Supplementary material

Model-Based Cost Optimization of Double-Effect Water-Lithium Bromide Absorption Refrigeration Systems

1. Modeling

1.1 Process model

The mathematical model employed to determine the optimal design and operation conditions includes: equality constraints for describing the mass and energy balances of the process units, design relationships, and correlations for estimating the physicochemical and thermodynamic properties of the process streams, as well as inequality constraints to specify minimum allowable temperature difference approximations and to avoid temperature crosses in the heat exchangers. The total annual cost (TAC, in $\cdot$year^{-1}), which accounts for capital and operating expenditures, is the objective function that is minimized.

1.2 Assumptions

The main model assumptions are the following:
- The system is at steady-state condition [1,2,3].
- Pressure drops are not considered in pipes and heat exchangers [2].
- Heat losses are not considered in the process units [1,2,3].
- The work by the solution pump is not considered in the total energy balance since it is negligible with respect to the heat loads in the process units [3].
- The refrigerant leaves the condenser and the evaporator at saturation conditions [1,2].
- The refrigerant leaves the low-temperature and high-temperature generators at superheated vapor conditions [4].
- The weak LiBr solution leaves the absorber at saturation condition [1].
- The strong LiBr solutions leave LTG and HTG at equilibrium at their corresponding pressure and temperature values [1].
- All the expansion valves perform isenthalpic [1,3].

1.3 Mathematical model formulation

– Definitions

Let PU be the set of the main process units vector k:

$$PU = \{ABS, HTG, LTG, COND, EVAP, HTSHE, LTSHE, EV1, EV2, EV3, EV4\} \tag{1}$$

Let PS be the set of the process streams vector i:

$$PS = \{1, \cdots i \cdots, 25\} \tag{2}$$

Let PC be the set of the system components vector j:

$$PC = \{LiBr, water, vapor\} \tag{3}$$

Let IN and OUT be the subsets of PS with the process streams entering and leaving the system, respectively. The following balances and relationships can now be formulated according to Fig. 1:
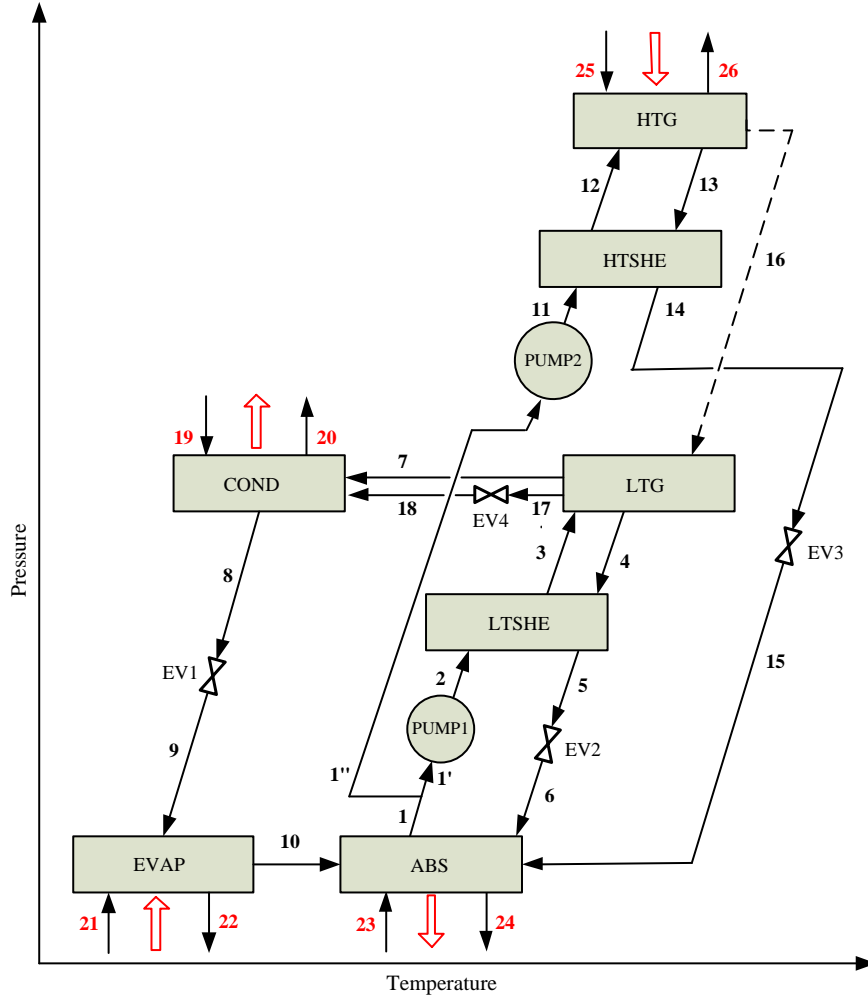
Figure 1. Schematic of the studied double-effect H_2O-LiBr ARS.

– Mass and energy balances for a process unit k

Total mass balance:

$$\sum_{i \in IN} M_{i,k} - \sum_{i \in OUT} M_{i,k} = 0, \quad \forall k \in PU \tag{4}$$

Component mass balance:

$$\sum_{i \in IN} M_{i,k} \cdot X_{j,i,k} - \sum_{i \in OUT} M_{i,k} \cdot X_{j,i,k} = 0, \quad \forall k \in PU, \quad j = LiBr \tag{5}$$

Energy balance:

$$Q_{u,k} - W_k + \sum_{i \in IN} H_{i,k} - \sum_{i \in OUT} H_{i,k} = 0, \quad \forall k \in PU$$

$$Q_{u,k} = \pm(H_{u,in,k} - H_{u,out,k}) \tag{6}$$

$$H_i = M_i \cdot h_i \tag{7}$$

M indicates mass flow rate ($kg \cdot s^{-1}$), X refers to mass fraction ($kg \cdot kg^{-1}$), Q is the heat load (kW), W refers to power (kW), H represents enthalpy flow rate (kW), and h is the specific enthalpy ($kJ \cdot kg^{-1}$).

– Heat transfer process units

Heat transfer area (HTA) of a process unit k:

$$Q_k = U_k \cdot HTA_k \cdot LMTD_k, \quad \forall k \in PU \tag{8}$$

where the logarithmic mean temperature difference (LMTD) in a process unit k is calculated as:

$$LMTD_k = \frac{\Delta T_k^H - \Delta T_k^C}{\ln \dfrac{\Delta T_k^H}{\Delta T_k^C}}, \quad \forall k \in PU \tag{9}$$

where ΔT^H_k and ΔT^C_k are the temperature differences at the hot and cold sides, respectively. Then, the total heat transfer area THTA (m²) is given by:

$$THTA = \sum_k HTA_k, \quad \forall k \in PU \tag{10}$$

Effectiveness factor of the solution heat exchangers LTSHE and HTSHE

The effectiveness factor ε is based on the strong solution side, and it is calculated by Eq. (11) and (12) for LTSHE and HTSHE, respectively:

$$\varepsilon_{LTSHE} = \frac{M_2 X_2 \left(T_3 - T_2 \right)}{M_6 X_6 \left(T_4 - T_2 \right)} \tag{11}$$

$$\varepsilon_{HTSHE} = \frac{M_{11} X_{11} \left(T_{12} - T_{11} \right)}{M_{13} X_{13} \left(T_{13} - T_{11} \right)} \tag{12}$$

Inequality constraints

The following inequality constraints are included to avoid temperature crosses in the process units by assigning a small positive value to δ (in this case δ=0.01).

High-temperature generator HTG:

$$T_{25} \geq T_{12} + \delta \tag{13}$$

$$T_{26} \geq T_{13} + \delta \tag{14}$$

High-temperature solution heat exchanger HTSHE:

$$T_{13} \geq T_{12} + \delta \tag{15}$$

$$T_{14} \geq T_{11} + \delta \tag{16}$$

Low-temperature generator LTG:

$$T_{16} \geq T_4 + \delta \tag{17}$$

$$T_{16} \geq T_{17} + \delta \tag{18}$$

Low-temperature solution heat exchanger LTSHE:

$$T_4 \geq T_3 + \delta \tag{19}$$

$$T_5 \geq T_2 + \delta \tag{20}$$

Absorber ABS:

$$T_{23} \geq T_{10} + \delta \tag{21}$$

$$T_6 \geq T_1 + \delta \tag{22}$$

$$T_6 \geq T_{24} + \delta \tag{23}$$

$$T_1 \geq T_{22} + \delta \tag{24}$$

Evaporator EVAP:

$$T_{21} \geq T_9 + \delta \tag{25}$$

$$T_{22} \geq T_9 + \delta \tag{26}$$

Condenser COND:

$$T_8 \geq T_{19} + \delta \tag{27}$$

– Physico-chemical property estimation

The model includes correlations to compute the physicochemical properties of the process streams (strong and weak LiBr solutions, water, and vapor). The LiBr solution enthalpy at 40%–70% concentration range is estimated by the correlation proposed by ASHRAE [5]. The crystallization conditions of the LiBr solution are considered using the correlation given by Gilani et al. [6] (Eq. 28)

$$X < 0.0809 \cdot T + 61.341 \tag{28}$$

Water and vapor properties are estimated through correlations given by Rogers and Mayhew [7].

1.4 Cost model

The total annual cost (TAC) is calculated using Eq. (29). This calculation takes into account the annualized capital expenditures (annCAPEX) and the operating expenditures (OPEX):

$$TAC = annCAPEX + OPEX \tag{29}$$

In Eq. (29), annCAPEX is calculated using Eq. (30) in terms of the capital recovery factor (CRF) and the investment (Z_k) for each process unit k, which are calculated using Eq. (31) and Eq. (32), respectively. The CRF is computed for an assumed project lifetime (n) of 25 years and an interest rate (i) of 10.33% [8].

$$annCAPEX = CRF \cdot \sum_k Z_k \tag{30}$$

$$CRF = \frac{i \cdot (1+i)^n}{(1+i)^n - 1} \tag{31}$$

$$Z_k = A_k \cdot \left(f \cdot HTA_k\right)^{B_k} + C_k \tag{32}$$

In Eq. (32), the investment Z_k is expressed in $ and HTA_k in m^2; the coefficient f=10.764 $ft^2 \cdot m^{-2}$ is the area conversion factor; and the numerical values of parameters A_k ($ $(ft^{-2})^{B_k}$), B_k (dimensionless), and C_k ($) are listed in Table 1 [9].

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Table 1. Parameter values for estimating process unit investment Z_k (see Eq. (32)).

Unit k	A_k	B_k	C_k
GEN	1800	0.8	24915
EVAP	5900	0.552	0
ABS	9.976	1.820	0
COND	2119	0.497	0
SHE	2774	0.700	0

The OPEX is calculated using Eq. (33). This equation includes costs associated with the heating (HU) and cooling (CU) utilities. The individual unit costs are $C_{HU} = 2.0$ \$ t^{-1} and $C_{CU} = 0.0195$ \$ t^{-1} [8].

$$OPEX = C_{HU} \cdot HU + C_{CU} \cdot CU \qquad (33)$$

Reference

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