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Stabilization of Stochastic Differential Equations Driven by G-Brownian Motion with Aperiodically Intermittent Control

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Abstract: The paper is devoted to studying the exponential stability of a mild solution of stochastic differential equations driven by G-Brownian motion with an aperiodically intermittent control. The aperiodically intermittent control is added into the drift coefficients, when intermittent intervals and coefficients satisfy suitable conditions; by use of the G-Lyapunov function, the p-th exponential stability is obtained. Finally, an example is given to illustrate the availability of the obtained results.

Keywords: exponential stability; aperiodically intermittent control; G-Brownian motion; stochastic differential equations

1. Introduction

In this paper, stochastic differential equations driven by G-Brownian motion (G-SDEs) are considered as follows

$$dx(t) = f(t, x(t))dt + g(t, x(t))d\langle B\rangle(t) + \sigma(t, x(t))dB(t),\ t \geq 0, \tag{1}$$

where the coefficients $f,\ g,\ \sigma : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$, $x(0) = x_0 \in \mathbb{R}^n$. $B(t)$ is G-Brownian motion, $\langle B\rangle(t)$ is usually called the quadratic variation process of G-Brownian motion. Due to the nonlinear properties of expectation with G-Brownian motion, G-SDEs are more general than classical SDEs driven by Brownian motion, and can be widely used in many fields. With the development of G-theory and related stochastic calculus ([1]), many interesting results of G-SDEs have been obtained, for instance, existence, uniqueness, boundedness ([2–7] and the references therein).

As we know, stability is one of the most interesting topics in dynamic behaviors. Regarding SDEs, many interesting works have been obtained on this issue (one can see [8,9]). Similarly, a lot of researchers have made great efforts on the subject of G-SDEs, for instance, exponential stabilization and quasi-sure exponential stabilization ([10]). However, the most relevent is how to make an unstable system stable. Recently, an aperiodically intermittent control has been presented to make systems stable ([11,12]). In particular, Yang et al. [13] investigated the stability of a solution of G-SDEs by constructing an aperiodically intermittent control which is set in diffusion coefficient. Meanwhile, based on the stability of G-SDEs, the stabilization of a stochastic Cohen–Grossberg neural networks driven by G-Brownian motion was established. A natural problem is whether one can stabilize the G-SDEs when an aperiodically intermittent control is added into the drift coefficients. As far as we know, there is no result on this topic. Taking the issue under consideration, we will investigate the stability of (1) with an aperiodically intermittent control added into the drift coefficient

$$dy(t) = f(t, y(t))dt + h(s)g(t, y(t))d\langle B\rangle(t) + h(s)\sigma(t, y(t))dB(t),\ t \geq 0, \tag{2}$$

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where the aperiodically intermittent control

$$h(t) = \begin{cases} -1, \ t \in [t_i, s_i), \\ 0, \ t \in [s_i, t_{i+1}), \end{cases}$$

$t_{i+1} - s_i$ is the rest width and $s_i - t_i$ is the control width. Let $\inf_i (s_i - t_i) = \mu > 0$, $\sup_i (t_{i+1} - t_i) = v < \infty$, $\psi_k = (t_{k+1} - s_k)(t_{k+1} - t_k)^{-1}$ and $\psi = \limsup_{k \to \infty} \psi_k > 0$.

Differing from Yang et al. [13], we investigate the stabilization problem of G-SDEs, whose drift coefficients are added with an aperiodically intermittent control. The main innovations and contributions of this paper are highlighted as follows.

- A new aperiodically intermittent control is designed to stabilize this class stochastic system, driven by G-Brownian motion. Moreover, the aperiodically intermittent control is added to the drift coefficient.
- The aperiodically intermittent interval satisfies
$$a_2 \psi < a_1(1 - \psi),$$
where $\psi_k = (t_{k+1} - s_k)(t_{k+1} - t_k)^{-1}$ and $\psi = \limsup_{k \to \infty} \psi_k > 0$, which can be easily realized.
- By the Lyapunov function satisfying suitable conditions, the p-th exponential stability is obtained. When $p = 2$, it is the exponential stability in mean square. Finally, an example is presented to show the efficiency of the obtained result .

The rest of the paper is arranged as follows. In the next section, some basic notions, preliminaries and lemmas are provided. In Section 3, we prove exponential stability for the solution of G-SDEs, whose drift coefficients are added to an aperiodically intermittent control. Finally, an example is presented to show the efficiency of the result.

2. Notations

In this section, some notations, with respect to G-Brownian motion and related stochastic calculus, are introduced. Ω denotes the collection of all continuous functions ω on $\mathbb{R}^n$ with $\omega_0 = 0$, and the distance in Ω is given by

$$\rho(\omega^1, \omega^2) = \sum_{i=1}^{\infty} 2^{-i} \left[\left(\max_{t \in [0,i]} |\omega_t^1 - \omega_t^2| \right) \wedge 1 \right].$$

$B_t(\omega)$ is the canonical process and is defined by $B_t(\omega) = \omega_t, \ t \geq 0$. The filtration $\mathcal{F}_t$ generated by $(B_t)_{t \geq 0}$ is given with $\mathcal{F}_t = \sigma(B_s, 0 \leq s \leq t)$, and $\mathcal{F} = \bigvee_{t \geq 0} \mathcal{F}_t$, $\mathbb{E}$ is a sublinear expectation defined on $(\Omega, \mathcal{F})$.

We denote $\mathcal{C}_{b,Lip}(\mathbb{R}^n)$ as the space of all bounded Lipschitz continuous functions on $\mathbb{R}^n$, and

$$\begin{aligned} \mathcal{L}_{Lip}(\mathcal{F}) \quad = \quad & \{\xi := \phi(B(t_1), B(t_2), \ldots, B(t_n)), n \geq 1, 0 < t_1 < t_2 < \cdots < t_n < \infty, \\ & \phi \in \mathcal{C}_{b,Lip}(\mathbb{R}^n) \Big\}. \end{aligned}$$

Definition 1. *A random variable X is G-normally distributed, denoted by $X \backsim N(0, [\bar{\sigma}, \underline{\sigma}])$, $0 \leq \bar{\sigma} \leq \underline{\sigma}$, if for any $\xi \in \mathcal{L}_{Lip}(\mathcal{F})$, the operator defined by $\mathbb{E}\left[\xi\left(x + \sqrt{t}X\right)\right] := u(t, x)$ is the viscosity solution of the following nonlinear heat equation*

$$\begin{cases} \dfrac{\partial u}{\partial t} - G\left(\dfrac{\partial^2 u}{\partial t^2}\right) = 0 \\ u(0, x) = \xi(x), \end{cases}$$

where $G(r) = \frac{1}{2}\left(\bar{\sigma}^2 r^+ - \underline{\sigma}^2 r^-\right), r \in \mathbb{R}$.

Definition 2. *The canonical process $B(t)_{t\geq 0}$ is called G-Brownian motion, if the following properties are verified*

(1) $B_0(\omega) = 0$;

(2) For each $t,\ s \geq 0$, the increment $B(t+s) - B(s) \backsim N(0, [\sqrt{s}\bar{\sigma},\ \sqrt{s}\underline{\sigma}])$ and is independent from $(B(t_1),\ B(t_2),\ \cdots,\ B(t_n))$, for any $0 \leq t_1 \leq t_2 \leq \cdots \leq t_n$.

Furthermore, the sublinear expectation $\mathbb{E}$ is called the G-expectation.

In the following part, we introduce the Itô integral with respect to the G-Brownian motion. Firstly, some space notations are prsented.

$L_G^p(\mathcal{F}_T)(p \geq 1)$ is the completion of $\mathcal{L}_{Lip}(\mathcal{F}_T)$ with the norm $\|X\| = \{\mathbb{E}|X|^p\}^{\frac{1}{p}}$, as well as, $L_G^p(\mathcal{F})$ is considered as the completion of $\mathcal{L}_{Lip}(\mathcal{F})$. Furthermore

$$\mathcal{M}_G^{p,0}([0,T]) = \left\{ g_t = \sum_{j=1}^{N} \xi_{t_j} I_{[t_{j-1},t_j)}(t);\ \xi_{t_j} \in L_G^p(\mathcal{F}_{t_j}),\ t_{j-1} < t_j,\ j = 1,\ 2, \ldots,\ N \right\}.$$

$\mathcal{M}_G^p([0,T])$ is denoted as the completion of $\mathcal{M}_G^{p,0}([0,T])$ satisfying

$$\|g\|_{\mathcal{M}_G^p([0,T])} = \left(\int_0^T \mathbb{E}\|g_s\|^p \mathrm{d}s\right)^{\frac{1}{p}}.$$

Definition 3. *(Itô Integral) For $g_t = \sum_{j=0}^{N-1} \xi_{t_j} I_{[t_j,t_{j+1})} \in \mathcal{M}_G^{p,0}([0,T])$, Itô Integral with respect to $B(t)$ is defined*

$$\int_0^T g_s \mathrm{d}B(s) := \sum_{j=0}^{N-1} \xi_{t_j}\big(B(t_{j+1}) - B(t_j)\big),$$

moreover, the quadratic variation process of the G-Brownian motion $B(t)$ is defined by

$$\langle B \rangle_t := \lim_{N\to\infty} \sum_{j=0}^{N-1} \left(B^N(t_{j+1}) - B^N(t_j)\right)^2 = B^2(t) - 2\int_0^t B(s)\mathrm{d}B(s).$$

Definition 4. *For any $g_t \in \mathcal{M}_G^{1,0}([0,T])$, define*

$$\int_0^T g_t \mathrm{d}\langle B \rangle(t) := \sum_{j=0}^{N-1} \xi_{t_j}\big[\langle B \rangle(t_{j+1}) - \langle B \rangle(t_j)\big].$$

3. Main Results

Definition 5. *Suppose there exist positive constants λ and C, such that the solution $X(t)$ of* (1) *satisfies*

$$\mathbb{E}\big|X(t)\big|^p \leq C\mathbb{E}|X(0)|^p e^{-\lambda t}, \text{ for any initial value } X(0),\ p \geq 2,$$

then, the mild solution $X(t)$ is said to be p-th exponentially stable.

If $V(t,x) \in \mathcal{C}^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n;\ \mathbb{R}^n)$, the Lyapunov operator $L : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ associated to the G-SDEs (1) is defined as below

$$LV(t,x) = V_t(t,x) + V_x(t,x)f(t,x) + G(\langle V_x(t,x), 2g(t,x)\rangle + \langle V_{xx}(t,x)\sigma(t,x), \sigma(t,x)\rangle),$$

where

$$V_t(t,x) = \frac{\partial V(t,x)}{\partial t},\ V_x(t,x) = \left(\frac{\partial V(t,x)}{\partial x_1}, \frac{\partial V(t,x)}{\partial x_2}, \dots, \frac{\partial V(t,x)}{\partial x_n}\right),$$
$$V_{xx}(t,x) = \left(\frac{\partial^2 V(t,x)}{\partial x_i \partial x_j}\right)_{n\times n}.$$

Theorem 1. *Assume that the function $V(t,x)$ associated with (2) is in $\mathcal{C}^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^n)$ and there exist positive constants c_1, c_2, a_1, a_2, γ, M such that*

$$c_1|y(t)|^p \le V(t,y(t)) \le c_1|y(t)|^p, \tag{3}$$

$$LV(t,y(t)) \le -a_1 V(t,y(t)), t \in [t_i, s_i), \tag{4}$$

and

$$LV(t,y(t)) \le a_2 V(t,y(t)), t \in [s_i, t_{i+1}). \tag{5}$$

Furthermore, suppose the aperiodically intermittent interval satisfies the inequality $a_2\psi < a_1(1-\psi)$, then

$$\mathbb{E}|y(t)|^p \le M\mathbb{E}|y(0)|^p e^{-\gamma t},\ \gamma \in (0, a_1(1-\psi) - a_2\psi).$$

Proof of Theorem 1. Taking Itô formula to $e^{a_1 t}V(t,x)$, we have

$$\begin{aligned} \mathrm{d}\big(e^{a_1 t}V(t,y(t))\big) &= e^{a_1 t}\{a_1 V(t,y(t)) + V_t(t,y(t)) + V_y(t,y(t))f(y(t))\}\mathrm{d}t \\ &\quad + e^{a_1 t}\langle V_y(t,y(t)), h(s)g(y(t))\rangle \mathrm{d}\langle B\rangle(t) \\ &\quad + e^{a_1 t}\langle V_y(t,y(t)), h(s)\sigma(y(t))\rangle \mathrm{d}B(t) \\ &\quad + \frac{1}{2}e^{a_1 t}\langle V_{yy}(t,y(t))h(s)\sigma(y(t)), h(s)\sigma(y(t))\rangle \mathrm{d}\langle B\rangle(t). \end{aligned} \tag{6}$$

If $t \in [t_0, s_0)$, it follows from the (6)

$$\begin{aligned} e^{a_1 t}V(t,y(t)) &= e^{a_1 t_0}V(t_0,y(t_0)) + \int_{t_0}^t e^{a_1 s}[a_1 V(s,y(s)) + LV(s,y(s))]\mathrm{d}s + M_t^{t_0} \\ &\quad + \int_{t_0}^t e^{a_1 s}\langle V_y(s,y(s)), h(s)\sigma(y(s))\rangle \mathrm{d}B(s), \end{aligned} \tag{7}$$

where

$$\begin{aligned} M_t^{t_0} &= \int_{t_0}^t e^{a_1 s}\langle V_y(s,y(s)), h(s)g(y(s))\rangle \mathrm{d}\langle B\rangle(s) \\ &\quad + \frac{1}{2}\int_{t_0}^t e^{a_1 s}\langle V_{yy}(s,y(s))h(s)\sigma(y(s)), h(s)\sigma(y(s))\rangle \mathrm{d}s \\ &\quad - \int_{t_0}^t e^{a_1 s}G\big(\langle V_y(s,y(s)), 2h(s)g(y(s))\rangle + \langle V_{yy}(s,y(s)), h(s)\sigma(y(s))\rangle\big)\mathrm{d}s. \end{aligned}$$

Following (4) and (7), it deduces

$$e^{a_1 t}V(t,y(t)) \le e^{a_1 t_0}V(t_0,y(t_0)) + M_t^{t_0} + \int_{t_0}^t e^{a_1 s}\langle V_y(s,y(s)), h(s)\sigma(y(s))\rangle \mathrm{d}B(s). \tag{8}$$

From [5], $\mathbb{E}M_t \le 0$, and by using G-expectation on (8), it shows

$$\mathbb{E}e^{a_1 t}V(t,y(t)) \le \mathbb{E}e^{a_1 t_0}V(t_0,y(t_0)). \tag{9}$$

When $t \in [s_0, t_1)$, as similar way, we can get

$$\begin{aligned} e^{a_1 t} V(t, y(t)) &= e^{a_1 s_0} V(s_0, y(s_0)) + \int_{s_0}^{t} e^{a_1 s}[a_1 V(s, y(s)) + LV(s, y(s))]\mathrm{d}s + M_t^{s_0} \\ &\quad + \int_{s_0}^{t} e^{a_1 s}\langle V_y(s, y(s)), h(s)\sigma(y(s))\rangle \mathrm{d}B(s). \end{aligned} \tag{10}$$

Again, taking the expectation on both sides of (11), and from (5), then

$$\mathbb{E}e^{a_1 t} V(t, y(t)) \leq \mathbb{E}e^{a_1 s_0} V(s_0, y(s_0)) + (a_1 + a_2)\int_{s_0}^{t} e^{a_1 s}\mathbb{E}V(s, y(s))\mathrm{d}s. \tag{11}$$

By means of the Gronwall inequality, we can claim that

$$\mathbb{E}e^{a_1 t} V(t, y(t)) \leq \mathbb{E}e^{a_1 s_0} V(s_0, y(s_0)) e^{(a_1+a_2)(t-s_0)} \leq \mathbb{E}e^{a_1 t_0} V(t_0, y(t_0)) e^{(a_1+a_2)(t-s_0)}.$$

For $t \in [t_1, s_1)$, we have

$$\mathbb{E}e^{a_1 t} V(t, y(t)) \leq \mathbb{E}e^{a_1 t_1} V(t_1, y(t_1)) \leq \mathbb{E}e^{a_1 t_0} V(t_0, y(t_0)) e^{(a_1+a_2)(t_1-s_0)}.$$

We consider the other case $t \in [s_1, t_2)$,

$$\begin{aligned} \mathbb{E}e^{a_1 t} V(t, y(t)) &\leq \mathbb{E}e^{a_1 s_1} V(s_1, y(s_1)) e^{(a_1+a_2)(t-s_1)} \\ &\leq \mathbb{E}e^{a_1 t_0} V(t_0, y(t_0)) e^{(a_1+a_2)(t_1-s_0)+(a_1+a_2)(t-s_1)}. \end{aligned}$$

Repeating the aforementioned procedure, we have

$$\mathbb{E}e^{a_1 t} V(t, y(t)) \leq \mathbb{E}e^{a_1 t_0} V(t_0, y(t_0)) e^{(a_1+a_2)\sum\limits_{k=0}^{i-1}(t_{k+1}-s_k)},\ t \in [t_i, s_i),$$

and

$$\begin{aligned} \mathbb{E}e^{a_1 t} V(t, y(t)) &\leq \mathbb{E}e^{a_1 t_0} V(t_0, y(t_0)) e^{(a_1+a_2)\sum\limits_{k=0}^{i-1}(t_{k+1}-s_k)+(a_1+a_2)(t-s_i)} \\ &\leq \mathbb{E}e^{a_1 t_0} V(t_0, y(t_0)) e^{(a_1+a_2)\sum\limits_{k=0}^{i}(t_{k+1}-s_k)},\ t \in [s_i, t_{i+1}). \end{aligned}$$

With regard to the definition of ψ, for $t \in [t_i,\ s_i)$

$$\begin{aligned} (a_1 + a_2)\sum_{k=0}^{i-1}(t_{k+1} - s_k) - a_1 t &= (a_1 + a_2)\sum_{k=0}^{i-1}\psi_k(t_{k+1} - t_k) - a_1 t \\ &\leq (a_1 + a_2)\psi t - a_1 t. \end{aligned}$$

Thus,

$$\mathbb{E}V(t, y(t)) \leq \mathbb{E}V(t_0, y(t_0)) e^{((a_1+a_2)\psi - a_1)t},\ t \in [t_i,\ s_i). \tag{12}$$

By the definition of ψ and v, for $t \in [s_i,\ t_{i+1})$

$$\begin{aligned} (a_1 + a_2)\sum_{k=0}^{i}(t_{k+1} - s_k) - a_1 t &= (a_1 + a_2)\sum_{k=0}^{i}[\psi_k(t_{k+1} - t_k)] - a_1 t \\ &\leq (a_1 + a_2)\psi t + (a_1 + a_2)\psi(t_{k+1} - t) - a_1 t \\ &\leq [(a_1 + a_2)\psi - a_1]t + (a_1 + a_2)\psi v. \end{aligned}$$

Then,

$$\mathbb{E}V(t, y(t)) \leq \mathbb{E}V(t_0, y(t_0))e^{(a_1+a_2)\psi v}e^{((a_1+a_2)\psi - a_1)t}, \; t \in [s_i, \; t_{i+1}). \tag{13}$$

From (12) and (13), we obtain

$$\mathbb{E}V(t, y(t)) \leq \mathbb{E}V(t_0, y(t_0))e^{(a_1+a_2)\psi v}e^{-\gamma t}, \; t \geq 0.$$

It follows from (3) that

$$\mathbb{E}|y(t)|^p \leq \frac{c_2}{c_1}\mathbb{E}|y(0)|^p e^{(a_1+a_2)\psi v}e^{-\gamma t}, \; t \geq 0.$$

Setting $M = \frac{c_2}{c_1}e^{(a_1+a_2)\psi v}$, we get the desired result. □

Example 1. *Consider the one-dimensional stochastic nonliear system driven by G-Brownian motion*

$$\mathrm{d}x(t) = x(t)\sin t\mathrm{d}t + (4+\sin t)x(t)\mathrm{d}\langle B\rangle(t) + \sqrt{2+\cos t}x(t)\mathrm{d}B(t), \; t \geq 0, \tag{14}$$

where $B(t)$ is $\mathbb{R}$-valued G-Brownian motion obeying $N(0, [\frac{1}{2}, \; 1])$. Assuming $V(t, \; x) = |x|^2$, we have

$$\begin{aligned} V_x(t, \; x(t))f(t, x(t)) &= 2|x(t)|^2\sin t, \\ V_x(t, \; x(t))g(t, x(t)) &= 2|x(t)|^2(4+\sin t), \\ V_{xx}(t, \; x(t))\sigma^2(t, x(t)) &= 2|x(t)|^2(2+\cos t) \end{aligned}$$

and

$$LV(t, \; x(t)) \geq -2|x(t)|^2 + G(12|x(t)|^2 + 2|x(t)|^2) = 5|x(t)|^2$$

Therefore, the (14) is not exponentially stable in mean square. Now, substituting the aperiodiacally intermittent controller $h(t)$ into the system (14), we assume the control of the following system

$$\mathrm{d}y(t) = y(t)\sin t\mathrm{d}t + h(s)(4+\sin t)y(t)\mathrm{d}\langle B\rangle(t) + h(s)\sqrt{2+\cos t}y(t)\mathrm{d}B(t), t \geq 0, \tag{15}$$

For any $t \in [t_i, s_i)$, we can conclude

$$LV(t, \; y(t))) \leq 2|y(t)|^2 + G(-12|y(t)|^2 + 6|y(t)|^2) = -|y(t)|^2.$$

If $t \in [s_i, t_{i+1})$, we can obtain

$$LV(t, \; y(t))) \leq 2|y(t)|^2.$$

Let $C_1 = 1, C_2 = 5, C_3 = \sqrt{2}, \; a_1 = 1, a_2 = 2$, then, $\psi \in (0, \; \frac{1}{3})$, from Theorem 1, then the system (15) is stable.

4. Conclusions

The paper studied the p-th exponential stability for the mild solution of stochastic differential equations driven by G-Brownian motion. By using an aperiodically intermittent control, added to the drift coefficients and G-Lyapunov function, the desired result is obtained under suitable conditions. Moreover, the length of intermittent intervals is given. Finally, an example is presented to introduce the effectiveness of the results.

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