0.0713	0.0679	0.0650
		0.95	0.1737	0.1141	0.0928	0.0813	0.0740	0.0689	0.0650	0.0619	0.0594
		0.975	0.1552	0.1026	0.0838	0.0737	0.0672	0.0626	0.0591	0.0564	0.0542
8	0.80	0.80	0.3772	0.2344	0.1845	0.1583	0.1418	0.1303	0.1217	0.1150	0.1096
		0.85	0.3491	0.2187	0.1730	0.1489	0.1337	0.1231	0.1151	0.1089	0.1039
		0.90	0.3149	0.1994	0.1586	0.1371	0.1235	0.1139	0.1068	0.1012	0.0967
		0.95	0.2671	0.1718	0.1379	0.1199	0.1085	0.1005	0.0944	0.0897	0.0858
		0.975	0.2291	0.1492	0.1207	0.1055	0.0958	0.0890	0.0839	0.0798	0.0765
	0.85	0.80	0.3491	0.2187	0.1730	0.1489	0.1337	0.1231	0.1151	0.1089	0.1039
		0.85	0.3250	0.2051	0.1628	0.1406	0.1265	0.1166	0.1092	0.1035	0.0988
		0.90	0.2952	0.1880	0.1501	0.1300	0.1173	0.1084	0.1017	0.0965	0.0922
		0.95	0.2528	0.1632	0.1314	0.1144	0.1037	0.0961	0.0904	0.0860	0.0823
		0.975	0.2184	0.1427	0.1157	0.1012	0.0920	0.0856	0.0807	0.0768	0.0737
8	0.90	0.80	0.3149	0.1994	0.1586	0.1371	0.1235	0.1139	0.1068	0.1012	0.0967
		0.85	0.2952	0.1880	0.1501	0.1300	0.1173	0.1084	0.1017	0.0965	0.0922
		0.90	0.2703	0.1735	0.1392	0.1209	0.1094	0.1012	0.0951	0.0904	0.0865
		0.95	0.2344	0.1522	0.1229	0.1074	0.0974	0.0905	0.0852	0.0811	0.0777
		0.975	0.2045	0.1342	0.1091	0.0957	0.0871	0.0811	0.0765	0.0729	0.0700
	0.95	0.80	0.2671	0.1718	0.1379	0.1199	0.1085	0.1005	0.0944	0.0897	0.0858
		0.85	0.2528	0.1632	0.1314	0.1144	0.1037	0.0961	0.0904	0.0860	0.0823
		0.90	0.2344	0.1522	0.1229	0.1074	0.0974	0.0905	0.0852	0.0811	0.0777
		0.95	0.2068	0.1355	0.1101	0.0965	0.0879	0.0817	0.0771	0.0735	0.0706
		0.975	0.1832	0.1211	0.0988	0.0870	0.0794	0.0740	0.0699	0.0668	0.0641
8	0.975	0.80	0.2291	0.1492	0.1207	0.1055	0.0958	0.0890	0.0839	0.0798	0.0765
		0.85	0.2184	0.1427	0.1157	0.1012	0.0920	0.0856	0.0807	0.0768	0.0737
		0.90	0.2045	0.1342	0.1091	0.0957	0.0871	0.0811	0.0765	0.0729	0.0700
		0.95	0.1832	0.1211	0.0988	0.0870	0.0794	0.0740	0.0699	0.0668	0.0641
		0.975	0.1645	0.1094	0.0897	0.0791	0.0724	0.0676	0.0640	0.0611	0.0588
9	0.80	0.80	0.3852	0.2403	0.1896	0.1631	0.1463	0.1347	0.1260	0.1192	0.1137
		0.85	0.3571	0.2246	0.1781	0.1537	0.1382	0.1274	0.1194	0.1131	0.1080
		0.90	0.3230	0.2053	0.1638	0.1419	0.1280	0.1183	0.1110	0.1054	0.1007
		0.95	0.2754	0.1777	0.1431	0.1247	0.1130	0.1048	0.0987	0.0939	0.0899
		0.975	0.2375	0.1553	0.1260	0.1104	0.1004	0.0934	0.0881	0.0840	0.0806
	0.85	0.80	0.3571	0.2246	0.1781	0.1537	0.1382	0.1274	0.1194	0.1131	0.1080
		0.85	0.3329	0.2109	0.1679	0.1453	0.1310	0.1209	0.1135	0.1076	0.1028
		0.90	0.3030	0.1937	0.1551	0.1347	0.1218	0.1127	0.1059	0.1006	0.0962
		0.95	0.2607	0.1690	0.1365	0.1191	0.1081	0.1004	0.0946	0.0900	0.0863
		0.975	0.2266	0.1486	0.1208	0.1060	0.0965	0.0899	0.0849	0.0809	0.0777
9	0.90	0.80	0.3230	0.2053	0.1638	0.1419	0.1280	0.1183	0.1110	0.1054	0.1007
		0.85	0.3030	0.1937	0.1551	0.1347	0.1218	0.1127	0.1059	0.1006	0.0962
		0.90	0.2781	0.1792	0.1441	0.1256	0.1138	0.1055	0.0993	0.0944	0.0904
		0.95	0.2421	0.1578	0.1279	0.1119	0.1018	0.0946	0.0893	0.0851	0.0816
		0.975	0.2123	0.1399	0.1140	0.1002	0.0914	0.0852	0.0806	0.0769	0.0739
	0.95	0.80	0.2754	0.1777	0.1431	0.1247	0.1130	0.1048	0.0987	0.0939	0.0899
		0.85	0.2607	0.1690	0.1365	0.1191	0.1081	0.1004	0.0946	0.0900	0.0863
		0.90	0.2421	0.1578	0.1279	0.1119	0.1018	0.0946	0.0893	0.0851	0.0816
		0.95	0.2143	0.1410	0.1149	0.1010	0.0921	0.0858	0.0811	0.0774	0.0744
		0.975	0.1906	0.1265	0.1036	0.0913	0.0835	0.0780	0.0739	0.0706	0.0679
9	0.975	0.80	0.2375	0.1553	0.1260	0.1104	0.1004	0.0934	0.0881	0.0840	0.0806
		0.85	0.2266	0.1486	0.1208	0.1060	0.0965	0.0899	0.0849	0.0809	0.0777
		0.90	0.2123	0.1399	0.1140	0.1002	0.0914	0.0852	0.0806	0.0769	0.0739
		0.95	0.1906	0.1265	0.1036	0.0913	0.0835	0.0780	0.0739	0.0706	0.0679
		0.975	0.1717	0.1147	0.0943	0.0834	0.0764	0.0715	0.0678	0.0649	0.0625

Table A1. *Cont.*

			$\varphi(m,P^*,\gamma)$								
m	P^*	γ	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	0.80	0.80	0.3915	0.2449	0.1937	0.1669	0.1500	0.1382	0.1294	0.1225	0.1170
		0.85	0.3635	0.2293	0.1822	0.1574	0.1418	0.1309	0.1228	0.1164	0.1113
		0.90	0.3294	0.2100	0.1679	0.1457	0.1316	0.1218	0.1144	0.1087	0.1040
		0.95	0.2819	0.1825	0.1472	0.1285	0.1167	0.1083	0.1021	0.0972	0.0932
		0.975	0.2442	0.1601	0.1302	0.1142	0.1041	0.0969	0.0916	0.0874	0.0839
	0.85	0.80	0.3635	0.2293	0.1822	0.1574	0.1418	0.1309	0.1228	0.1164	0.1113
		0.85	0.3392	0.2155	0.1720	0.1490	0.1345	0.1244	0.1168	0.1109	0.1061
		0.90	0.3093	0.1983	0.1592	0.1384	0.1253	0.1161	0.1092	0.1038	0.0995
		0.95	0.2671	0.1736	0.1405	0.1229	0.1117	0.1038	0.0979	0.0933	0.0895
		0.975	0.2330	0.1533	0.1249	0.1097	0.1001	0.0933	0.0882	0.0842	0.0809
10	0.90	0.80	0.3294	0.2100	0.1679	0.1457	0.1316	0.1218	0.1144	0.1087	0.1040
		0.85	0.3093	0.1983	0.1592	0.1384	0.1253	0.1161	0.1092	0.1038	0.0995
		0.90	0.2842	0.1837	0.1481	0.1292	0.1173	0.1089	0.1026	0.0976	0.0936
		0.95	0.2482	0.1623	0.1318	0.1156	0.1053	0.0980	0.0925	0.0883	0.0848
		0.975	0.2185	0.1444	0.1180	0.1039	0.0949	0.0886	0.0838	0.0801	0.0770
	0.95	0.80	0.2819	0.1825	0.1472	0.1285	0.1167	0.1083	0.1021	0.0972	0.0932
		0.85	0.2671	0.1736	0.1405	0.1229	0.1117	0.1038	0.0979	0.0933	0.0895
		0.90	0.2482	0.1623	0.1318	0.1156	0.1053	0.0980	0.0925	0.0883	0.0848
		0.95	0.2202	0.1454	0.1187	0.1045	0.0955	0.0891	0.0843	0.0805	0.0774
		0.975	0.1966	0.1308	0.1074	0.0949	0.0869	0.0812	0.0770	0.0737	0.0709
10	0.975	0.80	0.2442	0.1601	0.1302	0.1142	0.1041	0.0969	0.0916	0.0874	0.0839
		0.85	0.2330	0.1533	0.1249	0.1097	0.1001	0.0933	0.0882	0.0842	0.0809
		0.90	0.2185	0.1444	0.1180	0.1039	0.0949	0.0886	0.0838	0.0801	0.0770
		0.95	0.1966	0.1308	0.1074	0.0949	0.0869	0.0812	0.0770	0.0737	0.0709
		0.975	0.1775	0.1189	0.0980	0.0868	0.0797	0.0747	0.0709	0.0679	0.0654
15	0.80	0.80	0.4100	0.2586	0.2058	0.1781	0.1607	0.1485	0.1395	0.1324	0.1267
		0.85	0.3819	0.2429	0.1942	0.1686	0.1525	0.1413	0.1329	0.1263	0.1210
		0.90	0.3480	0.2237	0.1800	0.1569	0.1423	0.1321	0.1245	0.1185	0.1137
		0.95	0.3011	0.1965	0.1595	0.1399	0.1275	0.1188	0.1122	0.1071	0.1030
		0.975	0.2640	0.1745	0.1427	0.1258	0.1151	0.1075	0.1019	0.0974	0.0938
	0.85	0.80	0.3819	0.2429	0.1942	0.1686	0.1525	0.1413	0.1329	0.1263	0.1210
		0.85	0.3575	0.2290	0.1839	0.1601	0.1451	0.1346	0.1268	0.1207	0.1157
		0.90	0.3276	0.2119	0.1711	0.1495	0.1359	0.1263	0.1192	0.1136	0.1091
		0.95	0.2857	0.1873	0.1525	0.1340	0.1223	0.1141	0.1079	0.1031	0.0991
		0.975	0.2521	0.1672	0.1371	0.1210	0.1108	0.1037	0.0983	0.0941	0.0906
15	0.90	0.80	0.3480	0.2237	0.1800	0.1569	0.1423	0.1321	0.1245	0.1185	0.1137
		0.85	0.3276	0.2119	0.1711	0.1495	0.1359	0.1263	0.1192	0.1136	0.1091
		0.90	0.3023	0.1971	0.1599	0.1402	0.1277	0.1190	0.1124	0.1073	0.1031
		0.95	0.2662	0.1757	0.1435	0.1265	0.1157	0.1080	0.1023	0.0978	0.0942
		0.975	0.2368	0.1579	0.1298	0.1148	0.1054	0.0987	0.0936	0.0897	0.0865
	0.95	0.80	0.3011	0.1965	0.1595	0.1399	0.1275	0.1188	0.1122	0.1071	0.1030
		0.85	0.2857	0.1873	0.1525	0.1340	0.1223	0.1141	0.1079	0.1031	0.0991
		0.90	0.2662	0.1757	0.1435	0.1265	0.1157	0.1080	0.1023	0.0978	0.0942
		0.95	0.2379	0.1585	0.1302	0.1152	0.1057	0.0990	0.0939	0.0899	0.0867
		0.975	0.2141	0.1438	0.1188	0.1055	0.0970	0.0910	0.0865	0.0830	0.0801
15	0.975	0.80	0.2640	0.1745	0.1427	0.1258	0.1151	0.1075	0.1019	0.0974	0.0938
		0.85	0.2521	0.1672	0.1371	0.1210	0.1108	0.1037	0.0983	0.0941	0.0906
		0.90	0.2368	0.1579	0.1298	0.1148	0.1054	0.0987	0.0936	0.0897	0.0865
		0.95	0.2141	0.1438	0.1188	0.1055	0.0970	0.0910	0.0865	0.0830	0.0801
		0.975	0.1947	0.1317	0.1092	0.0973	0.0897	0.0843	0.0802	0.0771	0.0745
20	0.80	0.80	0.4190	0.2653	0.2117	0.1836	0.1659	0.1536	0.1445	0.1373	0.1315
		0.85	0.3909	0.2496	0.2001	0.1741	0.1578	0.1463	0.1378	0.1312	0.1258
		0.90	0.3571	0.2304	0.1859	0.1624	0.1476	0.1372	0.1295	0.1234	0.1185

Table A1. *Cont.*

m	P^*	γ	$\varphi(m,P^*,\gamma)$ 2	3	4	5	6	7	8	9	10
		0.95	0.3104	0.2034	0.1656	0.1455	0.1328	0.1239	0.1173	0.1121	0.1078
		0.975	0.2737	0.1816	0.1489	0.1316	0.1205	0.1128	0.1070	0.1025	0.0988
	0.85	0.80	0.3909	0.2496	0.2001	0.1741	0.1578	0.1463	0.1378	0.1312	0.1258
		0.85	0.3664	0.2357	0.1898	0.1656	0.1504	0.1397	0.1318	0.1255	0.1205
		0.90	0.3365	0.2185	0.1769	0.1550	0.1411	0.1314	0.1241	0.1184	0.1138
		0.95	0.2947	0.1940	0.1584	0.1395	0.1276	0.1192	0.1129	0.1079	0.1039
		0.975	0.2614	0.1741	0.1431	0.1266	0.1162	0.1088	0.1033	0.0990	0.0955
20	0.90	0.80	0.3571	0.2304	0.1859	0.1624	0.1476	0.1372	0.1295	0.1234	0.1185
		0.85	0.3365	0.2185	0.1769	0.1550	0.1411	0.1314	0.1241	0.1184	0.1138
		0.90	0.3111	0.2036	0.1657	0.1456	0.1329	0.1240	0.1173	0.1121	0.1078
		0.95	0.2751	0.1822	0.1493	0.1319	0.1208	0.1130	0.1072	0.1026	0.0989
		0.975	0.2459	0.1645	0.1357	0.1203	0.1106	0.1037	0.0986	0.0945	0.0912
	0.95	0.80	0.3104	0.2034	0.1656	0.1455	0.1328	0.1239	0.1173	0.1121	0.1078
		0.85	0.2947	0.1940	0.1584	0.1395	0.1276	0.1192	0.1129	0.1079	0.1039
		0.90	0.2751	0.1822	0.1493	0.1319	0.1208	0.1130	0.1072	0.1026	0.0989
		0.95	0.2466	0.1649	0.1359	0.1205	0.1108	0.1039	0.0987	0.0947	0.0914
		0.975	0.2228	0.1503	0.1245	0.1108	0.1021	0.0959	0.0913	0.0877	0.0848
20	0.975	0.80	0.2737	0.1816	0.1489	0.1316	0.1205	0.1128	0.1070	0.1025	0.0988
		0.85	0.2614	0.1741	0.1431	0.1266	0.1162	0.1088	0.1033	0.0990	0.0955
		0.90	0.2459	0.1645	0.1357	0.1203	0.1106	0.1037	0.0986	0.0945	0.0912
		0.95	0.2228	0.1503	0.1245	0.1108	0.1021	0.0959	0.0913	0.0877	0.0848
		0.975	0.2032	0.1380	0.1148	0.1025	0.0947	0.0891	0.0850	0.0817	0.0790
25	0.80	0.80	0.4243	0.2692	0.2152	0.1868	0.1690	0.1566	0.1474	0.1402	0.1344
		0.85	0.3962	0.2535	0.2036	0.1774	0.1609	0.1494	0.1408	0.1341	0.1286
		0.90	0.3624	0.2344	0.1894	0.1657	0.1507	0.1403	0.1324	0.1263	0.1214
		0.95	0.3160	0.2075	0.1691	0.1489	0.1360	0.1270	0.1203	0.1150	0.1107
		0.975	0.2794	0.1858	0.1526	0.1350	0.1238	0.1160	0.1101	0.1055	0.1017
	0.85	0.80	0.3962	0.2535	0.2036	0.1774	0.1609	0.1494	0.1408	0.1341	0.1286
		0.85	0.3716	0.2396	0.1932	0.1688	0.1535	0.1427	0.1347	0.1284	0.1234
		0.90	0.3418	0.2224	0.1804	0.1582	0.1442	0.1344	0.1271	0.1213	0.1167
		0.95	0.3001	0.1980	0.1619	0.1428	0.1307	0.1222	0.1158	0.1108	0.1068
		0.975	0.2670	0.1782	0.1467	0.1300	0.1194	0.1119	0.1063	0.1019	0.0984
25	0.90	0.80	0.3624	0.2344	0.1894	0.1657	0.1507	0.1403	0.1324	0.1263	0.1214
		0.85	0.3418	0.2224	0.1804	0.1582	0.1442	0.1344	0.1271	0.1213	0.1167
		0.90	0.3163	0.2075	0.1691	0.1488	0.1360	0.1270	0.1202	0.1150	0.1107
		0.95	0.2804	0.1861	0.1528	0.1351	0.1239	0.1160	0.1101	0.1055	0.1018
		0.975	0.2512	0.1685	0.1392	0.1236	0.1137	0.1067	0.1015	0.0974	0.0941
	0.95	0.80	0.3160	0.2075	0.1691	0.1489	0.1360	0.1270	0.1203	0.1150	0.1107
		0.85	0.3001	0.1980	0.1619	0.1428	0.1307	0.1222	0.1158	0.1108	0.1068
		0.90	0.2804	0.1861	0.1528	0.1351	0.1239	0.1160	0.1101	0.1055	0.1018
		0.95	0.2517	0.1688	0.1393	0.1237	0.1138	0.1068	0.1016	0.0975	0.0942
		0.975	0.2280	0.1541	0.1279	0.1140	0.1051	0.0989	0.0942	0.0905	0.0876
25	0.975	0.80	0.2794	0.1858	0.1526	0.1350	0.1238	0.1160	0.1101	0.1055	0.1017
		0.85	0.2670	0.1782	0.1467	0.1300	0.1194	0.1119	0.1063	0.1019	0.0984
		0.90	0.2512	0.1685	0.1392	0.1236	0.1137	0.1067	0.1015	0.0974	0.0941
		0.95	0.2280	0.1541	0.1279	0.1140	0.1051	0.0989	0.0942	0.0905	0.0876
		0.975	0.2083	0.1418	0.1182	0.1056	0.0977	0.0920	0.0878	0.0845	0.0818

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