

ELASTO-PLASTIC THERMAL STRESS ANALYSIS IN A THERMOPLASTIC COMPOSITE DISC UNDER UNIFORM TEMPERATURE USING FEM

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Abstract-The aim of this study was to investigate elasto-plastic thermal stresses in a thermoplastic composite disc that is reinforced by steel fibers, curvilinearly. Finite element method (FEM) was used to calculate the thermal stress distribution in the model of composite disc. The solution was performed by ANSYS software code. In order to evaluate the effects of uniform temperature, different values of it were carried out on the model of composite disc, uniformly. Radial and tangential stresses were calculated under a uniform temperature distribution which was selected from 60 C to 120 C. Because of the composite disc having different thermal expansions in radial and tangential directions, thermal stresses were produced in it by the applied uniform temperature values. The magnitude of the tangential stress component for elastic and elasto-plastic solutions was higher than the radial stress component. The radial stress components were obtained as compressive on the inner and outer surfaces. Besides the tangential stress components were calculated as compressive and tensile on the inner and outer surfaces, respectively. The absolute values of it were the highest on the inner surface both radial and tangential directions. The residual stress components also were calculated using elastic and elasto-plastic solution results. The obtained results showed that the positions of the improved thermal stresses and residual stresses were considerably affected increasing uniform temperature value.

Key words- Thermal Stress Analysis, Thermoplastic Disc, FEM, Residual Stress

1. INTRODUCTION

Construction engineering history may have reached a new stage with the advent and availability of the new, low-cost, high-performance structural composites. Inherit lightness, placement flexibility, corrosion resistance, and magnetic transparency of FRP composites make them attractive as logical substitutes for steel structural elements [1].

Sayman [2] investigated a thermal stress analysis on an aluminum metal-matrix orthotropic composite disc by using an analytical solution. Trende et al. [3] examined residual stresses and dimensional changes in the compression moulded glass-mat reinforced thermoplastic parts. Eraslan and Orcan [4] studied an analytical solution for the elastic-plastic stress distribution in rotating variable thickness solid discs. Jahed and Shirazi [5] investigated loading and residual stresses, and associated strains and displacements in thermoplastic rotating discs at elevated temperatures.

You et al. [6] improved a numerical method for the analysis of deformations and stresses in the elastic-plastic rotating discs with arbitrary cross-sections of continuously varying thicknesses and arbitrary variable density made of nonlinear strain-hardening materials. Bektas et al. [7] studied an elastic-plastic stress analysis of a thin aluminum metal-matrix composite disk under internal pressure. An analytical solution was performed for satisfying the elastic-plastic stress-strain relations and boundary conditions for small plastic deformations. Mackin et al. [8] studied thermal stresses as a result of cracking in the disc brakes.

This paper was concerned with the examination of the uniform temperature effect on the thermal stresses that is improved in a composite disc, curvilinearly. It is reinforced by steel fibers. Finite element method (FEM) was used to calculate elastic

and elasto-plastic thermal stress fields of the composite disc model. For this purpose, a computer code ANSYS was used both elastic and elasto-plastic thermal stress analyses. Residual stress distributions were obtained from using these analysis results. In addition, thermal stresses were calculated during different uniform temperature loading.

2. MATERIALS AND METHODS

Consider a hollow fiber-reinforced thermoplastic composite disc of inner radius $a=30$ mm, and outer radius $b=50$ mm is shown in Figure 1. It consists of low-density polyethylene as a thermoplastic matrix and steel fibers. Its mechanical properties are given in Table 1 [9].

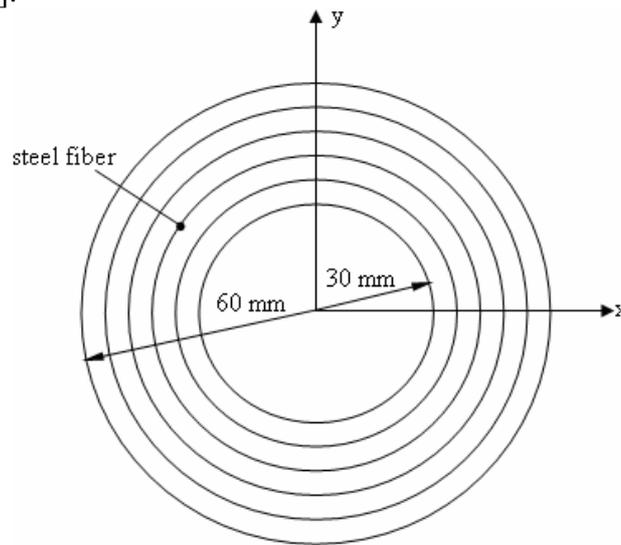


Figure 1. The fiber-reinforced thermoplastic composite disc

Table 1. Mechanical properties of the composite disc [9]

E_r (MPa)	E_θ (MPa)	G_{12} (MPa)	ν_{12}	X (MPa)	Y (MPa)	S (MPa)	K (MPa)	α_r ($1/^\circ\text{C}$)	α_θ ($1/^\circ\text{C}$)
260	11300	75	0.43	14.0	5.4	3.8	92.0	$130.0 \cdot 10^{-6}$	$12.8 \cdot 10^{-6}$

The distributions of thermal elastic and elasto-plastic stress components were calculated by FEM, therefore ANSYS software that is a general-purpose finite element code was used for solving this problem. Element type was chosen as PLANE 42 that is used for 2-D modeling of solid structures. The element can be used either as a plane element (plane stress or plane strain) or as an axisymmetric element. The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The geometry, node locations, and the coordinate system for this element are shown in Figure 2. Orthotropic material directions correspond to the element coordinate directions [10-11].

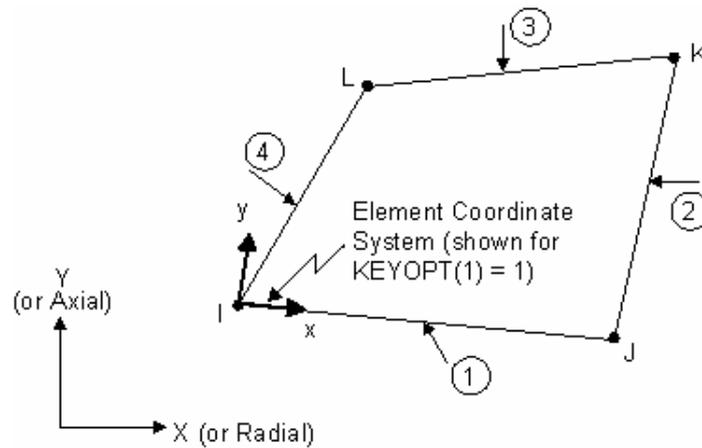


Figure 2. PLANE 42 element type in ANSYS

The finite element model and boundary conditions in ANSYS of the composite disc with hole is illustrated in Figure 3. It can be seen in this figure, mapped mesh shape is selected for the construction the disc model because of compared to a free mesh; a mapped mesh is limited in terms of the element shape it contains and the pattern of the mesh [10-11]. Furthermore, 2850 elements and 2976 nodes modeled the composite disc, after the mesh process.

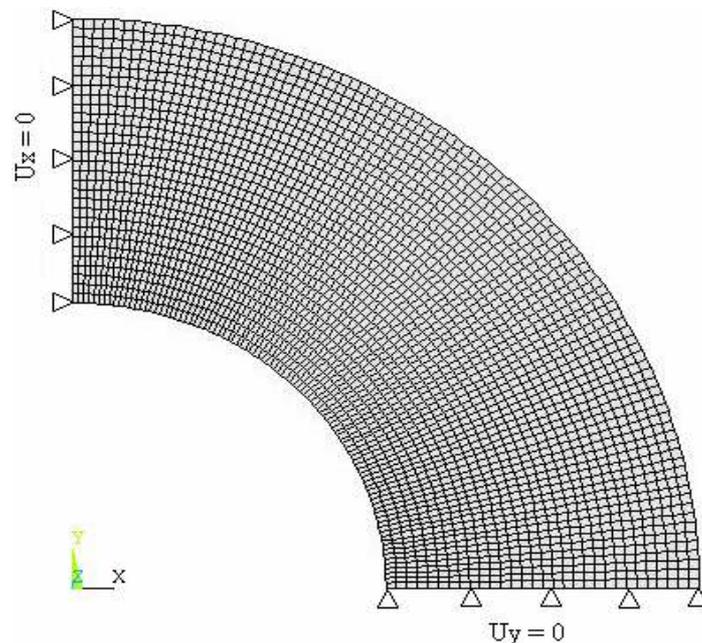


Figure 3. Finite element model and boundary conditions in ANSYS

Uniform temperature load is applied whole of the composite disc. Meanwhile, to investigate effect of increasing uniform temperature value on thermal stresses each model was performed under 60, 70, 80, 90, 100, 110 and 120 °C uniform temperatures loading. In addition, to examine residual stresses, elastic and elasto-plastic solutions were performed these models.

Firstly, elastic solution is carried out composite disc under the uniform temperature distribution. The thickness of it is assumed to be thin, so the solution is reduced to a plane stress case. The equations of the equilibrium without body force are written as [2],

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad (1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \quad (2)$$

The strain components of the thermal expansion for an orthotropic material can be written as,

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = a_{11}\sigma_x + a_{12}\sigma_y + \alpha_x T \\ \varepsilon_y &= \frac{\partial v}{\partial y} = a_{12}\sigma_x + a_{22}\sigma_y + \alpha_y T \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a_{66}\tau_{xy} + \alpha_{xy} T \end{aligned} \quad (3)$$

where α_x and α_y are the thermal expansion coefficients and α_{xy} is zero in the orthotropic materials. The compatibility equation is the relation between continuum displacement components,

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \quad (4)$$

Differentiating, the Eqs. (1) and (2) with respect to x and y axes, respectively gives the shear stress,

$$\frac{\partial^2 \tau_{xy}}{\partial x \partial y} = -\frac{1}{2} \left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2} \right) \quad (5)$$

If the strain components are substituted in the compatibility equation by writing $-(\sigma_{x,xx} + \sigma_{y,yy})/2$ as a replacement for of $\tau_{xy,xy}$ the governing differential equation is calculated,

$$a_{22} \frac{\partial^4 F}{\partial x^4} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} + a_{11} \frac{\partial^4 F}{\partial y^4} = -\alpha_x \frac{\partial^2 T}{\partial y^2} - \alpha_y \frac{\partial^2 T}{\partial x^2} \quad (6)$$

Secondly, elasto-plastic solution is performed on composite disc under the uniform temperature distribution. The Tsai-Hill theory is used as a yield criterion during the solution owing to the same yield strengths of the thermoplastic composite disc in tension and compression. According to this criterion the equivalent stress in the fiber direction (tangential direction) can be written as [9],

$$\bar{\sigma} = \sqrt{\sigma_\theta^2 - \sigma_r \sigma_\theta + \frac{\sigma_r^2 X^2}{Y^2}} \quad (7)$$

where X and Y are the yield strengths in fiber and transverse directions or in the tangential and radial directions. The material is assumed no work hardens. According to

the assumption the equivalent stress, ($\bar{\sigma}$) is equal to X. The strain increments are written as,

$$d\varepsilon_r = a_{rr}d\sigma_r + a_{r\theta}d\sigma_\theta + d\varepsilon_r^p + \alpha_r dT \tag{8}$$

$$d\varepsilon_\theta = a_{r\theta}d\sigma_r + a_{\theta\theta}d\sigma_\theta + d\varepsilon_\theta^p + \alpha_\theta dT$$

Finally, to calculate the thermal residual stresses it is necessary to superpose on the thermal stress system. The superposition of the elastic stresses and plastic stresses provides the residual stress values as,

$$\begin{aligned} (\sigma_r)_r &= (\sigma_r)_p - (\sigma_r)_e \\ (\sigma_\theta)_r &= (\sigma_\theta)_p - (\sigma_\theta)_e \end{aligned} \tag{9}$$

3. RESULTS AND DISCUSSION

Thermal elastic, elasto-plastic, equivalent plastic stress components and equivalent plastic strain values at the inner and outer surfaces of thermoplastic composite disc are presented in Table 2. According to the table, tangential stress values higher than radial stress values for same uniform temperature loading. This case are valid both tensile and compressive stress components.

Table 2. Thermal elastic, plastic, equivalent stress and equivalent plastic strain components at the inner and outer surfaces

Temperature (°C)	Surface	Elastic Stress		Plastic Stress		Equivalent Plastic Stress σ_{eqv}	Equivalent Plastic strain ($\times 10^{-6}$)	
		$(\sigma_r)_e$	$(\sigma_\theta)_e$	$(\sigma_r)_p$	$(\sigma_\theta)_p$		Radial Direct.	Tan gential Direct.
							$(\varepsilon_p)_r$	$(\varepsilon_p)_\theta$
60	Inner	-0,210	-13,745	-0,210	-13,745	0	0	0
	Outer	-0,082	10,303	-0,082	10,303	0	0	0
70	Inner	-0,245	-16,036	-0,230	-14,119	14,017	46,9	119
	Outer	-0,096	12,020	-0,096	12,018	14,000	0	0
80	Inner	-0,280	-18,326	-0,232	-14,179	14,050	138	350
	Outer	-0,110	13,737	-0,110	13,730	14,000	0	0
90	Inner	-0,315	-20,617	-0,233	-14,321	14,087	241	614
	Outer	-0,124	15,454	-0,116	13,953	14,016	60,5	109
100	Inner	-0,350	-22,908	-0,234	-14,282	14,129	357	907
	Outer	-0,138	17,171	-0,116	13,977	14,040	154	279
110	Inner	-0,385	-25,199	-0,234	-14,317	14,176	485	1233
	Outer	-0,152	18,888	-0,116	14,003	14,066	257	463
120	Inner	-0,421	-27,490	-0,235	-14,330	14,226	623	1585
	Outer	-0,165	20,605	-0,116	14,031	14,094	367	662

Although all radial stress components are obtained as compressive at inner and outer surfaces, tangential stress components are calculated as compressive and tensile at inner and outer surfaces, respectively. Moreover, plastic flow is started at inner surface during 70 °C uniform temperatures loading. The maximum value of plastic equivalent stress is calculated 14,226 MPa at inner surface and 14,094 MPa at outer surface under 120 °C uniform temperature effect. Besides, the magnitude of plastic yielding expands the

highest value with this loading (120 °C) inner and outer surfaces for radial and tangential directions.

The residual stress values for radial and tangential directions at inner and outer surfaces depending on applied uniform temperature loading are given in Figure 4 and 5, respectively. It can be seen in these figures, the magnitude of residual stresses for tangential direction are becomes higher than radial direction. Besides, residual stress values at 60 °C are zero both inner and outer surfaces. The highest value of residual stress is obtained 13,160 MPa as tensile at inner surface and -6,574 MPa as compressive at outer surface for tangential direction during 120 °C uniform temperatures loading.

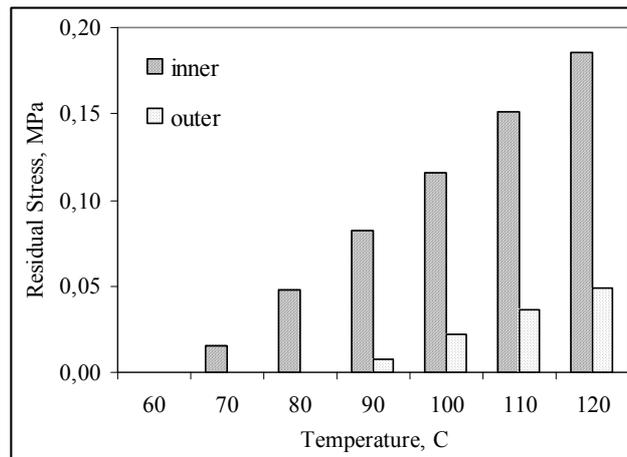


Figure 4. The residual stress values for radial direction on inner and outer surfaces depending on applied uniform temperature values (all stresses in MPa)

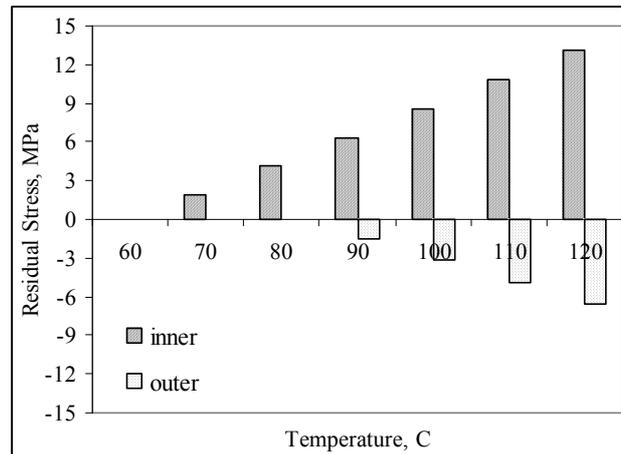


Figure 5. The residual stress values for tangential direction on inner and outer surfaces depending on applied uniform temperature values (all stresses in MPa)

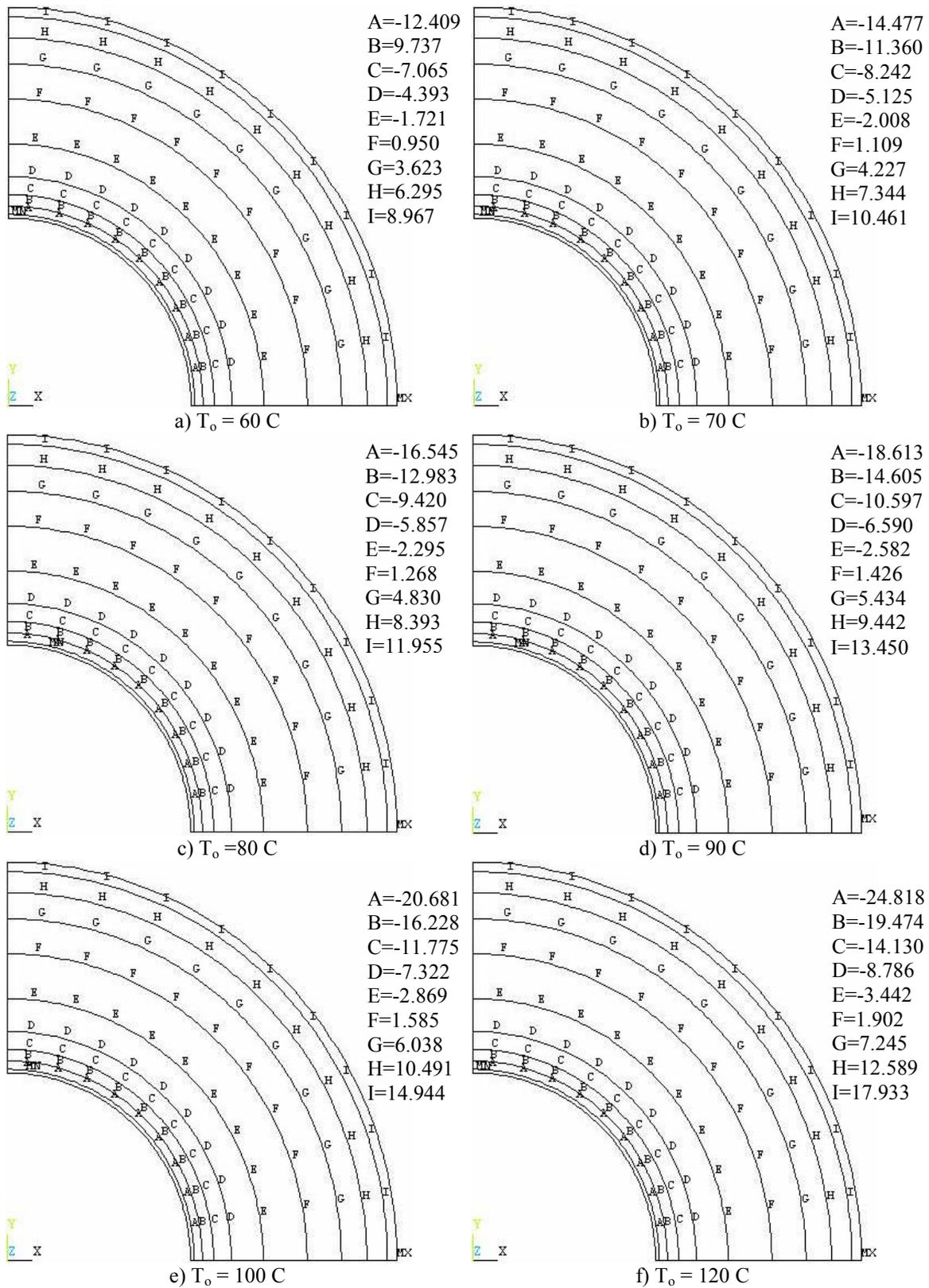


Figure 6. Elastic stress contours for tangential directions (all stresses in MPa)

Elastic stress contours for tangential directions depending on applied uniform temperature value are showed in Figure 6, because of seeing distributions of elastic stresses on the composite model, clearly. As seen in this figure, the value of elastic stress is increased when the uniform temperature loading is increased.

The equivalent plastic strain under 120 °C temperature effects is illustrated both radial and tangential directions in Figure 7. It is seen that this figure, the magnitude of equivalent plastic strains for tangential direction are bigger than radial direction at the inner and outer surfaces.

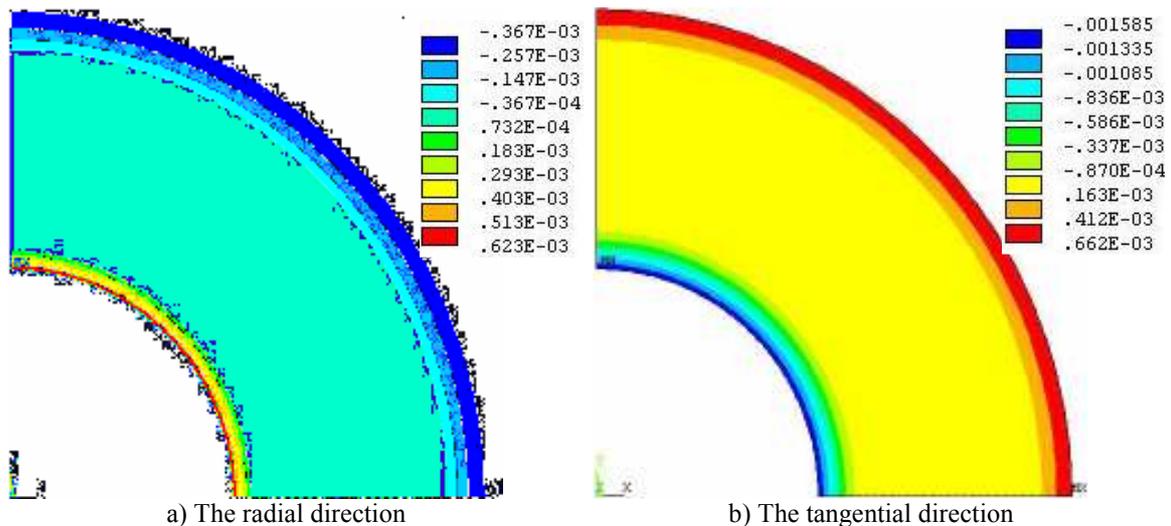


Figure 7. The equivalent plastic strain under 120 °C temperature effect

4. CONCLUSIONS

From the elastic solution and elasto-plastic solution results presented it can be concluded that:

1. Because of the different thermal expansion coefficients in principal material directions, thermal stresses may occur in the composite disc under uniform temperature loading.
2. Plastic flow comes into existence at the inner surface, firstly.
3. Plastic yielding enlarges both inner and outer surfaces while the uniform temperature value is arising.
4. The absolute value of tangential stress component is higher than radial stress component.
5. The residual stresses are produced to except 60 °C uniform temperatures loading.

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