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Refined Expected Value Decision Rules under Orthopair Fuzzy Environment

Yige Xue and Yong Deng *

Institute of Fundamental and Frontier Sciences, University of Electronic Science and Technology of China, Chengdu 610054, China; 2016200301019@std.uestc.edu.cn
* Correspondence: dengentropy@uestc.edu.cn or prof.deng@hotmail.com

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Abstract: Refined expected value decision rules can refine the calculation of the expected value and make decisions by estimating the expected values of different alternatives, which use many theories, such as Choquet integral, PM function, measure and so on. However, the refined expected value decision rules have not been applied to the orthopair fuzzy environment yet. To address this issue, in this paper we propose the refined expected value decision rules under the orthopair fuzzy environment, which can apply the refined expected value decision rules on the issues of decision making that is described in the orthopair fuzzy environment. Numerical examples were applied to verify the availability and flexibility of the new refined expected value decision rules model. The experimental results demonstrate that the proposed model can apply refined expected value decision rules in the orthopair fuzzy environment and solve the decision making issues with the orthopair fuzzy environment successfully.

Keywords: refined expected value; orthopair fuzzy environment; decision rules

1. Introduction

In the real world, there are many uncertainties and unreliabilities [1–4]. In order to deal with the uncertainties [5–7], many mathematical theories are proposed, such as Bayesian network [8], hyper structures [9], fuzzy sets (FS) [10–12], hesitant fuzzy subalgebras [13], D-S evidence theory [14–16], information quality [17,18], Z-number [19,20], D-number [21], entropy [22,23] and belief structure [24] and are applied in many fields [25–27]. Among these theories and models, the orthopair fuzzy set (OFS) [28–30] allows the membership degree, non-membership degree and hesitancy degree to be $[0, 1] \times [0, 1]$, which results in the orthopair fuzzy set generalization of the intuitionistic fuzzy set and Pythagorean fuzzy set and giving great freedom to the modelers of systems in order to capture human knowledge. In this way, the orthopair fuzzy set is able to deal with the uncertainties more flexibly and accurately, has been widely applied in many fields [31,32], such as uncertainty multi-attribute decision making [33], enterprise resource planning systems selection [34], potential evaluation of emerging technology commercialization [35], green suppliers selection [36], scheme selection of construction project [37], venture capital in real estate market [38], medical diagnosis [39] and so on.

Recently, Yager proposed the refined expected value decision rules, which has the promising aspect [40]. However, what the refined expected value decision rules for a given orthopair fuzzy set is still an open issue to be addressed.

This paper proposes refined expected value decision rules under orthopair fuzzy environment, which is an approach that can refine the expected values of alternatives under orthopair fuzzy environment and make decision. It means that if an issue of decision making is under an orthopair fuzzy environment, then the refined expected value decision rules can be applied to solve this kind of issue.

The remain of this paper is structured as follows. Section 2 introduces the preliminary details. Section 3 presents the refined expected value decision rules under the orthopair fuzzy environment. Section 4 illustrates the flexibility and accuracy of the refined expected value decision rules under an orthopair fuzzy environment. Section 5 summarizes the whole paper.

2. Preliminaries

To deal with uncertainty, many tools and models have been proposed [41–44], such as fuzzy sets [45,46], basic probability assignment [47,48], rough sets [49], ordered weighted average operator [50], entropy [51–54], game theory [55,56] and complex networks [57–62]. In this section, some relative definitions are briefly introduced, such as refined expected value [40], measure [63,64] and Choquet integral [65].

2.1. Aggregation Function

The definition of aggregation function is defined as follows:

Definition 1. *(Aggregation Function) [66]*
A mapping Agg: $[0,1]^n \to [0,1]$ *is an aggregation function if*

$$Agg(0,\ldots,0) = 0 \tag{1}$$

$$Agg(c_1,\ldots,c_n) \geq Agg(d_1,\ldots,d_n) \text{ if } 1 \geq c_j \geq d_j \geq 0 \text{ for all } j \tag{2}$$

$$Agg(1,\ldots,1) = 1 \tag{3}$$

Aggregation functions have many examples and models. For example, $Agg(c_1,\ldots,c_n) = \prod_n^{i=1} c_i$ *and* $Agg(c_1,\ldots,c_n) = Min_i[c_1,\ldots,c_n]$ *are aggregation functions.*

2.2. Measure

The real world is uncertain. So, how to measure the degree of uncertainty is a important issue. The measure has effective performance in this issue. Given a space Y, the definition of measure is defined as follows:

Definition 2. *(Measure) [67]*
Assume μ *on* Y *is a mapping* μ*:* $Y \to [0,1]$*. If* μ *satisfies the following conditions:*

$$\mu(\emptyset) = 0 \tag{4}$$

$$\mu(C) \geq \mu(D) \text{ if } D \subseteq C \tag{5}$$

$$\mu(Y) = 1 \tag{6}$$

where, C *and* D *are subsets of* Y*.*
Hence, μ *is a measure.*

As a highly effective tool to indicate uncertainty, measure has wide applications [68], which is very flexible and effective.

2.3. Dual

Given a measure μ on a space Y, the definition of its dual $\hat{\mu}$ is defined as follow:

Definition 3. *(Dual) [69]*

$$\hat{\mu}(F) = 1 - \mu(\bar{F}) \tag{7}$$

$\hat{\mu}$ is also a measure. The self-dual is a special measure such that $\hat{\mu}(F) = \mu(F)$.

2.4. Choquet Integral

Given a measure μ on a space $Y = \{y_1, \ldots, y_n\}$. The definition of Choquet integral $Choq_\mu$ related to μ is defined as follows:

Definition 4. *(Choquet Integral) [70]*

$$Choq_\mu(y_1, \ldots, y_n) = \sum_{j=1}^{n} (\mu(H_j) - \mu(H_{j-1})) y_{\alpha(j)} \tag{8}$$

where $\alpha(j)$ is an index function and $y_{\alpha(j)}$ is the jth largest element in 2^Y such that $y_{\alpha(n)} \leq \cdots \leq y_{\alpha(2)} \leq y_{\alpha(1)}$ and $H_j = \{y_{\alpha(i)}$ for $i = 1$ to $j\}$ is subset of 2^Y.

2.5. Primal Monotonic Function

Definition 5. *(Primal Monotonic Function) [71]*
A function $f : [0,1] \to [0,1]$ is called a primal monotonic function, PM function, if

$$f(0) = 0 \tag{9}$$

$$f(A) \leq f(B) \text{ if } 0 \leq A \leq B \leq 1 \tag{10}$$

$$f(1) = 1 \tag{11}$$

For example, $f(x) = x^r$ for $r \in (0, \infty)$ is a PM function.

2.6. Refined Expected Value

Assume a collection of alternatives is $B = \{B_1, B_2, \cdots, B_n\}$, each B_i with probability distribution $Prob_i$ on a space $Z = \{z_1, z_2, \cdots, z_m\}$ is an uncertain alternative and f is a PM function.

The definition of refined expected value related to alternative B_i is defined as follows:

Definition 6. *(Refined Expected Value) [40]*

$$EV_f(Prob_i) = \sum_{j=1}^{m} \left(f\left(Prob_i\left(H_j\right)\right) - f\left(Prob_i\left(H_{j-1}\right)\right)\right) z_j \tag{12}$$

Here z_j is the jth largest element in Z. Hence, $Prob_i(z_1) \geq Prob_i(z_2) \geq \cdots \geq Prob_i(z_n)$ and $H_j = \{z_k$ for $k = 1$ to $j\}$.

2.7. Generalized Orthopair Fuzzy Sets

Dealing with uncertainty is an open issue and many tools are presented to address this issue [72,73]. Generalized orthopair fuzzy sets have extended intuitionistic fuzzy sets [74] and Pythagorean fuzzy sets [75,76]. The orthopair fuzzy sets have advantages in representing uncertainties [77] and have been used in a wide scope of applications [78,79]. It is more flexible, practical and efficient than intuitionistic fuzzy sets and Pythagorean fuzzy sets in dealing with ambiguity and uncertainty [80,81].

Given an universe set $Z = \{z_1, z_2, \cdots, z_n\}$, the definition of q-rung orthopair fuzzy set E on Z is defined as follows:

Definition 7. *(Q-Rung Orthopair Fuzzy Set) [28]*

$$E = \left\{ \left\langle z_j, E^+ \left(z_j \right), E^- \left(z_j \right) \right\rangle_q : z_j \in Z \right\} \tag{13}$$

where $E\left(z_j\right) = \left\langle E^+ \left(z_j \right), E^- \left(z_j \right) \right\rangle_q$ *is a q-rung orthopair membership grade [28]. If* $q = 2$*, E will be a Pythagorean fuzzy set. If* $q = 1$*, E will be an intuitionistic fuzzy set.*

Since each orthopair fuzzy grade consists of three degrees, which are membership degree, non-membership degree and hesitancy degree. It is not easy for an orthopair fuzzy grade to compare with each other. Therefore, if we can change the three degrees of an orthopair fuzzy value into a degree, we can make decisions easily. So, assume $E(z) = \langle E^+(z), E^-(z) \rangle_q$ is a given q-rung orthopair fuzzy grade, Yager has proposed the following [82]:

$$V(z) = \left(\lambda \left(B_U(z) \right)^q + (1 - \lambda) \left(B_L(z) \right)^q \right)^{1/q} \tag{14}$$

where, $B_L(z) = E^+(z)$ and $B_U(z) = \left(1 - \left(E^-(z) \right)^q \right)^{1/q}$.

3. The Proposed Model

In this paper, the refined expected value decision rules under orthopair fuzzy environment is proposed. The refined expected value decision rules is a good tool to represent uncertainty, but it has been applied under orthopair fuzzy environment, which is still an open issue. In this section, the refined expected value decision rules under orthopair fuzzy environment has been proposed, which can solve the problem of decision making under the orthopair fuzzy environment. The refined expected value decision rules under orthopair fuzzy environment can get an interval value from an object or alternative with the aid of Choquet integral, primal monotonic function and refined expected value. It leads to the result that interval values can be obtained by membership degree, non-membership degree and hesitancy degree under orthopair fuzzy environment to indicate the uncertain information of an object. It means that the proposed model can use fully the information of orthopair fuzzy environment to make decision.

Definition 8. *(Refined expected value decision rules under orthopair fuzzy environment)*

Given an orthopair fuzzy set $E = \left\{ \left\langle z_j, E^+ \left(z_j \right), E^- \left(z_j \right) \right\rangle_q : z_j \in Z \right\}$ *with orthopair fuzzy grades* $E\left(z_j\right) = \langle E^+ \left(z_j \right), E^- \left(z_j \right) \rangle_q$ *such that* $\left(E^+ \left(z_j \right) \right)^q + \left(E^- \left(z_j \right) \right)^q \leq 1$*. Assume f is a PM function and* μ *is a measure on Z.*

The refined expected value decision rules under orthopair fuzzy environment is defined as follows:

$$\begin{aligned} E^+(z) &= Choq_\mu \left(E^+(z_1), \ldots, E^+(z_n) \right) \\ &= \sum_{j=1}^{n} \left(f\left(\mu \left(G_j \right) \right) - f\left(\mu \left(G_{j-1} \right) \right) \right) E^+ \left(z_{\alpha(j)} \right) \end{aligned} \tag{15}$$

$$E^-(z) = Neg_q \left(Choq_\mu \left(Neg_q \left(E^-(z_1) \right), \ldots, Neg_q \left(E^-(z_n) \right) \right) \right) \tag{16}$$

Then, $\langle E^+(z), E^+(z) \rangle$ *is an orthopair value.*

Where, $G_j = \{ z_{\alpha(1)}, z_{\alpha(2)}, \ldots, z_{\alpha(j)} \}$ *and* $Neg_q(d) = (1 - d^q)^{1/q}$.

Where μ is a cardinality-based measure and f is linear function, $f(x) = x$. Here, we have a set of parameters $0 = a_0 \leq a_1 \ldots \leq a_n = 1$ such that $\mu\left(G_j\right) = a_{|G_j|}$. The value of $\mu\left(G_j\right)$ is decided on the cardinality of the set G_j. We denote $v_j = a_j - a_{j-1}$ so that $w_j = f\left(\mu\left(G_j\right)\right) - f\left(\mu\left(G_{j-1}\right)\right) = \mu\left(G_j\right) - \mu\left(G_{j-1}\right) = a_j - a_{j-1} = v_j$. We see this is essentially an OWA aggregation [83]. Based on the above assumptions, we provide some important and interesting theorems about the refined expected value under orthopair fuzzy environment.

Theorem 1. *Given a q-rung orthopair fuzzy set* $E = \left\{ \langle z_j, E^+ (z_j), E^- (z_j) \rangle_q : z_j \in Z \right\}$ *with orthopair fuzzy grades* $E(z_j) = \langle E^+(z_j), E^-(z_j) \rangle_q$ *and* $E^+(z_j)$ *and* $E^-(z_j)$ *as defined above, then* $(E^+(z))^q + (E^-(z))^q \leq 1$.

Proof. Since the Choquet integral related to a cardinality-based measure is OWA aggregation [83].
Hence, we can get that $E^+(z)$ and $E^-(z)$ are OWA aggregation.
Since each $E(z)$ has $(E^+(z))^q + (E^-(z))^q \leq 1$
Then

$$E^+(z) = Agg\left(E^+(z_1), \ldots, E^+(z_n)\right)$$

$$E^-(z) = \left(1 - Agg\left(\left(1 - \left(E^-(z_1)\right)^q\right)^{1/q}, \ldots, \left(1 - \left(E^-(z_n)\right)^q\right)^{1/q}\right)^q\right)^{1/q}$$

Since $(E^+(z_i))^q \leq 1 - (E^-(z_i))^q$,so

$$E^+(z) \leq Agg\left(\left(1 - \left(E^-(z_1)\right)^q\right)^{1/q}, \ldots, \left(1 - \left(E^-(z_n)\right)^q\right)^{1/q}\right)$$

Here, we see that $(E^+(z))^q + (E^-(z))^q \leq 1$.
Hence, we know that $\langle E^+(z), E^-(z) \rangle$ is an q-rung orthopair fuzzy grade. □

When μ is a probability measure and f is linear function, $f(x) = x$, we provide some important and interesting theorems about the refined expected value under orthopair fuzzy environment.

Theorem 2. *If* $\langle E^+(z), E^-(z) \rangle$ *is a refined expected value under orthopair fuzzy environment, then*

$$\langle E^+(z), E^-(z) \rangle = \langle \sum_{i=1}^{n} p_i E^+(z_i), \left(1 - \left(\sum_{j=1}^{n} p_i \left(1 - E^-(z_i)^q\right)^{1/q}\right)^q\right)^{1/q} \rangle$$

Proof. Relying on Equation (15), Equation (16) and arising from the definition of the refined expected value decision rules under orthopair fuzzy environment, one has the following equation:

$$\begin{aligned} E^+(z) &= Choq_\mu\left(E^+(z_1), \ldots, E^+(z_n)\right) \\ &= \sum_{i=1}^{n} \left(f\left(\mu\left(G_i\right)\right) - f\left(\mu\left(G_{i-1}\right)\right)\right) E^+\left(z_{\alpha(i)}\right) \\ &= \sum_{i=1}^{n} p_{\alpha(i)} E^+\left(z_{\alpha(i)}\right) \\ &= \sum_{i=1}^{n} p_i E^+(z_i) \end{aligned}$$

$$
\begin{aligned}
E^{-}(z) &= Neg_q\left(Choq_\mu\left(Neg_q\left(E^{-}(z_1)\right),\ldots,Neg_q\left(E^{-}(z_n)\right)\right)\right)\\
&= \left(1-Choq_\mu\left(Neg_q\left(E^{-}(z_1)\right),\ldots,Neg_q\left(E^{-}(z_n)\right)\right)^q\right)^{1/q}\\
&= \left(1-Choq_\mu\left(\left(1-E^{-}(z_1)^q\right)^{1/q},\ldots,\left(1-E^{-}(z_n)^q\right)^{1/q}\right)^q\right)^{1/q}\\
&= \left(1-\left(\sum_{i=1}^{n}\left(f\left(\mu\left(K_i\right)\right)-f\left(\mu\left(K_{i-1}\right)\right)\right)\left(1-E^{-}\left(z_{\vartheta(i)}\right)^q\right)^{1/q}\right)^q\right)^{1/q}\\
&= \left(1-\left(\sum_{i=1}^{n}p_{\vartheta(i)}\left(1-E^{-}\left(z_{\vartheta(i)}\right)^q\right)^{1/q}\right)^q\right)^{1/q}\\
&= \left(1-\left(\sum_{i=1}^{n}p_i\left(1-E^{-}(z_i)^q\right)^{1/q}\right)^q\right)^{1/q}
\end{aligned}
$$

So, we have that

$$
\langle E^{+}(z),E^{-}(z)\rangle=\langle\sum_{i=1}^{n}p_iE^{+}(z_i),\left(1-\left(\sum_{j=1}^{n}p_i\left(1-E^{-}(z_i)^q\right)^{1/q}\right)^q\right)^{1/q}\rangle
$$

□

Theorem 3. *Assume $\langle E^{+}(z),E^{-}(z)\rangle$ is a refined expected value under orthopair fuzzy environment. Then, if $Prob=\{p_j|j=1,\ldots,n\}$ is a constant probability distribution such that $p_i=1$ for $j*$, then*

$$
\langle E^{+}(z),E^{-}(z)\rangle=\langle E^{+}\left(z_{j*}\right),E^{-}\left(z_{j*}\right)\rangle
$$

Proof. In Theorem 2, we note that

$$
\langle E^{+}(z),E^{-}(z)\rangle=\langle\sum_{i=1}^{n}p_iE^{+}(z_i),\left(1-\left(\sum_{i=1}^{n}p_i\left(1-E^{-}(z_i)^q\right)^{1/q}\right)^q\right)^{1/q}\rangle
$$

For the definition of constant probability distribution, one has the equation as follows:

$$
p_i=1 \text{ for } i=j*
$$

Based on Theorem 2, we get that

$$
E^{+}(z)=\sum_{i=1}^{n}p_iE^{+}(z_i)=E^{+}\left(z_{j*}\right)
$$

$$
\begin{aligned}
E^{-}(z) &= \left(1-\left(\sum_{i=1}^{n}p_i\left(1-E^{-}(z_i)^q\right)^{1/q}\right)^q\right)^{1/q}\\
&= \left(1-\left(p_{j*}\left(1-E^{-}\left(z_{j*}\right)^q\right)^{1/q}\right)^q\right)^{1/q}\\
&= E^{-}\left(z_{j*}\right)
\end{aligned}
$$

Above all, we see that $\langle E^{+}(z), E^{-}(z)\rangle = \langle E^{+}(z_{j*}), E^{-}(z_{j*})\rangle$. Obviously, the $\langle E^{+}(z), E^{-}(z)\rangle$ is also an orthopair fuzzy grade. □

Example 1. *Given an orthopair fuzzy set* $Z = \left\{\langle z_1, 1, 0\rangle_q, \langle z_2, 0.4, 0.3\rangle_q, \langle z_3, 0.8, 0.5\rangle_q\right\}$. *Here, we see that*

$$(1)^{q_1} + (0)^{q_1} \leq 1 \, for \; q_1 \geq 1$$

$$(0.4)^{q_2} + (0.3)^{q_2} \leq 1 \, for \; q_2 \geq 1$$

$$(0.8)^{q_3} + (0.5)^{q_3} \leq 1 \, for \; q_3 \geq 2$$

Thus, we see that $q = Max(q_1, q_2, q_3) = 2$. *Further more, we see that*

$$\alpha(1) = 1 \, , \, \alpha(2) = 3 \, , \, \alpha(3) = 2$$

$$\vartheta(1) = 1 \, , \, \vartheta(2) = 2 \, , \, \vartheta(3) = 3$$

Here, we see that

$$G_1 = \{z_1\} \, , \, G_2 = \{z_1, z_3\} \, , \, G_3 = \{Z\}$$

$$K_1 = \{z_1\} \, , \, K_2 = \{z_1, z_2\} \, , \, K_3 = \{Z\}$$

On space $Z = \{z_1, z_2, z_3\}$, *assume* μ *is a measure such that*
From Table 1, we know that

$$\mu(G_0) = 0 \, , \, \mu(G_1) = 0.5 \, , \, \mu(G_2) = 0.8 \, , \, \mu(G_3) = 1$$

$$\mu(K_0) = 0 \, , \, \mu(K_1) = 0.5 \, , \, \mu(K_2) = 0.7 \, , \, \mu(K_3) = 1$$

Based on Equations (15) and (16), one has the following equation:

$$\begin{aligned}
E^{+}(x) &= Choq_{\mu}\left(E^{+}(z_1), \ldots, E^{+}(z_n)\right) \\
&= \sum_{j=1}^{3}\left(f\left(\mu\left(G_j\right)\right) - f\left(\mu\left(G_{j-1}\right)\right)\right) E^{+}\left(z_{\alpha(j)}\right) \\
&= \sum_{j=1}^{3}\left(\mu\left(G_j\right) - \mu\left(G_{j-1}\right)\right) E^{+}\left(z_{\alpha(j)}\right) \\
&= (0.5)(1) + (0.3)(0.8) + (0.2)(0.4) \\
&= 0.82
\end{aligned}$$

$$
\begin{aligned}
E^{-}(x) &= \left(1-\left(\sum_{j=1}^{3}\left(f\left(\mu\left(K_j\right)\right)-f\left(\mu\left(K_{j-1}\right)\right)\right)\left(1-\left(E^{-}\left(z_{\vartheta(j)}\right)\right)^2\right)^{1/2}\right)^2\right)^{1/2} \\
&= \left(1-\left(\sum_{j=1}^{3}\left(\mu\left(K_j\right)-\mu\left(K_{j-1}\right)\right)\left(1-\left(E^{-}\left(z_{\vartheta(j)}\right)\right)^2\right)^{1/2}\right)^2\right)^{1/2} \\
&= \left(1-\left((0.5)\left(1-0^2\right)^{1/2}+(0.2)\left(1-0.3^2\right)^{1/2}+(0.3)\left(1-0.5^2\right)^{1/2}\right)^2\right)^{1/2} \\
&= 0.31
\end{aligned}
$$

Hence, $\langle E^{+}(z), E^{-}(z)\rangle = \langle 0.82, 0.31\rangle$. *We see that* $(0.82)^2 + (0.31)^2 < 1$. *Finally, we get an orthopair fuzzy grade by the proposed model with* $q = 2$.

The proposed model requires an alternative in the decision-making process as an orthopair fuzzy set. Each of the orthopair fuzzy grades of orthopair fuzzy set as a criteria, which consists of the hesitancy degree, non-membership degree and membership degree. Then, the Equations (15) and (16) are used to aggregate all orthopair fuzzy grades of an orthopair fuzzy set and get a orthopair fuzzy grade. Finally the Equation (14) is used to get a degree of the orthopair fuzzy grade, which can represent the calculation of refined expected value of this alternative.

Table 1. The information about this measure.

T	ϕ	$\{z_1\}$	$\{z_2\}$	$\{z_3\}$	$\{z_1, z_2\}$	$\{z_1, z_3\}$	$\{z_2, z_3\}$	$\{Z\}$
$\mu(T)$	0	0.5	0.3	0.4	0.7	0.8	0.8	1

4. Study Case

Consider the following: a library wants to purchase a coffee machine. The manager has to decide what kind should be chosen. Now, assume there are several possible choices $E_i\,(i = 1,2,3,4,5)$ can be chosen by the library and three attributes $y_1\,(price)$, $y_2\,(function)$ and $y_3\,(appearance)$ are taken into account. According to the experts, E_4 is the best choice. Now, we use the refined expected value decision rules [40] to make the decision.

Assume fuzzy sets will represent the alternatives $E_i\,(i = 1,2,3,4,5)$ as follows:

$$E_i = \left\langle y_j, E_i\left(y_j\right)\right\rangle \text{ and } j = 1 \text{ to } 3$$

where, $E_i\left(y_j\right)$ represent the degree to which E_i satisfies to y_j.

Then, according to the previous description, the five alternatives with fuzzy sets transformed into matrix as follows:

$$
E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} \langle y_1, 0.4\rangle & \langle y_2, 0.8\rangle & \langle y_3, 0.5\rangle \\ \langle y_1, 0.5\rangle & \langle y_2, 0.3\rangle & \langle y_3, 0.6\rangle \\ \langle y_1, 0.9\rangle & \langle y_2, 0.8\rangle & \langle y_3, 0.3\rangle \\ \langle y_1, 0.7\rangle & \langle y_2, 0.1\rangle & \langle y_3, 0.4\rangle \\ \langle y_1, 0.9\rangle & \langle y_2, 0.4\rangle & \langle y_3, 0.6\rangle \end{bmatrix}
$$

Furthermore, we see that matrices consists of indexes of five alternatives.

$$\alpha=\begin{bmatrix}\alpha_{11} & \alpha_{12} & \alpha_{13}\\ \alpha_{21} & \alpha_{22} & \alpha_{23}\\ \alpha_{31} & \alpha_{32} & \alpha_{33}\\ \alpha_{41} & \alpha_{42} & \alpha_{43}\\ \alpha_{51} & \alpha_{52} & \alpha_{53}\end{bmatrix}=\begin{bmatrix}2 & 3 & 1\\ 3 & 1 & 2\\ 1 & 2 & 3\\ 1 & 3 & 2\\ 1 & 3 & 2\end{bmatrix}$$

The probability of this matrix is as follows:

$$\begin{aligned}
&Prob\,(E_1=y_1)=0.5\,,\,Prob\,(E_1=y_2)=0.4\,,\,Prob\,(E_1=y_3)=0.1\\
&Prob\,(E_2=y_1)=0.2\,,\,Prob\,(E_2=y_2)=0.4\,,\,Prob\,(E_2=y_3)=0.4\\
&Prob\,(E_3=y_1)=0.2\,,\,Prob\,(E_3=y_2)=0.3\,,\,Prob\,(E_3=y_3)=0.5\\
&Prob\,(E_4=y_1)=0.6\,,\,Prob\,(E_4=y_2)=0.2\,,\,Prob\,(E_4=y_3)=0.2\\
&Prob\,(E_5=y_1)=0.3\,,\,Prob\,(E_5=y_2)=0.4\,,\,Prob\,(E_5=y_3)=0.3
\end{aligned}$$

Then, the probabilities of subsets of y_i as follows:

$$\begin{aligned}
&Prob_1\,(H_1)=0.4\,,\,Prob_1\,(H_2)=0.5\,,\,Prob_1\,(H_3)=1\\
&Prob_2\,(H_1)=0.4\,,\,Prob_2\,(H_2)=0.6\,,\,Prob_2\,(H_3)=1\\
&Prob_3\,(H_1)=0.2\,,\,Prob_3\,(H_2)=0.5\,,\,Prob_3\,(H_3)=1\\
&Prob_4\,(H_1)=0.6\,,\,Prob_4\,(H_2)=0.8\,,\,Prob_4\,(H_3)=1\\
&Prob_5\,(H_1)=0.3\,,\,Prob_5\,(H_2)=0.6\,,\,Prob_5\,(H_3)=1
\end{aligned}$$

Hence, we get the refined expected values of these alternatives as follows:

$$\begin{aligned}
EV_r\,(E_1)&=\sum_{i=1}^{3}\left((Prob_1\,(H_i))^r-(Prob_1\,(H_{i-1}))^r\right)y_i\\
&=\left((Prob_1\,(H_1))^r\right)y_2+\left((Prob_1\,(H_2))^r-(Prob_1\,(H_1))^r\right)y_3+\left((Prob_1\,(H_3))^r-(Prob_1\,(H_2))^r\right)y_1\\
&=(0.4)\,0.8+((0.5)-(0.4))\,0.5+((1)-(0.5))\,0.4\\
&=0.57
\end{aligned}$$

The same is true:

$$EV_r\,(E_2)=0.46\,,\,EV_r\,(E_3)=0.57\,,\,EV_r\,(E_4)=0.52\,,\,EV_r\,(E_5)=0.61$$

From the discussion above, one get that $V\,(E_5)>V\,(E_1)=V\,(E_3)>V\,(E_4)>V\,(E_2)$. Then, we can find the choice E_5 is the best choice in this problem. It means that the refined expected value decision rules can not make a true decision in this issue.

Now, we use the refined expected value decision rules under orthopair fuzzy environment to make the decision. Assume orthopair fuzzy sets will represent the alternatives $E_i\,(i=1,2,3,4,5)$ as follows:

$$E_i=\left\langle y_j,E_i^+\left(y_j\right),E_i^-\left(y_j\right)\right\rangle_q \text{ and } j=1 \text{ to } 3$$

where, $E_i^+\left(y_j\right)$ and $E_i^-\left(y_j\right)$ represent the degree to which E_i satisfies to y_j and that to which $E_i\,(i=1,2,3,4,5)$ does not satisfy y_j.

Then, according to the previous description, the five alternatives with orthopair fuzzy sets transformed into a matrix as follows:

$$E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} \langle y_1, 0.7, 0.5 \rangle & \langle y_2, 0.6, 0.2 \rangle & \langle y_3, 0.8, 0.4 \rangle \\ \langle y_1, 0.3, 0.3 \rangle & \langle y_2, 0.6, 0.2 \rangle & \langle y_3, 0.8, 0.1 \rangle \\ \langle y_1, 0.4, 0.6 \rangle & \langle y_2, 0.7, 0.4 \rangle & \langle y_3, 0.9, 0.2 \rangle \\ \langle y_1, 0.8, 0.6 \rangle & \langle y_2, 0.7, 0.1 \rangle & \langle y_3, 0.9, 0.4 \rangle \\ \langle y_1, 0.9, 0.3 \rangle & \langle y_2, 0.8, 0.2 \rangle & \langle y_3, 0.1, 0.7 \rangle \end{bmatrix}$$

It is easy to get that $q = 2$. Further more, we see that matrices consists of indexes of five alternatives.

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\vartheta = \begin{bmatrix} \vartheta_{11} & \vartheta_{12} & \vartheta_{13} \\ \vartheta_{21} & \vartheta_{22} & \vartheta_{23} \\ \vartheta_{31} & \vartheta_{32} & \vartheta_{33} \\ \vartheta_{41} & \vartheta_{42} & \vartheta_{43} \\ \vartheta_{51} & \vartheta_{52} & \vartheta_{53} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

On $Y = \{y_1, y_2, y_3\}$, assume μ is a measure such that

$$\begin{aligned} &\mu(\theta) = 0 \,,\, \mu(y_1, y_2) = 0.5 \\ &\mu(y_1) = 0.2 \,,\, \mu(y_1, y_3) = 0.4 \\ &\mu(y_2) = 0.4 \,,\, \mu(y_2, y_3) = 0.6 \\ &\mu(y_3) = 0.3 \,,\, \mu(Y) = 1 \end{aligned}$$

Here, relying on the matrix α and ϑ and μ, we see that

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \\ G_{41} & G_{42} & G_{43} \\ G_{51} & G_{52} & G_{53} \end{bmatrix} = \begin{bmatrix} \{y_2\} & \{y_2, y_3\} & \{Y\} \\ \{y_3\} & \{y_2, y_3\} & \{Y\} \\ \{y_3\} & \{y_2, y_3\} & \{Y\} \\ \{y_2\} & \{y_2, y_3\} & \{Y\} \\ \{y_1\} & \{y_1, y_2\} & \{Y\} \end{bmatrix}$$

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \\ K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \end{bmatrix} = \begin{bmatrix} \{y_2\} & \{y_2, y_3\} & \{Y\} \\ \{y_3\} & \{y_2, y_3\} & \{Y\} \\ \{y_3\} & \{y_2, y_3\} & \{Y\} \\ \{y_2\} & \{y_2, y_3\} & \{Y\} \\ \{y_2\} & \{y_1, y_2\} & \{Y\} \end{bmatrix}$$

Relying on Equations (15) and (16), one has the following equation:

$$\begin{aligned} E = \{ &\langle E_1, 0.67, 0.388 \rangle, \langle E_2, 0.54, 0.226 \rangle, \langle E_3, 0.64, 0.458 \rangle, \\ &\langle E_4, 0.77, 0.433 \rangle, \langle E_5, 0.47, 0.537 \rangle \} \end{aligned}$$

Relying on Equation (14) and assuming λ is 0.5, one has the following results.

From Table 2, one get that $V(E_4) > V(E_2) > V(E_1) > V(E_3) > V(E_5)$. Then, we can find that E_4 is the best choice in this problem. It means that the proposed model is validity in issue of decision making and that refined expected value decision rules under orthopair fuzzy environment is more efficient than the refined expected value decision rules.

Table 2. The information about this measure.

x	E_1	E_2	E_3	E_4	E_5
$V(x)$	0.641	0.667	0.593	0.676	0.467

5. Conclusions

This paper proposes the refined expected value decision rules under orthopair fuzzy environment. The proposed model requires an alternative in the decision-making process as an orthopair fuzzy set. Each of the orthopair fuzzy grades of this orthopair fuzzy set as a criteria, which consists of the membership degree, the non-membership degree and hesitancy degree. Then, some equations are used to aggregate all orthopair fuzzy grades of an orthopair fuzzy set and get a orthopair fuzzy grade. Finally the Equation (14) is used to get a degree of the orthopair fuzzy grade, which can represent the calculation of refined expected value of this alternative. The proposed model applies the refined expected value decision rules on decision making that is described by orthopair fuzzy environment, which means that the proposed model can enlarge the applied scope of the classical refined expected value decision rules. In conclusion, the proposed model can apply the refined expected value decision rules on orthopair fuzzy environment. Numerical examples verify the availability and flexibility of the proposed model.

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