Supplementary

Below, tableaus for the proposed families of methods (with s = 2, 3, 4) are provided in the form of analytical expressions. Tables include:

- Tables S1-S3: tableaus for Gs|Gs method;
- Tables S4-S6: tableaus for Gs|Gs+1 method;
- Tables S7-S9: tableaus for Ls|Ls method;
- Tables S10-S12: tableaus for Ls|Ls+1 method;
- Tables S13-S15: tableaus for Ls|Gs+1 method;
- Tables S16-S18: tableaus for eLs|Gs method;
- Tables S19-S21: tableaus for eLs|Gs+1 method.

Table S1. G2|G2 method.

π_1	π_2	π_1	π_2
π_2	π_1	π_2	π_1
$1/4$	α_2		
α_1	$1/4$		
$1/2$	$1/2$	$2\,\pi_2$	$2\,\pi_1$

$\pi_{1,2} = \frac{1}{12}\left(3 \pm \sqrt{3}\right)$, $\alpha_{1,2} = \frac{1}{12}\left(3 \pm 2\sqrt{3}\right)$

Table S2. G3|G3 method.

π_1	0	π_2	π_1	0	π_2
$1/9$	$4/9$	$1/9$	$1/9$	$4/9$	$1/9$
π_2	0	π_1	π_2	0	π_1
$5/36$	α_2	α_2'			
α_1''	$2/9$	α_2''			
α_1'	α_1	$5/36$			
$5/18$	$4/9$	$5/18$	$(5-\sqrt{15})/10$	$1/2$	$(5+\sqrt{15})/10$

$\pi_{1,2} = \frac{1}{36}\left(3 \pm \sqrt{15}\right)$, $\alpha_{1,2} = \frac{2}{9} \pm \frac{1}{\sqrt{15}}$, $\alpha'_{1,2} = \frac{5}{36} \pm \frac{1}{2\sqrt{15}}$, $\alpha''_{1,2} = \frac{1}{72}\left(10 \pm 3\sqrt{15}\right)$

Table S3. G4|G4 method.

$\pi_{1\|1\|1}$	$\pi_{2\|2\|2}$	$\pi_{2\|1\|2}$	$\pi_{1\|2\|1}$	$\pi_{1\|1\|1}$	$\pi_{2\|2\|2}$	$\pi_{2\|1\|2}$	$\pi_{1\|2\|1}$
$\pi'_{2\|1\|2}$	$\pi'_{1\|1\|1}$	$\pi'_{1\|2\|1}$	$\pi'_{2\|2\|2}$	$\pi'_{2\|1\|2}$	$\pi'_{1\|1\|1}$	$\pi'_{1\|2\|1}$	$\pi'_{2\|2\|2}$
$\pi'_{2\|2\|2}$	$\pi'_{1\|2\|1}$	$\pi'_{1\|1\|1}$	$\pi'_{2\|1\|2}$	$\pi'_{2\|2\|2}$	$\pi'_{1\|2\|1}$	$\pi'_{1\|1\|1}$	$\pi'_{2\|1\|2}$
$\pi_{1\|2\|1}$	$\pi_{2\|1\|2}$	$\pi_{2\|2\|2}$	$\pi_{1\|1\|1}$	$\pi_{1\|2\|1}$	$\pi_{2\|1\|2}$	$\pi_{2\|2\|2}$	$\pi_{1\|1\|1}$
$\frac{1}{24}(3-\sqrt{5/6})$	$\alpha_{1\|2\|2}$	$\alpha_{2\|1\|2}$	$\alpha''_{2\|2\|2}$				
$\alpha'_{1\|2\|1\|1}$	α'''_1	$\alpha''_{1\|2\|1}$	$\alpha'_{2\|2\|2\|1}$				
$\alpha'_{1\|1\|1\|2}$	$\alpha''_{1\|1\|1}$	α'''_1	$\alpha'_{2\|1\|2\|2}$				
$\alpha''_{2\|1\|2}$	$\alpha_{1\|2\|1}$	$\alpha_{2\|1\|1}$	$\frac{1}{24}(3-\sqrt{5/6})$				
$2\,\alpha'''_2$	$2\,\alpha'''_1$	$2\,\alpha'''_1$	$2\,\alpha'''_2$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\pi_{1,2|1,2|1,2} = \frac{1}{720}\left(15 \pm 5\sqrt{30} \pm \sqrt{15\left(45 \pm 8\sqrt{30}\right)}\right),\ \pi'_{1,2|1,2|1,2} = \frac{1}{144}\left(15 \pm 2\sqrt{30} \pm \sqrt{225 \pm 30\sqrt{30}}\right),$$

$$\alpha_{1,2|1,2|1,2} = \frac{630+35\sqrt{30}\pm 60\sqrt{45+6\sqrt{30}}\pm 39\sqrt{150+20\sqrt{30}}\pm 3\sqrt{70(585+106\sqrt{30})}}{5040},\ \alpha'_{1,2|1,2|1,2|1,2} = \frac{\pm 39\pm 6\sqrt{14}\pm 6\sqrt{30}\pm 3\sqrt{105}+7\sqrt{75+4\sqrt{30}}}{24\sqrt{630+84\sqrt{30}}},$$

$$\alpha''_{1,2|1,2|1,2} = \frac{1}{8} \pm \frac{\sqrt{\frac{5}{6}}}{24} \pm \frac{1}{24}\sqrt{\frac{90}{7} \pm \frac{52\sqrt{\frac{6}{5}}}{7}},\ \alpha'''_{1,2} = \frac{1}{144}\left(18 \pm \sqrt{30}\right),\ \gamma_{1,2|1,2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{35}\left(15 \pm 2\sqrt{30}\right)}$$

Table S4. G2|G3 method.

π_1	π_2	ω_1	$2/9$	ω_2
π_2	π_1	ω_2	$2/9$	ω_1
$\alpha_{1\|2}$	$\alpha_{2\|1}$			
$(2+\sqrt{3})/8$	$(2-\sqrt{3})/8$			
$\alpha_{1\|1}$	$\alpha_{2\|1}$			
$1/2$	$1/2$	$(5-\sqrt{15})/10$	$1/2$	$(5+\sqrt{15})/10$

$$\pi_{1,2} = \frac{1}{12}\left(3 \pm \sqrt{3}\right),\ \omega_{1,2} = \frac{1}{36}(5 \pm \sqrt{15}),\ \alpha_{1,2|1,2} = \frac{1}{20}(5 \pm \sqrt{3} \pm \sqrt{15})$$

Table S5. G3|G4 method.

π_1	0	π_2	$\omega_{1\|1\|1}$	$\omega_{2\|1\|2}$	$\omega_{2\|2\|2}$	$\omega_{1\|2\|1}$
$1/9$	$4/9$	$1/9$	ω'_2	ω'_1	ω'_1	ω'_2
π_2	0	π_1	$\omega_{1\|2\|1}$	$\omega_{2\|2\|2}$	$\omega_{2\|1\|2}$	$\omega_{1\|1\|1}$
$\alpha_{1\|2\|2\|1}$	$\alpha'_{2\|1}$	$\alpha_{2\|1\|2\|1}$				
$\alpha_{1\|1\|2\|2}$	$\alpha'_{2\|2}$	$\alpha_{2\|2\|2\|2}$				
$\alpha_{1\|1\|1\|2}$	$\alpha'_{1\|2}$	$\alpha_{2\|2\|1\|2}$				
$\alpha_{1\|2\|1\|1}$	$\alpha'_{1\|1}$	$\alpha_{2\|1\|1\|1}$				
$5/18$	$4/9$	$5/18$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\pi_{1,2} = \frac{1}{36}\left(3 \pm \sqrt{15}\right),\ \omega_{1,2|1,2|1,2} = \frac{1}{720}\left(30 \pm 3\sqrt{30} \pm \sqrt{30\left(75 \pm 4\sqrt{30}\right)}\right),\ \omega'_{1|2} = \frac{1}{90}\left(15 \pm 2\sqrt{30}\right), \tag{30}$$

$$\alpha_{1,2|1,2|1,2|1,2} = \frac{5}{36} \pm \frac{\sqrt{\frac{5}{3}}}{14} \pm \frac{1}{14\sqrt{2}} \pm \frac{1}{84}\sqrt{\frac{1}{21}\left(585 \pm 106\sqrt{30}\right)},\ \delta'_{1,2|1,2} = \frac{2}{9} \pm \frac{1}{21}\sqrt{\frac{2}{105}\left(1035 \pm 82\sqrt{30}\right)}, \tag{31}$$

$$\gamma_{1,2|1,2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{35}\left(15 \pm 2\sqrt{30}\right)} \tag{32}$$

Table S6. G4|G5 method. (33)

$\pi_{1\|1\|1}$	$\pi_{2\|2\|2}$	$\pi_{2\|1\|2}$	$\pi_{1\|2\|1}$	ω_1	ω'_1	$-8/225$	ω'_2	ω_2
$\pi'_{2\|1\|2}$	$\pi'_{1\|1\|1}$	$\pi'_{1\|2\|1}$	$\pi'_{2\|2\|2}$	$\omega''_{2\|1\|2}$	$\omega''_{1\|1\|1}$	$8/45$	$\omega''_{1\|2\|1}$	$\omega''_{2\|2\|2}$
$\pi'_{2\|2\|2}$	$\pi'_{1\|2\|1}$	$\pi'_{1\|1\|1}$	$\pi'_{2\|1\|2}$	$\omega''_{2\|2\|2}$	$\omega''_{1\|2\|1}$	$8/45$	$\omega''_{1\|1\|1}$	$\omega''_{2\|1\|2}$
$\pi_{1\|2\|1}$	$\pi_{2\|1\|2}$	$\pi_{2\|2\|2}$	$\pi_{1\|1\|1}$	ω_2	ω'_2	$-8/225$	ω'_1	ω_1
$\alpha 0_{2\|1} \cdot A11$	$C11$	$C21$	$\alpha 0_{2\|1} \cdot A21$					
$\alpha 0_{2\|2} \cdot A12$	$C12$	$C22$	$\alpha 0_{2\|2} \cdot A22$					
$\alpha'_{2\|1\|2}$	$\alpha'_{1\|1\|1}$	$\alpha'_{1\|2\|1}$	$\alpha'_{2\|2\|2}$					
$\alpha 0_{1\|2} \cdot A13$	$C13$	$C23$	$\alpha 0_{1\|2} \cdot A23$					
$\alpha 0_{1\|1} \cdot A14$	$C14$	$C24$	$\alpha 0_{1\|1} \cdot A24$					
ϵ_2	ϵ_1	ϵ_1	ϵ_2	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$1/2$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\pi_{1,2|1,2|1,2} = \frac{1}{720}\left(15 \pm 5\sqrt{30} \pm \sqrt{15\left(45 \pm 8\sqrt{30}\right)}\right)\ ,\ \pi'_{1,2|1,2|1,2} = \frac{1}{144}\left(15 \pm 2\sqrt{30} \pm \sqrt{225 \pm 30\sqrt{30}}\right), \tag{34}$$

$$\omega_{1,2} = \frac{749+49\sqrt{70} \pm \sqrt{568155+63924\sqrt{70}}}{25200},\ \omega'_{1,2} = \frac{749+49\sqrt{70} \pm \sqrt{568155+63924\sqrt{70}}}{25200}, \tag{35}$$

$$\omega''_{1,2|1,2|1,2} = \frac{1}{720}\left(43 \pm 4\sqrt{70} \pm 5\sqrt{177 \pm 678\sqrt{\frac{2}{35}}}\right), \tag{36}$$

$$\alpha 0_{1,2|1,2} = 21 \pm \sqrt{7\left(35 \pm 2\sqrt{70}\right)},\ A = 140\sqrt{30+4\sqrt{30}},\ B = 34\sqrt{525+70\sqrt{30}} + 108\sqrt{630+84\sqrt{30}}, \tag{37}$$

$$A11 = \left(A + B + 2310 + 378\sqrt{30} + 105\sqrt{70} + 210\sqrt{105+8\sqrt{21}+14\sqrt{30}+6\sqrt{70}} + 110\sqrt{7\left(35+2\sqrt{70}\right)} + 35\sqrt{350+20\sqrt{70}} + 18\sqrt{210\left(35+2\sqrt{70}\right)}\right), \tag{38}$$

$$A12 = \left(-A + B + 2310 + 378\sqrt{30} - 105\sqrt{70} + 210\sqrt{105-8\sqrt{21}+14\sqrt{30}-6\sqrt{70}} + 110\sqrt{7\left(35-2\sqrt{70}\right)} - 35\sqrt{10\left(35-2\sqrt{70}\right)} + 18\sqrt{210\left(35-2\sqrt{70}\right)}\right), \tag{39}$$

$$A13 = \left(-A + B + 2310 + 378\sqrt{30} - 105\sqrt{70} - 210\sqrt{105-8\sqrt{21}+14\sqrt{30}-6\sqrt{70}} - 110\sqrt{7\left(35-2\sqrt{70}\right)} + 35\sqrt{10\left(35-2\sqrt{70}\right)} - 18\sqrt{210\left(35-2\sqrt{70}\right)}\right), \tag{40}$$

$$A14 = \left(A + B + 2310 + 378\sqrt{30} + 105\sqrt{70} - 210\sqrt{105+8\sqrt{21}+14\sqrt{30}+6\sqrt{70}} - 110\sqrt{7\left(35+2\sqrt{70}\right)} - 35\sqrt{350+20\sqrt{70}} - 18\sqrt{210\left(35+2\sqrt{70}\right)}\right), \tag{41}$$

$$A21 = \left(A + B - 2310 - 378\sqrt{30} - 105\sqrt{70} + 210\sqrt{105+8\sqrt{21}+14\sqrt{30}+6\sqrt{70}} - 110\sqrt{7\left(35+2\sqrt{70}\right)} - 35\sqrt{350+20\sqrt{70}} - 18\sqrt{210\left(35+2\sqrt{70}\right)}\right), \tag{42}$$

$$A22 = \left(-A + B - 2310 - 378\sqrt{30} + 105\sqrt{70} + 210\sqrt{105-8\sqrt{21}+14\sqrt{30}-6\sqrt{70}} - 110\sqrt{7\left(35-2\sqrt{70}\right)} + 35\sqrt{10\left(35-2\sqrt{70}\right)} - 18\sqrt{210\left(35-2\sqrt{70}\right)}\right), \tag{43}$$

$$A23 = \left(-A + B - 2310 - 378\sqrt{30} + 105\sqrt{70} - 210\sqrt{105-8\sqrt{21}+14\sqrt{30}-6\sqrt{70}} + 110\sqrt{7\left(35-2\sqrt{70}\right)} - 35\sqrt{10\left(35-2\sqrt{70}\right)} + 18\sqrt{210\left(35-2\sqrt{70}\right)}\right), \tag{44}$$

$$A24 = \left(A + B - 2310 - 378\sqrt{30} - 105\sqrt{70} - 210\sqrt{105+8\sqrt{21}+14\sqrt{30}+6\sqrt{70}} + 110\sqrt{7\left(35+2\sqrt{70}\right)} + 35\sqrt{350+20\sqrt{70}} + 18\sqrt{210\left(35+2\sqrt{70}\right)}\right), \tag{45}$$

$$G=\frac{5}{486}\sqrt{\frac{1}{7}\left(15+2\sqrt{30}\right)},\ H=\frac{1}{36}\sqrt{\frac{5}{42}\left(15+2\sqrt{30}\right)},\ F=\frac{17\sqrt{150+20\sqrt{30}}}{3888},\ J=\frac{1}{108}\sqrt{45+6\sqrt{30}},\tag{46}$$

$$C11=\frac{1}{8}-H+G+J-F-\frac{1}{24}\sqrt{\frac{1}{7}(35+2\sqrt{70})}+\frac{5\sqrt{105+6\sqrt{70}}}{1944}-\frac{1}{648}\sqrt{\frac{5}{21}\left(11809+529\sqrt{70}-621\sqrt{7\left(35+2\sqrt{70}\right)}\right)},\tag{47}$$

$$C12=\frac{1}{8}+\frac{\sqrt{\frac{5}{6}}}{24}+H-G+J-F-\frac{23}{648}\sqrt{\frac{5}{42}(35-2\sqrt{70})}-\frac{1}{24}\sqrt{\frac{1}{7}(35-2\sqrt{70})}-\frac{5}{648}\sqrt{\frac{1}{3}\left(35-2\sqrt{70}\right)},\tag{48}$$

$$C13=\frac{1}{8}+\frac{\sqrt{\frac{5}{6}}}{24}+H-G+J-F+\frac{23}{648}\sqrt{\frac{5}{42}(35-2\sqrt{70})}+\frac{1}{24}\sqrt{\frac{1}{7}(35-2\sqrt{70})}+\frac{5}{648}\sqrt{\frac{1}{3}\left(35-2\sqrt{70}\right)},\tag{49}$$

$$C14=\frac{1}{8}-H+G+J-F+\frac{1}{24}\sqrt{\frac{1}{7}\left(35+2\sqrt{70}\right)}-\frac{5\sqrt{105+6\sqrt{70}}}{1944}+\frac{1}{648}\sqrt{\frac{5}{21}\left(11809+529\sqrt{70}+621\sqrt{7\left(35+2\sqrt{70}\right)}\right)},\tag{50}$$

$$C21=\frac{1}{8}+H-G-J+F-\frac{1}{24}\sqrt{\frac{1}{7}(35+2\sqrt{70})}+\frac{5\sqrt{105+6\sqrt{70}}}{1944}-\frac{1}{648}\sqrt{\frac{5}{21}\left(11809+529\sqrt{70}-621\sqrt{7\left(35+2\sqrt{70}\right)}\right)},\tag{51}$$

$$C22=\frac{1}{8}+\frac{\sqrt{\frac{5}{6}}}{24}-H+G-J+F-\frac{23}{648}\sqrt{\frac{5}{42}(35-2\sqrt{70})}-\frac{1}{24}\sqrt{\frac{1}{7}(35-2\sqrt{70})}-\frac{5}{648}\sqrt{\frac{1}{3}\left(35-2\sqrt{70}\right)},\tag{52}$$

$$C23=\frac{1}{8}+\frac{\sqrt{\frac{5}{6}}}{24}-H+G-J+F+\frac{23}{648}\sqrt{\frac{5}{42}(35-2\sqrt{70})}+\frac{1}{24}\sqrt{\frac{1}{7}(35-2\sqrt{70})}+\frac{5}{648}\sqrt{\frac{1}{3}\left(35-2\sqrt{70}\right)},\tag{53}$$

$$C24=\frac{1}{8}+H-G-J+F+\frac{1}{24}\sqrt{\frac{1}{7}\left(35+2\sqrt{70}\right)}-\frac{5\sqrt{105+6\sqrt{70}}}{1944}+\frac{1}{648}\sqrt{\frac{5}{21}\left(11809+529\sqrt{70}+621\sqrt{7\left(35+2\sqrt{70}\right)}\right)},\tag{54}$$

$$\alpha'_{1,2|1,2|2}=\frac{144\pm8\sqrt{30}\pm3\sqrt{2070\pm164\sqrt{30}}}{1152},\ \epsilon_{1,2}=\frac{1}{72}(18\pm\sqrt{30}),\ \gamma_{1,2|1,2}=\frac{1}{2}\pm\frac{1}{6}\sqrt{5\pm2\sqrt{\frac{10}{7}}}\tag{55}$$

Table S7. L2|L2 method. 56

1/3	1/6	1/3	1/6
1/6	1/3	1/6	1/3
0	0		
1/2	1/2		
1/2	1/2	0	1

Table S8. L3|L3 method. 57

2/15	1/15	−1/30	2/15	1/15	−1/30
1/15	8/15	1/15	1/15	8/15	1/15
−1/30	1/15	2/15	−1/30	1/15	2/15
0	0	0			
5/25	1/3	−1/24			
1/6	2/3	1/6			
1/6	2/3	1/6	0	1/2	1

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Table S9. L4|L4 method.

$1/14$	$\sqrt{5}/84$	$-\sqrt{5}/84$	$1/84$	$1/14$	$\sqrt{5}/84$	$-\sqrt{5}/84$	$1/84$
$\sqrt{5}/84$	$5/14$	$5/84$	$-\sqrt{5}/84$	$\sqrt{5}/84$	$5/14$	$5/84$	$-\sqrt{5}/84$
$-\sqrt{5}/84$	$5/84$	$5/14$	$\sqrt{5}/84$	$-\sqrt{5}/84$	$5/84$	$5/14$	$\sqrt{5}/84$
$1/84$	$-\sqrt{5}/84$	$\sqrt{5}/84$	$1/14$	$1/84$	$-\sqrt{5}/84$	$\sqrt{5}/84$	$1/14$
0	0	0	0				
α_1	α_2'	α_2''	α_1'''				
α_2	α_1''	α_1'	α_2'''				
$1/12$	$5/12$	$5/12$	$1/12$				
$1/12$	$5/12$	$5/12$	$1/12$	0	$(5-\sqrt{5})/10$	$(5+\sqrt{5})/10$	1

$\alpha_{1,2} = \frac{1}{120}\left(11 \pm \sqrt{5}\right)$, $\alpha'_{1,2} = \frac{1}{120}\left(25 \pm \sqrt{5}\right)$, $\alpha''_{1,2} = \frac{1}{120}\left(25 \pm 13\sqrt{5}\right)$, $\alpha'''_{1,2} = \frac{1}{120}\left(-1 \pm \sqrt{5}\right)$

Table S10. L2|L3 method.

$1/3$	$1/6$	$1/6$	$1/3$	0
$1/6$	$1/3$	0	$1/3$	$1/6$
0	0			
$3/8$	$1/8$			
$1/2$	$1/2$			
$1/2$	$1/2$	0	$1/2$	1

Table S11. L3|L4 method.

$2/15$	$1/15$	$-1/30$	$1/12$	ω_1	ω_2	0
$1/15$	$8/15$	$1/15$	0	$1/3$	$1/3$	0
$-1/30$	$1/15$	$2/15$	0	ω_2	ω_1	$1/12$
0	0	0				
α_2	α_2'	α_2''				
α_1	α_1'	α_1''				
$1/6$	$2/3$	$1/6$				
$1/6$	$2/3$	$1/6$	0	$(5-\sqrt{5})/10$	$(5+\sqrt{5})/10$	1

$\omega_{1,2} = \frac{1}{24}(1 \pm \sqrt{5})$, $\alpha_{1,2} = \frac{1}{300}(55 \pm \sqrt{5})$, $\alpha'_{1,2} = \frac{1}{75}(25 \pm 7\sqrt{5})$, $\alpha''_{1,2} = \frac{1}{300}(-5 \pm \sqrt{5})$

Table S12. L4|L5 method.

$1/14$	$\sqrt{5}/84$	$-\sqrt{5}/84$	$1/84$	$1/20$	ω_1	$-2/45$	ω_2	0
$\sqrt{5}/84$	$5/14$	$5/84$	$-\sqrt{5}/84$	0	ω_1'	$2/9$	ω_2'	0
$-\sqrt{5}/84$	$5/84$	$5/14$	$\sqrt{5}/84$	0	ω_2'	$2/9$	ω_1'	0
$1/84$	$-\sqrt{5}/84$	$\sqrt{5}/84$	$1/14$	0	ω_2	$-2/45$	ω_1	$1/20$
0	0	0	0					
α_1	$\alpha_{1\|2}'$	$\alpha_{2\|2}'$	α_1'''					
$17/192$	α_1''	α_2''	$-1/192$					
α_2	$\alpha_{1\|1}'$	$\alpha_{2\|1}'$	α_2'''					
$1/12$	$5/12$	$5/12$	$1/12$					
$1/12$	$5/12$	$5/12$	$1/12$	0	γ_2	$1/2$	γ_1	$\gamma_{1\|1}$

$$\omega_{1,2} = \frac{1}{180}(7+\sqrt{21}),\ \omega_{1,2}' = \frac{1}{72}(7+\sqrt{105}),\ \gamma_{1,2}' = \frac{1}{14}(7\pm\sqrt{21}) \tag{67}$$

$$\alpha_{1,2} = \frac{103+3\sqrt{21}}{1176},\ \alpha_{1,2|1,2}' = \frac{5(49\pm6\sqrt{5}\pm9\sqrt{21})}{1176},\ \alpha_{1,2}'' = \frac{5}{192}(8\pm3\sqrt{5})\ ,\ \alpha_{1,2}''' = \frac{-5\pm3\sqrt{21}}{1176} \tag{68}$$

Table S13. L2|G3 method.

$1/3$	$1/6$	ω_1	$2/9$	ω_2
$1/6$	$1/3$	ω_2	$2/9$	ω_1
α_2	α_2'			
$3/8$	$1/8$			
α_1	α_1'			
$1/2$	$1/2$	$(5-\sqrt{15})/10$	$1/2$	$(5+\sqrt{15})/10$

$$\omega_{1,2} = \frac{1}{36}(5\pm\sqrt{15})\ ,\ \alpha_{1,2} = \frac{1}{20}(6\pm\sqrt{15})\ ,\ \alpha_{1,2}' = \frac{1}{20}(4\pm\sqrt{15}) \tag{70}$$

Table S14. L3|G4 method.

$2/15$	$1/15$	$-1/30$	$\omega_{1\|1\|1}$	$\omega_{2\|1\|2}$	$\omega_{2\|2\|2}$	$\omega_{1\|2\|1}$
$1/15$	$8/15$	$1/15$	ω_2'	ω_1'	ω_1'	ω_2'
$-1/30$	$1/15$	$2/15$	$\omega_{1\|2\|1}$	$\omega_{2\|2\|2}$	$\omega_{2\|1\|2}$	$\omega_{1\|1\|1}$
$\alpha_{2\|2\|1}$	$\alpha_{2\|1}'$	$\alpha_{1\|2\|1}''$				
$\alpha_{1\|2\|2}$	$\alpha_{2\|2}'$	$\alpha_{2\|2\|2}''$				
$\alpha_{1\|1\|2}$	$\alpha_{1\|2}'$	$\alpha_{2\|1\|2}''$				
$\alpha_{2\|1\|1}$	$\alpha_{1\|1}'$	$\alpha_{1\|1\|1}''$				
$1/6$	$2/3$	$1/6$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\omega_{1,2|1,2|1,2} = \frac{1}{720}\left(30\pm3\sqrt{30}\pm\sqrt{30\left(75\pm4\sqrt{30}\right)}\right),\ \omega_{1,2}' = \frac{1}{90}\left(15\pm2\sqrt{30}\right), \tag{72}$$

$$\alpha_{1,2|1,2|1,2} = \frac{455\pm21\sqrt{30}\pm\sqrt{21(585\pm106\sqrt{30})}}{2940},\ \alpha_{1,2|1,2}' = \frac{1}{3}\pm\frac{1}{735}\sqrt{35595\pm3864\sqrt{30}}, \tag{73}$$

$$\alpha_{1,2|1,2|1,2}'' = \frac{35\pm21\sqrt{30}\pm\sqrt{21(585\pm106\sqrt{30})}}{2940},\ \delta_{1,2|1,2}' = \frac{2}{9}\pm\frac{1}{21}\sqrt{\frac{2}{105}(1035\pm82\sqrt{30})},\ \gamma_{1,2|1,2} = \frac{1}{2}\pm\frac{1}{2}\sqrt{\frac{1}{35}(15\pm2\sqrt{30})} \tag{74}$$

Table S15. L4|G5 method.

$1/14$	$\sqrt{5}/84$	$-\sqrt{5}/84$	$1/84$	ω_1	ω_1'	$-8/225$	ω_2'	ω_2
$\sqrt{5}/84$	$5/14$	$5/84$	$-\sqrt{5}/84$	ω_1''	ω_1'''	$8/45$	ω_2'''	ω_2''
$-\sqrt{5}/84$	$5/84$	$5/14$	$\sqrt{5}/84$	ω_2''	ω_2'''	$8/45$	ω_1'''	ω_1''
$1/84$	$-\sqrt{5}/84$	$\sqrt{5}/84$	$1/14$	ω_2	ω_2'	$-8/225$	ω_1'	ω_1
$\alpha_{2\|2\|1}$	$\alpha'_{1\|2\|2\|1}$	$\alpha'_{2\|1\|2\|1}$	$\alpha'''_{1\|2\|1}$					
$\alpha_{1\|1\|2}$	$\alpha'_{1\|1\|2\|2}$	$\alpha'_{2\|2\|2\|2}$	$\alpha'''_{2\|1\|2}$					
$17/192$	α_1''	α_2''	$-1/192$					
$\alpha_{1\|2\|2}$	$\alpha'_{1\|1\|1\|2}$	$\alpha'_{2\|2\|1\|2}$	$\alpha'''_{2\|2\|2}$					
$\alpha_{2\|1\|1}$	$\alpha'_{1\|2\|1\|1}$	$\alpha'_{2\|1\|1\|1}$	$\alpha'''_{1\|1\|1}$					
$1/12$	$5/12$	$5/12$	$1/12$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$1/2$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\omega_{1,2} = \frac{749+49\sqrt{70}\pm\sqrt{568155+63924\sqrt{70}}}{25200},\ \omega'_{1,2} = \frac{107-7\sqrt{70}\pm\sqrt{11595-9132\sqrt{\frac{10}{7}}}}{3600}, \tag{76}$$

$$\omega''_{1,2} = \frac{1}{720}\left(43-4\sqrt{70}\pm 5\sqrt{177-678\sqrt{\frac{2}{35}}}\right),\ \omega'''_{1,2} = \frac{301+28\sqrt{70}+\sqrt{105(2065+226\sqrt{70})}}{5040}, \tag{77}$$

$$\alpha_{1,2|1,2|1,2} = \frac{346\pm 8\sqrt{70}\pm\sqrt{7595\pm 164\sqrt{70}}}{4536},\ \alpha'_{1,2|1,2|1,2|1,2} = \frac{5(189\pm 19\sqrt{5}\pm 10\sqrt{14}\pm\sqrt{26915\pm 944\sqrt{70}})}{4536}, \tag{78}$$

$$\alpha''_{1,2} = \frac{5}{192}(8+3\sqrt{5}),\ \alpha'''_{1,2|1,2|1,2} = \frac{32\pm 8\sqrt{70}\pm\sqrt{7595\pm 164\sqrt{70}}}{4536},\ \gamma_{1,2|1,2} = \frac{1}{2}\pm\frac{1}{6}\sqrt{5\pm 2\sqrt{\frac{10}{7}}} \tag{79}$$

Table S16. eL2|G2 method.

$1/2$	$1/2$	$1/2$	$1/2$
α_2	α_2'		
α_1	α_1'		
$1/2$	$1/2$	γ_2	γ_1

$$\alpha_{1,2} = \frac{1}{12}(4\pm\sqrt{3}),\ \alpha'_{1,2} = \frac{1}{12}(2\pm\sqrt{3}),\ \gamma_{1,2} = \frac{1}{6}(3\pm\sqrt{3}) \tag{81}$$

Table S17. eL3|G3 method.

$1/6$	$1/3$	0	ω_1	$2/9$	ω_2
0	$1/3$	$1/6$	ω_2	$2/9$	ω_1
α_2	α_2''	α_2'			
$5/24$	$1/3$	$-1/24$			
α_1	α_1''	α_1'			
$1/6$	$2/3$	$1/6$	$(5-\sqrt{15})/10$	$1/2$	$(5+\sqrt{15})/10$

$$\omega_{1,2} = \frac{1}{36}\left(5\pm\sqrt{15}\right),\ \alpha_{1,2} = \frac{1}{300}\left(40\pm 3\sqrt{15}\right),\ \alpha'_{1,2} = \frac{1}{300}\left(10\pm 3\sqrt{15}\right),\ \alpha''_{1,2} = \frac{1}{3}\pm\frac{2\sqrt{\frac{3}{5}}}{5} \tag{83}$$

Table S18. eL4|G4 method.

$1/12$	π_1	π_2	0	$\omega_{1\|1\|1}$	$\omega_{2\|1\|2}$	$\omega_{2\|2\|2}$	$\omega_{1\|2\|1}$
0	$1/3$	$1/3$	0	ω_2'	ω_1'	ω_1'	ω_2'
0	π_2	π_1	$1/12$	$\omega_{1\|2\|1}$	$\omega_{2\|2\|2}$	$\omega_{2\|1\|2}$	$\omega_{1\|1\|1}$
$\alpha_{2\|2\|2}$	$\alpha'_{1\|2\|2\|1}$	$\alpha'_{2\|1\|2\|1}$	$\alpha''_{1\|2\|2}$				
$\alpha_{1\|1\|1}$	$\alpha'_{1\|1\|2\|2}$	$\alpha'_{2\|2\|2\|2}$	$\alpha''_{2\|1\|1}$				
$\alpha_{1\|2\|1}$	$\alpha'_{1\|1\|1\|2}$	$\alpha'_{2\|2\|1\|2}$	$\alpha''_{2\|2\|1}$				
$\alpha_{2\|1\|2}$	$\alpha'_{1\|2\|1\|1}$	$\alpha'_{2\|1\|1\|1}$	$\alpha''_{1\|1\|2}$				
$1/12$	$5/12$	$5/12$	$1/12$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\pi_{1,2} = \frac{1}{24}\left(1 \pm \sqrt{5}\right),\ \omega_{1,2|1,2|1,2} = \frac{1}{720}\left(30 \pm 3\sqrt{30} \pm \sqrt{30\left(75 \pm 4\sqrt{30}\right)}\right),\ \omega'_{1,2} = \frac{1}{90}\left(15 \pm 2\sqrt{30}\right), \tag{86}$$

$$\alpha_{1,2|1,2|1,2} = \frac{470 \pm 12\sqrt{30} \pm \sqrt{105(75 \pm 4\sqrt{30})}}{5880},\ \alpha'_{1,2|1,2|1,2|1,2} = \frac{245 \pm 39\sqrt{5} \pm 30\sqrt{6} \pm \sqrt{35595 \pm 3864\sqrt{30}}}{1176}, \tag{87}$$

$$\alpha''_{1,2|1,2|1,2} = \frac{20 \pm 12\sqrt{30} \pm \sqrt{105(75 \pm 4\sqrt{30})}}{5880},\ \gamma_{1,2|1,2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{35}(15 \pm 2\sqrt{30})} \tag{88}$$

Table S19. eL2|G3 method.

$1/2$	$1/2$	$5/18$	$4/9$	$5/18$
α_2	α'_2			
$3/8$	$1/8$			
α_1	α'_1			
$1/2$	$1/2$	$(5-\sqrt{15})/10$	$1/2$	$(5+\sqrt{15})/10$

$$\alpha_{1,2} = \frac{1}{20}\left(6 - \sqrt{15}\right),\ \alpha'_{1,2} = \frac{1}{20}\left(4 - \sqrt{15}\right) \tag{90}$$

Table S20. eL3|G4 method.

$1/6$	$1/3$	0	$\omega_{2\|1\|1}$	$\omega'_{1\|1\|2}$	$\omega'_{1\|2\|2}$	$\omega_{2\|2\|1}$
0	$1/3$	$1/6$	$\omega_{2\|2\|1}$	$\omega'_{1\|2\|2}$	$\omega'_{1\|1\|2}$	$\omega_{2\|1\|1}$
$\alpha_{2\|2\|1}$	$\alpha'_{2\|1}$	$\alpha''_{1\|2\|1}$				
$\alpha_{1\|2\|2}$	$\alpha'_{2\|2}$	$\alpha''_{2\|2\|2}$				
$\alpha_{1\|1\|2}$	$\alpha'_{1\|2}$	$\alpha''_{2\|1\|2}$				
$\alpha_{2\|1\|1}$	$\alpha'_{1\|1}$	$\alpha''_{1\|1\|1}$				
$1/6$	$2/3$	$1/6$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\omega_{1,2|1,2|1,2} = \frac{1}{720}\left(90 \pm 5\sqrt{30} \pm \sqrt{30\left(75 \pm 4\sqrt{30}\right)}\right), \omega'_{1,2|1,2|1,2} = \frac{1}{24}\left(3 \pm \sqrt{\frac{5}{6}} \pm \sqrt{\frac{5}{2} \pm 2\sqrt{\frac{2}{15}}}\right), \tag{92}$$

$$\alpha_{1,2|1,2|1,2} = \frac{455 \pm 21\sqrt{30} \pm \sqrt{21(585 \pm 106\sqrt{30})}}{2940},\ \alpha'_{1,2|1,2} = \frac{1}{3} \pm \frac{1}{735}\sqrt{35595 \pm 3864\sqrt{30}}, \tag{93}$$

$$\alpha''_{1,2|1,2|1,2} = \frac{35 \pm 21\sqrt{30} \pm \sqrt{21(585 \pm 106\sqrt{30})}}{2940},\ \gamma_{1,2|1,2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1}{35}(15 \pm 2\sqrt{30})} \tag{94}$$

Table S21. eL4|G5 method.

$1/12$	π_1	π_2	0	$\omega_{1\|1\|2}$	$\omega_{2\|1\|1}$	0	$\omega_{2\|2\|1}$	$\omega_{1\|2\|2}$
0	$1/3$	$1/3$	0	ω_2'	ω_1'	$64/225$	ω_1'	ω_2'
0	π_2	π_1	$1/12$	$\omega_{1\|2\|2}$	$\omega_{2\|2\|1}$	0	$\omega_{2\|1\|1}$	$\omega_{1\|1\|2}$
$\alpha_{2\|2\|1}$	$\alpha'_{1\|2\|2\|1}$	$\alpha'_{2\|1\|2\|1}$	$\alpha'''_{1\|2\|1}$					
$\alpha_{1\|1\|2}$	$\alpha'_{1\|1\|2\|2}$	$\alpha'_{2\|2\|2\|2}$	$\alpha'''_{2\|1\|2}$					
$17/192$	α_1''	α_2''	$-1/192$					
$\alpha_{1\|2\|2}$	$\alpha'_{1\|1\|1\|2}$	$\alpha'_{2\|2\|1\|2}$	$\alpha'''_{2\|2\|2}$					
$\alpha_{2\|1\|1}$	$\alpha'_{1\|2\|1\|1}$	$\alpha'_{2\|1\|1\|1}$	$\alpha'''_{1\|1\|1}$					
$1/12$	$5/12$	$5/12$	$1/12$	$\gamma_{2\|1}$	$\gamma_{2\|2}$	$1/2$	$\gamma_{1\|2}$	$\gamma_{1\|1}$

$$\pi_{1,2} = \frac{1}{24}\left(1 \pm \sqrt{5}\right),\ \omega_{1,2|1,2|1,2} = \frac{150 \pm 3\sqrt{70} \pm \sqrt{45570 \pm 984\sqrt{70}}}{3600},\ \omega'_{1,2} = \frac{1}{450}\left(43 \pm 4\sqrt{70}\right), \tag{97}$$

$$\alpha_{1,2|1,2|1,2} = \frac{346 \pm 8\sqrt{70} \pm \sqrt{7595 \pm 164\sqrt{70}}}{4536},\ \alpha'_{1,2|1,2|1,2|1,2} = \frac{5(189 \pm 19\sqrt{5} \pm 10\sqrt{14} \pm \sqrt{26915 \pm 944\sqrt{70}})}{4536}, \tag{98}$$

$$\alpha''_{1,2} = \frac{5}{192}(8 + 3\sqrt{5}),\ \alpha'''_{1,2|1,2|1,2} = \frac{32 \pm 8\sqrt{70} \pm \sqrt{7595 \pm 164\sqrt{70}}}{4536},\ \gamma_{1,2|1,2} = \frac{1}{2} \pm \frac{1}{6}\sqrt{5 \pm 2\sqrt{\frac{10}{7}}} \tag{99}$$